Communication and the Optimality of Hierarchy in Organizations

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Abstract

This paper studies optimal organization structures in multidivisional organizations with more than two divisions. The decisions of individual divisions need to be adapted to local conditions and also be coordinated. The needs for coordination are described by a coordination network. Information about local conditions is held by division managers who communicate strategically. Hierarchy is an organizational form in which divisions are organized into groups, and decisions are made by group managers. Under a circle coordination network, our central result is that when the needs for coordination are intermediate, hierarchy performs better than centralization or decentralization. We then compare M-form hierarchy (in which mutually more dependent divisions are grouped together) and U-form hierarchy (in which mutually less dependent divisions are grouped together). We also consider other coordination networks, in which different divisions have different network positions. Hierarchy, or some hybrid governance which combines hierarchy and centralization, remains optimal in some situations.

Keywords: Coordination, Cheap talk, Hierarchy, M-form, U-form

JEL classification: D23, D83, L23

1 Introduction

An important question in organization economics is what organizational form strikes a better balance between adaptation and coordination for a multidivisional organization. On the one hand, each division's activity needs to be adapted to its local condition. On the other hand, the activities of various divisions also need to be coordinated. Since individual division managers are usually best informed about their own local conditions, inducing them to communicate effectively is critical in achieving coordinated adaptation.

For an organization with more than two divisions (functional units), organizational forms other than centralization (in which headquarters make decisions for the divisions) and decentralization (in which division managers make the decisions) are possible. One such possibility is hierarchical governance: divisions are organized into several groups, and for each group a group manager is

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introduced to make decisions for the divisions within his group. Indeed, in the real world hierarchical governance is ubiquitous in large multi-divisional organizations. For instance, in addition to the headquarters, multinational firms typically set up groups or regional headquarters (in Asia, Europe, North America, etc.) to coordinate divisional activities.

Chandler (1977) is among the first to note the importance of hierarchical governance. He stated that "the existence of managerial hierarchy is a defining characteristic of the modern business enterprise." Specifically, Chandler (1962) identified three layers of management at the top level: departments (functional units), divisions (each of which has a number of departments), and headquarters (oversee all divisions). Moreover, he pointed out two typical ways in which large firms group functional units into divisions: M-form (multidivisional) or U-form (unitary). Under M-form, complementary functional units are grouped together into divisions (organized according to products or regions), while under U-form, similar functional units are grouped together.

Given the popularity of hierarchical governance in the business world, the following questions naturally arise. Why do large firms adopt hierarchical governance (i.e., introduce a middle layer of management) at the top level of management? What are the advantages and disadvantages of hierarchical governance relative to centralization and decentralization (i.e., without the middle layer of management), both in terms of achieving informative communication and coordinated adaptation? What are the relative advantages and disadvantages of M-form and U-form?

To address these issues, we consider a multidivisional organization with four divisions. Our model is an extension of the two-division models of Alonso et al. (2008; ADM henceforth) and Rantakari (2008).[1] Specifically, each division has a decision to make, which needs to be adapted to its local condition. Moreover, the decisions of different divisions also need to be coordinated, and the needs for coordination are described by a weighted coordination network. Information is dispersed within the organization: each division's local condition is only observed by its division manager, whose incentive is to maximize the payoff of his own division. Communication among division managers is strategic or in the form of cheap talk (Crawford and Sobel, 1982). Finally, the organization is only able to commit to ex ante allocation of decision rights.

The timing is as follows. First, the headquarter (HQ), whose incentive is to maximize the joint payoff of all four divisions, determines the allocation of decision rights, or the governance structure. Then all division managers simultaneously send public messages. Messages are public, in the sense that all relevant parties receive the same message from any given division manager. Finally, decisions are made simultaneously by the parties who have decision rights.

We first study a circle (coordination) network, which is typical in the real world. For instance, consider a multinational operating in two regions, the US and Asia; each region has two functional

[1] We refer to functional units as divisions in this paper to conform to the terminology used by ADM and Rantakari (2008).

divisions: production and marketing.[2] Thus, in total there are four divisions. The coordination needs of the two divisions within the same region are high (strong coordination link); the coordination needs of the same functional divisions across regions are low (weak coordination link); and different functional divisions in different regions do not need to be directly coordinated. We consider four governance structures. The first two are centralization, under which HQ makes all decisions, and decentralization, under which each division manager makes the decision for his own division. The other two are hierarchical governance, under which an intermediate layer of managers between the division managers and HQ is introduced. Furthermore, the four divisions are organized into two groups, and group managers are introduced. Each group manager has decision rights for the two divisions within his group, and his incentive is to maximize the joint payoff of the two divisions under his control. There are two possible groupings. The first one is M-form, which groups strongly linked divisions together (in the example, according to region—the US group and the Asia group). The second is U-form, which groups weakly linked divisions together (in the example, according to functions—the production group and the marketing group).

We show that communication is more informative under either hierarchical governance form than under centralization or decentralization. Intuitively, compared to the HQ under centralization, in a hierarchy the group managers' incentives are relatively more aligned with individual division managers, which leads to more informative communication. Overall, centralization is good for coordination but bad for adaptation (due to noisy communication), and decentralization is good for adaptation but bad for coordination. On the other hand, hierarchy achieves a better balance between adaptation and coordination: Relative to centralization, it improves adaptation because it improves communication; relative to decentralization, it improves coordination because it internalizes some coordination links. When the importance of coordination between the weak coordination link is not too high or too low, hierarchy actually performs better than centralization and decentralization.

Between the two hierarchical governance structures, we show that communication is more informative under U-form than under M-form, and thus U-form achieves better adaptation. Intuitively, relative to group managers under M-form, under U-form group managers' incentives are more aligned with individual division managers, as the weak coordination links are internalized under U-form. However, relative to U-form, M-form is better at coordination, as it internalizes the strong coordination links. Therefore, when all coordination links have similar intermediate weights, adaptation is relatively more important than coordination, and consequently U-form is optimal (performs better than M-form, centralization, and decentralization). On the other hand, when the weak coordination links have an intermediate weight—but the strong coordination links have a much higher weight—coordination becomes relatively more important, and M-form is optimal. Overall, the parameter space under which M-form is optimal is much larger than that under

[2] A more concrete example can be found in Section 4.

which U-form is optimal. This is consistent with what observed by Chandler (1962, 1977): Large US firms typically adopt M-form organization.

We then study a star coordination network, under which the center division has coordination links with each peripheral division, but three peripheral divisions do not have coordination links with each other. Our focus is on how the asymmetry of divisions' network positions affects their incentives to communicate and its implications for the optimal organizational form. In addition to centralization and decentralization, we also consider two governance structures of partial centralization/decentralization: The center division is centralized and the three peripheral divisions are decentralized, and vice versa. In addition, we consider a hybrid hierarchy that combines hierarchy and centralization: The center division is centralized, while the other three divisions are controlled by a group manager.

We found that the center division's communication could be more informative under decentralization than under centralization, which is not possible in ADM's or Rantakari's (2008) two-division models. For the optimal organizational form, partial centralization/decentralization can never be optimal. However, the hybrid hierarchy is optimal when the need for coordination is not too low or too high. In particular, the hybrid hierarchy always dominates partial centralization/decentralization under which the center division is centralized. Interestingly, the group of the three peripheral divisions internalizes no coordination link, yet it improves the overall performance of the organization. The underlying logic is similar to that under the circle network: Introducing a hierarchy to the decentralized divisions improves these divisions' communication significantly, which in turn improves coordination without sacrificing much of the adaptation of the three peripheral divisions.

Finally, we study a line coordination network, under which each of the two central divisions has two coordination links, while each of the two peripheral divisions has one coordination link. We consider various organizational forms, including hierarchy, partial centralization/decentralization, and hybrid hierarchical governance. Consistent with earlier results, partial centralization/decentralization can never be optimal, but hierarchy and hybrid hierarchy can be an optimal organizational form in some situations. Moreover, when a hybrid hierarchy is optimal, it is the two central divisions that are centralized, while the two peripheral divisions are controlled by a group manager, not vice versa.

1.1 Related literature

As mentioned earlier, this paper follows the work of ADM (2008) and Rantakari (2008), who study strategic communication within an organization in the presence of a trade-off between adaptation and coordination. Specifically, ADM mainly focus on a setting with two symmetric divisions in which the division manager's incentives could be more aligned. They show that decentralization performs better than centralization, no matter how important coordination is, as long as the incentives of the division managers are sufficiently aligned.[3] Rantakari (2008) focuses on a setting with two asymmetric divisions in which the division managers are completely selfish. He considers richer organization forms (partial centralization, for instance) and shows that the optimal governance structure could be asymmetric if the two divisions are too asymmetric. The main difference between our paper and theirs is that we consider more than two divisions. This not only adds realism to the model, but can also incorporate richer structures of coordination needs among divisions (i.e., coordination network). Moreover, more divisions also imply richer ways to allocate decision rights or richer organizational structures. In particular, hierarchy or intermediate layer of management becomes a possibility, which is not the case in a two-division framework. Further differences between our model and their framework will be pointed out as the paper proceeds.

Two recent papers study strategic communication in networks. Hagenbach and Koessler (2010) and Galeotti et al. (2013) consider a setting with multiple senders and multiple receivers. Specifically, there is a common state of the world, and each agent has binary private information. Agents have heterogenous but known biases, and they strategically communicate about their private information before taking action.[4] In Hagenbach and Koessler (2010), communication is private, in that an agent can send different messages to different agents; in Galeotti et al. (2013), in contrast, communication can be either private or public. They study who will communicate truthfully with whom in equilibrium, which forms a communication network. The focus of our paper is quite different from theirs. For one thing, the trade-off between adaptation and coordination is central to our model—it is not present in theirs. For another, they do not consider the allocation of decision rights or organizational structure.

Calvo-Armengol et al. (2015) also study communication within an organization.[5] In particular, agents' decisions need to be adapted to local conditions and be coordinated with each other; the relevant coordination needs are described by a coordination network. The main difference is that in their model, agents' communication is costly but verifiable, and thus not strategic; their focus is on agents' incentives to invest in bilateral communication links in order to increase the precision of communication. Again, optimal organizational structure is not considered.

Previous work has studied hierarchy from different perspectives. One approach (cf., Calvo and Wellisz (1978); Qian (1994)) emphasizes factors such as moral hazard and monitoring to determine the number of hierarchy tiers. Garicano (2000) develops a model of knowledge-based hierarchy: Lower-layer workers deal with common and easy-to-solve problems, while higher-layer workers

[3] Delegation of decision rights is also studied in Aghion and Tirole (1997), Melumad and Shibano (1991), and Dessein (2002).

[4] In Hagenbach and Koessler (2010) agents' private information is independent but their actions need to be coordinated with the average action among all agents. In Galeotti et al. (2013) agents' private information is correlated.

[5] There is an earlier literature that studies information transmission in organizations based on team theory: Marschak and Radner (1972), Aoki (1986), Radner (1993), Bolton and Dewatripont (1994), Prat (1997), Van Zandt (1999), and Hart and Moore (2005). In these models, information transmissions are not strategic, but are hampered by physical communication contraints.

specialize in rare and hard-to-solve problems. The main difference is that while these papers focus on hierarchy in organizing production, our paper studies hierarchy in top-level management.

Studies on M-form and U-form organizations were pioneered by Chandler (1962) and Williamson (1975). In terms of formal models, Aghion and Tirole (1995) study how overload considerations affect a firm's choice between M-form and U-form. Maskin, Qian, and Xu (2000) analyze the incentive issues related to M-form and U-form as they generate different information about managers' performance. Qian, Roland and Xu (2006) compare the performance between M-form and U-form organizations in terms of facilitating innovation and reform.[6] In contrast, the focus of our study is, how M-form and U-form organizations achieve a better balance between adaptation and coordination via endogenous communication. Another difference is that while in the previous studies the hierarchical structure is implicitly assumed, in our setting the optimality of hierarchy (relative to centralization and decentralization) emerges endogenously.

The rest of the paper is organized as follows: A general model of coordination network is laid out in Section 2, and the structure of equilibrium communication is presented in Section 3. Section 4 analyzes a circle network and studies the optimality of U-form and M-form. Section 5 further explores optimal organizational governance for a star coordination network and a line coordination network. We conclude in Section 6.

2 Model

There are $n = 4$ divisions or functional units within an organization.[7] Each division i, labeled as D_i, has a corresponding division manager i. For each D_i there is a decision to make, which is denoted as d_i. Moreover, each D_i has a local condition θ_i, which is uniformly distributed on $[-s, s]$, $s > 0$, with variance σ^2. The realization of θ_i is only observed by the division manager of D_i. All θ_is are independent from each other. The decisions of different divisions also need to be coordinated, the details of which will be specified shortly.

Specifically, the payoff function of division D_i is given by

$$\pi_i = -\delta_{ii}(d_i - \theta_i)^2 - \sum_{j \neq i} \delta_{ij}(d_i - d_j)^2. \tag{1}$$

In (1), all the $\delta_{ij} \geq 0$ and $\delta_{ii} > 0$. The first term is D_i's adaptation loss, resulting from the difference between d_i and its local condition θ_i. The parameter δ_{ii} captures the importance of D_i's adaptation loss. The second term is D_i's coordination loss, which is due to the difference between d_i and d_j. The parameter δ_{ij} captures the importance of d_i being coordinated with d_j for D_i. We will focus on the case in which $\delta_{ij} = \delta_{ji}$, or each coordination link is symmetric.[8] The $n \times n$ matrix

[6] Dessein et al. (2010) studied hybrid organizations in which some functions are centralized, while the decisions of other functions are decentralized.

[7] Our model can be extended to the case with more than four divisions, which will be discussed later in Section 6.

[8] When δ_{ij} and δ_{ji} are different, D_i and D_j value the coordination need between d_i and d_j differently.

of $[\delta_{ij}]$ defines a coordination network within the organization, which is undirected and weighted.[9] The model extends ADM's and Rantakari's (2008) two-division models to n divisions.

The objective of division manager i is to maximize his own division D_i's payoff.[10] The organization also has an HQ, whose objective is to maximize the joint payoff of all divisions, $\sum_i \pi_i$. We assume that contracts are highly incomplete, so that the organization is only able to commit to an ex ante allocation of decision rights. The game proceeds in three stages. In stage 0, the HQ allocates decision rights: for each D_i, who will make decision d_i. We call an allocation of decision rights a governance structure g. Once a particular g is determined, it becomes common knowledge. Then individual division managers observe the realized local conditions θ_is. In stage 1, all division managers simultaneously send *public* messages regarding their realized local conditions. Denote m_i as division manager i's message, and $m = (m_1, ..., m_n)$ as a message profile. Note that manager i cannot send different messages to different parties, as communication is public. In particular, all of the relevant parties hear the same message m_i.[11] Finally, in stage 2, all decisions are made simultaneously by the parties who have decision rights, as specified by g.

Given that $n = 4$, there are many possible ways to allocate decision rights, or many possible governance structures. We will mainly focus on the following. The first is centralization, under which the HQ retains the decision rights of all divisions. The second is decentralization, under which each division manager i makes decision d_i. These two governance structures serve as benchmarks. The third is hierarchical governance structures. Specifically, the HQ organizes divisions into groups, and for each group introduces an intermediate-level manager, who is responsible for making decisions for all divisions within the group (this will be elaborated on later sections). Finally, we also consider some hybrid governance structures. The first is partial centralization/decentralization. Under this organizational form, the HQ retains the decision rights of some divisions and the decision rights of other divisions are decentralized. The second hybrid governance is a combination of hierarchy and centralization or decentralization, which will be explained in more detail later.

Our equilibrium solution concept is Perfect Bayesian Nash equilibria (PBE). In particular, in stage 2 the decisions form a Bayesian Nash equilibrium given beliefs. In stage 1, each division manager's communication strategy is optimal, given other division managers' equilibrium communication strategies and the equilibrium decisions in stage 2. Moreover, beliefs are consistent with division managers' equilibrium communication strategies.

[9] To be more precise, the δ_{ii}s are the adaptation needs.

[10] In the real world, division managers could also care about other divisions' payoffs to some extent. We adopt this maximum conflict of interest, as in Rantakari (2008), to focus on optimal organizational governance. This assumption will be discussed in more detail in Section 6.

[11] Public communication is a reasonable assumption, as the communication is between top-level managers. One can think of communications taking place in a meeting with all division managers and other relevant parties present. Actually, when Alfred Sloan was the president of GM, he set up various committees that gave division managers opportunities to exchange ideas and information (Sloan, 1964).

3 Structure of Communication Equilibria

In this section we study communication equilibria in stage 1 for general coordination networks. The results are applicable to all specific coordination networks considered later.

By backward induction, we first solve for equilibrium decisions in the final stage, given governance structure g and messages m in earlier stages. Let d_i^g be the decision for D_i under g, and $\overline{m}_i = E[\theta_i|m_i]$ be the posterior belief of θ_i. In later sections, we will show that d_i^g has the following general form:

$$d_i^g = z_i^g \theta_i + \sum_j a_{ij}^g \overline{m}_j, \tag{2}$$

where $a_{ij}^g \in [0,1]$, $z_i^g \in [0,1]$, and $z_i^g + \sum_j a_{ij}^g = 1$. One can think of a_{ij}^g and z_i^g as endogenously determined decision weights. Using the terminology of Rantakari (2008), a_{ij}^g $(i \neq j)$ is the rate of accommodation of D_i to D_j, which measures how sensitive d_i is to m_j. When division manager i does not have the decision rights of D_i, it can be shown that $z_i^g = 0$. In this case, a_{ii}^g, which we call D_i's own-decision weight (reflecting how sensitive d_i is to m_i), captures the rate of adaptation of D_i. When D_i is decentralized, $z_i^g > 0$ can be considered as the rate of direct adaptation of D_i, and a_{ii}^g represents the rate of induced adaptation of D_i. Generally speaking, these weights depend on the governance structure g and the structure of the coordination network $[\delta_{ij}]$.[12] In later sections, we will investigate in detail how the weights are determined for specific networks.

Next, we consider the communication stage, given governance structure g. As in ADM, we use the following thought experiment to derive division manager i's incentive to misrepresent information. Suppose manager i, by sending message m_i, can successfully induce v_i as the posterior belief of θ_i among all other parties. Then he will choose v_i to maximize his expected payoff

$$\min_{v_i} E[\delta_{ii}(d_i^g - \theta_i)^2 + \sum_j \delta_{ij}(d_i^g - d_j^g)^2 | \theta_i],$$

where d_i^g and d_j^g are given by (2). The optimal v_i^* can be derived as

$$v_i^* - \theta_i = b_i^g \theta_i, \text{ where } b_i^g = \frac{\delta_{ii} a_{ii}^g (1 - z_i^g) + \sum_{j \neq i} \delta_{ij}(a_{ji}^g - a_{ii}^g) z_i^g}{\delta_{ii}(a_{ii}^g)^2 + \sum_{j \neq i} \delta_{ij}(a_{ii}^g - a_{ji}^g)^2} - 1. \tag{3}$$

This b_i^g represents manager i's endogenously determined communication bias: For a given θ_i, manager i would rather report $(1 + b_i^g)\theta_i$. It can be shown that $b_i^g > 0$. That is, manager i has an incentive to exaggerate his own state.[13]

Proposition 1 *Under any governance structure g, manager i's equilibrium communication is an interval equilibrium. In particular, the state space $[0, s]$ is partitioned into K intervals, where K*

[12] For connected networks, all a_{ij}s will be strictly positive. This is because, even if there is no direct coordination link between D_i and D_j $(\delta_{ij} = 0)$, D_i and D_j are still indirectly linked through the network.

[13] The direction of exaggeration is always away from 0, which is the unconditional mean of state θ_i.

is a positive integer. Let the partition points within $[0, s]$ *be* $x_i^g = (x_{i,0}^g, x_{i,1}^g, ..., x_{i,K}^g)$ $(x_{i,0}^g = 0$ *and* $x_{i,K}^g = s)$*. The equilibrium partition points* x_i^g *satisfy difference equation* $(x_{i,k+1}^g - x_{i,k}^g) - (x_{i,k}^g - x_{i,k-1}^g) = 4b_i^g\theta_i$*. The partition on the state space* $[-s, 0]$ *is symmetric. In the most informative equilibrium, K goes to infinity and* $E[(\overline{m}_i^g)^2] = \frac{3(1+b_i^g)}{3+4b_i^g}\sigma^2$*.*

Proposition 1 is a straightforward extension of the results of ADM and Rantakari (2008). For each individual division, the structure of communication equilibria is the same as that in ADM's and Rantakari's (2008) two-division models. This is true for two reasons. First, communication is public. Second, for any two different divisions D_i and D_j, $E_i[\overline{m}_j] = 0$, as θ_i and θ_j are independent. Adding more divisions, therefore, does not change the basic structure of communication equilibria. However, the communication bias of D_i, b_i^g, becomes more complicated, since it depends on both the structure of the coordination network and the organization form g.

One observation from (3) is that communication bias b_i^g mainly depends on a_{ii}^g, D_i's own-decision weight. In particular, the bigger the own-decision weight a_{ii}^g, the smaller the communication bias. In later sections, for specific networks, we will investigate in more detail how b_i^g is determined by the structure of the coordination network and the governance structure g.

4 U-form and M-form Hierarchy under a Circle Network

In this section we study a coordination network that is a circle. As illustrated in Figure 1, there are four coordination links. Two links are strong (12 and 34) and two links are weak (23 and 14). For each i, we normalize δ_{ii} to 1. Moreover, $\delta_{12} = \delta_{34} = \delta_H$, and $\delta_{23} = \delta_{41} = \delta_L$, $\delta_H \geq \delta_L$. More precisely, this network is an asymmetric circle, as coordination links have different weights. However, all divisions have symmetric network positions, with each division having a strong coordination link and a weak coordination link.

This coordination network structure is typical in the real world. For a concrete example, consider a multinational operating in two regions: the US and Asia. In each region, the multinational has two functional units: production and marketing. Thus there are four divisions in total: US production (D1), US marketing (D2), Asia marketing (D3), and Asia production (D4). The two divisions in the same region (D1 and D2, and D3 and D4) need close coordination, while the same functional divisions across two regions (D1 and D4, D2 and D3) need to be coordinated to some extent. On the other hand, different functional divisions in different regions (D1 and D3, D2 and D4) do not need to be directly coordinated.[14]

We will study four governance structures. The two benchmarks are centralization and decentralization, denoted C and D, respectively. In addition, we will study two hierarchical governance

[14] For a multiproduct firm, the two regions or two functional departments in the example can be replaced by two products.

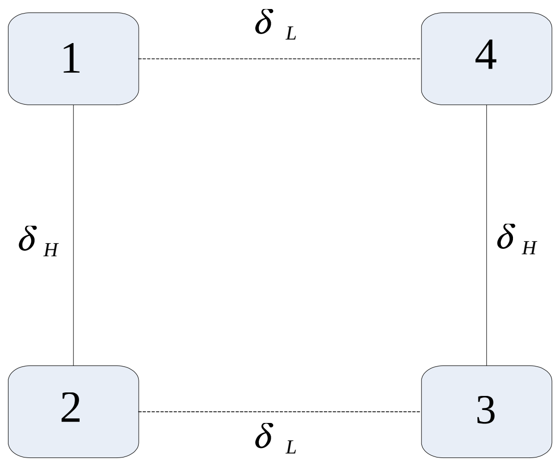

Figure 1: A Circle Network

structures. Specifically, the four divisions are organized into two groups, with each group consisting of two divisions. Moreover, a (new) group manager is introduced to each group, who is responsible for making decisions for the two divisions in his own group. The group manager's incentive is to maximize the joint payoff of his own two divisions.[15] However, the group manager does not observe the realized states of his divisions; they are still only observed by the relevant division managers, who are responsible for the day-to-day operations of their own divisions. This is a hierarchical governance, as a new layer of managers is introduced between the HQ and the division managers. Note that group managers are equally informationally disadvantaged as the HQ. Actually, the group managers differ from the HQ only in their incentives: Relative to the HQ, a group manager's incentive is closer to his own division managers', since a group manager just internalizes one coordination link.

We consider two possible groupings. In the first, the two more mutually dependent divisions (D1 and D2, D3 and D4) are grouped together. We call this governance M-form, denoted as M, since in the corresponding example groups are organized according to regions (or products). In the second case, the two weakly linked divisions (D1 and D4, D2 and D3) are grouped together. We call this governance U-form, denoted as U, since in the corresponding example groups are organized according to functions. Under each governance, first the division managers simultaneously send public messages, then the group managers simultaneously make decisions. The two hierarchical structures are depicted in Figure 2. Note that hierarchy does not change the flow of communication.[16]

[15] A group manager's pay, and/or his future promotion and career, could be tied to the overall performance of the two divsions under his supervision. More generally, this demonstrates that group managers exhibit own-group bias, echoing the fact that division managers exhibit own-division bias.

[16] An alternative flow of communication under hierarchy will be discussed in Section 6.

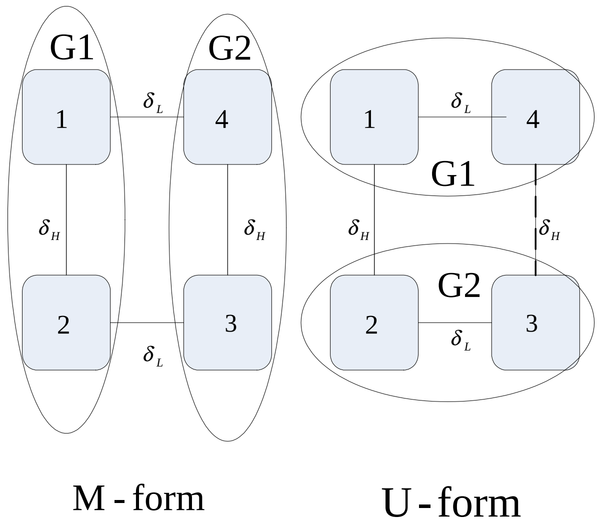

Figure 2: M-form and U-form

4.1 Equilibrium decisions

The four divisions are symmetric under each governance structure considered. As a result, the equilibrium decisions are also symmetric across divisions.[17] Here we only present the equilibrium decisions for D1. Specifically, under governance structure $g = C, D, M, U$, the equilibrium decisions for D1 take the following forms:

$$\begin{aligned} d_1^C &= \sum_j a_{1j}^C \overline{m}_j,\ d_1^D = z\theta_1 + \sum_j a_{1j}^D \overline{m}_j, \\ d_1^U &= \sum_j a_{1j}^U \overline{m}_j,\ d_1^M = \sum_j a_{1j}^M \overline{m}_j. \end{aligned}$$

The detailed derivation and exact expressions for the decision weights can be found in the Appendix. Note that d_1^C, d_1^U, and d_1^M have similar forms, as in each case the decision maker for D1 has no private information about θ_1. Moreover, a_{13}^g is strictly positive. This is because although there is no direct coordination link between D1 and D3, D3's message affects d_2 and d_4, which in turn affects D1's payoff.

For $i \neq j$, recall that a_{ij}^g can be interpreted as D$_i$'s accommodation of D$_j$ under g. To compare the decision weights across governance structures, denote s as D$_i$'s strongly linked division and w as D$_i$'s weakly linked division. For instance, for D1 the strongly linked division is D2 and the weakly linked division is D4.

[17] All the dicisions are symmetric in the sense that the weights a_{ij}^g are the same across i's. Specifically, a_{ij}^g only depends on the relative network positions of D$_i$ and D$_j$: They are strongly linked, weakly linked, or not directly linked.

Lemma 1 *The following relationships hold for the equilibrium decision weights: (i)* $a_{is}^M > a_{is}^C > a_{is}^U$*, and* $a_{iw}^U > a_{iw}^C > a_{iw}^M$*. (ii)* $a_{ii}^U > a_{ii}^C$*, and* $a_{ii}^M > a_{ii}^C$*. If* $\delta_H \delta_L \leq 0.5$ *and* $\delta_H + \delta_L \leq 1.5$ *(with at least one strict inequality), then* $a_{ii}^U > a_{ii}^M$*.*

Part (i) of Lemma 1 basically states that, compared to centralization, under hierarchy decision weight a_{ij} is bigger if D$_i$ and D$_j$ belong to the same group and smaller if D$_i$ and D$_j$ belong to different groups. This result is intuitive. Recall that each group manager only cares about his own two divisions, while the HQ cares about all four divisions. Group managers' own-group bias implies that under hierarchy, the across-division accommodations are higher for divisions within the same group, but lower for divisions across groups, relative to centralization. Since under M-form the strongly linked divisions are within the same group, while under U-form they belong to different groups, we have $a_{is}^M > a_{is}^C > a_{is}^U$. The opposite pattern holds for weakly linked divisions, which leads to $a_{iw}^U > a_{iw}^C > a_{iw}^M$.

Part (ii) says that the own-decision weights under either hierarchy are always larger than those under centralization ($a_{ii}^U > a_{ii}^C$, and $a_{ii}^M > a_{ii}^C$). This is again due to the group managers' own-group bias. Because of this bias, relative to the HQ under centralization, a group manager is less willing to accommodate the two divisions in the other group, and thus will put more weights on the messages of his own two divisions. With a mild condition, which is sufficient but not necessary, the own-decision weight under U-form is larger than the own-decision weight under M-form. The main reason for this result is that under U-form the weak coordination links are internalized, while under M-form the strong coordination links are internalized. When the internalized coordination link is weaker, a group manager will reduce the rate of accommodation between his own two divisions and increase their own decision weights. This leads to $a_{ii}^U > a_{ii}^M$.

4.2 Quality of Communication

Recall that under each governance g considered, all of the divisions are symmetric. Therefore, given g, each division's communication bias is the same, which is denoted as b^g. For $g = C, U, M$, the formula for communication bias, (3), can be simplified as

$$b^g = \frac{1}{a_{11}^g[1 + \delta_H(1 - \frac{a_{12}^g}{a_{11}^g})^2 + \delta_L(1 - \frac{a_{14}^g}{a_{11}^g})^2]} - 1. \tag{4}$$

By (4), the communication bias b^g is decreasing in own-decision weight a_{11}^g and increasing in the ratios of accommodation to adaptation, $\frac{a_{12}^g}{a_{11}^g}$ and $\frac{a_{14}^g}{a_{11}^g}$.[18] To understand the result, note that division manager 1's ideal decision of d_1 is θ_1. Without coordination concerns, division manager 1 would report $v_1 = \theta_1 / a_{11}^g$. Thus his incentive to exaggerate is decreasing in a_{11}^g. But exaggeration also leads to coordination losses. Due to the differences between a_{11}^g and the accommodations a_{12}^g and

[18] The ratios of $\frac{a_{12}^g}{a_{11}^g}$ and $\frac{a_{14}^g}{a_{11}^g}$ are smaller than 1.

a_{14}^g, more exaggeration leads to bigger differences between d_1 and d_2 and d_4, resulting in bigger coordination losses for D1. This cost of exaggeration is captured by the ratios of accommodation to adaptation, $\frac{a_{12}^g}{a_{11}^g}$ and $\frac{a_{14}^g}{a_{11}^g}$. When these ratios increase (move toward 1), the cost of exaggeration becomes smaller, and division manager 1 has a stronger incentive to exaggerate. Finally, we want to point out that a_{11}^g and the ratios of $\frac{a_{12}^g}{a_{11}^g}$ and $\frac{a_{14}^g}{a_{11}^g}$ typically move in opposite directions, as an increase in the adaptation weight a_{11}^g usually implies decreases in the accommodation weights a_{12}^g and a_{14}^g. Therefore, the communication bias b^g mostly depends on a_{11}^g: A bigger own-decision weight implies a smaller communication bias.

Proposition 2 *(i) Suppose $\delta_H \delta_L \leq 0.5$ and $\delta_H + \delta_L \leq 1.5$. The quality of communication under U-form is better than that under M form: $b^M > b^U$. (ii) If $\delta_H \leq 1$, then communication under M-form is more informative than that under centralization: $b^C > b^M$.*

Proposition 2 establishes that under some mild conditions,[19] the quality of communication improves when the governance structure moves from centralization to M-form or moves from M-form to U-form. As pointed out earlier, the communication bias is decreasing in own-decision weight. Thus, technically, the result that $b^C > b^M > b^U$ mainly follows the results in Lemma 1: $a_{ii}^U > a_{ii}^M > a_{ii}^C$. More intuitively, relative to the HQ under centralization, the interest of division manager i is more aligned with his group manager under either U-from or M-form. As a result, communication is more informative under either U-form or M-form than under centralization. Between U-form and M-form, the interest of division manager i is more aligned with his group manager under U-form than under M-form. This is because under U-form a group manager internalizes a weak coordination link, while under M-form a group manager internalizes a strong coordination link. As a result, communication is more informative under U-form than under M-form.

Figure 3 plots the communication biases under alternative organization forms when $\delta_L = 0.1$ and δ_H takes values on $[0.3, 2]$. Communication under decentralization is the least informative. This is not surprising, as a division manager's incentive is more aligned with the HQ than with other division managers. Under the other three organization forms, the quality of communication improves in the following order: centralization, M-form, and U-form.[20] Finally, as δ_H increases, communication under decentralization improves, but communication under the other three organizational form worsens. The reason for this pattern is as follows. Under decentralization, as δ_H increases division managers become more willing to accommodate each other, and thus the own-decision weight a_{ii}^D, which is induced adaptation, increases. Therefore, division managers' incentives to exaggerate are weakened. Under the other three organizational forms, as δ_H increases the HQ or group managers will lower the own-decision weight a_{ii}^g and increase the accommodation weight a_{ij}^g. As a result, division managers' incentives to exaggerate become stronger.

[19] The conditions listed in the proposition are sufficient, but not neccessary, for the ranking to hold.

[20] Note that for $\delta_H \in (1.4, 2]$, the conditions in Proposition 2 are not satisifed; however, the ranking of communication quality still holds.

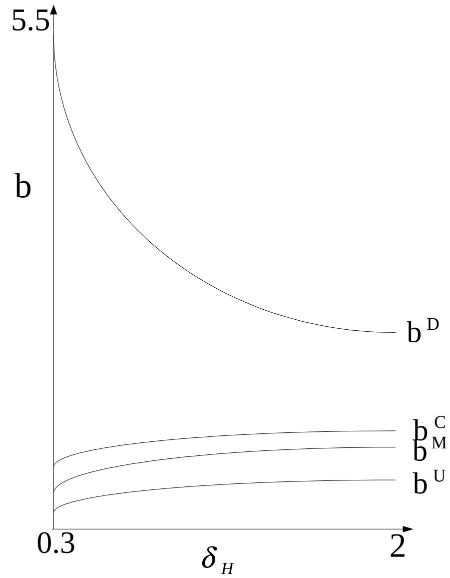

Figure 3: Communication Biases across Alternative Governance Structures

4.3 Relative performance

Given equilibrium decisions d^g and communication biases b^g, for each organizational form we can compute the ex ante expected performance under the most informative communication equilibrium, which is denoted as π^g (the detailed derivation is in the Appendix). Define the expected loss under g as L^g. Since the four divisions are symmetric, we have

$$-\pi^g = L^g = 4E[(d_1^g - \theta_1)^2 + \delta_H(d_1^g - d_2^g)^2 + \delta_L(d_1^g - d_4^g)^2]. \tag{5}$$

As in Rantakari (2008), we can decompose the expected losses under governance structure g (relative to the first best) into two components. The first component is due to distortions in decisions. Note that under centralization, there is no distortion in decisions. The second component of loss is due to noisy communication. Furthermore, this loss is affected by two things: the quality of communication and the value of communication. Under centralization or either form of hierarchical governance, communication is essential to achieve adaptation. On the other hand, under decentralization, communication is mainly to achieve coordination.

Figure 4 illustrates the optimal governance structure in the parameter space of $\delta_L \in [0.01, 0.5]$ and $\delta_H \in [\delta_L, 1]$. In the figure, we see that decentralization is optimal if both δ_L and δ_H are small. On the other hand, when both δ_L and δ_H are large, centralization is optimal. These results are not surprising, as centralization suffers from large adaptation losses (due to noisy communication) and decentralization suffers from large coordination losses (due to distortions in decisions). Thus centralization performs better than decentralization when the coordination needs (δ_L and δ_H) are

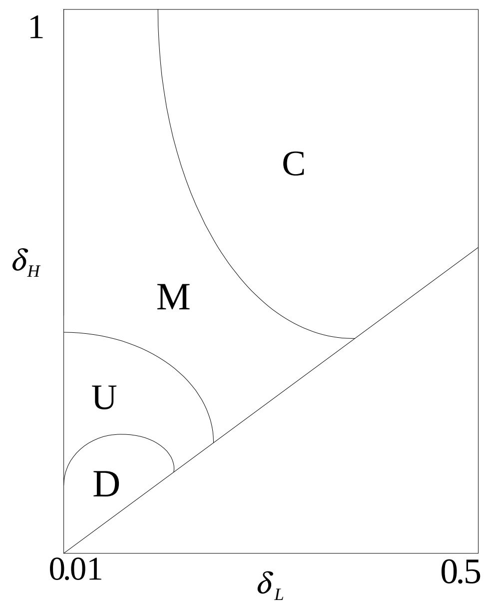

Figure 4: Optimal Governance under A Circle Coordination Network

high.

The interesting case is when δ_L is intermediate. In this case, one of the hierarchical governance structures is optimal. In particular, when δ_H is close to δ_L then U-form is optimal. On the other hand, when δ_H is large then M-form is optimal. These results show that there is an area of parameter space under which introducing an intermediate level of managers is beneficial, or hierarchy endogenously emerges.

To understand these results, let us first rank distortions in decisions across different governance structures. Recall that there is no distortion in decisions under centralization. Under both U-form and M-form, decisions are distorted due to group managers' own-group bias. Compared to decentralization, however, decisions under hierarchical governance are less distorted, as each group manager internalizes the coordination losses within his own group. Between U-form and M-form, distortions in decisions are smaller under M-form. This is because under M-form the strongly linked divisions are within the same group, while under U-form the strongly linked divisions are in different groups.

On the other hand, Proposition 2 shows that communication is more informative under hierarchy than under centralization. Therefore, relative to centralization, hierarchy leads to smaller adaptation losses. This improved communication under hierarchy makes it possible for hierarchy to outperform centralization.

Taken together, relative to decentralization, hierarchical governance leads to less distorted decisions, which implies smaller coordination losses. Relative to centralization, hierarchical governance achieves better adaptation due to more informative communication. Thus, under hierarchical governance coordination and adaptation are relatively more balanced, while centralization is good for coordination but bad for adaptation, and vice versa for decentralization. Starting from a very small δ_L, as δ_L (and hence δ_H) increases, relative to hierarchical governance, decentralization becomes worse because its distortions in decisions become more costly. On the other hand, starting from a large δ_L, as δ_L decreases, relative to hierarchical governance, centralization becomes worse because adaptation becomes relatively important. As a result, when δ_L is in an intermediate range, such that adaptation needs and coordination needs are roughly equally important, hierarchical governance is optimal since it better balances adaptation and coordination.

Proposition 3 *Suppose $\delta_H = \delta_L = \delta$ (M-form and U-form coincide). There exist a $\underline{\delta} \simeq 0.09$ and $\overline{\delta} \simeq 0.26$ such that decentralization is optimal when $\delta \in (0, \underline{\delta})$, centralization is optimal when $\delta > \overline{\delta}$, and hierarchy is optimal when $\delta \in (\underline{\delta}, \overline{\delta})$.*

Proposition 3 formally shows that hierarchical governance's being optimal does not require asymmetry of the coordination links. Even for a symmetric network ($\delta_H = \delta_L$, the diagonal line in the figure), hierarchical governance is optimal when the coordination need is intermediate. Intuitively, for an organization with more than two divisions, centralization sacrifices too much adaptation due to noisy communication, since the interest of the HQ is too diffused relative to individual division managers. On the other hand, decentralization sacrifices too much coordination, again because individual division managers' interests are far away from the HQ's. By introducing an intermediate layer of management, hierarchical governance is able to achieve a better balance between adaptation and coordination. Therefore, it is the multiple number of divisions that makes hierarchical governance potentially optimal. Applying this insight, we expect similar results to hold in organizations with more than four divisions.

Between U-form and M-form, the figure indicates that the region in which M-form is optimal is much larger than the region in which U-form is optimal. Moreover, more asymmetry among coordination links favors M-form over U-form: U-form is optimal only if δ_H is close enough to δ_L. The underlying reason for this pattern is as follows. Relative to M-form, the distortions in decisions are larger and the quality of communication is higher under U-form. Thus U-form is relatively good for adaptation but bad for coordination (closer to decentralization), while M-form is relatively good for coordination but bad for adaptation (closer to centralization).[21] As δ_H increases, overall coordination becomes more important. As a result, M-form becomes more likely to be optimal.

[21] Intuitively, decision-making under U-form is closer to decentralization, while decision-making under M-form is closer to centralization (because the the strong coordination links are internalized).

However, asymmetry in coordination links indeed makes M-form more likely to be optimal in some sense. To see this, fix the overall weight of coordination $\delta_L + \delta_H = \delta$. Starting from a point $(\delta/2, \delta/2)$ where centralization is optimal, as δ_H increases (and δ_L decreases), at some point M-form becomes the optimal governance. To understand why asymmetry in coordination links tends to make M-form optimal, let us compare M-form and centralization more closely. As δ_H increases and δ_L decreases, decisions under centralization and M-form are actually moving closer. This is because M-form internalizes the strong coordination links. As a result, the distortions in decisions under M-form become smaller; this effect tends to make M-form more likely to be optimal. Moreover, this is also why the region in which hierarchical governance is optimal mainly depends on the weight of the weak coordination link, δ_L.

Some anecdotal and empirical evidence is consistent with our prediction. Milgrom and Roberts (1992) state that firms initially define divisions to "minimize the connections among them" and "avoid the need for coordination across division boundaries." This essentially says that M-form is more likely to be better. Chandler (1962, 1977) cites firm diversification as one of the main reasons that M-form becomes popular among large firms; this pattern is verified in an empirical study by Mahoney (1992). This is consistent with our prediction. Firm diversification reduces coordination needs among different products (i.e., the weights of the weak coordination links). As a result, firms move from centralization to M-form.

5 Two Other Coordination Networks

Under the circle network studied earlier, the network positions of all divisions are symmetric. In this section we study two other coordination networks, i.e. the star and the line, under which different divisions have different network positions. In addition to centralization, decentralization, and hierarchy, we will consider other governance structures, such as partial centralization/decentralization and some hybrid organization forms. It turns out that hierarchy or a mixture of hierarchy and centralization remain the optimal governance forms for these coordination networks.

5.1 Star Network

In this subsection we study a star coordination network, which is illustrated in Figure 5. Specifically, there is a center division that has coordination links with each other division, while there is no coordination link between any pair of other divisions. Such coordination networks are common in the real world. For a concrete example, consider a firm consisting of one production division and three marketing divisions, each of which is responsible for selling products in a separate region. The decision of the production division must be coordinated with that of each marketing division, while the decisions of the marketing divisions do not need to be directly coordinated with each other, since they sell products in separate regions. In this example, the production division is the

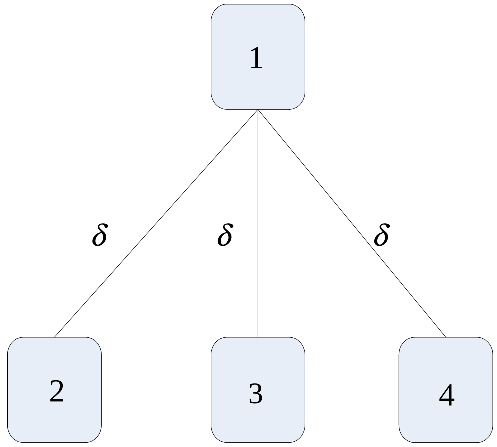

Figure 5: A Star Coordination Network

Notation	**Definition**
C	Centralization
D	Decentralization
$C1DO$	D1 is centralized, while D2, D3, and D4 are decentralized
$D1CO$	D1 is decentralized, while D2, D3, and D4 are centralized
$C1H$	D1 is centralized, while D2, D3, and D4 form a group

Table 1: Notations for Alternative Governance under A Star Coordination Network

center division, while the three marketing divisions are the peripheral divisions.[22]

Specifically, D1 is the center division. For each $i \neq 1$, $\delta_{1i} = \delta > 0$, and $\delta_{ii} = 1$. We set $\delta_{11} = \gamma > 0$, and will vary it as a parameter. Given the specification, the three peripheral divisions are symmetric, which we will label as D_i.[23]

We consider five governance structures, which are listed in Table 1. In addition to centralization and decentralization, we will study two partial centralization/decentralization governance modes: $C1DO$ and $D1CO$. The governance $C1H$ is a hybrid governance that combines centralization and hierarchy. Specifically, for the group manager of the three peripheral divisions for which he makes the decisions, his incentive is to maximize the joint payoff for the three divisions. Note that all these governance structures are quasi-symmetric, in that the allocation of decision rights is the same for divisions with the same network position.

For equilibrium decisions, we only present those under $C1DO$ and $C1H$. All of the detailed

[22] For another example, consider a diversified firm with one finance division and three product divisions, each of which sells unrelated products. In this case the finance division is the center division and the three product divisions are peripheral divisions, since they sell unrelated products and thus do not need to be directly coordinated with each other.

[23] Symmetry is imposed in order to reduce computational burden.

derivations and expressions of decision weights can be found in the Appendix. Specifically, under $C1DO$, equilibrium decisions are given by

$$\begin{aligned} d_1^{C1DO} &= a_{11}^{C1DO}\overline{m}_1 + \sum_{i\neq 1} a_{1i}^{C1DO}\overline{m}_i, \\ d_i^{C1DO} &= z_i^{C1DO}\theta_i + a_{i1}^{C1DO}\overline{m}_1 + \sum_{j\neq 1} a_{ij}^{C1DO}\overline{m}_j. \end{aligned}$$

And under $C1H$, equilibrium decisions are given by

$$\begin{aligned} d_1^{C1H} &= a_{11}^{C1H}\overline{m}_1 + \sum_{i\neq 1} a_{1i}^{C1H}\overline{m}_i, \\ d_i^{C1H} &= a_{i1}^{C1H}\overline{m}_1 + a_{ii}^{C1H}\overline{m}_i + \sum_{j\neq 1,i} a_{ij}^{C1H}\overline{m}_j. \end{aligned}$$

Due to the symmetry of the three peripheral divisions, under each governance structure g, $a_{12}^g = a_{13}^g = a_{14}^g$; for $i, j \neq 1$ and $i \neq j$, $a_{i1}^g = a_{j1}^g$, $a_{ii}^g = a_{jj}^g$, and $a_{ij}^g = a_{ji}^g$.

Lemma 2 *The following relationships hold for various decision weights. (i) Suppose $\gamma = 1$. Then $a_{11}^C < a_{ii}^C$, and $a_{11}^D > a_{ii}^D$. (ii) Across centralization and decentralization, $a_{11}^C > a_{11}^D$, and $a_{ii}^C > a_{ii}^D$. (iii) $a_{11}^{C1DO} < a_{11}^C$, and $a_{ii}^{D1CO} < a_{ii}^C$. (iv) $a_{ii}^{C1DO} > a_{ii}^D$, and $a_{11}^{D1CO} > a_{11}^D$. (v) $a_{ii}^{C1H} = z_i^{C1DO} + a_{ii}^{C1DO}$, and all other corresponding decision weights are the same across C1DO and C1H.*

Part (i) of Lemma 2 illustrates how a division's network position affects its own-decision weight. To facilitate the comparison, we normalize $\gamma = 1$ to make the need for adaptation the same across all divisions. Under centralization, the center division's own-decision weight is lower than peripheral division's ($a_{11}^C < a_{ii}^C$). However, under decentralization the relationship is the opposite ($a_{11}^D > a_{ii}^D$). To understand this result, note that under either centralization or decentralization, D1 and each D$_i$ are equally accommodating for each other.[24] Since D1 has three coordination links—while Di only has one—under centralization overall D1 accommodates more of other divisions, which leads to a smaller adaptation weight a_{11}^C. Under decentralization, a division's own message affects its own decision only by affecting the decisions of other divisions. In other words, a_{11}^D and a_{ii}^D are induced adaptation. Since D1 is the center, D1's message directly affects the three divisions' decisions, and thus its induced adaptation a_{11}^D is higher than a_{ii}^D.

Part (ii) shows that the own-decision weights are higher under centralization than under decentralization. This result is not surprising, as under decentralization a_{11}^D and a_{ii}^D represent induced adaptation, while under centralization a_{11}^C and a_{ii}^C represent adaptation. Part (iii) compares centralization and partial centralization/decentralization, and the results can be understood as follows. Compared to centralization, under partial centralization/decentralization the divisions that remain

[24]When $\gamma = 1$, $a_{1i}^C = a_{i1}^C$ and $a_{1i}^D = a_{i1}^D$, since each coordination link is symmetric.

centralized become more accommodating of the decentralized divisions, and their own-decision weights decrease. To understand the intuition, compare C1DO and centralization. Since under C1DO the three peripheral divisions are decentralized, they accommodate less of D1's decision. As a result, the HQ, which cares about all divisions, responds by increasing D1's accommodation of other divisions, thus reducing D1's own adaptation. The intuition for part (iv) is similar: Compared to decentralization, under partial centralization/decentralization the divisions that remain decentralized become less accommodating of the centralized divisions, and their own-decision weights increase.

Part (v) compares decision weights across C1H and C1DO. Between these two governance structures, the only difference is that under the former the three peripheral divisions form a group, while under the latter they are decentralized. Since the three peripheral divisions have no direct coordination link with each other, the group manager under C1H does not internalize any coordination link. In other words, the group manager's incentive is almost perfectly aligned with that of individual division managers. As a result, all decision weights are the same across C1H and C1DO, except that $a_{ii}^{C1H} = z_i^{C1DO} + a_{ii}^{C1DO}$ (under C1DO, D_i's adaptation weight equals to D_i's direct adaptation plus induced adaptation). The difference is due to the fact that under C1H the group manager does not directly observe local conditions.

The following proposition compares divisions' incentives to exaggerate under various governance structures.

Proposition 4 *(i) Under centralization, $b_1^C > b_i^C$ if and only if $\gamma < \widehat{\gamma}$, $\widehat{\gamma} > 1$; under decentralization, $b_1^D < b_i^D$ if and only if $\gamma > 1$. (ii) $b_1^{D1CO} < b_1^D$ and $b_i^{C1DO} < b_i^D$. (iii) $b_i^{C1H} < b_i^C$ and $b_i^{C1H} < b_i^{C1DO}$.*

Part (i) of Proposition 4 shows that there is a range of γ ($\gamma \in (1, \widehat{\gamma})$) such that, under centralization, relative to the peripheral divisions, the center division's communication is less informative, while under decentralization the center division's communication is more informative. This result is different from Rantakari (2008): In his setting, under either centralization or decentralization, the more dependant division's communication is always more noisy.[25] This result is closely related to the comparison of own-decision weights across divisions. As pointed out earlier (when $\gamma = 1$), under centralization the center division's own-decision weight is lower relative to the peripheral division's ($a_{11}^C < a_{ii}^C$). As a result, the center division has a stronger incentive to exaggerate its own state. Under decentralization, however, the center division's own-decision weight is higher relative to peripheral division's own-decision weight ($a_{11}^D > a_{ii}^D$). This implies that D1's incentive to exaggerate is relatively lower. Taken together, there is a range of γ such that D1's communication is relatively more informative than the peripheral divisions' communication under decentralization, but relatively less informative under centralization.

[25] In our model, the center division is relatively more dependent.

Part (ii) shows that, compared to decentralization, under partial centralization/decentralization the quality of communication is improved for the divisions that are still decentralized. This result is intuitive. When some division(s) are centralized, the HQ makes these divisions' decisions more accommodating. As a result, the decentralized divisions' induced adaptations increase, and their incentives to exaggerate decrease. For instance, as shown in Lemma 2, $a_{11}^{D1CO} > a_{11}^{D}$ and $a_{ii}^{C1DO} > a_{ii}^{D}$.

Part (iii) shows that peripheral divisions' communication bias is smaller under C1H than under centralization. Echoing a similar result in the last section, this is because the group manager's incentive under C1H, relative to the HQ under centralization, is more aligned with individual division managers'. Peripheral divisions' communication bias under C1H is also smaller than under C1DO. Technically, this is because the own-decision weight of the peripheral divisions under C1H, a_{ii}^{C1H}, is bigger than the induced adaptation under C1DO, a_{ii}^{C1DO} (part (v) of Lemma 2). Intuitively, under C1H, D_i's communication is essential to achieve D_i's adaptation, as the group manager does not observe local conditions. Under C1DO, however, D_i only achieves induced adaptation, which leads to a smaller communication bias under C1H. As mentioned earlier, under C1H the group manager's incentive is almost perfectly aligned with individual division managers. This implies that under C1H, peripheral divisions' communication is very informative, and the improvement in communication is significant relative to C1DO.[26]

Again, we denote the overall expected loss under g as L^g. The following proposition compares the relative performance of various governance structures when $\gamma = 1$.

Proposition 5 *Suppose $\gamma = 1$ and $\delta > 0$. (i) C1H always performs better than C1DO: $L^{C1H} < L^{C1DO}$. (ii) D1CO is always dominated by either centralization or decentralization: $L^{D1CO} > \min\{L^C, L^D\}$. (iii) There are $\delta_1 \simeq 0.08$ and $\delta_2 \simeq 0.31$ such that the optimal governance is decentralization if $\delta < \delta_1$, centralization if $\delta > \delta_2$, and C1H if $\delta \in (\delta_1, \delta_2)$.*

In broad terms, Proposition 5 shows that under the star coordination network, partial centralization/decentralization cannot be optimal. The optimal governance is decentralization when the need for coordination is low, centralization when the need for coordination is high, and (hybrid) hierarchy when the need for coordination is intermediate. This pattern is consistent with the pattern discovered under the circle network.

To understand why partial centralization/decentralization cannot be optimal, first compare C1H and C1DO. As mentioned earlier, under C1H the group manager does not internalize any coordination link. This means that the decisions of the three peripheral divisions under C1H are almost as distorted as those under C1DO. Compared to C1H, the adaptations of the three peripheral divisions under C1DO are better, since the division managers have the decision rights. However,

[26] When $\gamma = 1$, it can be shown that b_i^{C1H} is increasing in δ and b_i^{C1DO} is decreasing in δ. Even when $\delta = 1$, $b_i^{C1H} = 3/17$ is much smaller than $b_i^{C1DO} = 3$.

as shown in Proposition 4, compared to C1DO, under C1H the quality of communication of the peripheral divisions is much higher. This communication advantage has two layers of implication. First, relative to C1DO, the loss in adaptation of the peripheral divisions under C1H is small. Second, the improved communication under C1H can significantly reduce the coordination loss relative to C1DO. This is because under C1DO, each peripheral division's decision varies with its own local condition, but other divisions only observe a noisy message from that division; thus, noisy communication hurts coordination.[27] As a result, C1H always dominates C1DO.[28]

Next we compare the two governance structures of partial centralization/decentralization: D1CO and C1DO. It turns out that there is a region of δ such that C1DO outperforms D1CO, centralization, and decentralization. On the other hand, D1CO cannot outperform centralization and decentralization at the same time. This shows that C1DO is a better organizational form than D1CO. This result is in contrast to the one in Rantakari (2008), where the better partial centralization governance is to decentralize the more dependent division. In our model, D1 is the more dependent division, yet the better partial centralization/decentralization governance is to have D1 centralized. To understand the intuition for this result, note that under both governance structures, coordination losses are more or less the same. This is because for each coordination link, decentralizing the center division and decentralizing the peripheral divisions lead to similar coordination losses. However, under C1DO only the adaptation of D1 is sacrificed, while under D1CO the adaptations of the three peripheral divisions are sacrificed. Therefore, centralizing the center division D1 leads to a smaller overall adaptation loss, or C1DO performs better than D1CO.

Finally, the reason that C1H can outperform both centralization and decentralization at the same time is again that under C1H, coordination and adaptation are more balanced. Specifically, under C1H the three coordination links are also partially internalized (center division D1 is centralized). Relative to decentralization, C1H improves coordination but sacrifices the adaptation of D1. Relative to centralization, C1H improves the adaptation of the three peripheral divisions but sacrifices coordination. As a result, when coordination need δ is intermediate, CIH is optimal. This shows that introducing hierarchy is beneficial even if a group manager does not internalize any coordination link. The benefit of hierarchy is that it induces higher quality communication among the managers of the peripheral divisions.

Figure 6 illustrates the optimal governance structure in the parameter space of $\delta \times \gamma = [0.01, 2]^2$. For a given γ, the pattern is similar to the pattern when $\gamma = 1$: as δ increases, the optimal governance moves from decentralization to C1H and then to centralization. Moreover, partial centralization/decentralization can never be optimal. The figure also demonstrates the following

[27] Under C1H noisy communication does not affect coordination, since all decision makers have no private information.

[28] This holds even when $\delta \to 0$. The reason is that when $\delta \to 0$, under C1H $b_i^{C1H} \to 0$, but under C1DO $b_i^{C1DO} \to \infty$. Thus the adaptation advantage of C1DO disappears in the limit, while the coordination advantage of C1H still remains.

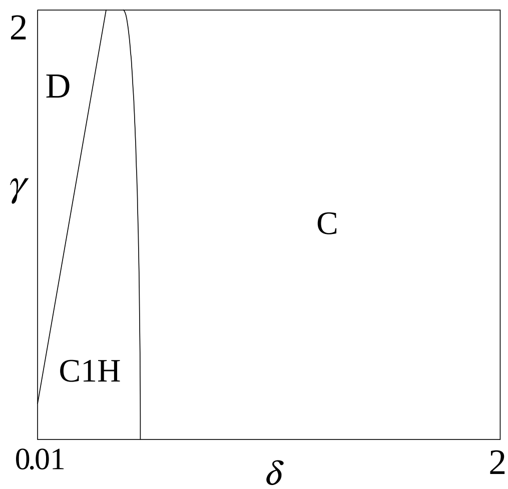

Figure 6: Optimal Governance under A Star Coordination Network

pattern: As γ increases, the range of δ under which decentralization is optimal expands, while the range of δ under which C1H is optimal shrinks. To see this, recall that relative to decentralization, under C1H the adaptation of D1 is sacrificed. As γ increases, the adaptation of D1 becomes more important. Consequently, decentralization performs relatively better than C1H.[29]

5.2 Line Network

Consider a line network, which is illustrated in Figure 7. For each division i, adaptation need δ_{ii} is normalized to 1. The coordination links 12 and 34 have the same weight: $\delta_{12} = \delta_{34} = \gamma$, while coordination link 23's weight is $\delta_{23} = \delta$. In the specification, D1 and D4 are symmetric and are relatively peripheral; D2 and D3 are symmetric, and relatively central.

In terms of network structure, a line network lies somewhere between a star and a circle: It is more symmetric than a star, but more asymmetric than a circle. Line coordination networks are also relevant in the real world. For a concrete example, consider a firm consisting of four divisions, each of which sells a different product. The relationships among the four products are described by a line network: products 1 and 2 are related (i.e., they share common components or marketing); products 2 and 3 are related; and products 3 and 4 are related. No other pairs are directly related.[30]

The main purpose of this subsection is for robustness check—that is, whether the insights

[29] We also study directional governance structures under which the division manager of D2 has the decision rights of D2 and D1, and D3 and D4 are either centralized or decentralized. Under such governance sturctures, the adaptation of D1 is completely sacrificed. It turns out that these directional governance structures can never be optimal. Intuitively, when there are more than two divisions, relative to the coordination network, directional goverance is too asymmetric to be optimal.

[30] In another example, the four divisions could sell the same product, but in different regions. The four regions geographically form a line, and the decisions of two neighboring regions need to be directly coordinated.

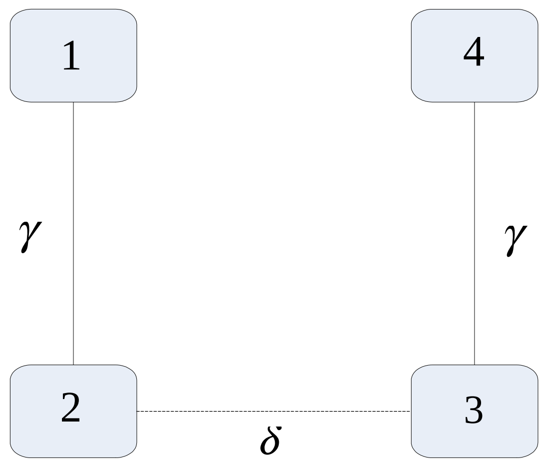

Figure 7: A Line Coordination Network

Notation	Definition
C	decisions are made by the HQ
D	decisions are made by the individual divisions
$D14C23$	D1 and D4 are decentralized, while D2 and D3 are centralized
$C14D23$	D2 and D3 are decentralized, while D1 and D4 are centralized
$H12$	D1 and D2 form a group, and D3 and D4 form another group
$H23$	D2 and D3 form a group, and D1 and D4 form another group
$D14G23$	D1 and D4 are decentralized, while D2 and D3 form a group
$G14D23$	D1 and D4 form a group, while D2 and D3 are decentralized
$G14C23$	D1 and D4 form a group, while D2 and D3 are centralized
$C14G23$	D1 and D4 are centralized, while D2 and D3 form a group

Table 2: Notations for Alternative Governance under A Line Coordination Network

obtained under the circle network and the star network also apply to the line network. In particular, we are interested in the comparison between partial centralization/decentralization and hierarchical governance structures.

We will consider 10 governance structures, which (along with the notations) are summarized in Table 2.

Note that all of the above governance structures are quasi-symmetric, as the allocation of decision rights is symmetric for divisions with the same network position. As a result, under each governance g considered, D1 and D4 are symmetric and D2 and D3 are symmetric, both in terms of decisions and communication. In the rest of the subsection, we will only compare the relative performance of various governance structures. The detailed analysis can be found in an online appendix.

The following figure illustrates the optimal organization structure in the parameter space $(\delta, \gamma) \in$

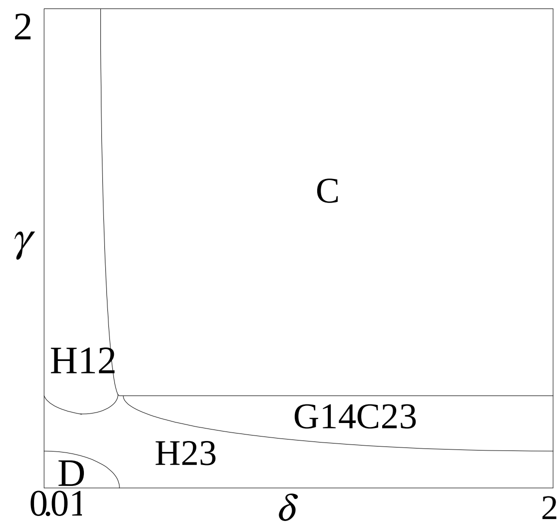

Figure 8: Optimal Governance under A Line Coordination Network

$[0.01, 2] \times [0.01, 2]$. Again, decentralization is optimal only when both γ and δ are very small, and centralization is optimal when both γ and δ are large. When δ is small but γ is large, hierarchical governance $H12$ is optimal. When γ is very small and δ is not too small, hierarchical governance $H23$ is optimal. When γ is small and δ is relatively large, hybrid governance $G14C23$ is optimal. Partial centralization/decentralization and other hybrid governance structures can never be optimal.

To understand the results, let's separate the parameter space roughly by the line $\gamma = \delta/2$. Above this line, coordination links 12 and 34 are relatively more important than coordination link 23; below this line, coordination link 23 is relatively more important. Abusing terminology, in the upper region (above the line), governance $H12$ can be considered as M-form, as the strong coordination links are internalized, while governance $H23$ is U-form as the weak coordination link is internalized. In the lower region, the pattern is reversed: H12 is U-form, while H23 is M-form. As shown in Section 4, relative to M-form, U-form improves the quality of communication, and thus adaptation, but worsens coordination. In the upper region, when γ is small, overall coordination is less important relative to adaptation, and thus U-form (H23) is optimal. When γ is large and δ is small, overall coordination becomes relatively important, and thus M-form (H12) is optimal. In the lower region, when γ is small and δ is small, U-form (H12) is optimal (a tiny region, which is not shown in the figure).[31] But for large δ, M-form (H23) dominates U-form (H12). This pattern is consistent with the results under the circle network.

The result that partial centralization/decentralization cannot be optimal is consistent with the result under the star network. The intuition is also the same. For a given organizational

[31] Roughly, H12 is optimal when $\gamma \in [0.01, 0.02]$ and $\delta \in [0.15, 0.22]$.

structure of partial centralization/decentralization, there is a corresponding hybrid (hierarchical) governance in which the two decentralized divisions form a group that performs better. The main reason is that introducing hierarchy for the two decentralized divisions improves these two divisions' communication, which in turn improves coordination.[32] For the same reason, hybrid governance that combines decentralization and hierarchy (G14D23 and D14G23) cannot be optimal either.

Finally, we compare $H23$, centralization, and two hybrid governance structures, $G14C23$ and $C14G23$, when δ is relatively large. Note that both $C14G23$ and $G14C23$ lie between $H23$ and centralization. In particular, under either $C14G23$ or $G14C23$, the coordination links 12 and 34 are partially internalized. When governance moves from $H23$ to $G14C23$ and to centralization, coordination becomes better overall. As a result, when the coordination links 12 and 34 become more important (γ increases), the optimal organization form changes from $H23$ to $G14C23$ and to centralization.

Between the two hybrid governance structures, $C14G23$ is never optimal, while $G14C23$ could be optimal. In other words, if an organization adopts a hybrid governance that combines centralization and hierarchy, then the central divisions—instead of the peripheral divisions—should be centralized. The underlying driving force for this result is the difference in communication quality.[33] Relative to $H23$, under $C14G23$ the communication quality of the two peripheral divisions D1 and D4 is significantly worsened. This is because there is no direct coordination link between D1 and D4, which implies that D1's and D4's incentives are aligned almost perfectly with their group manager's under $H23$. As a result, the adaptations of D1 and D4 are worsened significantly under $C14G23$. On the other hand, relative to $H23$, under $G14C23$ the communication quality of the two central divisions D2 and D3 is only slightly worsened. This is because the weight of coordination link 23, δ, is relatively large. This implies that even under H23, communication between D2 and D3 is quite noisy. Therefore, relative to H23, under $G14C23$ the adaptation of D2 and D3 is only slightly worsened. Taken together, $G14C23$ performs better than $C14G23$.

Some empirical evidence supports our prediction. In a study of multinational firms, Colombo and Delmastro (2004) find that the allocation of decision rights can be different across different subsidiaries. They also show that the degree of decentralization in an organization is negatively correlated with intra-firm interdependencies. Abernethy et al. (2004) show that delegation of decision rights is less likely for divisions that are more dependent. This is consistent with our last prediction, that central divisions are more likely to be centralized than peripheral divisions.

[32] If the two divisions have a direct coordination need, internalizing the coordination link can also improve coordination directly.

[33] Distortions in decisions under the two governance structures are roughly the same.

6 Concluding remarks

In this paper we study adaptation versus coordination in a multidivisional organization with endogenous communication, and compare relative performance across different organizational structures. Under a circle coordination network, we find that hierarchy emerges as an optimal governance structure when coordination needs are intermediate. Compared with centralization and decentralization, communication under hierarchy is more informative. Consequently, hierarchy achieves a better balance between adaptation and coordination, and thus outperforms both centralization and decentralization when coordination and adaptation are roughly equally important. Between M-form and U-form hierarchical organizations, U-form organization leads to more informative communication and adapts better to local conditions, but results in worse coordination among divisions. As the need for coordination of the strong coordination links increases, the optimal governance moves from U-form to M-form.

We further study star and line coordination networks, under which divisions have asymmetric network positions. One pattern we found is that partial centralization/decentralization can never be optimal, but hierarchy or hybrid hierarchy can be optimal. In particular, grouping divisions together and introducing a group manager is beneficial even if the divisions within the group have no direct coordination link with each other. This result is again driven by the fact that introducing hierarchy improves the quality of communication. Taken together, these results suggest that introducing a middle layer of management is beneficial to organizations, as the middle managers, via improved communication, are able to strike a better balance between adaptation and coordination.

In our analysis, we have restricted our attention to the case of four divisions. However, we expect that our main insights regarding hierarchy as an optimal mode of governance will continue to hold for organizations with more than four divisions. This is because grouping divisions together improves communication, as individual division managers' incentives are more aligned with those of group managers than with those of HQ or other division managers. Consequently, hierarchy can achieve a better balance between adaptation and coordination, and thus outperforms centralization and decentralization when the need for coordination is intermediate.

We have assumed that hierarchy does not change the flow of communication. Specifically, under hierarchical governance the communication is still one-round and public. We believe this is a reasonable assumption, because we focus on decision-making and communication at the top management level. Alternatively, one could assume that with hierarchy there will be two rounds of communication: Division managers communicate to their group manager in the first round, and then the two group managers communicate with each other in the second round. Although this is an interesting and realistic format for communication, it significantly complicates the model and makes it very hard to solve. The difficulty mainly comes from the fact that group managers could manipulate the information they received in the first stage when they send messages in the second stage. We leave this option for future research.[34]

Finally, we have assumed that the division managers are completely selfish or only care about the payoffs of their own divisions. In reality, division managers may care about the payoffs of other divisions to some degree. This is especially true when their payoff is somewhat tied to the performance of the whole organization. As shown by ADM, making division managers' incentives more aligned with the HQ's will improve the relative performance of decentralization, thus making it more likely to be the optimal organizational form. Following their logic, we expect that our main insight—i.e. that hierarchy performs better than centralization and decentralization when the needs for coordination are intermediate—will continue to hold as long as each division manager has an own-division bias. This is because when the division managers are biased, under hierarchy group managers' incentives will still be more aligned with division managers' relative to the HQ or other division managers.[35] This implies that communication is more informative under hierarchy, and thus it is able to achieve a better balance between adaptation and coordination.

[34] In the cheap talk literature, we are aware of no paper that deals with two rounds of communication, in which receivers in the first round act as senders in the second round.

[35] For instance, suppose division manager i's objective is to maximize $\lambda\pi_i + (1-\lambda)\sum_{j\neq i}\pi_j$, where $\lambda \in (1/2, 1]$. Similarly, suppose under hierarchy, group manager G's objective is to maximize $\lambda\sum_{i\in G}\pi_i + (1-\lambda)\sum_{j\notin G}\pi_j$. Given any λ, it is easy to verify that division manager i's incentive is more aligned with his group manager's than with the HQ's or other division managers'.

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Appendix

Proof of Proposition 1.

Proof. First note that in equilibrium $E[\overline{m}_i] = E[\theta_i] = 0$ for all i. Due to the independence of θ_i and θ_j for any $i \neq j$, we have $E[\overline{m}_i \overline{m}_j] = E[\theta_i \overline{m}_j] = E[\overline{m}_i \theta_j] = E[\theta_i \theta_j] = 0$.

Following a similar proof as that of Proposition 1 in ADM, we can show that all PBE must be interval equilibria. To characterize interval equilibria, let $\overline{m}_{i,k}$ be the posterior belief of state θ_i after hearing a message $m_{i,k} \in (x^g_{i,k-1}, x^g_{i,k})$. For $\theta_i = x^g_{i,k}$, division manager i must be indifferent between sending message $m_{i,k}$ (inducing $\overline{m}_{i,k}$) and sending message $m_{i,k+1}$ (inducing $\overline{m}_{i,k+1}$). That is, given the decisions of (2), $E_{-\theta_i}[\pi_i | x^g_{i,k}, \overline{m}_{i,k}] - E_{-\theta_i}[\pi_i | x^g_{i,k}, \overline{m}_{i,k+1}] = 0$. In particular, using earlier results, this indifference condition is equivalent to

$$
\begin{aligned}
& \delta_{ii}[(a^g_{ii}\overline{m}_{i,k})^2 - 2a^g_{ii}(1 - z^g_i)\overline{m}_{i,k}\theta_i] + \sum_{j \neq i} \delta_{ij}[((a^g_{ii} - a^g_{ji})\overline{m}_{i,k})^2 + 2z^g_i(a^g_{ii} - a^g_{ji})\overline{m}_{i,k}\theta_i] \\
= \; & \delta_{ii}[(a^g_{ii}\overline{m}_{i,k+1})^2 - 2a^g_{ii}(1 - z^g_i)\overline{m}_{i,k+1}\theta_i] + \sum_{j \neq i} \delta_{ij}[((a^g_{ii} - a^g_{ji})\overline{m}_{i,k+1})^2 + 2z^g_i(a^g_{ii} - a^g_{ji})\overline{m}_{i,k+1}\theta_i].
\end{aligned}
$$

Using $\overline{m}_{i,k} = (x^g_{i,k-1} + x^g_{i,k})/2$ and $\overline{m}_{i,k+1} = (x^g_{i,k} + x^g_{i,k+1})/2$ and rearranging, the above indifference condition can be simplified as

$$
(x^g_{i,k+1} - x^g_{i,k}) - (x^g_{i,k} - x^g_{i,k-1}) = 4b^g_i \theta_i, \tag{6}
$$

where b^g_i is given by (3).

The rest of the proof follows Proposition 2 in ADM. In particular, for each positive integer K (the number of partition elements), there is an equilibrium characterized by (6). Moreover, the informativeness of the communication equilibria is increasing in K, and in the most informative equilibrium, $K \to \infty$. The equilibrium partition points in the most informative equilibrium can be analytically solved, which yields $E[(\overline{m}^g_i)^2] = \frac{3(1+b^g_i)}{3+4b^g_i}\sigma^2$. ■

Equilibrium decisions under the circle network.

We only demonstrate how to derive the equilibrium decisions under M-form, as the derivations under other governance structures are similar. Specifically, under M-form manager G1 controls D1 and D2, and manager G2 controls D3 and D4. Given message m, manager G1 chooses d_1 and d_2 to maximize the joint payoff of D1 and D2. This yields G1's best responses as follows:

$$
d_1 = \frac{(1 + \delta_L + 2\delta_H)\, E[\theta_1|m] + 2\delta_H E[\theta_2|m] + \left(\delta_L + \delta_L^2 + 2\delta_H\delta_L\right) E[d_4|m] + 2\delta_H\delta_L E[d_3|m]}{4\delta_H + 2\delta_L + \delta_L^2 + 4\delta_H\delta_L + 1} \tag{7}
$$

$$
d_2 = \frac{(1 + \delta_L + 2\delta_H)\, E[\theta_2|m] + 2\delta_H E[\theta_1|m] + \left(\delta_L + \delta_L^2 + 2\delta_H\delta_L\right) E[d_3|m] + 2\delta_H\delta_L E[d_4|m]}{4\delta_H + 2\delta_L + \delta_L^2 + 4\delta_H\delta_L + 1} \tag{8}
$$

Similarly, manager G2's best responses are

$$d_3 = \frac{(1+\delta_L+2\delta_H)\,E[\theta_3|m] + 2\delta_H E[\theta_4|m] + \left(\delta_L+\delta_L^2+2\delta_H\delta_L\right)E[d_2|m] + 2\delta_H\delta_L E[d_1|m]}{4\delta_H+2\delta_L+\delta_L^2+4\delta_H\delta_L+1} \quad (9)$$

$$d_4 = \frac{(1+\delta_L+2\delta_H)\,E[\theta_4|m] + 2\delta_H E[\theta_3|m] + \left(\delta_L+\delta_L^2+2\delta_H\delta_L\right)E[d_1|m] + 2\delta_H\delta_L E[d_2|m]}{4\delta_H+2\delta_L+\delta_L^2+4\delta_H\delta_L+1} \quad (10)$$

Taking expectations of each d_i in the above equations, we have four equations with 4 unknowns: $E[d_i|m]$. Solving $E[d_i|m]$ simultaneously, and then substituting the relevant $E[d_i|m]$ into the best responses (7)-(10), we get the equilibrium decision weights.

Under centralization, the optimal decision weights a^C_{1j} are given by

$$\begin{aligned}
a^C_{11} &= \frac{16\delta_H^2\delta_L+8\delta_H^2+16\delta_H\delta_L^2+24\delta_H\delta_L+6\delta_H+8\delta_L^2+6\delta_L+1}{(4\delta_H+1)(4\delta_L+1)(4\delta_H+4\delta_L+1)},\\
a^C_{12} &= \frac{\delta_H\left(16\delta_H\delta_L+8\delta_H+16\delta_L^2+8\delta_L+2\right)}{(4\delta_H+1)(4\delta_L+1)(4\delta_H+4\delta_L+1)},\\
a^C_{13} &= \frac{8\delta_H\delta_L(2\delta_H+2\delta_L+1)}{(4\delta_H+1)(4\delta_L+1)(4\delta_H+4\delta_L+1)},\\
a^C_{14} &= \frac{\delta_L\left(16\delta_H^2+16\delta_H\delta_L+8\delta_H+8\delta_L+2\right)}{(4\delta_H+1)(4\delta_L+1)(4\delta_H+4\delta_L+1)}.
\end{aligned}$$

Under decentralization, the equilibrium decision weights a^D_{1j} are given by

$$\begin{aligned}
z &= \frac{1}{1+\delta_H+\delta_L},\\
a^D_{11} &= \frac{2\delta_H^3\delta_L+2\delta_H^3+4\delta_H^2\delta_L^2+2\delta_H^2\delta_L+\delta_H^2+2\delta_H\delta_L^3+2\delta_H\delta_L^2+2\delta_L^3+\delta_L^2}{(2\delta_H+1)(2\delta_L+1)\left(2\delta_H^2+4\delta_H\delta_L+3\delta_H+2\delta_L^2+3\delta_L+1\right)},\\
a^D_{12} &= \frac{\delta_H\left(2\delta_H+2\delta_L+2\delta_L^2+2\delta_H\delta_L+1\right)}{(2\delta_H+1)(2\delta_L+1)(2\delta_H+2\delta_L+1)},\\
a^D_{13} &= \frac{2\delta_H\delta_L(\delta_H+\delta_L+1)}{(2\delta_H+1)(2\delta_L+1)(2\delta_H+2\delta_L+1)},\\
a^D_{14} &= \frac{\delta_L\left(2\delta_H+2\delta_L+2\delta_H^2+2\delta_H\delta_L+1\right)}{(2\delta_H+1)(2\delta_L+1)(2\delta_H+2\delta_L+1)}.
\end{aligned}$$

The equilibrium decision weights under U-form, a^U_{1j}, are given by

$$\begin{aligned}
a^U_{11} &= \frac{4\delta_H^2\delta_L+2\delta_H^2+8\delta_H\delta_L^2+12\delta_H\delta_L+3\delta_H+8\delta_L^2+6\delta_L+1}{(2\delta_H+1)(4\delta_L+1)(2\delta_H+4\delta_L+1)},\\
a^U_{12} &= \frac{4\delta_H^2\delta_L+2\delta_H^2+8\delta_H\delta_L^2+4\delta_H\delta_L+\delta_H}{(2\delta_H+1)(4\delta_L+1)(2\delta_H+4\delta_L+1)},\\
a^U_{13} &= \frac{4\delta_H\delta_L(\delta_H+2\delta_L+1)}{(2\delta_H+1)(4\delta_L+1)(2\delta_H+4\delta_L+1)},\\
a^U_{14} &= \frac{2\delta_L\left(2\delta_H+4\delta_L+2\delta_H^2+4\delta_H\delta_L+1\right)}{(2\delta_H+1)(4\delta_L+1)(2\delta_H+4\delta_L+1)}.
\end{aligned}$$

Finally, under M-form the equilibrium decision weights a_{1j}^{M} are given by

$$\begin{aligned}
a_{11}^{M} &= \frac{8\delta_H^2\delta_L + 8\delta_H^2 + 4\delta_H\delta_L^2 + 12\delta_H\delta_L + 6\delta_H + 2\delta_L^2 + 3\delta_L + 1}{(4\delta_H + 1)(2\delta_L + 1)(4\delta_H + 2\delta_L + 1)}, \\
a_{12}^{M} &= \frac{8\delta_H^2\delta_L + 8\delta_H^2 + 4\delta_H\delta_L^2 + 4\delta_H\delta_L + 2\delta_H}{(4\delta_H + 1)(2\delta_L + 1)(4\delta_H + 2\delta_L + 1)}, \\
a_{13}^{M} &= \frac{4\delta_H\delta_L(2\delta_H + \delta_L + 1)}{(4\delta_H + 1)(2\delta_L + 1)(4\delta_H + 2\delta_L + 1)}, \\
a_{14}^{M} &= \frac{\delta_L\left(4\delta_H + 2\delta_L + 8\delta_H^2 + 4\delta_H\delta_L + 1\right)}{(4\delta_H + 1)(2\delta_L + 1)(4\delta_H + 2\delta_L + 1)}.
\end{aligned}$$

Proof of Lemma 1.

Proof. Part (i). Due to the symmetry among divisions, we only need to show that the results hold for D1. Note that for D1, the strongly linked division s is D2, and the weakly linked division w is D4. By the equilibrium decision weights, we have

$$\begin{aligned}
a_{12}^{C} - a_{12}^{U} &= \frac{\delta_H\left(8\delta_H^2 + 12\delta_H\delta_L + 6\delta_H + 8\delta_L^2 + 4\delta_L + 1\right)}{\left(8\delta_H^2 + 6\delta_H + 1\right)\left(8\delta_H^2 + 24\delta_H\delta_L + 6\delta_H + 16\delta_L^2 + 8\delta_L + 1\right)} > 0, \\
a_{12}^{M} - a_{12}^{C} &= \frac{4\delta_H\delta_L(2\delta_H + 3\delta_L + 1)}{\left(8\delta_L^2 + 6\delta_L + 1\right)\left(16\delta_H^2 + 24\delta_H\delta_L + 8\delta_H + 8\delta_L^2 + 6\delta_L + 1\right)} > 0, \\
a_{14}^{C} - a_{14}^{M} &= \frac{\delta_L}{8\delta_L^2 + 6\delta_L + 1}\frac{8\delta_H^2 + 12\delta_H\delta_L + 4\delta_H + 8\delta_L^2 + 6\delta_L + 1}{16\delta_H^2 + 24\delta_H\delta_L + 8\delta_H + 8\delta_L^2 + 6\delta_L + 1} > 0, \\
a_{14}^{U} - a_{14}^{C} &= \frac{4\delta_H\delta_L(3\delta_H + 2\delta_L + 1)}{\left(8\delta_H^2 + 6\delta_H + 1\right)\left(8\delta_H^2 + 24\delta_H\delta_L + 6\delta_H + 16\delta_L^2 + 8\delta_L + 1\right)} > 0.
\end{aligned}$$

Part (ii). By the equilibrium decision weights, we have

$$\begin{aligned}
a_{ii}^{M} - a_{ii}^{C} &= \frac{\delta_L}{8\delta_L^2 + 6\delta_L + 1}\frac{8\delta_H^2 + 12\delta_H\delta_L + 4\delta_H + 8\delta_L^2 + 6\delta_L + 1}{16\delta_H^2 + 24\delta_H\delta_L + 8\delta_H + 8\delta_L^2 + 6\delta_L + 1} > 0, \\
a_{ii}^{U} - a_{ii}^{C} &= \frac{\delta_H}{8\delta_H^2 + 6\delta_H + 1}\frac{8\delta_H^2 + 12\delta_H\delta_L + 6\delta_H + 8\delta_L^2 + 4\delta_L + 1}{8\delta_H^2 + 24\delta_H\delta_L + 6\delta_H + 16\delta_L^2 + 8\delta_L + 1} > 0.
\end{aligned}$$

To prove $a_{ii}^{U} > a_{ii}^{M}$, note that

$$\begin{aligned}
a_{ii}^{U} - a_{ii}^{M} &\propto -32\delta_H\delta_L\left(\delta_H^2 + \delta_H\delta_L + \delta_L^2\right) - 16\delta_H^2\delta_L^2 + 8\left(\delta_H^2 + \delta_H\delta_L + \delta_L^2\right) + 16\delta_H\delta_L + 6(\delta_H + \delta_L) + 1 \\
&\geq 16\delta_H\delta_L[1 - (\delta_H + \delta_L)^2] + 6(\delta_H + \delta_L) + 1 \geq 8[1 - (\delta_H + \delta_L)^2] + 6(\delta_H + \delta_L) + 1 \\
&= [3 - 2(\delta_H + \delta_L)][3 + 4(\delta_H + \delta_L)] > 0,
\end{aligned}$$

where the first two inequalities use condition $\delta_H\delta_L \leq 0.5$, and the last inequality follows $\delta_H + \delta_L \leq 1.5$. Note that the second inequality implicitly assumes that $(\delta_H + \delta_L) > 1$. If $(\delta_H + \delta_L) \leq 1$, then the result is trivially satisfied. ■

Proof of Proposition 2.

Proof. Again, we only need to show that the results hold for D1.

Part (i). To show $b_1^M > b_1^U$, we first determine the signs of $\frac{a_{11}^U - a_{12}^U}{a_{11}^U} - \frac{a_{11}^M - a_{12}^M}{a_{11}^M}$ and $\frac{a_{11}^U - a_{14}^U}{a_{11}^U} - \frac{a_{11}^M - a_{14}^M}{a_{11}^M}$. By earlier results, we have

$$\begin{aligned}
\frac{a_{11}^U - a_{12}^U}{a_{11}^U} &= \frac{(2\delta_H + 2\delta_L + 1)(4\delta_L + 1)}{4\delta_H^2\delta_L + 2\delta_H^2 + 8\delta_H\delta_L^2 + 12\delta_H\delta_L + 3\delta_H + 8\delta_L^2 + 6\delta_L + 1}, \\
\frac{a_{11}^M - a_{12}^M}{a_{11}^M} &= \frac{(4\delta_H + \delta_L + 1)(2\delta_L + 1)}{8\delta_H^2\delta_L + 8\delta_H^2 + 4\delta_H\delta_L^2 + 12\delta_H\delta_L + 6\delta_H + 2\delta_L^2 + 3\delta_L + 1}.
\end{aligned}$$

Take the difference,

$$\begin{aligned}
&\frac{a_{11}^U - a_{12}^U}{a_{11}^U} - \frac{a_{11}^M - a_{12}^M}{a_{11}^M} \propto \\
&32\delta_H^2\delta_L^2 + 48\delta_H^2\delta_L + 8\delta_H^2 + 24\delta_H\delta_L^3 + 72\delta_H\delta_L^2 + 46\delta_H\delta_L + 6\delta_H + 16\delta_L^4 + 24\delta_L^3 + 22\delta_L^2 + 9\delta_L + 1 > 0.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{a_{11}^U - a_{14}^U}{a_{11}^U} &= \frac{(\delta_H + 4\delta_L + 1)(2\delta_H + 1)}{4\delta_H^2\delta_L + 2\delta_H^2 + 8\delta_H\delta_L^2 + 12\delta_H\delta_L + 3\delta_H + 8\delta_L^2 + 6\delta_L + 1}, \\
\frac{a_{11}^M - a_{14}^M}{a_{11}^M} &= \frac{(2\delta_H + 2\delta_L + 1)(4\delta_H + 1)}{8\delta_H^2\delta_L + 8\delta_H^2 + 4\delta_H\delta_L^2 + 12\delta_H\delta_L + 6\delta_H + 2\delta_L^2 + 3\delta_L + 1}.
\end{aligned}$$

Take the difference,

$$\begin{aligned}
&\frac{a_{11}^U - a_{14}^U}{a_{11}^U} - \frac{a_{11}^M - a_{14}^M}{a_{11}^M} \\
\propto\ & -[16\delta_H^4 + 24\delta_H^3\delta_L + 24\delta_H^3 + 32\delta_H^2\delta_L^2 + 72\delta_H^2\delta_L + 22\delta_H^2 + 48\delta_H\delta_L^2 + 46\delta_H\delta_L + 9\delta_H + 8\delta_L^2 + 6\delta_L + 1] < 0.
\end{aligned}$$

From Lemma 1, we know that $a_{11}^U > a_{11}^M$. Thus to show $b_1^M > b_1^U$, by (4) the following condition is sufficient: $[\frac{a_{11}^U - a_{12}^U}{a_{11}^U} - \frac{a_{11}^M - a_{12}^M}{a_{11}^M}] + [\frac{a_{11}^U - a_{14}^U}{a_{11}^U} - \frac{a_{11}^M - a_{14}^M}{a_{11}^M}] \geq 0$. This is because in the formula of the communication bias, (4), the first term's (which is positive) coefficient δ_H is bigger than the second term's (which is negative) coefficient δ_L. Overall,

$$\begin{aligned}
&[\frac{a_{11}^U - a_{12}^U}{a_{11}^U} - \frac{a_{11}^M - a_{12}^M}{a_{11}^M}] + [\frac{a_{11}^U - a_{14}^U}{a_{11}^U} - \frac{a_{11}^M - a_{14}^M}{a_{11}^M}] \\
\propto\ & 8(\delta_H^4 - \delta_L^4) + (48\delta_H\delta_L + 6)(\delta_H^3 - \delta_L^3) + (16\delta_H^2\delta_L^2 + 46\delta_H\delta_L + 1)(\delta_H^2 - \delta_L^2) + (48\delta_H^2\delta_L^2 + 9\delta_H\delta_L)(\delta_H - \delta_L) \\
>\ & 0.
\end{aligned}$$

Therefore, we conclude that $b^M > b^U$.

Part (ii). In order to show $b_1^C > b_1^M$, we first determine the signs of $[\frac{(a_{11}^M - a_{12}^M)^2}{a_{11}^M} + a_{11}^M] - [\frac{(a_{11}^C - a_{12}^C)^2}{a_{11}^C} + a_{11}^C]$ and $\frac{a_{11}^M - a_{14}^M}{a_{11}^M} - \frac{a_{11}^C - a_{14}^C}{a_{11}^C}$. By earlier results,

$$\begin{aligned}\frac{\left(a_{11}^C - a_{12}^C\right)^2}{a_{11}^C} &= \frac{\left(4\delta_H + 2\delta_L + 1\right)^2 \left(4\delta_L + 1\right)}{\left(4\delta_H + 4\delta_L + 1\right)\left(16\delta_H^2\delta_L + 8\delta_H^2 + 16\delta_H\delta_L^2 + 24\delta_H\delta_L + 6\delta_H + 8\delta_L^2 + 6\delta_L + 1\right)\left(4\delta_H + 1\right)}, \\ \frac{\left(a_{11}^M - a_{12}^M\right)^2}{a_{11}^M} &= \frac{\left(4\delta_H + \delta_L + 1\right)^2 \left(2\delta_L + 1\right)}{\left(8\delta_H^2\delta_L + 8\delta_H^2 + 4\delta_H\delta_L^2 + 12\delta_H\delta_L + 6\delta_H + 2\delta_L^2 + 3\delta_L + 1\right)\left(4\delta_H + 2\delta_L + 1\right)\left(4\delta_H + 1\right)}.\end{aligned}$$

Taking the difference, we can show that

$$[\frac{\left(a_{11}^M - a_{12}^M\right)^2}{a_{11}^M} + a_{11}^M] - [\frac{\left(a_{11}^C - a_{12}^C\right)^2}{a_{11}^C} + a_{11}^C] > 0,$$

and

$$\begin{aligned}&\frac{a_{11}^M - a_{14}^M}{a_{11}^M} - \frac{a_{11}^C - a_{14}^C}{a_{11}^C} \\ \propto\ & \delta_L\left(8\delta_H^2 + 6\delta_H + 1\right)\left(8\delta_H^2 + 12\delta_H\delta_L + 4\delta_H + 8\delta_L^2 + 6\delta_L + 1\right) > 0.\end{aligned}$$

From Lemma 1, we have $a_{11}^M > a_{11}^C$. By (4), if $\frac{(a_{11}^M - a_{12}^M)^2}{a_{11}^M} \geq \frac{(a_{11}^C - a_{12}^C)^2}{a_{11}^C}$, then $\frac{a_{11}^M - a_{14}^M}{a_{11}^M} - \frac{a_{11}^C - a_{14}^C}{a_{11}^C} > 0$ is sufficient for $b^C > b^M$. But we have shown $\frac{a_{11}^M - a_{14}^M}{a_{11}^M} - \frac{a_{11}^C - a_{14}^C}{a_{11}^C} > 0$. Now consider the other case that $\frac{(a_{11}^M - a_{12}^M)^2}{a_{11}^M} < \frac{(a_{11}^C - a_{12}^C)^2}{a_{11}^C}$. Since $\delta_H \leq 1$, the following conditions are sufficient for $b^C > b^M$: $[\frac{(a_{11}^M - a_{12}^M)^2}{a_{11}^M} + a_{11}^M] - [\frac{(a_{11}^C - a_{12}^C)^2}{a_{11}^C} + a_{11}^C] > 0$ and $\frac{a_{11}^M - a_{14}^M}{a_{11}^M} - \frac{a_{11}^C - a_{14}^C}{a_{11}^C} > 0$. But they have been proved earlier. Therefore, $b^C > b^M$. ■

Performance of various organization forms under the circle network

Again we just demonstrate how to compute the expected loss under M-form. Since the four divisions are symmetric, we only need to compute the expected loss for D1. In particular, under M-form the expected adaptation loss and coordination losses are:

$$\begin{aligned}E[(d_1^M - \theta_1)^2] &= \left((a_{11}^M)^2 - 2a_{11}^M\right)E(\overline{m}_1^2) + \sigma^2 + (a_{12}^M)^2 E(\overline{m}_2^2) \\ &\quad + (a_{13}^M)^2 E(\overline{m}_3^2) + (a_{14}^M)^2 E(\overline{m}_4^2), &(11)\\ E[(d_1^M - d_2^M)^2] &= 2(a_{11}^M - a_{21}^M)^2 E(\overline{m}_1^2) + 2(a_{13}^M - a_{23}^M)^2 E(\overline{m}_3^2), &(12)\\ E[(d_1^M - d_4^M)^2] &= 2(a_{11}^M - a_{41}^M)^2 E(\overline{m}_1^2) + 2(a_{12}^M - a_{42}^M)^2 E(\overline{m}_2^2). &(13)\end{aligned}$$

In the last two equations, (12) and (13), we use the symmetry of the decision weights and $E(\overline{m}_1^2) = E(\overline{m}_j^2)$ for $j = 2, 3, 4$. Moreover, by Proposition 1, in the most informative equilibrium $E[(\overline{m}_i^M)^2] = \frac{3(1+b^M)}{3+4b^M}\sigma^2$. The overall expected loss for D1 under M-form can be computed from the components of (11)-(13) according to (5).

Proof of Proposition 3.

Proof. Denote the hierarchy governance as H. Setting $\delta_H = \delta_L = \delta$, we can simplify the equilibrium decision weights and the formula of the communication biases. In particular,

$$\begin{aligned} b^C &= 2\delta \frac{12\delta^2 + 8\delta + 1}{8\delta^3 + 24\delta^2 + 10\delta + 1}, \\ b^D &= \frac{1}{\delta}(3\delta + 1), \\ b^H &= \delta \frac{108\delta^4 + 148\delta^3 + 75\delta^2 + 15\delta + 1}{36\delta^5 + 212\delta^4 + 221\delta^3 + 91\delta^2 + 16\delta + 1}. \end{aligned}$$

Under the most informative equilibrium, the expected losses can be derived as

$$L^C = 4\frac{2\delta\left(384\delta^3 + 412\delta^2 + 100\delta + 7\right)}{960\delta^4 + 1208\delta^3 + 440\delta^2 + 62\delta + 3}, \tag{14}$$

$$L^D = 4\frac{2\delta\left(96\delta^3 + 146\delta^2 + 61\delta + 8\right)}{(2\delta+1)^2\left(60\delta^2 + 31\delta + 4\right)}, \tag{15}$$

$$L^H = 4\delta\frac{20\,736\delta^7 + 63\,120\delta^6 + 75\,584\delta^5 + 46\,152\delta^4 + 15\,724\delta^3 + 3029\delta^2 + 309\delta + 13}{25\,920\delta^8 + 82\,704\delta^7 + 106\,736\delta^6 + 73\,632\delta^5 + 29\,932\delta^4 + 7391\delta^3 + 1089\delta^2 + 88\delta + 3} \tag{16}$$

First, we compare hierarchy and centralization. In particular, by (14) and (16)

$$\begin{aligned} L^H - L^C \propto\ & [256\,512\delta^{10} + 876\,032\delta^9 + 1088\,768\delta^8 + 637\,888\delta^7 + 174\,640\delta^6 + 6208\delta^5] \\ & -[10\,104\delta^4 + 3152\delta^3 + 451\delta^2 + 33\delta] - 1. \end{aligned} \tag{17}$$

Define the RHS of (17) as $F_{HC}(\delta)$. It is easy to verify that $F_{HC}(\delta) > 0$ for any $\delta \geq 1$. Moreover, $F_{HC}(1) > 0$ and $F_{HC}(0) < 0$. Thus, by the continuity of $F_{HC}(\delta)$, there is some $\overline{\delta} \in (0, 1)$ such that $F_{HC}(\overline{\delta}) = 0$.

Now we show that such a $\overline{\delta}$ is unique. Denote δ_s as solutions to $F_{HC}(\delta_s) = 0$. And without loss, suppose $\overline{\delta}$ is the smallest δ_s. First, we show that $F'_{HC}(\delta_s) > 0$ for any δ_s. Denote the terms in the first bracket of (17) as $X(\delta)$, and the terms in the second bracket of (17) as $Y(\delta)$. Then $F_{HC}(\delta) = X(\delta) - Y(\delta) - 1$. Since $F_{HC}(\delta_s) = 0$, we have $X(\delta_s) > Y(\delta_s)$. Taking derivative with respect to δ, we have

$$F'_{HC}(\delta_s) = X'(\delta_s) - Y'(\delta_s) > 5\frac{X(\delta_s) - Y(\delta_s)}{\delta_s} > 0.$$

Next we show that there is no $\delta_s > \overline{\delta}$. Since $F'_{HC}(\overline{\delta}) > 0$ and $F_{HC}(\overline{\delta}) = 0$, the continuity of $F_{HC}(\delta)$ implies that for the next δ_s, $F_{HC}(\delta)$ must across 0 from above; in other words, $F'_{HC}(\delta_s) < 0$. But this contradicts the earlier result that $F'_{HC}(\delta_s) > 0$. Therefore, δ_s is unique, which is $\overline{\delta}$.

Since $\overline{\delta}$ is the unique solution to $F_{HC}(\delta) = 0$ and $F_{HC}(0) < 0$, the continuity of $F_{HC}(\delta)$ implies that $F_{HC}(\delta) > 0$ if $\delta > \overline{\delta}$, and $F_{HC}(\delta) < 0$ if $\delta < \overline{\delta}$. Therefore, $L^H - L^C > 0$ if $\delta > \overline{\delta}$, and $L^H - L^C < 0$ if $\delta < \overline{\delta}$. It can be computed that $\overline{\delta} \simeq 0.26$.

Next, we compare decentralization and hierarchy. Specifically, by (15) and (16)

$$L^D - L^H \propto 93888\delta^8 + 2473344\delta^7 + 2554080\delta^6 + 134160\delta^5 + 37988\delta^4 + 5249\delta^3 + 123\delta^2 - 49\delta - 4. \quad (18)$$

Define the RHS of (18) as $F_{DH}(\delta)$. By a similar proof to that of $F_{HC}(\delta)$, we can show that there is a unique $\underline{\delta} \in (0,1)$ such that $F_{DH}(\underline{\delta}) = 0$. Moreover, $F_{DH}(\delta) > 0$ if $\delta > \underline{\delta}$, and $F_{DH}(\delta) < 0$ if $\delta < \underline{\delta}$. Therefore, $L^D - L^H > 0$ if $\delta > \underline{\delta}$, and $L^D - L^H < 0$ if $\delta < \underline{\delta}$. It can be computed that $\underline{\delta} \simeq 0.09$.

Since $\overline{\delta} > \underline{\delta}$, the earlier comparisons lead to the following conclusion: among L^C, L^H, and L^D, L^D is the smallest when $\delta \in (0, \underline{\delta})$, L^C is the smallest when $\delta > \overline{\delta}$, and L^H is the smallest when $\delta \in (\underline{\delta}, \overline{\delta})$. ■

Equilibrium decisions under the star network

The equilibrium decision weights under various governance structures are as follows.

$$\begin{aligned}
a_{11}^C &= \frac{\gamma + 2\gamma\delta}{\gamma + 6\delta + 2\gamma\delta}, a_{1i}^C = \frac{2\delta}{\gamma + 6\delta + 2\gamma\delta}, \\
a_{ii}^C &= \frac{\gamma + 6\delta + 4\delta^2 + 2\gamma\delta}{(2\delta+1)(\gamma + 6\delta + 2\gamma\delta)}, a_{i1}^C = \frac{2\gamma\delta}{\gamma + 6\delta + 2\gamma\delta}, a_{ij}^C = \frac{4\delta^2}{(2\delta+1)(\gamma + 6\delta + 2\gamma\delta)}.
\end{aligned}$$

$$\begin{aligned}
z_1^D &= \frac{\gamma}{3\delta + \gamma}, a_{11}^D = \frac{3\gamma\delta^2}{\gamma^2\delta + \gamma^2 + 3\gamma\delta^2 + 6\gamma\delta + 9\delta^2}, a_{1i}^D = \frac{\delta}{\gamma + 3\delta + \gamma\delta}, \\
z_i^D &= \frac{1}{\delta + 1}, a_{i1}^D = \frac{\gamma\delta}{\gamma + 3\delta + \gamma\delta}, a_{ij}^D = \frac{\delta^2}{(\delta+1)(\gamma + 3\delta + \gamma\delta)}.
\end{aligned}$$

$$\begin{aligned}
a_{11}^{C1DO} &= \frac{\gamma(\delta+1)}{\gamma + 6\delta + \gamma\delta}, a_{1i}^{C1DO} = \frac{2\delta}{\gamma + 6\delta + \gamma\delta}, \\
z_i^{C1DO} &= \frac{1}{1+\delta}, a_{i1}^{C1DO} = \frac{\gamma\delta}{\gamma + 6\delta + \gamma\delta}, a_{ij}^{C1DO} = \frac{2\delta^2}{(\delta+1)(\gamma + 6\delta + \gamma\delta)}.
\end{aligned}$$

$$\begin{aligned}
z_1^{D1CO} &= \frac{\gamma}{\gamma + 3\delta}, a_{11}^{D1CO} = \frac{6\gamma\delta^2}{2\gamma^2\delta + \gamma^2 + 6\gamma\delta^2 + 6\gamma\delta + 9\delta^2}, a_{1i}^{D1CO} = \frac{\delta}{\gamma + 3\delta + 2\gamma\delta}, \\
a_{i1}^{D1CO} &= \frac{2\gamma\delta}{\gamma + 3\delta + 2\gamma\delta}, a_{ii}^{D1CO} = \frac{\gamma + 3\delta + 2\delta^2 + 2\gamma\delta}{(2\delta+1)(\gamma + 3\delta + 2\gamma\delta)}, a_{ij}^{D1CO} = \frac{2\delta^2}{(2\delta+1)(\gamma + 3\delta + 2\gamma\delta)}.
\end{aligned}$$

$$\begin{aligned}
a_{11}^{C1H} &= \frac{\gamma + \gamma\delta}{\gamma + 6\delta + \gamma\delta}, a_{1i}^{C1H} = \frac{2\delta}{\gamma + 6\delta + \gamma\delta}, a_{i1}^{C1H} = \frac{\gamma\delta + \gamma\delta^2}{\gamma + 6\delta + 6\delta^2 + 2\gamma\delta + \gamma\delta^2}, \\
a_{ii}^{C1H} &= \frac{\gamma + 6\delta + 2\delta^2 + \gamma\delta}{\gamma + 6\delta + 6\delta^2 + 2\gamma\delta + \gamma\delta^2}, a_{ij}^{C1H} = \frac{2\delta^2}{\gamma + 6\delta + 6\delta^2 + 2\gamma\delta + \gamma\delta^2}.
\end{aligned}$$

Here we just demonstrate how to derive the equilibrium decisions under C1DO, as the derivations under other governance structures are similar. Specifically, under C1DO the HQ chooses d_1 to maximize the joint payoff of all divisions, while each divisional manager i ($i \neq 1$) chooses d_i to maximize the payoff of D_i. The first order conditions are

$$d_1 = \frac{\gamma}{6\delta+\gamma}E[\theta_1|m] + \frac{2\delta}{6\delta+\gamma}\left(E[d_2|m]+E[d_3|m]+E[d_4|m]\right), \tag{19}$$

$$d_i = \frac{1}{\delta+1}\theta_i + \frac{\delta}{\delta+1}E[d_1|m]. \tag{20}$$

Taking expectations over (19) and (20) and solve for $E[d_i|m]$, we get

$$\begin{aligned} E[d_1|m] &= \frac{\gamma+\gamma\delta}{\gamma+6\delta+\gamma\delta}E[\theta_1|m] + \frac{2\delta\left(E[\theta_2|m]+E[\theta_3|m]+E[\theta_4|m]\right)}{\gamma+6\delta+\gamma\delta}, \\ E[d_i|m] &= \frac{\gamma+6\delta+2\delta^2+\gamma\delta}{\gamma+6\delta+6\delta^2+2\gamma\delta+\gamma\delta^2}E[\theta_i|m] + \frac{\gamma\delta^2+\gamma\delta}{\gamma+6\delta+6\delta^2+2\gamma\delta+\gamma\delta^2}E[\theta_1|m] \\ &\quad + \frac{2\delta^2}{\gamma+6\delta+6\delta^2+2\gamma\delta+\gamma\delta^2}\sum_{j\neq 1,i}E[\theta_j|m]. \end{aligned}$$

Substituting the expressions of $E[d_1|m]$ and $E[d_i|m]$ into (19) and (20), we get the equilibrium decision weights a_{ij}^{C1DO} as shown earlier.

Proof of Lemma 2.

Proof. Parts (i) and (ii) are immediate from the equilibrium decision weights.

Part (iii). By the equilibrium decision weights, we have

$$\begin{aligned} a_{11}^{C} - a_{11}^{C1DO} &= \frac{6\gamma\delta^2}{(\gamma+6\delta+2\gamma\delta)(\gamma+6\delta+\gamma\delta)} > 0, \\ a_{ii}^{C} - a_{ii}^{D1CO} &= \frac{2\gamma\delta^2}{(\gamma+3\delta+2\gamma\delta)(\gamma+6\delta+2\gamma\delta)} > 0. \end{aligned}$$

Part (iv). By the equilibrium decision weights, we have

$$\begin{aligned} a_{ii}^{C1DO} - a_{ii}^{D} &\propto \gamma(1+\delta) > 0, \\ a_{11}^{D1CO} - a_{11}^{D} &= \frac{3\gamma\delta^2}{(\gamma+3\delta+\gamma\delta)(\gamma+3\delta+2\gamma\delta)} > 0. \end{aligned}$$

Part (v). These results are immediate by comparing the equilibrium decision weights under C1H and those under C1DO. ■

Proof of Proposition 4.

Proof. Part (i). Under centralization, we have

$$b_1^C = \frac{(2\delta+1)(\gamma+6\delta+2\gamma\delta)}{\gamma(1+2\delta)^2+3\delta} - 1, \tag{21}$$

$$b_i^C = \frac{\left(\gamma+6\delta+4\delta^2+2\gamma\delta\right)(2\delta+1)(\gamma+6\delta+2\gamma\delta)}{\left(\gamma+6\delta+4\delta^2+2\gamma\delta\right)^2+\delta(\gamma+4\delta+2\gamma\delta)^2} - 1. \tag{22}$$

Taking derivative of (21) with respect to γ, we get $\frac{\partial b_1^C}{\partial \gamma} < 0$. As to the difference, (21) and (22) yield

$$b_1^C - b_i^C \propto \left(3+4\delta+\frac{(\gamma+4\delta+2\gamma\delta)^2}{\gamma+6\delta+4\delta^2+2\gamma\delta}\right) - (2\gamma+4\gamma\delta) \equiv \Delta(\gamma).$$

It can be shown that $\frac{\partial(\Delta\gamma)}{\partial\gamma} < 0$. Thus, there is a $\widehat{\gamma}$ such that $b_1^C - b_i^C > 0$ if and only if $\gamma < \widehat{\gamma}$. Moreover, it is straightforward to show that $\Delta(1) > 0$. Therefore, $\widehat{\gamma} > 1$.

Under decentralization, we have

$$b_1^D = \frac{1}{\gamma\delta}(\gamma+3\delta+\gamma\delta) - 1, \tag{23}$$

$$b_i^D = \frac{1}{\delta}(\gamma+3\delta+\gamma\delta) - 1. \tag{24}$$

The result is immediate from the expressions of (23) and (24).

Part (ii). The communication bias b_1^{D1CO} can be computed as

$$b_1^{D1CO} = \frac{1}{2\gamma\delta}(\gamma+3\delta). \tag{25}$$

By (23) and (25), we have $b_1^D - b_1^{D1CO} = \frac{\gamma+3\delta}{2\gamma\delta} > 0$.

Next we show $b_i^D - b_i^{C1DO} > 0$. The communication bias b_i^{C1DO} can be computed as

$$b_i^{C1DO} = \frac{1}{2\delta}(\gamma+4\delta+\gamma\delta). \tag{26}$$

By (24) and (26), we have

$$b_i^D - b_i^{C1DO} = \frac{\gamma(\delta+1)}{2\delta} > 0.$$

Part(iii). The communication bias b_i^{C1H} can be computed as

$$b_i^{C1H} = 2\delta^2 \frac{\gamma+4\delta+\gamma\delta}{\gamma^2\delta^2+2\gamma^2\delta+\gamma^2+12\gamma\delta^2+12\gamma\delta+4\delta^3+36\delta^2}. \tag{27}$$

Taking the difference between (26) and (27), we have

$$b_i^{C1DO} - b_i^{C1H} = \frac{1}{2\delta}\frac{(\gamma+4\delta+\gamma\delta)(\gamma+6\delta+\gamma\delta)^2}{\gamma^2\delta^2+2\gamma^2\delta+\gamma^2+12\gamma\delta^2+12\gamma\delta+4\delta^3+36\delta^2} > 0.$$

Similarly, we can show that $b_i^C - b_i^{C1H} > 0$. ■

Performance of various organization forms under the star network

Again we just demonstrate how to compute the expected losses under C1DO. In particular, the expected adaptation losses are

$$E[(d_1^{C1DO}-\theta_1)^2] = \left((a_{11}^{C1DO})^2-2a_{11}^{C1DO}\right)E(\overline{m}_1^2)+\sigma^2 + (a_{1i}^{C1DO})^2(E(\overline{m}_2^2)+E(\overline{m}_3^2)+E(\overline{m}_4^2)), \tag{28}$$

$$E[(d_i^{C1DO}-\theta_i)^2] = \left(\frac{\delta}{\delta+1}\right)^2\sigma^2+(a_{i1}^{C1DO})^2E(\overline{m}_1^2) + \left((a_{ii}^{C1DO})^2-\frac{2\delta}{\delta+1}a_{ii}^{C1DO}\right)E(\overline{m}_i^2)+(a_{ij}^{C1DO})^2\sum_{j\neq 1,i}E(\overline{m}_j^2). \tag{29}$$

And the expected coordination loss is

$$E[(d_1^{C1DO}-d_i^{C1DO})^2] = \left(\frac{1}{\delta+1}\right)^2\sigma^2+\left(a_{11}^{C1DO}-a_{i1}^{C1DO}\right)^2E(\overline{m}_1^2) + \left(a_{1i}^{C1DO}-a_{ii}^{C1DO}\right)^2\left(E(\overline{m}_2^2)+E(\overline{m}_3^2)+E(\overline{m}_4^2)\right) - \frac{2}{\delta+1}\left(a_{1i}^{C1DO}-a_{ii}^{C1DO}\right)E(\overline{m}_i^2). \tag{30}$$

Moreover, by Proposition 1, in the most informative equilibrium $E[(\overline{m}_i^{C1DO})^2]=\frac{3(1+b_i^{C1DO})}{3+4b_i^{C1DO}}\sigma^2$. The overall expected loss under C1DO can be computed from (28)-(30).

Proof of Proposition 5.

Proof. When $\gamma=1$, the expected losses L^g can be computed as follows:

$$L^C = \frac{2\delta\left(7680\delta^5+19\,984\delta^4+14\,088\delta^3+3976\delta^2+482\delta+21\right)\sigma^2}{4800\delta^6+14\,080\delta^5+12\,852\delta^4+5148\delta^3+981\delta^2+88\delta+3}, \tag{31}$$

$$L^D = \frac{6\delta\left(96\delta^4+274\delta^3+215\delta^2+69\delta+8\right)\sigma^2}{(\delta+1)^2\left(180\delta^3+153\delta^2+43\delta+4\right)}, \tag{32}$$

$$L^{D1CO} = \frac{3\delta\left(11\,280\delta^6+37\,532\delta^5+37\,342\delta^4+17\,382\delta^3+4230\delta^2+523\delta+26\right)\sigma^2}{10\,260\delta^7+37\,452\delta^6+44\,725\delta^5+26\,049\delta^4+8394\delta^3+1533\delta^2+149\delta+6}, \tag{33}$$

$$L^{C1DO} = \frac{2\delta\left(13\,600\delta^5+39\,814\delta^4+33\,085\delta^3+10\,705\delta^2+1456\delta+70\right)\sigma^2}{(\delta+1)^2\left(8099\delta^4+7681\delta^3+2017\delta^2+197\delta+6\right)}, \tag{34}$$

$$L^{C1H} = \frac{\delta\left(10\,640\delta^7+63\,434\delta^6+133\,568\delta^5+120\,764\delta^4+50\,147\delta^3+10\,271\delta^2+1017\delta+39\right)\sigma^2}{(\delta+1)^2\left(3276\delta^6+13\,509\delta^5+14\,638\delta^4+5745\delta^3+1043\delta^2+90\delta+3\right)} \tag{35}$$

Part (i). Taking the difference of (34) and (35), we get

$$\begin{aligned} L^{C1DO}-L^{C1H} &\simeq 419\,120\delta^{10}+4629\,886\delta^9+13\,646\,660\delta^8+19\,742\,256\delta^7+16\,822\,605\delta^6+9054\,579\delta^5 \\ &\quad +3122\,824\delta^4+676\,166\delta^3+87\,949\delta^2+6249\delta+186 \\ &> 0. \end{aligned}$$

Part (ii). Taking the difference of (33) and (31), we get

$$L^{D1CO} - L^C \propto 4838\,400\delta^{12} + 31\,593\,600\delta^{11} + 85\,044\,864\delta^{10} + 124\,402\,480\delta^9 + 109\,788\,264\delta^8 + 61\,348\,492\delta^7 + 22\,014\,422\delta^6 + 4962\,476\delta^5 + 639\,162\delta^4 + 29\,627\delta^3 - 3074\delta^2 - 471\delta - 18$$

By a proof similar to that of Proposition 3, there is a unique $\widehat{\delta}$ such that $L^{D1CO} - L^C > 0$ if $\delta > \widehat{\delta}$, and $L^{D1CO} - L^C < 0$ if $\delta < \widehat{\delta}$. It can be computed that $\widehat{\delta} \simeq 0.08$. But for $\delta < \widehat{\delta}$, it can be shown that $L^{D1CO} > L^D$.

Part (iii). Taking the difference of (35) and (31), we get

$$L^{C1H} - L^C \propto 752\,640\delta^{13} + 15\,222\,272\delta^{12} + 86\,764\,256\delta^{11} + 197\,780\,664\delta^{10} + 208\,993\,960\delta^9 + 106\,408\,598\delta^8 + 21\,268\,362\delta^7 - 3311\,302\delta^6 - 2928\,825\delta^5 - 763\,155\delta^4 - 109\,728\delta^3 - 9324\delta^2 - 441\delta - 9$$

By a proof similar to that of Proposition 3, there is a unique δ_2 such that $L^{C1H} - L^C > 0$ if $\delta > \delta_2$, and $L^{C1H} - L^C < 0$ if $\delta < \delta_2$. It can be computed that $\delta_2 \simeq 0.31$. Similarly, it can be shown that there is a unique δ_1 such that $L^D - L^{C1H} > 0$ if $\delta > \delta_1$, and $L^D - L^{C1H} < 0$ if $\delta < \delta_1$. It can be computed that $\delta_1 \simeq 0.08 < \delta_2$. Therefore, decentralization is optimal if $\delta < \delta_1$, centralization is optimal if $\delta > \delta_2$, and C1H is optimal when $\delta \in (\delta_1, \delta_2)$. ■