Entrepreneurial investment dynamics and the wealth distribution

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Abstract

Using restricted firm level data from the Kauffman Firm Survey, I document that (a) the distribution of average returns to capital amongst nascent entrepreneurial firms is left skewed, and (b) has higher persistence in the left tail than the right. I find that a model where the entrepreneur’s capital investment is illiquid can largely rationalize this data. On average, a continuing entrepreneur loses about 43% of the value of her capital stock upon downsizing, and faces an additional loss of 55% when exiting. These frictions on capital reallocation generates sizable distortions - A full removal of both frictions under the assumption of a closed economy can lead to average welfare gains of almost 23.1% in consumption equivalent variation, and aggregate TFP gains of 23.3%. Further decomposition of these welfare changes find that 89% of welfare losses are directly attributable to incomplete markets and financial frictions, and 11% to the illiquidity frictions. In addition, I also find that a model directly calibrated to firm level data is unable to replicate the level of wealth inequality observed in the real economy, contrary to prior papers which are calibrated to household survey data. This arises directly from the left-skewed distribution of returns to entrepreneurship observed in the KFS. In contrast, a counterfactual world with no illiquidity frictions is much more successful at replicating the empirical wealth distribution. (JEL D31, E21, E22, L26)

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1 Introduction

1.1 Motivation

Ownership of a business is highly correlated with being wealthy - a fact that has been well documented in the earlier literature. However, the returns to entrepreneurship, and more importantly, the return to investment into entrepreneurial undertakings, is much less well understood. As such, the question of whether entry into entrepreneurship is the driver of heterogeneity in wealth accumulation, or whether the wealthy simply disproportionately pursue entrepreneurship, remains an open question. The goal of this paper is to (a) provide direct empirical evidence of entrepreneurial investment behavior, and (b) rationalize their behavior in a macroeconomic model of entrepreneurship, and thus understand whether entrepreneurship is a key determinant in generating the large wealth dispersion in the real economy.

To that end, I utilize a restricted firm level data set from the Kauffman Firm Survey (KFS) to document the investment dynamics of young entrepreneurial firms. There, I find that the distribution of log average revenue product of capital (henceforth just ARPK) of young entrepreneurial firms is left skewed. Moreover, firms also exhibit large autocorrelation in ARPK, with greater persistence in the left tail of the cross-section distribution than the right. This finding is surprising for two reasons.

Firstly, a standard dynamic investment model with only a single period time to build as a “friction” predicts that the distribution of ARPK inherits exactly the distribution of the underlying productivity (i.e. TFPR) process. Given that ARPK is left-skewed, a naive interpretation would be that entrepreneurial productivity is left-skewed as well. This runs counter to conventional belief that entrepreneurial talent is right-skewed. Moreover, this would likely imply that entrepreneurship could not possibly explain much of the right tail of the wealth distribution.

In addition, while a hypothesized left-skewed productivity process could easily explain the left skewness of ARPK, it does not explain the left tail persistence. In fact, a standard investment model predicts no persistence in ARPK. As such, it is unlikely that a left-skewed TFPR process could rationalize these findings.

Secondly, the extant literature has primarily focused on financial constraints (in particular collateral constraints) as the primary source of distortion to entry into entrepreneurship or

\footnote{1c.f. Quadrini (1999), Cagetti and De Nardi (2006), Kuhn and Rios-Rull (2015)}

\footnote{2For instance, there is still a debate regarding whether entrepreneurial undertaking has higher or lower returns than labor work, c.f. Hamilton (2000), Manso (2015), Dillon and Stanton (2017)}

\footnote{3The complete proof is relegated to the appendix}
investment. However, under the framework of a standard dynamic investment model with collateral constraints, the distribution of ARPK is right-skewed\(^4\), with greater persistence in the right tail. This implies that collateral constraints cannot be the only (or even dominant) source of distortion to the entrepreneur’s investment choice.

In order to explain these findings, I construct a model that incorporates partial irreversibility of entrepreneurial capital\(^5\). I build on the framework of Cagetti and De Nardi (2006), and extend their endogenous occupational choice incomplete markets model by differentiating between liquid assets (bonds) and illiquid assets (entrepreneur’s capital). Due to an asymmetry in the purchase and resale price of capital, rather than downsizing or exiting instantly when hit by a bad shock, entrepreneurs adopt a “wait-and-see” attitude. This leads them to operate larger than optimal firm sizes\(^6\), generating a naturally left-skewed distribution of log ARPK. Moreover, this also creates persistence in the left tail as entrepreneurs wait out the transient bad shock.

To evaluate the quantitative significance of this friction, I calibrate the model to identifying micro-level moments drawn from the KFS. I find that the frictions are quantitatively large. For every unit of capital sold, the entrepreneur loses 43% of the underlying real value of the capital asset. Upon exit, the entrepreneur faces an additional 55% write down on her assets\(^7\). As a consequence, significant distortions to the extensive and intensive margin regarding entrepreneurial entry, exit and investment arise. In particular, in a counterfactual world where these frictions did not exist, aggregate TFP is 23.3% higher, and average welfare is 23.1% higher in consumption equivalent terms. Further analysis of these welfare changes show that, relative to a baseline economy with full insurance, household suffer an approximately 65% welfare loss in terms of permanent consumption, with about 89% directly attributable to the lack of consumption insurance and financial frictions\(^8\), and 11% to the distortions inflicted upon the households as a result of the illiquidity frictions.

An important observation from this model is that while financial frictions are undoubtedly important in influencing entrepreneurial investment, the primary distortion to the entrepreneur’s behavior in fact comes from the illiquid nature of the entrepreneur’s capital.

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\(^4\)Again, I defer the proof to latter section of the paper

\(^5\)Partial irreversibility could arise naturally because the resale price of “used” capital is lower than the purchase price of new capital, as discussed in Lanteri (2016)

\(^6\)“Optimal” here refers to the case where the (expected) marginal product of capital is set equal to the user cost

\(^7\)This is not the only source of friction in the model that makes entrepreneurial capital illiquid. To match the data, the model also includes a proportional investment fixed cost of about 3.4%. While these frictions might seem large, they are comparable to prior literature in both the households (c.f. Diaz and Luengo-Prado (2010), Kaplan and Violante (2014) and Berger and Vavra (2014)) and firm dynamics (c.f. Cooper and Haltiwanger (2006), Bloom (2009) literature.

\(^8\)These results might seem large, but is in fact comparable to Buera and Shin (2011)
The low resale value of capital reduces the collateral value of capital, making financially constrained entrepreneurs even worse off. Moreover, the low resale value also forces poorly performing entrepreneurs to operate their firms at suboptimal sizes for extended periods of time, leading to a mis-allocation of capital. Consequently, a proposed government policy that specifically reduces these frictions can often lead to greater welfare gains than standard policies of credit provision.

Finally, I find that entrepreneurship in a heterogeneous household model does not substantially improve the model fit to the empirical wealth distribution, when compared to a standard Aiyagari (1994) model of labor income risk. This finding runs contrary to earlier research such as that in Quadrini (1999) or Cagetti and De Nardi (2006), which report that macro-entrepreneurship models calibrated to household income surveys are able to match the empirical wealth distribution. Instead, I find that entrepreneurial returns to capital are too low to generate the right tail risk required to match the wealth distribution. The low returns to capital stems from two results: A small (relatively) returns to scale, as well as the aforementioned illiquid aspect of capital, which lowers the overall returns to capital. In particular, for a counterfactual world where illiquidity frictions did not exist, the Gini coefficient for the economy rises from 0.66 to 0.72 (and to 0.77 if we fix the interest rate to the benchmark economy). This suggests that entrepreneurship is an unlikely candidate for explaining the rates of returns heterogeneity observed by recent empirical research into wealth inequality\textsuperscript{9}, the latter of which appears to be the source of the stark wealth inequality in the data.

The rest of this paper is divided as follow. In the rest of this section, I briefly summarize the related literature. In section 2, I give a brief summary and description of the data and stylized moments. Following, in section 3, I describe the model, as well as give a short discussion on why standard investment models are unable to explain this data. In section 4, I discuss in detail the calibration strategy. Section 5 discusses the model results of the calibration exercise. Section 6 extends the model to study the impact of illiquidity frictions on fiscal policy instruments and dynamic outcomes, and show why modeling partial irreversibility in this framework is important. Section 7 concludes the paper.

1.2 Related literature

Firstly, this paper is most related to the research agenda framed by Quadrini (2000) and Cagetti and De Nardi (2006), which study the contribution of entrepreneurship to wealth inequality. In that framework, the entrepreneur’s business wealth and savings wealth are

\textsuperscript{9}See for instance: Saez and Zucman (2014), Bach et al (2016), and Fagereng et al (2016)
perfect substitutes. In this paper, I generalize their framework to one that distinguishes between business wealth and savings wealth. Business wealth (entrepreneurial capital) is illiquid and subject to transaction costs. This leads to distortion of the entrepreneur’s investment, savings and entry / exit decisions.

I also add to that research agenda by contributing direct empirical evidence regarding the investment behavior of entrepreneurs, as well as calibrating my model directly to the data. In contrast, Quadrini (2000) and Cagetti and De Nardi (2006) calibrate their model to match the income process of entrepreneurs identified from the Panel Survey of Income Dynamics (PSID) or Survey of Consumer Finances (SCF). In the latter strategy, one is generally unable to identify who constitutes entrepreneurs, as well as what portion of the entrepreneur’s income should be considered capital income, and what is consider labor income. Moreover, the book value of capital is not available in household surveys, making it difficult to assess the model’s ability in matching actual investment behavior.

This paper is also relevant to a broad range of research that uses the Cagetti and De Nardi (2006) framework to study issues such as capital taxation (c.f. Kitao (2008)), estate taxation (c.f. Cagetti and De Nardi (2009)), or financial frictions (c.f. Bassetto et al (2015)). This paper finds that the entrepreneur’s entry/exit and investment decision in a two asset framework differs starkly from the one asset model, thus suggesting that some of the results from the preceding literature might not carry over into a framework that more accurately replicates the micro-level decisions of entrepreneurs.

In addition, this paper connects itself to a growing literature on rates of returns heterogeneity and its effect on the wealth distribution. Recent empirical work has showed that highly persistent and heterogeneous returns to wealth can largely explain the income and wealth inequality in the economy\(^\text{10}\), while recent theoretical work by Benhabib et al (2015) provide the theoretical underpinning explain why this particular mechanism is so successful in matching the wealth distribution. This paper adds to the literature that uses entrepreneurship as a micro-foundation to understand heterogeneity in rates of return among households.

More broadly speaking, this paper also adds to the growing literature on using quantitative heterogeneous agent macroeconomic models to understand the determinants of the empirical wealth distribution. This literature, dating back to Aiyagari (1994), has largely documented that a model of only exogenous labor income risk cannot, in general, match the wealth distribution without resorting to a largely counterfactual labor income process\(^\text{11}\).

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\(^{10}\)c.f. Saez and Zucman (2014) for tax returns data from the United States, Bach et al (2016) for data on Sweden, and Fagereng et al (2016) for data on Norway

\(^{11}\)see in particular Castaneda et al (2003) and Benhabib et al (2016) for discussions on how calibrated labor income risk are typically counterfactual to the real amount of risk households face. Hurst et al (2010)
This paper adds to the research agenda of using entrepreneurship to augment the basic Aiyagari model to understand wealth inequality.

Finally, this paper also adds to the discussion on financial constraints and capital mis-allocation in economies with entrepreneurs. This paper finds that while financial constraints are indeed important in understanding entrepreneurial behavior, the dominant distortion results from the illiquidity of an entrepreneur’s capital assets. In particular, illiquidity turns out to provide much of the microfoundation for the existence of collateral constraints in this model. Moreover, these frictions also lead to capital mis-allocation when low productivity entrepreneurs operate large firms, or choose not to exit. This latter effect does not appear in models of financial frictions.

2 Data

The data comes from the restricted version of the Kauffman Firm Survey. I begin this section by introducing the reader to the data universe, followed by a discussion of how I constructed the data and present the results.

2.1 Data universe

The data is drawn primarily from the restricted version of the Kauffman Firm Survey (KFS). The KFS is a single cohort panel survey, consisting solely of entrepreneurial firms that were formed in the year 2004, and tracked through 2011. The universe of firms considered for survey inclusion was all newly registered firms in 2004 from the Dun and Bradstreet database. The subset of firms in this universe that was included in the survey must then satisfy the following conditions:

1. Business was started as independent business, or by the purchase of an existing business, or by the purchase of a franchise in the 2004 calendar year.

2. Business was not started as a branch or a subsidiary owned by an existing business, that was inherited, or that was created as a not-for-profit organization in the 2004 calendar year.

also provides a discussion on the importance of directly modeling business ownership when studying the wealth distribution.

3. Business had a valid business legal status (sole proprietorship, limited liability company, subchapter S corporation, C-corporation, general partnership, or limited partnership) in 2004.

4. Business reported at least one of the following activities:

   (a) Acquired employer identification number during the 2004 calendar year

   (b) Organized as sole proprietorships, reporting that 2004 was the first year they used Schedule C or Schedule C-EZ to report business income on a personal income tax return

   (c) Reported that 2004 was the first year they made state unemployment insurance payments

   (d) Reported that 2004 was the first year they made federal insurance contribution act payments

As one can observe, the inclusion criterion is very strict, and relates largely to the common idea of an entrepreneur. Robb and Robinson (2014) provides a very detailed summary of the broad characteristics of the data set, showing in particular the relevance of this dataset to the study of entrepreneurs.

2.2 Data construction

Following the literature, my key strategy is to document key moments for two measures of firm investment: the investment rate distribution, and the average revenue product of capital.

2.2.1 Capital stock

The KFS provides the researcher the balance sheet of the firm. In particular, it provides a breakdown of the type of capital asset that the entrepreneurial firm owns. As in most standard models, I consider only a single generic capital asset of interest. As such, in order render the results comparable, I construct a single asset, real capital stock, $K_{i,t}$, using the nominal value of capital assets as follows:

$$K_t = \sum_s \frac{K_{i,s,t}}{P_{s,t}}$$

where $P_{s,t}$ is the relative price of each capital type $s$ and vintage $t$. Subscript $i$ indexes the firm. The relative prices are taken from the BEA.
2.2.2 Average Revenue Product of Capital

To construct the (log) average revenue product of capital, I first construct real revenue by deflating nominal revenue by industry level output price deflators. Denote by $Y_{it}$ the real revenue of the firm; in that case, (log) average revenue product of capital is simply

$$
\log ARPK \equiv \log \left( \frac{Y_{it}}{K_{i,t-1}} \right)
$$

$t$ indicates the survey year. This timing convention is adopted as the KFS only surveys the firm at the start of the following year for information regarding the current year. That is to say, if the survey report is for the year 2004, it was in fact surveyed in 2005. Therefore, $K_{2004}$ is in fact the end of period capital stock, while $Y_{2004}$ is the revenue for 2004. The convention chosen here hence matches the standard timing convention to construct average products of capital as the ratio of the total revenue to the start of period capital stock.

For the main body of the paper, the moments I report are constructed using a pooled measure of $\log ARPK$. In practice, moments constructed using pooled log $ARPK$ might not be a good measure since there is likely to be large heterogeneity in capital share and returns to scale across industries. However, due to the small sample size of the KFS relative to the number of industries, there is simply insufficient statistical power to draw any useful inference if one restricted analysis only to the 6 digit NAICS industry level. Instead, I address this issue through two methods.

Firstly, the pooled $\log ARPK$ variables are residualized by regressing the raw log $ARPK$ measure on two digit NAICS industry level fixed effects and time dummies. The residuals of this regression then form my measure of $\log ARPK$ for analysis. This avoids the issue where heterogeneity in capital share and returns to scale across industries would distort the distribution, and thus introduce spurious correlations and results. Moreover, it also removes some of the common aggregate shocks that might distort the distribution of $\log ARPK$ over time. These moments of residualized $\log ARPK$, constructed at the aggregate level, is presented in this section.

Secondly, I also directly investigate the relevant moments at the two digit industry level. Due to the smaller sample size, only one industry showed statistically significant results; however, most industries showed economically significant results. The results at the industry level are reported in Appendix IV.

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13see for instance, Burnside (1993), who estimates the returns to scale for different industries in the United States

14There were 3140 firms in 2004 when the survey started; by 2011, there were only 1630 firms left. In contrast, there are 659 six digit level NAICS code industries.
2.2.3 Investment rates

As in Cooper and Haltiwanger (2006), I document the distribution of investment rates \( \frac{I_t}{K_t} \) at the firm level. However, unlike most data sets involving investment, where the capital stock is unavailable (and as such the capital stock is reconstructed using investment flows), the KFS dataset provides a reasonably finely grained detail of the firm’s capital stock, but does not provide the investment flow. As such, I construct investment using the perpetual inventory method

\[
I_{i,s,t} = \frac{K_{i,s,t}}{P_{s,t}} - (1 - \delta_s) \frac{K_{i,s,t-1}}{P_{s,t-1}}
\]

\[
I_{i,s,t} = \sum_s \frac{K_{i,s,t-1}}{\sum_s K_{i,s,t-1}} I_{i,s,t}
\]

\[
\hat{i}_{i,s,t} = \frac{I_{i,t}}{K_{i,t-1}}
\]

and I allow for depreciation of each type of capital \( (\delta_s) \) to differ according to the BEA depreciation schedule.

Gross investment at the firm level is then constructed as a weighted average of the firm’s investment for each capital type. Investment rate is then constructed by scaling the gross investment by the total lagged capital stock.

Just as in the case of \( \log ARPK \), using \( i_{it} \) “as is” poses a potential problem that the investment series is biased by unobserved aggregate shocks that are orthogonal to the idiosyncratic shocks or frictions that I am interested in studying. To purge the effect of aggregate shocks on investment, I construct a residualized investment series by regressing investment rates on year and two digit NAICS code fixed effects, and construct the relevant investment moments using the residuals.

2.3 Distribution and Dynamics of Average Revenue Product of Capital

2.3.1 Cross-sectional moments

Figure 1 below shows the distribution of \( \log ARPK \) in the sample population, while table 1 below documents some key cross-sectional moments of this distribution. Visually, one sees immediately that the distribution is left-skewed. Moreover, just as the prior literature has documented in the context of other industries or countries, \( \log ARPK \) is very dispersed.
Figure 1: Distribution of residualized log \( \frac{Y}{K} \). skew \( \log \left( \frac{Y}{K} \right) \) \( \approx -0.33 \)

<table>
<thead>
<tr>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.75</td>
<td>-0.49</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Table 1: Selected moments from distribution of log \( ARPK \)

2.3.2 Asymmetric persistence of log \( ARPK \)

The average revenue product is also highly persistent in the KFS sample. Moreover, the persistence is asymmetric, being greater on the left tail than the right. This fact is documented below in two ways: (1) Estimating a transition matrix for log \( ARPK \), and (2) estimating the conditional autocorrelation of log \( ARPK \).

**Transition matrix**  Here, I estimate the persistence in relative rankings by first binning the firms into quintiles, and then estimating the transition matrix \( M \) (across quintiles) for the entire sample. I report the estimated transition matrix below, with standard errors in parenthesis below the estimated value:

\[
\hat{M} = \begin{bmatrix}
0.614 & 0.241 & 0.092 & 0.034 & 0.020 \\
(0.017) & (0.015) & (0.011) & (0.007) & (0.005) \\
0.253 & 0.389 & 0.248 & 0.080 & 0.038 \\
(0.015) & (0.017) & (0.015) & (0.010) & (0.007) \\
0.089 & 0.208 & 0.378 & 0.252 & 0.072 \\
(0.010) & (0.014) & (0.017) & (0.015) & (0.008) \\
0.037 & 0.105 & 0.220 & 0.381 & 0.257 \\
(0.007) & (0.011) & (0.015) & (0.017) & (0.016) \\
0.028 & 0.058 & 0.098 & 0.247 & 0.568 \\
(0.006) & (0.008) & (0.011) & (0.015) & (0.018)
\end{bmatrix}
\]
The reader should keep in mind that the columns reflect “today’s” relative rankings, and the rows reflect “tomorrow’s” relative rankings. As such, entry \((i, j)\) reflect the probability that a firm that was in quantile \(i\) transitions to quantile \(j\) tomorrow.

To better clarify the import of these results, I first direct the reader to look at the values along the diagonal. The values along the diagonal reflect the probability of staying in quantile \(q\), given that you were in quantile \(q\) yesterday. As such, a null hypothesis of no persistence should reflect that this probability is 0.2. In contrast, looking across all the values along the diagonal, we see that all values are statistically different from (greater than) 0.20.

In addition, we also see that there is higher conditional persistence at the tails than in the center of the distribution, and that this difference is also statistically significant, indicating that there is greater churning around the center of the distribution than at the tails.

Finally, there is also a significant asymmetry in the persistence of \(\log ARPK\) in the left and right tail - the persistence of \(\log ARPK\), conditional on being in the first quintile, is statistically and economically significantly larger than the persistence of \(\log ARPK\), conditional on being in the last quintile.

**Conditional autocorrelation**  Next, I estimate the conditional autocorrelation of \(\log ARPK\).

A simple way to study this is to form a regression of the form

\[
\log ARPK_{i,t} = a + \sum_{q=1}^{5} \rho_q \log ARPK_{i,t-1} + \epsilon_{it}
\]

where \(a\) is the intercept term, and \(\rho_q\) is a coefficient that depends on the \(\log ARPK\) quantile \(q\) that the firm is in currently. As table 2 shows, \(\rho_1\), which is the autocorrelation of \(\log ARPK\) when the firm is in quantile 1, is much larger than \(\rho_5\). A Wald test also rejects the null that the two estimates are equivalent, as such supporting the evidence that there is greater persistence in \(\log ARPK\) at the bottom quintile relative to the top quintile.

<table>
<thead>
<tr>
<th>Dependent variable: (\log ARPK_{i,t})</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.075 (0.037)</td>
</tr>
<tr>
<td>(\rho_1)</td>
<td>0.643 (0.034)</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>0.897 (0.058)</td>
</tr>
<tr>
<td>(\rho_3)</td>
<td>0.672 (0.109)</td>
</tr>
<tr>
<td>(\rho_4)</td>
<td>0.697 (0.052)</td>
</tr>
<tr>
<td>(\rho_5)</td>
<td>0.443 (0.035)</td>
</tr>
</tbody>
</table>

Table 2: Regression result of \(\log ARPK_{i,t} = a + \rho_q \log ARPK_{i,t-1} + \epsilon_{it}\). \(q = 1\) refers to the first quintile, \(q = 2\) the second quintile, and so on. Standard errors in parentheses.
2.4 Distribution of investment rates

Figure 2 shows the distribution of investment rates, and Table 3 reports some of the key moments associated with this. The investment distribution presented in Figure 2 is winsorized at the 1st and 95th percentile for clarity of presentation, while the moments in Table 3 are constructed for the distribution winsorized at the 1st and 99th percentile. Following Doms and Dunne (1998), I also conduct an event study of investment, which is reported in Figure 3. The event is defined as the largest investment associated with each firm over its appearance in the survey, averaged over all firms. A 2 year window is constructed around the event.

![Distribution of investment rates](image)

Figure 2: Distribution of investment rates (winsorized at 5th and 95th percentile)

<table>
<thead>
<tr>
<th>Moments of $\frac{I}{K}$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>1.17</td>
</tr>
<tr>
<td>standard deviation</td>
<td>4.80</td>
</tr>
<tr>
<td>5th percentile</td>
<td>-0.866</td>
</tr>
<tr>
<td>95th percentile</td>
<td>5.739</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>-0.072</td>
</tr>
<tr>
<td>skewness</td>
<td>2.12</td>
</tr>
<tr>
<td>kurtosis</td>
<td>6.95</td>
</tr>
<tr>
<td>% of $\frac{I}{K} &gt; 0$</td>
<td>52.4%</td>
</tr>
<tr>
<td>% of $\frac{I}{K} &lt; \frac{1}{4} \times mean(\frac{I}{K})$</td>
<td>34.9%</td>
</tr>
</tbody>
</table>

Table 3: Selected moments from distribution of investment rates
Figure 3: Event study - Event is defined as the largest investment spike at the firm level

As Figure 2 shows, the distribution of investment rates is highly right skewed, along with a very long right tail. The 95th percentile of investment rates is around 5.7, that is, at the 95th percentile, firms invest up to 5.7 times their capital stock in the same year; in contrast, the 5th percentile of investment rates is around -0.87, which corresponds to the firm owner essentially selling all his capital stock. Across the entire sample of firm-year observations, 44.9% of firms are downsizing and 55.1% are increasing their capital stock. Broadly speaking, this observation is reflective of the results on investment moments found in the firm dynamics literature, such as the seminal paper by Cooper and Haltiwanger (2006); that is, that investment at the firm level is typically very lumpy.

One might notice that the investment rate distribution features a standard deviation that is much larger than that in the firm dynamics literature (for example, the standard deviation of investment rates in Cooper and Haltiwanger is 0.33). This is in fact unsurprising, given the scale that entrepreneurial firms operate at. The median capital stock is $23,288 (the mean is $190,729, reflecting the sizable right-skewness of the firm size distribution). At the 95th percentile, where $\frac{I}{K} \approx 5.7$, this means that the median firm is investing around $132,741. While sizable for many individuals, in terms of a business investment, this is small. For example, this could correspond to a small moving firm simply buying a couple of new trucks.
2.5 Summary of empirical results

As noted in the firm dynamics literature, the stark dispersion of ARPK implies that there is potentially widespread capital mis-allocation\textsuperscript{15}, with many low productivity firms operating inefficiently large firms and high productivity firms operating inefficiently small firms. Unlike the preceding literature, I also find that the distribution is left-skewed, implying that there might be substantially more firms that are large with low productivity, than small firms with high productivity. Moreover, the asymmetric persistence implies that the former operate at this inefficient level for longer periods of time than the latter, which then naturally gives rise to the left skewed cross-sectional distribution. This suggests that entrepreneurial firms are facing frictions in both capital accumulation and decumulation, but the latter friction appears to be stronger.

In addition, the investment distribution reveals that entrepreneurial firm investment is lumpy and infrequent, which suggests that non-convex adjustment costs play a potentially huge role in influencing entrepreneurial firm investment behavior.

Taken as a whole, the results are suggestive that partial irreversibility of capital, which induces an asymmetry in the purchase and resale value of capital, could rationalize the dynamic investment behavior observed here\textsuperscript{16}. In particular, partial irreversibility, which modeled as a proportional transaction cost, induces the (s,S) investment dynamics that is crucial in generating lumpy (dis)-investment. The lower resale value of used capital also induces entrepreneurs to adopt a wait-and-see attitude when a bad shock is realized, operating firms that are “too large” relative to their productivity. This leads to an elongated left tail of ARPK, generating a left skewness in the distribution of ARPK. In addition, this also leads to greater persistence in the left tail. These heuristics therefore provide the motivation to include partial irreversibility as a key mechanism in the model, which I discuss in the following section.

3 Baseline Model

3.1 Households, occupation, and wealth

There is a continuum of mass 0 households in this economy, each indexed by \( i \in [0,1] \). Households are infinitely lived, and time is discrete.

All households are endowed with identical time-separable CRRA utility function, value non-durable consumption only, and discount future utility at rate \( \beta \). In short, the household’s}

\textsuperscript{15}When compared to a static investment model

\textsuperscript{16}See for example, Caballero (1999) for a summary of models with partial irreversibility
lifetime expected utility can be written as

\[ V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}) \]

\[ = E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\gamma}}{1-\gamma} \]

The household’s objective is to maximize expected lifetime utility.

There are two types of occupations available to the households: To become an entrepreneur, or to become a worker. The key distinction between entrepreneurs and workers lie in their endowments, their way of generating income, and their way of accumulating wealth. The latter sections provide a more detailed description, but I briefly summarize the key differences between entrepreneurs and workers here:

**Endowments** Entrepreneurs receive a stochastic endowment of a private business productivity draw \( z \in Z \), while worker’s receive a labor efficiency draw \( \theta \in \Theta \). This productivity draw is exclusive. Workers also receive a stochastic signal of business prospects \( \psi^z \). \( \psi^z \) informs the worker about the potential business prospects that he could engage in if he switches to becoming a business owner. Entrepreneurs receive a stochastic signal of labor prospects \( \psi^\theta \). \( \psi^\theta \) informs the entrepreneur about the potential labor prospects that he could engage in if he switches to becoming a worker.

**Wealth** Entrepreneurs accumulate two forms of wealth: illiquid physical capital \( k \) (which they use in production), and liquid bonds \( b \), which pay off a fixed interest rate. Workers can only save in liquid bonds. Physical capital is illiquid, as adjustment of the capital stock will lead the owner to incur specific adjustment costs.

**Technology** All households are endowed with a fixed amount of labor supply \( \bar{l} \). Workers can combine their labor supply endowment with their labor productivity to produce effective labor \( \theta \bar{l} \). They supply effective labor to a centralized labor market, and earn the corresponding market wage \( w \), give them an income of \( w\theta \bar{l} \). Entrepreneurs have access to a home production function \( f(z, k, l) \), which allows them to combine illiquid capital, labor (including part of their labor supply endowment), and business productivity to produce a consumption good which they sell into a centralized market for income.
3.2 Sources of uncertainty and occupational choice

3.2.1 Sources of uncertainty

There are four sources of idiosyncratic (and uninsurable) uncertainty in this economy. They are:

1. A stochastic Markov process \( P_{z|z_{-1}} \equiv Pr(z|z_{-1}) \) over private business productivity, with support \( z \in Z \) and invariant distribution \( F_z \).

2. A stochastic Markov process \( P_{\theta|\theta_{-1}} \equiv Pr(\theta|\theta_{-1}) \) over labor productivity, with support \( \theta \in \Theta \) and invariant distribution \( F_{\theta} \).

3. An IID process of signals regarding business prospects, with support \( \psi^z \in \Psi^z \), and distribution \( F_{\psi^z} \). Conditional on drawing a signal \( \psi^z \), next period private business productivity is assumed to be drawn from the conditional probability \( P_{z|\psi^z_{-1}} \equiv Pr(z|\psi^z_{-1}) \).

4. An IID process of signals regarding labor prospects, with support \( \psi^\theta \in \Psi^\theta \), and distribution \( F_{\psi^\theta} \). Conditional on drawing a signal \( \psi^\theta \), next period labor productivity is assumed to be drawn from the conditional probability \( P_{\theta|\psi^\theta_{-1}} \equiv Pr(\theta|\psi^\theta_{-1}) \).

3.2.2 Occupational choice

The occupation of the households determine the types of uncertainty they face. Specifically, as described earlier, only entrepreneurs receive endowments of business productivity and labor prospect signal draws, and only workers receive endowments of labor productivity and business prospect signal draws.

Households are allowed to endogenously choose their occupation, but they have to select their occupation one period in advance. In other words, a worker today who chooses to stay a worker tomorrow must pursue his chosen occupation for the next period. Occupational choice is exclusive, so a household cannot be both a worker and an entrepreneur simultaneously. This combination of “time-to-build” and exclusivity makes occupational choice itself risky.

Since there is endogenous occupational choice, we can in fact group the households into four types. The exact classification of the household is important, as this determines the sources of uncertainty that they face. Specifically, we have the following:

1. **Entrant entrepreneurs.** Entrants entrepreneurs are entrepreneurs who were workers last period. They are endowed with a productivity \( z \) that is drawn from the conditional distribution \( P_{z|\psi^z_{-1}} \), where \( \psi^z_{-1} \) is the signal that the entrants had received in the
last period (as workers). They also receive a signal $\psi^\theta$ regarding next period's labor prospects.

2. *Incumbent entrepreneurs.* Incumbents entrepreneurs are entrepreneurs who were entrepreneurs last period. They are endowed with a productivity $z$ that is drawn from the conditional distribution $P_z(z|z_{-1})$, where $z_{-1}$ is last period productivity that they received as entrepreneurs. Like entrant entrepreneurs, they also receive a signal $\psi^\theta$ regarding next period's labor prospects.

Upon drawing $\psi^\theta$, if the entrepreneur chooses to exercise this option and becomes a worker, her next period labor productivity follows the same Markov process as the worker. Otherwise, she loses the signal and draws a fresh signal next period from the invariant distribution of signals.

3. *Entrant workers.* Entrants workers are workers who were entrepreneurs last period. They are endowed with a productivity $\theta$ that is drawn from the conditional distribution $P_\theta(\theta|\psi_{-1})$. In addition, they receive a signal $\psi^\theta$ regarding next period's business prospects.

4. *Incumbent workers.* Incumbents workers are workers who were workers last period. They are endowed with labor productivity $\theta$ that is drawn from the conditional distribution $P_\theta(\theta|\theta_{-1})$, where $\theta_{-1}$ is last period labor productivity that they received as workers. Like entrant workers, they also receive a signal $\psi^\theta$ regarding next period's business.

Upon drawing $\psi^\theta$, if the worker chooses to exercise this option and starts a business, her next period labor productivity follows the same Markov process as the entrepreneurs. Otherwise, she loses the signal and draws a fresh signal next period from the invariant distribution of signals.

### 3.3 Asset structure

Households have access to 2 types of assets to smooth inter-temporal consumption: Liquid bonds $b$ and illiquid physical capital $k$.

#### 3.3.1 Entrepreneurial physical capital

Only households who elect to become (or stay) entrepreneurs tomorrow can save in illiquid physical capital $k$. The primary purpose of the physical capital is as input for entrepreneurial production. However, it also serves a secondary purpose as consumption insurance. In
particular, entrepreneurs can use the illiquid asset to smooth consumption by selling off parts of his asset stock in bad times, or to use it as collateral to borrow for consumption.

Capital is illiquid because of frictions associated with adjusting the capital stock. Here, I assume 4 forms of frictions.

The first two forms of friction affects the investment and disinvestment of entrepreneurs who stay entrepreneurs. Firstly, entrepreneurs who choose to increase the capital stock face a proportional fixed adjustment cost $f_s k$. The proportional fixed cost parametrizes the disruptions associated with expansion. Secondly, entrepreneurs who choose to decrease the capital stock face a proportional transaction cost of the form $\lambda i_k$, where $i_k$ is the amount of disinvestment chosen.

The third friction involves exit. Specifically, entrepreneurs who choose to exit their business face a proportional selling cost on top of the transaction cost involved in downsizing. Specifically, for a level of depreciated capital stock $(1 - \delta) k$, the exiting entrepreneur only collects $(1 - \zeta)(1 - \lambda)(1 - \delta) k$ on his sale of his assets. This parameterizes the difficulty of exiting a business and having to conduct a fire sale of all the physical capital stock.\footnote{It is important to note that this does not necessarily say that every entrepreneur who exits a business has to incur a loss. Rather, it says that when an entrepreneur exits his business, the physical capital stock that he owns is typically worth less than even the actual depreciated value. On the other hand, this paper has nothing to say about the potential profits to be made on intangible capital, such as brand name or R&D capital.}

Finally, the last friction involves entry. Specifically, it is assumed that workers who want to enter entrepreneurship must start their business with a minimum level of capital $k_{wmin}$. Households who are already entrepreneurs do not face this friction. Rather, this assumption reflects the fact that entry typically requires some form of minimum capital investment.

### 3.3.2 Liquid bonds

All households can trade in bonds, either by buying bonds (i.e. saving), or selling them (i.e. borrowing). The total volume of bonds supplied composed of all the bonds issued by individual households, as well as equity issued by a corporate sector. The cost of a bond that pays off next period is 1 unit of consumption.

All bonds, regardless of whether it was issued by the corporate sector or individuals, pay off the same interest rate $r$. As the corporate sector is large and is in fact issuing claims against its profits, I assume that there is no spread between the cost at which the corporate sector borrows, and the interest rate received by savers. In contrast, individuals issuing bonds are essentially issuing unsecured debt, and hence are “riskier”. As such, I assume that the household must pay a per unit intermediation cost $\phi_d$ when they borrow. As a result, this induces a spread between the interest rate $r$ received by savers, and interest rate $r_d$ paid...
by borrowers, given by \( r_d = r + \phi_d \).

In addition, households who decide to invest in physical capital can also elect to use the liquid value of their physical capital as collateral to borrow in bonds. Specifically, I define the liquid value of capital as \((1 - \lambda)(1 - \delta)k\), and the collateral constraint is defined as

\[
b' \geq -\phi(1 - \lambda)(1 - \delta)k' - b
\]

where \( \phi \in (0, \infty) \), with \( \phi = 0 \) representing no collateralized borrowing, and \( \phi = \infty \) representing no collateral required for borrowing. Households are allowed to borrow up to the liquid value of depreciated capital. \( b \) represents an ad-hoc borrowing constraint that does not require any collateral. This reflects the nature that some households can acquire unsecured debt. As such, both workers and entrepreneurs can borrow to smooth consumption. The only difference is that entrepreneurs can borrow more.

### 3.4 Sectors of the economy

There are two sectors of production in this economy: a corporate sector and an entrepreneurial sector. Both sector produce a single homogeneous non-durable consumption good.

#### 3.4.1 Corporate sector

I assume that there exist a large corporate sector encompassing all non-entrepreneurial firms. This corporate sector is represented by a representative firm, which owns physical capital \( K^c \) and hires labor \( L^c \) from a centralized labor market. It has access to a standard Cobb-Douglas production technology of the form

\[
Y^c = A(K^c)^\alpha (L^c)^{1-\alpha}
\]

where \( A \) is aggregate TFP, and \( \alpha \) is the capital share. All households purchase equity in corporate firms, and are paid a dividend each period.

The corporate firm decides independently on how much physical capital to invest, and how much labor to hire at the prevailing wage \( w \). As such, the representative firm solves the standard recursive problem

\[
\Pi(K) = \max_{K'} \pi + \frac{1}{1 + r} \Pi(K')
\]

s.t.

\[
\pi = Y^c - \left( K^c' - (1 - \delta)K^c \right) - wL^c
\]
where \( \pi \) represents dividends paid out to the firms’ investors, and the firm discounts future profits at rate \( \frac{1}{1+r} \). Note that because of the size of the corporate sector, I assume here that there are no adjustment costs associated with the corporate sector. Moreover, unlike entrepreneurs, corporate firms are allowed to issue equity, which is in line with the real world observation of larger corporate firms being listed on a stock market. This market arrangement leads to the standard first order condition

\[
\begin{align*}
  r + \delta &= \alpha A \left( \frac{K^c}{L^c} \right)^{\alpha-1} \\
  w &= (1 - \alpha) A \left( \frac{K^c}{L^c} \right)^\alpha
\end{align*}
\]

### 3.4.2 Entrepreneurial sector

The entrepreneurial sector is composed of entrepreneurial households. They utilize the production function

\[
y_e = f(z, k, l) = z \left( k^{\alpha_e} l^{1-\alpha_e} \right)^\nu.
\]

As discussed earlier, the inputs to production are physical capital \( k \) and labor \( l \). \( \nu \in (0, 1) \) here denotes the span-of-control, capturing the fact that managerial skills become stretched over larger and larger projects.

Physical capital stock is chosen last period, and cannot be altered in the current period. \( z \), as discussed earlier, is stochastic productivity and realized in the same period. Entrepreneurs are endowed with labor supply \( \bar{l} \), and they can use a fixed fraction \( \iota \) to run their own business. If they choose to hire extra workers, they have to pay the prevailing market wage \( w \). As such, the profit of the entrepreneurial firm can be written as

\[
\pi(z, k) = z \left( k^{\alpha_e} l^{1-\alpha_e} \right)^\nu - w \left( l - \iota \bar{l} \right)
\]

Note that given this setup, the labor choice is a static decision and completely independent of the structure of the rest of the problem. Consequently, we see that optimal labor demand satisfies:

\[
l^* = \begin{cases} 
  \iota \bar{l} & \text{if } l^* \leq \iota \bar{l} \\
  \left[ \frac{1}{w} \right]^{\frac{1}{1-\alpha_e}} z^\frac{1}{1-\alpha_e} k^\frac{\alpha_e}{1-\alpha_e} & \text{if } l^* > \iota \bar{l}
\end{cases}
\]

and optimal profits is given by

\[
\pi^* = \begin{cases} 
  z \left( k^{\alpha \left( \iota \bar{l} \right)^{1-\alpha}} \right)^\nu & \text{if } l^* \leq \iota \bar{l} \\
  \left[ A(w) - w A(w)^\frac{1}{1-\alpha_e} \right] z^\Theta_k k^\Theta_k + w \bar{l} & \text{if } l^* > \iota \bar{l}
\end{cases}
\]
where $\mathcal{A}(w) \equiv \left[ \frac{(1-\alpha)\nu}{w} \right]^{\frac{(1-\alpha)\nu}{1-(1-\alpha)\nu}}$, $\Theta_z \equiv \frac{1}{1-(1-\alpha)\nu}$ and $\Theta_k \equiv \frac{\alpha\nu}{1-(1-\alpha)\nu}$.

### 3.5 Recursive formulation of the problem

The preceding problem can be recast compactly into recursive formulation. Denote by $V_e$ and $V_w$ the value functions of entrepreneurs and workers respectively, by $C(k', k, h', h)$ the adjustment cost function as described earlier, and $h$ the occupational state (with 1 denoting worker and 0 denoting entrepreneur). Denote by $'$ variables all next-period variables, and unprimed variables current variables. Given this, we have the following problem:

For entrepreneurs,

$$V_e \left( \psi^\theta, z, k, b \right) = \max_{h', k', b', c} U \left( c \right)$$

$$+ (1 - h') \times \beta \int_{\psi^\theta'} \int_{z'} V_e \left( \psi'^{\theta'}, z', k', b' \right) dP_{\psi^\theta'} dz dF_{\psi^\theta}$$

$$+ h' \times \beta \int_{\psi^z} \int_{\theta^z} V_w \left( \psi^z', \theta^z', b' \right) dP_{\theta^z}\psi^\theta dF_{\psi^z}$$

s.t.

$$c + k' + b' = zf \left( k, \bar{l} + l \right) - wl + \left( 1 + r \times 1_{\{b \geq 0\}} + r_d \times 1_{\{b < 0\}} \right) b - C \left( k', k, h', h \right)$$

$$k' \begin{cases} > 0 & \text{if } h' = 0 \\ = 0 & \text{if } h' = 1 \end{cases}$$

$$b' \geq -\varphi \left( 1 - \lambda \right) \left( 1 - \delta \right) k' - \underline{b}$$
For workers,

\[
V_w(\psi^z, \theta, b) = \max_{h', k', b'} U(c)
\]

\[
+ (1 - h') \times \beta \int_{\psi^z} \int_{z'} V_e(\psi^{z'}, z', k', b') dP_{z'|\psi^z} dF_{\psi^z}
\]

\[
+ h' \times \beta \int_{\psi^z} \int_{\theta \in \Theta} V_w(\psi^{z'}, \theta', b') dP_{\theta'|\theta} dF_{\psi^z}
\]

s.t.

\[
c + k' + b' = \theta w l + (1 + r \times 1_{\{b \geq 0\}} + r_d \times 1_{\{b < 0\}}) b
\]

\[
\begin{align*}
  k' &> 0 & \text{if } h' = 0 \\
  &= 0 & \text{if } h' = 1 \\
  b' &\geq -\varphi (1 - \lambda) (1 - \delta) k' - b
\end{align*}
\]

### 3.6 Equilibrium Definition

The state space of the model can be described by bond holdings \( b \in \mathbb{B} \), capital holdings \( k \in \mathbb{K} \), occupational choice \( h \in \mathbb{H} \), entrepreneurial productivity \( z \in \mathbb{Z} \), labor productivity \( \theta \in \Theta \), entrepreneurial productivity signal \( \psi^z \in \Psi^z \), and labor productivity signal \( \psi^\theta \in \Psi^\theta \). As such, the complete state space \( \mathbb{S} \) can be written as \( \mathbb{S} = \mathbb{B} \times \mathbb{K} \times \mathbb{H} \times \mathbb{Z} \times \Theta \times \Psi^z \times \Psi^\theta \).

A stationary competitive equilibrium of the model consist of the interest rate \( r \), wage rate \( w \), value functions of households and firms \( \{V^e, V^w, \Pi\} \), allocations \( \{k', b', l\} \) and distribution of agents \( \Lambda \) over the state space \( \mathbb{S} \) such that,

1. Taking \( r \) and \( w \) as given, the households’ and firms’ choices are optimal.

2. Markets clear,

   (a) Bonds: \( \int b'd\Lambda = K^e \)

   (b) Labor: \( \int \theta h d\Lambda = \int l d\Lambda + L^c \)

3. The distribution \( \Lambda \) is time-invariant, given by

\[
\Lambda = \Gamma(\Lambda)
\]

Where \( \Gamma \) is the one-period transition operator on the distribution.

The method by which I compute the solution to the individual’s problem, and the stationary equilibrium, is documented in the appendix.
3.7 Illiquid capital - Skewness and left tail persistence

As I discussed in the introduction, a standard model of dynamic investment choice, featuring at most collateral constraints, cannot in general generate both negative skewness and left tail persistence in ARPK. For this rest of this subsection, I discuss briefly the reason for this result. For reference, I consider a canonical model of firm production, operating a production function \( Y = zK^\alpha \), where \( z \) is firm level TFP, \( K \) is capital, \( Y \) is output, \( \alpha \) is the returns to scale, \( r \) is the interest rate and \( \delta \) is the user cost of capital (depreciation). I use this to discuss four common frameworks adopted in this literature: (1) A static model of investment with no frictions (as the benchmark); (2) A dynamic model with a single period time-to-build as a “friction”; (3) The standard static model of investment with collateral constraints resulting from limited commitment; and (4) A dynamic model of investment with collateral constraints.

3.7.1 A static model of investment with no frictions

The canonical firm investment model with static investment choices yield the first order condition for capital as

\[
\alpha \frac{Y}{K} \equiv \text{MRPK} = r + \delta \\
\log (\text{MRPK}) = \log (r + \delta) \\
\Rightarrow \log (\text{ARPK}) = r + \delta - \log \alpha
\]

In this framework, there is no dispersion in log ARPK (or log MRPK). Hsieh and Klenow (2009) uses this as their motivation for studying “wedges” that distort a firm’s investment decision, leading to dispersion in log ARPK (log MRPK).

3.7.2 Dynamic investment model with only time-to-build

A key point that Asker et al (2014) raises in their paper, using numerical examples, is that time-to-build is, in itself, sufficient to generate the wide dispersion in log MRPK (log ARPK) observed by Hsieh and Klenow (2009). As such, one might suspect that time-to-build is also sufficient to generate the left skewness and persistence of log ARPK, as observed in the KFS.

As I show in the appendix, using the standard frictionless framework with a persistent productivity process, such as that discussed in Asker et al (2014), log ARPK is given by an

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18 All the derivations hold if we include labor as an input, but I ignore it in the interest of algebraic clarity.
equation of the form (denoted by a superscript $TTB$ for “time-to-build”)

$$\log ARPK^{TTB} = \vartheta + \epsilon$$

where $\vartheta$ is a collection of parameters, and $\epsilon$ is any i.i.d innovation to productivity. Under the common assumption of Gaussian innovations, $\log ARPK$ therefore also has a normal distribution, and will not feature any skewness.

However, one could argue that $\epsilon$ could be drawn from a non-normal distribution. In fact, one of the more common assumptions is to have $\epsilon$ be drawn from a Pareto distribution - a modeling tool to replicate the long right tail of firm sizes. Note that this assumption would imply that $\log ARPK$ would itself then be right-skewed - a result counterfactual to the evidence in the KFS.

What if one simply assumes that KFS entrepreneurs have productivity that are very left-skewed? Unfortunately, this result still does not address the second issue where $\log ARPK$ features large conditional persistence in both relative rankings and absolute values. In fact, this result shows that $\log ARPK$ is i.i.d.

**Endogenous occupational choice** One might be concerned that this result, which was derived using a framework with no entry and exit, will not survive into a model with entry and exit. However, this result does in fact survive into a model with endogenous entry and exit, as detailed further in the appendix. The key intuition here is that with no adjustment frictions (besides time to build), all entrepreneurs (incumbents and entrants) target the same ARPK in expectation (i.e. $r + \delta \alpha$). Consequently, the resulting distribution of $\log ARPK$ is i.i.d (as long as the expectation errors are common across households, and not state dependent). Therefore, $\log ARPK$, under a model of endogenous entry and exit, will likewise inherit the distribution of the underlying innovations, and feature no persistence.

### 3.7.3 A static model of investment with collateral constraints

This class of models feature a static collateral constraint of the form

$$k \leq (1 + \varphi) a$$

where here, $k$ is working capital and $a$ are household assets. $\varphi \geq 0$ refers to the strength of the collateral constraint, with $\varphi = 0$ meaning no borrowing, and $\varphi \to \infty$ reflecting no constraints. As shown in the appendix, this form of constraint leads to a left-truncated
support for $\log ARPK$, with (denoted with a superscript $FF$ for financial frictions)

$$\log ARPK^{FF} \in [\log (r + \delta) - \log \alpha, \infty)$$

Naturally, this will arise in a right-skewed distribution, and hence will not replicate the left-skewness observed in the KFS.

**Endogenous occupational choice** An inclusion of endogenous occupational choice in this framework does not extend the left tail, which is still always truncated at $\log (r + \delta) - \log \alpha$. However, collateral constraints do lead to a lower probability of observing entrepreneurs with high $z$ draws. Specifically, for potential entrants and current incumbents who face a very tight collateral constraint, the former will delay entry as they try to accumulate sufficient wealth, while the latter will exit since they cannot hit a minimum firm size that makes operating a business more profitable than being a worker (despite their high $z$ draws). Taken together, the right tail will become thinner. However, while this effect will reduce the overall skewness of $\log ARPK$, it will remain right-skewed due to the left truncation.

### 3.7.4 A dynamic model of investment with collateral constraints

Another concern one might have is that this model features *dynamic* decisions, not the static one common to prior models. One might therefore be rightfully concerned that the dynamic model would overturn the previous result.

However, as I show in the appendix, when we abstract from the partial irreversibility and exit frictions in my model, this framework leads to an analytical (implicit) solution for $\log ARPK$, given by (denoted by a superscript $TTB, FF$ for “time-to-build with financial frictions”)

$$\log ARPK^{TTB, FF} = \log ARPK^{TTB} + \xi_{-1}$$

where $\xi_{-1}$ is a random variable that has a right-skewed distribution (and is pre-determined last period), and $\log ARPK^{TTB}$ is the log ARPK derived under a framework of no collateral constraints (as in section 3.7.2).

Recall that $\log ARPK^{TTB}$ has a distribution that is simply the distribution of the underlying innovations, translated by a constant. When the underlying innovations are normal, $\log ARPK^{TTB}$ is also normal, and has zero skewness; if the underlying innovations are left-skewed, then $\log ARPK^{TTB}$ is also left-skewed. Under both conditions, $\log ARPK$ will feature a higher right-skewness than $\log ARPK^{TTB}$ due to the contribution of $\xi$. 

25
What about when the innovations are right-skewed? In this case, \( \log \text{ARPK} \) could have a lower skewness than \( \log \text{ARPK}^{TTB} \). However, \( \log \text{ARPK} \) will still be right-skewed.

3.8 Parametric forms for productivity

Before moving forward to discussing the calibration strategy in the next section, I first close out the model by assigning functional forms to the productivity processes.

**Business productivity**  Business productivity is assumed to follow an AR(1) process of the form \( \log z' = (1 - \rho_z) \mu_z + \rho_z \log z + \sigma_z \epsilon_z' \), with \( \epsilon_z' \sim iid, N(0,1) \). The process is discretized with a 9 point Markov transition matrix using the Tauchen (1986) method.

**Business productivity signal**  The signal is assumed to be drawn from a distorted invariant distribution of the actual invariant distribution of business productivity. Note that business productivity has the invariant distribution \( N(\mu_z, \sqrt{\frac{1}{1-\rho_z^2}}\sigma_z) \). In the case of the signal, I assume worker households draw a signal from the "twisted" signal distribution \( N(\tilde{\mu}_z, \sqrt{\frac{1}{1-\rho_z^2}}\sigma_z) \), where the \( \tilde{\mu}_z \) parameter distorts the household’s perception of the true mean of the distribution of productivity. The primary effect of this is to vary the entry rate into entrepreneurship, and thus steady-state population of entrepreneurs.

**Labor productivity**  Labor productivity is assumed to follow an AR(1) process of the form \( \log \theta' = (1 - \rho_\theta) \mu_\theta + \rho_\theta \log \theta + \sigma_\theta \epsilon_\theta' \), with \( \epsilon_\theta' \sim iid, N(0,1) \). The process is discretized with a 5 point Markov transition matrix using the Tauchen (1986) method.

**Labor productivity signal**  The signal is assumed to be drawn from a distorted invariant distribution of the actual invariant distribution of labor productivity. Similar to the case of business productivity signals, labor productivity has the invariant distribution \( N(\mu_\theta, \sqrt{\frac{1}{1-\rho_\theta^2}}\sigma_\theta) \); I assume then that entrepreneurial households draw a signal from the “twisted” distribution \( N(\tilde{\mu}_\theta, \sqrt{\frac{1}{1-\rho_\theta^2}}\sigma_\theta) \), where \( \tilde{\mu}_\theta \) distorts the entrepreneur’s perception of the true mean of the distribution of labor productivity. The primary effect of this is to vary the exit rate.

4 Calibration

The model frequency is annual, which corresponds to the frequency in the KFS. As in the literature, many standard parameters (such as the corporate sector’s capital share, depreci-
ation rate, labor income process) are taken from the preceding literature. The exact numbers used and their rationale are relegated to the appendix.

Two parameters are inferred directly from the data: The depreciation rate of the entrepreneur’s capital stock, and the labor share. The depreciation rate for the entrepreneur’s capital stock differs from the corporate sector, as the composition of the aggregated “capital” stock is different for an entrepreneur as compared to that of a corporate firm. Consequently, I construct the depreciation rate for the entrepreneur’s capital as a weighted average of the individual depreciation rates of the components of the types of capital that make up an entrepreneur’s stock of capital. The labor share (defined as $\hat{\beta}l \equiv (1 - \alpha_e)\nu$) is constructed using a cost shares approach, such as that adopted in Asker et al (2014). The exact method from which these two parameters are constructed is relegated to the appendix.

The rest of the eleven parameters are inferred indirectly from the data by jointly calibrating them to identifying moments from the data. These parameters are: (i) Downsizing transaction cost $\lambda$, (ii) exit transaction cost $\zeta$, (iii) collateral constraint $\varphi$, (iv) returns to scale $\nu$, (v) autocorrelation $\rho_z$ and (vi) standard deviation $\sigma_z$ of entrepreneur’s productivity shock, (vii) amount of endowed labor $\bar{l}$, (viii) fixed investment cost $f_s$, (ix) mean of worker’s signal shock $\mu_z$, (x) mean of entrepreneur’s signal shock $\mu_{\theta}$, and (xi) the household’s discount factor $\beta$. They are calibrated to the following moments: (i) and (ii) Conditional persistence of log ARPK (Left tail: Sum of the probability that a firm in quintile 1 today stays in quintile 1 or moves to quintile 2 tomorrow; Right tail: Sum of the probability that a firm in quintile 5 today stays in quintile 5 or moves to quintile 4 tomorrow); (iii) skewness of log ARPK; (iv) fraction of investment that is positive; (v) the autocorrelation of investment rates; (vi) coefficient of variation of investment rates; (vii) the fraction of entrepreneurs that are employer firms; (viii) the coefficient in a regression of log revenue on log capital, restricted to the sub-population of employer firms; (ix) the exit rate in the KFS; (x) the fraction of households that are entrepreneurs; (xi) the exit and startup rate in the population; and finally (xii) the interest rate. The last two moments are the only moments not drawn from the KFS; instead, it is taken from household surveys (for (ix)) and from macroeconomic data (for (x)).

The mapping of data moments to parameter is summarized in Table 4, this is done with the understanding that variation in any parameter will inadvertently trigger changes in other moments. The idea here is that these moments are most strongly associated with their corresponding parameters. The calibration of $\lambda$, $\zeta$, $\varphi$, and $\nu$, which are of first order importance in deriving the key results of this paper, is discussed in more detail here using a series of comparative statics exercises. The calibration of the other parameters are relegated to the appendix. Finally, given $\nu$ and $\hat{\beta}l$, the entrepreneurial production function capital share $\alpha_e$ can be immediately backed out using $\alpha_e = (\nu - \hat{\beta}l)/\nu$. 

27
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Identifying moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>Resale transaction cost</td>
<td>Conditional persistence of log ARPK</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Exit friction</td>
<td>Skewness of log ARPK</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Collateral constraint</td>
<td>Skewness of log ARPK / Conditional persistence of log ARPK</td>
</tr>
<tr>
<td>( f_s )</td>
<td>Investment fixed cost</td>
<td>Rate of positive investment reported</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Returns to scale</td>
<td>Production function regression</td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>Autocorrelation of productivity shock</td>
<td>Autocorrelation of investment rates</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>Volatility of productivity shock</td>
<td>Coefficient of variation of investment, firm size distribution</td>
</tr>
<tr>
<td>( l )</td>
<td>Entrepreneur’s endowed labor</td>
<td>% of firms that are employers</td>
</tr>
<tr>
<td>( \mu_z )</td>
<td>Mean of entrepreneurial prospects signal shock</td>
<td>Fraction of households that are entrepreneurs in steady-state</td>
</tr>
<tr>
<td>( \mu_\theta )</td>
<td>Mean of labor prospects signal shock</td>
<td>Exit rate</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>Interest rate of between 3% to 4%</td>
</tr>
</tbody>
</table>

Table 4: Calibrated parameters

For the purposes of the calibration, I choose to do two separate calibrations: One which targets moments taken from the full KFS sample (henceforth labeled "FULL"), which I use as my primary benchmark example for this text; and one that targets moments taken only from the modal two digits NAICS industry - Professional, Scientific and Technical Services (NAICS code 54, henceforth labeled NAICS54). This choice is made for several reasons.

Most importantly, when calibrating to the full sample, I am essentially assuming that the model entrepreneur behaves as if she was an “average” entrepreneur in the KFS sample. However, the various industries do exhibit large heterogeneity in labor shares. As a result, to properly capture the full aggregate dynamics in the KFS, the model would also have to incorporate heterogeneity along these dimensions. Unfortunately, due to computational limitations, this is not feasible. As such, I have to compromise by choosing a single labor share.

Given the importance of the parameters in determining the wealth distribution, I choose to also calibrate the model to a specific industry as a robustness check. In this case, the Professional, Scientific and Technical Services industry is the modal choice of industry amongst KFS entrepreneurs - a full 24.7% of the sample includes entrepreneurs operating in this industry.

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20 This assumption is not necessarily extreme. For instance, Bloom (2009) also makes such an assumption in targeting moments from the full Compustat sample.

21 I would have to essentially rewrite the model to include full fixed effects for each industry. In its current form, the state space of the model already has 1.8 million idiosyncratic state.
dustry. In contrast, the next largest industry in this sample (manufacturing) only makes up 8.7% of the sample. This makes the Professional, Scientific and Technical Services industry a natural candidate for calibration as a robustness check. The results are given in table 5 below. Finally, table 6 below compares the data and model implied moments.

Besides the 11 model parameters already mentioned, I also explored including an additional noise parameter. Here, I assume that capital is observed with measurement error, where for any true capital holdings \( K \), the observed model counterpart is \( \hat{K} = \exp(\varepsilon)K \), where \( \varepsilon \sim N(0, \sigma_\varepsilon) \). I explore in greater detail in the appendix how this parameter helps in improving the model fit to the data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \lambda )</th>
<th>( \zeta )</th>
<th>( \varphi )</th>
<th>( f_s )</th>
<th>( \rho_z )</th>
<th>( \sigma_z )</th>
<th>( \mu_z )</th>
<th>( \ddot{l} )</th>
<th>( \mu_\theta )</th>
<th>( \beta )</th>
<th>( \alpha_e )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FULL</td>
<td>0.43</td>
<td>0.55</td>
<td>0.92</td>
<td>0.035</td>
<td>0.66</td>
<td>0.43</td>
<td>0.51</td>
<td>0.23</td>
<td>0.75</td>
<td>0.9265</td>
<td>0.63</td>
<td>0.79</td>
</tr>
<tr>
<td>NAICS54</td>
<td>0.53</td>
<td>0.75</td>
<td>0.035</td>
<td>0.23</td>
<td>0.66</td>
<td>0.43</td>
<td>0.55</td>
<td>0.23</td>
<td>0.75</td>
<td>0.94</td>
<td>0.50</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Table 5: Calibrated parameters
<table>
<thead>
<tr>
<th>Condition</th>
<th>FULL (Data)</th>
<th>FULL (Model)</th>
<th>NAICS54 (Data)</th>
<th>NAICS54 (Model)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conditional persistence of log ARPK (left tail)</strong></td>
<td>86</td>
<td>74</td>
<td>78</td>
<td>74</td>
</tr>
<tr>
<td><strong>Conditional persistence of log ARPK (right tail)</strong></td>
<td>78</td>
<td>64</td>
<td>69</td>
<td>66</td>
</tr>
<tr>
<td><strong>Skewness of log ARPK</strong></td>
<td>-0.485</td>
<td>-0.38741</td>
<td>-0.56</td>
<td>-0.46689</td>
</tr>
<tr>
<td><strong>% reporting positive investment</strong></td>
<td>53.9</td>
<td>0.30958</td>
<td>57</td>
<td>0.26888</td>
</tr>
<tr>
<td><strong>Coefficient of variation of investment rates</strong></td>
<td>4.05</td>
<td>2.4281</td>
<td>3.18</td>
<td>3.4908</td>
</tr>
<tr>
<td><strong>% Employer firms</strong></td>
<td>53</td>
<td>43</td>
<td>50</td>
<td>37</td>
</tr>
<tr>
<td><strong>Coefficient of production function regression</strong></td>
<td>0.59</td>
<td>0.57</td>
<td>0.69</td>
<td>0.55</td>
</tr>
<tr>
<td><strong>KFS exit rate</strong></td>
<td>10</td>
<td>27.2251</td>
<td>10</td>
<td>33.359</td>
</tr>
<tr>
<td><strong>% of households that are entrepreneurs</strong></td>
<td>8% to 20%</td>
<td>12</td>
<td>8% to 20%</td>
<td>8.8</td>
</tr>
<tr>
<td><strong>% Exit rate</strong></td>
<td>20% to 40%</td>
<td>0.28</td>
<td>20% to 40%</td>
<td>39</td>
</tr>
<tr>
<td><strong>% Startup rate</strong></td>
<td>3% to 10%</td>
<td>3.9</td>
<td>3% to 10%</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>Interest rate in steady-state</strong></td>
<td>2% to 4%</td>
<td>3.21</td>
<td>2% to 4%</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Table 6: Data and corresponding model moments

### 4.1 Downsizing frictions and collateral constraints: Skewness and asymmetric persistence

How important are the downsizing frictions in helping the model to match the skewness and asymmetric persistence noted in the data? To reiterate, the standard frictionless dynamic investment model predicts that the cross-sectional distribution of log ARPK inherits exactly the (mean shifted) cross-sectional distribution of its underlying productivity process. As such, the skewness of the log ARPK distribution in such a model will be exactly equal to the skewness of the underlying productivity process. Moreover, log ARPK also has no persistence under a frictionless model. In particular then, a standard model with log-normal Gaussian shocks predict no skewness or persistence. In addition, a similar model with
collateral constraints predict right skewness and right tail persistence.

The intuition is straightforward. In the frictionless world, all firms always target the same (expected) ARPK. Ex-post, log-normal Gaussian innovations will therefore generate a normal log ARPK distribution. For the model with collateral constraints, firm that are unconstrained behave just as the firms in the frictionless world. As such, the conditional distribution (of unconstrained firms) is also normal. For constrained firms, they must necessarily operate firm sizes that are “too small" relative to their optimal sizes - that is, their ARPK must be higher than the optimal ARPK (in expectation). Consequently, the ex-post conditional distribution of constrained firms will feature a right skew. The combined distribution of constrained and unconstrained firms is therefore right-skewed. Moreover, the tighter the collateral constraint, the more right-skewed the distribution will be. This is reflected in the last two columns of table 7, which report the skewness of the ARPK distribution for a given $\varphi$, holding all else constant (note that the result is reported in terms of $1 - \varphi$, as a larger $\varphi$ indicates looser borrowing constraints).

In contrast, downsizing frictions have a tendency to extend the left tail of the distribution. When hit by a bad shock, the options value of capital induced by the asymmetry in purchase and resale price of capital lead downsizing entrepreneurs to target a lower ARPK (in expectation) than the unconstrained ARPK. As a result, the left tail of the ARPK distribution becomes extended, potentially making the distribution more left-skewed. Both $\lambda$ and $\zeta$ will induce this effect.

However, only $\zeta$ has a monotone effect on the skewness, as reflected in the first four columns of table 7. This is because increases in $\lambda$ also generates an opposing effect to the skewness through the right tail. For entrepreneurs seeking to invest, the options value effect induced by $\lambda$ will lead entrepreneurs to target a small firm size than the unconstrained optimal; in other words, these entrepreneurs will have higher ARPK than the unconstrained optimal. As a result, the right tail also elongates, thus potentially inducing a right skew to the distribution. As such, in general, the skew of the distribution cannot be used to identify $\lambda$.

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>skewness</th>
<th>$\lambda$</th>
<th>skewness</th>
<th>$1 - \varphi$</th>
<th>skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.035</td>
<td>0.04</td>
<td>-0.487</td>
<td>0.08</td>
<td>-0.450</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.033</td>
<td>0.06</td>
<td>-0.463</td>
<td>0.10</td>
<td>-0.441</td>
</tr>
<tr>
<td>0.30</td>
<td>-0.289</td>
<td>0.08</td>
<td>-0.458</td>
<td>0.30</td>
<td>-0.439</td>
</tr>
<tr>
<td>0.50</td>
<td>-0.373</td>
<td>0.10</td>
<td>-0.452</td>
<td>0.50</td>
<td>-0.421</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.449</td>
<td>0.12</td>
<td>-0.453</td>
<td>0.70</td>
<td>-0.407</td>
</tr>
<tr>
<td>1</td>
<td>-0.453</td>
<td>0.14</td>
<td>-0.445</td>
<td>0.90</td>
<td>-0.394</td>
</tr>
</tbody>
</table>

Table 7: Effect of parameters on skewness of ARPK distribution
Fortunately, $\lambda$ can be identified by the persistence in the tails of the distribution. Recall that $\lambda$ induces a “wait-and-see” attitude only for firms who want to downsize; in contrast, entrepreneurs looking to expand do not face the same constraint. As a result, increases in $\lambda$ will result in a left tail that is becomes increasingly more persistent than the right. In contrast, $\varphi$ and $\zeta$ have no such clear predictions. A simple way to view this relationship is to compare the effect of changing these parameters on the ratio $\frac{\rho_1}{\rho_5}$, as reflected in table 8 below:

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$\rho_1/\rho_5$</th>
<th>$\lambda$</th>
<th>$\rho_1/\rho_5$</th>
<th>$1 - \varphi$</th>
<th>$\rho_1/\rho_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.26</td>
<td>0.04</td>
<td>0.874</td>
<td>0.08</td>
<td>1.20</td>
</tr>
<tr>
<td>0.05</td>
<td>3.72</td>
<td>0.06</td>
<td>0.874</td>
<td>0.10</td>
<td>1.19</td>
</tr>
<tr>
<td>0.30</td>
<td>2.90</td>
<td>0.08</td>
<td>0.979</td>
<td>0.30</td>
<td>1.17</td>
</tr>
<tr>
<td>0.50</td>
<td>1.68</td>
<td>0.10</td>
<td>1.06</td>
<td>0.50</td>
<td>1.20</td>
</tr>
<tr>
<td>0.75</td>
<td>1.23</td>
<td>0.12</td>
<td>1.09</td>
<td>0.70</td>
<td>1.20</td>
</tr>
<tr>
<td>1</td>
<td>0.976</td>
<td>0.14</td>
<td>1.14</td>
<td>0.90</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Table 8: Effect of parameters on $\frac{\rho_1}{\rho_5}$

Note that if we set $\lambda = \zeta = f_s = 0$, we return to the one asset framework where bonds and capital are perfect substitutes.

4.1.1 Using skewness and relative persistence to discriminate amongst sources of friction

The results here have important considerations for understanding the underlying frictions that impede efficient capital allocation amongst entrepreneurs, as well as for the broader economy. Specifically, we see that frictions that distort downsizing ($\lambda$) or exit ($\zeta$) produces log ARPK distributions that are increasing left skewed. Moreover, it also leads to greater persistence in the left tail of the distribution, and can lead to greater persistence in the left tail than right tail. In contrast, frictions that impede investment, such as financial frictions, lead to greater right skewness in the log ARPK distributions. As such, this paper provides a way to discriminate between these two types of frictions.

In contrast, prior papers that focus on misallocation of capital, such as Hsieh and Klenow (2009), Asker et al (2014), and Midrigan and Xu (2014), have focused primarily on studying the dispersion of log MRPK (and equivalently, log ARPK). Asker et al (2014), for instance, focuses on capital adjustment costs (equivalent to the $f_s$ and $\lambda$ parameters in this model) while ignoring financing constraints, while Midrigan and Xu (2014) focuses on financing constraints (equivalent to $\varphi$ in this model) but largely abstracting away from adjustment costs. In both cases, frictions can lead to observationally equivalent outcomes in the dis-
persion of log MRPK (or ARPK). This paper therefore merges these two frameworks but incorporating both adjustment and financial frictions, and providing a simple framework to discriminate between the two (i.e. using the skewness, relative persistence, and conditional autocorrelation).

4.1.2 Are entrepreneurs “financially constrained”?

In the benchmark calibration, $\varphi = 0.92$. $\varphi$ in this model refers to the classical limited commitment problem in the Kiyotaki-Moore framework; in this context, this means that entrepreneurs are able to collateralize up to 92% of their capital assets.

What does this mean? At face value, this suggests that early stage entrepreneurs are in fact less encumbered by financial frictions than one suspects. In prior research such as that in Evans and Jovanovic (1989), Cagetti and De Nardi (2006) and following papers, financial friction play a large role in determining the dispersion of wealth and firm sizes. On average, entrepreneurs are able to collateralized between 50% to 80% of their assets. The inability to fully borrow and fund their investment leads to large distortions in the economy, primarily by deterring high productivity potential entrants from entering, and forcing poorer entrepreneurs to run sub-optimally small firms.

In contrast, my results are more in line with that reported in Hurst and Lusardi (2004), Nanda (2011), and Robb and Robinson (2014), which find no evidence that regular entrepreneurs face severe financing constraints. In particular, Robb and Robinson (2014), using the same data set as this paper, find that most entrepreneurs are able to finance their investment by simply going to the bank.

Does this mean that entrepreneurs don’t face borrowing constraints? On the contrary, the calibration says that entrepreneurs face tight collateral constraints similar to that in Cagetti and De Nardi (2006). Note that while $\varphi$ determines the financial frictions that affect the limited commitment problem, the true collateral constraint is given by $\tilde{\varphi} \equiv (1-\delta) \times (1-\lambda) \times \varphi$, i.e. the entrepreneur can only collateralize the net depreciated resale value of entrepreneurial capital. When this effect is taken into account, the “net collateral constraint” parameter $\tilde{\varphi}$ is 0.44, i.e. 44% of the real value of entrepreneurial capital is collateralizable. Taking into account of depreciation (i.e. $\tilde{\varphi}/(1-\delta) = 0.52$), this corresponds squarely in the range estimated by Evans and Jovanovic (1989), as well as the calibration in Cagetti and De Nardi (2006).

The implication here then, is that rather than entrepreneurs simply facing tight financial constraints resulting from limited commitment (i.e small $\varphi$), a large portion of the observed “financial friction” can simply be attributed to the illiquid nature of an entrepreneur’s capital (i.e large $\lambda$). Banks and other financial intermediaries recognize that used entrepreneurial
capital have a low resale value, and hence adjust their lending according to this. The implication of this finding is non-trivial. Unlike issues of limited commitment, which might be intrinsically hard to address, the friction arising from illiquid entrepreneurial capital can be addressed by a policy as straightforward as simply having a government policy that purchases an entrepreneur’s capital at a higher price, or supporting leasing markets for capital. These policy options are further explored in section 6.

4.2 Returns to scale

Having chosen to normalize the mean of the entrepreneur’s productivity to 1, the returns to scale becomes the dominant parameter in determining the rate of return to entrepreneurship. In the prior literature, due to a lack of data in identifying this parameter, the returns to scale parameter is often chosen to match moments observed in the literature on firm dynamics (i.e. large, corporate firms), or simply the returns to scale of an entire industry. Unfortunately, entrepreneurial firms are predominantly not large firms, and certainly not whole industries. Consequently, there is a concern that the quantitative results drawn from this older calibration might not be suitable for studying household entrepreneurial investment behavior. Moreover, given the importance of the returns to scale in determining the returns to entrepreneurship, and hence wealth inequality, it becomes even more crucial to pin down this parameter in a disciplined way.

The KFS is therefore a useful source of data, as I could, in theory, directly identify the returns to scale using a production function approach\(^{22}\). Unfortunately, the structure of the KFS precludes the usage of these methods. For instance, the large measurement error associated with the reported capital stock implies that investment, which is constructed from the capital stock, is also contaminated with measurement error. This precludes using regression methods that utilizes investment as an instrument. The KFS also does not report the use of materials or other variable inputs, making methods which utilize variable inputs as proxies or instruments unsuitable as an estimation strategy.

As such, I choose to calibrate \( \nu \) using an indirect inference strategy similar to how the transaction cost parameters are calibrated. Specifically, first note that for the subset of entrepreneurs who are hiring employees, we can rewrite their production function as

\[
\log y = \log A(w) + \Theta_k \log k + \Theta_z \log z
\]

\[
\Theta_k \equiv \frac{\alpha e^{\nu}}{1 - (1 - \alpha_e) \nu}
\]

\(^{22}\)For instance, as in Olley and Pakes (1994), Levinsohn and Petrin (2003) and Ackerberg et al (2006)
In other words, the coefficient on log $k$ is directly influenced by the capital share and returns to scale. As such, $\Theta_k$ becomes a natural target for the returns to scale parameter to hit; specifically, I choose $\nu$ such that the regression performed using simulated data matches the regression done using the actual KFS data. Note that while $\nu$ might appear to be unidentified in this strategy, the capital share itself is “fixed” to the labor share taken from the data: $\alpha_e = (\nu - \hat{\beta}^l)/\nu$. As such, $\nu$ is in fact exactly identified by this moment.

To give the reader a sense of how $\nu$ varies with $\Theta_k$, I refer the reader to table 9 below:

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\Theta_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.68</td>
<td>0.563</td>
</tr>
<tr>
<td>0.69</td>
<td>0.577</td>
</tr>
<tr>
<td>0.70</td>
<td>0.584</td>
</tr>
<tr>
<td>0.71</td>
<td>0.584</td>
</tr>
<tr>
<td>0.72</td>
<td>0.597</td>
</tr>
<tr>
<td>0.73</td>
<td>0.607</td>
</tr>
<tr>
<td>0.74</td>
<td>0.623</td>
</tr>
<tr>
<td>0.75</td>
<td>0.650</td>
</tr>
</tbody>
</table>

Table 9: Effect of $\nu$ on $\Theta_k$

5 Discussion

The illiquid nature of the entrepreneur’s capital generates distortions along the extensive margin (entry into, and exit out of, entrepreneurship) and the intensive margin (investment and disinvestment of capital, as well as portfolio choice). The combined effect leads to a “mis-allocation” of capital and entrepreneurial talent, where in particular, we see a relative increase in the proportion of wealthier but low entrepreneurial talent households operating larger firms. As a consequence, TFP and welfare losses ensue. Interesting, the same distortion leads to lower wealth inequality in the economy, as high productivity entrepreneurs are less able to capitalize on their talent.

Before I discuss in greater detail this dimension of mis-allocation, I first elaborate on the model mechanisms at the individual level, followed by the aggregate effect of illiquidity along the extensive and intensive margins. I will finally return to discussing this dimension of mis-allocation, as well as the implications for the wealth distribution.

5.1 Model mechanism

There are two main channels through which the illiquid nature of capital generate this mis-allocation: An options value effect channel, and a collateral constraint channel.
5.1.1 Options value effect

When capital is more illiquid (an increase in $\zeta$ or $\lambda$), the value of owning capital and the value of running a business falls from the perspective of potential entrants (holding all else constant). This happens because of a rise in illiquidity risk. Potential entrants compare the value of staying a worker, which guarantee a rate of return of $r$ on their asset, against investing in a business where they run the risk of losing $(1 - \lambda)\zeta$ of their investment if they have to exit the next period. When $\lambda$ or $\zeta$ increases, the losses incurred by entrepreneurs become larger upon a bad shock. The value of entrepreneurship falls from the perspective of workers, thus raising the bar for entry into entrepreneurship: Given any wealth level, workers have to receive higher signal shocks ($\psi_z$) to be convinced to switch into entrepreneurship. Consequently, many marginal potential entrants (in terms of productivity signals) now choose to stay as workers. This generates a distortion along the entry margin, where the economy loses out on many potential entrants who should be entrepreneurs, but now choose to be workers instead. This effect plays out in Figure 4, where I plot the entry threshold in $(\psi_z, b)$-space for different values of $\zeta$ and $\lambda^{23}$. The space left of the threshold is the space where workers will choose to enter into entrepreneurship; conversely the space right of the threshold is where workers choose to stay as workers.

![Figure 4](image)

Figure 4: Entry policy in $(\psi_z, a)$ space for different values of $\zeta$ and $\lambda$ (low = solid line; high = dashed line)

As we can see, the threshold shifts right when capital is more illiquid, meaning that

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23 Figures 4 to 8 are drawn such that the net collateral constraint, $\varphi(1 - \lambda)$ is held constant. This avoids the confounding secondary effect where changing $\lambda$ also affects the collateral constraint. This latter effect is differed to the next subsection.
for any given wealth level, the signal draw must be higher for a worker to switch into entrepreneurship.

The options value effect also affect current incumbents both in terms of their choice of investment and disinvestment, as well as their occupational choice. Focusing on the second effect first, notice that higher transaction costs lowers the value of becoming a worker, since exercising that option will involve paying a higher cost. As such, the relative value of staying an entrepreneur increases when capital is more illiquid. This means that the threshold for exiting increases: Given a wealth level, an entrepreneur must receive a better signal if she is to sell her business. This is in fact reflected in figure 5 below, where I plot the exit threshold in \((\psi, b)\)-space for different values of \(\zeta\) and \(\lambda\). The space left of the threshold is the space where entrepreneurs choose to exit; conversely, the space to the right of the threshold is where entrepreneurs will stay in entrepreneurship. In both figures, we see clearly that the exit threshold shifts left when capital becomes more illiquid, meaning that for any capital firm size, the signal required to trigger exit must be higher.

![Figure 5: Exit policy in \((\psi, a)\) space for different values of \(\zeta\) and \(\lambda\) (low = solid line; high = dashed line)](image)

Investment and disinvestment behavior also changes when capital becomes more illiquid. The put option value of capital falls when capital becomes more costly to sell, while the call option value of capital rises. Consequently, incumbents looking to invest will target smaller firm sizes, holding all else constant. In contrast, incumbents who are looking to downsize will operate larger firms. This is reflected in figure 6 below.

![Figure 6: Exit policy in \((\psi, a)\) space for different values of \(\zeta\) and \(\lambda\) (low = solid line; high = dashed line)](image)
Moreover, the illiquid nature of capital means that households are less able to use their physical capital stock as a means to smooth consumption. In the case of a model where bonds and capital are perfect substitutes, the household does not have to worry about losing their investment in the event of a bad shock. In the worst case scenario where \( z \to 0 \), the return to investment is \(-\delta\) (i.e. the user cost of capital). In contrast, with \( \lambda > 0 \) and/or \( \zeta > 0 \), the household also loses more capital due to the transaction costs. As such, for entrepreneurs in an economy where capital is more illiquid, entrepreneurial households will demand more bond holdings to insure themselves against illiquidity risk. Taken together, entrepreneurial households who are expanding their business will, in general, tend to operate smaller firms, and hold more liquid assets, as we can see in figure 7 below.

Figure 6: Next period capital holdings policy for different values of \( \lambda \) (low = solid line; high = dashed line)
This effect doesn’t only affect current incumbents. Workers that are potential entrants, who are not directly affect by $\lambda$, also invest in smaller firms and save more, in anticipation of the illiquidity risk they have to bear upon entry, as we see in figure 8 below.

Figure 7: Entrepreneurs: Incumbent’s investment (left) and savings (right) for different values of $\lambda$ (low = solid line; high = dashed line)

Figure 8: Worker: Entrant’s Investment (left) and savings (right) for different values of $\zeta$ (low = solid line; high = dashed line)

(a) Next period capital holdings vs current capital stock    (b) Next period bond holdings vs current capital stock

(a) Next period capital holdings vs current capital stock    (b) Next period bond holdings vs current capital stock
5.1.2 Collateral constraint effect

In addition to the options value effect, which is the primary distortion to the economy, the illiquid nature of capital also reduces the collateral value of capital. Recall that households can only borrow up to $b' \geq -\varphi(1 - \lambda)(1 - \delta)k'$. Similar to the collateral constraint effect resulting from financial frictions (represented by $\varphi$ in this model), households now need to accumulate more wealth in order to enter into entrepreneurship when $\lambda > 0$. However, unlike financial frictions, this mechanism is not a result of financial contracting issues that $\varphi$ represents. Instead, this financing constraint simply reflects the fact that the value of capital has been lowered by $1 - \lambda$.

What is the difference between $\varphi$ and $\lambda$ with regards to their impact on financial constraints? Note that while both constraints make it harder for households to enter into entrepreneurship, $\varphi$ does not distort the financially unconstrained optimal firm size; the primary channel of $\varphi$ is to force households to operate businesses that are “too small" for extended periods of time while they try to accumulate enough capital. In contrast, $\lambda$ (and $\zeta$) decreases the financially unconstrained optimal firm size, due to the effect of illiquidity risk. That is, even for households who are financially unconstrained, they will choose to operate smaller businesses. For financially constrained households, higher $\lambda$ has a double whammy effect where it makes them even more constraint while simultaneously lowering their business income (since they operate smaller optimal firm sizes).

5.2 Illiquidity, Occupational and Portfolio Choice, and the Allocation of Capital

5.2.1 Extensive margin

Along the extensive margin, the lower resale value of capital leads to a decrease in the value of entrepreneurship for potential incumbents, as discussed earlier. Consequently, the illiquid nature of capital deters entry, and the more illiquid capital is, the lower the entry rate. In contrast, for entrepreneurs, the value of staying in business becomes higher than exiting, thus leading to a deterrence in exit. As such, the more illiquid capital is, the lower the exit rate. Consequently, the amount of reallocation in the form of entry and exit will decrease. This is reflected in figure 9b below, which plots the number of households who are entering entrepreneurship in steady-state, as a function of $\zeta$ (top figure) and $\lambda$ (bottom figure). The values are scaled to the first data point.
Unlike their impact on the entry and exit rates, which is monotone, the impact of capital’s illiquidity on the fraction of households that are entrepreneurs in steady state is ambiguous for two reasons. Firstly, the rate of job switching simply gives us an idea on how active reallocation is being pursued, but does not give us any information on the number of entrepreneurs in steady state. Secondly, since the exit and entry rate both simultaneously decrease, the steady state number of entrepreneurs depends on which of the two forces are stronger along the transition path towards the steady state.

To understand this point, suppose an economy starts with $\zeta = 0$ at time 0 (call this $\zeta_0$). The economy then receives a shock, where $\zeta$ increases, such that $\zeta > 0$. We can then consider the perfect foresight partial equilibrium path, where the economy converges to the new steady state.

From the earlier comparative statics exercise, we already know that this increase in $\zeta$ must lead to a decrease in the entry and exit rate. The resulting steady state number of entrepreneurs then depend on the speed at which the two rates adjust to the new steady state. If the exit rate falls at a slower rate than the entry rate, the number of entrepreneurs will decline along the transition path towards the new steady state; on the other hand, if the exit rate falls at a faster rate than the entry rate, the number of entrepreneurs will rise as the economy transitions to the new steady state. An example of this process is given below in figure 10, where I consider an economy that starts in steady state at $\zeta = 0$, and transitions to $\zeta = 0.75$, the model calibration. In this case, we see that the exit rate
falls much slower than the entry rate over the transition path. As a result, the number of entrepreneurs fall as $\zeta$ increases. In fact, for the benchmark calibration, the exit rate always falls slower than the entry rate, and as a consequence, the number of entrepreneurs always falls in steady state, as reflected in figure 9a (all values are scaled to their values at steady state).

![Path of $\zeta$](image)

**Figure 10:** Response of number of entrepreneurs, and entry and exit rates, to $\zeta$ shock

### 5.2.2 Intensive margin

Along the intensive margin, the asymmetric purchase and resale price of capital leads incumbents to become more cautious in investing and dis-investing. Entrepreneurs who receive a good productivity shock will invest less than the unconstrained optimum, as they want to avoid being trapped with too much capital in the event of a downturn in the following periods. In contrast, entrepreneurs who receive a bad shock will disinvest less than the unconstrained optimum. In this case, entrepreneurs are hedging against the event of an upturn in the following periods. Since newly purchased capital is more expensive than used capital, entrepreneurs would like to reduce their expenditure in the case of an upturn. This leads poorly performing entrepreneurs to hold on larger capital stocks than is optimal. Taken
together, this effect leads high productivity entrepreneurs to operate sub-optimally small firms, and low productivity entrepreneurs to operate sub-optimally large firms.

In addition, the illiquid nature of capital means that households are less able to use their physical capital stock as a means to smooth consumption. In the case of transitory bad shocks, entrepreneurs are unwilling to disinvest much of their capital due to the effect discussed earlier. Consequently, physical capital serves as a poor instrument to insure households. Holding all else constant, this drives households to accumulate relatively more bonds, thus leading to a portfolio with greater liquid assets.

In fact, taken together, this effect looks very similar to a financial constraint, such as that in Cagetti and De Nardi (2006). (High productivity) entrepreneurs operate firms that are “too small”, and tend to accumulate more bonds, as reflected in figures 7 and 8. However, unlike the literature on financial constraints, this effect is through a different channel. Where the small firm size in the Cagetti and De Nardi (2006) framework is an indication that the entrepreneur would like to expand if she could, the smaller firm sizes here is also a result of households simply having no desire to expand to the unconstrained optimum. Likewise, where wealth accumulation in the face of financial frictions results from households trying to save out of their constraints, higher (liquid) wealth accumulation here is simply a result of households trying to self-insure against illiquidity risk.

This suggests that we should see an increase in aggregate liquid savings in the economy, along with a decrease in the aggregate holdings of physical capital. However, this outcome is in fact not a given, as we can see in figure 11. While the marginal propensity to save at the individual level does increase when capital becomes more illiquid, it also means that highly productive entrepreneurs are operating smaller firms on average (see bottom left panels of figures 11a and 11b). This in turn means that the same entrepreneurs are making lower revenues, hence leading to lower overall income. Consequently, while propensity to save increases, the volume of savings, which depends on their income, does not, as reflected in the top right panels of figures 11a and 11b). Moreover, this also mean that the total wealth held by entrepreneurs fall when capital becomes more illiquid.
Figure 11: Effect of $\zeta$ and $\lambda$ on the wealth and portfolios of entrepreneurs

While the total bond holdings do fall, the greater propensity to save translates into a greater preference for liquid assets over illiquid assets. As such, the aggregate portfolio composition of assets will see a larger portion of assets coming from liquid assets.

In contrast, financial constraints such as collateral constraints predict a different outcome in terms of the household’s portfolio mix. Tightening financial constraints force constrained households to operate sub-optimally small firms; consequently, entrepreneurial income also falls, leading to lower wealth and bond holdings. In general, the effect of tightening collateral constraints on total wealth, bonds and capital are therefore similar qualitatively to increasing illiquidity, as reflected in the first three panels of figure 12.
Figure 12: Effect of $\varphi$ on the wealth and portfolios of entrepreneurs. Values are plotted against $1 - \varphi$, i.e. tightening collateral constraint.

However, recall that as one tightens the collateral constraint, the unconstrained optimal firm size does not change, unlike the case when capital becomes more illiquid. Moreover, since the transaction costs associated with capital stays constant, there is no increase in illiquidity risk. As a result, entrepreneur simply invest more of their liquid assets into their firms when the collateral constraint tightens. Unlike in the case of $\lambda$ or $\zeta$, there is no flight to liquidity - in fact, the portfolio of the entrepreneurs becomes more illiquid, as reflected in the last panel of figure 12.

The results here suggest that financing constraint shocks can have, qualitatively, much of the same impact as illiquidity shocks (i.e. lower investment, savings, startup rates). However, there is a key distinguishing feature of these two frictions: While tightening collateral constraints will force entrepreneurs to hold more illiquid assets in their portfolio, increasingly illiquid capital will force entrepreneurs to hold less illiquid assets in their portfolios. This finding is important, because the policy response to a financing constraint is very different to one where the value of capital has fallen (for instance, if one is to interpret $1 - \lambda$ and $1 - \zeta$ as resale prices of used capital). For instance, recent research studying the used capital
market have shown that the resale price of used capital can be pro-cyclical. As such, this suggests that a portion of the decline in entrepreneurial startup rate over the course of a recession, as well as investment, can potentially be attributed to a natural response to the decline in the resale price of capital. Findings that attribute the decline in entrepreneurial startup or investment solely to financing constraints could therefore be over-estimating the extent to which financial constraints have tightened. Moreover, simply looking at aggregates such as the startup rate, exit rate or investment is insufficient to help researchers distinguish between the two sources of distortion. Instead, the results here suggest that looking at the evolution of the economy’s portfolio of assets might be crucial.

Another important point relates to recent research that have attempted to estimate the existence and impact of financing constraints on entrepreneurial startup and investment, using exogenous regional variations in financing conditions as an identifying instrument. A potential concern that this paper raises is that, given that the resale price of capital is also highly correlated with exogenous financial conditions, the resulting estimation might suffer from omitted variable bias.

5.3 The wealth distribution

Table 10 below reports the wealth distribution in the US, and contrasts the model predicted wealth distribution when the calibrated is done to only a single sector (Professional, Technical and Scientific "NAICS 54"), and the entire KFS sample ("Full"). The model predicted wealth distribution includes the benchmark calibration, as well as the predicted wealth distribution under a model with no resale frictions ($\lambda = \zeta = 0$). For the model with no resale frictions, I reset the collateral constraint parameter to $\tilde{\phi}$, such that $\tilde{\phi} = (1 - \lambda)\phi$, thus keeping the net collateral constraint constant. The no resale friction model is solved under partial equilibrium, where I keep the interest rate $r$ at the benchmark calibration, as well as under general equilibrium. It also reports the Gini coefficient as a single summary statistic for the extent of wealth inequality predicted by the model.
Table 10: Wealth distribution

First, referring to rows (1) and (4), one notices right away that the model with entrepreneurs, regardless of calibration target (i.e. either full sample or subset), is strictly closer to the empirical wealth distribution than a plain vanilla model without entrepreneurs (row (7)). This observation has been made by the preceding literature, and is replicated here. Just as Benhabib et al (2015) note in their paper, the uninsurable, persistent and
highly volatile returns to capital risk leads households to accumulate more wealth, thus extending the right tail of the wealth distribution. Entrepreneurial returns, which is a form of capital income risk, bears this feature, and thus leads to a model economy that features a more unequal distribution of wealth.

However, the wealth distribution in both calibrations ("NAICS 54" and "Full") are in fact much more egalitarian than the actual US wealth distribution. For instance, the top 1% in "NAICS 54" only hold 17% of the economy’s wealth. The top 1% in the "Full" calibration hold about 26% of the economy’s wealth, which is substantially closer to the true distribution (about 30%), but still not exactly the same. Likewise, if we look at the Gini coefficient as a measure of dispersion, both model economies feature lower Gini coefficients than the true economy.

To understand this, we have to first understand what constitutes capital income risk in a model of entrepreneurship. Recall that gross capital income in a model of entrepreneurship is simply production income. For a simplified model without labor, gross capital income is then simply $y = zk^{\nu}$. In a static one asset model without any frictions, we know that optimal choice of $k$ is simply $(\frac{z}{r+\delta})^{\frac{1}{1-\nu}}$, so gross capital income is just $z^{1+\frac{\nu}{r+\delta}}(\frac{1}{r+\delta})^{\frac{\nu}{r+\delta}}$. The dispersion and persistence in (log) capital income is then:

$$
var(\log(y_t)) = var(\log(z^{1+\frac{\nu}{r+\delta}})) = \left(1 + \frac{\nu}{1-\nu}\right)^2 var(\log(z))
$$

$$
cov(\log(y_t), \log(y_{t-1})) = \left(1 + \frac{\nu}{1-\nu}\right)^2 cov(\log(z_t), \log(z_{t-1}))
$$

where we see right away that (i) the volatility of capital income depends on both the volatility of TFP as well as the returns to scale, and (ii) the persistence of capital income depends only on the persistency of TFP, but it’s covariance depends on both the persistency of TFP and returns to scale. Crucially, $\nu$ inflates the volatility and covariance of capital income at an exponential rate. For instance, consider a TFP process with a small unconditional variance of 0.0335 and a returns to scale of 0.88$^{24}$: The resulting volatility of capital income is inflated to almost 1.34, a nearly 40 times increase over the underlying productivity process. On one level, this is in fact an important reason why models of entrepreneurship have been so successful in matching the wealth distribution: For even small values of underlying productivity risk, a positive returns to scale can greatly inflate the dispersion of capital income, thus also increasing the dispersion in wealth.

$^{24}$these numbers in fact correspond to the calibration chosen by Cagetti and De Nardi (2006)
As an example of the importance $\nu$ plays in determining the dispersion of wealth, I report in figure 13 below a comparative statics exercise that reports the Gini coefficient for every $\nu \in (0.68, 0.85)$. Solid lines refer to the calibration done under the “FULL” calibration, and dashed lines refer to the “NAICS54” calibration. The vertical lines are reference lines for the calibrated $\nu$ (solid for “FULL”, dashed for “NAICS54”) Notice that the returns to scale for the benchmark calibration (both "NAICS54" and "FULL"), are too small (relative to their respective calibrations) to generate the high empirical wealth inequality.

![Figure 13: Effect of $\nu$ on wealth inequality.](image)

5.3.1 Wealth inequality and illiquidity of assets

However, as important as the returns to scale is in determining the ability of the model to match the wealth distribution, it is not the only factor in this model that affects the returns to investment. Instead, the extent to which capital is illiquid can also affect the returns to capital. Indeed, if one looks back to table 10, one sees that a full removal of all illiquidity frictions (rows (2), (3), (5) and (6)) all lead to an increase in the extent of wealth inequality. Moreover, when the interest rate is not allowed to adjust, the model wealth distribution
almost approaches that of the empirical wealth distribution (rows (2) and (5)). This result is reflected in figure 14 below, where we see that the Gini coefficient for the whole economy (solid line) increases as capital becomes more liquid (i.e. as $\lambda$ or $\zeta$ decreases).

![Figure 14: Gini coefficient for whole population (solid line) and entrepreneurs only (dashed line), against $\lambda$ and $\zeta$](image)

How does capital illiquidity affect returns to investment?

Firstly, the transaction costs directly impact investment returns by simply increasing the user cost of capital. Since every entrepreneur has a non-zero probability of exit, this implies that the user cost of capital is larger than the depreciation rate. Consequently, the net return to investment decreases.

Secondly, recall that in section 5.1.1, I showed that the higher transaction costs lead entrepreneur to operate smaller than optimal firm sizes (if they are investing); moreover, they devote a larger portion of their portfolio to buying bonds. This diminishes the ability of the entrepreneur to accumulate wealth. Smaller firm sizes mean that the entrepreneur is making less business income than she could have; while a larger liquid asset stock means that the entrepreneur could strictly be better off moving some bonds into his firm. The gross return on asset, which is $zf(k, l) + rb$, is thus lower.

Finally, section 5.1.1 also showed that many entrepreneurs are operating larger than optimal firm sizes. Just as when firms are too small, firm sizes that are too larger also reduce the gross return on asset. In this case, the entrepreneur could be strictly better off transferring her firm’s assets into bonds.

The combined effect leads entrepreneurs to earn lower lifetime incomes when capital is
more illiquid, due to the depressed average returns to capital. As a result, the overall income
distribution becomes more compressed, and overall wealth dispersion also falls.

5.3.2 Frictions and the distribution of wealth and capital

As discussed earlier, the illiquid nature of capital leads entrepreneurs to accumulate (rela-
tively) more liquid assets as a buffer stock of savings. Moreover, this also drives low produc-
tivity entrepreneurs to operate larger-than-optimal businesses due to the options value effect.
As a result, entrepreneurs who are wealthy also tend to operate larger firms that are less
productive. Consequently, in a distributional sense, we will see that wealthier entrepreneurs
are also operating firms that have lower average revenue product of capital.

This result is reflected in figure 15 below, which plots the bivariate kernel density estimate
(i.e. joint distribution) of ARPK and wealth (defined as sum of total assets) for two different
values of $\lambda$ (figure 15a) and $\zeta$ (figure 15b). The plots are contour plots, with each continuous
line representing equal densities. In general, we see very clearly that for economies with
higher illiquidity, there is a much higher density of the entrepreneur population that is
wealthy and has low ARPK.

Figure 15: Joint distribution of ARPK and wealth for different $\zeta$ and $\lambda$. The two solid lines
are reference lines.

Another way to view this effect is to directly consider the correlation between wealth and
ARPK. For instance, we could fit a regression of the form

$$\log ARPK = \beta_0 + \beta_1 \log(b)$$

(1)

, where $b$ is the total bond holdings (including debt), and $\beta_1$ would reflect the strength of the correlation between bond holdings and ARPK. This is reflected in figure 16, where we see as the extent of illiquidity increases in the economy (i.e. $\lambda$ or $\zeta$ increases), wealth becomes increasingly negatively correlated with ARPK (and significantly so).

![Figure 16: Effect on $\beta_1$ for different $\lambda$ and $\zeta$. The dashed lines are 95\% confidence intervals.](image)

How does this contrast with an economy where the frictions are driven by changes borrowing conditions? When capital mis-allocation is driven by borrowing constraints, only low wealth entrepreneurs and potential entrants are affected. This happens because these entrepreneurs are unable to raise enough funds to target the optimal firm size, or in the case of potential entrants, operate at a size that is sufficiently profitable to warrant entry. In contrast, wealthy entrepreneurs and new entrants are completely unaffected by the borrowing constraints, and would see no correlation between their wealth and their firm’s average revenue product of capital. As a result, when we consider the entire population of entrepreneurs, we should see that tightening collateral constraints would increase the correlation between wealth and ARPK. In contrast, when we consider only the sub-population of wealthy entrepreneurs, we should see no change in correlation between wealth and ARPK when collateral constraints change.
In fact, this is exactly what we see run the same regression (1) as above for different values of $\varphi$, the collateral constraint parameter. In figure 17a, we see that when we run the regression above for the whole population, wealth becomes less negatively autocorrelated with ARPK. In contrast, when we focus only on entrepreneurs with savings (i.e. $b > 0$), the correlation between wealth and ARPK does not change (in a statistical sense).

5.4 Aggregate welfare and productivity

In this section, I discuss the effect of the distortions on two key aggregate outcomes: Welfare and Productivity. Prior to beginning this section, I first provide some definitions to give the reader a consistent reference to the terms I am referring to.

Welfare

Welfare here is measured as the consumption equivalent variation between a reference economy (given by a superscript “R”) and the current economy of interest (given by a superscript “N”). In particular, for any household $i$, the consumption equivalent variation $\mu_i$ solves the following problem:

$$V((1 + \mu_i)c^R_i) = V(c^N)$$

25 the correlation is still negative, as the comparative statics exercises are done with all other parameters held at their benchmark calibration, i.e. in particular $\lambda > 0$ and $\zeta > 0$
To compute $\mu_i$ for each household, I then simply solve the preceding equation for each point in the state space $\mathbb{S}$. Given this definition, $\mu_i > 0$ implies that the household prefers the new economy over the reference economy.

Having derive the entire distribution of $\mu_i$, I then compute aggregate (average) welfare as

$$\hat{\mu} = \int \mu_i d\Lambda$$

Moreover, I am also able to compute welfare changes for subsets of the economy $\Lambda_s \subseteq \Lambda$. For instance, if I wish to compute the average welfare change for only workers, I can compute

$$\hat{\mu}^w = \frac{\int \mu_i \times \mathbb{1}_w d\Lambda}{\int \mathbb{1}_w d\Lambda}$$

where $\mathbb{1}_w$ is an indicator function that equates to 1 if that household is a worker, and 0 otherwise. The denominator is necessary in order to re-normalize the distribution (i.e. compute the conditional average), in order to avoid conflating the changes in the measure of workers (which is endogenous) and actual changes in the agent’s welfare (or distribution of CEV).

**Aggregate TFP**

“Aggregate TFP” here refers specifically to the aggregate TFP in the entrepreneur sector. Since the corporate sector is composed of representative firms, there cannot be any TFP losses with respect to that sector, and hence I choose not to discuss it here\(^{26}\). In contrast, we are more interested in how distortions affect the aggregate TFP of the entrepreneurial sector.

Here, my concept of TFP stems from the perspective of a statistician who only observes the aggregate capital stock, labor input, and output of the entrepreneur sector. In this case, we can derive TFP $Z$ as

$$Z \equiv \frac{Y^e}{K^e_\alpha^\nu L^{(1-\alpha_e)^\nu}}$$

**Average productivity of entrepreneur**

As the model exhibits endogenous entry and exit, the steady-state distribution of observed

\(^{26}\)This does not mean that the corporate sector is not important in this model. In fact, the amount of (effective) capital allocated to the corporate sector reflects the change in the precautionary savings behavior of the households, which is an important dimension of this model relative to models of complete markets.
idiosyncratic TFP is different from the underlying distribution of shocks. The selection effects can either increase (positive selection) or decrease (negative selection) the average productivity of entrepreneurs, and in turn, can have powerful effects on aggregate TFP. For this paper, I define the average productivity of an entrepreneur as

\[
\bar{Z} = \frac{\int z_i \times \mathbb{1}_e d\Lambda}{\int \mathbb{1}_e d\Lambda}
\]

(6)

where \( \mathbb{1}_e \) is an indicator function that equates to 1 if that household is an entrepreneur, and 0 otherwise.

Having clarified these definitions, we can now examine the impact of these distortions on the aggregate economy.

### 5.4.1 Welfare and productivity

Table 11 below reports the aggregate TFP, average productivity of entrepreneurs, and the average welfare change for the two asset economy, as well as the counterfactual economy with no resale frictions. Both the PE and GE results are reported for the counterfactual example. The results computed are steady-state values.

<table>
<thead>
<tr>
<th></th>
<th>Two assets, GE</th>
<th>No Frictions, fixed r</th>
<th>No Frictions, GE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.876</td>
<td>1.2</td>
<td>1.08</td>
</tr>
<tr>
<td>Average productivity, entrepreneur</td>
<td>1.14</td>
<td>1.25</td>
<td>1.24</td>
</tr>
<tr>
<td>average welfare (entrepreneur)</td>
<td>0</td>
<td>0.2</td>
<td>0.166</td>
</tr>
<tr>
<td>average welfare (worker)</td>
<td>0</td>
<td>0.583</td>
<td>0.688</td>
</tr>
<tr>
<td>average welfare (economy wide)</td>
<td>0</td>
<td>0.248</td>
<td>0.231</td>
</tr>
</tbody>
</table>

Table 11: Changes in TFP, average productivity of entrepreneur, and welfare, for one and two asset economy

As we can see, there are substantial losses to TFP and welfare, regardless of whether we take into account of equilibrium effects or not (even though the GE effects do ameliorate the losses). This happens exactly because of the distortionary effects discussed in the earlier section. Along the external margin, there is a selection towards low skilled entrepreneurs due to the wait-and-see effect induced by the adjustment costs. In fact, this is seen mostly starkly in figure 18 below. In both figures, I plot the distribution of entrepreneurs productivity.
under the benchmark calibration, and under the counterfactual calibration. The left figure plots the actual density, while the right plots the normalized density (i.e. normalized to the endogenous measure of entrepreneurs). In addition, I also plot the true underlying distribution of $z$ for figure 18b. The key takeaway is that (1) we see from figure 18a that the number of entrepreneurs is greatly reduced in the high friction world, with most of the selection being against the moderate-to-high productivity entrepreneurs; and (2) that even after renormalizing the distribution, there is still evident selection against moderate-to-high productivity entrepreneurs. In fact, the second row of table 11 reports this latter number (i.e. on average, the entrepreneur in the no friction world is 8.8% more productive than the benchmark entrepreneur).

![Figure 18: Effect of illiquidity frictions on the distribution of $z$.](image)

Distortions along the external margin are also not the only cause of this. Along the internal margin, we also see a substantial mis-allocation of wealth. Recall that the illiquidity frictions force entrepreneurs to run sub-optimally large firms if they are hit by bad productivity shocks, while they run sub-optimally small firms if they are looking to expand. In fact, the former result, which dominates in equilibrium, is exactly what drives the left skewness of ARPK in this model, as discussed in the earlier section.

As a result, we should see that low wealth entrepreneurs should be particularly productive, while high wealth entrepreneurs would be much less productive. This is in fact exactly the case, as we see in figure 19 below. Here, I plot bar charts of the average productivity of entrepreneurs in each (5%) quantile of the wealth distribution, for the benchmark calibration, as well as the PE and GE versions of the counterfactual example. In general, we see
exactly that poor entrepreneurs in the bottom quantiles are largely much more productive in the benchmark model than the counterfactual examples, but much less productive as they get richer.

Figure 19: Average productivity of entrepreneurs for each 5% quantile

5.4.2 Decomposing welfare losses: Incomplete markets and misallocation

Given the large welfare losses, it is important to consider how much of the loss in welfare can be attributed to solely to the incompleteness of the market, and how much is due to the market distortions introduced by the frictions. In particular, understanding this would allow us to appreciate the importance of modeling entrepreneurial investment in this two asset framework.

To do this, one would need to create a complete market baseline of this model for direct comparison; unfortunately, the numerous non-convex adjustment cost in this model means that exact aggregation is not possible, and as such, a social planning problem cannot be directly obtained.

Instead of doing a direct comparison, I therefore follow Beura and Shin (2011) doing
a two step analysis. Firstly, I compute the steady-state welfare of an equivalent worker / entrepreneur economy without any market distortions (i.e no partial irreversibility, fixed costs, financing constraints etc). Following this, I compute the welfare loss of market incompleteness by computing the welfare loss going from this economy to the economy with no frictions\(^{27}\), as well as the welfare loss of market incompleteness AND distortions by computing the welfare loss going from the complete markets baseline to the benchmark economy. The difference in welfare losses can then be interpreted as the welfare loss due to distortions.

What is the complete markets baseline for this economy? In this economy, one of the key aspects of this model is that each agent is endowed with a pair of productivities in steady-state: \(\theta\) and \(z\). As such, we can formulate a social planning problem, where the job of the social planner is to efficiently allocate agents according to their talents to an occupational choice, as well as to efficiently allocate physical resources (i.e. capital and labor) to the entrepreneurial and corporate sectors. Denoting each agent by \(i\) and the distribution of agents by \(\Lambda\) (and the distribution of productivities \(\theta\) and \(z\) as \(f_\theta\) and \(f_z\) respectively), this amounts to solving the following problem:

\[
V(\{k_i, h_i\}, K_c; \Lambda, f_\theta, f_z) = \max_{\{k_i', h_i'\}, K'_c} U(C') + \beta V(\{k_i', h_i'\}, K'_c, f_\theta, f_z)
\]

s.t.

\[
C + \int k_i'd\Lambda + K'_c = \int y_i^e d\Lambda + Y^c + (1 - \delta_k) \int k_i d\Lambda) + (1 - \delta_c K_c)
\]

\[
y_i^e = z_i k_{i,e}^{\alpha_k} l_{i,e}^{1-\alpha_k}
\]

\[
Y^c = K_c^{\alpha_c} L_c^{1-\alpha_c}
\]

\[
Y = Y^c + \int y_i^e d\Lambda
\]

\[
Le = \int l_e \times \mathbb{I}(h_i = 0) d\Lambda
\]

\[
L_c + Le = \int \theta_i \times \mathbb{I}(h_i = 1) d\Lambda
\]

where \(h_i\) is the occupation of agent \(i\), with \(h_i = 1\) representing a worker (and 0 entrepreneur); \(k_i\) representing the capital allocated by the social planner to agent \(i\); \(\{k_i, h_i\}\) therefore representing the distribution of occupational types and entrepreneur capital; \(K_c\)

\(^{27}\)as in the economy reported in the last column of table 11. It is important to note that this economy still suffers from financial frictions, and so is not necessarily a “frictionless” economy per se. However, the object of interest here is to understand how the irreversibility plays a role in affecting welfare. For more context on how financial frictions affect welfare, I refer the reader to Beura and Shin (2011)
representing aggregate capital stock allocated to the corporate sector; \( L_c \) and \( L_e \) representing aggregate labor demand from the corporate and entrepreneurial sectors; \( \int \theta_i \times \mathbb{I}(h_i = 1)d\Lambda \) therefore simply evaluating to the total labor supply in the economy; and finally, \( Y \) is simply GDP. In short, the problem of the social planner is to solve the trade-off between allocating more agents to working as entrepreneurs, which would increase the total productivity in the entrepreneurial sector, but at a cost of lowering the number of workers available to provide labor to the two sectors.

Note that this problem, as written, is essentially intractable, since I have to solve for the joint distribution of \( k_i \) and \( h_i \). However, all hope is not lost. There are two important things to note here. Firstly, given any distribution of occupational types (and their associated productivities), the capital and labor allocation choice amounts to solving a pair of equations in the individual “firm’s” problem (i.e. where the marginal product of capital equates the user cost of capital, and the marginal product of labor equates the implied “wage” coming from the corporate sector’s first order condition). Essentially, we are taking advantage of the fact that in the first best allocation, the marginal products of capital must equate across all production units, and also equal to the corporate sector’s marginal product of capital (and likewise for labor). More importantly, the individual production unit does not need to (directly) care about the actual distribution. Instead, when allocated some productivity \( z \), the individual production unit simply picks \( k \) and \( l \) satisfying\(^28\)

\[
\left( \frac{\alpha_k \nu z}{r + \delta_k} \right)^{\frac{1}{1-\alpha_k
\nu}} \times \frac{(1-\alpha_k)^\nu}{\sigma_k^\nu} = k_e \\
\left( \frac{(1 - \alpha_k)\nu z}{w} \right)^{\frac{1}{\alpha_k^\nu}} \times \frac{1-(1-\alpha_k)^\nu}{\alpha_k^\nu} = k_e \\
\int l_e \times \mathbb{I}(h_i = 0)d\Lambda \leq \int \theta_i \times \mathbb{I}(h_i = 1)d\Lambda
\]

If the last equation holds with equality, this implies that no labor is allocated to the corporate sector \( (L_c = 0) \). Moreover, to ensure that marginal products equate across production units, the individual labor demand now satisfies

\[
\int l_e \times \mathbb{I}(h_i = 0)d\Lambda = \omega \tilde{L}_e \\
\omega \leq 1
\]

\(^28\)again, \( r \) and \( w \) here are simply implicitly defined by the corporate firm’s first order conditions
where \( \bar{L}_e \) is the unconstrained labor hiring. In turn, \( k_e \) is scaled with the appropriate \( \omega \):

\[
k_e = \omega \frac{1-(1-\alpha_k)^\nu}{\alpha_k^\nu} \tilde{k}_e
\]

where \( \tilde{k}_e \) is the choice of capital under the unconstrained labor choice.

Now given the set of optimal capital and labor allocation, the social planner only needs to solve for the allocation of talent. Recall that \( \theta \) and \( z \) are independent continuous distributions that are stationary over time. As such, instead of solving for the entire distribution of capital and talent allocations, we can simply for a threshold function of the follow:

\[
h(z, \theta) = \begin{cases} 
1, & z < \bar{z}(\theta) \\
0, & z \geq \bar{z}(\theta)
\end{cases}
\]

Simply put, for any given \( \theta \), assign the agent to a labor occupation if her \( z \) is low; otherwise, assign her to an entrepreneurial occupation.

The earlier problem can now be written as

\[
V(\{k_i, h_i\}, K_c, \Lambda, f_\theta, f_z) = \max_{\bar{z}(\theta), K'_c} U(C) + +\beta V(\{k'_i, h'_i\}, K'_c; f_\theta, f_z)
\]

\[
s.t.
C + \int k'_i d\Lambda + K'_c = \int y_i^e d\Lambda + Y^c + (1 - \delta_k) \int k_i d\Lambda) + (1 - \delta_c K_c)
\]

\[
y_i^e = z_i \tilde{k}_e^{\alpha_k^\nu} L_1^{1-\alpha_k^\nu}
\]

\[
Y^e = \int \mathbb{E}[y_i^e(z, k, l) | z > \bar{z}(\theta)] f_\theta d\theta
\]

\[
Y^c = K_c^{\alpha_c} L_c^{1-\alpha_c}
\]

\[
Y = Y^c + Y^e
\]

\[
Le = \int \mathbb{E}[\theta | z < \bar{z}(\theta)] f_\theta d\theta
\]

\[
L_c + Le = \int \theta_i \times I(h_i = 1) d\Lambda
\]

with individual production units choosing capital and labor satisfying the earlier constraints. In this formulation, the problem is now numerically tractable once we exploit the
monotonicity property of $\bar{z}(\theta)^{29}$.

To compute relevant comparisons, I then compute the steady-state welfare of the complete markets economy. Letting $C^{ss}$ denote the steady-state consumption in this economy, the steady-state welfare is simply

$$V_{ss} = (\frac{1}{1-\beta})U(C^{ss})$$

As we can see from table 12, which reports the decomposed welfare losses, a substantial amount of the welfare loss simply comes from the incompleteness of the markets. Essentially, households are willing to forgo, on average, about 58% of their lifetime consumption to obtain full insurance. When taking into account of the full impact of the illiquidity frictions, the total welfare loss (relative to complete markets) is almost 65% of lifetime consumption; that is to say, the frictions alone accounts for about 11% of the welfare loss. This is clearly a non-trivial amount.

<table>
<thead>
<tr>
<th>Welfare losses, in %</th>
<th>Due to incomplete markets</th>
<th>Full impact</th>
<th>Implied losses from friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>-57.7</td>
<td>-64.8</td>
<td>-7.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 12: Welfare losses, decomposed in incomplete markets effect and effect from reallocation frictions.

5.5 Entrepreneurship and wealth inequality?

For this final subsection, I discuss the findings in this paper in the context of two concurrent and relevant strands of literature regarding entrepreneurship and uninsurable capital income risk.

5.5.1 Entrepreneurship and the rates of return literature

This paper suggests that, when a model is calibrated to the entrepreneur’s investment dynamics, this model is unlikely to generate wealth concentrations at the level observed in the real economy. This result in fact mirrors the empirical findings in Aghion et al (2016), where they find that entrepreneurship increases top income inequality, but has little impact on broader measures of inequality.

However, this does not necessarily imply that entrepreneurship, or more specifically, business ownership, is not a good explanation for the high wealth dispersion and skewness.

---

29To make the comparisons between this economy and the benchmark model, I solve for $\bar{z}(\theta)$ using the same grid as the benchmark model.
As suggested by the existing empirical wealth literature, rates of return heterogeneity is the strongest driver of wealth inequality, of which entrepreneurship and business ownership is the most direct micro-foundation. The key issue here is that the broader class of entrepreneurs, as captured by the KFS sample, has on average, returns to entrepreneurship that are too low to provide substantial power in matching the empirical wealth distribution. Due to both sample and computational limitations, this paper is unable to simultaneously capture other dimensions of heterogeneity (such as returns to scale, ability to issue equity) that are probably equally important in capturing the lifetime income process of superstar entrepreneurs. Instead, I focused solely on understanding how the average entrepreneur’s investment choices relate to the rates of return to wealth, and find that illiquidity has an economically significant impact on the allocation of capital amongst entrepreneurs, and overall welfare in the economy.

5.5.2 Entrepreneurship and the capital income taxation and redistribution literature

A large literature uses this framework of entrepreneurial risk as a way to study capital income taxation, its impact on capital accumulation, and whether it is effective as a form of wealth redistribution (in a welfare sense). This paper does not directly add to this literature, as I do not explicitly study the optimal capital income tax problem. In contrast, the key contribution of this paper is to sound a cautionary tale to researchers who wish to use this framework of entrepreneurship risk to study capital income taxation.

What do I mean? As I showed, a model of entrepreneurship, when built from the ground up using micro-data, generates wealth inequality that is far lower than that in the actual economy. This is, in fact, not a surprising result. The model entrepreneur, framed as an individual exposed to uninsurable investment risk, has more in the common with the broad definitions of small scale entrepreneurship rather than high growth entrepreneurship. The former group is exactly what the sample in the KFS captures, and exactly what my model matches.

In contrast, high growth entrepreneurship is a more likely suspect for generating high wealth inequality. The typical concern of issues regarding capital taxation is whether capital taxation distorts the investment decision of high growth entrepreneurs, leading to suboptimal outcomes. However, high growth entrepreneurs are also more likely to eventually diversify away their risk by issuing equity (such as taking the firm public or raising funds through venture capital equity injection). As such, these individuals do not correspond very well to this framework of entrepreneurship.

However, this is not to say that this framework is ineffective for studying capital income
taxation. In fact, small scale entrepreneurship forms the bulk of business ownership, with around 99.7% of firms classified as “small firms”. Clearly, 99.7% is a non-trivial number, and this framework is very useful if one wishes to study the impact of capital taxation on the vast majority of business owners. In the next section, I will detail two policy experiments that will exemplify the usefulness of this framework in understanding taxation and government intervention.

6 Policy implications

The majority of macroeconomic models that study the effect of fiscal policy in an entrepreneurship framework generally abstract away from adjustment frictions modeled in this paper. The key aim of this section is to document that ignoring these frictions can lead to very misleading results regarding the efficacy of certain fiscal policies. Moreover, ignoring these frictions can also lead us to neglect certain policies that can be relatively straightforward to implement. To provide an exposition into this, I consider three policy experiments that are financed by the same government budget: In section 6.1, I study a policy of credit expansion, where the government tries to lower the cost of borrowing; in section 6.2, I study a policy of increasing the resale value of capital by decreasing the transaction cost of selling capital (i.e. lowering $\lambda$); finally, in section 6.3, I study a policy of reducing the cost of exit (i.e. reducing $\zeta$).

Why do I choose to study these three policies?

In the case of a credit expansion policy, this is in fact a common policy tool used by the government to spur entrepreneurial startup, or help small businesses expand. For instance, the popular 7(a) and CDC/504 lending programs of the Small Business Administration corresponds to policies that aim to reduce the spread between the savings interest rate and the borrowing interest rate. With respect to this model, this would correspond to a policy that commits a fixed budget to decreasing the spread $\phi_d$. Given the large extant literature that finds severe financial constraints for entrepreneurs, as well as the literature documenting the costs of financial distortions, a good case can indeed be made for such credit provision.

However, one aspect of entrepreneurship that prior models and research have neglected is the distortion that illiquidity frictions (as a result of partial irreversibility) introduces to entrepreneurial entry, exit and investment. This paper finds that the distortions arising from these frictions can be large. As such, this naturally brings to mind a simple policy: Have the government act as a buyer of last resort. Here, the government offers a higher resale price $q \geq 1 - \lambda$ for capital, where $q$ is the endogenous resale price offered by the government. It is assumed that the government buys all used capital at $q$, and resells it at the market price.
1 – \( \lambda \), where \( \lambda \) is the partial irreversibility parameter from before. In section 6.2, I explore the efficacy of this policy. In section 6.3, I explore a very similar policy, where instead of supporting a market for all used capital, the government only increases the resale price of capital that is off-loaded by exiting entrepreneurs. While \( \lambda \) distorts both the internal and external margins in terms of allocation of capital and talent, \( \zeta \) primarily distorts the external margin. As such, this makes comparing the two economies useful in considering how much of the distortion within the internal margin is alleviated by the first (i.e. decreasing \( \lambda \)) policy.

An important point to note is that I assume that the frictions in place are real, corresponding to a “technological” friction that cannot be simply bypassed by the government. As such, the government always has to levy some taxes to fund its activities. As a parallel to the literature studying capital income taxation, I assume here that the government can only raise income through taxing returns on savings (bonds). In a sense, this intuitively corresponds to a policy where the government is trying to address some inherent asymmetry between the corporate sector and the entrepreneurial sector, by taxing the corporate sector and transferring surplus to the entrepreneurial sector.

### 6.1 A credit fueled “expansion”

In this example, the government addresses the financial friction introduced by the intermediation cost \( \phi_d \), by helping borrowers pay part of this intermediation cost. For simplicity, I assume that the government borrows from the financial intermediaries at interest rate \( r_d = r + \phi_d \) (i.e. the cost households had to pay), and then lends to households at a lower interest rate \( r_g = r + \phi_g \), where \( 0 \leq \phi_g < \phi_d \). \( \phi_g \) has a lower bound at 0; that is the government will not lend to households at interest rates below the market (saving) interest rate.

The net result of this policy boils down to the government helping to pay a portion (or all) of the intermediation fee. That is, for total borrowing \( B^G \), the government has to pay \( B^G (\phi_d - \phi_g) \). To finance this, the government levies a proportional tax \( \tau \) on capital gains accrued from bond savings. That is, for total savings \( B^S \), the government’s receipt is \( \tau r B^S \). I assume that the government has a budget \( G \) that is fixed and always balanced. I take \( G \) to be a parameter.

#### 6.1.1 Equilibrium Definition

The equilibrium in this extension is similar to that in section 2.6: The main differences now are that the interest rate for households who are borrowing is \( r_g \), an endogenous object, rather than \( r_d \), which was a parameter; and that now households are faced with an endogenous tax
rate \( \tau \).

Using the same notation as before, the equilibrium can be defined as follows:

**Definition of equilibrium** A stationary competitive equilibrium of the model consist of the interest rate \( r \) for households who are saving, interest rate \( r_g \) for households who are borrowing, wage rate \( w \), tax rate \( \tau \), value functions of households and firms \( \{V^e, V^w, \Pi\} \), allocations \( \{k', b', l\} \) and distribution of agents \( \Lambda \) over the state space \( \mathcal{S} \) such that,

1. Taking \( r, r_g, w \) and \( \tau \) as given, the households’ and firms’ choices are optimal.

2. The government’s budget, where \( G \) is a parameter, balances:
   \[
   G = \tau r \int b \times 1_{\{b \geq 0\}} d\Lambda = (r_d - r_g) \int b \times 1_{\{b < 0\}} d\Lambda
   \]

3. Markets clear,
   (a) Bonds: \( \int b'd\Lambda = K^c \)
   (b) Labor: \( \int \theta hd\Lambda = \int ld\Lambda + L^c \)

4. The distribution \( \Lambda \) is time-invariant, given by
   \[\Lambda = \Gamma(\Lambda)\]

Where \( \Gamma \) is the one-period transition operator on the distribution

**6.2 Governmental action to address illiquidity I**

In this example, the government serves as a buyer of last resort. What do we mean? Note that in the baseline model, the partial irreversibility can be interpreted as a parameter on resale prices. Specifically, every unit of capital sold fetches \( 1 - \lambda \) in consumption goods: as such, \( 1 - \lambda \) can be interpreted as the resale price of capital. In this example, the government commits to buying used capital at a price \( q \geq (1 - \lambda) \). After the purchase is made, the government can resell the capital at the actual resale price \( 1 - \lambda \). Note that this means that the government’s program costs \( q - 1 + \lambda \) per unit of capital transacted. Like before, I assume that the government sets a fixed budget \( G \) for this policy, which is paid for by a proportional tax \( \tau \) on capital gains accrued from bond savings.
6.2.1 Equilibrium Definition

The equilibrium in this extension is also similar to that in section 2.6 and 6.1.1: The main difference is $q$, the resale price, is now an endogenous object, as opposed to $1 - \lambda$, which was purely a parameter. Moreover, because $q$ is now the resale price, households face a collateral constraint $b' \geq -\varphi (1 - \delta) q k'$: that is, the government’s policy also relaxes the borrowing constraint.

Using the same notation as before, the equilibrium can be defined as follows:

**Definition of equilibrium** A stationary competitive equilibrium of the model consist of the interest rate $r$, wage rate $w$, used capital resale price $q$, tax rate $\tau$, value functions of households and firms $\{V^e, V^w, \Pi\}$, allocations $\{k', b', l\}$ and distribution of agents $\Lambda$ over the state space $S$ such that,

1. Taking $r$, $w$, $q$ and $\tau$ as given, the households’ and firms’ choices are optimal.

2. The government’s budget, where $G$ is a parameter, balances:

$$G = \tau r \int b \times 1_{\{b \geq 0\}} d\Lambda = (q - 1 + \lambda) \int (k' - (1 - \delta) k) \times 1_{\{k' - (1 - \delta) k < 0\}} d\Lambda$$

3. Markets clear,

   (a) Bonds: $\int b' d\Lambda = K^e$

   (b) Labor: $\int \theta h d\Lambda = \int ld\Lambda + L^e$

4. The distribution $\Lambda$ is time-invariant, given by

$$\Lambda = \Gamma (\Lambda)$$

Where $\Gamma$ is the one-period transition operator on the distribution

6.3 Governmental action to address illiquidity: II

This policy is very similar to the preceding one, but instead of buying any capital that is sold by entrepreneurs, the government only buys it from exiting entrepreneurs at a promised price that is higher than the true resale price. Recall that if an exiting entrepreneur sells $k$ units of capital, she only receives $(1 - \zeta)(1 - \lambda)k$. In this example, the government commits to decreasing $\zeta$. Specifically, for every $k$ sold, the government commits to paying the entrepreneur $(1 - \tilde{\zeta})(1 - \lambda)$, where $\tilde{\zeta} < \zeta$. The cost of the program is then $(\zeta - \tilde{\zeta}) K^{exit}$,
where $K^{\text{exit}}$ is the total volume of capital sold by exiting entrepreneurs. This cost is paid for by a proportional tax on capital gains accrued from bond savings, as in the previous two sections.

### 6.3.1 Equilibrium Definition

The equilibrium in this extension is very similar to that in section 6.2. Using the same notation as before, the equilibrium can be defined as follows:

**Definition of equilibrium** A stationary competitive equilibrium of the model consist of the interest rate $r$, wage rate $w$, tax rate $\tau$, government policy $\tilde{\zeta}$, value functions of households and firms $\{V^e, V^w, \Pi\}$, allocations $\{k', b', l\}$ and distribution of agents $\Lambda$ over the state space $S$ such that,

1. Taking $r, w, \tilde{\zeta}$ and $\tau$ as given, the households’ and firms’ choices are optimal.

2. The government’s budget, where $G$ is a parameter, balances:

$$G = \tau r \int b \times 1_{\{b \geq 0\}} d\Lambda = \left(\zeta - \tilde{\zeta}\right) \int (1 - \delta) k \times 1_{\{h' = 1\}} d\Lambda$$

3. Markets clear,

   (a) Bonds: $\int b' d\Lambda = K^c$

   (b) Labor: $\int \theta h d\Lambda = \int l d\Lambda + L^c$

4. The distribution $\Lambda$ is time-invariant, given by

$$\Lambda = \Gamma (\Lambda)$$

Where $\Gamma$ is the one-period transition operator on the distribution.

### 6.4 Results

As in section 5, I first compute the consumption equivalent cost for each agent, $\mu_i$, of switching from the baseline model to the policy of interest, and then compute aggregated consumption equivalent variation for different subsets of the population, as well as the entire population. In addition, I also compute the TFP associated with the new policy. The results are presented in tables 13a and 13b respectively.
(a) Welfare changes. "w to w" is for sub-population of workers who will be workers next period; "w to e" for workers to entrepreneurs; "w" for all workers; "e to w" for entrepreneurs to workers; "e to e" for entrepreneurs to entrepreneurs; "e" for all entrepreneurs; "All" for the entire economy. I also report the tax rate and interest rate. All values in % terms.

<table>
<thead>
<tr>
<th>Policy</th>
<th>w to w</th>
<th>w to e</th>
<th>w all</th>
<th>e to w</th>
<th>e to e</th>
<th>e all</th>
<th>All</th>
<th>τ</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit</td>
<td>0.167</td>
<td>0.229</td>
<td>0.169</td>
<td>0.244</td>
<td>-0.101</td>
<td>-0.00632</td>
<td>0.147</td>
<td>0.76</td>
<td>3.08</td>
</tr>
<tr>
<td>Resale I</td>
<td>0.0712</td>
<td>0.156</td>
<td>0.0745</td>
<td>0.191</td>
<td>0.0434</td>
<td>0.084</td>
<td>0.0756</td>
<td>0.741</td>
<td>3.16</td>
</tr>
<tr>
<td>Resale II</td>
<td>0.0648</td>
<td>0.147</td>
<td>0.068</td>
<td>0.183</td>
<td>0.0487</td>
<td>0.0856</td>
<td>0.0702</td>
<td>0.738</td>
<td>3.16</td>
</tr>
<tr>
<td>Fire sale</td>
<td>0.0656</td>
<td>0.168</td>
<td>0.0696</td>
<td>0.231</td>
<td>0.0518</td>
<td>0.101</td>
<td>0.0735</td>
<td>0.74</td>
<td>3.17</td>
</tr>
</tbody>
</table>

\[ r_g = 0.0286, 1 - q = 0.396 \text{ (Resale I), } 1 - q = 0.395 \text{ (Resale II), } \zeta = 0.504 \]

(b) TFP

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Credit</th>
<th>Resale I</th>
<th>Resale II</th>
<th>Fire sale</th>
</tr>
</thead>
<tbody>
<tr>
<td>levels</td>
<td>0.8898</td>
<td>0.8975</td>
<td>0.8965</td>
<td>0.8952</td>
<td>0.8942</td>
</tr>
<tr>
<td>relative to benchmark</td>
<td>1</td>
<td>1.009</td>
<td>1.008</td>
<td>1.006</td>
<td>1.005</td>
</tr>
</tbody>
</table>

Table 13: Effect of different policies on welfare and TFP. "Credit" = Credit policy as in 6.1, "Resale I" = Resale policy as in 6.2. "Resale II" = Resale policy as in 6.2, but holding the net collateral constraint fixed. "Fire sale" = Resale policy for exiting entrepreneurs, as in 6.3.
Let us first focus on the "Credit", "Resale I" and "Fire sale" policies. Here, we see that in general, these policies increase welfare by about 0.073% to 0.15% in consumption equivalent variation terms, and increase TFP by about .5% to .9%. Given the huge distortions discussed earlier, this is not surprising. With the new policies, entrepreneurial talent and capital is now better allocated, despite the distortionary tax used to finance the program.

However, it might seem surprising that worker households also benefit from the policy. Recall that workers in this economy cannot borrow, and do not own capital. This result derives directly from the fact that this is an endogenous occupational choice model. In equilibrium, policies that benefit entrepreneurs directly also benefit workers, since an increase in the value of being an entrepreneur also increases the option value of being an entrepreneur for the worker.

Notice, however, that the policies have a very much heterogeneous effect on households depending on their type (i.e. occupational, as well as occupational choice, for next period) and wealth. First focusing on welfare changes along occupational types, we see in table 13a that workers overwhelmingly prefer the credit policy over the other two policies. In contrast, entrepreneurs overwhelmingly prefer the resale price policy over the credit policy. To understand why this is so, we need to first consider who actually benefits from a credit policy, and who from a resale price policy. In particular, we will focus on the impact of these policies along the wealth distribution first, then focus on their impact on households depending on their occupational choice. Most of the discussion below will reference figures 21 and 22; note that the quantile bins used to create these bar charts are the wealth quantiles of the aggregate wealth distribution.

The wealth distribution

The effect along the aggregate wealth distribution is relatively straightforward. In general, the welfare gains for households decrease with wealth; moreover the top 10% richest households suffer a welfare loss (see figure 20). This result is easily understood when we put into context who the taxpayers are - the richest households in the population. While both poor and rich households have the potential to benefit from a credit subsidy or resale price subsidy, poor households do not pay their proportional share of the tax bill. Consequently, the welfare gains accrue most to the poor, and least to the rich.

\[30\] Since the effect is similar for both \( \zeta \) and \( \lambda \) policies, I will discuss the two together. However, they are not exactly the same, since \( \lambda \) also has a collateral constraint effect that \( \zeta \) does not. I will defer the discussion to later.
Having surmised the impact of these policies along the wealth distribution, we now look at their impact across occupational types.

**Entrepreneurs**

Entrepreneurs, by-and-large, should in theory directly benefit from both policies, since they are the ones who are directly involved in the credit and capital markets. However, the vast majority of entrepreneurs in fact *do not* borrow, as they are also the wealthiest in the population. In figure 21, we see that the number of entrepreneurs is increasing along the wealth distribution, with over 50% of entrepreneurs belonging to the top decile. As such, in the case of the credit policy, these entrepreneurs are largely funding the program while
never having the chance to benefit from it.

In the case of a resale policy however, entrepreneurs will benefit from it regardless of their position in the wealth distribution. Recall that the primary distortion generated by the resale frictions is that it forces entrepreneurs to persist in their occupation even when they are low productivity (or persist in a sub-optimally large scale). As a result, regardless of her rank in the wealth distribution, an entrepreneur will have the opportunity to benefit from the resale policy when the time to downsize arrives.\footnote{Since this is an infinite horizon model with mean reversion in productivity, there will always be a non-zero probability that a high performing entrepreneurial household will be forced to downsize.}

Indeed, as it turns out, entrepreneurial households in the bottom 90% of the wealth distribution overwhelmingly support the policy (see figure 22). However, one might notice that the top 10% do suffer welfare losses from this policy. This happens because the cost of the program, as borne by the rich, is so large that it overwhelms the gains from the program. However, note that the welfare losses are very small relative to the gains by the rest of the population. Consequently, entrepreneurs gain from the resale policy on average.

**Workers**

Moving on to the workers, we notice that workers are relatively poorer than entrepreneurs. While workers might not directly benefit from the credit policy (they do not borrow if they stay workers), they have the opportunity to borrow if they switch occupations. This greatly increases the value of entrepreneurship to workers, who are likely to enter into entrepreneurship as credit-constrained middle class households. Consequently, workers like the credit policy. Moreover, since workers are generally not as wealthy, they are also paying less for the program (on average).

Why do workers also like the resale program? As discussed above, $\lambda$ and $\zeta$ collectively reduce the option value of being an entrepreneur. As a result, an policy that reduces $\lambda$ or $\zeta$ will inadvertently increase the option value of entrepreneurship, and thus indirectly increasing the welfare of workers. However, as it turns out, the direct reduction to their cost of borrowing (through reducing $r_d$) is much more powerful than the indirect effect of the resale policy. In fact, this is not very surprising. Referring back to table 13a, we see that the interest rate paid by debtors is actually lower than the interest rate received by savers. In other words, the government subsidizing debtors. This is a large reason why workers prefer the credit policy.
Figure 21: Size of population, sorted by occupational type and wealth quantile, for the benchmark economy and alternatives.
Figure 22: Relative welfare change, sorted by occupational type and wealth quantiles

6.4.1 Collateral constraint effect

[...]

7 Conclusion

This paper studied extensively the investment dynamics of entrepreneurs, and in particular, the effect of illiquidity on an entrepreneur’s investment/savings decision, as well as the entry/exit decision of potential entrants and incumbent entrepreneurs. The illiquidity friction
is modeled as a set of non-convex adjustment costs that the entrepreneur has to incur when adjusting his physical capital stock. In particular, the entrepreneur has to incur a per unit transaction cost applicable to the amount of capital that he sells, which reflects a difficulty of entrepreneurs in recouping their original investment outlay. This transaction cost is interpreted as a resale price.

To the best of my knowledge, this paper is the first to utilize a set of micro-data, the Kauffman Firm Survey, to understand empirically the entrepreneur’s investment dynamics, and to discipline key processes and parameters in a Bewley model of heterogeneous households. In particular, the TFPR process that partly governs the level of risk entrepreneurs are exposed to, the returns to scale parameter, as well as the non-convex adjustment cost parameters, are calibrated using an auxiliary model in the spirit of indirect inference. This paper makes three contributions to the empirical and theoretical understanding of entrepreneurial investment dynamics, as well as its relation to the wealth distribution.

Firstly, I document new facts about an entrepreneur’s investment behavior. I found that the distribution of log average revenue product of capital is left skewed, and that it shows significant persistence in relative rankings and absolute values. The persistence is also higher on the left tail than the right tail. I show that existing macroeconomic models of entrepreneurship, which focuses on collateral constraints, cannot match this fact.

Secondly, I construct a new macroeconomic model of entrepreneurship, building on the framework in Cagetti and De Nardi (2006). I use this model to infer the extent of illiquidity entrepreneurs face. I find that, on average, when entrepreneurs wish to expand, they have to pay 3.5% of their capital stock in fixed costs; along the same parallel, when they downsize, they lose about 43% of their capital. Moreover, when they exit, they incur an additional 55% write down on the value of resale capital. When interpreted as a resale price, this implies that the resale price of entrepreneurial capital is 43% lower than its purchase price for incumbents, and 55% lower than its purchase price when the entire firm is liquidated.

This illiquidity friction is economically significant at the household level. In the case of the investment fixed cost, entrepreneurs are forced to operate their firm at sub-optimal levels (relative to the unconstrained entrepreneur) for long periods of time, leading a loss of lifetime consumption. In the case of the lower resale price, the impact is even larger. Households face tight borrowing constraints, which hampers their ability to invest and reach the optimal firm size. Moreover, due to the options value effect induced by the asymmetric purchase / resale price, low productivity entrepreneurs end up maintaining larger-than-efficient firm sizes, and also stay in business for longer when they should simply exit.

This framework also lends itself to a simple experiment of whether a government policy of credit provision or as a buyer of last resort is economically efficient, and if so, which is more
efficient. A policy of credit provision is akin to the SBA’s 7(a) or CDC/504 programs, which is designed to reduce borrowing cost; a program of buyer of last resort does not exists, but is similar to the case when private equity funds buy out failing firms and reallocate capital to successful firms. In this paper, I find that all three strategies are effective in improving welfare and aggregate TFP. However, the policies have very heterogeneous impacts across the wealth distribution, as well as occupational types. As such, the preferred policy instrument depends on the policy outcome desired by a policy maker,

Finally, this paper also adds to the research agenda of whether entrepreneurship can serve as a micro-foundation for the rates of return heterogeneity observed in the data, and as such match the wealth distribution. Unfortunately, this paper finds that when key parameters in the model are calibrated to match the micro-level data on entrepreneurial investment behavior, the model is not successful in replicating the wealth distribution. Generally, this paper finds that the returns to entrepreneurship is too low to deliver the right-skewed wealth distribution. Compared to the preceding literature, entrepreneurship has lower returns to scale and economically significant illiquidity frictions that greatly reduce the returns to entrepreneurship.

Given the importance of illiquidity frictions at the microeconomic level, I also explore its implication for the wealth distribution. I find that increasing levels of illiquidity can greatly reduce the overall returns to entrepreneurship and investment. Consequently, while increasing illiquidity does lead entrepreneurs to accumulate relatively more liquid wealth, the net effect impoverishes the entrepreneur by reducing her income. As such, this leads to a decrease in the wealth inequality in the whole economy.
References


