Asymmetric Inventory Dynamics and Product Market Search

Linxı Chen

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“...indeed, to a great extent, business cycles are inventory fluctuations.”

Alan Blinder, 1981

“This is a sad day. The only people I have met who are doing well are those who are not doing anything.”

André Gide
Importance of Inventory Investment

Real GDP Decomposition, 1954–2007

Figure: Importance
Introduction

- Document two stylized facts:
  1. Inventory investment accounts for much larger share of GDP change in recessions but not in expansions.
  2. Inventory-Sales ratio: lagging GDP for 4 quarters.

- Fact 1:
  - That’s what makes inventory relevant.
  - Partially known to the lit. but is not accounted for.

- Fact 2:
  - Important conclusions drawn from countercyclicality (Bils Kahn 2000, Midrigran Krytsov 2013, Sarte et al. 2015)
  - Not countercyclical for the last 3 decades.
  - Stylized fact to discipline most models.
Introduction

- Standard inventory models (e.g. Wen 2011) generates no such asymmetry (positive skewness) in importance of inventory investment and the lagging relationship.
- This paper:
  - Based on stockout-avoidance motive for inventory (Kahn 1987).
  - Augment with product market search
  - Disciplined by micro empirical findings on both product market search and inventory.
  - Matches the two facts while consistent with inventory stylized facts.
  - Consistent with data asymmetry: sym. output, neg. skewed employment, pos. skewed markup
HH’s search for variety introduce endogenous stock-out risk:

- HH’s procyclical demand for varieties enhances nonlinearity in firms’ trade-off between markup and buffer stock size.
- In recessions, firms are more inclined to lower markup than to expand inventory, vice versa in expansions.
- HH responds to lowered income by reduce varieties first then reduce.
- Prob. of matching and inventory holding return prolonged firms’ hump-shaped stocking response.
Asymmetric Importance

Figure: Asymmetry
Asymmetric Importance

Billions of 2009 Dollars, Trough to Peak 1981–90

Figure: Asymmetry
## Peak-to-trough Declines in All Postwar Recessions

<table>
<thead>
<tr>
<th>Peak</th>
<th>Trough</th>
<th>CIPI Decline</th>
<th>GDP Decline</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948:4</td>
<td>1949:4</td>
<td>-40.66</td>
<td>-30.68</td>
<td>132%</td>
</tr>
<tr>
<td>1953:2</td>
<td>1954:2</td>
<td>-24.47</td>
<td>-62.77</td>
<td>39%</td>
</tr>
<tr>
<td>1957:3</td>
<td>1958:2</td>
<td>-21.19</td>
<td>-84.98</td>
<td>25%</td>
</tr>
<tr>
<td>1969:4</td>
<td>1970:4</td>
<td>-35.84</td>
<td>-7.19</td>
<td>498%</td>
</tr>
<tr>
<td>1973:4</td>
<td>1975:1</td>
<td>-80.06</td>
<td>-169.95</td>
<td>47%</td>
</tr>
<tr>
<td>1980:1</td>
<td>1980:3</td>
<td>-67.26</td>
<td>-142.02</td>
<td>47%</td>
</tr>
<tr>
<td>1981:3</td>
<td>1982:4</td>
<td>-120.51</td>
<td>-169.73</td>
<td>71%</td>
</tr>
<tr>
<td>1990:3</td>
<td>1991:1</td>
<td>-46.87</td>
<td>-118.38</td>
<td>39%</td>
</tr>
<tr>
<td>2001:1</td>
<td>2001:4</td>
<td>-24.09</td>
<td>-40.20</td>
<td>59%</td>
</tr>
<tr>
<td>2007:4</td>
<td>2009:2</td>
<td>-213.07</td>
<td>-636.23</td>
<td>33%</td>
</tr>
</tbody>
</table>

Avg*: 72%

**Table:** Peak-to-Trough Declines in All Postwar Recessions.

**Note:** Units in billions of 2009 dollar, annualized quarterly rate.
## Trough-to-peak

<table>
<thead>
<tr>
<th>Trough</th>
<th>Peak</th>
<th>CIPI Increase</th>
<th>GDP Increase</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949:4</td>
<td>1953:2</td>
<td>33.28</td>
<td>588.80</td>
<td>6%</td>
</tr>
<tr>
<td>1954:2</td>
<td>1957:3</td>
<td>20.94</td>
<td>345.24</td>
<td>6%</td>
</tr>
<tr>
<td>1958:2</td>
<td>1960:2</td>
<td>22.89</td>
<td>320.36</td>
<td>7%</td>
</tr>
<tr>
<td>1961:1</td>
<td>1969:4</td>
<td>30.15</td>
<td>1613.21</td>
<td>2%</td>
</tr>
<tr>
<td>1970:4</td>
<td>1973:4</td>
<td>76.25</td>
<td>754.12</td>
<td>10%</td>
</tr>
<tr>
<td>1975:1</td>
<td>1980:1</td>
<td>33.01</td>
<td>1232.47</td>
<td>3%</td>
</tr>
<tr>
<td>1980:3</td>
<td>1981:3</td>
<td>115.93</td>
<td>279.97</td>
<td>41%</td>
</tr>
<tr>
<td>1982:4</td>
<td>1990:3</td>
<td>84.35</td>
<td>2490.81</td>
<td>3%</td>
</tr>
<tr>
<td>1991:1</td>
<td>2001:2</td>
<td>7.16</td>
<td>3844.74</td>
<td>0.1%</td>
</tr>
<tr>
<td>2001:4</td>
<td>2007:4</td>
<td>120.34</td>
<td>2286.52</td>
<td>5%</td>
</tr>
</tbody>
</table>

**Avg:** 8%

**Table:** Trough-to-Peak Increases in All Postwar Expansions

**Note:** Units in billions of 2009 dollar, annualized quarterly rate.
Asymmetric Importance

Figure: Asymmetry, Skewness = -0.33
Stylized No More?

Figure: HP Filtered, 40 Quarters Moving Window

corr = -0.24 before 1992, corr = 0.23 after 1992
Figure: HP Filtered (1600)
Model

- Intermediate good producer: identical good produced with labor only.
- Variety good producer: differentiated good, produced with intermediate goods.
- Final good producer: pack many varieties into consumption good.
- Household: “love for variety”
Matching

- Search and match protocol similar to labor search framework (Pissrides 1994).
- Measure 1 of HH (search intensity $d$) matches with measure 1 of varieties.
- Each variety is produced by one monopolistic firm.
- Generate $x$ matches (varieties consumed)

$$x = M(d, 1)$$

and thus the rate at which HH finds varieties

$$\Psi_D \equiv \frac{x}{d} = M(1, \frac{1}{d})$$
Household

- Solves the following Bellman’s equation:

\[
H(a, x) = \max_{c, d, n, a', x} u(x^\rho c, d, n) + \beta \mathbb{E} H(a', x')
\]

s.t. \( a' = Wn + a(1 + \Pi) - \bar{P}cx \)
\( x = \Psi_D d \)

- Consume \( x \) varieties with average level \( c \), search for varieties with effort \( d \), work for wage \( W \), receive profit from all firms \( \Pi \), save with stock purchase \( a' \) (numeraire).

- \( \Psi_D \) exogenous variety finding rate.
Final Good Producer

- Perfectly competitive. Unit measure.
- Produces final good with $x$ varieties dictated by HH, subject to good availability $z_i$ and aggregation technology with idiosyncratic demand shock $v_i$ (to the variety producer)

\[
\max_{c_i} \ P x c - \int_0^x P_i c_i di \\
\text{s.t.} \quad c_i \leq z_i \\
\]

\[
c = \left( \frac{1}{x} \int_0^x v_i \frac{1}{\rho} \ c_i^\rho \ di \right)^\rho
\]

- Demand for variety i:

\[
c_i = \min \left\{ z_i, v_i \left( \frac{P_i}{\bar{P}} \right)^{1-\rho} \ c \right\}
\]
Variety Producer

- Monopolistic competitive, unit measure.
- Faces discrete shock demand $x$ and continuous demand shock $v_i$.
- $v_i$ is i.i.d. across time and across varieties. Drawn once by all final good producers.
- Have to decide on pricing and production before knowing these shocks.
- Thus generate the incentive to hold inventories.
- With prob. $x$, the variety producer have access to final good producer’s demand ("matched").
Variety Producer

- Solves the following problem:

\[
\mathcal{V}(e_i) = \max_{y_i, p_i, n_i, e_i} \left( -P_M y_i + x \int \left\{ c_i p_i + \mathbb{E} m' \mathcal{V}(e'_i) \right\} F^v(dv_i) \right) \\
+ (1 - x) \mathbb{E} m' \mathcal{V}(e'_i)
\]

s.t. 

\[
c_i = \min \left( v_i \left( \frac{p_i}{P} \right)^{\frac{1}{1-\rho}} C, z_i \right) \\
z_i = e_i + y_i
\]

\[
e'_i = \begin{cases} (1 - \delta_e) [e_i + F(n_i) - c_i] & \text{"matched"} \\ (1 - \delta_e) [e_i + F(n_i)] & \text{"unmatched"} \end{cases}
\]

- \( z_i \) is the amount of good \( i \) made available to buyers. Inventory + new orders

- \( P_M \) price of intermediate goods, \( y_i \) the order
Variety Producer

- Pricing Decision:

\[ p_i = \frac{\epsilon_i}{\epsilon_i - 1} (1 - \delta_e) \mathbb{E} m' P'_M \]

where the price elasticity of expected sales is given by:

\[ \epsilon_i = \frac{\rho}{1 - \rho} \frac{\int_0^{v_i^*} c_i(p_i, n_i, v_i) F^v(dv_i)}{\int_0^{v_i^*} c_i(p_i, n_i, v_i) F^v(dv_i) + [1 - F^v(v_i^*)] [e_i + F(n_i)]} \]

- The cut-off point of stockout \( v_i^* \) is given by:

\[ c_i(p_i, n_i, v_i^*) = z_i \]
Variety Producer

- Availability decision:

\[(b - r^I)X(1 - F^v(v^*_i)) = 1 - r^I\]

where

\[b = \frac{p_i}{mc_i} = \frac{p_i}{W/F'(n_i)}\]

and

\[r^I \equiv (1 - \delta_e) \mathbb{E}m' \frac{P'_M}{P_M}.\]

- X is endogenous unlike traditional stockout model (constant therein).
- Timing and info. structure -> all variety producers choose the same \(z_i\) and \(p_i\).
Intermediate Producer

- Perfectly competitive, unit measure
- Solves the static problem:
  \[
  \max_n P^M F(n) - Wn
  \]
- Helps with exact aggregation despite heterogeneity in variety producers.
Functional Forms

Production Function:

\[ F(n) = An^{1-\alpha} \]

Utility (Generalized GHH 1988, search behavior):

\[ u(cx^\rho, d, n) = \log \left( cx^\rho - \zeta \frac{n^{1+\nu_n}}{1+\nu_n} - \xi d \right) \]

Distribution of idio. demand shock Pareto(\(v_{min}, \sigma_v\)):

\[ F_v(v) = 1 - \left( \frac{v_{min}}{v} \right)^{\sigma_v} \]

Matching function (den Hann et al. 2000):

\[ M(D, 1) = \frac{D}{(D^l + 1^l)^{1/l}} \]
## Calibration By Steady State

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Target</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_n$</td>
<td>Labor Elasticity</td>
<td>0.75</td>
<td>Frish Elas.</td>
<td>Chetty 2011</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Labor Disutility</td>
<td>1.5</td>
<td>1/3 time worked</td>
<td>ATUS</td>
</tr>
<tr>
<td>$\nu_{min}$</td>
<td>Loc. $\nu_i$</td>
<td>0.04</td>
<td>Mean 1</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\nu}$</td>
<td>Shape $\nu_i$</td>
<td>1.05</td>
<td>S.O. Prob=5%</td>
<td>Bils 2004</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Elas. of Subs.</td>
<td>1.17</td>
<td>20% markup</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>Deprec. Inven.</td>
<td>0.015</td>
<td>6% annual</td>
<td>Wen 2011</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Match Elasticity</td>
<td>1.18</td>
<td>0.35 elas.</td>
<td>Broda et al 2011</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Search Disutility</td>
<td>0.01</td>
<td>1 hr shopping</td>
<td>ATUS</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>TFP Pers.</td>
<td>0.96</td>
<td></td>
<td>SF-FED TFP</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>TFP Vola.</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table:** Calibration
### Performance

<table>
<thead>
<tr>
<th>Stat.</th>
<th>Data</th>
<th>Model</th>
<th>Wen 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(II/output, output)</td>
<td>0.66</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>AR(1) of IS ratio</td>
<td>0.75</td>
<td>0.89</td>
<td>0.77</td>
</tr>
<tr>
<td>corr(ISratio, output)</td>
<td>-0.43</td>
<td>-0.30</td>
<td>-0.68</td>
</tr>
<tr>
<td>skew(II/output)</td>
<td>-0.30</td>
<td>-0.46</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**Table:** Inventory Performance
Performance

- Examine peak-trough share of inventory investment.
- Dating turning points:
  - Treat the model as generating demeaned growth rates
  - Two consecutive periods with GDP growth < -0.8% (Data)
  - Matches share of recessions in data.
- Model depress avg. 4 quarters and expands 13 quarters.
- Peak to trough, inventory investment = 54% of output decline (data 72%).
- Trough to peak, 25% of output expansion (data 8%).
Cycles ($< -0.8\%$, 20\% of Periods )

**Figure:** Yellow Denotes Recession
Lead-Lag

Figure: IS Ratio Lags Output by 5 Quarters (Model)
Cross-correlogram

Figure: IS Ratio Lags Output by 5 Quarters (Data)
Figure: IS Ratio Lags Output by 5 Quarters (Data)
Role of Product Market Friction

<table>
<thead>
<tr>
<th>Stat.</th>
<th>$\xi = 0.006$</th>
<th>$\xi = 0.010$</th>
<th>$\xi = 0.012$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}$</td>
<td>0.91</td>
<td>0.88</td>
<td>0.75</td>
</tr>
<tr>
<td>P2T Share</td>
<td>0.37</td>
<td>0.92</td>
<td>0.95</td>
</tr>
<tr>
<td>T2P Share</td>
<td>2.21</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>skew(II/output)</td>
<td>-0.21</td>
<td>-0.46</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

**Table:** Inventory Performance
Intuition: Asymmetric Response
Intuition: Demand Curve

Higher \( v \)

\[
P^* \quad \frac{P^*}{\bar{P}}
\]

Sales

Price
Intuition: Optimal Markup

\[ b = \frac{\epsilon}{\epsilon - 1} r^l \]
Intuition: Optimal Buffer

\[(1 - F_v) = (\downarrow \text{ in markup, } \uparrow \text{ in buffer size})\]

\[x(1 - F_v)(b - r^l) = 1 - r^l\]
Intuition: Increases X

\[ x(1 - F_v)(b - r^I) = 1 - r^I \]
Intuition: Markup and Buffer TS

In Response to Positive TFP Shock

Markup
Buffer Size

Periods From Shock

-0.02
-0.01
0
0.01
Pct From SS

In Response to Positive TFP Shock

-1
0
1
2
Markup
Buffer Size
Intuition: Increase X at Trough
Intuition: Increase X at Peak
Intuition: Higher $r'$ at Peak
How Product Search Matters

- Nonlinearity exists in stock-out model, but unexplored.
- Allowing movement in $x$ enhances the nonlinearity as peak and trough are further away along markup decision curve.
- Generated quantitatively stronger asymmetry in stocking decision.
- With product search, expansion of varieties peaks first then the inventory holding return peak later.
- Prolonged impact on buffer stock, thus inventories.
Conclusion

- Two new stylized facts poses challenges to popular DSGE inventory models.
- In turn, questions implications based on unstable stylized facts.
- Product market search friction improve a off-the-shelf inventory model’s ability to be consistent with these two facts.
- Additionally consistent with household shopping empirics:
  - Procyclical search effort
  - Expansion of varieties and expenditure.
- Consistent with observed business cycle asymmetry.
- Bridged production market friction with inventory data.
Backup

Figure: Wen 2011
Related Literature

Broda et al. 2010:

1. UPC level data of HH consumption varieties (nondurable, 60% of CPI basket).
2. Large turnover of varieties HH consumes (75% common good in 4 year period).

Suggests that substantial risk of “out-of-favor” for producers when deciding inventory.