Recovery of 1933*

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Abstract

When Roosevelt abandoned the gold standard in April 1933, he converted what had been effectively real government debt into nominal government debt to open the door to *unbacked fiscal expansion*. We argue that he followed a state-contingent fiscal rule that ran nominal-debt-financed primary deficits until the price level rose and economic activity recovered. Theory suggests that government spending multipliers can be substantially larger when fiscal expansions are unbacked than when they are tax-backed. VAR estimates find that primary deficits made quantitatively important contributions to raising both the price level and real GNP from 1933 through 1937. The evidence does not support the conventional monetary explanation that gold revaluation and gold inflows, which were permitted to raise the monetary base, drove the recovery independently of fiscal actions.

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1 Introduction

Franklin D. Roosevelt's monetary and fiscal policies helped to pull the United States out of the Great Depression. His first step was monetary: America reduced the gold content of the dollar, abandoned the promise to convert dollars to gold, and abrogated the gold clause on all current, past, and future contracts. This paper emphasizes his second, fiscal, step: his administration expanded government spending, financed that spending with nominal bonds, and convinced people that the bonds would not be fully backed by future taxes until the economy recovered. Because the monetary components—devaluing the dollar and revoking convertibility—were necessary for the fiscal step to work, this narrative is about *joint* monetary-fiscal actions.

When Roosevelt shucked off the gold standard's straightjacket, he was freed to exploit the nominal nature of government debt. If dollars are convertible to gold, even dollar-denominated government liabilities are *real* obligations. Credibility of the gold standard rested on government standing ready to raise real taxes to acquire the requisite gold [Bordo and Kydland (1995)]. By revoking convertibility, Roosevelt enhanced his policy options. He could decide to continue the orthodox policy that new debt begets new taxes or to break from the past and allow prices to revalue outstanding bonds. Early in his presidency, Roosevelt chose the latter option.

Our thesis challenges the conventional wisdom that recovery had little to do with fiscal policy. Scholars from Brown (1956) to Romer (1992) to Fishback (2010) maintain that fiscal deficits during Roosevelt's first term were too small to close the gaping gap in output.[1] Those economists base their conclusion on a narrowly construed fiscal transmission mechanism. The government raises real spending, directly increasing real aggregate demand. Higher demand propagates through higher real expenditures and income, eventually to raise output by a multiple of the initial fiscal expansion. We call this mechanism "Keynesian hydraulics," to use Coddington's (1976) evocative label.

Nominal debt doubled before the end of Roosevelt's second term. Under Keynesian hydraulics, the resulting expansion in *nominal* demand provides no additional economic stimulus. Brown (1956) and the studies that followed explicitly exclude government borrowing from their analyses. Keynesian hydraulics implicitly assume that higher taxes extinguish all wealth effects from higher nominal debt. That assumption effectively continues to treat government debt as a real obligation, denying that the suspension of gold convertibility fundamentally altered the nature of government debt and the fiscal options available to policy makers after 1933.

We broaden the perspective on fiscal transmission to include both Keynesian hydraulics and a vehicle by which government debt dynamics affect economic activity. When nominal government debt expands without raising expected taxes, private-sector wealth and aggregate demand increase via a conventional Pigou-Keynes-Patinkin effect. Roosevelt exercised this option—"unbacked fiscal expansion"—to implement a state-contingent policy: run debt-financed fiscal deficits until the American economy recovers.

Our perspective complements and elaborates Eichengreen's (2000) conclusion that "...the fundamental change in policy making in the 1930s was not the Keynesian revolution, but the

[1] See also Chandler (1971), Peppers (1973), Beard and McMillin (1991), Raynold, McMillin, and Beard (1991), Eichengreen (2000), Steindl (2004), and Hausman (2016).

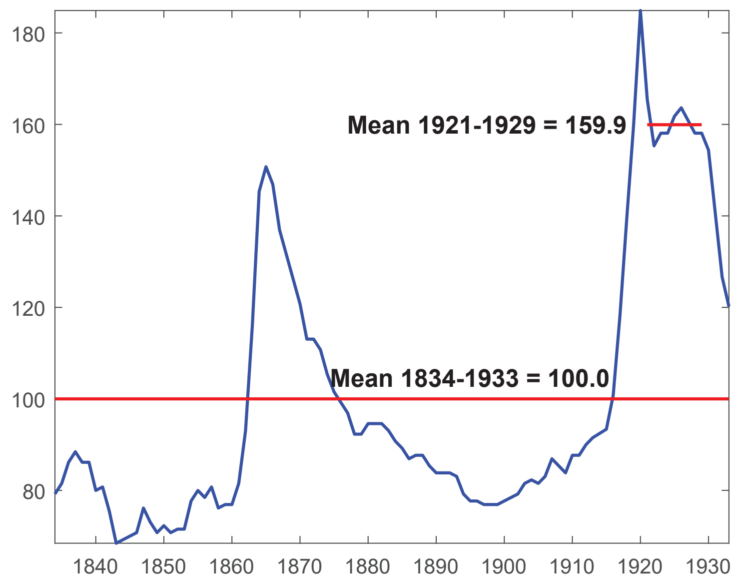

Figure 1: Consumer price index since the 1834 Coinage Act set the price of one ounce of gold at $20.67. Rescaled to make mean from 1834–1933=100. Source: Officer and Williamson (2018) and authors' calculations.

'nominal revolution'—the abandonment of the gold standard for managed money." To reach our perspective, broaden "money" to "nominal government liabilities." Nothing compels policy makers to back expansions in either component of nominal liabilities—base money or bonds—with higher taxes. When they don't, debt-financed fiscal expansion becomes a potent policy tool.

1.1 The Policy Problem

By the time Roosevelt was sworn in as the 32nd president of the United States in March 1933, the economy had been declining for over three years. Relative to the third quarter of 1929, real GNP was 36 percent lower while current-dollar GNP was 57 percent smaller; industrial production had fallen by half; unemployment had increased 22 percentage points; and government debt had grown from 16 percent to over 40 percent of output. Although his first acts salvaged a banking system left reeling by three consecutive banking crises, Roosevelt's focus never strayed far from those macroeconomic facts.

One fact figured prominently in his thinking: the precipitous decline in overall prices bankrupted the farmers and homeowners who had incurred nominal debts at elevated price levels. Those citizens were also among Roosevelt's strongest supporters. Figure 1 encapsulates the policy problem. FDR felt that the key to economic recovery lay in returning overall prices to their 1920s levels, to achieve "...the kind of a dollar which a generation hence will have the same purchasing power and debt-paying power as the dollar we hope to attain in the near future" [Roosevelt (1933c)]. The problem was that in the 1920s the price level was 60 percent above the long-run average to which it had to revert to maintain gold convertibility at the parity that prevailed over the previous century.

Roosevelt's objective to return the price level permanently to that high level was inconsistent with remaining on the gold standard at the historical conversion rate. FDR pursued a triple-barreled approach to the problem. The executive branch—with Congressional approval—took control of monetary policy from a Federal Reserve that by all accounts had been "inept" since the depression started.[2] The monetary component sharply reduced the gold content of the dollar; it then evolved into complete abandonment of the gold standard and abrogation of gold clauses on all public and private contracts.

The second barrel ran "emergency" fiscal deficits financed by new issuances of nominal Treasury bonds. Emergency spending served two purposes. It provided much-needed relief through a vast array of works programs and infrastructure projects. But the modifier "emergency" also communicated the temporary and state-contingent features of the fiscal program.

Political strategy, which was crucial to establish the unprecedented fiscal program was credible, composed the third barrel. Roosevelt made recovery *the* policy priority; higher, for example, than the last century's fiscal orthodoxy. The president found innovative ways to persuade the people the stakes of recovery were unprecedentedly high. On the domestic front, he feared "agrarian revolution" and "amorphous resentment" of economic institutions.[3] Internationally, FDR conjured images of European fascism. In advisor George F. Warren's words, Roosevelt faced "a choice between a rise in price or a rise in dictators."[4] The president framed economic recovery as "a war for the survival of democracy" [Roosevelt (1936a)].[5] Jalil and Rua (2017) present evidence that in the second quarter of 1933 inflation expectations picked up rapidly. That evidence suggests the third barrel succeeded to convince people that Roosevelt would experiment with selling bonds that do not portend higher taxes, at least temporarily.

1.2 What We Do

The paper places FDR's policy actions in the political and intellectual context of the times. That context drives the narrative. Desperate times can engender creative measures. Despite running for office on his belief in sound finance, Roosevelt was at root a pragmatist, willing to experiment with the economic levers at his disposal—and even some levers that were not.

Several theoretical results underpin our narrative:

1. Under a classical gold standard with fixed parity, monetary and fiscal policies are not free to achieve any desired price level.

[2]Friedman and Schwartz (1963, p. 407) characterize their adjective "inept" for monetary policy as a "plain description of fact." Also see Wicker (1965) and Meltzer (2003) for similar assessments.

[3]In October 1933, FDR told a group of financial advisors that the gold-buying policy the Administration pursued averted "an agrarian revolution in this country" [Blum (1959, p. 72)]. Leuchtenburg's (1963) aptly-titled chapter, "Winter of Despair," documents that by the winter of 1932–33, economic despair transformed into "amorphous resentment" of the economic institutions that people blamed for the depression.

[4]This quotation is found in Rauchway (2014, p. 4) and Rauchway (2015, ch. 5), who lays out Warren's influence in context. See also Sumner (2001).

[5]Authorities other than the president communicated the high stakes. As early as February 1933, Marriner Eccles, in his capacity as a private banker, testified to the Senate Finance Committee that in the absence of federal government intervention into the economy, "we can only expect to sink deeper in our dilemma and distress, with possible revolution, with social disintegration, with the world in ruins, the network of its financial obligations in shreds, with the very basis of law and order shattered" [Eccles (1933, p. 705)].

2. Unbacked fiscal expansion is infeasible under a classical gold standard.

3. Unbacked fiscal expansion permanently raises the price level.

4. Government spending and transfer impacts from unbacked fiscal expansion generally exceed those from tax-backed fiscal expansion.

We bring both informal and formal empirical evidence to bear on the thesis. Surprise inflation significantly reduced the value of government debt. Over the seven years after America left the gold standard, nominal debt rose 30 percent more than real debt. Negative real returns on the government bond portfolio became more prevalent in that period. Government debt, which was 16.4 percent of GNP in the last quarter of 1929, rose to 42.3 percent by the first quarter of 1933. Although nominal debt doubled over the next seven years, it averaged only 41.6 percent of GNP to belie the critics' hysteria about fiscal sustainability. Nominal economic growth stabilized debt.

Identified VAR evidence finds that temporary fiscal expansions produce persistent increases in output, the price level, the monetary base, the market value of nominal government debt, and the monetary gold stock. Fiscal disturbances are also important sources of fluctuations in those variables and account for significant fractions of the forecasts errors in real GNP and the price level. Although the VAR recovers the patterns of correlation that underlie conventional monetary explanations of the recovery, the VAR points to fiscal, rather than monetary or gold, shocks as the genesis of those comovements.

2 Political and Intellectual Context

Roosevelt's decision to leave the gold standard and reflate arose against a backdrop of growing political and intellectual consensus that higher retail and wholesale prices were critical to recovery of wages, employment, investment, and consumption. The banking crisis of February–March 1933 heightened expectations of a dollar devaluation as political pressure mounted against maintaining gold convertibility at the existing parity.[6] To avoid capital losses from the banking panic, foreign depositors in U.S. banks liquidated their dollar balances and converted them to gold, pushing gold reserves close to their statutory minimums, particularly at the New York Fed. Rebuilding dwindling gold reserves required the bank to raise its discount rate in the middle of a banking panic. To avoid further strain on the beleaguered financial sector, Senator Elmer Thomas advocated issuing unbacked currency to raise the price level to its 1920s level and Senator Tom Connally proposed reducing the gold content of the dollar by one-third. Financial and political forces were aligning against the gold standard.

Those realignments were echoed by a camp of economists who agitated for reflation. Irving Fisher's (1932; 1933b) debt-deflation theory argued that when the private sector is over-indebted, a falling price level triggers a sequence of events—lower asset prices, higher real interest rates, contraction of bank deposits, decrease in profits, reduction in output, rising unemployment, bank runs, and so on—that drives the economy into depression. Viewing nominal income through the equation of exchange, Fisher advocated government policies designed to raise the money supply and velocity.

[6] This exposition draws on Eichengreen (1992), particularly chapter 11.

Fisher carried on extensive correspondence with the president and met with him several times to discuss his economic proposals. In an April 30, 1933 letter to Roosevelt, Fisher (1933a) wrote, "No one is happier than I over the prospect of the passage of the reflation legislation," referring to the Agricultural Adjustment Act, which included the Thomas Amendment giving the president unprecedented powers to reflate. George F. Warren, though, had the ear of the president. "George F. Warren was the first person who ever advised a President of the United States to raise the price of gold," begins Pearson, Meyers, and Gans (1957, p. 5598), a detailed description of Warren's role in Roosevelt's inner circle.

Keynes (1933) wrote an open letter to Roosevelt, published in the *New York Times*, calling for the U.S. government "...to create additional current incomes through the expenditures of borrowed or printed money." Although today Keynesian stimulus often is narrowly construed as the real mechanisms of Keynesian hydraulics, Keynes's emphasis in this letter is on "governmental loan expenditure" as "the only sure means of obtaining quickly a rising output at rising prices." Keynes prescribed unbacked fiscal expansion: nominal-liability-financed deficits with no promise to raise future taxes to pay off the debt.

We do not claim that Roosevelt consciously engineered an unbacked fiscal expansion. Nor do we believe that he had in mind the precise economic mechanisms that we identify as critical to the recovery. There were false starts, such as the National Industrial Recovery Act of 1933, which in addition to being ruled to contain unconstitutional features, likely slowed recovery [Cole and Ohanian (2004)]. But his "try anything" macroeconomic approach contained the essential ingredients for an unbacked fiscal expansion: suspension of the gold standard, a commitment to run debt-financed emergency deficits until specified parts of the state of the economy improved, and a policy decision not to sterilize gold inflows, which permitted the monetary base to grow without further increases in government indebtedness for monetary reasons.

The paper does not try to use a formal model to reproduce recovery-period data, as Cole and Ohanian (2004) and Eggertsson (2008) do. In that tumultuous period, economic agents confronted an entirely new and ever-evolving economic structure. Interpretations that rely on modeling conventions like well-understood policy rules and rational expectations are difficult to align with the historical facts. Instead, we use theory to frame the issues and to interpret the history and the data.

3 Contacts with Literature

Our argument that the *joint* monetary-fiscal mix that underlies an unbacked fiscal expansion spurred recovery in the 1930s contrasts with existing explanations, which frequently attribute diminished roles to both monetary and fiscal policy. Existing studies argue that the combination of dollar devaluation, the departure from the gold standard, regime change, expansion of the monetary base, and rising inflation expectations account for the recovery. Our unbacked fiscal expansion interpretation broadly agrees with many of these arguments, but links them to the monetary and fiscal policies of the 1930s.

Another distinction concerns the view that monetary policy made no substantive contribution to the recovery. Friedman and Schwartz (1963), for example, conclude the immediate recovery "owed nothing to monetary expansion" [p. 433]. Wicker (1965) attributes Fed inaction to a leadership vacuum and the Fed's incomplete understanding of how monetary policy affects the economy and the price level. Meltzer (2003, p. 273) flatly declares that "...in the middle and late thirties, just as in the early thirties, the Federal Reserve did next to nothing to foster recovery."

We argue that by ensuring short-term interest rates did not rise with inflation throughout the 1930s, the Fed permitted unbacked fiscal expansion to reflate the economy. Expansions in nominal debt that do not portend higher future taxes raise household wealth at prevailing prices and interest rates. Bond holders convert higher wealth into higher aggregate demand. Some of the increased demand shows up in aggregate price levels, but if prices do not adjust instantaneously, some demand raises real economic activity. If interest rates are pegged, monetary policy prevents the nominal debt expansion from raising debt service enough to put debt on an explosive path. In this manner, Federal Reserve policy performed a critical role: it permitted higher price levels to bring the real market value of debt in line with the expected present value of the primary surpluses that back debt.[7] Monetary and fiscal policy are equal partners in successful unbacked fiscal expansion.

The economic consequences of the unbacked fiscal expansion that began in 1933 rationalize why concerns that expanding federal debt would threaten the U.S. government's creditworthiness were not realized. Studenski and Krooss (1952, p.428) summarize a key feature of unbacked fiscal expansion:

> "In its early years, the New Deal administration itself believed that the public credit could not sustain continuous budgetary deficits and increases in the public debt. But in practice this also proved incorrect. The public credit did not collapse under the burden of increased public debt. On the contrary, government credit grew stronger, interest rates on new government borrowing declined steadily, and the Treasury found it increasingly easy to finance its operations."

Unbacked expansions raise prices and real GNP to ensure that higher nominal debt does not transform into a higher debt-output ratio.

The initial impetus for recovery came from dollar devaluation and departure from the gold standard, which signaled a change in policy regime that raised inflation expectations, according to the consensus view. We agree that these elements all contributed to the recovery, particularly in commodity prices, but argue they cannot account for the rapid pick up in the price level and output in isolation. Temin and Wigmore (1990) offer evidence that dollar devaluation in 1933 signaled that Roosevelt had abandoned the deflation associated with adherence to the gold standard and that the lower dollar directly increased aggregate demand and indirectly raised prices and production throughout the economy. Hausman (2013) supports Temin and Wigmore's hypothesis by showing that increased agricultural incomes bolstered auto sales in rural areas. Hausman, Rhode, and Wieland (2019) offer evidence that the redistribution of income to constrained farmers in the spring of 1933 links devaluation to higher agricultural incomes and aggregate demand. Their framework abstracts away from macro policies to focus instead on the role of microeconomic channels through which the nominal effects of monetary and fiscal policies operated. Romer (1992), however, makes a forceful case that the dollar depreciation after April 1933 cannot account

[7] Leeper (1991), Sims (1994), Cochrane (1999), and Woodford (2001) describe this mechanism. Bianchi and Melosi (2017, 2018) formally model unbacked fiscal expansion.

for the sustained increases in subsequent price levels. We agree with Romer and point out—as do Jalil and Rua (2017)—that both Britain and France experienced similar depreciations in their currencies after leaving gold, yet prices and output did not rise as they did in the United States.

Our work complements Jalil and Rua's narrative evidence on the role of rising inflation expectations in the recovery of 1933. We ground those expectations in the prevailing monetary-fiscal policy mix.

Our narrative shares some elements with Eggertsson (2008), but the economic mechanisms differ in important ways. Eggertsson emphasizes a regime change in policy dogmas from Hoover to Roosevelt and relies on new Keynesian mechanisms for escaping from the lower bound on the nominal interest rate, with expectations anchored on an eventual return to the conventional active monetary/passive fiscal policy mix.[8] Eggertsson's story rests on the coordinated action of monetary and fiscal policy to maximize household utility. In the presence of distorting taxes, higher deficits provide an incentive for the Fed to keep interest rates low for an extended period of time, to manage the value of outstanding debt. Monetary policy mitigates tax distortions by committing to generate inflation when the Fed has the freedom to do so—that is, once the zero lower bound ceases to bind. In this way, the time-consistent policy generates the same stimulatory mechanisms that Eggertsson and Woodford's (2003) optimal commitment policy delivers.

This interpretation faces two difficulties. First, it requires substantial policy coordination. Eccles (1951) describes a highly decentralized Federal Reserve, both in its operations and in its objectives, an account that Wicker (1966), Wheelock (1991), and Meltzer (2003) confirm. Federal Reserve officials frequently voiced concerns about the prospect of inflation, even during the deflationary years in the early 1930s [Meltzer (2003, p. 280)].[9] Second, Eggertsson's mechanism leans heavily on rational expectations at a time when the entire monetary system had no precedent. It is difficult to square that history with Eggertsson's sophisticated and single-mindedly inflationary Fed behavior. Unbacked fiscal expansion does not require rational expectations, as Eusepi and Preston (2012) and Sims (2016a) show. In this important sense, our mechanism is far less demanding than is Eggertsson's.

History was not nearly as linear as our unbacked fiscal expansion interpretation makes it seem. Disparate viewpoints about the depression battled for "the soul of FDR," in Stein's (1996, ch. 6) memorable phrase. A 1932 "Memorandum" written by three young Harvard economists nicely distills those disparate views. The document denounces "the failure on the part of the government to adopt other than palliative measures" to combat the depression [Currie, White, and Ellsworth (2002, p. 534)]. Viewpoints Roosevelt contended with included: (1) economists who believe the depression cannot be stopped and any efforts to do

[8] Leeper (1991) defines an active policy authority as free to pursue its objective, while a passive authority is constrained by the behavior of the active authority and optimizing private behavior. In a model like Eggertsson's, a determinate bounded rational expectations model requires either an active monetary policy with a passive fiscal policy or vice versa.

[9] Some academic economists backed those voices. Eleanor Lansing Dulles (1933, p. V) at the University of Pennsylvania drew on the French experience in the 1920s to write in November 1933 that the United States faced "serious dangers" from inflation: "Inflation takes many forms, Government debt is the most insidious. . . ." Oliver Sprague, a Harvard professor and advisor to Roosevelt's first treasury secretary William Woodin, opposed leaving gold, warning that America was "in great danger of a great inflation such as German had" [Pearson, Meyers, and Gans (1957, p. 5616)].

so interfere with the "natural" functions of the economy; (2) those who believe the economy is so poorly understood that government efforts are likely to make matters worse; (3) some who adopt the view that depressions are cleansing and purge inefficiencies; (4) a group, like the Memorandum's authors, who "believe that recovery can and should be hastened thru [sic] adoption of proper measures."[10]

Roosevelt clearly sided with the fourth group, at least in the early years of the recovery.

4 Why Unbacked Fiscal Expansion?

Contemporary supporters and critics understood that Roosevelt's price-level objective entailed a permanent increase in prices to 60 percent above their long-run average. But a permanent revaluation of the dollar price of gold required leaving the gold standard.

Result 1. *Under the gold standard with a fixed parity—the classical gold standard—monetary and fiscal policies cannot achieve any desired price level.*

Straightforward economic logic underlies this result.[11] Private holdings of gold, which standard asset-pricing reasoning determines, establish the goods value of gold—the aggregate price level. The Euler equation for private gold demand implies that

$$\frac{P_t^g}{P_t} = E_t \sum_{T=t}^{\infty} q_{t,T} \frac{u_{G,T}}{u_{c,T}} \tag{1}$$

where P_t^g is the dollar price of gold, P_t is the price level, $q_{t,T}$ is the stochastic discount factor, $u_{G,T}$ is the marginal utility of gold holdings, and $u_{c,T}$ is the marginal utility of consumption. When the dollar price of gold is fixed at $P_t^g = \bar{P}^g$, expression (1) implies that the marginal rate of substitution between gold and consumption uniquely determines the equilibrium price level.

Monetary policy must passively adjust to accommodate the price level consistent with the pegged price of gold. Fiscal policy must passively adjust primary surpluses to provide gold backing for outstanding government debt at that price level. This establishes that leaving the gold standard and abandoning convertibility were necessary to achieve FDR's price-level objective.

Definition 2. *Unbacked fiscal expansion increases government expenditures on purchases or transfers, issues nominal bonds to cover the deficit, and persuades people that surpluses will not rise to finance the bonds.*

Simple theory makes this definition precise and illustrates the price-level consequences of unbacked fiscal expansion. A representative household receives a constant endowment, derives utility from consumption and real money balances, and holds initial nominal wealth in the form of nominal money and bonds, $A_0 \equiv M_{-1} + B_{-1}$. Nominal bonds at t sell at

[10] Two authors went on to play critical roles in policy: Currie at the Federal Reserve Board, Treasury, and the White House; White at the Treasury where, together with Keynes, he created the Bretton Woods system.

[11] See Barro (1979) or Goodfriend (1988) for details. Appendix B provides a complete model, calculations, and proofs for all the theoretical results in the paper.

price $1/(1+i_t)$ and pay \$1 at $t+1$; money earns no interest. The household's intertemporal budget constraint at time 0 is

$$E_0 \sum_{t=0}^{\infty} q_{0,t} \left[c_t + \frac{i_t - 1}{i_t} m_t \right] = \frac{A_0}{P_0} + E_0 \sum_{t=0}^{\infty} q_{0,t} \left[y_t - \tau_t \right] \tag{2}$$

$q_{0,t}$ is the stochastic discount factor for the date-0 value of goods at t, m_t is real money balances, and τ_t is lump-sum taxes net of transfers. Money demand yields the liquidity preference schedule $m_t = L(i_t, c_t)$.

To close the model, we assume the central bank pegs the nominal interest rate, $i_t = \bar{i}$, to approximate Federal Reserve behavior after 1933. Fiscal policy sets $\tau_t = \bar{\tau} + \varepsilon_t$, where $E_t \varepsilon_{t+j} = 0$ for $j > 0$, and government purchases are zero. Applying these policy rules, imposing goods- and bond-market clearing on (2), and evaluating expectations yields the equilibrium condition

$$\frac{M_{-1} + B_{-1}}{P_0} = L(\bar{i}, \bar{y}) + \tau_0 + \frac{\beta}{1-\beta} \bar{\tau} \tag{3}$$

The real value of government liabilities equals the expected present value of seigniorage revenues plus primary surpluses.

Lower τ_0 financed by newly issued B_0 is an unbacked fiscal expansion. Higher transfers with no offsetting future taxes shift resources from the government to households. This positive wealth effect induces households to attempt to raise their consumption paths. Higher demand for goods raises their price, P_0, which reduces the real value of the household's initial nominal assets, A_0/P_0. This negative wealth effect must be large enough to eliminate the excess demand for goods at time 0, and make households happy to consume their endowments.

Corollary 3. *Unbacked fiscal expansion is infeasible under a classical gold standard.*

Unbacked fiscal expansion requires active fiscal behavior; the government does not use future surpluses to stabilize debt. Condition (3) uniquely determines the price level as a function of the expected present value of primary surpluses including seigniorage revenues—the right side—and outstanding nominal government liabilities. Asset-pricing condition (1) determines the price level as a function of the gold price, $\bar{P}^g$, and prevailing conditions in the gold market. These two price levels will generally be different.

When the price level consistent with $\bar{P}^g$ is too low to satisfy (3), the real value of debt exceeds its real backing. Households will over-accumulate government bonds to violate their optimality conditions. When the price level under the gold standard is too high, households will refuse to buy bonds, and the government will violate its budget constraint. By either outcome, no equilibrium exists.

Result 4. *Unbacked fiscal expansion permanently raises the price level.*

A one-time unbacked fiscal expansion raises P_0 in equilibrium condition (3). To see that this increase is permanent, examine how nominal government liabilities at time 0 change. Both real money balances, $M_0/P_0 = L(\bar{i}, \bar{y})$, and real debt, $B_0/P_0 = \bar{\tau}/(1-\beta)$, remain unchanged because they do not depend on τ_0. With the change in price level, ΔP_0, given by the equilibrium condition, both M_0 and B_0 expand in proportion to ΔP_0. In the absence of any further disturbances, nominal liabilities remain at those permanently higher levels, as does the price level.[12]

These theoretical points establish that an appropriately scaled unbacked fiscal expansion could, in principle, achieve FDR's price-level objective and that ending convertibility of dollars for gold was a necessary first step. But why did Roosevelt opt for a fiscal, rather than a monetary, solution?

4.1 Monetary Policy

In the wake of the Federal Reserve's "inactivity" in the worst years of the depression, Congress feared that any recovery would be stymied by continued Fed inaction [Meltzer (2003, p. 459)]. The Thomas Amendment of May 1933 granted the Executive unprecedented monetary powers, which included fixing the gold value of the dollar, issuing greenbacks, and ordering the Fed to buy Treasury securities. This was a first step to ensure the Fed would not act to thwart the stimulative impacts of fiscal expansion.

Enter Klüh and Stella (2018) who argue that the Gold Reserve Act of 1934 undermined the Fed's ability to reverse the stimulus through open-market operations. The Act gave to the Treasury legal title to all monetary gold. Treasury bought gold by issuing gold certificates, which could be held only by the Fed and were redeemable *in dollars* only at the Treasury's discretion. Treasury gold purchases raised the Fed's monetary liabilities—new Treasury deposits at the Fed—without commensurate increases in liquid assets. By the end of 1936, the Fed's total monetary liabilities were \$10.89 billion, of which only \$2.43 billion were liquid: over 80 percent of the Fed's monetary liabilities were irredeemable gold certificates.[13]

Klüh and Stella (2018, p. 4) observe that Fed officials "understood they could not win a war of attrition with the Treasury." The Treasury could undertake gold purchases to expand reserves without limit, secure in the knowledge that it was infeasible for the Fed to sterilize them.

Operational factors combined with institutional features of the Federal Reserve in the early 1930s to reduce the Fed to "impotence," according to Eccles (1951). At the time, there was no single Federal Reserve policy; there was a policy for each regional Reserve Bank and the Board of Governors. Eccles emphasizes that Reserve Banks were beholden to their directors, who acted in the private interests of bankers. Before accepting the nomination to chair the Federal Reserve Board, Eccles insisted on institutional reforms that consolidated decision-making power in Washington, D.C. The Banking Act of 1935, among other things, changed the decision-making process at the Fed, which Eccles describes:

> "...before a uniform decision could be reached...there had to be a complete meeting of the minds between the governors of the 12 Reserve banks and the 108 directors of those banks, plus the FRB in Washington. A more effective way

[12] Because the expansion in M_0 depends on $L(\bar{i}, \bar{y})$, rather than directly on the size of the deficit, this is not conventional money financing of deficits, as in Sargent and Wallace (1981). Instead, the money supply expands passively to ensure the money market continues to clear at the pegged nominal interest rate $\bar{i}$.

[13] Board of Governors of the Federal Reserve System (1937). Total monetary liabilities are Federal Reserve and Federal Reserve Bank notes outstanding plus bank reserves; total liquid assets are gold reserves plus U.S. Treasuries. Gold certificates remain on the Fed's balance sheet today.

of diffusing responsibility and encouraging inertia and indecision could not very well have been devised." Eccles (1951, p. 170)

While the Fed could not sterilize the Treasury's gold purchases, monetary policy also did little to advance Roosevelt's economic agenda. After only minor actions in 1933, the Fed conducted no open-market operations after November 1933. This inactivity occurred against a backdrop of current and former Fed officials publicly expressing concerns about run-away inflation. After leaving his position as Fed Chairman on May 10, 1933, Eugene Meyer wrote that "...the mere fact that the Administration has assumed responsibility for defining our monetary policies and fixing our price goal, indicates a subordinate role for the Federal Reserve System" [Meyer (1934)]. Adolph Miller, one of the original governors of the Federal Reserve System, who served until 1936, was vociferous in calling for a return to gold, fearing the discretion that underlies a "managed currency," which he called "human nature money" [Miller (1936, p. 4)].

At a practical level, it was not clear that monetary stimulus would be effective. There was no assurance, particularly on the heels of sequential banking crises, that higher reserves would lead to higher bank deposits. Nor was it certain that higher deposits, if they were forthcoming, would ultimately increase bank loans to finance new investment.[14]

As it happened, banks, worried about the Federal Reserve's failure to fulfill its lender-of-last-resort function, behaved conservatively and expanded holdings of government bonds, rather than loans to the private sector. From March 1933 to June 1940, annual growth rates of narrow money far outstripped those of broad money: reserves (23.1 percent), base (12.8 percent), M1 (7.7 percent), and M2 (5.2 percent). This was a very different pattern from the 1920s when M2 averaged 3.2 percent annual growth and reserves averaged 2.8 percent.

4.2 Fiscal Policy

Unbacked fiscal expansion served several of FDR's objectives. Given his strong support in Congress, particularly from "inflationists" like Senators Thomas and Connally, fiscal policy was largely under the president's direct control. Federal Reserve actions, to FDR's frustration, were beyond his control.

Fiscal policy also served political objectives. By providing immediate relief to the unemployed, farmers, and the "forgotten man," federal expenditures tamped down domestic unrest. Direct relief was a highly visible indicator that the federal government had the common man's interests at heart, helping to re-establish confidence in policy institutions. Finally, economists and politicians alike understood that deflation had redistributed wealth from debtors to creditors. Reflation, and the fiscal actions underlying it, were deliberate efforts to reverse that redistribution.[15] Roosevelt's attitudes toward redistribution shone through in a letter to Secretary of the Treasury Woodin: "I wish our banking and economist

[14] Although Bordo and Sinha (2018) argue that the $1 billion open-market operation in 1932 was stimulative.

[15] Fisher (1934, ch. VI) thoughtfully discusses how to arrive at a "just" price level that balances the losses of borrowers and creditors. Eccles (1933) pointed to the redistribution of wealth as a source of the prolonged depression: "During the period of the depression the creditor sections have acted on our system like a great suction pump, drawing a large portion of the available income and deposits in payment of interest, debts, insurance and dividends...."

friends would realize the seriousness of the situation from the point of view of the debtor classes—i.e., 90 percent of the human beings in this country—and think less from the point of view of the 10 percent who constitute creditor classes" [Roosevelt (1933a)].

Roosevelt walked a fine line on fiscal policy, seeming to maintain contradictory positions. During the 1932 campaign for president, he harshly criticized Hoover's deficits and took a "Pittsburgh pledge" to balance the budget by reducing expenditures [Roosevelt (1932a)]. Just six months earlier he delivered his famous speech about "the forgotten man at the bottom of the economic pyramid" [Roosevelt (1932b)]. That speech characterized the depression as a "more grave emergency" than World War I and called on government to restore the purchasing power of farmers and rural communities and assistance to homeowners and farmers facing foreclosure.

Six days after taking office, Roosevelt sent to Congress a proposal to cut federal spending by an amount equal to nearly 14 percent of total expenditures. Cuts eliminated government agencies, reduced federal worker pay, and, most critically in light of the politics of the time, shrank veterans' benefits by half. When the Economy Act of 1933 was finally signed into law, the spending cuts amounted to a little under seven percent of expenditures, but Roosevelt could point to the legislation to help establish his bona fides as a "sound finance" man.

Just 20 days into his administration, Roosevelt created fresh fiscal nomenclature in a press conference. Asked when it might be possible to balance the budget, the president replied, "...it depends entirely on how you define the term, 'balance the budget"' [Roosevelt (1933b, p. 13)]. His reply spawned the distinction between "ordinary" and "emergency" expenditures, which became institutionalized in Treasury Reports.[16]

FDR was more comfortable with deficits by 1936. In the face of precipitous declines in tax receipts, he argued that "To balance our budget in 1933 or 1934 or 1935 would have been a crime against the American people" [Roosevelt (1936b)]. And in response to budget director Lewis W. Douglas's argument that the only way to project a balanced budget in 1936 was to cut spending, Roosevelt replied, "No, I do not want to taper off [spending programs] until the emergency is passed" [Rosen (2005, p. 85)]. On the other hand, he supported tax hikes in 1935 and 1937.

Why did FDR waffle so on fiscal policy? It is possible, as Stein (1996) suggests, that Roosevelt was tentative and uncertain about fiscal stimulus. But the waffling may have been deliberate. His distinction between "ordinary" and "emergency" government expenditures was central to communicating that unbacked fiscal expansion was state-contingent. Linking the state-contingent emergency expenditures tightly to the economic emergency—through both their timing and their labels—Roosevelt drove home their temporary nature. At the same time, by demonstrating fiscally responsible ordinary spending, he could reassure his critics, particularly bankers, that once the crisis passes, he would balance the budget. Roosevelt's

[16] The reply continued: "What we are trying to do is to have the expenditures of the Government reduced, or, in other words, to have the normal regular Government operations balanced and not only balanced, but to have some left over to start paying the debt. On the other hand, is it fair to put into that part of the budget expenditures that relate to keeping human beings from starving in this emergency? I should say probably not... You cannot let people starve, but this starvation crisis is not an annually recurring charge. I think that is the easiest way of illustrating what we are trying to do in regard to balancing the budget. I think we will balance the budget as far as the ordinary running expenses of the Government go" [Roosevelt (1933b, pp. 13–14)].

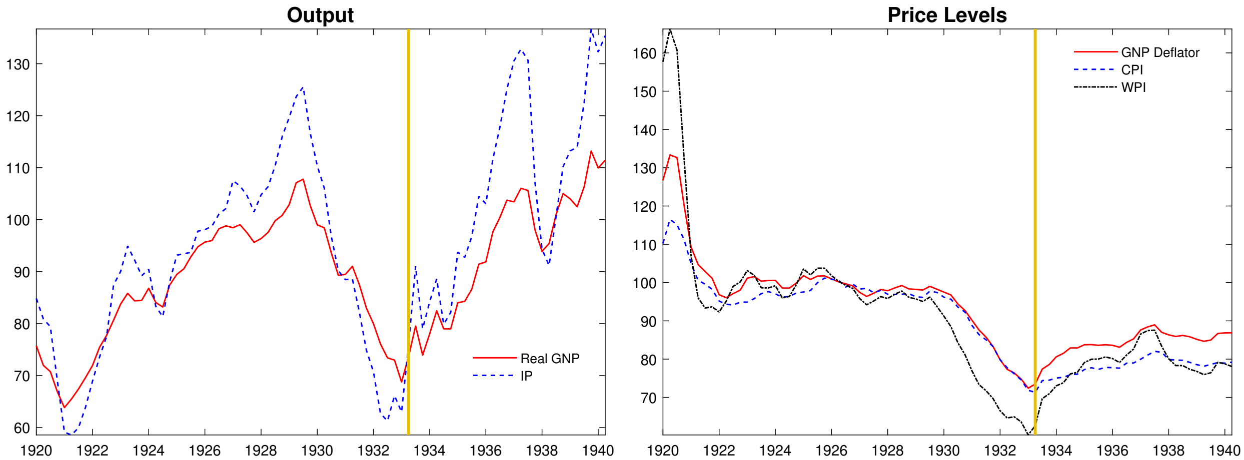

Figure 2: Measures of real economic activity and price levels. All series use 1926 base year. Vertical line marks when the United States abandoned the gold standard. Sources: Balke and Gordon (1986), Federal Reserve Board, BEA and BLS from NBER Macrohistory Database.

January 1936 budgetary address made this point explicit when he said, "... it is the deficit of today which is making possible the surplus of tomorrow" [Roosevelt (1936c)].

5 Empirical Facts and Theoretical Interpretations

This section presents a variety of facts about the state of the U.S. economy throughout the 1920s and 1930s. It offers some evidence that corroborates the interpretation that unbacked fiscal expansion spurred recovery. In the figures that follow, we contrast the performance of economic variables during the "gold standard" (January 1920 to March 1933) to their behavior during the "unbacked fiscal expansion" (April 1933 to June 1940). Data are quarterly. Vertical bars in the figures at April 1933 mark America's departure from the gold standard.

The section also lays out a new Keynesian model to contrast fiscal multipliers under Keynesian hydraulics and unbacked fiscal expansion. We then use the theory to interpret time series on government debt and on the returns to debt.

5.1 Macroeconomic Indicators

The price level, however measured, decreased by roughly 30 percent from the stock market crash in October 1929 to its trough in April 1933 when the United States abandoned the gold standard (right panel figure 2). Although consumer and wholesale prices and the GNP deflator rose through most of the 1930s, they never regained the 1920s levels that various policymakers desired.

Like prices, output also plunged after the stock market crash and rebounded with the abandonment of the gold standard. The left panel of figure 2 shows that real GNP fell by roughly 25 percent from peak to trough, as measured on an annual basis. GNP hits its trough in the first quarter of 1933. Industrial production dropped 45 percent from peak to trough and, like consumer and wholesale prices, began a sustained recovery in April 1933. Unlike prices, GDP and industrial production eventually surpassed their pre-recession peaks.

The left panel of figure 3 shows the dollar-sterling and dollar-franc exchange rates. The first vertical line marks when the United Kingdom left gold in September 1931, which triggered a very large dollar appreciation that was reversed in April 1933. Note that sterling's depreciation against the dollar is roughly comparable to its subsequent appreciation.

Figure 3's right panel plots the level of the GNP deflator along with two interest rates—the commercial paper rate for New York and the New York Fed's discount rate. Although during the gold standard period interest rates generally followed the decline in the price level, there are also several distinct deviations when rates rose sharply despite a flat or declining price level. For example, in October 1931, concerns about gold outflows induced most Federal Reserve banks to raise their discount rates after Britain left the gold standard, even though prices were in free fall. The Federal Reserve banks aimed to mitigate the gold outflows that dollar appreciation vis-à-vis the pound triggered. Meltzer (2003, p. 280) claims that Federal Reserve policy decisions were mostly consistent with the Riefler-Burgess and real bills doctrines.[17] But these interest-rate hikes were clear attempts by the Federal Reserve to follow the gold standard's "rules of the game" [p. 273].

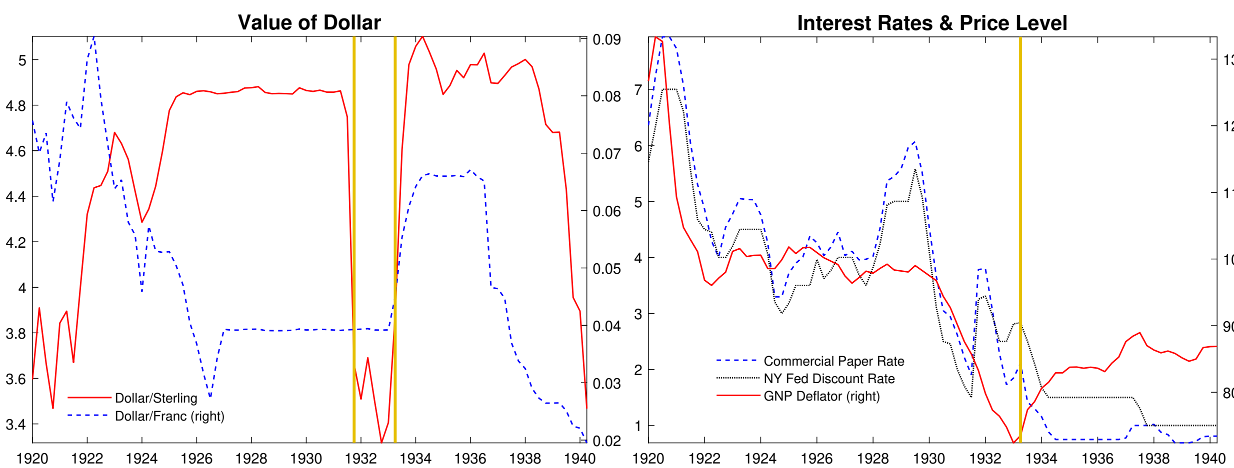

Figure 3: Exchange rates, inflation, and interest rates. Exchange rates in dollars per foreign currency; inflation is annual (quarter over four quarters prior). First vertical line marks when the United Kingdom abandoned the gold standard; second line marks when the United States abandoned the gold standard. Sources: Federal Reserve Board (1943).

After the abandonment of the gold standard in April 1933, the Federal Reserve pegged the discount rate, changing it infrequently. Meltzer (2003, p. 413) notes that the Federal Reserve made few changes to its market portfolio and discount rates from 1933 to 1941. If anything, rates moved against the price level: the Fed was not adjusting policy to combat

[17] Meltzer (2003, p. 282) elaborates that under the Riefler-Burgess framework, policymakers focused on borrowed reserves and short-term market interest rates as key signals of bank demand.

higher prices; instead, it was permitting price-level rises to devalue outstanding government bonds.

The top panel of figure 4 plots the monetary base and the monetary gold stock; the bottom panel plots the gold cover ratio. The large jumps in gold stock and the ratio in 1934 stem from the revaluation of gold to $35 an ounce. Steady increase in the two monetary measures during the unbacked fiscal expansion period reflects the Roosevelt Administration's decision not to sterilize gold inflows. That decision was reversed in 1937, reducing the growth rate of the base [Irwin (2012)] (see appendix C for more details on sterilization).

For a couple of years before the gold revaluation, the cover ratio was precariously low, imposing a severe constraint on the level of the monetary base. Eichengreen (1992) recounts events during February and March 1933 when the New York Fed was at its statutory 40 percent minimum gold cover ratio, which prevented it from rediscounting bills. Initially, other reserve banks discounted bills on New York's behalf. By March 3 the Chicago Fed, which held the bulk of the System's excess gold, refused to provide further assistance to New York for fear that it would be unable to help banks in the Chicago district. These tensions, which stemmed from the absence of a coherent *national* monetary policy, exacerbated the already tenuous state of commercial banks and raised doubts about the credibility of the System's commitment to gold parity.

Official revaluation of gold in January 1934, by almost 60 percent, increased the cover ratio sharply and it remained close to 0.90 for the remainder of the decade. Gold no longer constrained policy behavior as it had before April 1933, a point that is central to the theory of unbacked fiscal expansion.

Table 1 reports the highs and lows of GNP and its components. By 1937, constant-dollar GNP exceeded its 1929 high, but investment remained below its high. In current dollars, GNP and its components did not regain their 1929 peaks. Monetary aggregates fell in the early 1930s as financial unrest lead to contractions in deposits and cash hoarding by the public. Total deposits in all banks fell 30 percent between 1929 and the low point in 1932–33. Deposits bounced back to their pre-depression levels by 1937. Loans, which declined over 50 percent never regained their previous level. Bank holdings of U.S. government obligations largely filled the asset void left by loans, tripling between 1929 and 1937.

5.2 Policy Behavior

Many authors have noted that adherence to the gold standard imposed severe constraints on monetary and fiscal policies by focusing policy authorities on international considerations at the expense of domestic conditions [see Wicker (1966) for discussions of monetary policy constraints]. Eichengreen (2000) argues that the gold standard prevented governments from reflating: "So long as the gold standard remained in place, the commitment to defend the central bank's gold reserves and stabilise the gold parity was an insurmountable obstacle to the adoption of expansionary policies." Apropos of fiscal policy under the gold standard, when taxes must back government debt, is Eichengreen's statement: "Deficit spending could not be used... if deficit spending could not be financed."

Figure 5 illustrates precisely the constraint on monetary policy that Eichengreen has in mind. Dashed lines are interest rates and the solid line is the growth rate of the gold stock. A shrinking gold stock usually induced Federal Reserve banks to raise interest rates

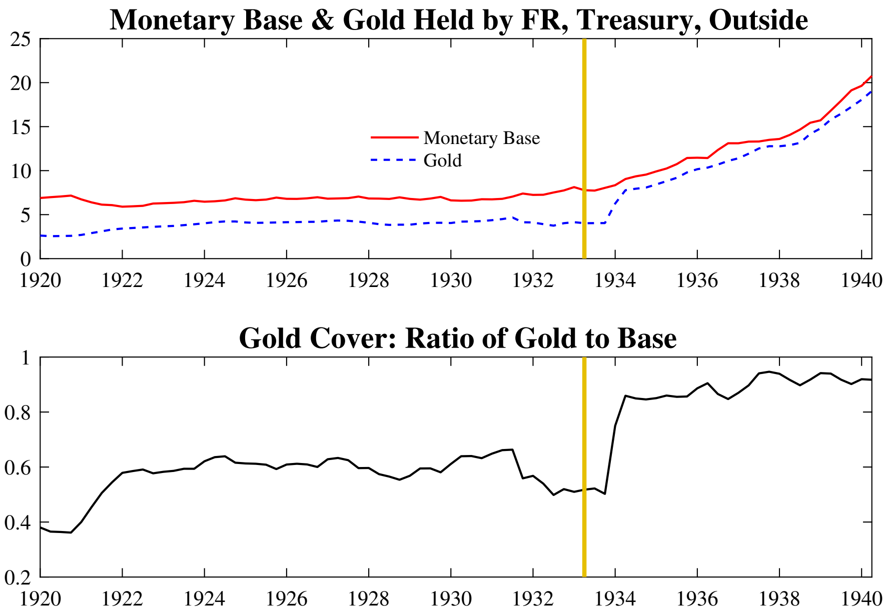

Figure 4: Monetary base and gold held by Federal Reserve banks. Monetary base is currency in circulation plus non-borrowed reserves. Vertical line marks when the United States abandoned the gold standard. Source: Federal Reserve Board (1943) from NBER Macrohistory Database.

	1929 High	1932-33 Low	1937 High
Annual data			
In 1939 prices, billions of dollars			
GNP	85.9	61.5	87.9
Gross domestic investment	14.9	1.1	11.4
In current prices, billions of dollars			
GNP	103.8	55.8	90.2
Gross domestic investment	15.8	0.9	11.4
Consumption	78.8	46.3	67.1
Biannual data			
All banks, billions of dollars			
Total deposits	59.8	41.5	59.2
Loans	41.9	22.1	22.1
U.S. government obligations	5.5	8.2	17.0

Table 1: Sources: Gordon (1952, p. 390) and Federal Reserve Board (1943).

to attract gold from abroad, which arrived with a lag. And when Federal Reserve banks lowered interest rates, gold would flow out of the United States. But in the 1920s, as figure 3 shows, these interest-rate movements occurred in the face of a steadily falling price level. The Fed's actions aimed to stabilize exchange rates at the expense of domestic prices.

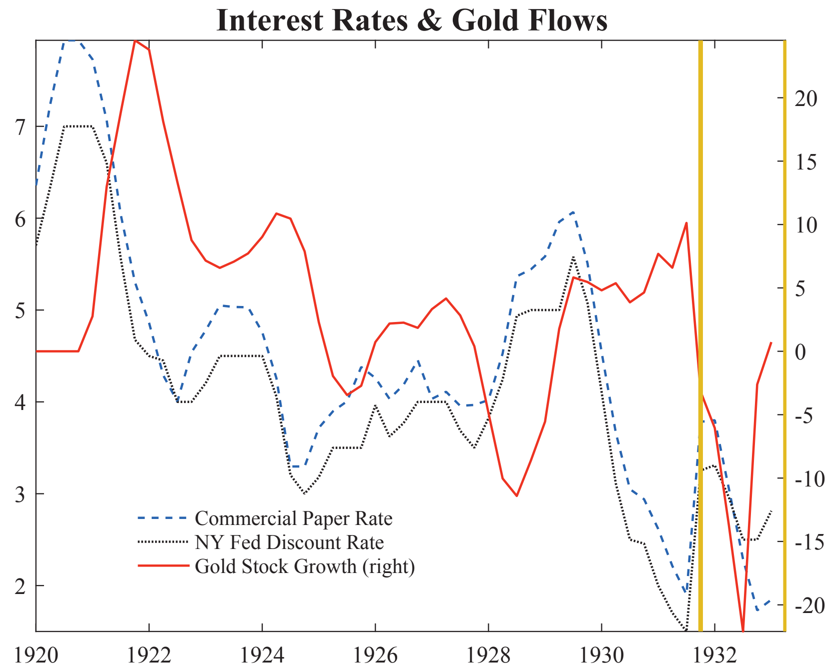

Figure 5: Interest rates and growth rate of monetary gold stock. Growth rate annual (quarter over four quarters prior). The vertical line marks when the United Kingdom abandoned the gold standard. Sources: Federal Reserve Board (1943).

5.2.1 Emergency Spending

Our interpretation of the 1930s recovery relies on a joint monetary-fiscal policy mix that was possible only after abandoning the gold standard. The top panel of figure 6 plots three measures of the federal budget surplus: gross, primary, and "ordinary," defined as total receipts less what are labeled "ordinary" expenditures. All three measures deteriorated sharply as economic activity contracted in the early 1930s. Falling surpluses stemmed from declining revenues due to lower corporate and income tax receipts and rising expenditures due to increased public works spending. Although Roosevelt touted the evils of deficits and was more outspoken than President Herbert Hoover in his promise to cut expenditures, until the second half of the decade he did little to convert primary deficits to primary surpluses.[18]

Deficits remained sizeable until 1936, despite growing receipts from 1934 onward [table 2]. To reassure the public that fiscal finances were "sound," Roosevelt's Treasury drew a clear line between "ordinary" and "emergency" government expenditures.[19] With the exception of 1936, when large veterans' bonuses were paid out, Roosevelt could claim that he balanced

[18] Stein (1996, p. 87) notes that, at least initially, Roosevelt was able to "rise above" his belief in reducing expenditures to do what he considered necessary, which was increasing spending.

[19] The annual reports of the Treasury categorize federal government expenditures chargeable against ordinary receipts in "general" and "emergency" categories in fiscal years 1934 and 1935. From 1936 to 1939, these categories are labeled "general" and "recovery and relief." Prior to Roosevelt taking office, expenditure were divided into "ordinary" and another category to retire public debt. In his annual budgetary messages in 1934 and 1935 and in a fireside chat on Monday, July 24, 1933, Roosevelt refers to non-emergency expenditures as "regular" expenditures. We use the terms regular and ordinary interchangeably to refer to non-emergency expenditures.

the "ordinary" budget [left panel of figure 6].

	1929	**1930**	**1931**	**1932**	**1933**	**1934**	**1935**	**1936**	**1937**
Total Receipts	4033	4178	3317	2121	2080	3116	3801	4116	5294
Total Expenditures									
(excluding debt retirements)	3299	3440	3780	4594	4681	6745	6802	8477	8001
Regular	3299	3440	3780	4594	4681	2741	3148	5186	5155
Emergency	0	0	0	0	0	4004	3655	3301	2847
Regular Deficit	−734	−738	463	2473	2601	−375	−653	1070	−139
Deficit	−734	−738	463	2473	2601	3629	3001	4361	2707

Table 2: Millions of current dollars. "Emergency" expenditures are variously labeled as "emergency organization expenditures," "major expenditures due to or affected by the depression," "recovery and relief," or "public works." Designations of types of spending as "regular" or "emergency" changed over time. A negative deficit is a surplus. Source: Department of the Treasury (various).

From 1934 to 1937, emergency expenditures ranged from one-third to over one-half of total federal expenditures [table 2]. Emergency expenditures fall into three categories—public works, relief, and other spending. Table 3 shows that public works expenditures—public highways, Hoover dam, reclamation projects, improvements of rivers and harbors, flood control, and the Tennessee Valley Authority—doubled under Roosevelt. But Hoover also doubled public expenditures from historical averages. Roosevelt's public works program included \$3.3 billion in funds allocated to the Public Works Administration (PWA) which also made loans for private construction in addition to federal projects.

Roosevelt's relief spending, a mixture of direct relief and works projects, was the largest emergency expenditure category in most years. The Federal Emergency Relief Administration (FERA) established in May 1933 replaced many of the relief efforts of the Reconstruction Finance Corporation (RFC) implemented under Hoover in January 1932. While the RFC

	Hoover				Roosevelt			
	1930	**1931**	**1932**	**1933**	**1934**	**1935**	**1936**	**1937**
Regular Operating	2927.5	3028.4	3231.3	2879.4	2348.7	2676.9	4743.2	3746.0
Social Security	0	0	0	0	0	0	28.4	447.7
Total Emergency	414.4	642.5	1509.7	1801.9	4396.5	4125.4	3704.6	3802
Public Works	256.5	404.1	478.7	458.7	613.1	762.7	912.5	1079.4
Relief	0	0	0	359.5	1852.8	2360.9	2342.4	2466.8
Other	157.9	238.4	1031.0	983.7	1930.6	1001.8	449.7	255.8
Total Expenditures	3341.9	3670.9	4741.0	4681.3	6745.2	6802.3	8476.3	7995.7

Table 3: Federal expenditures by category, millions of dollars. Total expenditures exclude debt and railroad retirements. Emergency expenditures in this table contain some regular expenditures resulting in totals that are 10 to 30 percent higher than the official emergency expenditures listed in Table 2. The "other emergency expenditures" category includes net loans, subscriptions to stock and surplus, and the agricultural adjustment program. Source: Annual Report of the Secretary of the Treasury (p. 354, 1937).

lent primarily to institutions such as states, public entities, or distressed financial institutions, the FERA made direct grants to states totaling \$3.1 billion from 1934 to 1936. States used proceeds from the FERA for direct relief along with relief programs including sanitation improvements, repair or construction of public buildings, national park improvements, and loans and grants to financially troubled farmers. [Studenski and Krooss (1952, pp. 374, 411) and Fishback, Kantor, and Wallis (2003)].

In contrast to FERA, the Civil Works Administration (CWA) operated directly under the federal government and focused on works projects such as building and improving sewer pipes, roads, schools, playgrounds, and airports as well providing work to teachers, writers, and artists.[20] From 1934 to 1935, the CWA encompassed \$1 billion in federal relief expenditures and employed 4 million workers. The Works Progress Administration (WPA) followed the CWA and also focused on work relief by spending \$8.1 billion between 1936 and 1940 and employing 2.2 million workers per year on average [Studenski and Krooss (1952, p. 412)]. Projects included highways, slum clearance, reforestation, and rural rehabilitation.

The Civilian Conservation Corps (CCC) spent \$2.5 billion to employ 3 million young men from 1933 to 1942 on works projects relating to the conservation and development of natural resources [Annual Report of the Secretary of the Treasury (1940, p. 27)]. Projects included planting trees and constructing and upgrading trails and facilities at more than 800 parks nationwide. The "other emergency spending" category includes grants to the Agricultural Adjustment Administration for farm subsidies aimed at raising agricultural prices, Reconstruction Finance Corporation loans, and other farm and housing assistance including the Home Owners' Loan Corporation created in June 1933 to assume mortgage debt of distressed homeowners amounting to \$3.1 billion [Studenski and Krooss (1952, p. 417)].

Regular operating expenditures declined under Roosevelt relative to Hoover, except in 1936 due to the payout of the veterans' bonus [Hausman (2016)]. Most of the decline in regular expenditures can be attributed to the shifting of existing RFC and public works expenditures to emergency categories starting in 1934 [Annual Report of the Secretary of the Treasury (1934, p. 5). Although the Economy Act of 1933 cut \$243 million of regular operating expenditures by reducing the pay of civilian and military federal workers by 15 percent and decreasing veterans' benefits by 10 percent, Congress eventually restored 95 percent of the pay cuts essentially undoing the budgetary savings [Studenski and Krooss (1952, p. 404)].

5.2.2 Measuring Fiscal Impulses

Unbacked fiscal expansion changes the relevant measure of fiscal impulse from the surplus-output ratio to the surplus-debt ratio. In expression (3), the ultimate impact on aggregate demand, and the price level in that example, depends on total real backing—right side—relative to oustanding nominal liabilities—left side. Consider the flow government budget constraint written in nominal terms

$$B_t + S_t = (1 + i_{t-1})B_{t-1}$$

[20] https://slate.com/news-and-politics/2009/01/four-million-jobs-in-two-years-fdr-did-it-in-two-months.html

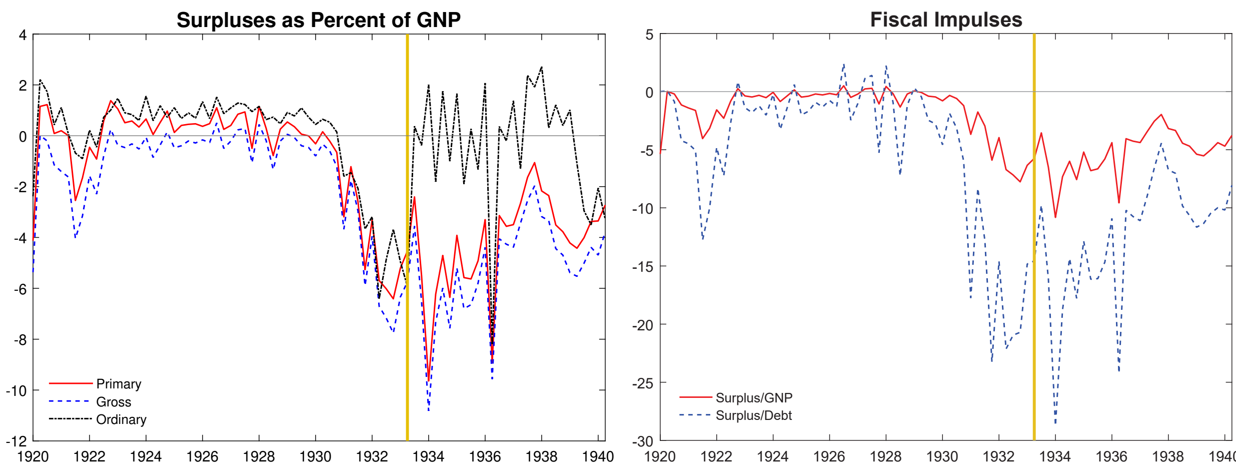

Figure 6: Surpluses defined as total receipts less expenditures, ordinary or total. Primary surplus is gross surplus less net interest payments. Vertical line marks when the United States abandoned the gold standard. Fiscal impulses defined as surplus-GNP, as in Keynesian hydraulics, and surplus-debt, as in unbacked fiscal expansion. Sources: Federal Reserve Board (1943) from NBER Macrohistory Database, and Balke and Gordon (1986). See Appendix A for more details on the data series.

where S is the primary surplus. Then the surplus-debt ratio is

$$\frac{S_t}{B_{t-1}} = (1 + i_{t-1}) - \frac{B_t}{B_{t-1}} \tag{4}$$

Declines in surpluses relative to debt raise nominal debt growth. Higher debt growth without correspondingly higher surpluses raises nominal wealth and nominal demand. Equation (4) highlights a key point: lower debt, B_{t-1}, implies larger nominal debt growth from a given deficit.

Fiscal stimulus from unbacked expansion hinges on the amount of fiscal backing relative to outstanding debt. Keynesian hydraulics focuses narrowly on the size of deficits relative to the economy, leading to Brown's (1956) oft-cited conclusion: "Fiscal policy, then, seems to have been an unsuccessful recovery device in the 'thirties—not because it did not work, but because it was not tried" [pp. 863–866].

Figure 6, right panel, contrasts the two measures of fiscal impulse. Data to the right of the vertical line shows that once government debt expansion *could* be unbacked, deficits were very large relative to debt. Between April 1933 and June 1940, primary deficits averaged 5.2 percent of GNP, but 12.5 percent of debt, almost two-and-a-half times larger.[21]

5.3 Keynesian Hydraulics vs. Unbacked Fiscal Expansion

Economic theory supports the unbacked fiscal expansion narrative. Unbacked fiscal expansions generally have much larger fiscal spending and tax multipliers than those that arise

[21] Aside from World War II, only the decades of the 1970s and 1980s exhibit comparable deficit-debt ratios.

under Keynesian hydraulics. This section applies the reasoning in Beck-Friis and Willems (2017) to contrast fiscal impacts under the two monetary-fiscal regimes.[22]

Consider a new Keynesian model given by the aggregate demand and supply relations

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1} - r_t^n\right) \\ \pi_t &= \kappa x_t + \beta E_t \pi_{t+1} \end{aligned}$$

where x_t is the output gap; π_t inflation, i_t the nominal interest rate; and r_t^n the natural real rate of interest. The intertemporal elasticity of substitution and discount factor satisfy $\sigma > 1$ and $0 < \beta < 1$. Giving focus to fiscal disturbances, the natural rate of interest is

$$r_t^n = \frac{1}{\sigma + \omega^{-1}} F_t$$

where F_t is an *i.i.d.* exogenous government purchases shock and $\omega > 0$ the inverse Frisch elasticity.

The government issues one-period nominal debt to satisfy

$$b_t = (F_t - T_t) + \beta^{-1}\left(b_{t-1} - \delta \pi_t\right) + \delta i_t$$

where

$$\delta = \frac{bP^b}{y}$$

is the steady-state ratio of the market value of debt to GDP; b_t a measure of the real value of debt outstanding; and T_t taxes. We close the model with monetary and tax policy rules

$$\begin{aligned} i_t &= \phi_\pi \pi_t \\ T_t &= \phi_T b_{t-1} + \tau_t \end{aligned}$$

where $\phi_\pi, \phi_\tau > 0$ and τ_t is an *i.i.d.* exogenous process.

Result 5. *Government spending and transfer impacts from unbacked fiscal expansions typically exceed those from Keynesian hydraulics alone.*

Keynesian Hydraulics. The standard Keynesian view of fiscal expansions assumes

$$|\phi_\pi| > 1$$

and

$$\left|\beta^{-1} - \phi_T\right| < 1.$$

This assignment of policy implies monetary policy actively stabilizes inflation, while tax policy passively stabilizes debt. Equilibrium inflation and debt dynamics are

$$\begin{aligned} \pi_t &= \frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi + 1\right)\left(\sigma + \omega^{-1}\right)} F_t \\ b_t &= (F_t - \tau_t) + \beta^{-1} b_{t-1} - \delta\left(\beta^{-1} - \phi_\pi\right)\pi_t. \end{aligned}$$

[22] To make the exposition transparent, we log-linearize the model around its deterministic steady state. Appendix B provides details, extensions to long-duration debt, and numerical examples.

Inflation rises when government purchases increase. This is the conventional multiplier effect of government spending associated with Keynesian hydraulics. Inflation is independent of debt and tax/transfer policy.

Define the impulse response function for inflation in response to a one percent of GDP *i.i.d.* increase in spending or transfers in period 0 under Keynesian hydraulics as $KH_F^{\pi}(j)$ and $KH_T^{\pi}(j)$ for $j \geq 0$. Then the impact effects are

$$\begin{aligned} KH_F^{\pi}(0) &= \frac{\kappa\sigma}{(\kappa\sigma\phi_\pi + 1)(\sigma + \omega^{-1})} \\ KH_T^{\pi}(0) &= 0 \end{aligned}$$

in period 0 and

$$\begin{aligned} KH_F^{\pi}(j) &= 0 \\ KH_T^{\pi}(j) &= 0 \end{aligned}$$

in subsequent periods $j > 0$.

Unbacked Fiscal Expansion. An unbacked fiscal expansion places the restrictions

$$|\phi_\pi| < 1$$

and

$$\left|\beta^{-1} - \phi_T\right| > 1$$

on the policy parameters. Assume taxes are purely exogenous so that $\phi_T = 0$. Then aggregate dynamics are

$$\pi_t = \delta^{-1}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1} + (1-\beta\lambda_2)(F_t - \tau_t)\right)\left(\beta^{-1} - \phi_\pi\right)^{-1} + \beta\lambda_2 KH_F^{\pi}(0)F_t \quad (5)$$

$$b_t = \lambda_2 b_{t-1} + \beta\lambda_2(F_t - \tau_t) + \delta\left(\left(\phi_\pi - \beta^{-1}\right)\right)\beta\lambda_2 KH_F^{\pi}(0)F_t \quad (6)$$

where $0 \leq \lambda_2 < 1$ is the model's only eigenvalue inside the unit circle. In contrast to Keynesian hydraulics, an unbacked fiscal expansion makes inflation depend on all aspects of fiscal policy. Debt has monetary consequences, and the timing of taxes and transfers matter.

Define the impulse response function for inflation in response to a one percent of GDP *i.i.d.* increase in spending or transfers in period 0 in an unbacked fiscal expansion to be $UB_F^{\pi}(j)$ and $UB_T^{\pi}(j)$ for $j \geq 0$. Then the dynamic impacts of a reduction in taxes or increase in transfers on inflation are

$$\begin{aligned} UB_T^{\pi}(0) &= \delta^{-1}(1-\beta\lambda_2)\beta \\ UB_T^{\pi}(j) &= \lambda_2^j\beta\lambda_2 \end{aligned}$$

for $j > 0$. The fact that the impact effect depends on the inverse of the debt-to-GDP ratio is relevant for later results—the smaller is outstanding debt relative to GDP the larger will be the impacts. Equivalently, the larger are deficits relative to outstanding debt, the larger the expansionary effect. Early in the recovery, deficits were almost 30 percent of debt [figure 6]. This measure of impulse is fundamentally different to standard Keynesian hydraulics logic.

We can now express the effects of government purchases on inflation as

$$UB_F^{\pi}(j) = KH_F^{\pi}(j) + \left[1 - \delta\left(\beta^{-1} - \phi_{\pi}\right) KH_F^{\pi}(0)\right] UB_T^{\pi}(j) \tag{7}$$

for $j \geq 0$. The effects of government purchases on inflation—and output, though not shown—in an unbacked fiscal expansion decompose into two components. The first comes from the effects of spending in the standard Keynesian story, $KH_F^{\pi}(j)$. This is the effect of spending when taxes back debt. The second term comes from the fact that spending does not herald future tax increases: it reflects the wealth effects of nominal debt issuance on aggregate demand. This is why the term depends on the tax multiplier: it captures the implicit reduction in taxes, of magnitude equal to the spending impulse, under an unbacked fiscal expansion.

A sufficient condition for larger impacts from an unbacked fiscal expansion than under Keynesian hydraulics is

$$\delta < \frac{\beta\left(\sigma + \omega^{-1}\right)}{\kappa\sigma}$$

which standard parameter values easily satisfy. For this same reason, Keynesian hydraulics are small relative to the effects of reduced taxes. This captures the effect of nominal debt dynamics arising from the commitment not to raise taxes—an effect theoretically and empirically large. For example, taking standard parameter values in the new Keynesian model, and a debt-to-GDP ratio of 40 percent in annual terms, the present discounted value of output resulting from a one percent *i.i.d.* increase in spending is 4.14 in an unbacked fiscal expansion versus 0.977 under Keynesian hydraulics. Prices rise by 0.65 in the long run, some 42 times the size of response under Keynesian hydraulics. While only meant to be suggestive, such differences are a robust feature of the model. There are reasons to think these magnitudes are understated relative to the environment of the 1930s. Infrastructure spending and transfers to constrained households with high marginal propensities to consume were important features of FDR's fiscal strategy, and both would deliver larger multipliers [table 3].

5.4 Government Debt and Unbacked Fiscal Expansion

The theory of unbacked fiscal expansion just described informs how we interpret government debt dynamics in the 1930s. The expression for equilibrium real debt, equation (6), implies that debt-financed fiscal expansion is consistent with stable debt, even when that expansion does not eventually bring forth higher taxes. Theory explains why significant growth in nominal debt did not undermine public credit, as many contemporaneous critics predicted.

Figure 7 summarizes key features of unbacked fiscal expansion. The figure uses the model in section 5.3 to plot responses to an *i.i.d.* increase in government purchases, with the nominal interest rate pegged in the left panel and responding weakly to inflation in the right panel. Fiscal policy sets taxes independently of debt. By pegging the nominal rate, monetary policy also pegs bond prices. All debt revaluation arises from a jump in the price level, so even on impact real debt rises less than nominal debt. Higher government spending raises expected inflation, so the real interest rate falls. The combination of surprise inflation and lower real debt service retire real debt back to steady state.

When monetary policy raises the nominal rate with inflation, bond prices fall to introduce an additional debt-stabilizing mechanism. The market value of real debt rises less than the par value. Because monetary policy raises the nominal rate, real interest rates decline less than under a pegged nominal rate.

With no response in future primary surpluses to the fiscal expansion, how can the value of debt rise, as it does in both panels of figure 7? The answer lies in real interest rates. Lower real rates raise the present value of the fixed stream of surpluses to increase the current value of real backing and, therefore, the value of debt.

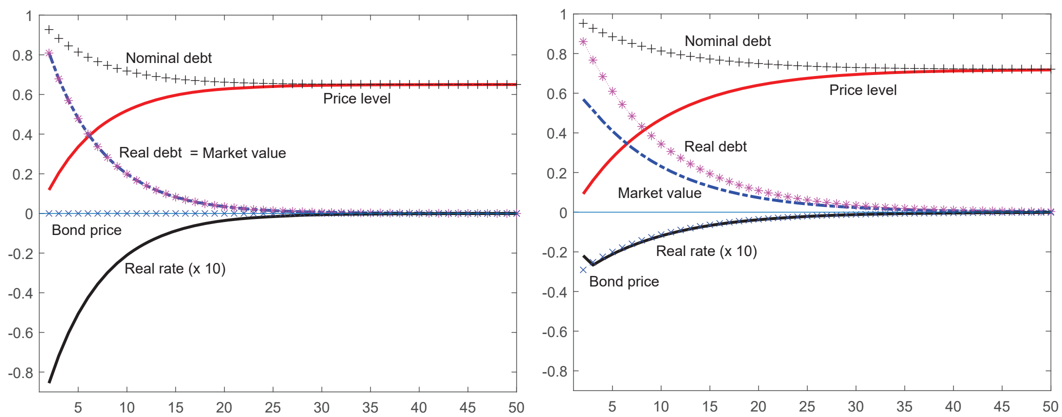

Figure 7: Responses to an *i.i.d.* increase in government purchases under unbacked fiscal expansion in the model of section 5.3. Left panel assumes monetary policy does not respond to inflation; right panel assumes monetary policy responds to inflation with a coefficient of 0.5. Taxes are independent of debt. Market value of debt is $P_t^l b_t$; see appendix B for details.

5.4.1 Developments in Government Debt

If FDR had intended to engineer an unbacked fiscal expansion, growth in government liabilities suggests he was successful. Nominal gross debt doubled during his first seven years in office. By comparison, seven fiscal years after the financial crisis in 2008, U.S. gross federal debt increased by a factor of 1.8.

The left panel of figure 8 plots index numbers for nominal and real federal debt. Taken together, the two panels highlight central features of unbacked fiscal expansions: despite increases in nominal debt, real debt rises less dramatically and there may be no increase at all in debt as a share of income. The index equals 100 in 1932Q2 to 1933Q1, the year leading up to America's departure from the gold standard. After declining for a decade, nominal debt began to rise in 1931; real debt started to increase a year earlier, due to deflation. From 1933Q2 until 1940Q2, the par value of nominal debt rose 112 percent, while real debt rose 82 percent. The ratio of these indexes reached its nadir when the country left gold and then rose 19 percent by 1940Q2, but 22 percent just before the 1937–1938 recession. Those changes in the ratio measure how much debt was devalued by a higher price level.[23]

[23] These numbers are nearly identical when measured in terms of the market value of debt.

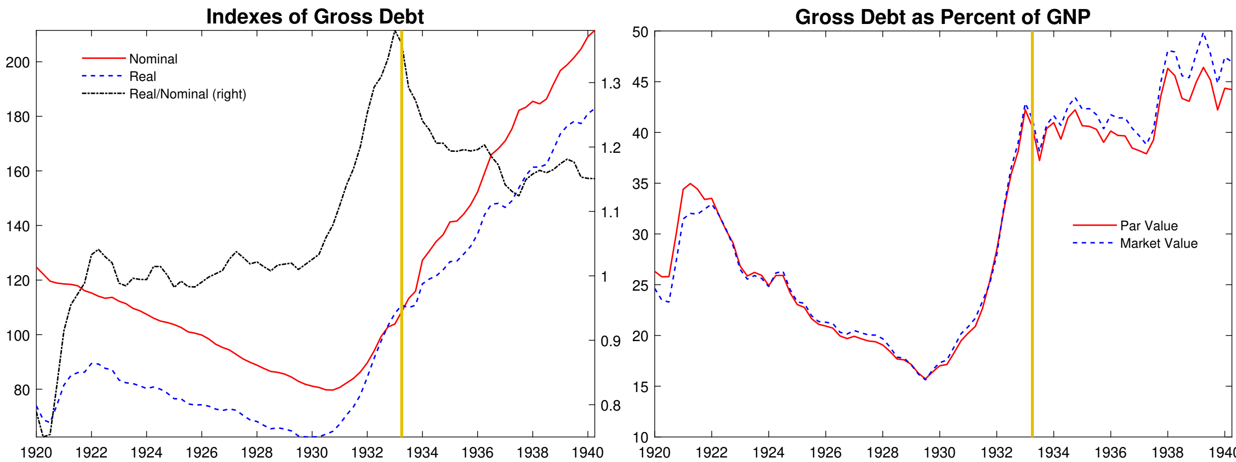

Figure 8: Par value of U.S. gross debt, real debt is par value deflated by GNP deflator. Converted to index numbers 100=1932Q2–1933Q1 (year before departure from gold standard). Nominal/Real is ratio of the two index numbers converted to percent. Par and market values of debt as percentage of nominal GNP. Vertical line marks when the United States abandoned the gold standard. Sources: Authors' calculations, Balke and Gordon (1986).

More striking is the right panel of the figure. The debt-GNP ratio, whether measured at par or market value of debt, rose sharply from 15 percent in 1930 to 42 percent at the time gold was abandoned. Then it hovered around 40 percent for the next six years, until the recession raised the ratio. In the last few years of the decade, when Roosevelt abandoned the unbacked fiscal expansion policy, the debt-GNP ratio rose.

Figure 9 performs the accounting exercise that breaks the growth rate of the debt-GNP ratio in figure 8, B_t/P_tY_t, into growth rates of the three components. All three drove debt-output in the three years before Roosevelt took office. From the first quarter of 1933 on, nominal debt contributed to driving the ratio higher. That influence, though, was offset by higher prices and real GNP, with the exception of the recession of 1937–38.

5.4.2 RETURNS ON TREASURY BOND PORTFOLIO

To interpret data related to the government's bond portfolio, we require some notation.[24] With a complete and general maturity structure, the government's budget identity is

$$\sum_{j=0}^{\infty}\big(Q_t^D(t+j)+IP_t(t+j)\big)B_{t-1}(t+j)=P_ts_t+\sum_{j=1}^{\infty}Q_t^D(t+j)B_t(t+j) \tag{8}$$

where $Q_t^D(t)\equiv 1$ and $IP_t(t+j)$ is the interest payable on bonds outstanding at t that mature in $t+j$. $Q_t^D(t+j)$ is the dirty price of bonds, defined as the clean price plus accrued interest.

[24] Appendix A.3 details the definitions and calculations that follow.

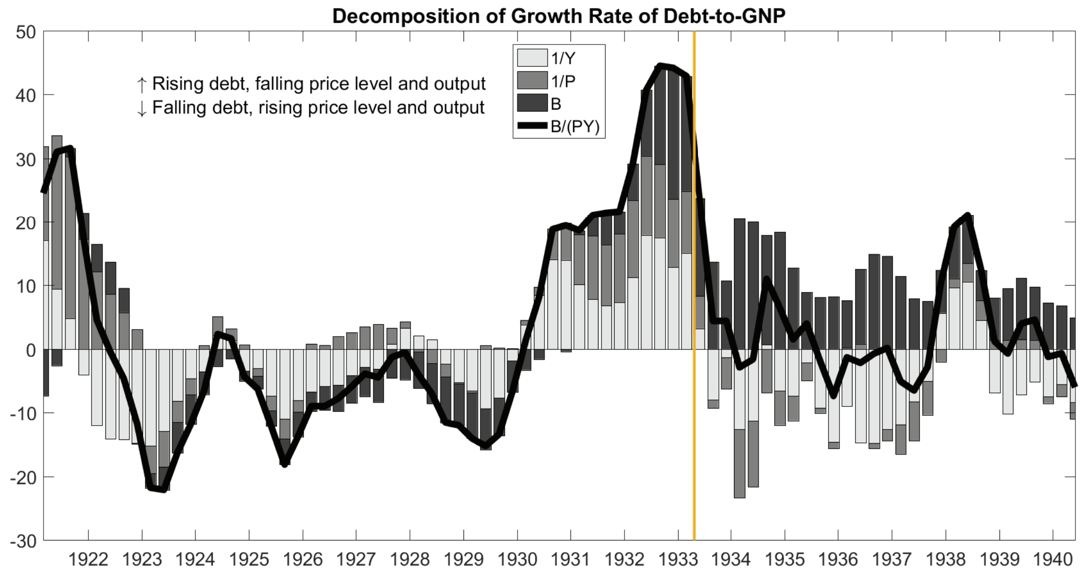

Figure 9: The four-quarter percentage change in debt-GNP ratio (solid line) decomposed into percentage changes of its components: nominal debt, the inverse of the price level, and the inverse of real GNP. Sources: Balke and Gordon (1986), Hall and Sargent (2015), and authors' calculations.

The market value of debt outstanding in period t is

$$P_t^M B_t^M \equiv \sum_{j=1}^{\infty} Q_t^D(t+j) B_t(t+j) \tag{9}$$

so the budget identity may be rewritten as

$$R_t^M P_{t-1}^M B_{t-1}^M = P_t s_t + P_t^M B_t^M \tag{10}$$

or, in real terms

$$r_t^M P_{t-1}^M b_{t-1}^M = s_t + P_t^M b_t^M \tag{11}$$

where $b_t^M \equiv B_t^M / P_t$ is the real par value of debt outstanding at t. The nominal and real rates of return on the portfolio—R_t^M and r_t^M—reflect *ex-post* returns.

With B_t^M the par value of debt and $P_t^M B_t^M$ the market value, $P_t^C B_{t-1}^M$ is the carry-over market value of debt. The growth rate in the market value of debt may be written as

$$\frac{P_t^M B_t^M}{P_{t-1}^M B_{t-1}^M} \equiv \underbrace{\frac{P_t^C B_{t-1}^M}{P_{t-1}^M B_{t-1}^M}}_{\substack{\text{nominal} \\ \text{rate of return}}} \cdot \underbrace{\frac{P_t^M B_t^M}{P_t^C B_{t-1}^M}}_{\text{size ratio}} \tag{12}$$

where P_t^C, defined in the appendix, reflects intermediate coupon payments and is the carry-over price of the portfolio. The first ratio on the right side of (12) is the nominal return, R_t^M, in (10). An *ex-post* real return simply deflates the nominal return by the inflation rate between $t-1$ and t to give r_t^M in (11).

	Gold Standard		Unbacked Fiscal Expansion	
	Monthly	*Annual*	*Monthly*	*Annual*
Nominal	0.24	2.91	0.23	2.72
Real	0.66	7.86	0.10	1.20
Surprise Real	0.40	4.81	−0.06	−0.76

Table 4: Returns on government bond portfolio at monthly and annual rates.

The surprise component in the real return on the bond portfolio is

$$\eta_t \equiv r_t^M - E_{t-1} r_t^M \tag{13}$$

This innovation can be decomposed into surprise capital gains and losses on the bond portfolio due to inflation and bond prices as

$$\eta_t = R_t^M \underbrace{(1/\pi_t - 1)}_{\text{due to price level}} + R_t^M \underbrace{\left(\frac{\sum_{j=0}^{\infty} \big(Q_t(t+j) - Q_{t-1}(t+j)\big) B_{t-1}(t+j)}{P_t^C B_{t-1}^M} \right)}_{\text{due to bond prices}} \tag{14}$$

Because η_t is the surprise revaluation on bonds carried into period t, its dollar magnitude is given by $\eta_t P_{t-1}^M B_{t-1}^M$. To gauge the quantitative importance of these revaluations, we compute them as a percentage of the market value of debt at the end of period t, $P_t^M B_t^M$.

Revaluation effects on nominal debt are a distinctive feature of unbacked fiscal expansion. Several patterns emerge from returns data in table 4. First, nominal returns are comparable across the gold standard and unbacked fiscal expansion period.[25] Second, real returns are substantially higher in the gold standard period than in the later period (average annual real returns of 7.86 percent versus 1.20 percent). Finally, on average, surprises in real returns are strongly positive in the early period (4.81 percent), but negative during the unbacked fiscal expansions (−0.76 percent).[26] These patterns are fully consistent with surprise inflation devaluing government debt during Roosevelt's administration.

Surprise real returns on government debt are quantitatively important. Figure 10 shows that as a percentage of the market value of outstanding debt, these revaluations—computed as $\eta_t P_{t-1}^M B_{t-1}^M / P_t^M B_t^M$—are a central feature of fiscal financing (left panel). After leaving the gold standard, surprise revaluations are both large and frequently negative. With debt at 40 percent of GNP, the revaluations are several percentage points of output, a substantial fraction of primary deficits.[27]

[25] Return data start in 1926, so "gold standard" refers to 1926Q1 to 1933Q1.

[26] Romer (1992, p. 778) estimates the *ex-ante* real commercial paper rate to find that it is negative nearly the entire unbacked fiscal expansion period except the 1937–1938 recession.

[27] Sims (2013) computes surprise capital gains and losses on U.S. government bonds since World War II to argue that these revaluation effects are important—the same order of magnitude as annual fluctuations in primary surpluses. Of course, *any* stochastic model with monetary and fiscal policy in which inflation and interest rates fluctuate will generate revaluation effects. This holds regardless of the monetary-fiscal policy regime, so merely finding revaluation effects during the recovery of the 1930s does not imply that the United States experienced an unbacked fiscal expansion. Such an inference requires identifying assumptions, to which we turn in section 6.

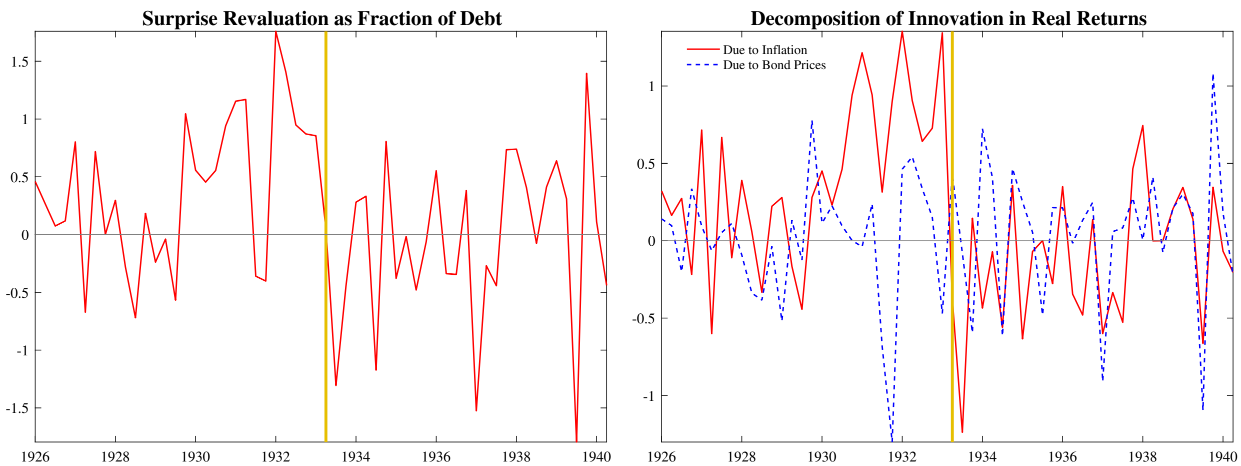

Figure 10: Left panel is surprises in real returns on bond portfolio as percentage of market value of outstanding debt, computed as $\eta_t P^M_{t-1} B^M_{t-1} / P^M_t B^M_t$. Right panel is Decomposition of surprises in real returns on bond portfolio into components due to unanticipated inflation and unanticipated bond prices. See appendix A.3 for details. Vertical line marks when the United States abandoned the gold standard. Source: Hall and Sargent (2015), CRSP, and authors' calculations.

The decomposition of surprise real returns, graphed in the right panel of figure 10, confirms that before leaving the gold standard, high realized real returns were driven by low inflation. The negative spike due to bond prices in 1931Q4 was created by the Fed's efforts to defend the gold parity by sharply raising discount rates. In the period of unbacked fiscal expansions, again with the exception of the jump in early 1938, surprise devaluations of debt from inflation dominate the surprise real returns.

The last informal piece of empirical evidence about the unbacked fiscal expansion appears in figure 11, which plots the relative price of the bond portfolio. This relative price is computed as the real market value of debt over the par value of debt, which yields P^M_t / P_t, the goods-price of government bonds. Bonds became increasingly costly in terms of goods throughout the gold standard period, reaching a peak in 1933Q1. With the departure from gold came a steady devaluation of the bond portfolio, bottoming out in the middle of 1937 when the 1937–1938 recession began. This cheapening of bonds is consistent with bondholders substituting out of debt and into buying goods and services—an increase in aggregate demand triggered by unbacked fiscal expansion.

6 Structural VAR Analysis

We turn now to more formal analysis of fiscal and monetary impacts over the period of unbacked fiscal expansions. Because the identified VAR methodology is well understood, we review it only briefly here.[28]

[28] See Leeper, Sims, and Zha (1996) or Christiano, Eichenbaum, and Evans (1999) for detailed surveys.

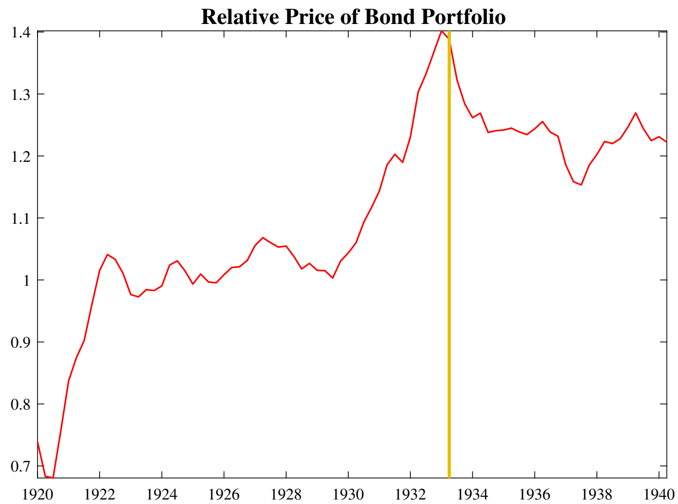

Figure 11: Relative price of the bond portfolio is the ratio of the real market value of debt to the par value of debt, roughly equivalent to the real "price" of the bond portfolio. Vertical line marks when the United States abandoned the gold standard. Source: authors' calculations.

6.1 VAR Methods

If y_t is a $k \times 1$ vector of time series, the economic structure is

$$A_0 y_t = A_+(L) y_{t-1} + \varepsilon_t \tag{15}$$

where $E\varepsilon_t\varepsilon_t' = I$ and ε_t is uncorrelated with y_s for $s < t$. The ε_t's are economically interpretable exogenous disturbances. The reduced-form is

$$y_t = B(L) y_{t-1} + u_t \tag{16}$$

where, assuming that A_0 is invertible, $B(L) = A_0^{-1}A_+(L)$, $u_t = A_0^{-1}\varepsilon_t$, and $Eu_tu_t' = A_0^{-1}(A_0^{-1})' = \Sigma$.

6.2 Data and Identification

We estimate a seven-variable monthly VAR from April 1933 to June 1940. The seven variables are: the commercial paper rate, i, (NSA), the monetary base, M, (NSA), federal primary surplus, S, (SA), the market value of nominal gross federal government debt, B, (NSA), the monetary gold stock, G, (NSA), monthly interpolated GNP deflator, P, (100 = 1926), and monthly interpolated real GNP, Y.[29]

[29] Primary surpluses were seasonally adjusted using the X-11 procedure in RATS. The deflator and real GNP were interpolated from Balke and Gordon's (1986) quarterly series using the Chow and Lin (1971) algorithm. Monthly series used to interpolate the deflator included M2, the consumer price index, the wholesale price index, the long-term yield on Treasury bonds (NBER Macrohistory Database, m13033a), and index composite wages (NBER Macrohistory Database, m08061c); series used to interpolate real GNP included industrial production, composite index of six roughly coincident series (NBER Macrohistory Database, m16003a); index of factory employment, total durable goods (NBER Macrohistory Database, m08146a), and production worker employment, manufacturing (NBER Macrohistory Database, m08010b). Appendices A.1 and A.2 describe fiscal data in detail and compare our series to three widely used sources—NBER Macrohistory Database, Firestone (1960), and Romer (1992).

VAR estimates employ the Sims and Zha (1998) prior, which allows for unit roots and cointegration, and probability bands are computed as in Sims and Zha (1999). All variables except the primary surplus and the interest rate are logged; the interest rate is divided by 100 to put it in percentage units. We include six lags and a constant.[30]

The identification aims to be consistent with actual policy behavior in the post-gold standard period of the 1930s. We impose restrictions only on A_0, the contemporaneous interactions among innovations in variables, leaving lags unrestricted. With monthly time series, this means every variable responds to past values of every other variable.

Money Supply: The supply of monetary base, M^s, depends on the short-term nominal interest rate, R, and the monetary gold stock, G. The decision about whether or not to sterilize gold inflows lay with the Treasury during this period, but in the case when inflows were not sterilized, there was a direct impact of G on M^s. In addition, the Federal Reserve might decide to adjust supply in order to influence interest rates, so we have the money supply rule

$$a_1 M_t^s = a_2 i_t + a_3 G_t + \varepsilon_t^{MP} \tag{17}$$

Money Demand: The demand for base money is a derived demand. Demand for nominal money balances, M^d, depends on the short-term nominal interest rate, the price level, P, and income, Y

$$a_4 M_t^d = a_5 P_t + a_6 i_t + a_7 Y_t + \varepsilon_t^{MD} \tag{18}$$

Fiscal Policy: Fiscal policy chooses the primary surplus, S. Revenues are procyclical and an unindexed tax code makes revenues depend on the price level. Because surplus movements in the period were dominated by FDR's "emergency spending" programs, which were a reaction to prevailing economic conditions, there was little contemporaneous reaction of fiscal choices to variables other than measures of the price level and real economic activity. We also permit a contemporaneous response of surpluses to the nominal market value of debt, B. This leads to the fiscal rule

$$a_8 S_t = a_9 B_t + a_{10} P_t + a_{11} Y_t + \varepsilon_t^{PS} \tag{19}$$

Government Debt: We measure government debt as the nominal market value of gross federal debt. Because bond prices react immediately to all shocks in the economy, B is an "information variable," in Leeper, Sims, and Zha's (1996) terminology. The debt equation is

$$a_{12} B_t = a_{13} i_t + a_{14} M_t + a_{15} S_t + a_{16} G_t + a_{17} P_t + a_{18} Y_t + \varepsilon_t^B \tag{20}$$

Gold: With the passage of the Gold Reserve Act in January 1934, the Treasury bought all gold supplied at the price chosen by the Treasury and the President, which was $34.00 an ounce. This made the demand for gold perfectly elastic at that price. Supply of gold to America, on the other hand, was driven by both exogenous political conditions in Europe and endogenous factors within the United States. Among those endogenous factors were the relative strength of the U.S. recovery, U.S. willingness to buy unlimited quantities of gold at a high price, increased sale of U.S. merchandise abroad as the dollar depreciated, the

[30] In notation analogous to that in Sims and Zha (1998), these results set the hyperparameters for the prior as $\mu_1 = 0.6, \mu_2 = 0.3, \mu_3 = 1.0, \mu_4 = 1.75, \mu_5 = 2.0, \mu_6 = 2.0$. The prior was chosen based on the model's marginal data density. See figure 40 in appendix D for the model's unconditional forecasts under this prior.

inflow of capital to the United States, and foreign-owned capital sent to U.S. to build up dollar balances or to purchase American securities [Paris (1938)]. We model the supply of monetary gold as a function of the nominal interest rate and goods-market conditions:

$$a_{19}G_t = a_{20}i_t + a_{21}P_t + a_{22}Y_t + \varepsilon_t^G \tag{21}$$

Goods Market: We refer to the remaining variables in the VAR—the price level and real GNP—as "goods market variables." We treat these as inertial variables that are predetermined and obey a recursive ordering. The limitation in this assumption is that we do not distinguish between the two "goods market shocks," treating them simply as disturbances unrelated to the behavior identified in other equations

$$a_{23}P_t = a_{24}Y_t + \varepsilon_t^P \tag{22}$$

$$a_{25}Y_t = \varepsilon_t^Y \tag{23}$$

Predeterminedness of goods market variables is not a stringent restriction: it says that the price level and output do not respond to non-goods-market shocks within the month, as assumption that Romer (1992) employs with annual data.

The identification determines the money stock, the nominal interest rate, the gold stock, nominal debt, and the primary surplus simultaneously, with P and Y predetermined.

With 28 distinct moments in the covariance matrix of innovations and 25 freely estimated parameters, the system is overidentified. If data strongly reject the overidentifying restrictions, the estimated exogenous disturbances may not be mutually uncorrelated, muddling the economic interpretations of the shocks.

Table 5 reports posterior modes and 68-percent probability intervals for the estimated parameters. The money supply rule is consistent with the central bank expanding high-powered money in response to surprise increases in the nominal interest rate. Contemporaneous interactions between gold and the base are weak. Money demand has a negative but weak interest elasticity, but positive and significant short-run income and price elasticities. A negative contemporaneous relation between debt and surpluses likely emerges from the government's flow budget constraint. Primary surpluses are weakly connected to goods market innovations, although over longer horizons real economic activity does affect surpluses. Real income innovations raise the monetary gold stock, which is consistent with the U.S. economic recovery inducing gold inflows from abroad, which are met by an elastic demand for gold by the Treasury. Finally, the nominal market value of government bonds is significantly associated with contemporaneous innovations in variables, reflecting the responsiveness of asset prices to news. Those contemporaneous relationships make good economics sense: a surprisingly high market value of bonds is associated with negative innovations in the interest rate, money stock, primary surpluses, and the price level, but positive innovations in gold. Strong effects are associated with the interest rate and inflation, as theory would suggest.[31]

6.3 Primary Surplus Impacts

Figure 12 reports the dynamic impacts of a surprise decrease in the real primary surplus during the unbacked fiscal expansion period. The one standard deviation initial shock raises

[31] Appendix D reports that the exogenous shocks in this model are mutually uncorrelated [see table 10].

$$
\begin{aligned}
\underset{(1.133,2.101)}{1.989R} &= \underset{(-.014,.037)}{.022M^s} + \underset{(-.006,.010)}{.004G} + \varepsilon^{MP} \\
\underset{(.061,.080)}{.073M^d} &= \underset{(-1.155,.318)}{-.688R} + \underset{(.004,.044)}{.028P} + \underset{(.001,.016)}{.009Y} + \varepsilon^{MD} \\
\underset{(.0045,.0054)}{.0050S} &= \underset{(-.039,-.006)}{-.023B} - \underset{(-.040,.001)}{.020P} - \underset{(-.009,.006)}{.001Y} + \varepsilon^{PS} \\
\underset{(.013,.019)}{.018G} &= \underset{(-1.262,.602)}{-.457R} + \underset{(-.010,.030)}{.013P} + \underset{(.001,.017)}{.010Y} + \varepsilon^{G} \\
\underset{(.077,.094)}{.087B} &= \underset{(-1.061,-.563)}{-.826R} - \underset{(-.035,-.018)}{.027M} - \underset{(-.0019,.0004)}{.0008S} + \underset{(.003,.007)}{.005G} \\
&\quad - \underset{(-.048,-.008)}{.028P} + \underset{(-.001,.014)}{.007Y} + \varepsilon^{B} \\
\underset{(.159,.186)}{.172P} &= \underset{(.007,.022)}{.015Y} + \varepsilon^{P} \\
\underset{(.060,.070)}{.065Y} &= \varepsilon^{Y}
\end{aligned}
$$

Table 5: Posterior mode estimates of parameters in the A_0 matrix. 68-percent probability intervals appear in parentheses base on 500,000 draws from the posterior distribution. Coefficients and probability intervals in the table are divided by 1000.

the primary deficit by \$0.21 billion, which is about half of the average annualized monthly deficit in the sample. Because the deficit decays rapidly, the total increase over the three-year forecast horizon is only \$0.51 billion. This is a relatively small and transitory fiscal impulse. Higher deficits do not bring forth higher future surpluses, lending support to the interpretation that fiscal expansion is unbacked.

Higher deficits produce Keynesian impacts. Prices and output, which the identification prevents from rising contemporaneously, steadily increase and significantly so. Monetary policy makes no effort to offset the inflationary consequences of the fiscal expansion, suggesting the Fed behaves passively. Nominal interest rates fall slightly in the short run. The lower nominal rates, together with higher expected inflation, drive *ex-ante* real rates lower. Lower real rates induce households and firms to shift demand for goods into the present.

New nominal bonds finance the higher deficits. Debt jumps on impact and remains elevated. Economic recovery encourages gold to flow into the United States. By choosing not to sterilize gold inflows, the Treasury allows the monetary base to expand to accommodate rising demand for money.

Looking down the column in figure 12 it is easy to see the conventional monetary narrative of the recovery that Friedman and Schwartz (1963), Romer (1992), and Steindl (2004) recount.[32] The initial revaluation of gold, together with the steady inflows of gold largely

[32]Friedman and Schwartz give this narrative a different twist than Romer. Friedman and Schwartz (1963, p. 499) write that "...the rise in the money stock [from 1933 to 1937] was produced not by the monetary authorities but by gold inflow. Though accidental gold inflows served the same economic function as compliant monetary authorities would have, it occurred despite rather than because of the actions of unions, business organizations, and government in pushing up prices." Romer, in contrast, attributes much of the growth in base money to a policy choice—the Treasury's decision not to sterilize the inflows.

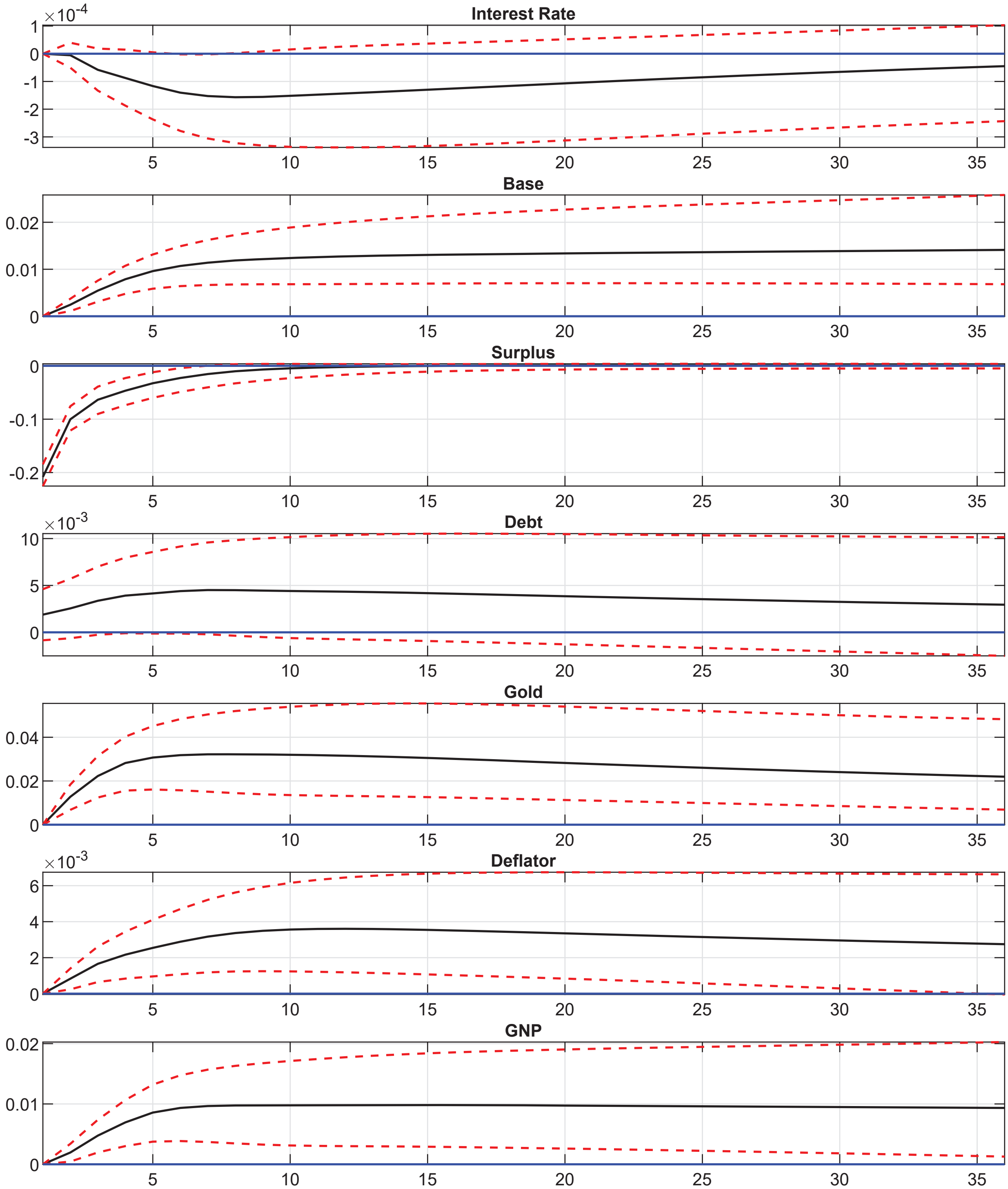

Figure 12: Responses to an unanticipated decrease in the primary surplus in the unbacked fiscal expansion period (April 1933 to June 1940). Solid lines are modes and dashed lines are 68 percentile probability bands based on 500,000 draws from the posterior distribution of all the VAR parameters.

due to political uncertainty in Europe, were permitted by the Treasury to steadily increase the monetary base. Expansion in high-powered money stimulated real activity and raised prices. At the same time, enhanced confidence in banks after the early 1930s crises reduced cash hoarding and raised the income velocity of money to reinforce the expansionary effects of the growth in the base.

But the impulse responses create a problem for this conventional narrative. How does one reconcile monetary-induced economic recovery with the sharp short-run declines in primary surpluses and the persistent increase in nominal government debt? Existing literature does not address this question, primarily because the fiscal dimensions have not been fully integrated with the monetary interpretations of the recovery.

6.4 Monetary Impacts

Higher deficits generate positive comovements among output, the price level, the monetary base, and the gold stock. But that interpretation ascribes to fiscal policy a causal role. Perhaps those fiscally-induced correlations are but a small part of the story about the recovery. Perhaps other disturbances, unrelated to fiscal policy, generate the same comovements, but account for the bulk of fluctuations in output and prices, as the conventional monetary narrative maintains.

We address these concerns by examining four other impulse response functions. Figure 13 reports the dynamic impacts of four shocks related to the monetary sector—monetary policy, money demand, "government debt," and the supply of gold. Our identification does not attach any distinct behavioral interpretation to the shock in the equation for debt, but does to the other three disturbances.[33]

From early 1933 until December 1936, the Treasury opted not to sterilize gold inflows, which permitted the monetary base to expand along with the gold stock. We view figure 13 with an eye toward shocks that move base money strongly and persistently. The first two columns—a monetary policy contraction and an increase in money demand—generate such movements, but only monetary policy moves the gold stock appropriately, but then does so only insignificantly. In any case, neither shock has significant impacts on the price level or real GNP.

Shocks to the supply of gold—the fourth column of the figure—are the genesis of the conventional monetary narrative. They are an important source of gold-stock fluctuations, but little else. Positive innovations in gold are followed by a higher monetary base, although not significantly higher; if anything, though, higher monetary gold leads to lower prices and real GNP. The prime candidate for the monetary narrative shock does not deliver the required comovements of macro variables.

Innovations to the market value of debt are followed by a higher gold stock, monetary base (insignificantly), and price level (briefly significantly). But the debt innovation also creates a transitory decline in the surplus that arises from the coefficient on B in the third equation in table 5. As we found in figure 12, excursions in variables that look like the conventional monetary narrative arise only in conjunction with significant movement in primary surpluses.[34]

[33] Figure 41 in appendix D reports the full moving average representation.

[34] An identification that excludes B from the S equation does not produce a significant response of the

Only disturbances to the primary surplus generate the full set of movements in assets, the price level, and real GNP that would seem to align with existing monetary explanations of the recovery. Figure 12's responses to a shock that raises the primary deficit are fully consistent with what the theory predicts for the consequences of an unbacked fiscal expansion. We turn now to how important these fiscal disturbances are in generating fluctuations in the variables of interest.

6.5 Quantitative Importance

Variance and historical decompositions help to assess the quantitative importance of fiscal policy for the economic recovery. Those decompositions measure how important each exogenous shock is for future movements in the variables in the VAR. This section also reports output multipliers for several VAR specification.

6.5.1 Variance Decompositions Table 6 reports variance decompositions of the seven variables in the VAR at 6- and 36-month horizons. These statistics record how important disturbances in each exogenous shock are for explaining fluctuations in the variables, on average over the estimated sample.

Looking first at the goods market variables, P and Y, in the first two panels, aside from own shocks, the only disturbance that accounts for an important fraction of error variance in those variables comes from fiscal policy. A bit under 20 percent of goods market variables' fluctuations arise from shocks to the primary surplus. Monetary disturbances—monetary policy, money demand, and gold flows—jointly explain at most only 5.5 percent.

Money market shocks together account for substantial fractions of error variances in the monetary base (49 percent) and the commercial paper rate (87 percent). But primary surpluses explain almost all the remaining variance in base money (39 percent), suggesting a strong endogenous response of money to fiscal disturbances.

Primary surpluses—the fifth panel—are largely exogenous, with own shocks accounting for 92 percent of surplus movements at all horizons. This finding is consistent with Roosevelt's "emergency spending" driving fiscal policy in the period. Of course, this spending was most decidedly *not exogenous* in the usual meaning of the term because the spending was an explicit response to economic conditions in the preceding years.

Primary surplus disturbances explain 27 percent of the forecast error variance in gold. This finding belies the argument by Friedman and Schwartz (1963) and others that gold inflows were almost entirely due to European political turmoil and gold discoveries. Of course, a substantial fraction (60 percent) of fluctuations in gold are due to exogenous shocks in demand and supply for gold, which may reflect the factors that Friedman and Schwartz emphasize. As figure 13 reports, though, positive shocks to gold supply do not produce the higher price levels and output associated with recovery.

6.5.2 Historical Decompositions Decompose the vector of variables in the VAR, y_t, into the forecast conditional only on initial conditions using estimated VAR parameters,

surplus to a debt innovation, and it also does not produce strong responses in gold, money, or the price level. We do not report that result because the identification produces a non-zero correlation between the shocks to surpluses and money demand.

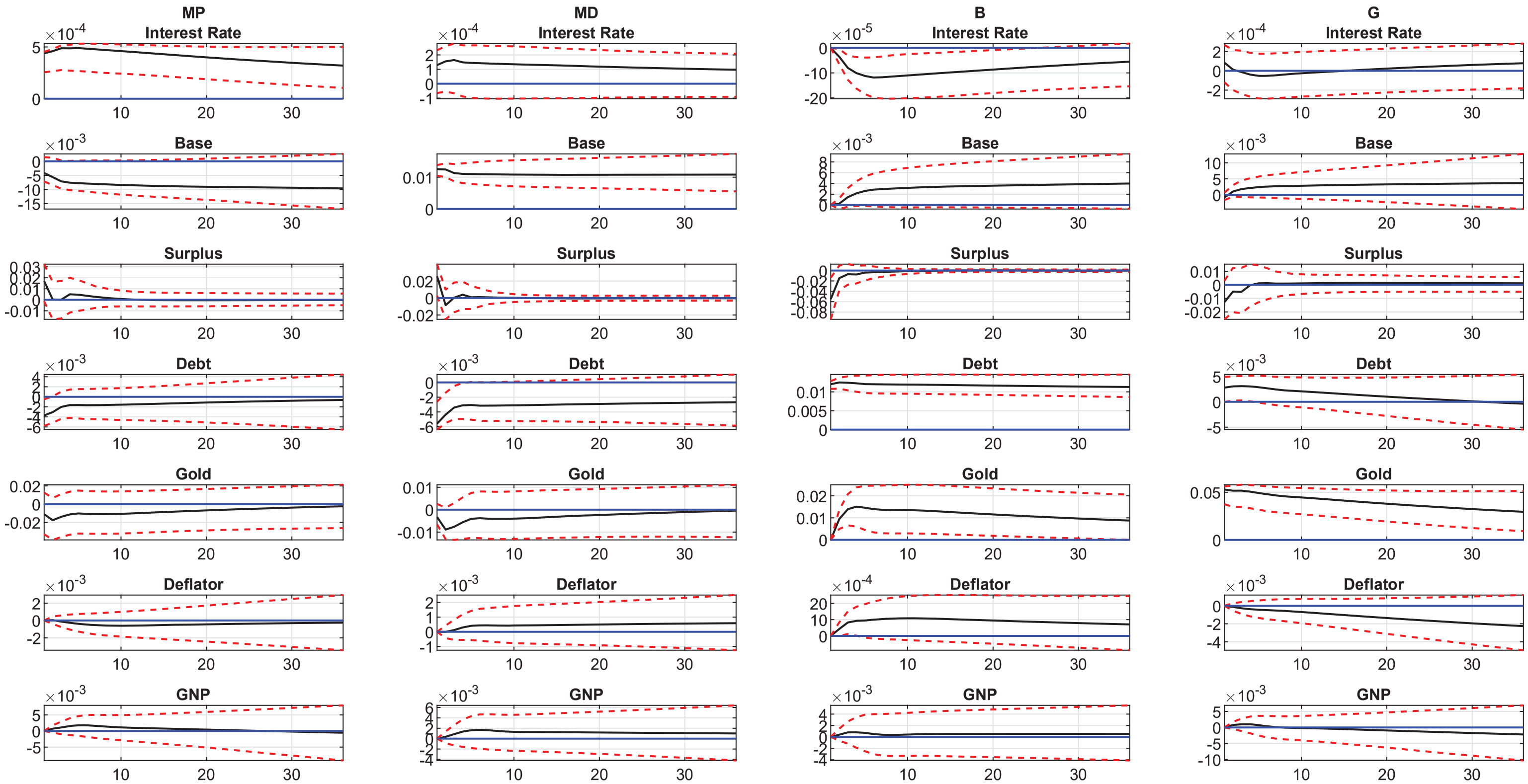

Figure 13: Responses to unanticipated shocks in the "monetary sector," which includes monetary policy (MP), money demand (MD), government debt (B), and the gold stock (G). Unbacked fiscal expansion period (April 1933 to June 1940). Solid lines are modal estimates; dashed lines are 68 percentile probability bands based on 500,000 draws from the posterior distribution of all the VAR parameters.

	Percent of P Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	0.2	0.2	7.9	1.2	0.2	82.4	8.0
36	0.3	0.4	16.3	1.4	3.4	70.0	8.2
	Percent of Y Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	0.4	0.3	8.9	0.1	0.1	0.2	90.0
36	0.1	0.3	17.8	0.1	0.3	0.0	81.4
	Percent of M Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	17.7	50.9	19.4	1.4	1.4	1.4	7.9
36	18.9	29.8	39.1	2.8	2.4	0.7	6.3
	Percent of R Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	82.8	8.0	2.7	2.7	0.9	1.1	1.7
36	79.9	7.2	5.2	3.5	1.0	2.1	1.2
	Percent of S Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	0.5	1.1	92.2	5.1	0.3	0.2	0.5
36	0.5	1.1	92.1	5.1	0.4	0.2	0.5
	Percent of G Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	4.3	1.0	15.3	4.1	69.3	2.2	3.8
36	2.5	0.4	26.9	4.9	59.3	3.3	2.6
	Percent of B Due to Shocks in						
Months	**MP**	**MD**	**FP**	**B**	**G**	**P**	**Y**
6	3.0	7.6	6.2	75.1	4.3	1.7	2.2
36	1.2	5.5	8.1	79.9	1.6	1.4	2.2

Table 6: Percentage of forecast error variance in GNP deflator (P), real GNP (Y), primary surplus (S), monetary base (M), commercial paper rate (R), monetary gold supply (G), and nominal market value of debt (B) attributable to shocks to each equation. Columns may not sum to 100 due to rounding.

$E_0 y_t$, and the sum of the realized exogenous shocks, ε_t, as

$$y_t = \sum_{s=0}^{t-1} C_s \varepsilon_{t-s} + E_0 y_t \tag{24}$$

Group the shocks into three bins: fiscal policy, $\varepsilon_t^F = \varepsilon_t^{PS}$, goods markets, $\varepsilon_t^M = (\varepsilon_t^P, \varepsilon_t^Y)$, and other, $\varepsilon_t^O = (\varepsilon_t^{MP}, \varepsilon_t^{MD}, \varepsilon_t^G, \varepsilon_t^B)$, with associated moving-average coefficients C^F, C^M, and C^O. Then (24) for variable j in period t may be written as

$$y_{jt} = E_0 y_{jt} + \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^F + \sum_{i=1}^{t} C_j(i)\varepsilon_t^M + \sum_{i=1}^{t} C_j(i)\varepsilon_t^O \tag{25}$$

where each summation is the cumulative impact of exogenous shocks on variable j from period 1 to period t.

Figures 14 and 15 decompose the historical paths of the price level and real GNP into three of the components in (25). After accounting for lags in the VAR estimation, forecasts run from October 1933 through June 1940. Solid lines are actual values, y_{jt}, and solid dotted lines are forecasts, $E_0 y_{jt}$. The remaining three lines are actual values less the contributions of primary surplus disturbances for the model in table 5 and two recursive orderings.

Forecasts of both the price level and real GNP rise monotonically over the period, suggesting that in the absence of shocks, deterministic dynamics over the sample would raise prices and output. The marginal contribution of surpluses appears as the vertical distance between the actual value and the value less that group's addition. A consistent pattern across both figures and all three identifications is that absent fiscal shocks, prices and output would have been lower over most of the sample, except around 1939.[35] Goods market disturbances are the biggest contributors to macroeconomic activity, but their impacts can be positive or negative, depending on the period.

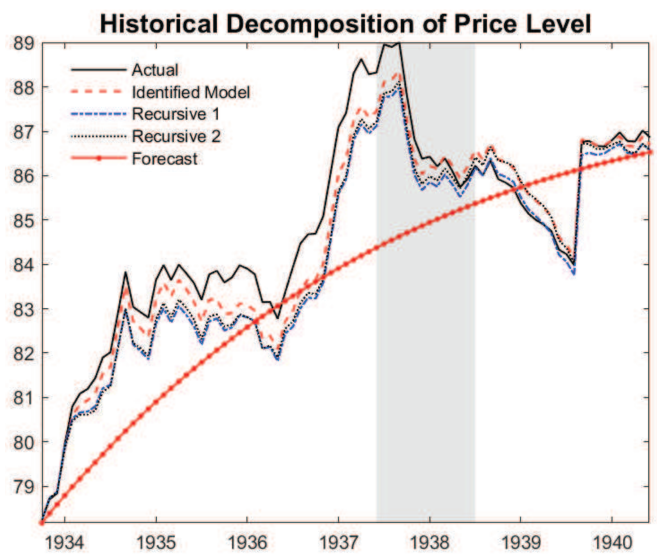

Figure 14: Contribution of primary surplus shocks to the path of the price level. In terms of components of equation (25), Actual is y_{jt}, Forecast is $E_0 y_{jt}$, other lines are $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^F$. Identified Model is from table 5; Recursive 1 orders variables P, Y, S, R, M, G, B; Recursive 2 orders variables P, Y, R, M, G, S, B.

[35] Figures 38 and 39 in appendix D show that the four shocks that constitute the "other" group—monetary policy, money demand, gold, and debt—have small effects that run counter to Roosevelt's economic objectives: prices and output would be a bit higher in the absence of those disturbances.

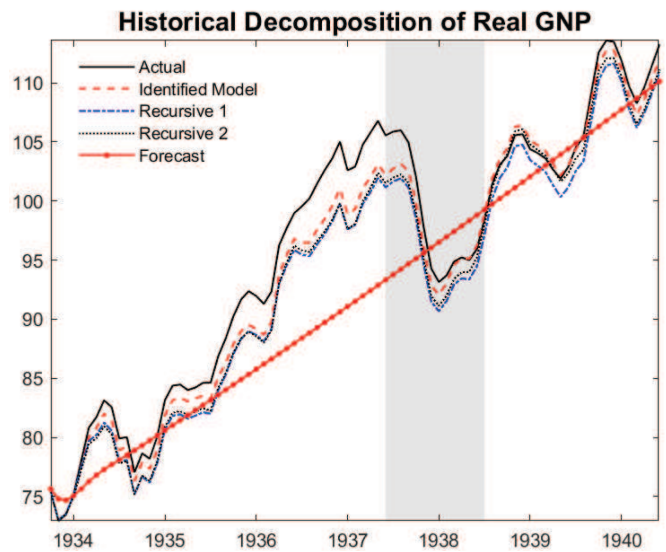

Figure 15: Contribution of primary surplus shocks to the path of real GNP. In terms of components of equation (25), Actual is y_{jt}, Forecast is $E_0 y_{jt}$, other lines are $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^F$. Identified Model is from table 5; Recursive 1 orders variables P, Y, S, R, M, G, B; Recursive 2 orders variables P, Y, R, M, G, S, B.

6.5.3 Output Multipliers

VAR estimates imply sizable output multipliers from increases in primary deficits. Figure 16 reports multipliers k periods after an increase in the deficit at time t, calculated as $\Delta Y_{t+k}/\Delta S_t$, as in Blanchard and Perotti (2002). In the VAR, real GNP is in logs, while the surplus is in real dollars, so we scale the impulse response by the mean of real GNP. Figure 15 makes clear that GNP grew over the sample period, so we compute the multipliers using two different measures of the mean—the full sample period and the first year of the sample.

Output multipliers are large and persistent. Taking the average of output over the full sample—top panel—the multiplier peaks at 4.5 after a year and remains close to that level. Credible sets expand over the forecast horizon, but remain above zero over the three-year horizon in the figure. The peak multiplier falls to 3.6 when the mean of real GNP is based on the first year of the sample. Multipliers are not appreciably different under the recursive orderings.[36]

7 Lessons for Today

We have argued that unbacked fiscal expansion was the critical policy action that contributed to recovery from the Great Depression. Roosevelt's "try anything" policies produced debt-financed primary deficits that remained in place until recovery was underway. Monetary policy combined with that fiscal policy to stabilize debt by preventing nominal interest rates from rising with inflation. The paper offers a variety of evidence that debt-financed deficits generated gold inflows and expanded the monetary base at the same time that they raised

[36] Appendix D reports results for a recursively ordered eight-variable VAR that splits the primary surplus into expenditures net of interest payments and tax receipts. Multipliers for government expenditures are comparable to those in figure 16, though less precisely estimated. Tax multipliers are highly uncertain.

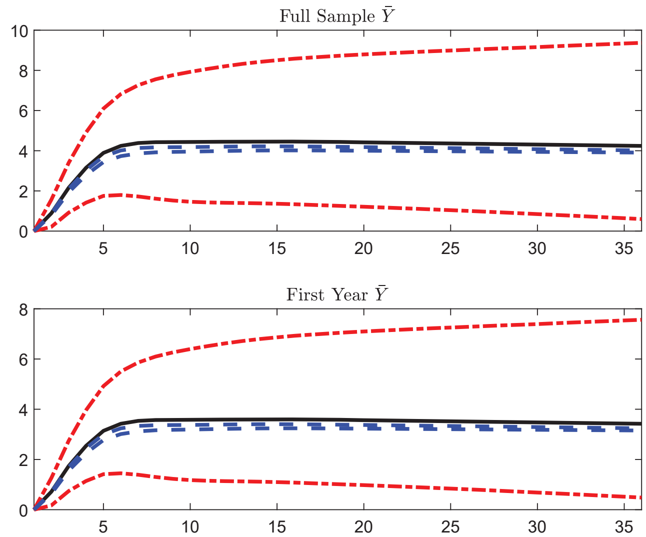

Figure 16: Output multipliers from a \$1 increase in the primary deficit, calculated as $\Delta Y_{t+k}/\Delta S_t$ at horizon k. Solid line is posterior mode from the identified model in table 5, dotted-dashed lines are 68 percent credible sets for that model, and dashed lines are posterior modes from recursive orderings used in figures 14 and 15. Top panel takes the mean of real GNP over the full sample, April 1933 to June 1940; bottom panel takes the mean over the first year of the sample.

prices and output. Gold inflows and higher base money that are not associated with higher deficits and nominal debt contain little predictive power for the GNP deflator and real GNP. Despite rapid growth in nominal debt between 1933 and 1937, the debt-GNP ratio was stable at about 40 percent, the level it had reached before the United States abandoned gold. This leads to the conclusion that unbacked fiscal expansion helped to lift the U.S. economy out of the depression without endangering the creditworthiness of the country.

Roosevelt's successful, if incomplete, reflation carries two important lessons for policymakers today. Many countries now suffer from low—below-target—inflation rates and tepid economic growth. Rather than relying on a joint monetary-fiscal attack on the problem, as Roosevelt did, these countries are leaning entirely on monetary policy. Central banks in the Euro Area, Sweden, Switzerland, and Japan have set policy interest rates below zero and undertaken large-scale asset purchases in an effort to reduce real interest rates and stimulate aggregate demand and inflation. This policy relies on intertemporal substitution induced by low real rates, rather than the wealth effects of an unbacked fiscal expansion. Fiscal policies in those areas, meanwhile, have lacked Roosevelt's initial single-minded goal to stimulate the economy, fluctuating between fiscal stimulus and fiscal austerity. Despite the Herculean efforts of monetary authorities for several years, reflation has been slow to come to those countries.

Ironically, those same countries and the United Kingdom, like the United States in the 1930s, are well positioned to undertake unbacked fiscal expansions. Monetary policies are already passive and central banks are on board to achieve higher inflation rates.[37]

[37]Because individual Euro Area countries do not control their monetary policy, it would require a co-

A second lesson from the Roosevelt policies is that fiscal stimulus and fiscal sustainability need not be in conflict. When the aim is to raise inflation and economic growth, higher nominal government debt—if people are convinced it does not portend higher future taxes—can achieve both the macroeconomic objectives and the goal of stabilizing debt. The two goals go hand-in-hand: higher inflation reduces the real value of the debt and higher economic growth raises surpluses and reduces debt-output ratios. But to engineer an unbacked fiscal expansion, governments must understand that rapid growth in *nominal* debt need not threaten fiscal sustainability, just as it didn't in 1930s America.

In the current atmosphere of what Sims (2016b) calls "hyper-Ricardian" beliefs about policy in which the public sees higher debt as bringing forth much higher surpluses in the future, it may be difficult for policymakers to credibly commit to an unbacked fiscal expansion. Here, too, FDR may have something to teach. Roosevelt never claimed to be aiming for what even he might have regarded as "irresponsible" fiscal policy. But his communications and actions made clear that he was willing to do whatever it took to bring the country out of the depression. Roosevelt was also agnostic, willing to experiment, even with what at the time seemed to be radical policies. He kept the public's attention on the policy *objectives*, objectives over which there was nearly universal agreement, rather than on the policy tools. And he made policy actions explicitly state-dependent. Today, state-dependence is *de rigueur* among central banks. It rarely enters the fiscal calculus.

ordinated unbacked fiscal expansion across member nations together with the ECB's pegging of interest rates.

Appendices

A Data

A.1 Net Interest

A.1.1 Interest Receipts This section details our sources and calculation of monthly net interest. Interest receipts are only available on a yearly basis in the *Annual Report of the Secretary of the Treasury on the State of the Finances.* From 1928 to 1940, we use the total of series called "Interest, exchange, and dividends on capital stock" or "Total interest, exchange, dividends" computed from the unrevised daily Treasury statements.[38] Disaggregated components of this series are available in tables based on warrants issued or revised daily Treasury statements.[39]

[38] From 1928 to 1933, interest receipts are split into general and special funds categories. We use total interest receipts.

[39] On Page 389 of the 1928 Annual Report, daily Treasury statements (unrevised) are defined as figures compiled "from the latest daily reports received by the Treasurer of the United States, from Treasury officers, and public depositaries holding Government funds. The daily Treasury statement, therefore, is a current report compiled from latest available information, and, by reason of the promptness with which the information is obtained and made public, it has come into general use as reflecting the financial operations of the Government covering a given period, and gives an accurate idea of the actual condition of the Treasury as far as it is ascertainable from day to day. This is known as 'current cash basis,' according to daily Treasury statements (unrevised)." Revised Treasury statements reflect actual transactions during the period under review. Page 373 of the 1929 annual report explains that receipts and expenditures are revised "on account of the distance of some of the Treasury offices and depositaries from the Treasury, it is obvious that the report from all officers covering a particular day's transactions can not be received and assembled in the Treasury at one time without delaying for several days the publication of the Treasury statement." Warrants issued (receipts) are defined based on Section 305 of the Revised Statues as, "receipts for all moneys received by the Treasurer of the United States shall be indorsed upon warrants signed by the Secretary of the Treasury, without which warrants, so signed, no acknowledgment for money received into the Public Treasury shall be valid. The issuance of warrants by the Secretary of the Treasury, as provided by law, represents the formal covering of receipts into the Treasury." Warrants issued (expenditures) are defined by the fact that, "The Constitution of the United States provides that no money shall be drawn from the Treasury but in consequence of appropriations made by law. Section 305 of the Revised Statutes requires that the Treasurer of the United States shall disburse the moneys of the United States upon warrants drawn by the Secretary of the Treasury. As the warrants are issued by the Secretary they are charged against the appropriate appropriations provided by law. Some of these warrants do not represent actual payments to claimants, but are merely advances of funds to be placed to the credit of disbursing officers of the Government with the Treasurer of the United States for the payment of Government obligations. The disbursing officer then issues his check on the Treasurer in payment of such obligations. As far as the appropriation accounts are concerned, the warrants issued and charged thereto constitute expenditures, but it will be observed that such expenditures necessarily include unexpended balances to the credit of the disbursing officers. Under normal conditions these balances over a period of several years fluctuate very little in the aggregate, and the difference between the total expenditures on a warrant basis and a cash basis (revised) is immaterial.

Table 1.—*Receipts and expenditures for the fiscal year 1928, classified according to funds*

[On basis of daily Treasury statements (revised), see p. 389]

	General funds	Special funds (various acts) [1]	Trust funds (various acts)[1]	District of Columbia (act June 29, 1922) [2]	Total
Ordinary receipts:					
Revenue receipts—					
Customs	$568, 154, 301. 86	$2, 291. 06			$568, 156, 592. 92
Internal revenue	2, 791, 799, 268. 52	393, 843. 15			2, 792, 193, 111. 67
Interest, premium, and discount	[3] 43, 706, 736. 12	136, 388, 871. 69			[3] 180, 095, 607. 81

[1] Exclusive of District of Columbia special and trust funds.
[2] Includes District of Columbia special and trust funds.
[3] Includes $11,212,165.68 receipts credited direct to appropriations.

Figure 17: 1928 Annual Report, page 391

Table 1.—*Receipts and expenditures for the fiscal year 1929, classified according to funds*

[On basis of daily Treasury statements (revised), see p. 373]

	General funds	Special funds (various acts)[1]	Trust funds (various acts)[1]	District of Columbia (act June 29, 1922)[2]	Total
ORDINARY RECEIPTS					
Revenue receipts:					
Customs	$602, 813, 939. 84	$6, 216. 58			$602, 820, 156. 42
Internal revenue	2, 939, 629, 903. 78	414, 582. 31			2, 940, 044, 486. 09
Miscellaneous taxes	6, 217, 857. 20	2, 842, 745. 01			9, 060, 602. 21
Interest, exchange, and dividends on capital stock	32, 783, 750. 49	137, 786, 535. 06			170, 570, 285. 55

Figure 18: 1929 Annual Report, page 374

In 1927, interest receipts are only available based on warrants issued.[40] Although the aggregate total of "Interest, premium, and discount" is no longer provided, the disaggregated elements of this total are included. We continue to included dividends, premiums, discounts, and exchanges to be consistent with the years when only the aggregate series is available.

[40] See footnote 39 for a description of warrants versus unrevised cash basis.

Table 4.—*Comparison of detailed receipts for the fiscal years 1927 and 1926*

[On basis of warrants issued, see p. 421]

	1927	1926	Increase, 1927	Decrease, 1927
Ordinary receipts:				
Customs—				
Duties	$603, 426, 552. 67	$577, 891, 561. 18	$25, 534, 991. 49	
Tonnage tax	2, 245, 912. 51	1, 825, 049. 44	420, 863. 07	
	605, 672, 465. 18	579, 716, 610. 62	25, 955, 854. 56	
Internal revenue—				
Income tax	2, 219, 952, 443. 72	1, 974, 104, 141. 33	245, 848, 302. 39	
Miscellaneous internal revenue taxes	648, 230, 548. 89	862, 252, 303. 79		$214, 021, 754. 90
Collections under enforcement of national prohibition act	[1] 501, 891. 11	[2] 415, 336. 63	86, 554. 48	
	2, 868, 684, 883. 72	2, 836, 771, 781. 75	245, 934, 856. 87	214, 021, 754. 90
Public lands (included in public domain receipts below).				
Miscellaneous—				
Interest, premium, and discount—				
Interest on bonds of foreign governments under funding agreements	139, 826, 159. 14	139, 804, 662. 99	21, 496. 15	
Interest on unfunded obligations of foreign governments	20, 563, 440. 76	19, 556, 925. 99	1, 006, 514. 77	
Interest on miscellaneous obligations	1, 092, 143. 04	989, 520. 80	102, 622. 24	
Interest on overpayments under section 209, transportation act, 1920, as amended	5, 244. 48	17, 811. 46		12, 566. 98
Interest on farm loan bonds	670, 060. 92	3, 648, 139. 22		2, 978, 078. 30
Interest on public deposits	4, 707, 706. 25	4, 530, 081. 48	177, 624. 77	
Interest on advance payments to contractors	44, 551. 39	194, 161. 69		149, 610. 30
Dividends on capital stock of the Panama Railroad owned by the United States	350, 000. 00	350, 000. 00		
Final dividend of the U. S. Sugar Equalization Board	(3)			
Gain by exchange	1, 707, 203. 70	24, 418. 98	1, 682, 784. 72	

[3] On July 15, 1926, the unexpended balance to the credit of the checking account of the United States Sugar Equalization Board on the books of the Treasurer of the United States amounting to $11,370,621.39 was transferred to the warrant account, $5,000,000 of which was covered into the Treasury to the credit of the appropriation as a repayment of capital stock originally advanced therefrom; the remainder, $6,370,621.39, was covered into the Treasury as "Miscellaneous Receipts—final dividends of United States Sugar Equalization Board." Since this transfer of funds from one account to another is merely an adjustment between accounts in this fiscal year of cash transactions occurring in prior fiscal years, the items have not been included in the receipts or expenditures as they did not affect the cash in the Treasury during the current fiscal year.

Figure 19: 1927 Annual Report, page 431

Starting in 1922, interest receipts, premium, discounts, and exchanges are no longer given as separate categories. The components of federal receipts are listed alphabetically.[41]

Comparison of receipts, fiscal years 1922 and 1921, on the basis of warrants issued (net).

	1922	1921	Increase, 1922.	Decrease, 1922.
Customs	$357,544,712.40	$308,025,102.17	$49,519,610.23	
Internal revenue:				
Income and profits taxes	2,086,918,464.85	3,228,137,673.75		$1,141,219,208.90
Miscellaneous	1,121,239,843.45	1,351,835,935.31		230,596,091.86
Sales of public lands	895,391.22	1,530,439.42		635,048.20
Alaska fund	136,053.10	174,329.90		38,276.80
Assessments on Federal reserve banks for salaries, etc.	3,067,169.36	4,819,339.72		1,752,170.36
Assessments on national banks for expenses of examiners	2,012,600.00	1,583,037.11	429,562.89	
Consular fees	6,707,058.72	5,676,850.61	1,030,208.11	
Customs fees, fines, penalties, services of officers, etc.	1,032,589.34	1,173,285.63		140,696.29
Commerce collections	239,432.57	305,904.84		66,472.27
Donation of royalty on machine guns		520,266.12		520,266.12
Depredations on public lands	60,149.90	68,646.25		8,496.35
Deposits for surveying public lands	68,461.03	62,324.51	6,136.52	
District of Columbia general receipts	14,777,218.19	14,439,985.93	337,232.26	
District of Columbia sources	457,798.25	561,106.29		103,308.04
Discount on bonds, notes, and certificates purchased	3,436,145.91	10,675,194.55		7,239,048.64
Earnings on radio service	369,735.67	666,371.84		296,636.17
Federal land banks, liquidation of capital stock	1,057,830.00	954,835.00	102,995.00	
Food Administration		37,078,988.55		37,078,988.55
Forest Service, cooperative fund	1,394,826.71	1,946,041.18		551,214.47
Fees on letters patent	2,875,013.15	2,696,502.46	178,510.69	
Forest reserve fund	5,125,668.20	2,591,297.93	2,534,370.27	
Franchise tax (surplus earnings of Federal reserve banks)	59,974,465.64	60,724,742.27		750,276.63
Funds contributed for river and harbor improvements	2,930,051.68	3,774,947.68		844,896.00
Gain by exchange	7,245,624.49	19,008.08	7,226,616.41	
Grain Corporation, decrease of capital stock	25,000,000.00	100,000,000.00		75,000,000.00
Housing Corporation, operations and disposal of properties	4,523,207.53	4,240,055.17	283,152.36	
Farm loan bonds:				
Principal	44,400,000.00		44,400,000.00	
Interest	8,611,170.08	8,306,075.00	305,095.08	
Foreign loans:				
Principal	49,114,107.46	83,678,223.38		34,564,115.92
Interest	6,607,723.54	18,327,306.91		11,719,583.37
Interest on foreign obligations, sale of surplus property, War Department	21,107,317.25	12,701,508.93	8,405,808.32	
Interest on public deposits	7,388,278.07	5,668,852.42	1,719,425.65	
Interest on loans to railroad companies	[1] 3,000.00	[2] 84,000.00		$1,000.00
Interest on advance payments to contractors	14,300.29	667,383.05		653,082.76
Immigrant fund	2,517,823.19	5,767,893.69		3,250,070.50
Judicial fees, fines, penalties, etc	5,132,937.71	4,382,676.51	750,261.20	
Land fees	1,139,880.25	1,753,759.83		613,879.58

[1] Exclusive of $12,906,960.89 interest received on account of loans to railroads under section 210 of the transportation act of 1920, and $27,324,181.14 interest collected under the provisions of the Federal control act of Mar. 21, 1918, which amounts were credited, respectively, to the revolving funds, "Loans to railroads" and "Federal control of transportation systems."

[2] Exclusive of $4,369,007.49 interest received on account of loans to railroads under sec. 210 of the transportation act of 1920, and $26,415,163.88 interest collected under the provisions of the Federal control act of Mar. 21, 1918, which amounts were credited, respectively, to the revolving funds, "Loans to railroads" and "Federal control of transportation systems."

Figure 20: 1922 Annual Report, page 107

Interest receipts on foreign obligations – a subset of total interest receipts – are available on an unrevised cash basis. This data is also available at a monthly frequency for fiscal years 1929 to 1931 and 1936 to 1940. The location of these data is included in Table 7.

[41] Net warrants issued includes unexpended balances to the credit of disbursing officers at the end of the year, but not expenditures under such unexpended balances at the beginning of the year.

Table name	Year	Basis	Page number
Comparison of receipts, fiscal years 1920 and 1919	1920	warrant	262/263
Comparison of receipts, fiscal years 1921 and 1920	1921	warrant	140
Receipts and expenditures for fiscal years 1920 and 1921 (int. on foreign obligations)		unrevised	152
Comparison of receipts, fiscal years 1922 and 1921	1922	warrant	107
Receipts and expenditures for fiscal years 1921 and 1922 (int. on foreign obligations)		unrevised	100
Comparison of receipts, fiscal years 1923 and 1922	1923	warrant	114
Receipts and expenditures for fiscal years 1922 and 1923 (int. on foreign obligations)		unrevised	107
Comparison of receipts, fiscal years 1924 and 1923	1924	warrant	131
Receipts and expenditures for fiscal years 1923 and 1924 (int. on foreign obligations)		unrevised	123
Comparison of receipts, fiscal years 1925 and 1924	1925	warrant	150
Receipts and expenditures for fiscal years 1924 and 1925 (int. on foreign obligations)		unrevised	141
Comparison of receipts, fiscal years 1926 and 1925	1926	warrant	429
Receipts and expenditures for fiscal years 1925 and 1926 (int. on foreign obligations)		unrevised	176
Comparison of receipts, fiscal years 1927 and 1926	1927	warrant	431
Receipts and expenditures for fiscal years 1926 and 1927 (int. on foreign obligations)		unrevised	30
Receipts and expenditures for the fiscal year 1928	1928	revised	391
Receipts and expenditures for the fiscal year 1928 (int. on foreign obligations)		unrevised	19
Receipts and expenditures for the fiscal year 1929	1929	revised	375
Receipts and expenditures for the fiscal year 1929 (int. on foreign obligations)		unrevised	20
Ordinary Receipts (monthly) (foreign obligations)		unrevised	535
Receipts and expenditures for the fiscal year 1930	1930	revised	469
Receipts and expenditures for the fiscal year 1930 (int. on foreign obligations)		unrevised	35
Ordinary Receipts (monthly) (foreign obligations)		unrevised	631
Receipts and expenditures for the fiscal year 1931	1931	warrant	426
Receipts and expenditures for the fiscal year 1931 (int. on foreign obligations)		unrevised	25
Receipts and Expenditures, by months (foreign obligations)		unrevised	575
Receipts and expenditures for the fiscal year 1932	1932	warrant	341
Receipts and expenditures for the fiscal year 1932 (int. on foreign obligations)		unrevised	27
Details of receipts by sources and funds, for the fiscal year 1933	1933	warrant	310
Receipts and expenditures for the fiscal year 1933 (int. on foreign obligations)		unrevised	19
Details of receipts by sources and funds, for the fiscal year 1934	1934	warrant	276
Receipts and expenditures for the fiscal year 1934 (int. on foreign obligations)		unrevised	20
Details of receipts by sources and funds, for the fiscal year 1935	1935	warrant	296
Receipts and expenditures for the fiscal year 1935 (int. on foreign obligations)		unrevised	32
Details of receipts by sources and funds, for the fiscal year 1936	1936	warrant	314
Receipts and expenditures for the fiscal year 1935 (int. on foreign obligations)		unrevised	35
Classified receipts and expenditures, monthly		unrevised	339/344
Actual receipts for the fiscal year 1937	1937	warrant	380
Classified receipts and expenditures for the fiscal years 1932 to 1937		unrevised	338
Classified receipts and expenditures, monthly (int. on foreign obligations)		unrevised	320/326
Actual receipts for the fiscal year 1937	1938	warrant	457
Classified receipts and expenditures for the fiscal years 1932 to 1938		unrevised	401
Classified receipts and expenditures, monthly (int. on foreign obligations)		unrevised	379/387
Details of receipts, by sources and accounts	1939	warrant	314
Classified receipts and expenditures, monthly (int. foreign obligations)		unrevised	337/345
Details of receipts, by sources and accounts.	1940	warrant	587
Classified receipts and expenditures, monthly (int. foreign obligations)		unrevised	612/619

Table 7: Table names and page numbers from the *Annual Reports of the Secretary of the Treasury* for interest receipts

A.1.2 Interest Expenditures Interest expenditures are available on a monthly basis starting in January 1922. For July 1919 to December 1921, interest expenditures are available on a quarterly frequency. We divide the quarterly data by three to interpolate monthly data for this time period.

Table name	Year	Basis	Page number
Preliminary Statement Showing Classified Expenditures (quarterly)... Receipts and expenditures of the Government for fiscal (yearly)...	1920	unrevised unrevised	see 1921 357 see 1926 448
Preliminary Statement Showing Classified Expenditures (quarterly)... Receipts and expenditures of the Government for fiscal (yearly)...	1921	unrevised unrevised	357 see 1926 448
Preliminary Statement Showing Classified Expenditures (monthly)... Receipts and expenditures of the Government for fiscal (yearly)...	1922	unrevised unrevised	103 see 1926 448
Preliminary Statement Showing Classified Expenditures (monthly)... Receipts and expenditures for fiscal years 1922 and 1923 (yearly)	1923	unrevised unrevised	110 107
Preliminary Statement Showing Classified Expenditures (monthly)... Receipts and expenditures for fiscal years 1923 and 1924 (yearly)	1924	unrevised unrevised	127 123
Preliminary Statement Showing Classified Expenditures (monthly)... Receipts and expenditures for fiscal years 1924 and 1925 (yearly)	1925	unrevised unrevised	145 142
Expenditures of the Government, by months for the fiscal year 1926 Receipts and expenditures of the Government for fiscal years (yearly)	1926	unrevised unrevised	452 450
Expenditures by months, classified according to... Ordinary receipts, expenditures chargeable against... (yearly)	1927	unrevised unrevised	463 448
Expenditures by months, classified according to... Receipts and expenditures for the fiscal year 1928	1928	unrevised unrevised	425 19
Expenditures by months, classified according to... Receipts and expenditures for the fiscal year 1929 (yearly)	1929	unrevised unrevised	414 20
Expenditures by months, classified according to... Receipts and expenditures for the fiscal year 1930 (yearly)	1930	unrevised unrevised	510 35
Expenditures by months, classified according to... Ordinary receipts, expenditures chargeable against... (yearly)	1931	unrevised unrevised	464 446
Expenditures by months, classified according to... Receipts and expenditures for the fiscal year 1932 (yearly)	1932	unrevised unrevised	371 27
Expenditures by months, classified according to... Receipts and expenditures for the fiscal year 1933 (yearly)	1933	unrevised unrevised	313 280
Expenditures by months, classified according to... Receipts and expenditures for the fiscal year... (yearly)	1934	unrevised unrevised	308 305
Expenditures by months, classified according to... Expenditures by months, classified according to (yearly)...	1935	unrevised unrevised	330 334
Classified receipts and expenditures, monthly Classified receipts and expenditures, monthly (yearly)	1936	unrevised unrevised	337 339
Classified receipts and expenditures, monthly Classified receipts and expenditures, monthly (yearly)	1937	unrevised unrevised	322/328 328
Classified receipts and expenditures, monthly Classified receipts and expenditures, monthly (yearly)	1938	unrevised unrevised	381/389 389
Classified receipts and expenditures, monthly Classified receipts and expenditures, monthly (yearly)	1939	unrevised unrevised	339/347 347
Classified receipts and expenditures, monthly Classified receipts and expenditures, monthly (yearly)	1940	unrevised unrevised	614/621 621

Table 8: Table names and page numbers from the *Annual Reports of the Secretary of the Treasury* for interest expenditures

A.1.3 Calculating Monthly Net Interest Because interest receipts are only available on a yearly basis, we are only able to calculate net interest on a yearly basis. We then use the yearly net interest series to impute monthly net interest. We first calculate the ratio of yearly interest receipts to yearly interest expenditures and then multiply this ratio by monthly interest expenditures to impute monthly interest receipts. Let the expression for imputed interest receipts in month t be given as:

$$\text{Imputed Monthly Interest Receipts}_t = \frac{\text{Yearly Interest Receipts}}{\text{Yearly Interest Expenditures}} * \text{Monthly Interest Expenditures}_t$$

Monthly net interest is then calculated as:

$$\text{Imputed Monthly Net Interest}_t = \text{Monthly Interest Expenditures}_t - \text{Imputed Monthly Interest Receipts}_t$$

A.2 Federal Receipts and Expenditures

This section details how our series of monthly federal receipts and expenditures from July 1919 to June 1940 from the *Annual Reports of the Secretary of the Treasury on the State of Finances* differ from other sources. We use data for receipts and expenditures that was revised in 1933 to "cover all expenditures of the Reconstruction Finance Corporation, including payments against credits established for the corporation through the purchase of its notes under section 9 of the Reconstruction Finance Corporation Act."[42] We use data on an unrevised cash basis for receipts and expenditures.[43]

Our three main sources of comparison are data from the NBER Macro History Database (NBER)[44], Firestone's (1960) book, and Romer (1992) who uses receipts and outlays[45] from the 1979 *Statistical Appendix to the Annual Report*, table 2, pp. 4-11 [Romer (1992)].

A.2.1 Federal Receipts Receipts from Firestone correspond to our series except for fiscal years 1931, 1932, and 1940. On page 80, Firestone explains that trust fund receipts were eliminated from internal revenue after June 1932 and his series take into a account this revision back to July 1930. Firestone (page 82) also deducts net transfers from the Federal Old-Age and Survivors Insurance Trust Fund from receipts to obtain lower monthly receipts for fiscal year 1940. The NBER receipts data is split into three receipt series a, b, and c. NBERa matches our series up to fiscal year 1932. NBERb matches Firestone for fiscal years 1931 and 1932 – also taking into account the elimination of trust fund receipts – and then tracks our series through fiscal year 1940. NBERc (not shown) also deducts net transfers from the Federal Old-Age and Survivors Insurance Fund and thus tracks Firestone for fiscal year 1940.

[42] Footnote 1, Table 6, page 312 of *Annual Report of the Secretary of the Treasury on the State of the Finances for Fiscal year ended June 30, 1933*

[43] See footnote 39 for an explanation of accounting conventions.

[44] Accessed via the NBER's Macrohistory Database, Chapter 15

[45] Starting in 1968, the Department of the Treasury (various) introduced new unified budget concepts including outlays. On page 8, the report explains that federal outlays include loans and expenditures.

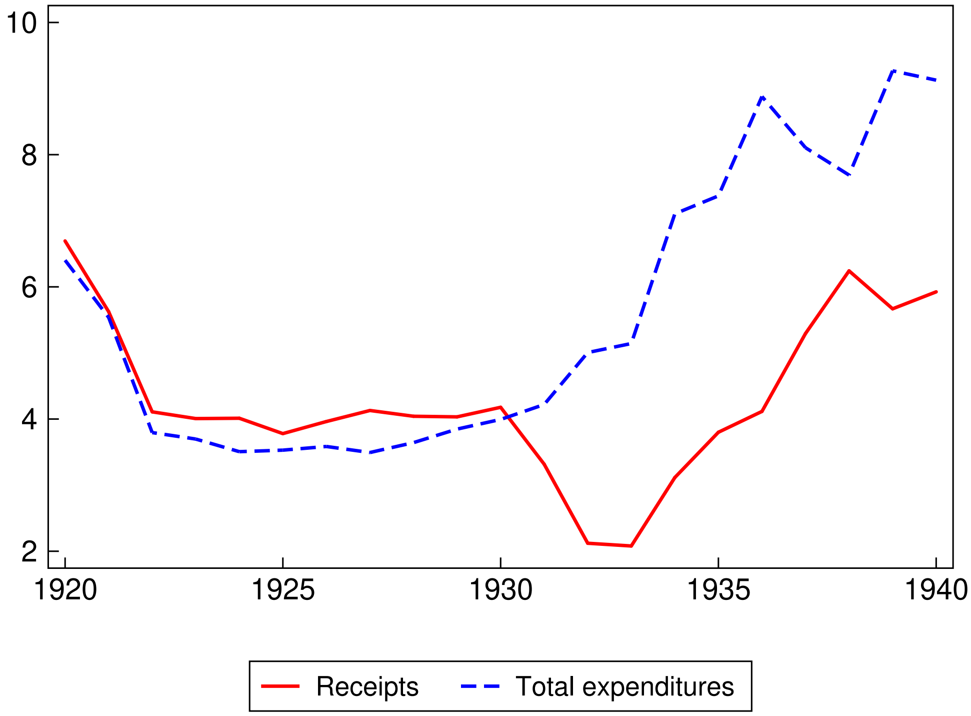

Figure 21: Fiscal year totals of monthly receipts and total expenditures, billions of dollars. Source: Department of the Treasury (various). See table 9 for details.

Figure 22: Fiscal year totals of monthly receipts and total expenditures, billions of dollars. Source: Department of the Treasury (various). See table 9 for details; Firestone (1960); NBER Macrohistory database (m15004b,m15004c).

Our yearly totals of monthly receipts data do not always match the yearly totals in other tables in the annual reports. Although the yearly data is revised throughout various annual reports, the monthly is not. The yearly receipts data is unrevised from fiscal years 1920 to

1935. In 1936, the data is revised starting in 1931. Our series of annual totals of monthly receipts data matches the yearly data until fiscal year 1933 when our series turns slightly lower.

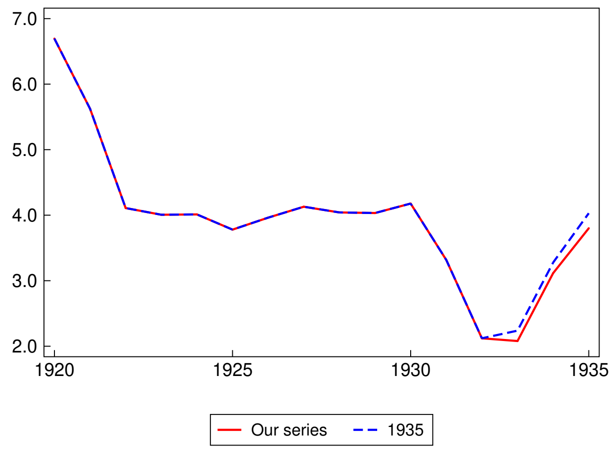

Figure 23: Fiscal year totals of monthly receipts and receipts by fiscal year, billions of dollars. Source: Department of the Treasury (various). See table 9 for details.

Annual receipts data remains unrevised from fiscal years 1936 to 1939. In 1939, receipts were mostly revised downwards for fiscal years 1931 through 1935. This revised series matches our series from fiscal years 1933 through 1939. In 1940, receipts data was revised downwards for fiscal years 1937 through 1940.[46]

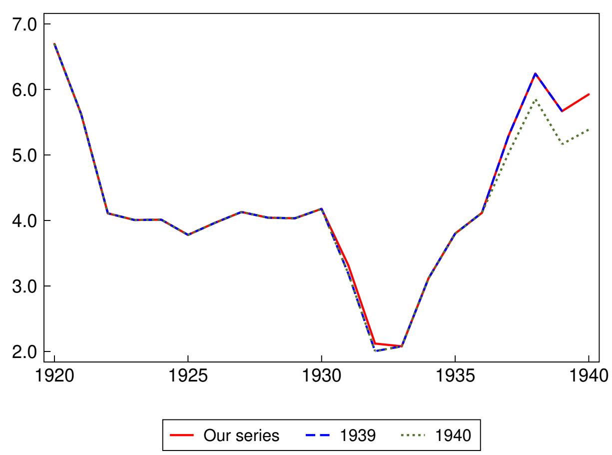

Figure 24: Fiscal year totals of monthly receipts and receipts by fiscal year, billions of dollars. Source: Department of the Treasury (various). See table 9 for details.

[46]Footnote 14 on Page 649 of the 1940 Annual Report explains that: "In the fiscal year 1941 amounts representing appropriations equal to 'Social Security-Unemployment taxes' collected and deposited as provided under sec. 201 (a) of the Social Security Act Amendments of 1939, less reimbursements to the General Fund for administrative expenses, are deducted on the daily Treasury statement from total receipts. Such net amounts are reflected under trust account receipts as net appropriations to the Federal old-age and survivors insurance trust fund. The fiscal years 1937, 1938, and 1939, have been revised in this statement to reflect similar treatment. Fiscal year 1940 figures are also on this revised basis."

A.2.2 FEDERAL EXPENDITURES Firestone and the NBER use ordinary expenditures for their expenditure series starting in December 1920 through fiscal year 1933 (June 1933). Romer uses ordinary outlays through fiscal year 1933.[47] Ordinary expenditures are a subset of total expenditures and exclude public debt retirements. For fiscal years 1920 through 1926, ordinary expenditures exclude purchases of obligations of foreign governments in addition to public debt retirements. Starting in fiscal year 1934, the *Annual Report of the Secretary of the Treasury* divides total expenditures into general and emergency categories.[48] Starting in 1934, Firestone, the NBER, and Romer begin using total expenditures for their expenditures series. We use total expenditures throughout the entire sample. Prior to fiscal year 1934, total expenditures are on average roughly 13 percent higher than ordinary expenditures.

The expenditure series from Firestone matches our series of ordinary expenditures from 1922 through fiscal year 1930. Firestone explains on page 82 that starting in fiscal year 1931, trust fund transactions were eliminated from ordinary expenditures chargeable against ordinary receipts. Trust fund expenditures were, however, still included in ordinary receipts through 1933 for comparison purposes. Our yearly totals of monthly ordinary expenditures diverge from Firestone's from fiscal years 1931 to 1933. Firestone's data for January 1932 to June 1933 matches that of NBERc (not shown). Our series of ordinary expenditures matches NBERb up to fiscal year 1933. Romer's series of ordinary outlays is almost always lower than our series and those given by the NBER and Firestone.

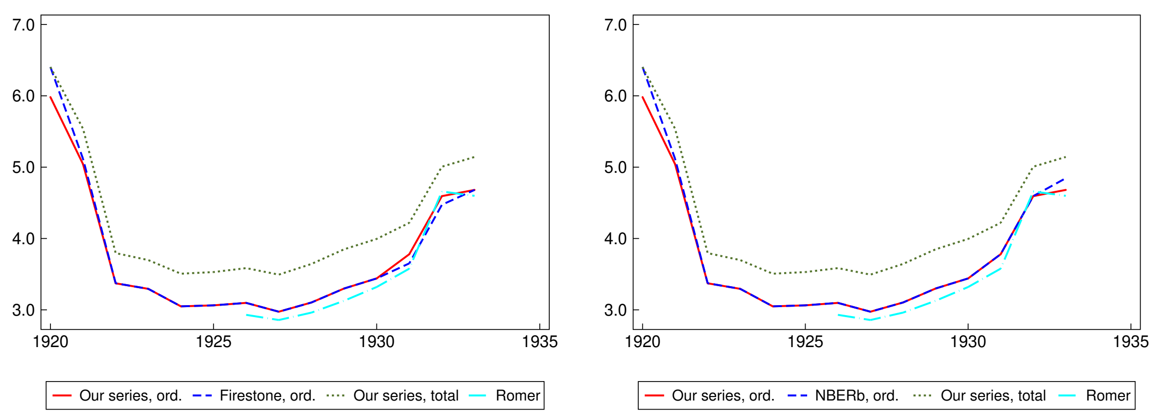

Figure 25: Fiscal year totals of monthly ordinary expenditures, billions of dollars. Source: Department of the Treasury (various). See table 9 for details; Firestone (1960); NBER Macrohistory database (m15004b,m15004c).

The total expenditure series from Firestone matches NBERc from fiscal year 1934 through fiscal year 1937. From fiscal year 1937 through 1939, Firestone's data matches NBERd. Firestone explains on page 84 that under an act of February 1938, the Secretary of the

[47]See footnote 45 for the difference between outlays and expenditures.

[48]Table 6 Footnote 6 on page 316 from the *Annual Report of the Secretary of the Treasury on the State of the Finances for Fiscal year ended June 30, 1934* explains that "Emergency expenditures prior to the fiscal year 1934 (except Reconstruction Finance Corporation) are included in general expenditures, the classification of which emergency expenditures is not available for comparison with emergency expenditures for the fiscal year 1934. Therefore, neither the totals of general expenditures nor the totals of emergency fiscal expenditures for the fiscal year 1934 are comparable with the total of prior fiscal years."

Treasury canceled $2.7 billion of obligations purchased from the RFC which the RFC could not repay to the Treasury. As a consequence, budget expenditures show only amounts spent from funds allocated by the RFC for purposes for which no provisions for repayment to the Treasury were made.The series from Firestone matches NBERe (not shown) for fiscal year 1940. Our series is larger than Firestone's and NBERc from 1934 through 1938. Although the gap shrinks from 1938 through 1940, our series is slightly higher than the other three series. Romer's series of total outlays is below our series and those given by the NBER and Firestone for most years.

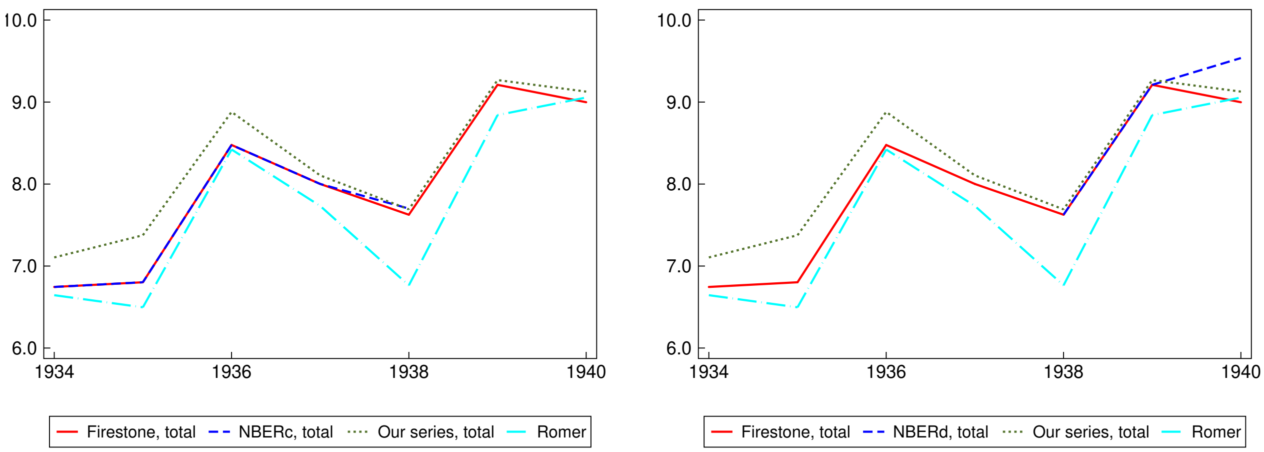

Figure 26: Fiscal year totals of monthly total expenditures, billions of dollars. Source: Department of the Treasury (various). See table 9 for details; Firestone (1960); NBER Macrohistory database (m15004b,m15004c).

As with the receipts series, our series for total and ordinary expenditure do not always match yearly data given elsewhere in the annual reports. From fiscal year 1922 to fiscal year 1931 our series of yearly totals of monthly expenditures data match yearly totals given elsewhere in the annual reports on an unrevised cash basis. In the 1927 annual report, ordinary expenditures are revised upwards. In the 1933 annual report, total and ordinary expenditures are revised for fiscal years 1932 and 1933. These revisions differ from revisions covering the expenditures of the Reconstruction Finance Corporation in 1933.

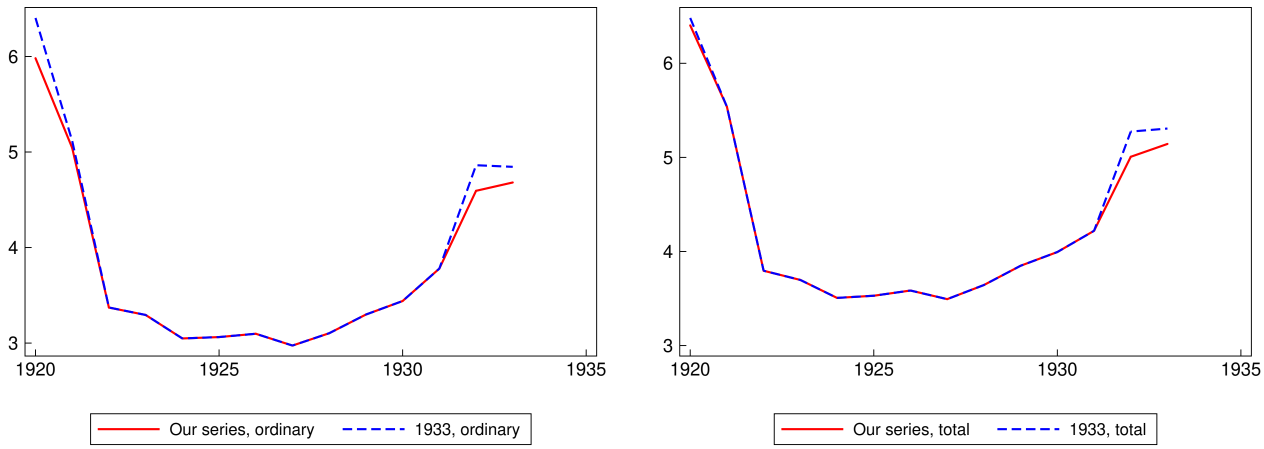

Figure 27: Fiscal year totals of monthly ordinary and total expenditures and ordinary and total expenditures by fiscal year, billions of dollars. Source: Department of the Treasury (various). See table 9 for details.

As mentioned previously, starting in 1934 until 1939, monthly expenditures are split into general and emergency expenditures categories rather than ordinary and total expenditures categories. Tables of yearly totals continue to categorize expenditures into ordinary and total even though the monthly series does not maintain this distinction. Our yearly totals of monthly ordinary expenditures stop in 1934 and we instead compute general expenditures for 1934-1939. Yearly ordinary and total expenditure series in the table are not revised from 1933 to 1935. Starting in 1936, the yearly ordinary and total expenditure series are revised back to 1930. Our series of total expenditures is lower than the 1935 and 1936 yearly series.

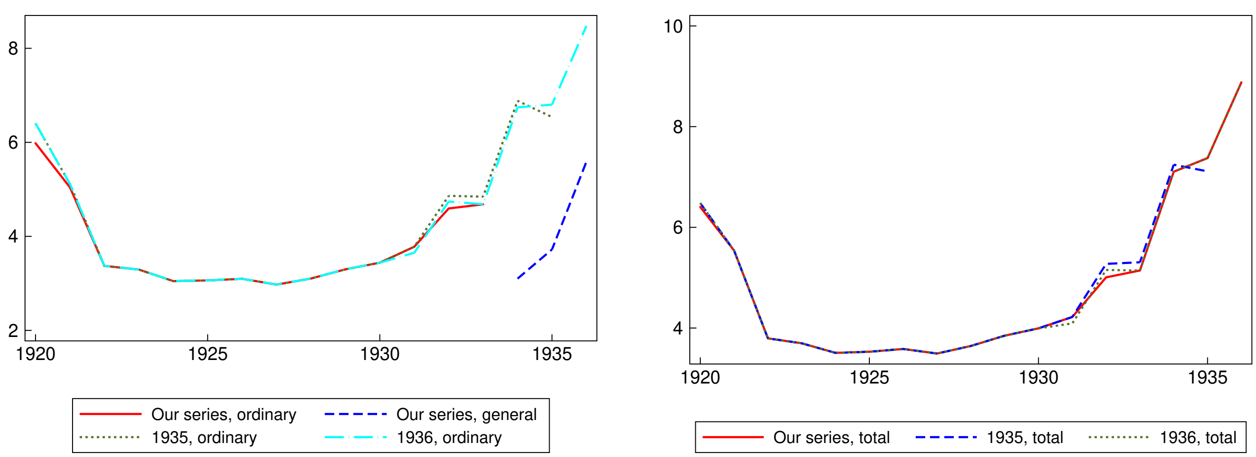

Figure 28: Fiscal year totals of monthly ordinary and total expenditures and ordinary and total expenditures by fiscal year, billions of dollars. Source: Department of the Treasury (various). See table 9 for details.

Yearly ordinary and total expenditures are revised in 1937, 1939, and 1940. The 1937 total expenditure series matches our series of yearly totals of monthly data the best.

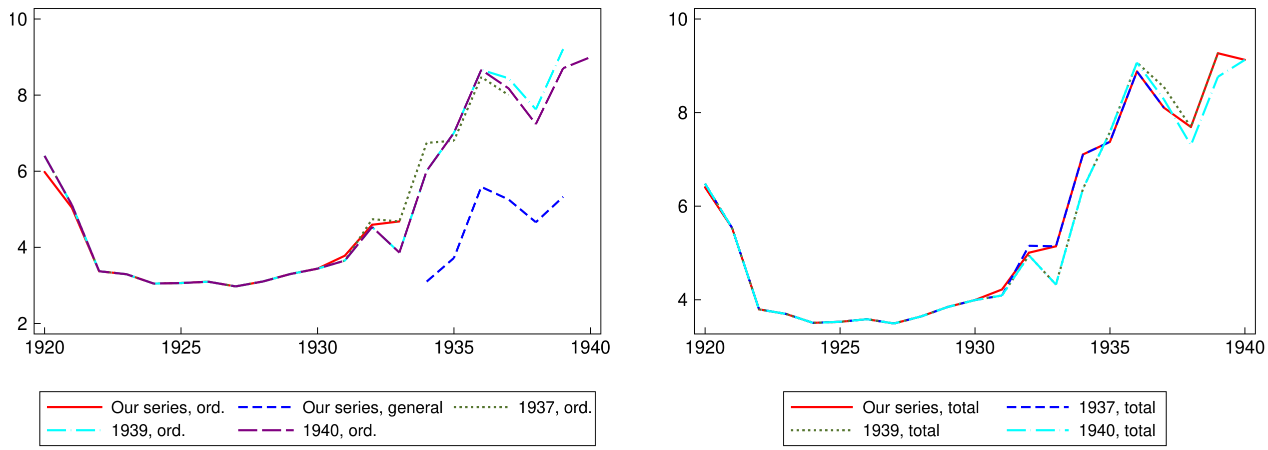

Figure 29: Fiscal year totals of monthly total expenditures, billions of dollars. Source: Department of the Treasury (various). See table 9 for details.

Table name	Year	Receipts page	Expenditures page
Ordinary receipts, and expenditures chargeable against (monthly)	1920	see 1921	
Ordinary receipts, and expenditures chargeable against (yearly)		see 1922	
STATEMENT SHOWING CLASSIFIED RECEIPTS...	1921	240	241
Ordinary receipts, and expenditures chargeable against (yearly)		see 1922	
Ordinary receipts, and expenditures chargeable against (monthly)	1922	270	271
Ordinary receipts, and expenditures chargeable against (yearly)		270	271
Ordinary receipts, and expenditures chargeable against (monthly)	1923	512	513
Receipts and expenditures of the United States Government...		512	513
Ordinary receipts, and expenditures chargeable against (monthly)	1924	378	379
Receipts and expenditures of the United States Government...		378	379
Ordinary receipts, and expenditures chargeable against (monthly)	1925	472	474
Receipts and expenditures of the United States Government...		472	474
Ordinary receipts, and expenditures chargeable against (monthly)	1926	445	447
Ordinary receipts, and expenditures chargeable against (yearly)		443	443
Ordinary receipts, and expenditures chargeable against (monthly)	1927	462	462
Ordinary receipts, and expenditures chargeable against (yearly)		445	445
Summary of ordinary receipts, expenditures chargeable (monthly)...	1928	424	424
Summary of ordinary receipts, expenditures chargeable (yearly)...		407	407
Summary of ordinary receipts, expenditures chargeable (monthly)...	1929	412	412
Summary of ordinary receipts, expenditures chargeable (yearly)...		394	394
Summary of ordinary receipts, expenditures chargeable (monthly)...	1930	506	506
Summary of ordinary receipts, expenditures chargeable (yearly)....		488	488
Summary of ordinary receipts, expenditures chargeable (monthly)...	1931	462	462
Ordinary receipts, expenditures chargeable against (yearly)...		448	448
Summary of ordinary receipts, expenditures chargeable (monthly)...	1932	370	370
Receipts and expenditures for the fiscal years 1789 to...		365	369
Summary of ordinary receipts, expenditures chargeable (monthly)...	1933	312	312
Receipts and expenditures for the fiscal years 1789 to...		306	310
Summary of ordinary receipts, expenditures chargeable (monthly)...	1934	306	306
Receipts and expenditures for the fiscal years 1789 to...		301	305
Summary of ordinary receipts, expenditures chargeable (monthly)...	1935	328	328
Receipts and expenditures for the fiscal years 1789 to...		323	327
Classified receipts and expenditures, monthly...	1936	337	339/342
Receipts and expenditures for the fiscal years 1789 to...		359	363
Classified receipts and expenditures, monthly...	1937	320	322/324
Receipts and expenditures for the fiscal years 1789 to...		349	353
Expenditures by major functions for the fiscal years 1930-1937			354
Classified receipts and expenditures, monthly...	1938	379	381/384
Receipts and expenditures for the fiscal years 1789 to...		413	417
Expenditures by major functions for the fiscal years 1931-1938			418
Classified receipts and expenditures, monthly...	1939	337	339/342
Receipts and expenditures for the fiscal years 1789 to...		361	365
Expenditures by major functions for the fiscal years 1931-1939			367
Classified receipts and expenditures, monthly...	1940	612	615/616
Receipts and expenditures for the fiscal years 1789 to...		645	649
Expenditures by major functions for the fiscal years 1933-1940			653
Receipts in general and special accounts, by major sources...		651	

Table 9: Table names and page numbers from the *Annual Reports of the Secretary of the Treasury* for federal receipts and expenditures

A.3 Market Value and Returns

The following section details our calculation of market value and return on the United States's bond portfolio. We use data from Hall and Sargent (2015), provided to us by the authors, as well as CRSP to obtain the quantity, price, accrued interest, interest rate, and coupon frequency of each government security outstanding in a given month.

Let $B_{it}(t+j)$ denote the dollar value of type i bonds outstanding in period t that mature in period $t+j$ and $Q^D_{it}(t+j)$ be the dirty price (price+accrued interest) of such bonds. Because the number of types of bonds of a certain maturity each period can vary over time, we let $N_t(t+j)$ represent the number of such bonds in period t.

Let $B_t(t+j)$ denote the dollar value of all bonds outstanding in period t that mature in period $t+j$, defined as

$$B_t(t+j) = \sum_{i=1}^{N_t(t+j)} B_{it}(t+j) \tag{26}$$

Then the par value of all debt outstanding at the end of period t—the face value of the bond portfolio—is

$$B^M_t = \sum_{j=1}^{\infty} \sum_{i=1}^{N_t(t+j)} B_{it}(t+j) = \sum_{j=1}^{\infty} B_t(t+j) \tag{27}$$

Define $\nu_i(t+j)$ as the share of security of type i that is outstanding at t and matures at $t+j$

$$\nu_i(t+j) = \frac{B_{it}(t+j)}{\sum_{i=1}^{N_t(t+j)} B_{it}(t+j)} = \frac{B_{it}(t+j)}{B_t(t+j)} \tag{28}$$

where $\sum_{i=1}^{N_t(t+j)} \nu_i(t+j) = 1$. Then the weighted dirty price of bonds outstanding at t that mature in $t+j$ is

$$Q^D_t(t+j) = Q_t(t+j) + AI_t(t+j) = \sum_{i=1}^{N_t(t+j)} \Big(Q_{it}(t+j) + AI_{it}(t+j)\Big)\nu_i(t+j) \tag{29}$$

where $Q_t(t+j)$ is the clean price of bonds outstanding at t that mature in $t+j$, $AI_t(t+j)$ is the accrued interest on bonds outstanding at t that mature in $t+j$. For zero-coupon bonds, the dirty price is equal to the clean price.

We also define $\mu_t(t+j)$ as the share of the total par value of bonds outstanding at the end of t that matures in $t+j$

$$\mu_t(t+j) = \frac{B_t(t+j)}{B^M_t} \tag{30}$$

where $\sum_{j=1}^{\infty} \mu_t(t+j) = 1$. This permits us to define the nominal price of the bond portfolio, P^M_t, as

$$P^M_t = \sum_{j=1}^{\infty} Q^D_t(t+j)\mu_t(t+j) \tag{31}$$

With a complete and general maturity structure, the government's budget identity is

$$\sum_{j=0}^{\infty}\big(Q_t^D(t+j)+IP_t(t+j)\big)B_{t-1}(t+j)=P_t s_t+\sum_{j=1}^{\infty}Q_t^D(t+j)B_t(t+j) \tag{32}$$

Where $Q_t^D(t)\equiv 1$ and $IP_t(t+j)$ is the interest payable on bonds outstanding at t that mature in $t+j$. Interest payable is an government expense in period t and is thus included in the government budget identity.

The market value of debt outstanding in period t is

$$P_t^M B_t^M \equiv \sum_{j=1}^{\infty} Q_t^D(t+j)B_t(t+j) \tag{33}$$

so that the comparable expression at $t-1$ is

$$P_{t-1}^M B_{t-1}^M \equiv \sum_{j=1}^{\infty} Q_{t-1}^D\big((t-1)+(j+1)\big)B_{t-1}\big((t-1)+(j+1)\big)=\sum_{j=1}^{\infty}Q_{t-1}^D(t+j)B_{t-1}(t+j) \tag{34}$$

The carry-over market value uses the same bonds as the market value for period $t-1$ but using period t dirty prices and intermediate coupon payments. The carry-over price, P_t^C, reflects coupon payments that were paid between periods $t-1$ and t. The carry-over market value is defined as

$$P_t^C B_{t-1}^M \equiv \sum_{j=0}^{\infty}\Big(Q_t^D(t+j)+IP_t(t+j)\Big)B_{t-1}(t+j) \tag{35}$$

$IP_t(t+j)$ is the interest payable on bonds outstanding at t that mature in $t+j$. P_t^C differs from its dirty-price analog only when there is a coupon payment in month t. Figure 30 illustrates the timing of coupon payments.

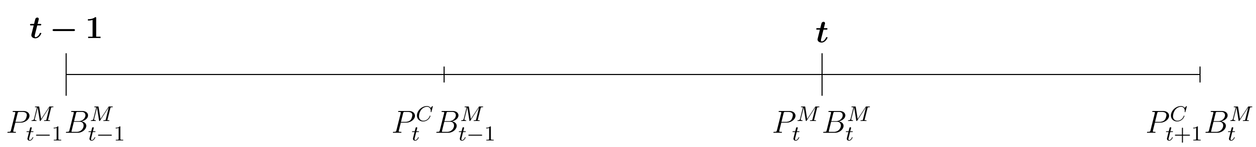

Figure 30: Timing of actual and carry-over market value

Using the definitions of market value and carry over market value, (32) can be written as:

$$P_t^C B_{t-1}^M = P_t s_t + P_t^M B_t^M \tag{36}$$

Multiplying and dividing the left hand side by last period's market value allow the government budget identity to be expressed in terms of the rate of return on government debt:

$$\underbrace{\frac{P_t^C B_{t-1}^M}{P_{t-1}^M B_{t-1}^M}}_{\text{rate of return}} P_{t-1}^M B_{t-1}^M = P_t s_t + P_t^M B_t^M \tag{37}$$

The rate of return can also be derived by decomposing changes in market value into rates of return and changes in size. We start by expanding the ratio of period t to period $t-1$ market value

$$\frac{P_t^M B_t^M}{P_{t-1}^M B_{t-1}^M} \equiv \underbrace{\frac{P_t^C B_{t-1}^M}{P_{t-1}^M B_{t-1}^M}}_{\text{rate of return}} \cdot \underbrace{\frac{P_t^M B_t^M}{P_t^C B_{t-1}^M}}_{\text{size ratio}} \tag{38}$$

The expression for the rate of return is the same as (37) and can be expressed as

$$\frac{P_t^C B_{t-1}^M}{P_{t-1}^M B_{t-1}^M} = \frac{\sum_{j=0}^{\infty} \Big(Q_t(t+j) + AI_t(t+j) + IP_t(t+j)\Big) B_{t-1}(t+j)}{\sum_{j=1}^{\infty} \Big(Q_{t-1}(t+j) + AI_{t-1}(t+j)\Big) B_{t-1}(t+j)} \tag{39}$$

This rate of return reflects the percentage change in the value of the bond portfolio between period $t-1$ and t, holding the bond portfolio fixed.

The size ratio can be expressed as

$$\frac{P_t^M B_t^M}{P_t^C B_{t-1}^M} = \frac{\sum_{j=1}^{\infty} \Big(Q_t(t+j) + AI_t(t+j)\Big) B_t(t+j)}{\sum_{j=0}^{\infty} \Big(Q_t(t+j) + AI_t(t+j) + IP_t(t+j)\Big) B_{t-1}(t+j)} \tag{40}$$

Changes in size incorporates new issues, redemptions, and coupon payments that occur between periods $t-1$ and t. The size ratio reflects the percentage change in the value of the bond portfolio that arises from changes in the bond portfolio itself, including any changes in maturity structure.

The real return can then be defined from (39) with the expression for weighted dirty price:

$$r_t^M = \frac{P_t^C B_{t-1}^M / P_t}{P_{t-1}^M B_{t-1}^M / P_{t-1}} = \frac{\sum_{j=0}^{\infty} \Big(Q_t^D(t+j) + IP_t(t+j)\Big) B_{t-1}(t+j)/P_t}{\sum_{j=1}^{\infty} Q_{t-1}^D(t+j) B_{t-1}(t+j)/P_{t-1}} \tag{41}$$

The identity (37) can also be expressed in real terms as:

$$r_t^M P_{t-1}^M b_{t-1}^M = s_t + P_t^M b_t^M \tag{42}$$

where $b_t^M \equiv B_t^M / P_t$ is the real par value of debt outstanding at t.

The surprise component in the real return on the bonds portfolio is:

$$\eta_t \equiv r_t^M - E_{t-1} r_t^M \tag{43}$$

Using $E_{t-1}\left[\sum_{j=0}^{\infty} \Big(Q_t^D(t+j) + IP_t(t+j)\Big)/P_t\right] = \sum_{j=0}^{\infty} \big(Q_{t-1}(t+j) + AI_t(t+j) + IP_t(t+j)\big)/P_{t-1}$, then the expectation is of no real capital gain or loss on the portfolio. Accrued interest, $AI_t(t+j)$, and interest payable, $IP_t(t+j)$, of bonds outstanding in period t that mature in period $t+j$ is known in period $t-1$. Hence, $E_{t-1}[AI_t(t+j)+IP_t(t+j)] = AI_t(t+j)+IP_t(t+j)$. The surprise in the real return becomes

$$\eta_t = \Bigg(\frac{\sum_{j=0}^{\infty} \big(Q_t(t+j)+AI_t(t+j)+IP_t(t+j)\big)B_{t-1}(t+j)/P_t}{\sum_{j=0}^{\infty} \big(Q_{t-1}(t+j)+AI_t(t+j)+IP_t(t+j)\big)B_{t-1}(t+j)/P_{t-1}} - .... \\ ...1 \Bigg) \frac{\sum_{j=0}^{\infty} \big(Q_{t-1}(t+j)+AI_t(t+j)+IP_t(t+j)\big)B_{t-1}(t+j)/P_{t-1}}{P^M_{t-1}B^M_{t-1}/P_{t-1}} \tag{44}$$

Real returns can be scaled by components isolating changes in the price level and changes in bond prices. Re-writing (44) as:

$$\eta_t = \underbrace{\frac{P^c_t B^M_{t-1}/P_t}{P^M_{t-1}B^M_{t-1}/P_{t-1}}}_{r^M_t} - \underbrace{\frac{P^c_t B^M_{t-1}}{P^M_{t-1}B^M_{t-1}}}_{R^M_t} + \underbrace{\frac{P^c_t B^M_{t-1}}{P^M_{t-1}B^M_{t-1}}}_{R^M_t} \frac{\sum_{j=0}^{\infty} \big(Q_t(t+j)-Q_{t-1}(t+j)\big)B_{t-1}(t+j)}{P^C_t B^M_{t-1}} \tag{45}$$

Which can be further re-arranged to:

$$\eta_t = R^M_t \underbrace{(1/\pi_t - 1)}_{\text{due to price level}} + R^M_t \underbrace{\left(\frac{\sum_{j=0}^{\infty} \big(Q_t(t+j)-Q_{t-1}(t+j)\big)B_{t-1}(t+j)}{P^C_t B^M_{t-1}} \right)}_{\text{due to bond prices}} \tag{46}$$

If there are no changes in the price level between periods $t-1$ and t, i.e. $\pi = 1$ and weighted changes in bond prices sum to zero $\sum_{j=0}^{\infty} Q_t(t+j) - Q_{t-1}(t+j) = 0$, then $\eta_t = 0$ indicating no capital gains or losses. If there is no change in the price level ($\pi_t = 1$) then $R^M_t(1/\pi_t - 1) = 0$ then capital gains or losses can be interpreted as the weighted change in bond prices as a share of market value scaled by nominal returns. If the weighted changes in bond prices sum to zero, $\big(\sum_{j=0}^{\infty}\big(Q_t(t+j) - Q_{t-1}(t+j)\big) = 0$, then capital gains or losses are changes in the price level scaled by nominal returns.

Because $Q_t(t) = 1$ for all t, then then when $j = 0$, $Q_t(t) - Q_{t-1}(t) = 0$ and (46) can be written as:

$$\eta_t = R^M_t \underbrace{(1/\pi_t - 1)}_{\text{due to price level}} + R^M_t \underbrace{\left(\frac{\sum_{j=1}^{\infty} \big(Q_t(t+j)-Q_{t-1}(t+j)\big)B_{t-1}(t+j)}{P^C_t B^M_{t-1}} \right)}_{\text{due to bond prices}}$$

Real and nominal returns are denominated in percentage points of market value outstanding at B_{t-1}

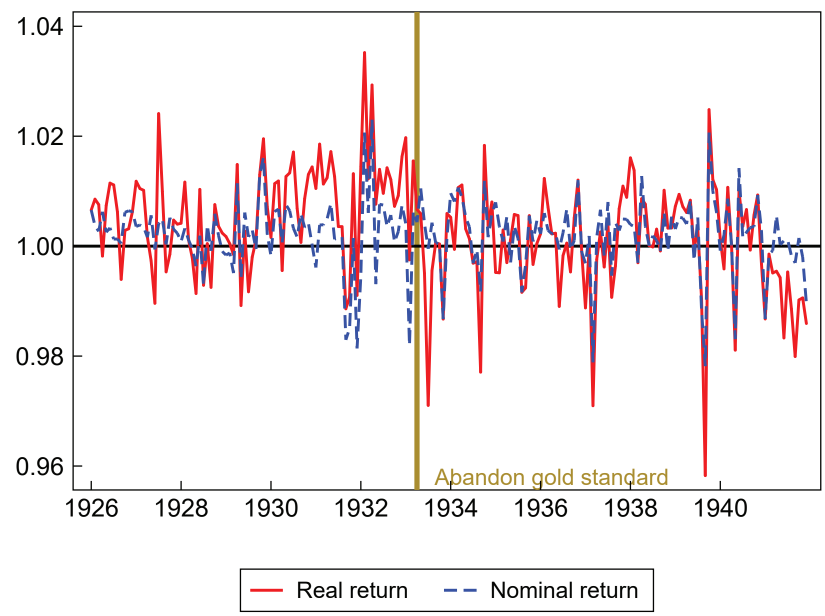

Figure 31: Real and nominal price returns

Real returns to U.S. debt show a much larger drop than nominal returns to U.S. debt after the departure form the gold standard.

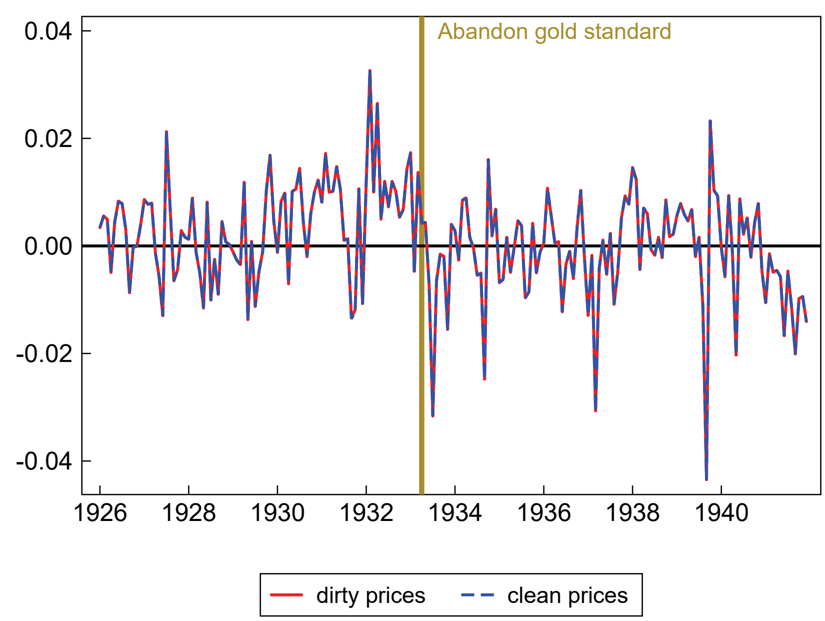

Figure 32: Real innovations to price returns with clean and dirty prices

Innovations show large losses after the abandonment of the gold standard.

Innovations capture the unexpected losses or gains on U.S. debt due to bond prices or the price level. We multiplying innovations by the beginning of period market value $(P_{t-1}^M B_{t-1}^M)$ to capture the dollar amount of the difference between real and expected real returns to holding U.S. debt. We then take this dollar amount as ratio of the current period market value $(P_t^M B_t^M)$ to capture surprise capital gains or losses as a percent of market value. Figure

33 is thus:

$$\eta_t^D \frac{P_{t-1}^M B_{t-1}^M}{P_t^M B_t^M} * 100 \tag{47}$$

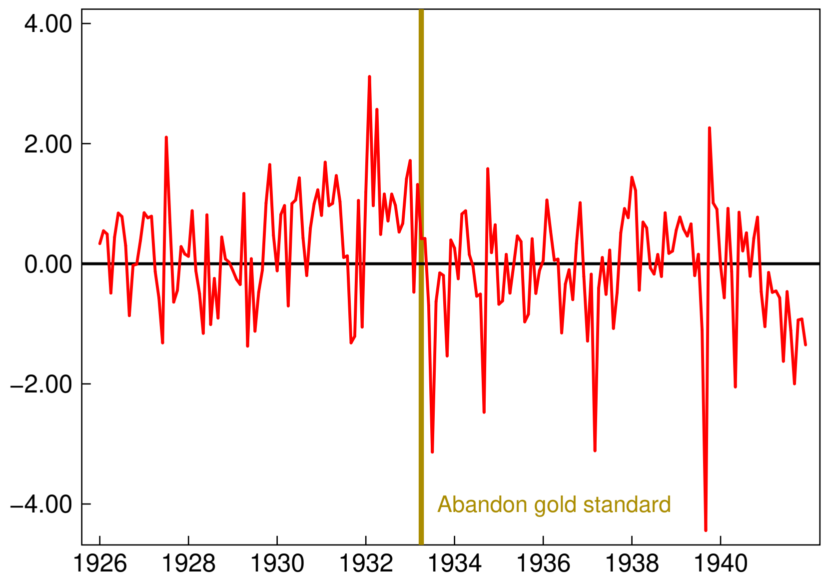

Figure 33: Capital gains and loss as a percent of market value (47)

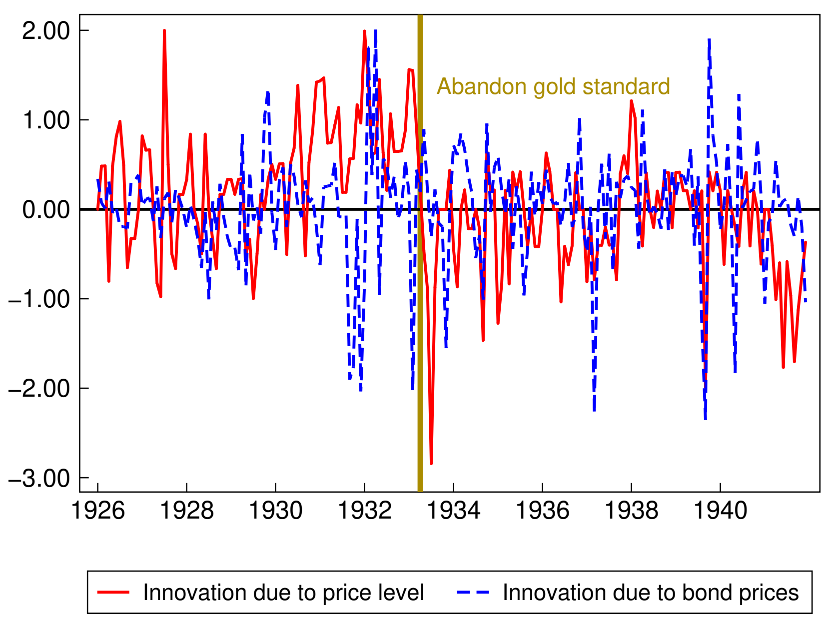

Figure 34: Innovations to price returns decomposed into changes from bond prices and changes from the price level (46)

After the abandonment of the gold standard, the price level is largely responsible for the capital loss on holding government debt.

B Model and Theoretical Results

B.1 Basic Environment

What follows builds directly on the Barro (1979) and Goodfriend (1988) models of the gold standard.

Households. A continuum of households maximize

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[u\left(C_T, M_T/P_T, G_T^p; \zeta_T\right) - \int_0^1 v\left(h_T\left(i\right); \zeta_T\right) di \right] \tag{48}$$

where $0 < \beta < 1$ and the consumption index C_t and associated price P_t are given by the constant elasticity of substitution aggregators

$$C_t = \left[\int_0^1 c_t\left(i\right)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \quad \text{and } P_t = \left[\int_0^1 p_t\left(i\right)^{1-\theta} di \right]^{1/(1-\theta)}.$$

Assume utility over consumption, real money balances and gold are additively separable. Maximization occurs subject to the flow budget constraint

$$\int_0^1 p_t\left(i\right) c_t\left(i\right) di + P_t D_t + P_t T_t = \int_0^1 w_t\left(i\right) h_t\left(i\right) di + \int_0^1 \Pi_t\left(i\right) di + P_t W_t$$

where

$$\begin{aligned} D_t &= \frac{M_t}{P_t} + \frac{P_t^s B_t^s}{P_t} + \frac{P_t^l B_t^l}{P_t} + \frac{P_t^g G_t^p}{P_t} \\ W_t &= \frac{M_{t-1}}{Pt} + \frac{B_{t-1}^s}{P_t} + \frac{\left(1 + \rho_b P_t^l\right) B_{t-1}^l}{P_t} + \frac{P_t^g G_{t-1}^p}{P_t} \end{aligned}$$

denote end of period and beginning of period wealth respectively. Households can hold wealth in money balances, M_t, gold G_t^p, one-period nominal government debt, B_t^s, and long-duration nominal government debt B_t^l; each with prices $1/P_t$, P_t^g, P_t^s and P_t^l. Rearranging provides

$$\begin{aligned} & P_t^{-1} \int_0^1 p_t\left(i\right) c_t\left(i\right) di + \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{P_t^s B_t^s}{P_t} - \frac{B_{t-1}^s}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{P_t^l B_t^l}{P_t} + \frac{P_t^g G_t^p}{P_t} - \frac{P_t^g G_{t-1}^p}{P_t} \\ = \; & P_t^{-1} \int_0^1 w_t\left(i\right) h_t\left(i\right) di + P_t^{-1} \int_0^1 \Pi_t\left(i\right) di - T_t + \frac{\left(1 + \rho_b P_t^l\right) B_{t-1}^l}{P_t} \frac{P_{t-1}}{P_{t-1}} \end{aligned}$$

in any period t. We can write this as

$$\begin{aligned} & C_t + m_t - m_{t-1}\pi_t^{-1} + P_t^s b_t^s - b_{t-1}^s \pi_t^{-1} + P_t^l b_t^l + \frac{P_t^g}{P_t}\left(G_t^p - G_{t-1}^p\right) \\ = \; & P_t^{-1} \int_0^1 w_t\left(i\right) h_t\left(i\right) di + P_t^{-1} \int_0^1 \Pi_t\left(i\right) di - T_t + \left(1 + \rho_b P_t^l\right) b_{t-1}^l \pi_t^{-1} \end{aligned}$$

where $m_t \equiv M_t/P_t$ is real money balances, $\pi_t = P_t/P_{t-1}$ inflation, and $b_t = B_t^j/P_t$ a measure of real debt for $j \in \{s, l\}$.

Taking first-order conditions provides

$$\begin{aligned}
u_{c,t} &= \lambda_t \\
u_{m,t} &= \lambda_t - \beta E_t \lambda_{t+1} \pi_t^{-1} \\
u_{G^p,t} &= \lambda_t \frac{P_t^g}{P_t} - E_t \lambda_{t+1} \frac{P_{t+1}^g}{P_{t+1}} \\
v_{h,t}(i) &= \lambda_t w_t(i) P_t^{-1} \\
\lambda_t P_t^s &= \beta E_t \lambda_{t+1} \pi_{t+1}^{-1} \\
\lambda_t P_t^l &= E_t \lambda_{t+1} \left(1 + \rho_b P_{t+1}^l\right) \pi_{t+1}^{-1}
\end{aligned}$$

along with the transversality condition. Eliminating the marginal value of wealth gives

$$\begin{aligned}
1 &= \beta E_t (1 + i_t) \frac{u_{c,t+1}}{u_{c,t}} \frac{P_t}{P_{t+1}} \\
1 &= \beta E_t \frac{\left(1 + \rho_b P_{t+1}^l\right)}{P_t^l} \frac{u_{c,t+1}}{u_{c,t}} \frac{P_t}{P_{t+1}} \\
\frac{u_{c,t}}{P_t} &= \beta E_t \frac{u_{c,t+1}}{P_{t+1}} + \frac{u_{m,t}}{P_t} \\
u_{c,t} \frac{P_t^g}{P_t} &= \beta E_t u_{c,t+1} \frac{P_{t+1}^g}{P_{t+1}} + u_{G^p,t} \\
\frac{v_{h,t}}{u_{c,t}} &= \frac{w_t}{P_t}
\end{aligned}$$

where $1 + i_t = (P_t^s)^{-1}$.

Firms. A continuum of firms solve a standard Calvo (1983) price setting problem. Because the gold sector will be determined exogenously it has no implications for the pricing of goods. We therefore state the implications of this well known model without derivation. See Woodford (2003) for details.

Policy. The model is closed with a specification of monetary and fiscal policy. Here monetary policy includes a description of the supply of money balances and monetary gold. We follow Goodfriend and specify the "gold block" by two assumptions. The dollar price of gold is fixed so that

$$P_t^g = \bar{P}^g$$

and the money supply is backed by the monetary gold stock, G_t^m, in fixed proportion so that

$$\frac{\bar{P}^g G_t^m}{M_t} = \frac{\bar{P}^g G_t^m}{P_t m_t} = \alpha.$$

Because the quantity of gold supply, G_t, is exogenously determined, market clearing in the gold sector requires

$$G_t^m + G_t^P = G_t.$$

Fiscal policy is given by a set of equations specifying the portfolio of issued debt, spending and taxes. For simplicity we assume only long-duration debt is issued in positive supply to avoid determining the relative supply of each type of security. For generality, we retain one-period debt in some expressions that follow. Government purchases F_t are determined exogenously. The flow budget constraint of the government is then

$$m_t - m_{t-1}\pi_t^{-1} + P_t^s b_t^s - b_{t-1}^s \pi_t^{-1} + P_t^l b_t^l + \frac{P_t^g}{P_t}\left(G_t^m - G_{t-1}^m\right) = F_t - T_t + \left(1 + \rho_b P_t^l\right) b_{t-1}^l \pi_t^{-1}.$$

Taxes are given by the rule

$$T_t = b_t^{\phi_\tau} - \Xi_t$$

where

$$\Xi_t = m_t - m_{t-1}\pi_t^{-1} + \frac{P_t^g}{P_t}\left(G_t^m - G_{t-1}^m\right)$$

so that taxes, in addition to being responsive to outstanding debt, facilitate the appropriate transfer to the public of money balances and monetary gold.

These policy assumptions are sufficient to close the model.

B.2 The Steady State

We study the model in the neighborhood of a steady state in which

$$\frac{P_t^g}{P_t} = 1 \text{ and } \frac{P_t}{P_{t-1}} = 1$$

and all remaining variables are constant over time. The steady state then satisfies the following restrictions. The short-bond Euler equation implies

$$\beta(1+i) = 1$$

The long-bond Euler equation

$$\frac{\left(1 + \rho_b P^l\right)}{P^l} = 1 + i = \beta^{-1}$$

which implies the steady state price of long-term debt

$$P^l = \frac{\beta}{1 - \beta\rho_b}.$$

The asset price equation determining money balances provides

$$(1-\beta) = \frac{u_m}{u_c}$$

while that for private gold holdings gives

$$(1-\beta) = \frac{u_{G^p}}{u_c}.$$

Hence in steady state

$$u_m = u_{G^p}.$$

Note we could approximate the model for

$$\frac{P_t^g}{P_t} = \frac{P^g}{P}$$

not equal to unity. This would then give

$$(1-\beta)\frac{P^g}{P} = \frac{u_{G^p}}{u_c}$$

which would drive a wedge between real money balances and gold holdings. But relative steady state price drops out of the approximation to the first order so is not relevant for calculations that follow. It would matter for model calibration.

B.3 The Log-linear Model

For any variable z_t with mean z define

$$\hat{z}_t = \log\left(\frac{z_t}{z}\right) \text{ and } \tilde{z}_t = \frac{z_t - z}{z}$$

with the exception of

$$\begin{aligned}
\hat{p}_t^g &= \log\left(\frac{P_t^g}{P_t}\right) = \log(P^g) - \log(P_t) \\
\pi_t &= \log\left(\frac{P_t}{P_{t-1}}\right) \\
\hat{\imath}_t &= \log\left(\frac{R_r}{R}\right).
\end{aligned}$$

A first order approximation of the model gives

$$\begin{aligned}
\pi_t &= \kappa(\hat{y}_t - Y_t^n) + \beta E_t \pi_{t+1} \\
\hat{y}_t &= E_t \hat{y}_{t+1} - \sigma E_t(\hat{\imath}_t - \pi_{t+1}) + E_t(g_t - g_{t+1}) \\
\beta E_t \pi_{t+1} + \varphi(1-\beta)\hat{m}_t &= \sigma^{-1}(\hat{y}_t - g_t) - \sigma^{-1}\beta E_t(\hat{y}_{t+1} - g_{t+1}) \\
\hat{p}_t^g &= \beta E_t \hat{p}_{t+1}^g - \nu(1-\beta)\hat{G}_t^p + \sigma^{-1}E_t((\hat{y}_t - g_t) - \beta(\hat{y}_{t+1} - g_{t+1})) \\
\hat{G}_t^p &= (1+\theta_m)\hat{G}_t - \theta_m \hat{G}_t^m \\
\hat{p}_t^g + \hat{G}_t^m &= \hat{m}_t \\
\pi_t &= -\hat{p}_t^g + \hat{p}_{t-1}^g \\
\hat{P}_t^l &= \beta\rho E_t \hat{P}_{t+1}^l - \hat{\imath}_t
\end{aligned}$$

where

$$\begin{aligned}
\hat{y}_t &= s_c \hat{c}_t + \tilde{F}_t \\
x_t &= \hat{y}_t - Y_t^n \\
Y_t^n &= \frac{\sigma^{-1} g_t + \omega q_t}{\sigma^{-1} + \omega} \\
g_t &= \tilde{F}_t + s_c \bar{c}_t \\
q_t &= \left(1 + \omega^{-1}\right) a_t + \omega^{-1} v \bar{h}_t
\end{aligned}$$

define the output gap and exogenous disturbances ($\bar{c}_t$ and $\bar{h}_t$ exogenous variation in the desire to consumption and the disutility of labor supply — see Woodford (2003) for definitions) and a_t technology used in production of differentiated goods by firms. Finally

$$\sigma = s_c \sigma_c$$

where

$$\sigma_c = -\frac{u_c}{u_{cc} c} > 0$$

The model describes the evolution of

$$\left\{ y_t, \pi_t, \hat{i}_t, m_t, p_t^g, G_t^p, G_t^m, \hat{P}_t^l \right\}$$

which must be solved as a function of exogenous disturbances

$$\left\{ Y_t^n, g_t, \hat{G}_t \right\}.$$

Note that using these definitions the aggregate Euler equation can be written in terms of the output gap

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$

where

$$r_t^n = \sigma^{-1} \left[\left(g_t - Y_t^n \right) - E_t \left(g_{t+1} - Y_{t+1}^n \right) \right].$$

We'll make use of this notation in the calculation of fiscal multipliers.

B.4 Implications of the Classical Gold Standard

This section establishes the first result of the paper: under a classical gold standard with balanced-budget rule for tax policy (or tax policy as a rule of the kind written earlier which ensures a Ricardian equilibrium), there is a unique bounded rational expectations equilibrium for all maintained parameter values. An implication is the price of goods is a stationary variable. Several corollaries are then presented.

B.4.1 Requirements for a unique bounded equilibrium Because the disturbances are irrelevant to the properties of uniqueness and boundedness, drop them for the time being. Proceed by simplifying the system. The policy rules and inflation definition permit

$$-\hat{p}_t^g + \hat{p}_{t-1}^g = \kappa \hat{y}_t + \beta E_t \left(-\hat{p}_{t+1}^g + \hat{p}_t^g\right) \tag{49}$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \sigma E_t \left(\hat{\imath}_t + \hat{p}_{t+1}^g - \hat{p}_t^g\right) \tag{50}$$

$$\beta E_t \left(-\hat{p}_{t+1}^g + \hat{p}_t^g\right) + \varphi \left(1-\beta\right) \hat{m}_t = \sigma^{-1} \hat{y}_t - \sigma^{-1} \beta E_t \hat{y}_{t+1} \tag{51}$$

$$\hat{p}_t^g = \beta E_t \hat{p}_{t+1}^g + \nu \left(1-\beta\right) \theta_m \left(\hat{m}_t - \hat{p}_t^g\right) + \sigma^{-1} E_t \left(\hat{y}_t - \beta \hat{y}_{t+1}\right).$$

Note that the final two equations imply the linear restriction

$$\left(1 + \nu\theta_m\right) \hat{p}_t^g = \left(\nu\theta_m + \varphi\right) \hat{m}_t$$

or

$$\hat{m}_t = \frac{\left(1+\nu\theta_m\right)}{\left(\varphi + \nu\theta_m\right)} \hat{p}_t^g. \tag{52}$$

Using this to eliminate money balances in (51), and combining with (49), provides two equations

$$-\hat{p}_t^g + \hat{p}_{t-1}^g = \kappa y_t + \beta E_t \left(-\hat{p}_{t+1}^g + \hat{p}_t^g\right) \tag{53}$$

and

$$\sigma^{-1} \beta E_t \hat{y}_{t+1} - \beta E_t \hat{p}_{t+1}^g = \sigma^{-1} y_t - \beta \hat{p}_t^g - \varphi \left(1-\beta\right) \frac{\left(1+\nu\theta_m\right)}{\left(\varphi+\nu\theta_m\right)} \hat{p}_t^g. \tag{54}$$

which constitute a linear rational expectations model of $\{\hat{y}_t, \hat{p}_t^g\}$. This system can be solved using standard methods. Given a solution for $\{\hat{y}_t, \hat{p}_t^g\}$, the variables $\left\{\pi_t, \hat{m}_t, \hat{G}_t^p, \hat{G}^m, \hat{\imath}_t, \hat{P}_t^l\right\}$ follow directly.

Proposition 1. *Under the gold standard with fixed parity — the classical gold standard — the price level is a stationary process with constant mean.*

The dynamics of $\{\hat{y}_t, \hat{p}_t^g\}$ can be written as the system

$$\begin{bmatrix} \sigma^{-1}\beta & -\beta & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} E_t \begin{bmatrix} y_{t+1} \\ p_{t+1}^g \\ p_t^g \end{bmatrix} = \begin{bmatrix} \sigma^{-1} & \frac{-\varphi(1-\beta)(1+\nu\theta_m)}{(\varphi+\nu\theta_m)} - \beta & 0 \\ \kappa & 1+\beta & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ p_t^g \\ p_{t-1}^g \end{bmatrix}$$

which gives

$$\begin{aligned} E_t \begin{bmatrix} y_{t+1} \\ p_{t+1}^g \\ p_t^g \end{bmatrix} &= \begin{bmatrix} \frac{\sigma}{\beta} & \frac{\sigma}{\beta} & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma^{-1} & \frac{-\varphi(1-\beta)(1+\nu\theta_m)}{(\varphi+\nu\theta_m)} - \beta & 0 \\ \kappa & 1+\beta & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ p_t^g \\ p_{t-1}^g \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{\beta} + \frac{\sigma\kappa}{\beta} & \frac{\sigma}{\beta}\left(\beta+1\right) - \frac{\sigma}{\beta}\left(\beta - \frac{\varphi}{\varphi+\nu\theta_m}\left(\beta-1\right)\left(\nu\theta_m+1\right)\right) & -\frac{\sigma}{\beta} \\ \frac{\kappa}{\beta} & \frac{1}{\beta}\left(\beta+1\right) & -\frac{1}{\beta} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ p_t^g \\ p_{t-1}^g \end{bmatrix} \end{aligned}$$

or

$$E_t \begin{bmatrix} y_{t+1} \\ p^g_{t+1} \\ p^g_t \end{bmatrix} = M \begin{bmatrix} y_t \\ p^g_t \\ p^g_{t-1} \end{bmatrix} \tag{55}$$

Model properties depend on the eigenvalues of the matrix M. For a unique bounded rational expectations equilibrium, we require two eigenvalues outside, and one inside, the unit circle. This is because there are two non-predetermined, and one predetermined variable. In the background is the assumption of passive fiscal policy. The fiscal block contains two eigenvalues: one from the debt dynamics in the flow budget constraint of the government; the second from the price of government debt which takes the value $\rho_b \beta$. As there is one predetermined (the quantity) and one non-predetermined (the price) variable, then passive fiscal policy implies we require two eigenvalues outside the unity circle corresponding to the non-predetermined variables $\{\hat{y}_t, \hat{p}^g_t\}$.

The above system has characteristic polynomial

$$P(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0$$

where

$$\begin{aligned} A_2 &= -\frac{\sigma}{\beta}\left(\kappa + 2\sigma^{-1} + \sigma^{-1}\beta\right) \\ A_1 &= \frac{\left(\kappa\varphi + \sigma^{-1}\varphi + 2\sigma^{-1}\beta\varphi + \sigma^{-1}\nu\theta_m + \kappa\beta\nu\theta_m + 2\sigma^{-1}\beta\nu\theta_m + \kappa\nu\varphi\theta_m - \kappa\beta\nu\varphi\theta_m\right)}{\sigma^{-1}\beta^2\left(\varphi + \nu\theta_m\right)} \\ A_0 &= -\frac{1}{\beta^2} \end{aligned}$$

Then for there to be two eigenvalues outside, and one eigenvalue inside the unit circle, one of the following three cases needs to be satisfied [Woodford (2003, appendix C)]. There are three cases to consider.

Case I. The first case requires

$$1 + A_2 + A_1 + A_0 < 0$$

and

$$-1 + A_2 - A_1 + A_0 > 0$$

The first condition gives

$$-\frac{\sigma\kappa}{\beta^2}\frac{\varphi}{\varphi + \nu\theta_m}(\beta - 1)(\nu\theta_m + 1) > 0$$

so case 1 violated.

Case II: The second case requires satisfaction of

$$1 + A_2 + A_1 + A_0 > 0 \tag{56}$$

and

$$-1 + A_2 - A_1 + A_0 < 0 \tag{57}$$

and

$$A_0^2 - A_0 A_2 + A_1 - 1 > 0. \tag{58}$$

The first condition is satisfied from the calculation in Case I. The second condition

$$-1 - \frac{1}{\sigma^{-1}\beta}\left(\kappa + 2\sigma^{-1} + \sigma^{-1}\beta\right)$$
$$-\frac{1}{\sigma^{-1}\beta^2\left(\varphi + \nu\theta_m\right)}\left(\kappa\varphi + \sigma^{-1}\varphi + 2\sigma^{-1}\beta\varphi + \sigma^{-1}\nu\theta_m + \kappa\beta\nu\theta_m + 2\sigma^{-1}\beta\nu\theta_m + \kappa\nu\varphi\theta_m - \kappa\beta\nu\varphi\theta_m\right) - \frac{1}{\beta^2}$$

is obviously negative.

The final condition provides

$$\frac{1}{\sigma^{-1}\beta^4\left(\varphi + \nu\theta_m\right)}\left(\beta - 1\right)\times$$
$$\left(\sigma^{-1}\beta^2\varphi - \sigma^{-1}\varphi - \sigma^{-1}\beta^3\varphi + \sigma^{-1}\beta\varphi - \sigma^{-1}\nu\theta_m + \sigma^{-1}\beta^2\nu\theta_m - \sigma^{-1}\beta^3\nu\theta_m + \sigma^{-1}\beta\nu\theta_m + \kappa\beta\left(\varphi + \nu\theta_m\left(\beta\right.\right.\right.$$

To sign this, focus on the final bracketed term. Suppose κ equals zero then

$$\begin{aligned}\left(\beta^2 - 1 - \beta^3 + \beta\right)\left(\sigma^{--11}\varphi + \sigma\nu\theta_m\right) &= \left(\beta^2\left(1-\beta\right) - \left(1-\beta\right)\right)\left(\sigma^{-1}\varphi + \sigma^{-1}\nu\theta_m\right) \\ &= \left(1-\beta\right)\left(\beta^2 - 1\right)\left(\sigma^{-1}\varphi + \sigma^{-1}\nu\theta_m\right) < 0\end{aligned}$$

which means the final condition (58) is satisfied. With general κ we have

$$\beta^2\varphi - \varphi - \beta^3\varphi + \beta\varphi - \nu\theta_m + \beta^2\nu\theta_m - \beta^3\nu\theta_m + \beta\nu\theta_m + \sigma\kappa\beta\left(\varphi + \beta\nu\theta_m + \nu\theta_m - \beta\nu\varphi\theta_m\right)$$

Whether this is positive or negative doesn't matter. The terms involving κ are

$$\sigma\kappa\beta\left(\varphi + \beta\nu\theta_m + \nu\theta_m - \beta\nu\varphi\theta_m\right).$$

Depending on the values of different elasticities this could be positive or negative. And if negative, could violate (58). So consider Case III.

Case III: But note that for case III (56) and (57) must hold along with

$$A_0^2 - A_0 A_2 + A_1 - 1 < 0$$

and

$$|A_2| > 3. \tag{59}$$

Because (58) holds for small κ as its value grows this condition is either positive, or switching to negative. But in that case $|A_2|$ is independent of the terms in κ so the requirements are satisfied.

Stated differently, the variables that could make (58) negative, such as ν, φ and θ_m do not affect the final condition (59). Hence we have a unique bounded rational expectations equilibrium for $\{\hat{y}_t, \hat{p}_t^g\}$. Which means that $\log P_t$ is stationary with mean $\log \bar{P}^g$. These conditions hold for all maintained parameter values.

B.4.2 Implications of Alternative Fiscal Policies The assumption of a balanced budget rule, or more generally, fiscal policy which generates a Ricardian equilibrium, is crucial for the statistical properties of goods prices and inflation — specifically prices are a stationary time series. We now show that for other fiscal policies, goods prices must jump to ensure intertemporal solvency of the government accounts. This drift in the price level is inconsistent with the assumptions of a Classical Gold Standard. We consider a class of fiscal policy defined as follows.

Definition 6. *An unbacked fiscal expansion is an increase in government expenditures financed initially by sales of nominal government bonds, with no expectation of higher taxes or lower expenditure in the future to service payment of the bonds.*

Formally, taxes exhibit weak sensitivity to outstanding debt: fiscal policy is passive. An immediate implication of Proposition 1 follows.

Corollary 7. *An unbacked fiscal expansion is infeasible under a classical gold standard.*

The proof of proposition 1 establishes a unique bounded equilibrium exists for all maintained parameter values given the assumption of passive fiscal policy. Because the system in

$$\left\{y_t, p_t^g, p_{t-1}^g, b_{t-1}^l, P_t^l\right\}$$

has three non-predetermined and two predetermined variables, an active fiscal policy requires the subsystem (55) to have two eigenvalues inside the unit circle. But Proposition 1 shows the matrix M has two eigenvalues outside the unit circle for all parameter values. This means an UBFE is never feasible under the classical gold standard. If we alter the conduct of tax policy to render the debt dynamics explosive, all remaining eigenvalues remain unaltered in so far as they remain either inside or outside the unit circle independently of the specification of tax policy. Because there are four eigenvalues outside the unit circle and only thee non-predetermined variables $\left\{\hat{y}_t, \hat{p}_t^g, P_t^l\right\}$ there exists no bounded equilibrium for these variables. In this sense, the UBFE is infeasible under the classical gold standard.

B.4.3 Properties of an UBFE Central to our thesis is that departure from the Gold Standard permitted the execution of a UBFE. We now characterize formally the properties of the model under that policy configuration. Specifically, we abandon the gold standard assumption that

$$P_t^g = \bar{P}^g \text{ for all } t\,.$$

This is the only new assumption, along with the specification of fiscal policy. The model still has a steady state in which

$$\frac{P_t^g}{P_t} = \frac{P_t}{P_{t-1}} = 1$$

so we continue to look for a unique bounded equilibrium in which $\hat{p}_t^g$ stationary, forever in a bounded neighborhood of unity. However, we now show the statistical properties of $\log(P_t)$ are fundamentally different. We can maintain the assumption of a fixed coverage ratio of money supply (for which there is some evidence the Fed followed more closely after the abandonment of the gold standard) or specify policy for the nominal interest rate. The results don't depend on which is assumed, though the proof for subsequent results make use of a conventional Taylor rule.

Appending a debt evolution equation to model above permits the following result.

Proposition 2. *An unbacked fiscal expansion permanently raises the price level without permanently raising the inflation rate.*

Details of the proof are deferred until subsequent sections which develop model implications for fiscal multipliers. However, the proof proceeds in the same manner as before. The model permits study of a subsystem in the variables

$$\left\{\pi_t, y_t, b_t^l, P_t^l\right\}.$$

The proof shows that under an active fiscal policy (and implicitly passive interest rate policy), this system has three eigenvalues outside the unit circle and one inside, giving a unique bounded rational expectations equilibrium. Inflation defined as

$$\log\left(\frac{P_t}{P_{t-1}}\right)$$

is therefore a stationary time series with constant mean. This in turn implies that $\log(P_t)$ is a unit root process with constant drift. Because the solutions are a function of all exogenous disturbances, an increase in government spend F_t results in a permanent increase in the price level, but no permanent impact on inflation. This is an example of the fiscal theory of price level.

B.5 The Complete Model

With interest rates determined by a Taylor rule the variables $\{y_t, \pi_t, i_t\}$ comprise a subsystem independent of the dynamics of $\left\{m_t, \hat{p}_t^g, \hat{G}_t^p\right\}$. Given a solution to the former, the dynamics of the latter immediately follow. We therefore drop the variables $\left\{m_t, \hat{p}_t^g, \hat{G}_t^p\right\}$ from most subsequent discussion, giving focus to the fiscal effects on inflation and output. The demand and supply equations are given by

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma\left(i_t - E_t\pi_{t+1} - r_t^n\right) \\ \pi_t &= \kappa x_t + \beta E_t \pi_{t+1} \end{aligned}$$

where

$$\begin{aligned} x_t &= \hat{y}_t - Y_t^n \\ r_t^n &= \sigma^{-1}\left[\left(g_t - Y_t^n\right) - E_t\left(g_{t+1} - Y_{t+1}^n\right)\right] \\ Y_t^n &= \frac{\sigma^{-1} g_t + \omega q_t}{\sigma^{-1} + \omega} \\ g_t &= \tilde{F}_t + s_c \bar{c}_t \\ q_t &= \left(1 + \omega^{-1}\right) a_t + \omega^{-1} v \bar{h}_t \end{aligned}$$

and

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\omega+\sigma^{-1}}{1+\omega\theta}.$$

This permits thinking about a range of disturbances, though the focus below will be on the dynamics in response to an exogenous variation in government purchases, F_t. Variables are defined as deviations from steady state values, with the exception of fiscal spending and interest rates which satisfy

$$\tilde{F}_t = \frac{F_t - F}{Y} \text{ and } \hat{\imath} = \log\left(\frac{1+i_t}{1+i}\right).$$

For simplicity, all hats and tildas are dropped in the above. Finally note that the terms $\bar{c}_t$ and $\bar{h}_t$ are disturbances to the utility of consumption and the disutility of labor supply. And a_t is a technology shock.

The government issues both short and long term debt. The flow budget constraint of the government is given by

$$\frac{M_T}{P_T} + \frac{P_T^s B_T^s}{P_T} + \frac{P_T^l B_T^l}{P_T} + \frac{P_T^g G_T^p}{P_T} + T_T = F_T + \frac{M_{T-1}}{P_T} + \frac{B_{T-1}^s}{P_T} + \frac{\left(1+\rho_b P_T^l\right) B_{T-1}^l}{P_T} + \frac{P_T^g G_{T-1}^p}{P_T}.$$

Rearranging provides

$$\frac{M_T}{P_T} - \frac{M_{T-1}}{P_{T-1}}\frac{P_{T-1}}{P_T} + \frac{P_T^s B_T^s}{P_T} - \frac{B_{T-1}^s}{P_{T-1}}\frac{P_{T-1}}{P_T} + \frac{P_T^l B_T^l}{P_T} + \frac{P_T^g G_T^p}{P_T} - \frac{P_T^g G_{T-1}^p}{P_T} = F_T - T_T + \frac{\left(1+\rho_b P_T^l\right) B_{T-1}^l}{P_T}\frac{P_{T-1}}{P_{T-1}}$$

in any period T. We can write this as

$$m_t - m_{t-1}\pi_t^{-1} + P_t^s b_t^s - b_{t-1}^s \pi_t^{-1} + P_t^l b_t^l + \frac{P_t^g}{P_t}\left(G_t^p - G_{t-1}^p\right) = F_t - T_t + \left(1+\rho_b P_t^l\right) b_{t-1}^l \pi_t^{-1}$$

where $m_t \equiv M_t/P_t$ is real money balances, $\pi_t = \log\left(P_t/P_{t-1}\right)$ inflation, and $b_t = B_t/P_t$ a measure of real debt. Take a first order approximation, defining any variable that can take zero value in deviations from steady state output (T_t, F_t, b_t^s, b_t^l) denoted with a tilda, and remaining variables in log deviations from steady state, denoted with a hat. Hence

$$\tilde{T}_t = \frac{T_t - T}{Y}; \tilde{F}_t = \frac{F_t - F}{Y} \text{ and } \tilde{b}_t^j = \frac{b_t^j - b^j}{Y}$$

for $j \in (s, l)$ and

$$\hat{m}_t = \log\left(\frac{m_t}{m}\right); \hat{G}_t^p = \log\left(\frac{G_t^p}{G^p}\right)$$

and so on. To a first order

$$\begin{aligned}
LHS &= m\hat{m}_t - m\left(\hat{m}_{t-1} - \pi_t\right) + P^s y \tilde{b}_t^s + b^s P^s \hat{P}_t^s - y\tilde{b}_{t-1}^s + b^s \pi_t + P^l y \tilde{b}_t^l + P^l P_t^l b^l + \frac{P^g G^p}{P}\left(\hat{G}_t^p - \hat{G}_{t-1}^p\right) \\
RHS &= y\tilde{F}_t - y\tilde{T}_t + \left(1+\rho_b P^l\right)\left(y\tilde{b}_{t-1}^l - \pi_t\right) + b\rho_b P^l \hat{P}_t^l. \\
&= y\tilde{F}_t - y\tilde{T}_t + \left(1+\rho_b P^l\right) b\left(\frac{y}{b}\tilde{b}_{t-1}^l - \hat{\pi}_t\right) + b\rho_b P^l \hat{P}_t^l
\end{aligned}$$

where

$$P^l = \frac{\beta}{1-\beta\rho_b}$$

Dividing by the steady state level of income gives

$$\begin{aligned}
& \frac{m}{y}\hat{m}_t - \frac{m}{y}\left(\hat{m}_{t-1} - \hat{\pi}_t\right) + P^s\tilde{b}^s_t + \frac{b^sP^s}{y}\hat{P}^s_t - \tilde{b}^s_{t-1} + \frac{b^s}{y}\hat{\pi}_t + P^l\tilde{b}^l_t + \frac{P^lb}{y}\hat{P}^l_t + \frac{P^gG^p}{Py}\left(\hat{G}^p_t - \hat{G}^p_{t-1}\right) \\
= \; & \tilde{F}_t - \tilde{T}_t + \left(1+\rho_bP^l\right)\frac{b}{y}\left(\frac{y}{b}\tilde{b}^l_{t-1} - \hat{\pi}_t\right) + \frac{b}{y}\rho_bP^l\hat{P}^l_t.
\end{aligned}$$

Let's specialize to a model without money or gold (which is essentially our environment post abandonment of the gold standard). Let's also assume there is only long debt. This gives

$$P^l\tilde{b}^l_t + \frac{P^lb^l}{y}\hat{P}^l_t = \tilde{F}_t - \tilde{T}_t + \left(1+\rho_bP^l\right)\frac{b}{y}\left(\frac{y}{b}\tilde{b}^l_{t-1} - \hat{\pi}_t\right) + \frac{b}{y}\rho_bP^l\hat{P}^l_t$$

or

$$\tilde{b}^l_t + \frac{b^l}{y}\hat{P}^l_t = \left(\tilde{F}_t - \tilde{T}_t\right)\frac{1}{P^l} + \frac{\left(1+\rho_bP^l\right)}{P^l}\frac{b}{y}\left(\frac{y}{b}\tilde{b}^l_{t-1} - \hat{\pi}_t\right) + \frac{b}{y}\rho_b\hat{P}^l_t$$

or

$$\tilde{b}^l_t = \left(\tilde{F}_t - \tilde{T}_t\right)\frac{1}{P^l} + \beta^{-1}\frac{b}{y}\left(\frac{y}{b}\tilde{b}^l_{t-1} - \hat{\pi}_t\right) + \frac{b}{y}\left(\rho_b - 1\right)\hat{P}^l_t.$$

If there were only one period debt then $\rho_b = 0$ and $P^l = \beta$.

Dropping all hats and tilda's, the complete model is

$$\begin{aligned}
x_t &= E_tx_{t+1} - \sigma\left(i_t - E_t\pi_{t+1} - r^n_t\right) \\
\pi_t &= \kappa x_t + \beta E_t\pi_{t+1} \\
b^l_t &= \left(F_t - T_t\right)\frac{1}{P^l} + \beta^{-1}\frac{b}{y}\left(\frac{y}{b}b^l_{t-1} - \pi_t\right) + \frac{b}{y}\left(\rho_b - 1\right)P^l_t \\
P^l_t &= \rho_b\beta E_tP^l_t - i_t \\
i_t &= \phi_\pi\pi_t \\
T_t &= P^l\phi_Tb_{t-1}
\end{aligned}$$

Furthermore, assume there are only *i.i.d.* fiscal spending shocks so that

$$\begin{aligned}
r^n_t &= \frac{1}{\sigma+\omega^{-1}}F_t \\
Y^n_t &= \frac{\sigma^{-1}}{\sigma^{-1}+\omega}F_t
\end{aligned}$$

In this case

$$\begin{aligned}
y_t &= E_ty_{t+1} + \frac{\sigma^{-1}}{\sigma^{-1}+\omega}F_t - \sigma\left(i_t - E_t\pi_{t+1} - \frac{1}{\sigma+\omega^{-1}}F_t\right) \\
&= E_ty_{t+1} + \left(\frac{1}{1+\sigma\omega} + \frac{\omega\sigma}{\omega\sigma+1}\right)F_t - \sigma\left(i_t - E_t\pi_{t+1}\right) \\
&= E_ty_{t+1} + F_t - \sigma\left(i_t - E_t\pi_{t+1}\right)
\end{aligned}$$

and

$$\begin{aligned}
\pi_t &= \kappa x_t + \beta E_t \pi_{t+1} \\
&= \kappa \left(y_t - Y_t^n\right) + \beta E_t \pi_{t+1} \\
&= \kappa \left(y_t - \frac{\sigma^{-1}}{\sigma^{-1} + \omega} F_t\right) + \beta E_t \pi_{t+1} \\
&= \kappa y_t + \beta E_t \pi_{t+1} - \frac{\kappa \sigma^{-1}}{\sigma^{-1} + \omega} F_t \\
&= \kappa y_t + \beta E_t \pi_{t+1} - \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\omega + \sigma^{-1}}{1 + \omega\theta} \frac{\sigma^{-1}}{\sigma^{-1} + \omega} F_t .
\end{aligned}$$

B.5.1 SOLVING THE MODEL UNDER REGIME F The final goal is to compute fiscal multipliers under a UBFE. There are two properties to establish, which are relevant to our interpretation of the recovery from the Great Depression.

1. Many argue that the size of deficits, and the size of government, captured by the quantity of outstanding debt, were too small for fiscal policy to have been an important contributor to the recovery. We show that the smaller is the debt to income ratio, the larger are fiscal multipliers; and

2. That these multipliers are more likely to be large for economies of the kind represented by the US economy in 1933. Specifically for economies pursuing an UBFE as compared to a policy configuration in which monetary policy is active; and fiscal policy is passive, as proposed by Eggertsson (2007).

To establish these results, we solve the model under each policy configuration. We do this under the assumption of long-duration debt, which is a contribution to the literature. Substituting for the policy rules permits the first four relations to be written

$$\begin{aligned}
x_t &= E_t x_{t+1} - \sigma \left(\phi_\pi \pi_t - E_t \pi_{t+1} - r_t^n\right) \\
\pi_t &= \kappa x_t + \beta E_t \pi_{t+1} \\
b_t^l &= \frac{1}{P^l} F_t + \beta^{-1} \left(1 - \phi_T \beta\right) b_{t-1}^l - \beta^{-1} \frac{b}{y} \pi_t + \frac{b}{y} \left(\rho_b - 1\right) P_t^l \\
P_t^l &= \rho_b \beta E_t P_{t+1}^l - \phi_\pi \pi_t
\end{aligned}$$

This leaves four equations in four variables. To map this representation back into Proposition 2, note that $x_t = y_t$ in absence of exogenous disturbances; and that conditional on the solution for $\left\{y_t, \pi_t, b_t^l, P_t^l\right\}$ one can solve for a unique bounded equilibrium in $\{i_t, p_t^g, m_t, G_t^m\}$.

For a unique bounded equilibrium we require three eigenvalues outside the unit circle and one inside — real debt is the only predetermined variable. Write the subsystem that is independent of debt as

$$E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \\ P_{t+1}^l \end{bmatrix} = \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa & 0 \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1 + \kappa\sigma\beta^{-1} & 0 \\ \phi_\pi / \left(\rho_b \beta\right) & 0 & \left(\rho_b \beta\right)^{-1} \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ P_t^l \end{bmatrix} + \begin{bmatrix} 0 \\ -\sigma \\ 0 \end{bmatrix} r_t^n .$$

If we are looking for a fiscal theory equilibrium then we have one explosive eigenvalue in the debt equation. Formally this requires

$$\left|\beta^{-1} - \phi_T\right| > 1.$$

This means we need one stable eigenvalue in this subsystem. Since $(\rho_b \beta)^{-1}$ is outside the unit circle, one of the remaining two eigenvalues must be in the unit circle. The system has characteristic polynomial

$$\begin{aligned} P_3(\lambda) &= \frac{1}{\beta}\left(\lambda - (\beta\rho_b)^{-1}\right)\left(-\lambda - \lambda\beta + \lambda^2\beta - \lambda\kappa\sigma + \kappa\sigma\phi_\pi + 1\right) \\ &= \left(\lambda - (\beta\rho_b)^{-1}\right)\left(\lambda^2 - \lambda\left(\beta^{-1} + 1 + \beta^{-1}\kappa\sigma\right) + (\kappa\sigma\phi_\pi + 1)\beta^{-1}\right) \end{aligned}$$

Hence once eigenvalue is $(\beta\rho_b)^{-1}$. The remaining are determined by

$$P_2(\lambda) = \lambda^2 - \lambda\left(\beta^{-1} + 1 + \beta^{-1}\kappa\sigma\right) + (\kappa\sigma\phi_\pi + 1)\beta^{-1}$$

which satisfies

$$\begin{aligned} P_2(0) &= (\kappa\sigma\phi_\pi + 1)\beta^{-1} \\ P_2(1) &= \beta^{-1}\kappa\sigma(\phi_\pi - 1). \end{aligned}$$

This implies that for one eigenvalue inside the unit circle we must have $\phi_\pi < 1$ as is well known under the fiscal theory.

Because the subsystem in inflation and output is independent of the debt price, work with

$$E_t\begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1 + \kappa\sigma\beta^{-1} \end{bmatrix}\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\sigma \end{bmatrix} r_t^n.$$

Compute the eigenvector associated with the explosive eigenvalue as

$$\begin{bmatrix} 1 & d_1 \end{bmatrix}\begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1 + \kappa\sigma\beta^{-1} \end{bmatrix} = \begin{bmatrix} 1 & d_1 \end{bmatrix}\lambda_1$$

implying that

$$-\beta^{-1}\kappa + d_1\left(1 + \kappa\sigma\beta^{-1}\right) = d_1\lambda_1$$

or

$$\begin{aligned} d_1 &= \frac{\beta^{-1}\kappa}{1 + \kappa\sigma\beta^{-1} - \lambda_1} \\ &= \frac{\kappa}{\beta\lambda_2 - 1} \end{aligned}$$

since

$$1 + \kappa\sigma\beta^{-1} + \beta^{-1} = \lambda_1 + \lambda_2.$$

Transforming the system with the unstable eigenvector gives

$$E_t\bar{z}_{t+1} = \lambda_1\bar{z}_t + \frac{\kappa}{1 - \lambda_2\beta}\sigma r_t^n$$

where

$$\bar{z}_t = \begin{bmatrix} 1 & d_1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}.$$

Solving forward provides

$$\begin{aligned} \bar{z}_t &= \left(\frac{1}{\lambda_1}\right) E_t \bar{z}_{t+1} - \left(\frac{1}{\lambda_1}\right) \frac{\kappa}{1-\lambda_2 \beta} \sigma r_t^n \\ &= -\left(\frac{1}{\lambda_1}\right) \frac{\kappa}{1-\lambda_2 \beta} \sigma r_t^n \end{aligned}$$

Which implies

$$\pi_t = -d_1 x_t - \left(\frac{1}{\lambda_1}\right) \frac{\kappa}{1-\lambda_2 \beta} \sigma r_t^n$$

on the balance growth path. Hence

$$x_t = -\frac{1}{d_1} \left(\left(\frac{1}{\lambda_1}\right) \frac{\kappa}{1-\lambda_2 \beta} \sigma r_t^n + \pi_t \right).$$

Now using this expression with the aggregate supply curve provides

$$\begin{aligned} E_t \pi_{t+1} &= \left(\pi_t + \frac{\kappa}{d_1} \left(\frac{1}{\lambda_1} \frac{\kappa}{1-\lambda_2 \beta} \sigma r_t^n + \pi_t \right) \right) \beta^{-1} \\ &= \beta^{-1} \left(1 + \frac{\kappa}{d_1} \right) \pi_t - \beta^{-1} \frac{1}{\lambda_1} \kappa \sigma r_t^n \end{aligned}$$

where

$$\beta^{-1} \left(1 + \frac{\kappa}{d_1} \right) = \beta^{-1} \left(1 + \frac{\kappa}{\frac{\kappa}{\beta \lambda_2 - 1}} \right) = \lambda_2$$

so that

$$E_t \pi_{t+1} = \lambda_2 \pi_t - \beta^{-1} \frac{1}{\lambda_1} \kappa \sigma r_t^n$$

and for subsequent use

$$\begin{aligned} E_t \pi_{t+2} &= E_t \left(\lambda_2 \pi_{t+1} - \beta^{-1} \frac{1}{\lambda_1} \kappa \sigma r_{t+1}^n \right) = \lambda_2 E_t \pi_{t+1} \\ E_t \pi_{t+3} &= E_t \lambda_2 \pi_{t+2} = (\lambda_2)^2 E_t \pi_{t+1}. \end{aligned}$$

Hence in general

$$E_t \pi_{t+j} = \lambda_2^j \pi_t - (\lambda_2)^{j-1} \beta^{-1} \frac{1}{\lambda_1} \kappa \sigma r_t^n.$$

Return to the debt dynamics

$$\begin{aligned} b_t^l &= \frac{1}{P^l} F_t + \beta^{-1} (1 - \phi_T \beta) b_{t-1}^l - \beta^{-1} \frac{b}{y} \hat{\pi}_t + \frac{b}{y} (\rho_b - 1) P_t^l \\ P_t^l &= \rho_b \beta E_t P_{t+1}^l - \phi_\pi \pi_t. \end{aligned}$$

The bond price equation gives

$$\begin{aligned} P_t^l &= \rho_b \beta E_t P_{t+1}^l - \phi_\pi \pi_t \\ &= -\phi_\pi E_t \sum_{T=t}^{\infty} (\rho_b \beta)^{T-t} \pi_T \\ &= -\phi_\pi E_t \sum_{T=t}^{\infty} (\rho_b \beta)^{T-t} \pi_T \end{aligned}$$

in a bounded equilibrium. Using the earlier derived expression to evaluate expectations gives

$$\begin{aligned} P_t^l &= -\phi_\pi E_t \sum_{T=t}^{\infty} (\rho_b \beta)^{T-t} \pi_T \\ &= -\phi_\pi \pi_t - \phi_\pi E_t \sum_{T=t+1}^{\infty} (\rho_b \beta)^{T-t} \pi_T \\ &= -\phi_\pi \pi_t - \phi_\pi \rho_b \beta E_t \sum_{T=t+1}^{\infty} (\rho_b \beta)^{T-t-1} \left(\lambda_2^{T-t} \pi_t - (\lambda_2)^{T-t-1} \beta^{-1} \frac{1}{\lambda_1} \kappa \sigma r_t^n \right) \\ &= -\phi_\pi \pi_t - \phi_\pi \rho_b \beta E_t \sum_{T=t+1}^{\infty} (\rho_b \beta)^{T-t-1} \lambda_2^{T-t-1} \left(\lambda_2 \pi_t - \beta^{-1} \frac{1}{\lambda_1} \kappa \sigma r_t^n \right) \\ &= -\phi_\pi \pi_t - \frac{\phi_\pi \rho_b \beta \lambda_2}{1 - \rho_b \beta \lambda_2} \left(\pi_t - \beta^{-1} \frac{1}{\lambda_1 \lambda_2} \kappa \sigma r_t^n \right). \end{aligned}$$

Using this in the debt dynamics equation provides

$$\begin{aligned} b_t^l &= \frac{1}{P^l} F_t + \beta^{-1} (1 - \phi_T \beta) b_{t-1}^l - \beta^{-1} \frac{b}{y} \pi_t + \frac{b}{y} (\rho_b - 1) P_t^l \\ &= \frac{1}{P^l} F_t + \beta^{-1} (1 - \phi_T \beta) b_{t-1}^l - \beta^{-1} \frac{b}{y} \pi_t + \frac{b}{y} (1 - \rho_b) \left(\phi_\pi \pi_t + \frac{\phi_\pi \rho_b \beta \lambda_2}{1 - \rho_b \beta \lambda_2} \left(\pi_t - \beta^{-1} \frac{1}{\lambda_1 \lambda_2} \kappa \sigma r_t^n \right) \right) \\ &= \beta^{-1} (1 - \phi_T \beta) b_{t-1}^l + \frac{1}{P^l} F_t + \left(\frac{b}{y} (1 - \rho_b) \left(\frac{\rho_b \beta \lambda_2 \phi_\pi}{1 - \rho_b \beta \lambda_2} + \phi_\pi \right) - \beta^{-1} \frac{b}{y} \right) \pi_t \\ &\quad - \frac{b}{y} (1 - \rho_b) \phi_\pi \frac{\rho_b \beta \lambda_2}{1 - \rho_b \beta \lambda_2} \frac{1}{\beta \lambda_2 \lambda_1} \kappa \sigma r_t^n \\ &= \beta^{-1} (1 - \phi_T \beta) b_{t-1}^l + \frac{1}{P^l} F_t + \left(\frac{b}{y} (1 - \rho_b) \left(\frac{\phi_\pi}{1 - \rho_b \beta \lambda_2} \right) - \beta^{-1} \frac{b}{y} \right) \pi_t \\ &\quad - \frac{b}{y} (1 - \rho_b) \phi_\pi \frac{\rho_b \beta \lambda_2}{1 - \rho_b \beta \lambda_2} \frac{1}{\beta \lambda_2 \lambda_1} \kappa \sigma r_t^n. \end{aligned}$$

Consider purely exogenous tax rules only. Solving the debt equation forward

$$\begin{aligned}
b^l_{t-1} &= \beta b^l_t - \frac{\beta}{P^l}F_t - \beta\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\pi_t + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= \beta E_t\sum_{T=t}^{\infty}(\beta)^{T-t}\left[-\frac{1}{P^l}F_t - \left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\pi_T + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{1}{\beta\lambda_2\lambda_1}\kappa\sigma r^n_t\right] \\
&= -\frac{\beta}{P^l}F_t - \beta\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\sum_{T=t}^{\infty}(\beta)^{T-t}E_t\pi_T + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= -\frac{\beta}{P^l}F_t - \beta\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\left[\pi_t + \frac{\beta\lambda_2}{1-\beta\lambda_2}\left(\pi_t - \beta^{-1}\frac{1}{\lambda_1\lambda_2}\kappa\sigma r^n_t\right)\right] \\
&\quad + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= -\frac{\beta}{P^l}F_t - \beta\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\left[\frac{1}{1-\beta\lambda_2}\pi_t - \frac{\beta\lambda_2}{1-\beta\lambda_2}\frac{1}{\beta\lambda_1\lambda_2}\kappa\sigma r^n_t\right] \\
&\quad + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t
\end{aligned}$$

which implies inflation is given as

$$\begin{aligned}
\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\frac{\beta}{1-\beta\lambda_2}\pi_t &= -b^l_{t-1} - \frac{\beta}{P^l}F_t \\
&\quad + \left(\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\frac{\beta\lambda_2}{1-\beta\lambda_2} + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\right) \\
&\quad \times\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t
\end{aligned}$$

or

$$\begin{aligned}
\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)\pi_t &= -\frac{(1-\beta\lambda_2)}{\beta}b^l_{t-1} - \frac{(1-\beta\lambda_2)}{P^l}F_t \\
&\quad + \left(\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right) + \frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t
\end{aligned}$$

Rearranging gives

$$
\begin{aligned}
\pi_t &= \left(-\frac{(1-\beta\lambda_2)}{\beta}b^l_{t-1} - \frac{(1-\beta\lambda_2)}{P^l}F_t\right)\left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)^{-1} \\
&\quad + \left(\lambda_2 + \left(\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\frac{b}{y}\right)^{-1}\frac{b}{y}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= \frac{y}{b}\left(\frac{(1-\beta\lambda_2)}{\beta}b^l_{t-1} + \frac{(1-\beta\lambda_2)}{P^l}F_t\right)\left(\beta^{-1} - \frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\
&\quad + \left(\lambda_2 + \left(\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= \frac{y}{b}\left(\frac{(1-\beta\lambda_2)}{\beta}b^l_{t-1} + \frac{(1-\beta\lambda_2)}{P^l}F_t\right)\left(\beta^{-1} - \frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\
&\quad + \left(\lambda_2 + \left(\frac{\beta}{(1-\rho_b)\,\phi_\pi\beta - (1-\rho_b\beta\lambda_2)}\right)(1-\rho_b)\,\phi_\pi\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= \frac{y}{b}\left(\frac{(1-\beta\lambda_2)}{\beta}b^l_{t-1} + \frac{(1-\beta\lambda_2)}{P^l}F_t\right)\left(\beta^{-1} - \frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\
&\quad + \left(\lambda_2 + \left(\frac{(1-\rho_b)\,\phi_\pi}{(1-\rho_b\beta\lambda_2)} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t.
\end{aligned}
$$

Given this the debt dynamics are

$$
\begin{aligned}
b^l_t &= \beta^{-1}b^l_{t-1} + \frac{1}{P^l}F_t + \left(\frac{b}{y}\left(\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right) - \beta^{-1}\frac{b}{y}\right)\pi_t - \frac{b}{y}(1-\rho_b)\,\phi_\pi\frac{\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{1}{\beta\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= \beta^{-1}b^l_{t-1} + \frac{1}{P^l}F_t - \left(\frac{(1-\beta\lambda_2)}{\beta}b^l_{t-1} + \frac{(1-\beta\lambda_2)}{P^l}F_t\right) \\
&\quad + \left(\left(\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right) - \beta^{-1}\right)\frac{b}{y}\left(\lambda_2 + \left(\frac{(1-\rho_b)\,\phi_\pi}{(1-\rho_b\beta\lambda_2)} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&\quad - \frac{b}{y}(1-\rho_b)\,\phi_\pi\frac{\rho_b\beta\lambda_2}{1-\rho_b\beta\lambda_2}\frac{1}{\beta\lambda_2\lambda_1}\kappa\sigma r^n_t \\
&= \lambda_2 b^l_{t-1} + \frac{1}{P^l}\beta\lambda_2 F_t \\
&\quad + \frac{b}{y}\left(\left(\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)\lambda_2 - (1-\rho_b)\,\phi_\pi\frac{\beta\lambda_2\rho_b\lambda_2}{1-\rho_b\beta\lambda_2}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r^n_t.
\end{aligned}
$$

B.5.2 Solving the Model under Regime M

Now the eigenvalue in the debt equation is in the unit circle, which implies the remaining eigenvalues must be outside the unit circle. From earlier discussion the eigenvalue of the price of long debt satisfies the requirement. Let's work again with the subsystem in inflation and output, written as

$$
E_t\begin{bmatrix}\pi_{t+1}\\ x_{t+1}\end{bmatrix} = \begin{bmatrix}\beta^{-1} & -\beta^{-1}\kappa \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1+\kappa\sigma\beta^{-1}\end{bmatrix}\begin{bmatrix}\pi_t\\ x_t\end{bmatrix} + \begin{bmatrix}0\\ -\sigma\end{bmatrix}r^n_t
$$

which has two eigenvalues outside the unit circle so long as the Taylor principle is satisfied — see earlier calculations. Compute the eigenvector associated with the explosive eigenvalues as

$$\begin{bmatrix} 1 & d_1 \end{bmatrix} \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1+\kappa\sigma\beta^{-1} \end{bmatrix} = \begin{bmatrix} 1 & d_1 \end{bmatrix} \lambda_1$$

implying that

$$-\beta^{-1}\kappa + d_1\left(1+\kappa\sigma\beta^{-1}\right) = d_1\lambda_1$$

or

$$\begin{aligned} d_1 &= \frac{\beta^{-1}\kappa}{1+\kappa\sigma\beta^{-1}-\lambda_1} \\ &= \frac{\kappa}{\beta\lambda_2 - 1} \end{aligned}$$

since

$$1+\kappa\sigma\beta^{-1}+\beta^{-1} = \lambda_1+\lambda_2.$$

And the second

$$\begin{bmatrix} 1 & d_2 \end{bmatrix} \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1+\kappa\sigma\beta^{-1} \end{bmatrix} = \begin{bmatrix} 1 & d_2 \end{bmatrix} \lambda_2$$

gives

$$\begin{aligned} d_2 &= \frac{\beta^{-1}\kappa}{1+\kappa\sigma\beta^{-1}-\lambda_2} \\ &= \frac{\kappa}{\beta\lambda_1 - 1}. \end{aligned}$$

Transforming the system with the unstable eigenvector gives

$$\begin{aligned} E_t\bar{z}_{1,t+1} &= \lambda_1\bar{z}_{1,t} - d_1\sigma r_t^n \\ &= \lambda_1\bar{z}_{1,t} + \frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n \end{aligned}$$

where

$$\bar{z}_{1,t} = \begin{bmatrix} 1 & d_1 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}.$$

Solving forward provides

$$\begin{aligned} \bar{z}_{1,t} &= \left(\frac{1}{\lambda_1}\right) E_t\bar{z}_{1,t+1} + \left(\frac{1}{\lambda_1}\right) d_1\sigma r_t^n \\ &= \left(\frac{1}{\lambda_1}\right) E_t\bar{z}_{1,t+1} - \left(\frac{1}{\lambda_1}\right) \frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n \\ &= -\left(\frac{1}{\lambda_1}\right) \frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n \end{aligned}$$

Which implies

$$\pi_t = -d_1 x_t - \left(\frac{1}{\lambda_1}\right)\frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n \tag{60}$$

on the balance growth path. Similarly

$$E_t \bar{z}_{2,t+1} = \lambda_2 \bar{z}_{2,t} - \frac{\kappa}{1-\lambda_1\beta}\sigma r_t^n$$

which implies

$$\bar{z}_{2,t} = \left(\frac{1}{\lambda_2}\right)\frac{\kappa}{1-\lambda_1\beta}\sigma r_t^n$$

or

$$\pi_t = -d_2 x_t - \left(\frac{1}{\lambda_2}\right)\frac{\kappa}{1-\lambda_1\beta}\sigma r_t^n \tag{61}$$

Using (60) and (61) it follows that

$$-d_1 x_t - \left(\frac{1}{\lambda_1}\right)\frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n = -d_2 x_t - \left(\frac{1}{\lambda_2}\right)\frac{\kappa}{1-\lambda_1\beta}\sigma r_t^n$$

or

$$\begin{aligned}
\left(d_2 - d_1\right) x_t &= \left(\left(\frac{1}{\lambda_1}\right)\frac{\kappa}{1-\lambda_2\beta} - \left(\frac{1}{\lambda_2}\right)\frac{\kappa}{1-\lambda_1\beta}\right)\sigma r_t^n \\
&\left(\frac{\kappa\left(\beta\lambda_2 - 1\right)}{\left(\beta\lambda_1 - 1\right)\left(\beta\lambda_2 - 1\right)} - \frac{\kappa\left(\beta\lambda_1 - 1\right)}{\left(\beta\lambda_1 - 1\right)\left(\beta\lambda_2 - 1\right)}\right) x_t \\
&= \left(\left(\frac{1}{\lambda_1\lambda_2}\right)\frac{\lambda_2\kappa\left(1-\lambda_1\beta\right)}{\left(1-\lambda_2\beta\right)\left(1-\lambda_1\beta\right)} - \left(\frac{1}{\lambda_1\lambda_2}\right)\frac{\lambda_1\kappa\left(1-\lambda_2\beta\right)}{\left(1-\lambda_1\beta\right)\left(1-\lambda_2\beta\right)}\right)\sigma r_t^n \\
\left(\frac{\kappa\beta\left(\lambda_2 - \lambda_1\right)}{\left(\beta\lambda_1 - 1\right)\left(\beta\lambda_2 - 1\right)}\right) x_t &= \left(\left(\frac{1}{\lambda_1\lambda_2}\right)\frac{\lambda_2\kappa\left(1-\lambda_1\beta\right) - \lambda_1\kappa\left(1-\lambda_2\beta\right)}{\left(1-\lambda_2\beta\right)\left(1-\lambda_1\beta\right)}\right)\sigma r_t^n
\end{aligned}$$

which gives

$$x_t = \frac{\sigma}{\beta\lambda_2\lambda_1} r_t^n$$

Now the product of the eigenvalues is

$$\begin{aligned}
\left(\kappa\sigma\phi_\pi + 1\right)\beta^{-1} &= \lambda_1\lambda_2 \\
1 + \kappa\sigma\beta^{-1} + \beta^{-1} &= \lambda_1 + \lambda_2
\end{aligned}$$

which provides

$$x_t = \frac{\sigma}{\kappa\sigma\phi_\pi + 1} r_t^n.$$

It follows that inflation is given by

$$\begin{aligned}
\pi_t &= -d_2 x_t - \left(\frac{1}{\lambda_2}\right)\frac{\kappa}{1-\lambda_1\beta}\sigma r_t^n \\
&= -d_2\frac{\sigma}{\beta\lambda_2\lambda_1}r_t^n - \left(\frac{1}{\lambda_2}\right)\frac{\kappa}{1-\lambda_1\beta}\sigma r_t^n \\
&= \left(\frac{1}{(1-\beta\lambda_1)}\frac{1}{\beta\lambda_2\lambda_1} - \left(\frac{1}{\beta\lambda_1\lambda_2}\right)\frac{\beta\lambda_1}{1-\lambda_1\beta}\right)\kappa\sigma r_t^n \\
&= \left(\frac{1-\beta\lambda_1}{(1-\beta\lambda_1)}\frac{1}{\beta\lambda_2\lambda_1}\right)\kappa\sigma r_t^n \\
&= \frac{\kappa\sigma}{(\kappa\sigma\phi_\pi+1)}r_t^n
\end{aligned}$$

Under F we had

$$\begin{aligned}
\pi_t &= \frac{y}{b}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1}^l + \frac{(1-\beta\lambda_2)}{P^l}F_t\right)\left(\beta^{-1} - \frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\
&\quad + \left(\frac{1}{1-\beta\lambda_2} + \left(\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)\frac{(1-\beta\lambda_2)}{\beta\lambda_2\lambda_1}\kappa\sigma r_t^n.
\end{aligned}$$

Recall

$$\beta\lambda_2\lambda_1 = \kappa\sigma\phi_\pi + 1.$$

B.6 Computing Multipliers

Inflation dynamics are

$$\begin{aligned}
\pi_t &= \frac{y}{b}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1}^l + \frac{(1-\beta\lambda_2)}{P^l}F_t\right)\left(\beta^{-1} - \frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\
&\quad + \left(\lambda_2 + \left(\frac{(1-\rho_b)\,\phi_\pi}{(1-\rho_b\beta\lambda_2)} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n
\end{aligned}$$

Debt dynamics are

$$\begin{aligned}
b_t^l &= \lambda_2 b_{t-1}^l + \frac{1}{P^l}\beta\lambda_2 F_t \\
&\quad + \frac{b}{y}\left(\left(\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)\lambda_2 - (1-\rho_b)\,\phi_\pi\frac{\beta\lambda_2\rho_b\lambda_2}{1-\rho_b\beta\lambda_2}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n
\end{aligned}$$

Let's add purely exogenous tax shocks to give

$$\begin{aligned}
\pi_t &= \frac{y}{bP^l}\left(\frac{(1-\beta\lambda_2)}{\beta}P^l b_{t-1}^l + (1-\beta\lambda_2)(F_t-\tau_t)\right)\left(\beta^{-1} - \frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\
&\quad + \left(\lambda_2 + \left(\frac{(1-\rho_b)\,\phi_\pi}{(1-\rho_b\beta\lambda_2)} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\,\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n.
\end{aligned}$$

and

$$\begin{aligned} P^l b_t^l &= \lambda_2 P^l b_{t-1}^l + \beta\lambda_2 (F_t - \tau_t) \\ &\quad + \frac{bP^l}{y}\left(\left(\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)\lambda_2 - (1-\rho_b)\phi_\pi \frac{\beta\lambda_2\rho_b\lambda_2}{1-\rho_b\beta\lambda_2}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n \end{aligned}$$

(Excepting the natural rate terms, simply replace F_t with $F_t - \tau_t$ in all previous calculations, since this is only a consequence of the flow budget constraint of the government). To facilitate comparison across maturity structures, it is helpful to use a different definition of the debt variable, which is the real market value rather than the real face value in steady state. Let

$$b_t = P^l b_t^l \text{ and } \delta = \frac{b^l P^l}{y}$$

then

$$\begin{aligned} \pi_t &= \delta^{-1}\left(\frac{(1-\beta\lambda_2)}{\beta} b_{t-1} + (1-\beta\lambda_2)(F_t - \tau_t)\right)\left(\beta^{-1} - \frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\ &\quad + \left(\lambda_2 + \left(\frac{(1-\rho_b)\phi_\pi}{(1-\rho_b\beta\lambda_2)} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n \end{aligned}$$

and

$$\begin{aligned} b_t &= \lambda_2 b_{t-1} + \beta\lambda_2 (F_t - \tau_t) \\ &\quad + \delta\left(\left(\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)\lambda_2 - (1-\rho_b)\phi_\pi\frac{\beta\lambda_2\rho_b\lambda_2}{1-\rho_b\beta\lambda_2}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n. \end{aligned}$$

Some special cases follow.

B.6.1 Flexible Prices

Recall $\beta\lambda_2\lambda_1 = \kappa\sigma\phi_\pi + 1$.

With flexible prices and one period debt we have

$$\pi_t = \delta^{-1}\left(b_{t-1} + \beta(F_t - \tau_t)\right) + \frac{\beta}{\sigma + \omega^{-1}} F_t$$

$$b_t = \phi_\pi b_{t-1} + \beta\phi_\pi (F_t - \tau_t) + \delta\left(\phi_\pi - \beta^{-1}\right)\frac{\beta}{\sigma+\omega^{-1}} F_t$$

$$r_t^n = \frac{1}{\sigma + \omega^{-1}} F_t$$

B.6.2 Interest Rate Peg

What if $\phi_\pi = 0$? In this case we know that there are no effects operating through the price of debt and

$$\Phi(\rho_b, \phi_\pi) = 0$$

at all times. Recall

$$(\kappa\sigma\phi_\pi + 1)\beta^{-1} = \lambda_1\lambda_2$$

$$\pi_t = \delta^{-1}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1} + (1-\beta\lambda_2)(F_t - \tau_t)\right)\beta + \frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

and

$$b_t = \lambda_2 b_{t-1} + \beta\lambda_2 (F_t - \tau_t) - \delta\beta^{-1}\frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

Keynesian hydraulics drive the final terms in each equation.

B.6.3 ONE PERIOD DEBT What if $\rho_b = 0$. When we have one-period debt

$$\pi_t = \delta^{-1}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1} + (1-\beta\lambda_2)(F_t - \tau_t)\right)\left(\beta^{-1} - \phi_\pi\right)^{-1} + \frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

and

$$b_t = \lambda_2 b_{t-1} + \beta\lambda_2 (F_t - \tau_t) + \delta\left(\left(\phi_\pi - \beta^{-1}\right)\right)\frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

When will debt impact effects be positive for purchases shocks? Depends on the sign of

$$\beta + \delta\left(\left(\phi_\pi - \beta^{-1}\right)\right)\frac{1}{\lambda_2\lambda_1}\frac{\kappa\sigma}{(\sigma + \omega^{-1})}$$

For the impact effect on debt to be positive requires

$$1 > \delta(1-\beta\phi_\pi)\frac{1}{\kappa\sigma\phi_\pi + 1}\frac{\kappa\sigma}{(\sigma+\omega^{-1})}$$

$$\delta < \frac{(\kappa\sigma\phi_\pi + 1)(\sigma+\omega^{-1})}{\kappa\sigma(1-\beta\phi_\pi)} = \frac{\left(\phi_\pi + \frac{1}{\kappa\sigma}\right)(\sigma+\omega^{-1})}{(1-\beta\phi_\pi)}$$

For standard slopes of Phillips curves the RHS will be a large number — well over standard debt to GDP ratios. For an interest rate peg (the RHS is smallest for this value of policy coefficient) at the ZLB we have

$$\delta < \frac{1}{\kappa\sigma}\left(\sigma + \omega^{-1}\right).$$

It is clear for a range of values that are standard – i.e $\sigma = \omega = 1$ and $\kappa \in (0, 0.1)$ — then $\frac{b}{y}$ must be at least 10 (250% debt to GDP ratio). For smaller values of κ which are highly common, this quantity becomes even greater. Hence it must be highly likely that the impact effect on debt is positive. It follows that

$$\delta < \frac{(\kappa\sigma\phi_\pi + 1)(\sigma+\omega^{-1})}{\kappa\sigma(1-\beta\phi_\pi)}$$

gives positive impact effects.

What are the dynamic effects? The impact effects are

$$\begin{aligned}\pi_0 &= \left(\beta^{-1} - \phi_\pi\right)^{-1}(1-\beta\lambda_2)\delta^{-1} + \beta\lambda_2\frac{\kappa\sigma}{(\kappa\sigma\phi_\pi + 1)(\sigma+\omega^{-1})} \\ b_0 &= \beta\lambda_2 + \delta\left(\phi_\pi - \beta^{-1}\right)\beta\lambda_2\frac{\kappa\sigma}{(\kappa\sigma\phi_\pi + 1)(\sigma+\omega^{-1})}\end{aligned}$$

Note that the impact effect in regime M takes the form

$$\frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}.$$

Hence the impact effect on inflation is a function of the regime M multiplier. For the impact effect to be larger under F than under M (at given common coefficients) we must have

$$\left(\beta^{-1}-\phi_\pi\right)^{-1}\left(1-\beta\lambda_2\right)\delta^{-1}+\beta\lambda_2\frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}>\frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}$$

or

$$\left(\beta^{-1}-\phi_\pi\right)^{-1}\delta^{-1}>\frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}$$

or

$$\frac{\beta\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}{\left(1-\phi_\pi\beta\right)\kappa\sigma}>\delta$$

which is almost the same condition for the impact effect on debt to be positive — they differ by a factor β. So this seems like a sufficient condition to get larger multipliers. (Note that assuming ϕ_π is the same in either regime is a little crazy, but it makes the regime M multiplier bigger than what it would be when $\phi_\pi>1$. So this is a conservative calculation.

Subsequent effects are

$$\begin{aligned}
\pi_j &= \left(\beta^{-1}-\phi_\pi\right)^{-1}\frac{\left(1-\beta\lambda_2\right)}{\beta}\delta^{-1}b_{j-1}\\
b_j &= \lambda_2 b_{j-1}=\lambda_2^j b_0\\
&= \lambda_2^j\left(\beta\lambda_2+\delta\left(\phi_\pi-\beta^{-1}\right)\beta\lambda_2\frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}\right).
\end{aligned}$$

What are the effects of an increase in transfers?

$$\pi_t=\delta^{-1}\left(\frac{\left(1-\beta\lambda_2\right)}{\beta}b_{t-1}+\left(1-\beta\lambda_2\right)\left(F_t-\tau_t\right)\right)\left(\beta^{-1}-\phi_\pi\right)^{-1}+\frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

and

$$b_t=\lambda_2 b_{t-1}+\beta\lambda_2\left(F_t-\tau_t\right)+\delta\left(\left(\phi_\pi-\beta^{-1}\right)\right)\frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

Then

$$\begin{aligned}
\pi_0 &= \delta^{-1}\left(1-\beta\lambda_2\right)\left(\beta^{-1}-\phi_\pi\right)^{-1}\\
b_0 &= \beta\lambda_2
\end{aligned}$$

$$\begin{aligned}
\pi_0 &= \frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}+\left[1-\frac{\delta\left(\beta^{-1}-\phi_\pi\right)\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}\right]\left(\beta^{-1}-\phi_\pi\right)^{-1}\left(1-\beta\lambda_2\right)\delta^{-1}\\
b_0 &= \beta\lambda_2+\delta\left(\phi_\pi-\beta^{-1}\right)\beta\lambda_2\frac{\kappa\sigma}{\left(\kappa\sigma\phi_\pi+1\right)\left(\sigma+\omega^{-1}\right)}.
\end{aligned}$$

Afterwards

$$\begin{aligned}\pi_j &= \delta^{-1}\frac{(1-\beta\lambda_2)}{\beta}b_{j-1} \\ &= \delta^{-1}(1-\beta\lambda_2)\lambda_2^{j-1}\beta\lambda_2.\end{aligned}$$

Hence a reduction in taxes, or an increase in transfers, unambiguously raise inflation. In regime M, the effects are always zero.

B.6.4 Consol Debt What if $\rho_b = 1$? With consol debt

$$\pi_t = \delta^{-1}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1} + (1-\beta\lambda_2)(F_t - \tau_t)\right)\beta + \frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

and

$$b_t = \lambda_2 b_{t-1} + \beta\lambda_2(F_t - \tau_t) - \delta\beta^{-1}\frac{\lambda_2}{\lambda_2\lambda_1}\kappa\sigma r_t^n$$

which is almost identical to the pegged interest rate case, which makes sense. No price effects on the portfolio in either case (to a first-order approximation).

B.6.5 General Case Now consider the general case. Dynamics are

$$\begin{aligned}\pi_t &= \delta^{-1}\left(\frac{(1-\beta\lambda_2)}{\beta}b_{t-1} + (1-\beta\lambda_2)(F_t-\tau_t)\right)\left(\beta^{-1} - \frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\ &\quad + \left(\lambda_2 + \left(\frac{(1-\rho_b)\phi_\pi}{(1-\rho_b\beta\lambda_2)} - \beta^{-1}\right)^{-1}\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\frac{(1-\beta\lambda_2)}{\beta}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n\end{aligned}$$

and

$$\begin{aligned}b_t &= \lambda_2 b_{t-1} + \beta\lambda_2(F_t-\tau_t) \\ &\quad + \delta\left(\left(\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2} - \beta^{-1}\right)\lambda_2 - (1-\rho_b)\phi_\pi\frac{\beta\lambda_2\rho_b\lambda_2}{1-\rho_b\beta\lambda_2}\right)\frac{1}{\lambda_2\lambda_1}\kappa\sigma r_t^n.\end{aligned}$$

We now derive a generalization of the results of Beck-Friis and Willems (2017) to permit long term debt. Introduce the following notation. Define the impulse response functions for inflation in response to a one percent of GDP increase in spending or transfers under regime F as $UB_F^\pi(j)$ and $UB_T^\pi(j)$ for $j \geq 0$. Similarly, define the impulse response functions for inflation in response to a one percent of GDP *i.i.d.* increase in spending or transfers under regime M as $KH_F^\pi(j)$ and $KH_T^\pi(j)$ for $j \geq 0$. From the above results we have the impulse response functions for transfer changes in regime M (Keynesian hydraulics)

$$\begin{aligned}KH_T^\pi(j=0) &= 0 \\ KH_T^\pi(j>0) &= 0\end{aligned}$$

and regime G (unbacked fiscal expansion)

$$\begin{aligned}UB_T^\pi(j=0) &= \delta^{-1}(1-\beta\lambda_2)\left(\beta^{-1} - \frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\ UB_T^\pi(j>0) &= \lambda_2^j\beta\lambda_2.\end{aligned}$$

The effects of government purchases under Keynesian hydraulics can then be written

$$\begin{aligned} KH_F^{\pi}(j=0) &= \frac{\kappa\sigma}{(\kappa\sigma\phi_\pi+1)(\sigma+\omega^{-1})} \\ KH_F^{\pi}(j>0) &= 0. \end{aligned}$$

Finally, effects of government purchases in an unbacked fiscal expansion can be written

$$\begin{aligned} UB_F^{\pi}(j=0) &= \delta^{-1}(1-\beta\lambda_2)\left(\beta^{-1}-\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1} \\ &\quad+\left(\beta\lambda_2+\left(\frac{(1-\rho_b)\phi_\pi}{(1-\rho_b\beta\lambda_2)}-\beta^{-1}\right)^{-1}\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2(1-\beta\lambda_2)\right)\frac{\kappa\sigma}{(\kappa\sigma\phi_\pi+1)(\sigma+\omega^{-1})} \\ &= \frac{\kappa\sigma}{(\kappa\sigma\phi_\pi+1)(\sigma+\omega^{-1})}+\delta^{-1}(1-\beta\lambda_2)\left(\beta^{-1}-\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)^{-1}\times \\ &\quad\left[1-\delta\left(\left(\beta^{-1}-\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)+\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\right)\frac{\kappa\sigma}{(\kappa\sigma\phi_\pi+1)(\sigma+\omega^{-1})}\right] \\ &= KH_F^{\pi}(0)+\left[1-\delta\left(\left(\beta^{-1}-\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\right)+\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}\rho_b\beta\lambda_2\right)KH_F^{\pi}(0)\right]UB_T^{\pi}(0) \\ &= KH_F^{\pi}(0)+\left[1-\delta\beta^{-1}(\beta\phi_\pi\rho_b-\beta\phi_\pi+1)KH_F^{\pi}(0)\right]UB_T^{\pi}(j=0) \end{aligned}$$

and

$$\begin{aligned} UB_F^{\pi}(j>0) &= \lambda_2^j\left[\beta\lambda_2+\delta\left(\left(\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}-\beta^{-1}\right)\lambda_2-(1-\rho_b)\phi_\pi\frac{\beta\lambda_2\rho_b\lambda_2}{1-\rho_b\beta\lambda_2}\right)\frac{\beta\kappa\sigma}{(\kappa\sigma\phi_\pi+1)(\sigma+\omega^{-1})}\right] \\ &= \left[1+\delta\left(\left(\frac{(1-\rho_b)\phi_\pi}{1-\rho_b\beta\lambda_2}-\beta^{-1}\right)-(1-\rho_b)\phi_\pi\frac{\beta\lambda_2\rho_b}{1-\rho_b\beta\lambda_2}\right)\frac{\beta\kappa\sigma}{(\kappa\sigma\phi_\pi+1)(\sigma+\omega^{-1})}\right]\beta\lambda_2\lambda_2^j \\ &= KH_F^{\pi}(j>0)+\left[1-\delta\beta^{-1}(\beta\phi_\pi\rho_b-\beta\phi_\pi+1)KH_F^{\pi}(j=0)\right]UB_T^{\pi}(j>0). \end{aligned}$$

Hence we can write

$$UB_F^{\pi}(j)=KH_F^{\pi}(j)+\left[1-\delta\beta^{-1}(\beta\phi_\pi\rho_b-\beta\phi_\pi+1)KH_F^{\pi}(0)\right]UB_T^{\pi}(j)$$

for any j.

The effects of government purchases in an unbacked fiscal expansion decompose into two components. The first comes from the standard effects of spending in the standard Keynesian story, $KH_F^{\pi}(j)$. This is the effect of spending when backed by taxes. The second term comes from the fact that spending does not herald future tax increases — it represents the effects of nominal debt issuance on wealth. It is for this reason that this term is given by the tax multiplier: it captures the implicit reduction in taxes, of magnitude equal to the spending change, under an unbacked fiscal expansion.

For the effects to be larger in an unbacked fiscal expansion than under Keynesian hydraulics we require

$$1-\delta\beta^{-1}(\beta\phi_\pi\rho_b-\beta\phi_\pi+1)KH_F^{\pi}(0)>0$$

when
$$\delta\beta^{-1}\left(\beta\phi_\pi\rho_b - \beta\phi_\pi + 1\right) K H_F^\pi(0) < 1$$
or
$$\delta < \frac{\beta}{\left(\beta\phi_\pi\rho_b + 1 - \beta\phi_\pi\right) K H_F^\pi(0)} = \frac{\beta\left(\kappa\sigma\phi_\pi + 1\right)\left(\sigma + \omega^{-1}\right)}{\left(\beta\phi_\pi\rho_b + 1 - \beta\phi_\pi\right)\kappa\sigma}.$$
For standard parameterization, the RHS will be large. For example, the denominator will be largest when $\rho_b = 0$. Then we have
$$\beta\left(\phi_\pi + \frac{1}{\kappa\sigma}\right)\left(\sigma + \omega^{-1}\right)$$
which is well above 10 for unitary elasticity of substitution in consumption and Frisch elasticity, and $\kappa = 0.1$ (which is high by modern standards). This requires debt to GDP ratio to be less than 250% on an annual basis.

B.7 Output Effects

Keep in mind the output effects are inferred from
$$\begin{aligned} x_t &= -\frac{1}{d_1}\left(\left(\frac{1}{\lambda_1}\right)\frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n + \pi_t\right) \\ &= -\frac{\beta\lambda_2 - 1}{\kappa}\left(\left(\frac{1}{\lambda_1}\right)\frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n + \pi_t\right) \end{aligned}$$

$$\begin{aligned} r_t^n &= \frac{1}{\sigma + \omega^{-1}} F_t \\ Y_t^n &= \frac{\sigma^{-1}}{\sigma^{-1} + \omega} F_t \end{aligned}$$

So
$$\begin{aligned} y_t &= Y_t^n - \frac{\beta\lambda_2 - 1}{\kappa}\left(\left(\frac{1}{\lambda_1}\right)\frac{\kappa}{1-\lambda_2\beta}\sigma r_t^n + \pi_t\right) \\ &= \frac{\sigma^{-1}}{\sigma^{-1} + \omega} F_t + \left(\frac{1}{\lambda_1}\right)\frac{\sigma}{\sigma + \omega^{-1}} F_t + \frac{1 - \beta\lambda_2}{\kappa}\pi_t \end{aligned}$$
so that the impact effects are always positive on output if the impact effects on inflation are also positive for all parameter values. It is immediate that similar decomposition holds for output as inflation since this expression holds in both regime M and F.

B.8 Simple Quantitative Examples

We conclude with some simple quantitative examples to illustrate that unbacked fiscal expansions will generally have more stimulatory effects than standard Keynesian hydraulics. The examples are just that — they are not an exhaustive treatment of the issue, nor meant to definitively establish the size of multipliers or their relative magnitudes across different environments. Having said that, there are reasons to think the numbers in both regimes would be understated relative to the 1930s environment of the United States. Infrastructure spending, and transfers to constrained households, were both important features of the fiscal strategy. And both will serve to increase multipliers. Indeed, capital investment would lead to permanent increases in output, consistent with out VAR results. See also **?** for a discussion of household heterogeneity and its implications for recovery from the Great Depression.

Make the following parametric assumptions. The discount factor, β, is 0.99, the Calvo parameters, α, is 0.66, the Frisch elasticity and intertemporal consumption elasticities are equal to unity, the elasticity of demand across differentiated goods, θ, is 10, and the average maturity of debt during the Great Depression was 8.7 years, giving $\rho_b = 0.98$ (shorter maturities give rise to larger effects). In the subsequent discussion the benchmark average debt to GDP ratio is 40% on an annual basis.

Consider the impulse response functions for the level of output, goods prices and debt to a 1% of GDP *i.i.d.* spending shock. Figure 1 gives the results under an unbacked fiscal expansion. Figure 2 gives the results under Keynesian hydraulics. There are substantial differences across policy regimes. The impact effect of spending on output is 1.53 under an unbacked expansion, while only 0.977 under Keynesian hydraulics. Because the effects of the former are persistent, the present discounted multipliers, which compute the present discounted flow of output from the *i.i.d.* spending shock, are 4.14 versus 0.977. The effects on the price level also differ markedly by a factor of 42 with prices rising by 0.65 and 0.0155 respectively.

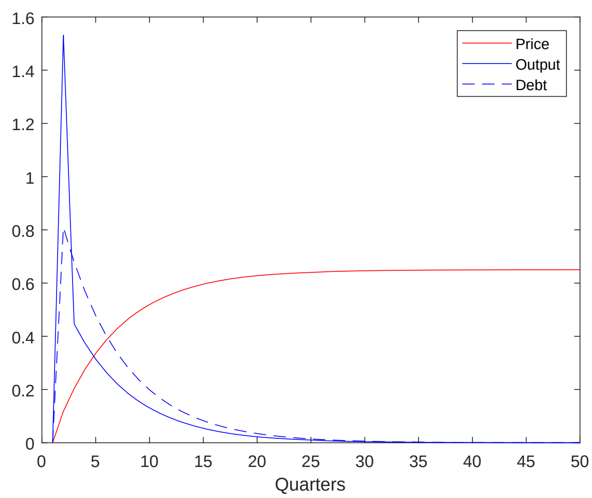

Figure 35: Impulse response functions under an unbacked fiscal expectations in response to a 1% *i.i.d.* spending shock.

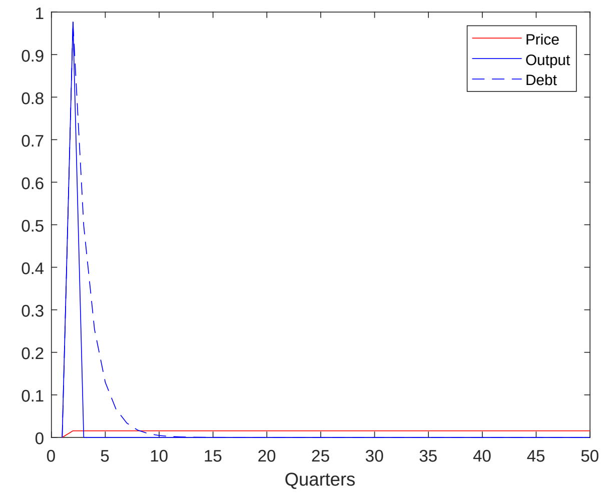

Figure 36: Impulse response functions under Keynesuan hydraulics in response to a 1% *i.i.d.* spending shock.

Figures 3 computes the present discounted multipliers for output as a function of the debt to GDP ratio in response to a 1% increase in spending and also a 1% increase in transfers. Because Keynesian hydraulics are independent of debt, the effects of spending and transfer innovations are constant, and equal to zero in the case of transfers (therefore not shown). For debt levels observed in the Great Depression the effects are large, and indeed infinite as the average level of debt goes to zero. And over all debt levels, the multipliers are larger under an unbacked fiscal expansion.

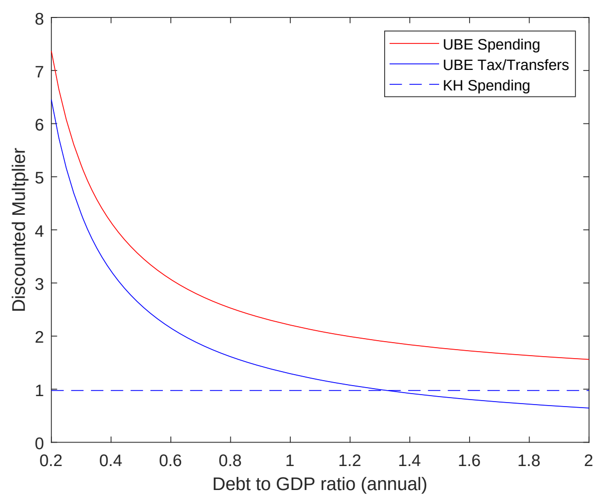

Figure 37: Present discounted multipliers as a function of the debt to GDP ratio. Results are shown for 1% spending and transfer shocks. The effects of the latter are zero under Keynesian hydraulics.

C Fiscal Implications of Gold Sterilization

C.1 Gold Sterilization during the Great Depression

Gold imports have the potential to increase the monetary base of an economy following the classical gold standard or the gold exchange standard. Policymakers can counteract the increase in the monetary base by sterilizing gold inflows which entails paying for imported gold in government securities rather than bank reserves. Prior to 1933, the Federal Reserve conducted gold import operations and sterilization decisions. By June of 1934, these responsibilities shifted to the Treasury as the result of a series of presidential proclamations, executive orders, joint-resolutions, and Acts that culminated in an embargo on gold exports[49] and the Treasury seizing the entire monetary gold stock including coins and bullion held by private citizens, business, and the Federal Reserve banks.[50]

Massive gold imports more than tripled the monetary gold stock from $4.25 billion at the start of 1933 to $14.42 billion at the end of 1938. Meltzer (2003, p. 459) notes that the Treasury purchased more than $4 billion of gold from 1934-1936. Friedman and Schwartz (1963, p. 545) attribute the gold inflows throughout this period to the depreciation of the dollar, Hitler's rise to power, and the outbreak of war in Europe. Studenski and Krooss (1952, p. 394) include the Treasury's $35 an ounce purchase price for gold, favorable trade balances, and the creditor position of the United States as additional factors that increased gold imports. To our knowledge, the Gold Reserve Act of 1934's ban on private citizens holding monetary gold required banks to sell newly imported gold to the Treasury.[51] With gold inflows pushing up excess reserves, policymakers feared that the growing monetary base could ignite inflationary forces [Jaremski and Mathy (2016)]. To curb the growth of excess reserves and hence the monetary base, the Treasury sterilized gold imports from December 1936 to April 1938.

Expanding on the example provided by Johnson (1939, p. 144), we illustrate the effects of the Treasury's non-sterilized and sterilized gold purchases on the balance sheets of the Treasury, the Federal Reserve, and member banks.

[49]Executive Order 6111 on Transactions in Foreign Exchange was implemented on April 20, 1933. See http://www.presidency.ucsb.edu/ws/index.php?pid=14621

[50]See Bordo, Humpage, and Schwartz (2015, pp. 56–57) for a detailed time line of events. Jaremski and Mathy (2016, p. 6) report that most gold imports came through New York City's gold market and New York City banks continued to sell their gold to the Federal Reserve Bank of New York who acted as fiscal agent to the Treasury, the ultimate purchaser of the gold.

[51]Bordo, Humpage, and Schwartz (2015, p. 65) explain that the Treasury issued special licenses for commercial banks to obtain gold for customers. This suggests that banks were not allowed to keep gold on their balance sheets.

1. Gold Imported by Member Banks Member banks import \$1,000 worth of gold and fund it by issuing \$1,000 worth of deposits. Member bank assets and liabilities rise by \$1,000.

Treasury		Federal Reserve		Member Banks	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
				+\$1,000 gold	+\$1,000 deposits
				+\$1,000	+\$1,000

2. High Powered Money Creation Member banks sell their imported gold to the Federal Reserve for \$1,000. The Federal Reserve pays for the gold by issuing reserves to member banks which increases high-powered money by \$1,000. For member banks, gold is swapped for reserves and their aggregate asset position is unchanged –both assets and liabilities remain elevated by the original \$1,000 injection. If the Federal Reserve did not want to sterilize and they were responsible for sterilization decision, this would be the final step. Skip to step 2b at the end of this Appendix for the effects of Federal Reserve sterilization.

Treasury		Federal Reserve		Member Banks	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
		+\$1,000 gold	+\$1,000 reserves	-\$1,000 gold +\$1,000 reserves	\$1,000 deposits
		+\$1,000	+\$1,000	\$1,000	\$1,000

3. Gold Transferred to Treasury Under the Gold Act of 1934, gold could not be exported and any imported gold had to be turned over to the Treasury. As noted by Jaremski and Mathy (2016, p. 6), the Federal Reserve would then transfer the gold to the Treasury who paid for the gold by drafting on its balances at the Federal Reserve. Although the aggregate value of the balance sheets of the Treasury and the Federal Reserve are unchanged, the composition of their balance sheets change.

Treasury		Federal Reserve		Member Banks	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
+\$1,000 gold -\$1,000 due from Fed		-\$1,000 gold	\$1,000 reserves -\$1,000 due to Treasury	\$1,000 reserves	\$1,000 deposits
				\$1,000	\$1,000

4a. No Sterilization Under the Treasury: The Treasury replenishes its balances at the Federal Reserve by issuing gold certificates and depositing them at the Federal Reserve as the final payment for gold purchases. Non-sterilized gold imports ultimately increase the balance sheets of the Treasury, the Federal Reserve, and member banks and leave the amount of free-gold at the Treasury unchanged. The Treasury does not offset the creation of high powered money by retiring the newly created reserves as will be the case with sterilization.

Importantly, in the case of no sterilization, there is no increase in Treasury indebtedness to the private sector because the Treasury creates "money" through gold certificates.

Treasury		Federal Reserve		Member Banks	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
+\$1,000 due from Fed	+\$1,000 gold certificates to Treasury	+\$1,000 gold certificates from Treasury	\$1,000 reserves	\$1,000 reserves	\$1,000 deposits
+\$1,000	+\$1,000	+\$1,000	+\$1,000	\$1,000	\$1,000

4b. Sterilization Under the Treasury: When sterilizing gold imports, the Treasury replenishes balances at the Federal Reserve by selling government securities to member banks rather than issuing gold certificates and depositing them at the Federal Reserve. The Federal Reserve again settles the transaction between the Treasury and member banks through reserves. Member banks pay for security sales by retiring reserves outstanding at the Federal Reserve. The Federal Reserve then offsets this transaction by crediting their balance due to the Treasury/debiting the Treasury's balances held at the Federal Reserve. Sterilization increases the aggregate balance sheets of the Treasury and member banks, but not the Federal Reserve.

In this case, there is an increase in Treasury indebtedness to the private sector and there is no increase in bank reserves.

Treasury		Federal Reserve		Member Banks	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
+\$1,000 due from Fed	+\$1,000 gov't securities		-\$1,000 reserves +\$1,000 due to Treasury	-\$1,000 reserves +\$1,000 gov't securities purchased from Treasury	\$1,000 deposits
+\$1,000	+\$1,000			\$1,000	\$1,000

2b. Sterilization Under the Federal Reserve When sterilizing gold imports, the Federal Reserve pays for gold by selling government securities to member banks rather than creating reserves as seen in step 2. Sterilization leaves the aggregate balance sheets of the Federal Reserve and the Treasury unchanged while the balance sheet of member banks is expanded. In the case of Federal Reserve sterilization, there is no increase in Treasury indebtedness. Because security sales by the Federal Reserve prevent the creation of reserves, sterilization by the Federal Reserve is equivalent to contractionary open market operations.

Treasury

Assets	Liabilities

Federal Reserve

Assets	Liabilities
+\$1000 gold -\$1000 gov't securities	

Member Banks

Assets	Liabilities
-\$1000 gold +\$1000 gov't securities	\$1000 deposits
\$1000	\$1000

D Additional VAR Results

This appendix reports a more complete set of VAR results than those in the text. Table 10 reports the posterior correlations among exogenous shocks in the identified model. Figures 38 and 39 reports the historical decompositions of the price level and real GNP into the right-hand-side components of equation (25). Figure 40 reports actual data and unconditional forecasts for the seven series in the VAR. Figure 41 shows the full moving average representations for the seven-variable VAR estimated over the unbacked fiscal expansion period (April 1933 to June 1940).

Correlations Among Exogenous Shocks							
Shock	ε^{MP}	ε^{MD}	ε^{PS}	ε^{B}	ε^{G}	ε^{P}	ε^{Y}
ε^{MP}	1.0	$(-.09, .13)$	$(-.07, .10)$	$(-.11, .11)$	$(-.10, .12)$	$(-.02, 13)$	$(-.09, .11)$
ε^{MD}		1.0	$(-.05, .15)$	$(-.14, .08)$	$(-.14, .06)$	$(-.10, .12)$	$(-.13, .08)$
ε^{PS}			1.0	$(-.16, .06)$	$(-.04, .12)$	$(-.16, .08)$	$(-.15, .08)$
ε^{B}				1.0	$(-.12, .10)$	$(-.13, .09)$	$(-.14, .08)$
ε^{B}					1.0	$(-.13, .09)$	$(-.14, .09)$
ε^{P}						1.0	$(-.10, .10)$
ε^{Y}							1.0

Table 10: Correlations computed from 500,000 draws from the posterior distribution of the model that table 5 reports.

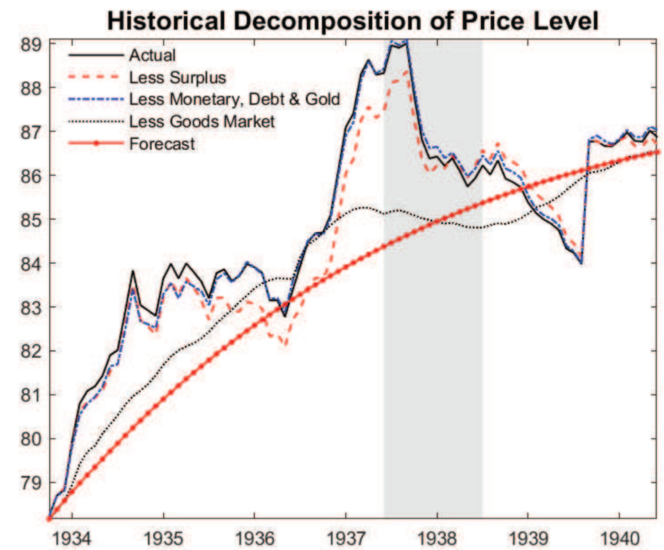

Figure 38: Historical decomposition of the price level. In terms of components of equation (25), Actual is y_{jt}, Forecast is $E_0 y_{jt}$, less surpluse is $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^F$, less monetary, debt & gold is $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^O$, less goods market is $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^M$. Uses the identification in table 5.

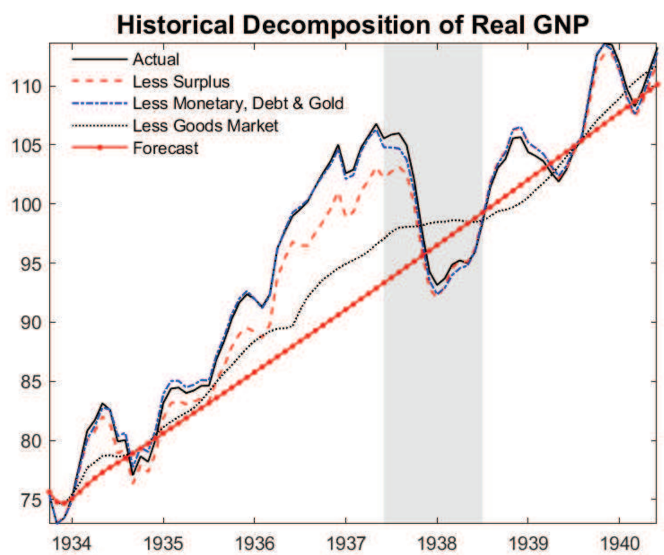

Figure 39: Historical decomposition of real GNP. In terms of components of equation (25), Actual is y_{jt}, Forecast is $E_0 y_{jt}$, less surpluse is $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^F$, less monetary, debt & gold is $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^O$, less goods market is $y_{jt} - \sum_{i=1}^{t} C_j^F(i)\varepsilon_i^M$. Uses the identification in table 5.

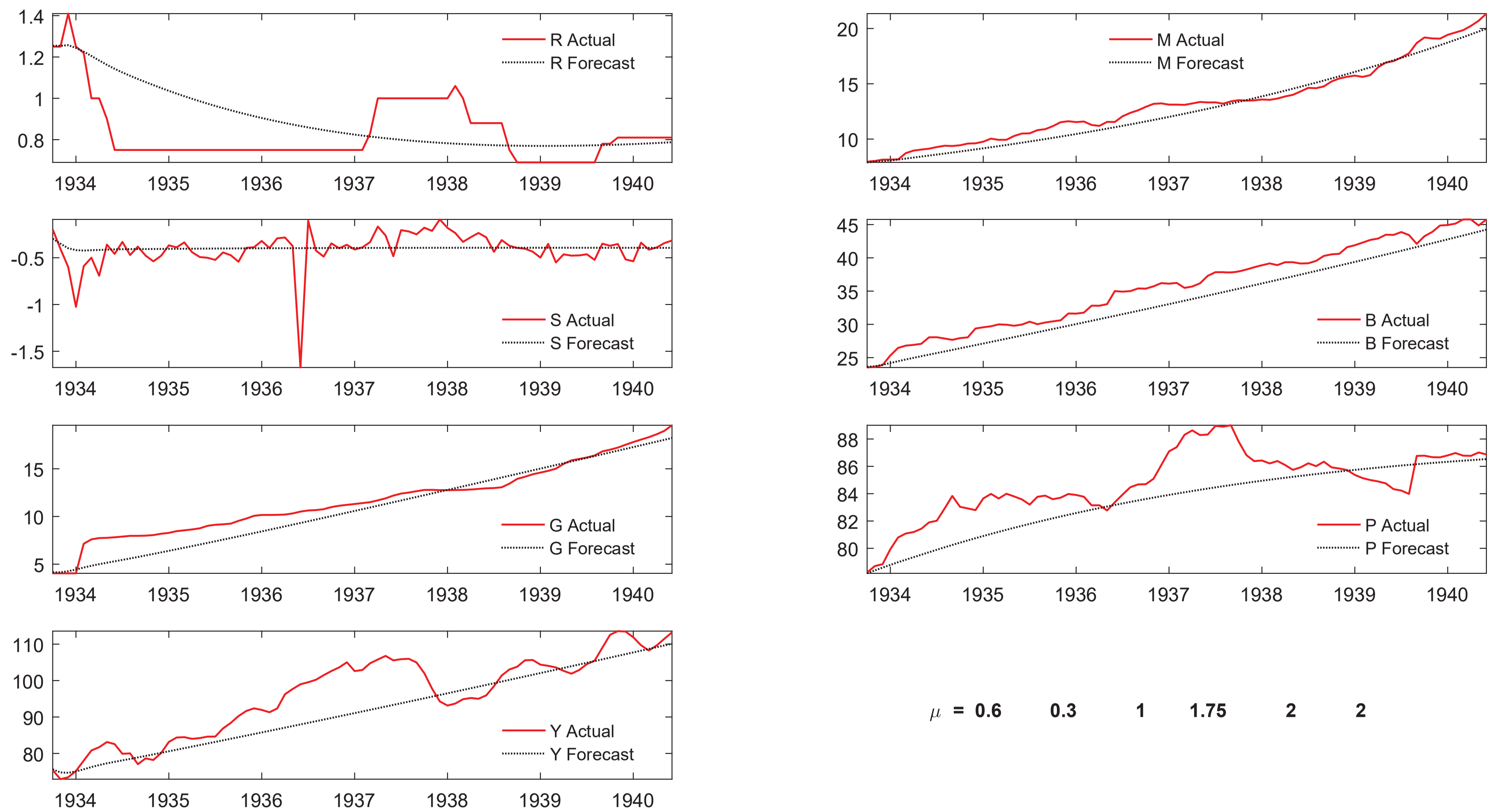

Figure 40: Actual and unconditional forecasts of variables in VAR using the hyperparameters $\lambda_0 = 0.6, \lambda_1 = 0.3, \lambda_3 = 1.0, \lambda_4 = 1.75, \mu_5 = \mu_6 = 2.0$, in the notation of Sims and Zha (1998).

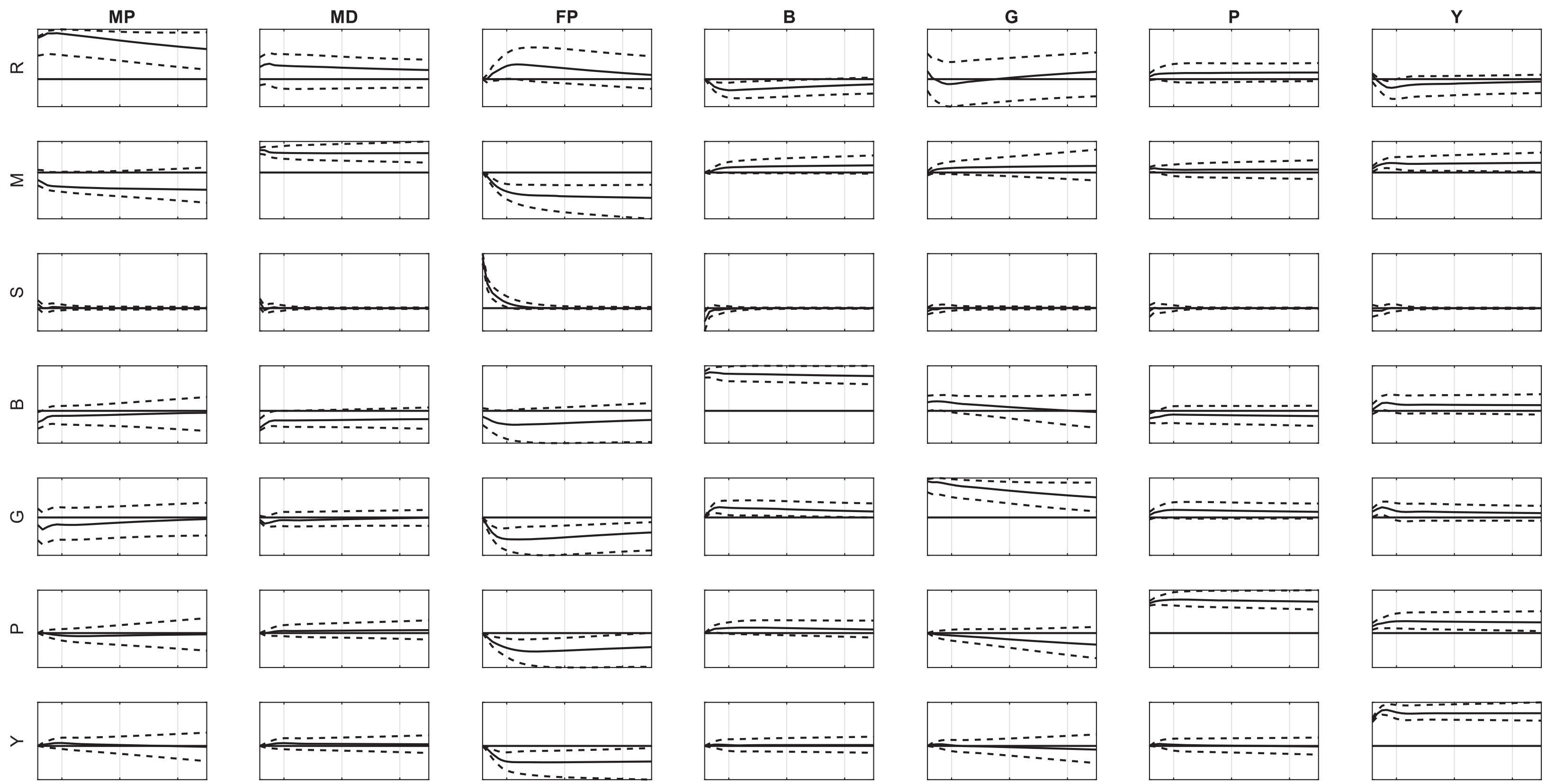

Figure 41: Full moving average representation of the identified VAR estimated over the period April 1933 to June 1940. Identification from table 5. Solid lines are maximum likelihood estimates; dashed lines are 68 percentile probability bands based on 500,000 draws from the posterior distribution of all the VAR parameters.

We also estimated a VAR specification in which primary surpluses are split into expenditures less interest payments and tax receipts. For this specification, we order the variables recursively: P, Y, F, T, R, M, G, B. Contemporaneous correlation between expenditures, F, and tax receipts, T, is nearly zero, so the order of the two fiscal variables has no effect on their dynamic impacts.

Figures 42 and 43 report impact multipliers for spending and taxes, while figure 44 reports the full moving average representation.

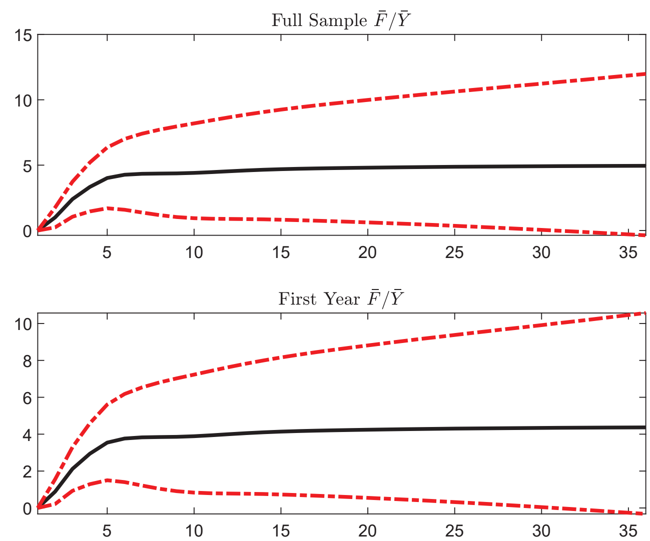

Figure 42: Blanchard and Perotti (2002) output multipliers for government expenditures net of interest payments. Solid lines are posterior modes; dashed lines are 68 percent credible sets. Top panel takes mean of expenditures to output from full sample; bottom panel takes mean from first year of sample.

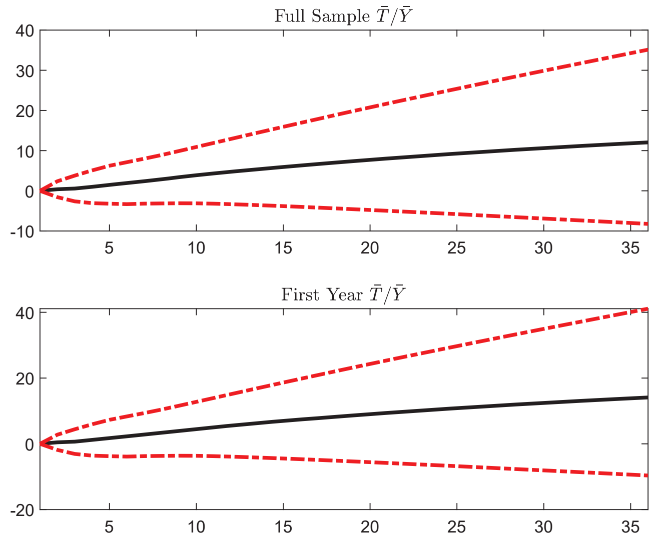

Figure 43: Blanchard and Perotti (2002) output multipliers for government tax receipts. Solid lines are posterior modes; dashed lines are 68 percent credible sets. Top panel takes mean of receipts to output from full sample; bottom panel takes mean from first year of sample.

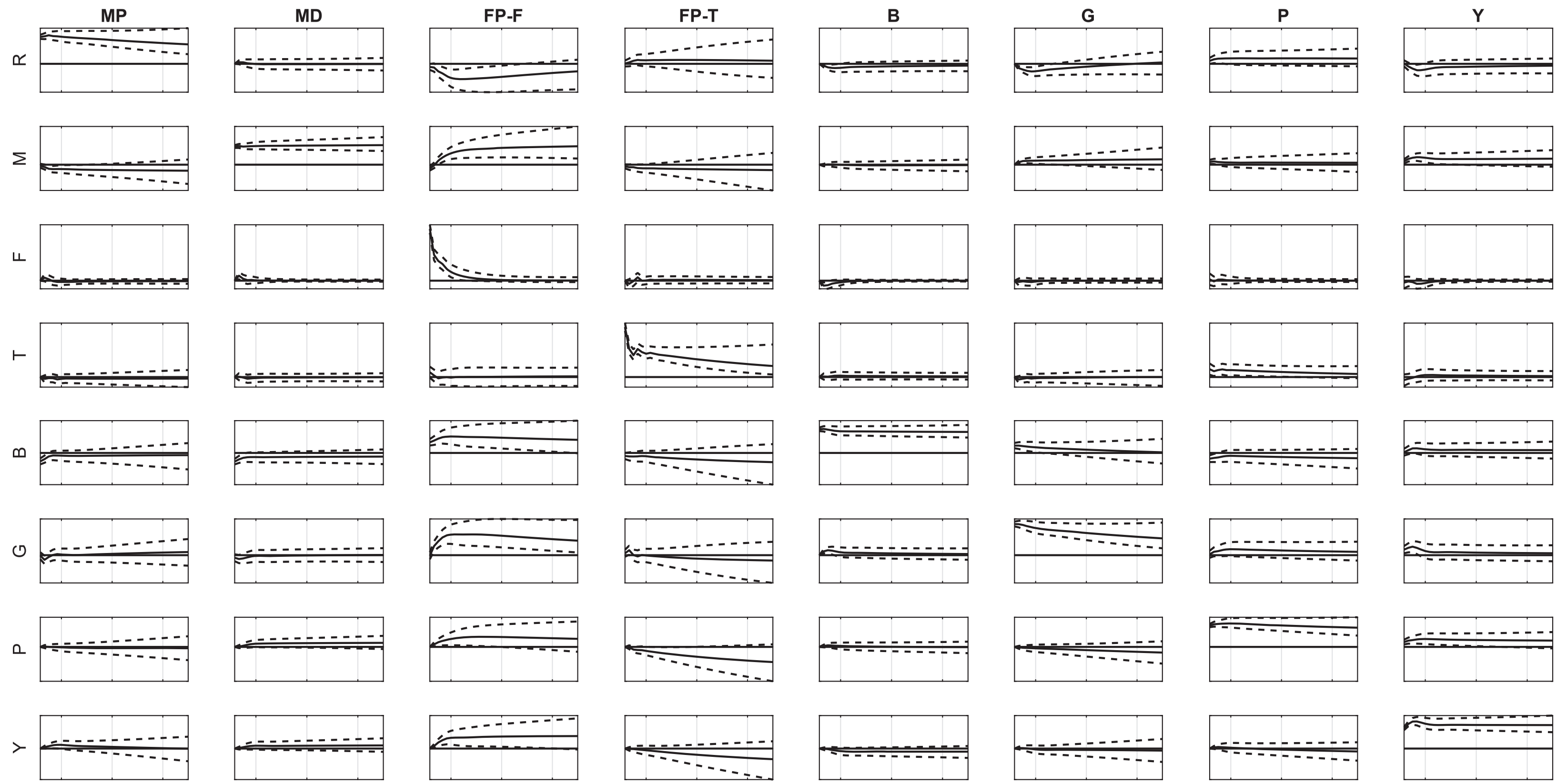

Figure 44: Full moving average representation of 8-variable VAR ordered recursively—P, Y, F, T, R, M, G, B—estimated over the period (April 1933 to June 1940). F is government expenditures net of interest payments and T is tax receipts. Solid lines are maximum likelihood estimates; dashed lines are 68 percentile probability bands based on 500,000 draws from the posterior distribution of all the VAR parameters.

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