A Dynamic Theory of Lending Standards*

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Abstract

We develop a dynamic model of credit markets in which both lending standards and the quality of potential borrowers are endogenous. Competitive lenders privately decide on their lending standards: whether to pay a cost to screen out some unprofitable borrowers or instead to lend (or not) to all potential borrowers. Lending standards – defined as the degree of screening – have externalities and are dynamic strategic complements: tighter screening worsens the future pool of borrowers, increasing the incentive to screen in the future. We show that lending standards can amplify and propagate fluctuations, can be tighter than socially optimal, and can lead to amplified and long-lasting effects on lending volume, credit spreads, and default rates in response to temporary adverse changes in fundamentals. We characterize constrained optimal policy which can be implemented as government loan insurance program. Further, we show that limits on lending such as from capital constraints can also cause lending standards to be inefficiently tight. Finally, we discuss market institutions that may arise to mitigate the negative externality of tight lending standards, focusing on the limited role provided by existing credit bureaus.

JEL codes: D82, G21, G01, G10

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1 Introduction

Through the allocation of external financing, lending standards play an important role in the economy, determining which entrepreneurs get funding, which consumers buy houses, and which firms grow. Figure 1 plots a measure of lending standards in the market for commercial and industrial (C&I) loans from banks. Lending standards, by this metric and many others, are highly cyclical, tightening in recessions and loosening in booms. For example, the recent credit boom-bust cycle was associated with relatively lax lending standards in the lending boom of the mid-2000's, when credit spreads and default rates were low, and relatively tight lending standards during credit crunch and recession that followed, when spreads and rates were high. Notably, while the volume of lending has increased since the crisis, lending standards have not loosened much.

In this paper we develop a dynamic model of lending standards and analyze whether lending standards amplify and propagate fluctuations, whether they are efficiently set by private markets, and, if not, whether government policy can improve the allocation of credit and real economic outcomes. In our model, a credit market consists of a mass of competitive banks and a pool of potential borrowers who are initially identical conditional on public or readily-available information (e.g. identical within credit score brackets). Each instant, borrowers are drawn from the pool and approach banks in search of a loan to fund an investment project. Projects differ by the type of borrower, and can either be of high or low quality, implying a positive or negative net present value for the investment project and thus for the loan. Lenders choose lending standards.

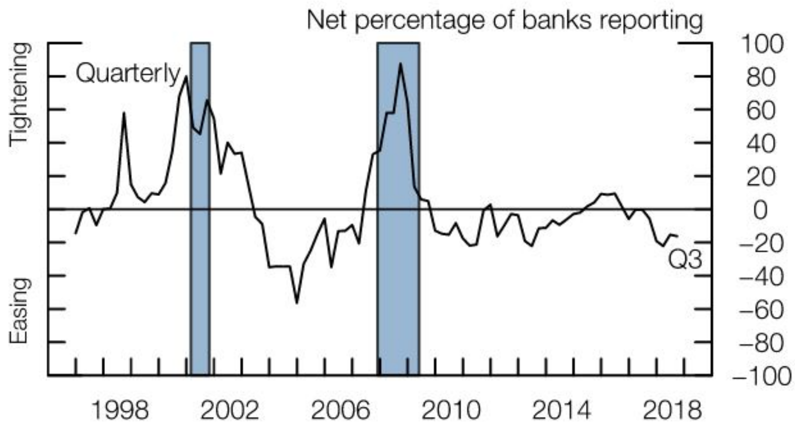
Lending standards have two dimensions. First, lenders can lend less or condition the terms of a loan on public information. For example, a lender might deny loans to all borrowers with credit scores below some threshold. In our model, a bank can lend to all applicants with a given set of observable characteristics or to only a fraction of applicants, or the bank can freeze credit to this market and make no loans.

Our main focus however is on the second dimension of lending standards: lenders can acquire additional information about the future payoff of lending to a potential borrower before deciding whether or not to originate the loan.[1] For example, lenders might interview a potential borrower, conduct a detailed valuation of their business plan, or verify reported information such as employment or income. In our model, lenders can tighten lending standards by expending costly effort to create private information about the future payoff of lending to that borrower and condition lending on this information. Importantly, the information is private and non-verifiable, and borrowers whose loan applications are declined at one bank may apply at another bank later.

The dynamics of our model are determined by the the interplay between lending standards and borrower quality. By rejecting borrowers found to be low-quality, a bank with tighter lending standards worsens the pool of potential borrowers for all banks in the future. Thus, the current

[1]These two types of lending standards are not completely distinct. Loosely, imposing tight lending standards is like writing costly contracts that screen low quality borrowers. Section 7.1 discusses alternative assumptions and screening with contracts.

Figure 1: Change in Bank Lending Standards for C&I Loans



Note: Figure shows the shows the share of banks reporting tightening less the share reporting loosening. Banks can also report no change in standards. *Source:* Senior Loan Officer Opinion Survey on Bank Lending Practices, Board of Governors of the Federal Reserve System (2018).

quality of potential borrowers reflects past lending standards. At the same time, lending standards depend on the current quality of potential borrowers. The key to understanding the dynamics of lending standards is the implication that lending standards are *dynamic strategic complements*. When one bank tightens lending standards, other banks are later confronted with more adversely selected potential borrowers, which increases their incentive to follow suit and tighten standards as well.

The key to understanding our normative results is that lending standards have *negative externalities.* Lending standards that lead to the rejection of some potential borrowers raise the share of low-quality applicants that apply for loans in the future which wastes banks' resources on screening more low-quality projects and/or increases the share of low-quality loans made. Since this cost that is not internalized by any individual bank, government intervention to prevent excessively tight lending standards can be beneficial. We characterize constrained-efficient policy and its implementation. Among other results, we show conditions under which policies such as government-subsidized loan guarantees can increase welfare, and under which the same policy can be detrimental to welfare if instead implemented after a delay.

Our first main result is that credit markets exhibit hysteresis. Markets with the same set of fundamentals may see persistently different lending volume, credit spreads, default rates, and lending standards depending on their specific history (i.e. their initial conditions). While at any point in time, the equilibrium of our model is unique, there are multiple steady states in the single state variable in our model—the share of high-quality potential borrowers in the pool, the *pool quality*. When the pool quality is high, banks do not find it worth the effort to check the quality of all applicants just to avoid the occasional low-quality borrower, and normal lending standards are optimal, which do not reduce the quality of the pool. When the pool quality is low, however, banks find it worthwhile to screen borrowers and so reject many low-quality borrowers which contributes to the low average quality of the pool. Accordingly, in the steady state with normal lending standards ("pooling steady state"), the volume of lending is high and, both because lending involves no screening costs and because the average quality of borrowers is high, loan spreads are low. In the steady state with tight lending standards ("screening steady state"), the volume of lending is low and, both because lending involves screening costs and because these costs are in equilibrium born only by the borrowers who are funded, loan spreads are high.

As a result of the multiplicity of steady states, temporary changes in market fundamentals—e.g. shifts in the payoff structure of borrowers' projects or the share of good borrowers entering the pool—can set in motion a self-reinforcing dynamic culminating in a *permanent* shift in the credit market equilibrium. Thus, absent interventions or changes in market fundamentals of the opposite sign, the endogenous response of lending standards can amplify and propagate fluctuations in fundamentals. Section 3.2 contains an example that illustrates these properties. This feature of our model is consistent with the limited relaxation of lending standards following the Great Recession documented in Figure 1.

Our second main result is that government intervention can improve private market outcomes, and in particular, that policies that relax lending standards can increase efficiency. These results follow in part from the fact that the two steady states in our model are Pareto-ranked: the planner always prefers the pooling equilibrium to the screening equilibrium. This ranking is a result of the negative externality associated with tight lending standards.[2] To characterize optimal policy in general, we solve a dynamic planning problem for the socially optimal path of lending standards. Optimal lending standards are concisely characterized by a *social* threshold of average borrower quality that always lies strictly below the private threshold. Thus, for intermediate levels of average borrower quality, normal lending standards are socially optimal but banks find it privately optimal to impose tight lending standards.

We show that for these intermediate levels of borrower quality, optimal policy can be implemented by a loan guarantee program funded by lump sum taxes. Alternatively, the government could tax loan payments and rebate the funds to lenders on a per-loan basis. The former policy would reduce interest rate spreads, the latter raise them, but both relax lending standards and increase lending volume. It is important to note that because the pool of potential borrowers is a common resource, there is no way for individual banks to recover the short-term losses from later profits absent collective, i.e. government, actions.

A further implication of this difference between private and socially optimal lending standards is that for small declines in the pool quality due to a temporarily higher share of low-quality borrowers among new borrowers, no intervention is needed, as the private threshold is not crossed. However, for large enough temporary declines in pool quality, the optimal policy response is an

[2] It is worth noting that in our model, lending standards do not influence the quality of new potential borrowers entering the pool of potential borrowers (as in Hu (2018), discussed below). However, because interest rates are lower, the incentive for borrowers to be higher quality is greater when lending standards are normal than when they are tight, as we discuss in Section 8.

intervention that ensures that banks do not screen and that credit standards remain normal. Because such support for lending markets involves short-run costs and long-run benefits, *early timing* of interventions can be critical. In fact, delayed policy interventions are more costly and can even even become so costly that it is optimal not to intervene at all, if the pool quality falls below the social threshold. Section 5 contains an example that illustrates these properties. This normative feature of our model is consistent with government support for lending markets during or following temporary downturns.

It is important to note that optimal support for lending improves outcomes, but does not reach the first-best. While inconsistent with our modeling assumptions, the best policy would be to would eliminate the externality associated with screening, for example by making the outcome of any valuation public. We discuss the existence of credit registries in practice and in theory and why they may not correct the externality of evaluation in Section 7.2.

Our third main result is that binding capital constraints naturally incentivize banks to tighten lending standards and can set in motion declines in pool quality that can lead to hysteresis and suboptimal market outcomes, and/or to the government support of lending markets just discussed. We characterize credit market dynamics when market-wide capital constraint binds and relaxes exogenously over time. When bank capital constrains the amount of loans that banks can make, market power shifts from borrowers to banks and so banks charge higher interest rates make greater profits on lending. Both because of limited capacity to lend and because high interest rates increase the relative profitability of lending to high-quality borrowers, capital constraints raise the profitability of tight lending standards. Thus, starting from a situation in which normal lending standards would prevail absent a binding constraint, a binding constraint on bank capital can lead the market to impose tight standards and ultimately to converge to a steady state with tight lending standards even when the constraint has relaxed, absent government recapitalization or other policies to avoid tight lending standards.

While we focus on the decision to acquire costly information and screen borrowers – what we call the second dimension of lending standards – we also show that there are situations in which banks impose tight lending standards of the first type – restricting lending without screening. For example, if the quality of the average borrower in the pool is low enough, banks do not lend at all because they can neither cover the costs of screening nor lend profitably without screening. Because this type of lending standard does not have the externality associated with screening, the private threshold for these tight lending standards can be inefficiently high or low, depending on incidental modeling choices.

More interestingly, for some parameter values, there is an intermediate range of pool quality in which banks impose tighter lending standards by rationing credit instead of screening, a situation we refer to as *slow thawing*. The logic behind credit rationing in our model is quite different from the typical credit rationing due to adverse selection (Stiglitz and Weiss (1981), Mankiw (1986)). At these levels of pool quality, banks lend without screening, but if banks lent to all borrowers, the pool quality would improve so rapidly over time that borrowers would choose to delay borrowing and wait for lower interest rates in the future. Previously-rejected borrowers would not delay however, making it unprofitable for banks to lend. In equilibrium, banks constrain lending to the point where not-previously-rejected borrowers are indifferent between delaying and not. Section 3.3 contains an example of this phenomenon, in which growth of lending volume and decline in interest rates is non-monotonic, slow, then rapid, then slow again as the market converges to the pooling steady state.

We discuss extensions to our model, as well as the relative importance of different modeling assumptions, in Section 8.

Related literature In our analysis lenders may face adverse selection. This is because the pool of borrowers may have been "cream skimmed," with good borrowers being systematically removed from the pool by lenders who employ tight lending standards. This source of adverse selection is also the central feature in Fishman and Parker (2015) and Bolton et al. (2016) and there gives rise to a strategic complementarity with regard to information acquisition that is conceptually similar to ours. That is, lenders find it more profitable to acquire information about potential borrowers if other lenders are acquiring such information. Theirs are static models where the complementarity leads to multiple equilibria, e.g., one with information acquisition and one without. And since these models are static, they cannot address the evolution, over time, of the quality of the borrower pool which is the focus of the dynamic model developed here. The intertemporal aspect of our model is key, shaping equilibrium behavior, e.g., the multiplicity of steady states but uniqueness of equilibrium, as well as the policy prescriptions, e.g., the importance of the timing of interventions.

Ruckes (2004) and Dell'Ariccia and Marquez (2006) also analyze static models of lending standards but neither features the strategic complementarity we emphasize. In Ruckes (2004) lending standards are strategic substitutes. This is because lenders simultaneously acquire information about borrowers and if a borrower is rejected or quoted a high interest rate, the borrower cannot then seek out other potential lenders. In Dell'Ariccia and Marquez (2006) there is cream skimming by informed lenders but these lenders are endowed with their information.

Hu (2018) analyzes a dynamic model and, like in our analysis, banks choose lending standards and the quality of the borrower pool evolves over time. A key difference is that in Hu (2018), lending standards are strategic substitutes, due to the positive response of the average quality of newly-entering borrowers to tighter lending standards. With this specification, Hu (2018) finds a number of interesting results regarding economic recoveries, e.g., his model may exhibit double dip recoveries.

Here, the incentive to collect information regarding an asset is higher if agents in the past have collected information, and hence there has been cream skimming. Asriyan et al. (2017) and Zou (2019) analyze models in which the incentive to collect information is higher if agents in the future are expected to collect information. This is because a buyer of the asset today may want to sell the asset in the future. There is no motive to resell a loan in our analysis and so this effect is not present in our analysis.

In our model, when lenders determine whether to lend they do not observe borrowers' prior activity, e.g., how long the borrowers have been in the market for a loan or whether the borrowers have been previously denied credit. Since bank borrowing rates are publicly observed, such information would be informative regarding whether has been denied credit and is thus a type-L and not worth financing. Similarly, in Daley and Green (2012, 2016) the history of offers received by the informed party is not observable. By contrast, in Chari et al. (2014) the history of the privately-informed party is observed by potential trading partners; their equilibrium features a partial separation of the high- and low-quality types.

In Axelson and Makarov (2019), borrowing rates are not publicly observed (borrowers make private offers to banks). So knowing that a borrower's offer was rejected does not necessarily indicate a bad borrower; rather the borrower may have made a low offer. Another key difference is that in their analysis, acquiring information on a borrower is costless and hence the information acquisition choice is trivial. These features of their model lead to the following interesting result: introducing a credit registry that tracks borrowers' loan application histories, but not the borrowing rates offered, can lead to more adverse selection and quicker market break down.

A number of papers study dynamic adverse selection models without information acquisition. Daley and Green (2012, 2016) and Malherbe (2014) analyze models where current markets can break down when high-quality sellers remain absent, waiting for market prices to improve over time. This behavior is related to, but distinct from, our slow thawing dynamics. In these models, the path of market prices over time separates good sellers from bad. In our slow thawing dynamics, the equilibrium *composition* of borrowers does not change, only the *speed* of lending is reduced. Zryumov (2015) and Caramp (2017) study models where bad sellers strategically enter when market prices are good. This, in and of itself, does not lead to a market shutdown (lower prices positively select entrants), but as Caramp (2017) emphasizes, the bigger presence of bad sellers can raise the likelihood of adverse selection induced market failures in the future.

In Gorton and He (2008) lending standards vary over time because of a different sort of strategic interaction among lenders. Theirs is a repeated games model of tacit collusion among banks. In equilibrium no bank defects from the collusive arrangement but a "punishment phase" can be triggered. The punishment phase entails tight lending standards which in turn implies less lending and lower bank profits and is interpreted as a credit crunch. So even though the quality of borrowers does not vary over time, lending standards do vary over time.

Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), and Dang et al. (2015), among others, analyze how the trading of debt securities minimizes adverse selection problems. Their results follow because the payoff on debt is less sensitive to the condition of the underlying assets as compared to other securities. Our analysis also features debt securities, though ours is a trivial security design setting. Debt would remain optimal, however, even with a more general setting.

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2 A Model of Lending Standards

Time is continuous and runs from t to infinity, $t \in [0, \infty)$. There are two sets of agents: a unit mass of potential borrowers who have no capital and are looking to fund projects and a large mass $\mathcal{J}$ of competitive banks. All agents are risk neutral and have discount rate $\rho > 0$. The main state variable in the model is the quality composition of the pool of borrowers, defined below and denoted by x_t, which both determines and is influenced by the main control variables, banks' lending standards, denoted by z_{jt}.

Borrowers and banks

Borrowers. At Poisson rate $\kappa > 0$, a potential borrower receives an investment opportunity. This opportunity is a project that requires an up-front investment of 1. Borrowers have no capital and must fund the investment externally. If the borrower raises the funds and makes the up-front investment at time t, then the project returns both a pledgeable cash flow at time $t + T$ and a non-pledgeable private benefit $u > 0$ (in present value) to the borrower. With this private benefit all borrowers will have the incentive to finance their project, even if they know they will receive no monetary benefit.

To capture differences in borrower quality, we assume that there are two types of borrowers: type H ("high quality") and type L ("low quality"). Type-H borrowers always have positive NPV investment opportunities. The pledgeable cash flow of a type-H borrower's project is D_H, with net excess return $r_H \equiv e^{-\rho T} D_H - 1 > 0$.[3] Type-$L$ borrowers always have negative NPV projects, with pledgeable cash flow D_L and net excess return $r_L \equiv e^{-\rho T} D_L - 1 < 0$. A borrower's type is permanent, always type H or always type L. We refer to $r^\Delta \equiv \frac{r_H - r_L}{-r_L} > 0$ as the (normalized) *return difference* between the investments opportunities of the two types.

When an investment opportunity arises, borrowers choose whether or not to apply to the competitive banking sector for a unit of funding to implement their project. A borrower that applies for funding is either approved or denied depending on whether she satisfies the bank's lending standards. If a borrower is funded, she invests in her project and exits the pool to run the project. Alternatively, if the borrower does not apply for funding or is denied funding, she returns to the pool where at rate $\kappa > 0$ a new investment opportunity arises.

New borrowers arrive as type H with exogenous probability λ and as type L with probability $1 - \lambda$. Potential borrowers in the pool "die" and leave the pool at Poisson rate $\delta > 0$. One can interpret 'dying' borrowers as ones who will no longer receive investment opportunities.

We make the following assumptions about the inflows borrowers so that the size of the pool is constant at 1: both potential borrowers that die and borrowers that are funded are immediately replaced in the pool by new potential borrowers. Partly as a result of this assumption, it will suffice to keep track of the fraction of type-H borrowers in the pool of potential borrowers at time t,

[3] It is a net excess return because the per-period log return $\frac{1}{T} \ln(1 + r_H) = \frac{1}{T} \ln(D_H) - \rho$.

$x_t \in [0,1]$ which makes our main results more transparent. However, the assumption that funded borrowers are immediately replaced by new borrowers is not particularly realistic. It is however inconsequential for all of our results.[4]

Finally, we assume throughout that the average project of a borrower entering the pool has positive NPV based on the pledgeable cash flow.

Assumption 1. *The average investment project has a positive net present value $\lambda r_H + (1-\lambda) r_L > 0$.*

Note that while we have not explicitly modeled collateral, it is straightforward to interpret the loan as a collateralized loan and u as the private benefit net of the loss of collateral (e.g. u is the net benefit of purchasing and living in a house until foreclosure; see Section 7.1). One can also re-interpret the model as capturing a secondary-market in which 'borrowers' are selling assets of unknown value in order to raise funds to make an investment or stay solvent.

Banks and lending standards. Banks make two decisions. First, they decide whether to be *active* or *inactive*. Second, conditional on being active, they choose their *lending standard*, that is, how aggressively to screen potential borrowers.

At any instant t, a bank may choose to be *active*, in which case it enters a competitive lending market, where it may receive a loan application by a borrower. Alternatively a bank may choose to be inactive in which case it makes no loans and consequently receives no loan applications. Let θ_{jt} denote the probability that bank j is active at time t. While generally all banks are active in our model, e.g. at all steady-states where $x = const$, there may be a region in the state space with equilibrium *credit rationing*, $\theta_{jt} < 1$, where banks offer fewer loans than borrowers demand.

An active bank j also chooses a *lending standard* $z_{jt} \in [0, \bar{z}]$, where $\bar{z} \in (0,1]$. With lending standard z_{jt}, a type-L borrower is identified as such with probability z_{jt}, in which case her loan is denied.[5] Otherwise, the borrower's loan is approved. A bank's cost of utilizing the lending standard z_{jt} is $\tilde{c} z_{jt}$, where $c \equiv \frac{\tilde{c}}{-r_L} > 0$ is the (normalized) marginal cost. The most lax lending standard corresponds to $z_{jt} = 0$, in which case all loan applications are deemed to meet the lending standards of bank j. Banks choose lending standards to maximize expected profit. Given a lending standard z_{jt}, banks offer to lend 1 in exchange for a promised loan payment at time $t+T$ equal to D_{jt}.

Due to symmetry and competition, it is without loss of generality to assume that all banks choose the same probability of being active, θ_t, the same lending standard z_t, and the same required loan payment D_t, paid at $t+T$. With a loan face value of D_t, repayment is $\min\{D_t, D\}$, where D is the payoff on the investment, D_L or D_H depending on borrower type. Since type-L borrowers

[4] The potential problem with this assumption is that it creates a social benefit of normal lending standards when there are many type$-L$ borrowers: more lending increases the flow of new average borrowers into the borrower pool. A more natural assumption is that the inflow to the pool is constant. Appendix ?? shows that all of Propositions except those in Section 3.3 apply under this alternative assumption.

[5] A type-H borrower is never misidentified as a type L, a modeling assumption important for tractability, as discussed in Section **??**.

have negative NPV investments, $D_t > D_L$ for a bank to break even in expectation. Thus, type-L borrowers always default. The repayment D_t is without loss of generality bounded above by D_H since any higher D_t will not generate additional repayment. So if a loan is made (meaning the bank can break even) then type-H borrowers will not default. We define $r_t \equiv e^{-\rho T} D_t - 1$ as the *credit spread* charged by the bank since $\rho + \frac{1}{T}\ln(1 + r_t)$ is per-period (log) return on a loan that does not default. We note that r_t always lies in (r_L, r_H).

Information structure. Borrowers have no information about their type when they enter the pool. And for as long as a borrower has no such private information, we call her an *average borrower*. Some type-L borrowers learn that they are type-L after being denied funding by a bank because of a failure to meet the lending standard. We call these borrowers *rejected borrowers*.[6] Whether a borrower is average or rejected, a bank cannot distinguish between a type-H and a type-L borrower unless it is probabilistically revealed by paying zc.[7] The shares of average and rejected borrowers are endogenously determined. For instance, the lower the lending standard z_t, the fewer rejected borrowers will be in the pool. All agents have common knowledge of the structural parameters of the lending market and the initial fraction of type-H borrowers in the pool, $x_0 \in [0, \lambda]$.[8] Also, all agents can infer past, current, and future x_t.

A borrower's problem

Taking the path of credit spreads $\{r_t\}$ as given, borrowers with investment opportunities choose whether to apply for a loan at each time t. Let φ_t^a denote the probability that an average borrower with an investment opportunity applies for a loan—as opposed to waiting in hope of an improvement in borrowing opportunities. Let φ_t^r denote the probability that a rejected borrower with an investment opportunity applies for a loan. Letting J_t^a and J_t^r denote the value functions of an average borrower and a rejected borrower, respectively, the optimal strategies for the two satisfy the following Hamilton-Jacobi-Bellman equations:

$$\rho J_t^a = \max_{\varphi_t^a \in [0,1]} \kappa \theta_t \varphi_t^a \left\{ \lambda \left(r_H - r_t + u \right) + (1-\lambda)(1-z_t)u + (1-\lambda) z_t J_t^r - J_t^a \right\} + \dot{J}_t^a - \delta J_t^a \tag{1a}$$

$$\rho J_t^r = \max_{\varphi_t^r \in [0,1]} \kappa \theta_t \varphi_t^r \{(1-z_t)(u - J_t^r)\} + \dot{J}_t^r - \delta J_t^r, \tag{1b}$$

where J_t^a and J_t^r satisfy the transversality conditions $\lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^a = \lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^r = 0$. For an average borrower, (1a) reflects three possible outcomes that may occur when she has an investment opportunity, is matched with an active bank, and chooses to apply for financing:

[6]Thus there are three types of borrowers in the pool at any time: average borrowers who are actually type H, average borrowers who are actually type L, and rejected borrowers (always type L). We show however that optimal behavior depends only on the share of type-H borrowers, x_t, because the behavior of all type-L borrowers is the same.

[7]Observe that for each individual bank, previously screened loan applicants will represent a zero mass in the pool of borrowers and can therefore be ignored.

[8]Here, $x_0 = \lambda$ corresponds to a pool consisting entirely of average borrowers.

with probability λ she is type H and is funded, receiving a monetary payoff of $r_H - r_t$ and u in private benefits; with probability $(1-\lambda)(1-z_t)$, she is type L but satisfies the lending standard, receiving a payoff of u in private benefits; and with probability $(1-\lambda)z_t$, she is type L and does not satisfy the lending standard (is rejected), receiving a payoff of J_t^r. For a rejected borrower who has an investment opportunity and is matched with an active bank, (1b) reflects the fact that with probability $1 - z_t$, she satisfies the lending standard and receives a payoff of u in private benefits; otherwise, she continues as a rejected borrower.

With strategies $\{\varphi_t^a, \varphi_t^r\}$, there is a flow of

$$\kappa_{Ht} \equiv \kappa \varphi_t^a x_t \tag{2}$$

type-H borrowers applying for loans. Note that all of the type-H borrowers belong to the sub-pool of average borrowers. There is a flow of

$$\kappa_{Lt} \equiv \kappa \varphi_t^a \frac{1-\lambda}{\lambda} x_t + \kappa \varphi_t^r \frac{\lambda - x_t}{\lambda} \tag{3}$$

type-L borrowers applying for loans. For the derivation of (3), let A_t denote the share of average borrowers at time t, with $1 - A_t$ being the share of rejected borrowers at time t. The fraction of type-H borrowers in the whole pool is $x_t = A_t \lambda$. The flow of type-L borrowers equals $\kappa \varphi_t^a A(1 - \lambda) + \kappa \varphi_t^r (1 - A)$. Substituting in $A_t = x_t / \lambda$ yields (3). In equilibrium, it will be the case that $\varphi_t^a = \varphi_t^r = 1$, so that $\kappa_{Ht} = \kappa x_t$ and $\kappa_{Lt} = \kappa(1 - x_t)$.

A bank's problem

Since there is a flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications by borrowers at time t, it is without loss to assume that there are at most a flow of $\kappa_{Ht} + \kappa_{Lt}$ active banks at time t. As will be seen below, there are cases where some banks remain inactive in equilibrium, leaving only $\theta_t\,(\kappa_{Ht} + \kappa_{Lt})$ active banks, with $\theta_t \in [0, 1]$. A fraction θ_t of the flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications is then received by the $\theta_t(\kappa_{Ht} + \kappa_{Lt})$ active banks.

Conditional on flows κ_{Ht}, κ_{Lt} and credit spread r_t, an active bank's lending standard z solves

$$\Pi_t(r_t) \equiv \max_{z \in [0, \bar{z}]} \kappa_{Ht} r_t + \kappa_{Lt}(1 - z) r_L - (\kappa_{Ht} + \kappa_{Lt}) \tilde{c} z. \tag{4}$$

Taking z as given, Bertrand competition among banks then determines r_t by

$$\Pi_t(r_t) = 0. \tag{5}$$

Whenever this cannot be satisfied by any finite r_t, no bank will find it profitable to lend. In this case, we set $r_t = \infty$ and $\theta_t = 0$.

Evolution of the borrower pool

The evolution of the quality of the borrower pool is given by

$$\dot{x}_t = \theta_t \kappa_{Lt}(1 - z_t)\lambda - \theta_t \kappa_{Ht}(1 - \lambda) + \delta(\lambda - x_t), \tag{6}$$

which is the combination of three distinct forces: the first term accounts for the $\theta_t \kappa_{Lt}(1 - z_t)$ type-L borrowers who are funded and replaced with a fraction λ of type-H borrowers; the second term accounts for the $\theta_t \kappa_{Ht}$ type-H borrowers who are funded and replaced with a fraction $1 - \lambda$ of type-L borrowers; and the third term accounts for the $\delta\lambda$ type-H borrowers being born and δx_t borrowers dying each instant.

Equilibrium

We define an equilibrium as follows:

Definition 1. Given an initial share of type-H borrowers $x_0 \in [0, 1]$ in the pool, an *equilibrium* consists of a path of the fraction of type-H borrowers $\{x_t\}$, credit spreads $\{r_t\}$, shares of active banks $\{\theta_t\}$, borrowers' application decisions $\{\varphi_t^a, \varphi_t^r\}$, implied application flows of type-H and type-L borrowers $\{\kappa_{Ht}, \kappa_{Lt}\}$, and screening choices $\{z_t\}$ such that

- $\{\varphi_t^a, \varphi_t^r\}$ solve each type's maximization problem (1) given $\{r_t, z_t, \theta_t\}$,
- $\{\kappa_{Ht}, \kappa_{Lt}\}$ are determined by (2) and (3),
- z_t solves the bank's maximization problem (4) given $\{r_t, \kappa_{Ht}, \kappa_{Lt}\}$,
- r_t is determined by the zero profit condition for banks (5) given κ_{Ht}, κ_{Lt} whenever possible; if not, $r_t = \infty$ if $\kappa_{Ht} = 0$,
- $\{x_t\}$ follows the law of motion (6),
- at no time t can a bank raise its profit Π_t by being active, charging a rate $\tilde{r} < r_t$ that average borrowers would weakly prefer to waiting, and lending to the entire set of borrower applicants (a flow of κx_t type-H and a flow of $\kappa(1 - x_t)$ type-L borrowers).

A *steady state (equilibrium)* is an equilibrium in which all equilibrium objects $\{x_t, r_t, \theta_t, \varphi_t^a, \varphi_t^r, z_t\}$ are constant over time.

To study variation in lending standards, we make the following assumption on parameters throughout the remainder of this paper.

Assumption 2. *The cost of bank screening c is not too low or too high:*

$$1 - \lambda < c < 1 - x^s + \bar{z}^{-1} \min\left\{x^s r^\Delta - 1, 0\right\},$$

where $x^s = \lambda - \lambda \frac{(1-\lambda)\bar{z}}{(1-\lambda\bar{z})+\delta\kappa^{-1}}$.

The first inequality in Assumption 2 ensures that the bank screening cost $\tilde{c}$ is high enough that tight lending standards do not strictly dominate normal lending standards. The second inequality ensures that there can exist a steady state in which tight lending standards are optimal.

3 Equilibrium characterization

The model's tractability allows for an analytical characterization of the set of equilibria, starting with steady-state equilibria.

3.1 Steady-state equilibria

Borrowers' and banks' behavior simplifies significantly in a steady-state equilibrium.[9] Borrowers, facing the same interest rate r at all times, have no incentive to wait and therefore choose $\varphi^a = \varphi^r = 1$. In fact, they strictly prefer borrowing to waiting,

$$\lambda\left(r_H - r + u\right) + (1-\lambda)(1-z)u + (1-\lambda)zJ^r - J^a > 0, \tag{7}$$

because, for any bank to be active, the loan rate needs to be weakly below the highest pledgeable payoff, $r \leq r_H$, but then $J^a > 0$, which is equivalent to (7) in a steady state. Under these conditions, all banks are active in a steady state, $\theta = 1$.

The steady-state quality of the pool x and the steady-state lending standard z are jointly determined, by the interaction of two forces. On the one hand, the law of motion of x, (6), implies that when $\dot{x} = 0$,

$$x = \lambda - \lambda \frac{(1-\lambda)\, z}{(1-\lambda z) + \delta\kappa^{-1}}. \tag{8}$$

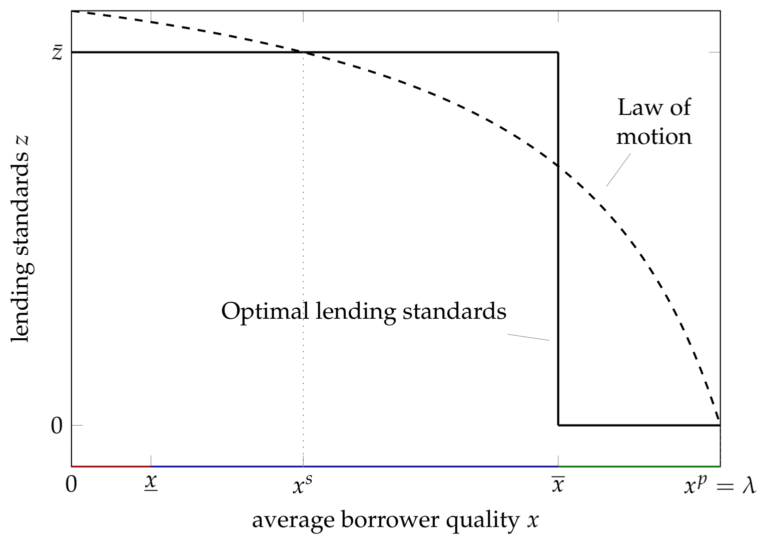
This equation highlights that tighter lending standards—higher z—are associated with a lower steady-state quality of the pool of borrowers x, as more low-quality borrowers are rejected by banks. This effect is greater when the effects of lending standards on the pool are more persistent (low death rate δ) or when opportunities to invest arise more frequently (high κ) and so potential investors are evaluated more frequently.

On the other hand, banks solve (4) and choose tighter lending standards z precisely when the pool is more adversely selected,

$$z = \begin{cases} 0 & \text{if } x > \bar{x} \\ [0, \bar{z}] & \text{if } x = \bar{x} \\ \bar{z} & \text{if } x < \bar{x} \end{cases}, \quad \text{where } \bar{x} \equiv 1 - c. \tag{9}$$

[9] Since prices and quantities are constant, we drop the time subscripts for this subsection.

Figure 2: The two forces shaping steady-state equilibria.



Note: This figure shows two curves whose intersections yield the steady-state pool quality x and the steady-state lending standard z. The solid line represents the optimal choice of the lending standard, (9). The dashed line represents the pool quality x that is caused by any given lending standard z through the law of motion.

The combination of the two equations (8) and (9) is illustrated in Figure 2. Both represent downward-sloping relationships between x and z, and given Assumption 2 admit three intersections, each of which represents a steady-state equilibrium. This logic is summarized in the following proposition.

Proposition 1 (Steady state equilibria)**.** *There exist three steady-state equilibria:*

(i) A pooling *steady state with* $z = 0$ *and* $x^p = \lambda$*.*

(ii) A screening *steady state with* $z = \bar{z}$ *and* $x = x^s \equiv \lambda - \lambda \frac{(1-\lambda)\bar{z}}{(1-\lambda\bar{z})+\delta\kappa^{-1}}$.

(iii) A mixed *steady state with* $z = \frac{\lambda-\overline{x}}{\lambda-\lambda\overline{x}}\left(1+\delta\kappa^{-1}\right) \in (0, \bar{z})$ *and* $x = \overline{x}$*.*

The root of the multiplicity is a *dynamic strategic complementarity* among banks. According to (9), banks naturally respond to a lower quality pool by tightening their lending standards; however, according to (8), tighter lending standards worsen the pool itself, creating an even bigger incentive for banks to tighten their standards in the future. This reasoning rationalizes the existence of the pooling and screening equilibria, see Figure 2. The mixed steady state formally exists but will turn out to be unstable and therefore play no role in the remainder of the analysis.

The pooling and screening steady states have the following important characteristics.

Corollary 1 (Quality of funded borrowers)**.** *In the pooling steady state,*

1. *the credit spread R is lower.*
2. *more projects are funded: κ relative to $\kappa x + \kappa(1-x)(1-z)$ in the screening steady state.*

3. *the default rate is higher: the share of funded borrowers who are of type L is λ relative to $\frac{(1-x^s)(1-z)}{x^s+(1-x^s)(1-z)} < \lambda$ in the screening steady state.*

The first point follows from the observation that a lower pool quality, ceteris paribus, hurts banks' profits, and therefore requires larger credit spreads for banks to break even. This is true even though banks choose tight lending standards because credit spreads rise partly due to greater screening costs.[10] The second point follows from the fact that screening reduces the flow of borrowers that receive funding. The third point has the following subtlety. In fact, when $\delta = 0$ (no birth and death from the pool of potential borrowers), the default rate, which in our model equals the share of type L borrowers among all funded borrowers, is equal to $1 - \lambda$ in *any* steady state. Indeed, in the screening steady state, the imposition of tight lending standards exactly balances the low average project quality in the pool. This leads to the same share of bad projects being funded as in the pooling equilibrium.[11]

3.2 Transitional dynamics

An important factor that simplifies the steady state analysis is the fact that banks are always active in a steady state, $\theta = 1$. This is no longer true in equilibria with dynamics. In particular, there are now up to two regions in which banks may choose to remain inactive. Naturally, this is the case when the quality of the pool x is very low, so that even a break-even loan rate at its maximum of $r = r_H$ is not enough to recoup the losses incurred from lending to the many type-L borrowers in the pool. This occurs when x is so low that $\Pi(r_H) < 0$, or equivalently,

$$\theta(x) = \begin{cases} 0 & \text{if } x < \underline{x} \\ [0,1] & \text{if } x = \underline{x} \end{cases}, \quad \text{where } \underline{x} \equiv \frac{1 - \overline{z} + c\overline{z}}{r^{\Delta} - \overline{z}}. \tag{10}$$

Somewhat more surprisingly, however, banks may also remain inactive in a subset of the pooling region, where inactivity can endogenously reduce the volume of lending and slow down the speed of convergence to the pooling steady state. We refer to this region as the "slow-thawing" region and it is described in detail in Section 3.3. Until then, we assume parameters are such that there is no such region:

Assumption 3 (No slow thawing)**.** *Assume that there is no slow-thawing region, that is, $\theta(x) = 1$ for all $x \geq \underline{x}$.*

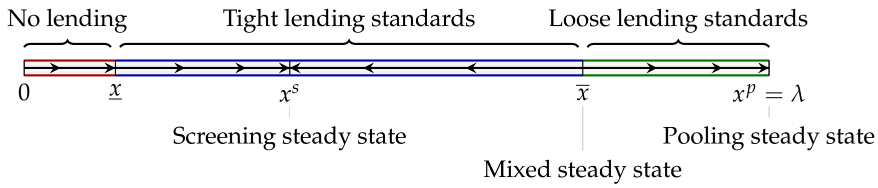
For the sake of exposition, this assumption is stated in terms of endogenous objects. The analytical condition is stated in the next section.

Under Assumption 3, the equilibrium transitional dynamics are as follows.

[10] Because r must be continuous and decreasing in x. A complete characterization of interest rate spreads at all x with formal proof is provided in Proposition 3.

[11] The results in Corollary 1 are robust to alternative assumptions on the dynamics of the borrower pool, e.g. assuming a constant inflow, rather than a constant pool size.

Figure 3: State space and banks' optimal strategies.



Proposition 2 (Transitional dynamics without slow thawing)**.** *Suppose Assumption 3 holds and $x_0 \in [0, \lambda]$ is the initial fraction of type-H borrowers in the pool. There is a unique equilibrium, in which banks' activity policy satisfies* (10) *for $x \leq \underline{x}$, their lending standards are given by* (9)*, and borrowers never wait, $\varphi_t^a = \varphi_t^r = 1$. As $t \to \infty$, the credit market converges to*

(i) the screening steady state, $x_t \to x^s$, if $x_0 < \overline{x}$.

(ii) the mixed steady state, $x_t \to \overline{x}$, if $x_0 = \overline{x}$

(iii) the pooling steady state, $x_t \to x^p$, if $x_0 > \overline{x}$

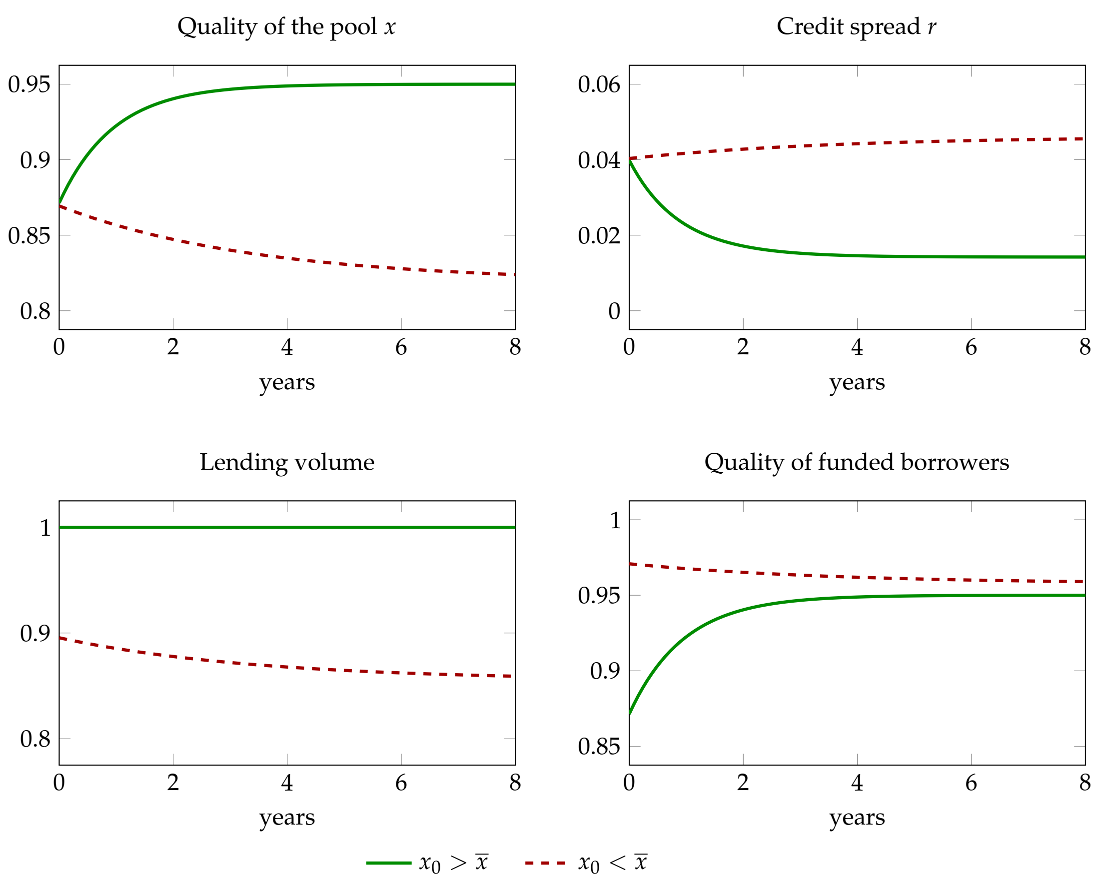
Proposition 2 provides a complete characterization of the equilibrium transitional dynamics of x. Despite the multiplicity of steady states (Proposition 1), there is a unique equilibrium for any $x \neq \bar{x}$, giving unambiguous model predictions..

The model predictions can be seen in Figure 3, which illustrates the state space of the credit market and highlights the transitional dynamics in the three different regions of bank behavior: the "no lending" region for low pool qualities, where banks are inactive ($\theta_t = 0$) and the pool quality improves only due to death and birth; the "tight lending standards" region, where banks screen borrowers $z_t = \overline{z}$ and the market approaches the screening steady state; and the "normal lending standards" region where banks choose $z_t = 0$ and the market returns to the pooling steady state.

A crucial part of the diagram is at $x = \overline{x}$. This point represents a sharp boundary between the tight and normal lending standards regions and gives rise to an important model prediction, a "bifurcation" property: when x_0 lies above $\overline{x}$, the credit market converges to the pooling steady state; when x_0 lies below $\overline{x}$, however, the self-reinforcing nature of tight lending standards pushes the market to the screening steady state.

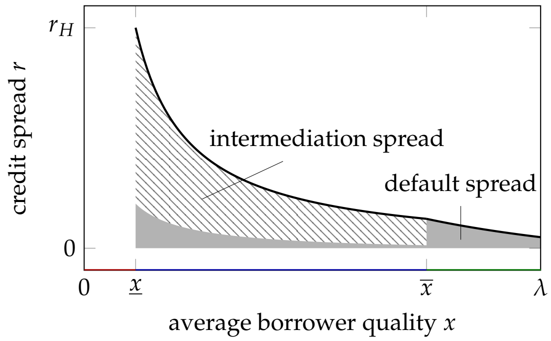
The bifurcation property also comes out in Figure 4 where we simulate the credit market with two different initial values for x_0, one just above $\overline{x}$ (green, solid) and one just below $\overline{x}$ (red, dashed). As can be seen, this small difference in initial conditions leads to quite different evolutions of pool qualities x, credit spreads r, and lending volumes $\kappa_{Ht} + \kappa_{Lt}(1-z)$. The final panel of Figure 4 shows the evolution of the quality of funded borrowers, $\kappa_{Ht}/\left(\kappa_{Ht} + \kappa_{Lt}(1-z)\right)$, which is one minus the default rate. The market with the relatively *lower* initial pool quality initially has a much lower lending volume and default rate, as banks are imposing tight lending standards. Interesting, the two markets initially have quite similar credit spreads. Over time, and foreshadowing our results

Figure 4: The self-reinforcing property of lending standards.



Note. This figure shows two sets of transitional dynamics in a credit market without slow thawing. Green and solid is a market starting at $x_0 = \overline{x} + \epsilon$ and therefore banks have normal lending standards; red and dashed is a market starting at $x_0 = \overline{x} - \epsilon$ and therefore banks impose tight lending standards. The parameters used for this simulation are as follows: TBD.

Figure 5: Break-even credit spread as function of pool quality x.



Note: Grey is the component of the credit spread that is due to default risk (the *default spread*). Hatched is the component of the credit spread that is due to intermediation costs (the *intermediation spread*).

on efficiency and optimal policy, the slightly lower initial pool quality causes convergence to a steady-state with much higher credit spreads and lower lending volume but quite similar default rates (see the discussion of Corollary 1).

Interestingly, we can characterize both equilibrium credit spreads r_t across credit markets and the relative roles of both expected default and intermediation costs. First, how do credit spreads vary with pool quality x_t? Markets with higher x_t have lower default rates for any given lending standard, but also are more likely to have normal lending standards, and so higher default rates and higher bank funding costs. The second effect suggests that a higher x could be associated with a higher default rate and so a higher credit spread, but this turns out not to be the case. As the following proposition formally proves, r_t is still inversely related to x_t.

Proposition 3 (Equlibrium credit spread)**.** *The equilibrium credit spread $r_t = r(x)$ is decreasing in the fraction of type-H borrowers, x, and is given by*

$$r_t = r(x) = \begin{cases} \infty & \text{if } x < \underline{x} \\ (-r_L)x^{-1}\left\{c\overline{z} + (1-\overline{z})(1-x)\right\} & \text{if } \underline{x} \leq x < \overline{x} \\ (-r_L)x^{-1}\left\{1-x\right\} & \text{if } x \geq \overline{x} \end{cases}. \tag{11}$$

Using (11), we can decompose $r(x)$ into a *default spread*, $-r_L x^{-1}(1-z(x))(1-x) > 0$ where $z(x)$ is the optimal screening choice given x; and into an *intermediation spread* $-r_L x^{-1} c z(x) > 0$. Figure 5 plots the credit spread $r(x)$ and these two components over the state space, illustrating the inverse relationship of $r(x)$ with pool quality x. The shaded areas in Figure 5 highlight that the default spread changes discretely at $x = \overline{x}$ as banks switch between screening and not screening, but this change is offset by an equally large change in the spread due to the costs of intermediation. The spread rises significantly due to intermediation costs at lower pool qualities x. The decoupling of credit spreads and credit risk in this region of the state space provides a new rationale for why, at times, credit spreads may appear to be high given the credit risk. He and Milbradt (2014) attribute such high credit spreads to low liquidity. Alternatively one might rely on risk aversion as an explanation. Here the high credit spread derives from the costs of intermediation.

The monotonicity of $r(x)$ is also reflected in Figure 4, with a rising loan rate for the credit market with the lower quality of potential borrowers and a falling loan rate for the market with higher quality. The falling loan rate raises an obvious question: wouldn't average borrowers have an incentive to wait for lower loan rates? The answer is yes in certain cases. Credit can be restricted even with normal lending standards, so that both lending volume and the improvement of the pool of borrowers are slowed, and credit markets recover ("thaw") much more slowly than otherwise.

3.3 Slow thawing

When x_0 is just above $\overline{x}$, it is possible that if all banks were active and $r(x)$ were as defined in Proposition 3, then the increase in pool quality, x_t, over time would lead to so rapid a decline in r_t that average borrowers would strictly prefer not to accept loans but would instead prefer to wait for lower credit spreads ($\varphi^a = 0$). This, however, would cause a market shutdown because no bank is willing to lend to rejected borrowers only, and therefore cannot be an equilibrium. Instead, the equilibrium must exhibit a slower speed of transition so that the improvement in the pool of potential borrowers and the decline in rates both occur more slowly and average borrowers are willing to accept loans in equilibrium. For this transition to be slower, it must be that not all banks are active ($\theta_t < 1$), which can only be the case if there are no profits to be made from making a new loan (see Definition 1). This is precisely the case when borrowers are also indifferent between waiting and applying for loans. The following proposition proves that these strategies are indeed an equilibrium.

Proposition 4 (Slow thawing)**.** *There exists a threshold $\hat{x} \in (0, x^p)$, such that: (i) if $\hat{x} \leq \overline{x}$, there is no slow thawing region; if (ii) $\hat{x} > \overline{x}$, then for any $x \in [\overline{x}, \hat{x})$, a positive fraction of banks are inactive*[12]

$$\theta(x) = \frac{(\rho + \delta)(r_H - r(x))}{-\kappa r'(x)(\lambda - x)} - \delta\kappa^{-1} < 1 \tag{12}$$

where $r(x) = -r_L x^{-1} \{1 - x\} > 0$. Borrowers are indifferent and apply for loans, $\varphi_t^a = \varphi_t^r = 1$, and $\hat{x}$ is determined as the unique solution to $\theta(\hat{x}) = 1$ in $(0, x^p)$.

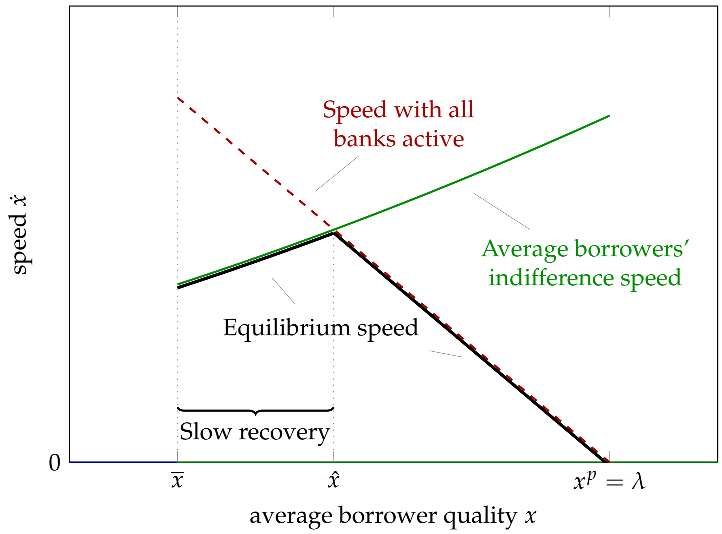
The intuition for the expression in (12) comes directly from the indifference condition of average borrowers. Still focusing on the instructive special case where $u \to 0$, the HJB of an average borrower is given by

$$\rho J_t^a = \max_{\varphi_t^a \in [0,1]} \kappa\theta_t\varphi_t^a \{\lambda(r_H - r_t) - J^a\} + \dot{J}^a - \delta J^a$$

with indifference between applying for a loan or not requiring that $J^a(x) = \lambda(r_H - r(x))$. Substituting this back into the HJB yields an equation for the speed $\dot{x}$ at which the pool needs to improve for

[12] For the sake of readability, we assume here that the utility benefit from running a project is comparatively small, $u \to 0$.

Figure 6: Slowly thawing credit markets.



Note. This figure illustrates when there exists a region with "slow thawing" where credit markets recover only very slowly from a crisis. The green solid line represents the speed at which the pool quality needs to improve for average borrowers to be exactly indifferent between applying for loans (strictly preferred below the curve) and waiting (strictly preferred above). The red dashed line represents the speed of improvement when all banks are active. The equilibrium speed (black solid line) is the minimum of both curves.

average borrowers to be indifferent,

$$\underbrace{-\lambda r'(x)\dot{x}}_{\text{benefit of waiting}} = \underbrace{(\rho+\delta)\lambda\left(r_H - r(x)\right)}_{\text{opportunity cost of waiting}}. \tag{13}$$

When is $\dot{x}$ the equilibrium speed? Precisely when θ_t is such that $\dot{x}$ satisfies the law of motion of x, (6). Together, (13) and (6) give (12).

Figure 6 schematically illustrates this logic. The green solid line represents the speed $\dot{x}$ at which average borrowers are indifferent between borrowing now and waiting for the pool to improve. This is an increasing line as the benefit of waiting declines the closer x is to the pooling steady state. The red dashed line represents the speed at which the pool quality improves when all banks choose to be active. Clearly, where this line lies below the green solid line of indifference, it is also equal to the equilibrium speed, shown in black thick solid line. However, for $x < \hat{x}$, the growth of $\dot{x}$ with all banks active lies above the solid green indifference curve so that borrowers would prefer to wait which cannot be an equilibrium. In this region, a fraction $1-\theta(x)$ of banks choose to be inactive, bringing down the equilibrium speed to match the one along the green solid indifference curve. This leads to a hump-shaped thawing speed: initially little lending due to the threat of average borrowers waiting, a period of slow thawing as lending volume and the pool quality accelerate, followed by a period of normal convergence to the steady state.[13]What determines how likely or how strong this period of slow thawing is? The following corollary reveals the roles of interest rates, project payoffs, and meeting frequencies.

Corollary 2 (Credit market recovery with loose lending standards)**.** *Fix a quality of the borrower pool $x \in (\overline{x}, x^p)$ and let $\dot{x}$ denote the speed of improvement in the pool's quality. Then:*

1. *Worse projects always slow down the recovery: $\dot{x}$ falls with lower r_L, r_H.*
2. *Low aggregate interest rates, ρ, can backfire: if x is in the slow-thawing region, $x < \hat{x}$, $\dot{x}$ falls with lower ρ.*
3. *Easy access to banks does not speed up the recovery in the slow thawing region: for $x < \hat{x}$, $\dot{x}$ does not rise with greater meeting frequencies κ; outside of the slow thawing region, easy access to banks speeds the recovery: for $x > \hat{x}$, $\dot{x}$ rises with κ.*

Most noteworthy are the second and third comparative statics. Lower ρ (or δ), holding r_L, r_H fixed, makes average borrowers more willing to wait, shifting down the indifference curve in Figure 6 and slowing down the recovery. This channel suggests that typical expansionary monetary policy – reducing bank funding rates to raise lending volume and stimulate credit markets – can backfire, or at least be less effective, in aiding the recovery from a financial crisis.

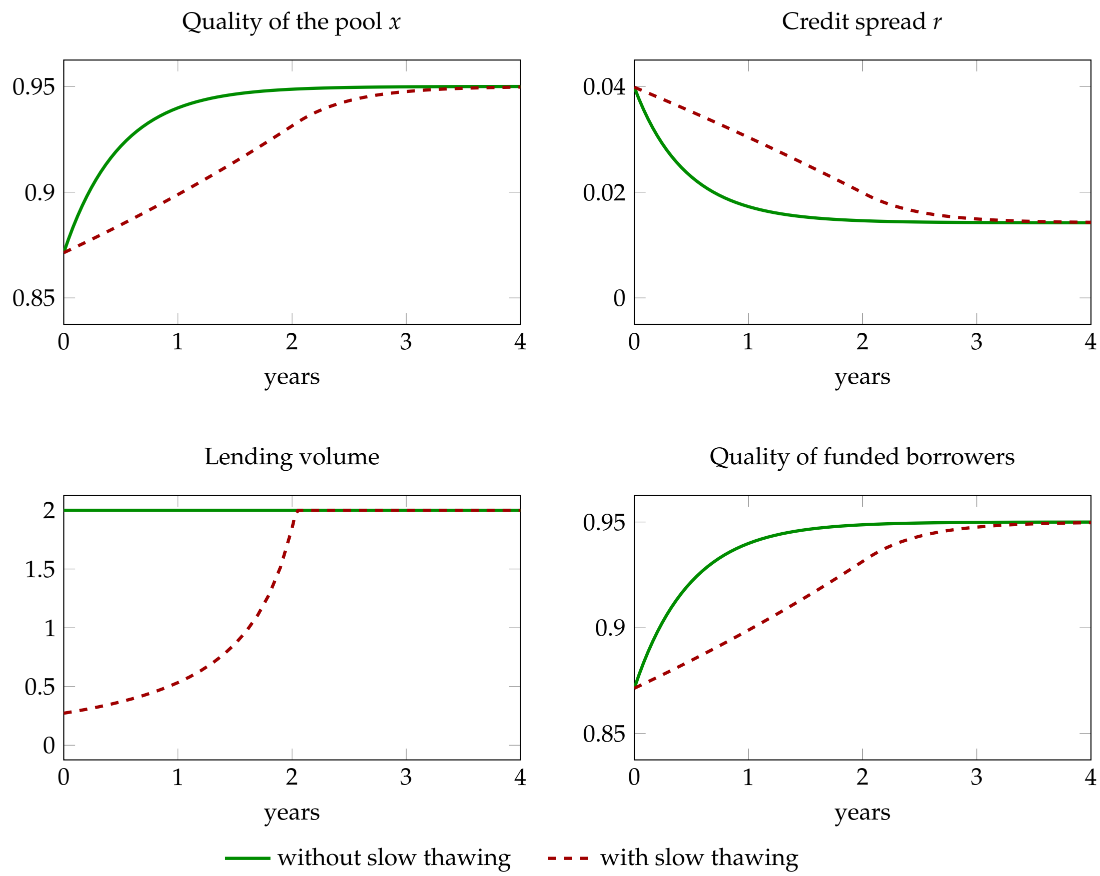
When the meeting frequency κ of borrowers and banks increases, the red line in Figure 6 increases. This naturally increases the speed of the recovery towards the steady state outside the slow-thawing region. Inside that region, however, it has no effect. In fact, even when $\kappa \to \infty$, the transition towards the pooling steady state is slow and entirely determined by the indifference condition (13).

Figure 7 juxtaposes the transitional dynamics with slow thawing (solid green line) and the transitional dynamics without slow thawing (dashed red line). The latter was computed by ruling out slow thawing by assumption, imposing $\varphi_t^a = 1$, $\theta_t = 1$, and dropping equilibrium equation 7 and instead.assuming that potential borrowers are myopic in the sense that (and only in the sense that) when they have the opportunity to invest, they approach the competitive banking sector and accept the loan and invest rather than optimally choosing whether instead to wait for their next opportunity to borrow. As is visible in the first panel, slow thawing can greatly slow the transition back to the pooling steady state and lead to a relatively low lending volume.

In closing, it is important to note that a similar region with slow thawing can also appear in the region between $\underline{x}$ and x^s and slow down the convergence to the screening steady state from the left.

[13]Note that Figure 6 does not show $\dot{x}$ just to the left of $\bar{x}$ because it is negative. By Proposition 3, $\dot{x} < 0$ implies $\dot{r} < 0$. With spreads decreasing over time, there is no incentive to delay and so no region of slow thawing.

Figure 7: Slowly thawing credit markets.



Note. The plots compare two transitions back to the pooling steady state. Green solid is a transition without "slow thawing", where average borrowers always accept current loan offers and banks do not ration credit; red dashed is a transition with slow thawing, where banks ration credit in equilibrium.

4 Efficiency

At the heart of the positive model predictions is a dynamic strategic complementarity: when current banks operate tight lending standards and screen out low-quality borrowers, future banks prefer tight lending standards as well. We next characterize the (constrained) efficient outcomes.[14]

4.1 Constrained efficient policy

In our concept of constrained efficiency, we allow the planner to control banks' activity and screening decisions, subject to borrowers' application decisions, so as to maximize the sum of agents' utilities.[15] To keep the derivations and exposition clear, we focus on the case where the private benefit from running the project u is vanishingly small, $u \to 0$. In this section, it is further assumed that the planner can set the path of market interest rates $\{r_t\}$, and therefore prevent average borrowers from waiting (i.e. there is no slow thawing). We discuss relaxing this assumption below.

The constrained efficient planning problem is then given by

$$\max_{z_t \in [0,\overline{z}], \theta_t \in [0,1]} \int_0^\infty e^{-\rho t} \kappa \theta_t \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt \tag{14}$$

subject to the law of motion of x_t, (6). The solution to this problem can be characterized as follows.

Proposition 5 (Second-best policy)**.** *There exists a threshold $\overline{x}^* \in [0, \overline{x})$ such that the second-best planner sets:*

$$z_t = \begin{cases} \overline{z} & \text{if } x_t < \overline{x}^* \\ 0 & \text{if } x_t > \overline{x}^* \end{cases} \tag{15}$$

The threshold $\overline{x}^*$ is the largest x that satisfies

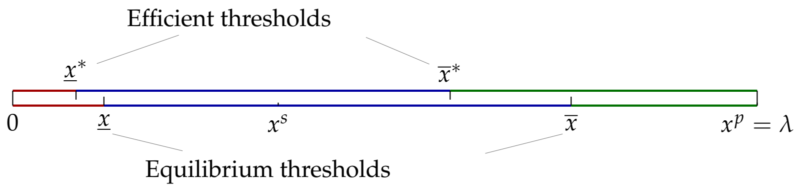
$$\underbrace{\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} r_H + (1 - \overline{z}) \left(1 - \frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} \right) r_L - \tilde{c} \overline{z}}_{\text{Average social benefit of screening}} \geq \underbrace{\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} \right) r_L}_{\text{Average social benefit of pooling}} \tag{16}$$

where $\alpha^p = \kappa + \delta$, $\alpha^s = \kappa + \delta - \overline{z} \lambda \kappa$. In particular, for any $x_t \in (\overline{x}^*, \overline{x})$, equilibrium lending standards are (second-best) inefficiently tight.

[14] The unconstrained efficient allocation (first-best) in our model would allow the planner to fund only average borrowers, which, given Assumption 2, would be done without screening.

[15] Since borrowers and banks are risk-neutral, this is without loss when the planner has access to transfers between agents.

Figure 8: Constrained efficient vs. equilibrium lending standards.



For any $\overline{x}^* > x^s$*, the optimal policy for bank activity is given by*

$$\theta_t = \begin{cases} 0 & \textit{if } x_t < \underline{x}^* \\ 1 & \textit{if } x_t > \underline{x}^* \end{cases}$$

for some $\underline{x}^* \in [0, \overline{x}^*)$*.*

Proposition 5 reveals that the optimal policy is similar in spirit to the equilibrium: when the quality of the pool is relatively high, $x > \overline{x}^*$, normal lending standards, $z = 0$, are optimal; when it is not, tight lending standards are optimal. But, importantly, the cutoffs for the optimal policy and for the market equilibrium differ: There exists a region in the state space, $(\overline{x}^*, \overline{x})$, where equilibrium lending standards are too tight relative to the constrained-efficient outcome.

To develop an intuition for this finding, imagine the current pool quality is x and banks operate normal lending standards, $z = 0$, in all periods from now on so that the credit market ultimately converges to the pooling steady state x^p. In (16), one can think of $\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$ as the time-averaged fraction of type-H borrowers funded. The weight on current x is ρ, as with greater discounting the present becomes relatively more important; the weight on the (long-run) steady state x^p is $\alpha^p = \kappa + \delta$, which is the speed at which x converges to x^p. The average social benefit of screening is therefore the weighted average surplus from lending to each type of borrower,

$$\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L.$$

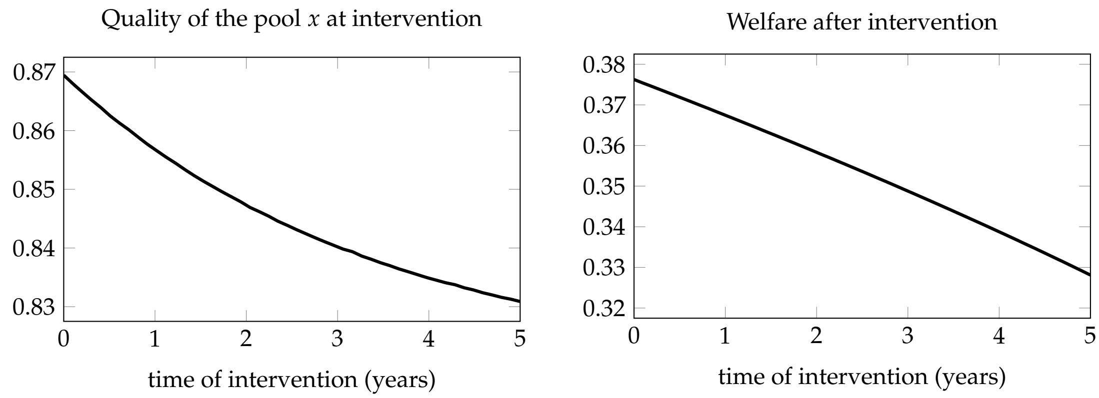
An analogous expression describes the social benefit of screening, where we additionally account for both the costs of screening and the fact banks successfully screen a fraction $\bar{z}$ of of low-quality borrowers, giving rise to (16).

In contrast, the private cut-off, $\overline{x}$, is the largest value satisfying

$$\underbrace{xr + (1 - \overline{z})(1 - x) r_L - \tilde{c}\overline{z}}_{\text{Average private benefit of screening}} \quad \geq \quad \underbrace{xr + (1 - x) r_L}_{\text{Average private benefit of pooling}} \tag{17}$$

which is calculated using the *current* fraction x of type-H borrowers and entirely ignores the dynamic

Figure 9: Early interventions dominate late ones.



Note. This figure shows how intervention policies affect a credit market that is transitioning towards the screening steady state. The horizontal axis shows the time at which an intervention starts (where 0 corresponds to the immediate, constrained efficient intervention).

consequences from screening and pooling. In particular, since in the relevant region it holds that

$$\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} < x < \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$$

agents privately ignore the dynamic costs from screening relative to pooling. Therefore, $\overline{x}^* < \overline{x}$. The private and social thresholds are shown in Figure 8.

Another way to highlight the differential dynamic consequences of pooling and screening is to compare steady states.

Corollary 3. *When both steady states exist (a result of Assumption 2), the screening steady state has strictly lower welfare than the pooling steady state.*

If $\delta = 0$, this result would be a simple consequence of the fact that screening potential borrowers is costly and the quality of funded borrowers is independent of the steady state (Corollary 1). But with $\delta > 0$, screening borrowers has a social benefit because a share of them are never funded. Still, the corollary shows that, as long as the cost c of screening is not too low (in which case the pooling steady state ceases to exist), welfare of the pooling steady state always dominates welfare of the screening steady state.

There are two important practical implications from the existence of a non-empty interval $(\overline{x}^*, \overline{x})$ where the market equilibrium diverges from the constrained optimal allocation.

1. Intervention timing matters. Figure 9 illustrates the welfare consequences of intervention in a credit market that starts at a given $x_0 \in (\overline{x}^*, \overline{x})$ for various times when an intervention starts (on the horizontal axis). The later the time of intervention is, the lower is the quality of the borrow pool when the policy switches from screening to pooling (left panel). Later intervention times thus increase the short-run losses incurred at the start of the intervention and are therefore welfare-inferior to early interventions. In fact, after a sufficiently long time, if x_t has fallen below $\overline{x}^*$, intervening may even be welfare-dominated by not intervening at all and allowing the market to converge to the screening steady state, despite its having lower welfare than the pooling steady state. That is, a late intervention may be worse than a policy of not intervening at all, a result that underscores the importance of the timing of interventions in our model. However, it may instead be the case that even at the screening steady state, it is still optimal to intervene and relax lending standards. In this case, intervention is always optimal (when the quality of the pool is weakly above that in the screening steady state).

2. Better screening technology may be detrimental to welfare. Suppose the cost $\tilde{c}$ of operating tight lending standards falls. While it is clear that such a reduction in costs necessarily raises efficiency in any steady-state equilibrium, it can decrease welfare because it raises thresholds both for the market convergence to a screening equilibrium and for the efficient intervention, $\overline{x}$ and $\overline{x}^*$. Therefore, if a market is just recovering from a crisis, with x_0 just above $\overline{x}$, such a technological improvement may cause $\overline{x}$ to rise above x_0 and thereby prevent a recovery and lead to a reduction in welfare. If $\overline{x}^*$ also rises above x_0 then it is too costly for policy to mitigate this decline in welfare.

A decrease in costs $\tilde{c}$ represents an improvement in private information technology. What happens if instead public information technology (e.g. credit reporting) improves? A crude way to capture such a change is as an increase in δ, an increase in the probability that rejected borrowers die. While the death of average borrowers has no effect on equilibrium as they are replaced in the pool by an equal measure of new average borrowers, a greater death rate of rejected borrowers does matter for equilibrium. A larger δ increases (decreases) the speed of convergence when x is increasing (decreasing), and raises the pool quality in the screening steady state, so therefore unambiguously increasing welfare. Thus, the welfare effects of improving public information are unambiguously positive, a point that leads naturally into a discussion of the first-best policy.

4.2 First-best policy

While inconsistent with our modeling assumptions, the first-best policy would be to would eliminate the externality associated with screening, for example by making the outcome of any valuation public. One might expect therefore that market participants might design institutions to track and make public such information in situations where the externalities associated with lending standards are severe. We discuss in Section 7.2 the extent to which credit bureaus in some countries partly mitigate this externality by reporting credit checks. But credit bureaus do not track rejections.Our model suggests why eliminating the externality is difficult: it is not privately optimal to report rejection because making bad information public destroys private value.

4.3 Implementation of the constrained optimum

There are several ways in which a government or a regulating authority could implement the constrained efficient outcome, that is, normal lending standards when $x \in (\overline{x}^*, \overline{x})$. We continue to assume that there is no region of slow thawing. Since such an intervention entails short-run losses (see Figure 9) and the model's banking sector is competitive, either the government or type-H borrowers have to bear these losses.

An example of a policy in the first category is a government-funded loan insurance program in which the government provides an insurance benefit $b > 0$ to be paid whenever a borrower defaults. Letting b be in present value terms, this policy incentivizes banks to pool as long as

$$\frac{b}{-r_L} > 1 - \frac{c}{1-x}.$$

This condition is satisfied for $b = 0$ in the region $x > \overline{x}$ where pooling is privately optimal. It requires nonzero insurance benefits $b = b(x) > 0$ when $x < \overline{x}$. As function of the pool quality, $b(x)$ is decreasing in x. This means, a typical intervention starting from some $x_0 < \overline{x}$ requires large insurance benefits early on, which are then phased out over time, until they disappear entirely.

Our model is thus consistent with the ability of government loan guarantees to increase the efficiency of credit markets by decreasing lending standards and interest rates. Examples of such loan guarantees in the US include mortgage markets (e.g. the FHA), student loans, and credit for international trade. All these loan guarantees require that the borrower meet eligibility requirements and/or condition rates on readily-available, verifiable information. Also notably, the retraction of a loan guarantee arguably characterized the start of the US financial crisis of 2008-2009. By allowing Lehman Brothers to fail, the US government retracted an implicit guarantee of the short-term debt of large financial institutions (as exhibited in the resolution of Bear Stearns in March 2018). In response, lenders (buyers of short-term commercial paper of financial institutions) tightened lending standards and interest rates rose and lending volumes declined. The government responded with a set of policies – explicit guarantees, implicit guarantees, and liquidity provision – that decreased the probability of losses and so reduced interest rate spreads and increased lending volumes to large financial institutions. [16]

Because this implementation entails a subsidy from the government to the banking sector, this policies reduces credit spreads. But a policymaker could implement the optimum instead by taxing future interest payments and refunding to all lenders (the equivalent of a loan guarantee funded from future bank interest earnings from loans of the same cohort). Such a policy would not entail a subsidy from the taxpayer so the burden clearly falls upon type-H borrowers. As a result, the policy increases credit spreads.

A second way to implement the constrained optimum would be to require that, whenever $x \in (\overline{x}^*, \overline{x})$, all loans made at each point in time when by placed into a common pool from which each lender receives a proportionate payout as the loans mature. Such mandated securitization

[16] A subset of specific examples from the U.S. government response to the financial crisis include the withdrawal of funds by retail investors from money market mutual funds (MMMFs) and the subsequent government Treasury guarantee program, the Public Private Investment Partnership that limited losses on newly purchased assets of unkown value (which fits our model if sold by large financial institutions to stay solvent), 'ring-fencing' of losses from take-overs of failing financial instituions, and a large set of actions (and verbal commitments) to keep large financial institutions from failing so that their short-term debt is almost guaranteed.

requires only that a loan origination is observable and contractible, not that a rejection is observable. Under this policy, no individual bank has the incentive to tighten lending standards when $x \in (\overline{x}^*, \overline{x})$ since they receive no benefits from placing a higher-quality loan into the securitized pool.

We assume that the private sector cannot observe which borrowers are rejected. But if the government could monitor banks and measure either lending standards or rejections (but, like the private sector, not observe the identity of the rejected agent), then another policy that implements the constrained optimum is to tax lending standards or rejections at a high enough rate to ensure normal lending standards (when $x \in (\overline{x}^*, \overline{x})$). This policy is the most direct: taxing the activity – tight lending standards – that has the negative externality. Interestingly, this policy increases equilibrium spreads even though no tax is collected because the profits from type-H borrowers have to cover the losses from lending to more type-L projects.

Note finally, that all such policies are not privately optimal without market-wide collusion. Thus, there is a role for government in a competitive market.

4.4 Limits to constrained efficiency

In practice, regulations or policies like government-funded loan subsidy or insurance programs are rarely undertaken for the entire financial sector, but instead usually apply only to certain types of institutions, such as traditional banks but not money market mutual funds or shadow banks for example.[17] We consider a situation where the government can only affect the lending decisions of some fraction $\eta \in [0, 1)$ of banks. We refer to these banks as *government-controlled banks*; they maybe owned by the government or they may simply be the subset of lending institutions not outside the government's legal or technological ability to control. We now ask what the optimal policy is under such limiting circumstances. For this section, we focus on the case without slow thawing or inactivity, i.e. $\theta = 1$, and further assume that government-owned banks always charge the same market interest rates as their private competitors.

To state the new planning problem for a given x_0, denote by $z^p(x)$ the optimal screening action of a private bank, that is, $z^p(x) = 0$. Then, the planner solves the same objective as before,

$$\max_{z_t} \int_0^\infty e^{-\rho t} \kappa \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt$$

subject to the same law of motion of x_t,

$$\dot{x}_t = (\kappa + \delta) (\lambda - x_t) - \kappa z_t \lambda (1 - x_t),$$

[17] For example, in the U.S., bank deposits are insured, but not MMMFs. In the financial crisis of 2008-2009, the government extended deposit guarantees from traditional banks to MMMFs, but did not extend the guarantees to other short-term debt markets. Similarly, the government took over the government-sponsored mortgage lending agencies and traditional banks but allowed private label securitizers and mortgage brokers to fail.

with the exception that z_t is now subject to an additional constrained,

$$z_t \in [(1-\eta)z^p(x_t), (1-\eta)z^p(x_t) + \eta\bar{z}], \tag{18}$$

rather than the entire interval $[0, \bar{z}]$. This is owed to the fact that only a fraction η of overall lending standards can be controlled by the government.

Constraint (18) significantly changes optimal policy. When $x \in (\bar{x}^*, \bar{x})$, normalizing lending standards for the subset of banks controlled by the government leads to slower improvement in credit conditions, because those banks not controlled by the government continue to apply tight lending standards and reduce $\dot{x}$. As we show in Appendix **??**, the optimal policy still takes a threshold form but the threshold $\bar{x}^*(\eta)$ depends on the share of government-owned banks η. For low levels of η, $\bar{x}^*(\eta) = \bar{x}$, the planner does not want to implement any other allocation than the competitive equilibrium. Only when the government controls a large enough share of the lending sector,

$$\eta > 1 - \frac{(\kappa+\delta)(\lambda - \bar{x})}{\kappa\bar{z}\lambda(1-\bar{x})}, \tag{19}$$

does the planner find it optimal to intervene in some region of the state space, $\bar{x}^*(\eta) < \bar{x}$.

Why is not optimal for the planner to intervene for low η? The planner's motivation for intervention is to shift away from convergence to the screening steady state to convergence to the pooling steady state. If, however, η is below the threshold in (19), the planner is not able to relax overall lending standards enough to induce the state x_t to move towards pooling for values of x_t close to the private threshold $\bar{x}$. Further above $\bar{x}$, achieving $\dot{x} > 0$ may be possible but credit conditions might improve too slowly to make it worthwhile. Thus, the government can lack the "firepower" to get to pooling and optimally choose not to intervene at all. Note that this policy, like a loan guarantee program, requires government funds to cover losses at the government-controlled banks. And, a government that intervenes when the privately-controlled banks are imposing tight lending standards and it is not optimal bears increasingly large losses, a point that may have relevance for the failure of the government sponsored mortgage agencies in the US in 2008.

5 A credit boom-bust cycle

One of the most salient features of the credit market is that it appears to come in "boom-bust cycles." In our model, booms and lending standards interact; neither is solely exogenously driving the other. We now demonstrate how our model can give rise to boom-bust dynamics. While such dynamics can be driven by exogenous movements in either lending standards or inflows to the pool of potential borrowers, in this section we study a credit boom shock driven by changes in the pool of borrowers.

The credit boom

We feed into the model a "market size shock" which allows a flow rate μ of new borrowers to enter the pool until time T. The new borrowers are assumed to have a lower fraction of type-H borrowers, $\underline{\lambda} < \lambda$, capturing the idea that these are not borrowers that usually believe they are able to get a loan.[18] Thus, the total size of the pool now evolves according to

$$\dot{N}_t = \mu 1_{\{t \leq T\}} - \delta(N_t - 1). \tag{20}$$

Our model in Section 2 involves a fixed pool size $N = 1$ and therefore needs to be amended to allow for dynamics in N. However, as we show in Appendix **??**, since the law of motion of N is entirely exogenous in the model, the model still applies to a "normalized" version of the credit market, where all absolute quantities (total volume of loans, total welfare, profit, etc) are to be thought of as normalized by N. The only adjustment that then needs to be made is that the fraction of type-H borrowers, x, now evolves according to

$$\dot{x}_t = \theta_t \kappa_{Lt}(1 - z_t)\lambda_t - \theta_t \kappa_{Ht}(1 - \lambda_t) + N_t^{-1}\left(\delta + \mu 1_{\{t \leq T\}}\right)(\lambda_t - x_t)$$

where λ_t is the average quality of new borrowers entering into the pool,

$$\lambda_t \equiv \lambda - \frac{\mu}{\mu + \delta} 1_{\{t \leq T\}}\left(\lambda - \underline{\lambda}\right).$$

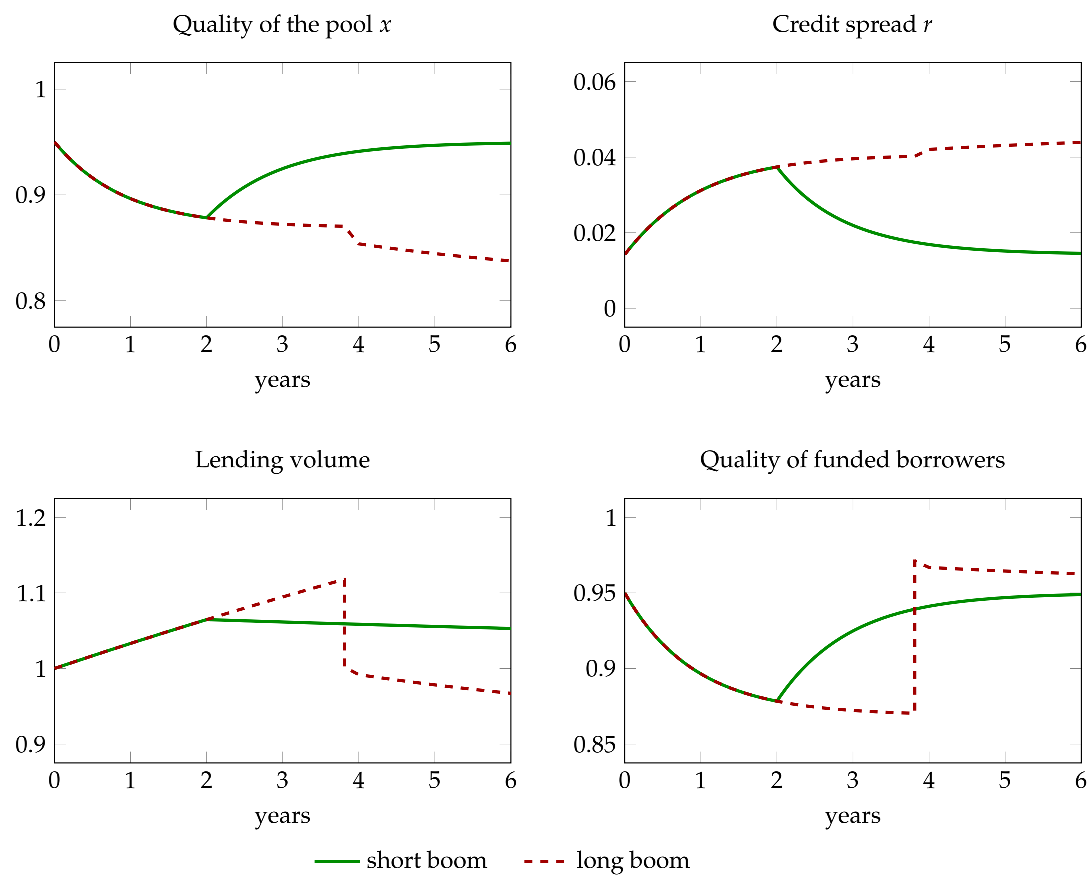
For the simulations in this section, we calibrate the model parameters as follows: TBD. Under this parameterization, the market turns out not to have a region with slow thawing.

The boom-bust cycle

We simulate the response to two different boom lengths, with $T_0 = 2$ years and $T_1 = 4$ years. The results are shown in Figure 10. As the solid green line shows, the short boom goes hand in hand with an increasing lending volume and a decline in the quality of borrowers, and ends in a soft decline ultimately converging back to the original steady state. Contrast this with the long boom (dashed red line). This boom ends in an abrupt decline in volume, an increase in lending standards, and a permanent transition away from the original steady state.

[18] We allow entering borrowers to know their average quality $\underline{\lambda}$.

Figure 10: The boom-bust cycle.



Note. This figure shows a credit market in response to two shocks: the green solid line represents a 2-year credit boom, the red dashed line a 4-year credit boom. A "credit boom" is modeled as an inflow of relatively less creditworthy borrowers. The transitions were computed using these parameters: TBD.

6 Capital constraints and lending standards

So far, our model has ignored bank balance sheets. However, as we illustrate next, there is an important interaction between banks' balance sheet capacity and lending standards.

6.1 Balance sheet constraints

We begin with a stylized model of bank balance sheet constraints. We assume a simple exogenous capital constraint, $\bar{V}_t$, which restricts the amount of lending proportionally by each bank, which grows over time so that the sector eventually becomes unconstrained. Let $V_t \equiv \theta_t\{\kappa_{Ht} + \kappa_{Lt}(1 - z_t)\}$ denote total bank lending. We impose

$$V_t \leq \bar{V}_t. \tag{21}$$

A benefit of the specification in (21) is that, because the evolution of the constraint is exogenous, the bank optimization problem remains static. Moreover, to simplify the analysis, we assume V_t grows sufficiently quickly such that, once banks are unconstrained, they remain unconstrained thereafter. We can thus separately analyze periods in which the constraint (21) binds and periods in which (21) is slack. Whenever the constraint stopped binding, the equilibrium is as described in Sections 2 and 3. When the constraint binds, each bank can charge the interest rate that lets average borrowers be indifferent as to whether to accept the loan or not. This implies that banks charge the interest rate that makes average borrowers indifferent between taking a loan or returning to the pool to wait for a future borrowing opportunity at a lower rate, which we denote by $\tilde{r}_t$:

$$r_t = \tilde{r}_t \tag{22}$$

where $\tilde{r}_t$ is determined by the indifference condition for average borrowers[19]

$$J_t^a = \lambda\left(r_H - \tilde{r}_t\right)$$

This equation follows straight from (1a). Indifference implies $\dot{J}_t^a = (\rho + \delta)J_t^a$. $\tilde{r}_t$ then evolves according to

$$\dot{\tilde{r}}_t = -(\rho + \delta)(r_H - \tilde{r}_t) \tag{23}$$

with the terminal condition that at the time T banks become unconstrained, $\tilde{r}_T = r(x_T)$ with $r(x)$ as in Proposition 11. Equation (22) then replaces the bank zero-profit condition in our definition of equilibrium. Since (23) implies that $\tilde{r}_t$ will always lie strictly below r_H, all borrowers accept loans in equilibrium and $\kappa_{Ht} = \lambda x_t$ and $\kappa_{Lt} = (1 - \lambda)x_t$.

[19] As in Section 3.3, we work here in the limit $u \to 0$ to simplify the exposition.

To understand how capital constraints affect lending standards, consider first the effect of lending standards on the profits from making one and only one loan which is given by

$$\Pi_t^{\text{single}}(z,r) \equiv \frac{x_t r + (1-x_t)(1-z)r_L}{x_t + (1-x_t)(1-z)} - \frac{\tilde{c}z}{x_t + (1-x_t)(1-z)} \tag{24}$$

The first term is the expected return on the loan, where $\frac{x_t}{x_t+(1-x_t)(1-z)}$ is the probability that the borrower is type-H and thus that the lender earns r on the loan, and $\frac{(1-x_t)(1-z)}{x_t+(1-x_t)(1-z)}$ is the probability that the borrower is type-L and the lender loses r_L. The second term is the expected cost of lending standards and exceeds $\tilde{c}$ because several potential borrowers may have to be checked before finding one that passes the lending standard. The bank chooses tight lending standards if $\Pi_t^{\text{single}}(\bar{z},r) > \Pi_t^{\text{single}}(0,r)$ which collapses to the condition

$$x_t(1-x_t)(r_t - r_L) > \tilde{c}. \tag{25}$$

Given the ability to make one loan, the bank prefers tight lending standards when costs of screening are low, when the information uncovered has high variance (when $x_t(1-x_t)$ is large), and when the difference in returns between lending to type-H and type-L borrowers is large. Most importantly, the propensity to impose tight lending standards is greater the higher r_t. Thus binding capital constrains—which endogenously lead to greater interest rates—also incentivize tighter lending standards.

The arguments of the previous paragraph apply when the capital constraint is tight in the sense that it binds even when lending standards are tight, and so lending volume is low. When the capital constraint is at an intermediate level so that it would not bind if $z = \bar{z}$ but would bind if $z = 0$, banks have three possible strategies for lending standards. First, they can impose tight standards, which gives profits

$$\kappa\left(x_t + (1-x_t)(1-\bar{z})\right)\Pi_t^{\text{single}}(\bar{z}, r_t).$$

Since the capital constraint does not bind when banks impose tight standards, the interest rate is equal that in the competitive economy. Thus tight lending standards can only occur for $x < \bar{x}$. Second, banks can impose normal lending standards in which case total profits are

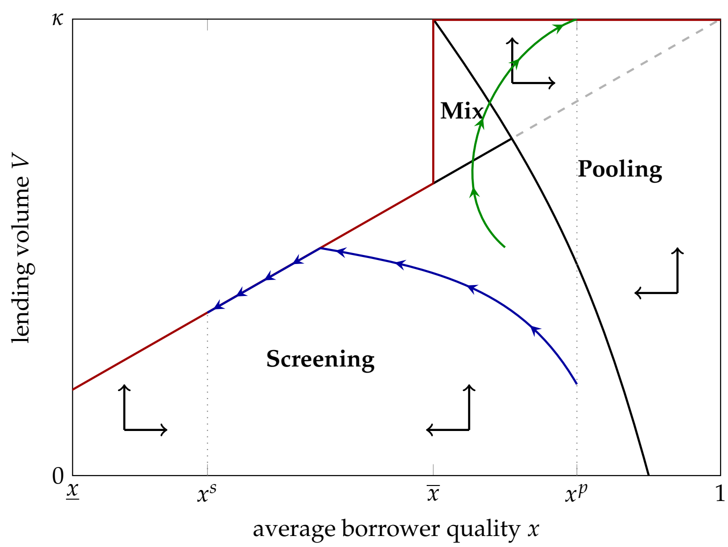
$$\overline{V}_t \Pi_t^{\text{single}}(0, r_t).$$

Finally, banks can set intermediate lending standards, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$, such that applying these standards to all potential borrowers leads banks to make $\overline{V}_t$ loans.[20] At this intermediate lending standard, bank profits are

$$\kappa x_t r + (\overline{V}_t - \kappa x_t) r_L - \frac{1-\overline{V}_t/\kappa}{1-x_t}\tilde{c}.$$

[20] This strategy has the same payoff as applying the lending standard $\bar{z}$ to a share $\frac{1-V_t/\kappa}{(1-x_t)\bar{z}}$ of borrowers and lending to the remaining borrowers at $z = 0$.

Figure 11: Optimal lending standards with capital constraints



Optimal lending standards are then characterized as follows:

Proposition 6 (Optimal lending standard with capital constraints)**.**

1. If $\overline{V}_t > \kappa$, the capital constraint does not bind: all current and future variables are identical to those in the market without constraints.

2. If $\overline{V}_t < \kappa x_t + \kappa(1 - x_t)(1 - \overline{z})$, the capital constraints binds: There exists a threshold

$$\bar{x}_t^C = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right) \geq \bar{x}$$

such that banks impose tight lending standards, $z = \bar{z}$, if $x_t < \bar{x}_t^C$, and impose normal lending standards, $z = 0$, if $x_t > \bar{x}_t^C$.

3. For $\kappa x_t + \kappa(1 - x_t)(1 - \overline{z}) < \overline{V}_t < \kappa$:

   - For $x_t > \bar{x}_t^C$ banks lend with normal lending standards and the constraint binds
   - For $x_t \in \left(\bar{x}, \bar{x}_t^C\right)$ banks impose intermediate lending standards, $z_t = \frac{1 - \overline{V}_t/\kappa}{1 - x_t}$, and the capital constraint binds.
   - For $x < \bar{x}_t$, banks impose tight lending standards and the constraint does not bind.

We illustrate Proposition 6 in Figure 11. The figure shows the volume banks lend at on the vertical axis and the quality of the pool on the horizontal. The red line at the top of the figure illustrates where the equilibrium typically evolves, without capital constraints: to the right of $\overline{x}$, the economy has volume $V_t = \kappa$ and moves towards the pooling steady state; to the left of $\overline{x}$, the

Figure 12: Balance sheet constraints and lending standards

TBD.

economy has volume $V_t = \kappa x_t + \kappa(1 - x_t)(1 - \overline{z})$ and moves towards the screening steady state. The latter is a transition along which the lending volume shrinks as more type-L borrowers accumulate in the pool.

With binding capital constraints, the economy finds itself in the region strictly below the red curve. As shown in Proposition 6, the threshold between screening and pooling is strictly above $\overline{x}$ in that region. This implies that even starting from the pooling steady state x^p, severe balance sheet constraints can lead to screening and thus to convergence to the screening steady state x^s.

6.2 Booms and busts in lending standards and balance sheets

We now run the following experiment. We start the economy just right of $\overline{x}$. After some time T, we assume that an unanticipated negative shock hits, leading to wide-spread defaults among type-L borrowers that have received loans in the past. This motivates a path for the lending volume constraint $\overline{V}_t$ that begins at $\overline{V}_T$ at time T and monotonically increases thereafter. Given that the longer normal lending standards continue, the more type-L loans accumulate on banks' balance sheets, it is natural that $\overline{V}_T$ declines in T.

Figure 12 solves for this transition for $T = 1$ and $T = 3$. When the "short boom", $T = 1$, bursts, the exposure to the aggregate shock $\overline{V}_T$ is not yet that large, so a relatively short period with screening follows, with subsequent recovery (as in the green line in Figure 11). When the "long boom", $T = 3$, bursts, however, the capital constraint $\overline{V}_T$ is tighter, leading to a sufficiently long period with screening, that ultimately pushes the pool quality below $\overline{x}$ and thus switches the steady state the economy ends up in to the screening steady state (as in the blue line in Figure 11).

These experiments ultimately emphasize two crucial insights on the interaction of balance sheet constraints and lending standards. First, tight balance sheet constraints lead banks to raise their lending standards. This is because tight lending standards are a way for banks to reduce current lending volumes by sacrificing partial repayments in the future. Thus, they represent a resource transfer from a bank's perspective from the future to the present and are particularly attractive when a bank is constrained and its implicit discount rate is high.

Second, the experiments illustrate that a richer version of our model, which links $\overline{V}_T$ directly to the accumulated pile of type-L loans on banks' balance sheets, would imply that the longer an economy is in the pooling region, the more constrained banks will be upon impact of the shock.

7 Markets institutions and the externality from lending standards

Our assumptions rule out two ways in which markets sometimes deal with the negative externality at the center of our model.

7.1 Screening with contracts

While not the focus of our model, tight lending standards lead to asymmetric information in the credit market. Some potential borrowers know that they have been rejected in the past, while banks are unaware which borrowers have been rejected. Our assumptions imply that banks cannot use contract terms like fees or covenants to screen these low-quality potential borrowers and instead lend to all potential borrowers at the same terms. In this subsection, we briefly discuss relaxing our assumptions so that banks can screen with contracts, and the implications of this alternative for the negative externality of screening and for observed credit markets.

Consider endowing potential borrowers with funds in excess of the private benefit of getting a loan u. In this case, an optimal lending contract can deter borrowers who have been previously screened and rejected, and thus an optimal contract can eliminate the negative externality from tight lending standards. The optimal contract takes the form of a down payment. Banks collect a fee from borrowers which is reimbursed (with interest) only if the loan is paid off when the project matures.[21] Because the fee exceeds the private benefit, only borrowers that have not been previously rejected apply for and take loans.

In this alternative model, under the optimal contract, lending standards are still dynamic strategic complements. Because an optimal contract does not fund low-type borrowers, tight lending standards worsen the pool quality in the future which makes tight lending standards optimal. Thus tight lending standards perpetuate themselves and the credit market can exhibit multiple steady states with different lending standards. However in this case, because previously-rejected borrowers are never evaluated for loans or funded, there is no negative externality associated with tight lending standards. As a result, the banks private cutoff $\bar{x}$ is also the socially optimal cutoff between normal and tight lending standards.

However, if employing screening contracts is costly – due to processing, costs of enforcement, and/or legal fees for example – then there is again a negative externality associated with tight lending standards and $\overline{x}^* < \overline{x}$. Further, there exists a cost of screening with contracts such that the steady-state with tight lending standards that also has screening with contracts has the same welfare as the steady-state of the original model, so that the steady-state equilibrium with normal lending standards is more efficient than the steady-state with tight lending standards. These results follow not because when x is low bad projects are evaluated and worse loans are funded, as in our original model, but rather because loan origination becomes technologically more costly at low

[21]Banks do not charge application fees to because a bank these cannot exist in equilibrium due to the following deviation. A bank can post a low interest rate, collect fees, and reject all borrowers with normal (unobservable and therefore non-contractable) lending standards and make profits.

value of x when there is sufficient asymmetric information in the market. While we do not address them in this paper, other model extensions can lead to other solutions to asymmetric information that look like other observed contract terms such bond covenants or loan terms that might partly address the negative externality of tight lending standards that we study.

7.2 Credit bureaus

In contrast to screening with loan terms, the negative externality of tight lending standards is eliminated if it is common knowledge that a given potential borrower has been rejected by a bank, as we noted in Section 4.2. Perhaps credit bureaus provide this information and eliminate the externality? In this subsection we describe what credit bureaus typically do, and how the information they provide is incomplete and so unlikely to eliminate completely the externality associated with lending standards.

Credit bureaus, as opposed to credit registries, track potential borrowers and provide information about them to potential lenders.[22] When a potential borrower approaches a lender that is a member of a credit bureau, the lender can perform a *credit check* before making a loan, which involves getting a *credit report* from the bureau. Credit reports provide information on potential borrowers including existing credit and payment histories. In addition, many credit bureaus keep track of information about past credit checks and include this information on credit reports. Table 1 describes credit reports for credit bureaus in different countries around the world.

In most countries, a bank that conducts a credit check can generally observe past credit checks and whether the potential borrower subsequently did or did not receive loans. The information in the bureaus tends to be available only to entities in the bureau's network, although some countries' bureaus sell the information to entities outside the credit market. In some countries like Japan and Germany, bureau members are required to report in exchange for access, but in other countries reporting is voluntary or only required by bureau members (second column of Table 1,). Most credit bureaus, like consumer bureaus in the US, delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring. Records of credit inquiries stay in credit report from 2 months in Taiwan to 24 months in the U.S. to 60 months in Ireland.

Importantly, however, none of the bureaus that we investigated report whether credit was denied or turned down by the potential borrower (final column of Table 1).[23] Further, credit bureaus

[22]Credit registries are more widespread than credit bureaus, but registries only track the history of outstanding credit and/or loan payments and delinquencies. In our model, and probably in reality, outstanding loans do not assist banks in discriminating among borrowers who have recently been rejected. Credit registries instead seem to serve the purpose of providing information to assist a bank in setting loan terms, such as loan amount and interest rate based on payments-to-income ratio and/or pre-existing liens on collateral. For firms, registries tend rate large firms, while for people, registries try to cover everyone.

[23]With the possible exception of Experian Italy. For example, Experian UK states "Here's what our role doesn't involve: - We aren't told which applications are successful or refused. - We don't know why you may have been refused credit." [Experian consumer]

Table 1: Data captured by credit bureaus

	Coverage? (consumers/firms)	Reporting required?	Credit checks: On report for ≤ months	Credit checks: Hard check labeled?	Credit checks: Who requested?	Credit checks: Purpose?	Rejections reported?
Advanced Economies							
Australia	Both		60	Yes	Yes	Yes	No
Canada	Both		72	Yes	Yes	No	No
European Union (AnaCredit)	Firms	By law	0	NA	No	No	No
France	Firms	By law	0	NA	No	No	No
Germany	Both	For access	12	Yes	No	No	No
Ireland	Both	By law	60	Yes	Yes	Yes	No
Italy	Both		6	Yes		Yes	No
Japan	Consumers	For access	6	Yes	Yes	Yes	No
Singapore	Both		24	Yes	Yes	Yes	No
South Korea	Both		0	NA	No	No	No
Taiwan	Both		3	Yes	Yes	Yes	No
United Kingdom	Both		24	Yes	Yes	Yes	No
United States	Both	Voluntary	24	Yes	Yes	Yes	No
Emerging Economies							
China	Both	By law	24	Yes	Yes	Yes	No
India	Both		24	Hard only	Yes	Yes	No
Malaysia	Both		12	Hard only	Yes	No	No

Blank cells are missing data.
Note: All information is from consumer credit reports and Bureau FAQs, except for EU and France, see Appendix for sources.

generally contain only rudimentary information about the initiator of previous credit checks, such as whether they were banks, mortgage brokers, utilities, etc., and some in some countries, such as South Korea and Malaysia, even this information is not recorded (fifth column). And only about half of the credit bureaus report the purpose of previous credit checks (sixth column), so that a credit card issuer for example does not know if a previous credit check was associated with an application for a credit card or a mortgage or a car loan.

Thus, credit bureaus provide only a noisy measure of whether a given borrower will or will not pass a given banks lending standard. A past credit check without a subsequent loan does not indicate that a given borrower failed a past lending standard. The borrower may have simply decide not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). As important in practice, lenders can evaluate potential borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for borrowers that pass the lending standard. Thus many applications are not recorded. This behavior is consistent with a feature of our model: it is not privately optimal for a borrower-lender pair to report a rejection since it destroys joint surplus.

Given the negative externality in our model, why do credit bureaus not track rejections? Our theoretical model suggests that bureaus do not track credit rejections because it is incentive incompatible for banks to report rejections.[24] In practice, the reasons that credit bureaus provide for not

[24]However, such behavior might be attainable in an extension of our model in which banks contribute information on rejections in exchange for access to the history of rejections for any potential borrower. For example, one could assume a low-cost monitoring technology for all banks so that non-reporting could be caught and then enforce the reporting and

tracking rejections are consistent with this interpretation. First, bureaus state that they want to avoid arbitrating arguments over rejections. Rejection is easy to hide (e.g. just offer unfavorable loan terms) and hard to verify (consistent with our assumptions). Second, bureaus store only verifiable information due to privacy and legal concerns. Credit checks are hard information, rejections are not. Every credit bureau lists data verification and correction measures on their website. These arguments are consistent with bureaus not being able to enforce the reporting of information that it is not privately optimal to report.

The primary implications of our model also apply in two real-world extensions which undermine the usefulness of credit bureau information on hard checks. First, credit bureaus track borrowers not their collateral. If tight lending standard evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending standards. Second, lenders may differ in the payout they receive from a given loan. For example, some banks may be better at recovery in delinquency, more able to hold or sell collateral, better able to observe profitability and adjust loan terms, or more able to provide small business advice and aid profitability.[25] In this case rejection by a one bank may not imply that the borrower is also a type-L borrower when borrowing from another bank. Thus, past rejections are not fully informative.

We conclude that mature credit markets in legal environments with low-cost enforcement mechanisms may exhibit various mechanisms for mitigating, but maybe not eliminating, the negative externality associated with tight lending standards.

8 Discussion and Extensions

In this section, we discuss the importance of a few of our assumptions and several extension of our model.

8.1 Borrower effort to raise project quality

An interesting extension could allow borrowers to exert effort to improve the quality of their projects. For instance, suppose when an average borrower is matched with a lender but before he is screened, the borrower privately chooses effort, where more effort increases the likelihood of being a type-H borrower and entails greater personal cost. Will the borrower have a bigger incentive to exert effort to become a type-H borrower when the market is in the screening range, $x_t < \bar{x}$, or in the pooling range, $x_t > \bar{x}$?

For $x_t > \bar{x}$ let $r_t^p(x_t)$ denote the borrowing rate when lenders are pooling. With lenders pooling, the difference between the type-H and type-L borrower payoffs is $r_H - r_t^p(x_t)$ (a type-H receives

the full-disclosure equilibrium through a standard folk theorem argument for infinitely repeated games.

[25] All credit bureaus that we investigated state that loan denials are bank-specifc and that each bank should evaluate applicants themselves rather than relying on evaluations by previous banks. For example, the German bureau SCHUFA states "Depending on your line of business and target group, it may make sense to use a tailored [credit] score focused to the needs of your company."

a cash payoff $r_H - r_t^p(x_t)$ and a private benefit u while a type-L receives only u). For $x_t < \bar{x}$ let $r_t^s(x_t)$ denote the borrowing rate when lenders are screening. With lenders screening, the difference between the type-H and type-L borrower payoffs is $r_H - r_t^s(x_t) + z(u - J_t^r)$ (a type-H receives a cash payoff $r_H - r_t^s(x_t)$ and a private benefit u while a type-L receives a private benefit u with probability $1 - z$ and a continuation payoff J_t^r with probability z). From the preceding analysis - before introducing borrower effort - we have $r_t^p(x') < r_t^s(x'')$, for $x' > \bar{x} > x''$. And since u is small, $u - J_t^r$ is small. Therefore, $r_H - r_t^p(x') > r_H - r_t^s(x'') + z(u - J_t^r)$ for $x' > \bar{x} > x''$ and a borrower's marginal incentive to exert effort is greater when the market is in the pooling range. The key to this result is that the borrowing rate is lower in the pooling range, $r_t^p(x') < r_t^s(x'')$. This logic implies that incorporating borrower effort into the model may speed up the convergence to the pooling steady state as well as to the screening steady state.

Moreover, within the pooling range, as x_t increases toward the pooling steady state, the borrowing rate $r_t^p(x_t)$ decreases. Therefore, the difference in payoffs between type-H and type-L borrowers, $r_H - r_t^p(x_t)$, is increasing. This implies that borrowers will exert more effort as x_t gets closer to the pooling steady state. In turn, over this range interest rates will drop faster when we incorporate borrower effort. Similarly, within the screening range, as x_t decreases toward the screening steady state, the borrowing rate $r_t^s(x_t)$ increases. Therefore, the difference in payoffs between type-H and type-L borrowers, $r_H - r_t^s(x_t) + z(u - J_t^r)$, is decreasing. This implies that borrowers will exert less effort as x_t decreases toward the screening steady state. In turn, over this range interest rates will rise faster when we incorporate borrower effort.

8.2 Non-essential assumptions and robustness

Next, we note that tractability motivates some of our specific assumptions about the inflow of new potential borrowers. Our assumptions imply that the pool of potential borrowers has a constant size, which implies that x is our sole state variable. One could instead for example assume that the inflow of new potential borrowers is constant, which would imply that the size of the pool would vary. While the choice of screening would still depend only on x, the dynamics of the market would in this alternative depend on the size of the pool as well as on x. While this would affect the dynamics of the market, this would not change many of the main lessons of the model, and in particular, the market would still exhibit two steady states (Corollary 1 would hold for a different range of parameter values) and the results on their relative efficiency, Corollary 3 derived in the Section 4.1, would still hold.

Turning to the banking sector, we have assumed that it is competitive so that banks make no profits in equilibrium. We conjecture that the qualitative features of the steady-states, dynamics and welfare results would all remain if banks shared the surplus of a match with a given potential borrower. The analysis is simplified by assuming that potential borrowers have all the bargaining power (or equivalently that banks compete by posting lending terms) as we have done.

Do our main results rely on our specific screening technology? For example, our screening technology never mistakes a type H borrower for a type L borrower. Such mistakes would imply that potential borrowers who are screened do not learn their type with certainty, and so there would be a distribution of beliefs among potential borrowers, with beliefs depending on the number of times a borrower had been screened and rejected for a loan. Such complexity would change the exact formula for $\dot{x}$. And it would complicate the analysis of the slow thawing region by potentially making possible regions with different speeds of slow thawing. Apart from this region however, since potential borrowers with different beliefs behave identically in the model (outside of any slow thawing region), our main results would remain intact.

Other changes to the screening technology are even less consequential. Our model can easily incorporate a screening technology is non-linear in cost. Concavity replicates our current results. Convexity would imply that rather than necessarily screening at a level of $\bar{z}$, banks might choose a lower level instead that equated marginal benefits of screening and marginal costs which would then necessarily depend on, and be increasing in, x. We have assumed that screening produces a binary signal, and it would be inconsequential to instead assume a continuous signal (banks would simply choose a cutoff value for their binary decision). Finally, if screening were correlated across banks, this would increase the strategic complementarity at the heart of our model since when one bank screens and rejects a potential borrower, it makes it easier for the next bank to detect that borrower as bad and so raises the private value to screening.

We have discussed the model as model of lending with a debt contract. But in fact, in our model there is an equity contract that delivers exactly the same payoffs to banks and borrowers of each type. This equivalence arises because of the model has only two types of investors so that all good. With more types, our model could become significantly more complex in general. While the exact degree of complexity would depend on how well the screening technology detected different types, the extensions we have considered have all involved more state variables, which raises the possibility of non-linear dynamics that can occur in such systems.

9 Concluding Remarks

The implications of our analysis all follow from the dynamic strategic complementarity associated with lending standards. If yesterday's lenders employed tight lending standards then it will be optimal for today's lenders to do the same. This is because yesterday's tight standards adversely affect the quality of today's pool of borrowers; that is, the pool will be cream skimmed. The adverse selection problem created by this cream skimming leads today's lenders to employ the same tight lending standards as yesterday's lenders. And so on for tomorrow's lenders. Alternatively, if yesterday's lenders employed normal lending standards then it will again be optimal for today's lenders to do the same. With normal lending standards, the pool of borrowers will evolve toward a pool comprised of average borrowers; and this is a pool (given our assumptions) that calls for normal lending standards.

Even if tight lending standards are not socially optimal, they may be the unique equilibrium outcome. For that case, we considered the possibility of government intervention that replaces an inefficient unique equilibrium with an alternative more efficient unique equilibrium; one with normal lending standards. Capitalizing on our dynamic model, we showed that while there are interventions that can put the market on the path to the socially efficient steady state outcome, the timing of the intervention is crucial for determining whether the intervention is value enhancing. If the intervention is delayed too long, it will not be value enhancing. An intervention considered involves a temporary subsidized (partial) loan guarantee program. Temporarily offering loan guarantees can eliminate lenders' incentives to employ tough lending standards. This in turn leads to an improvement of the borrower pool and then subsequent lenders will have the incentive to employ normal lending standards even without loan guarantees.

We considered the possibility that the borrower pool temporarily expands through the addition of below-average borrowers. By assumption, even though they are below-average borrowers, they are still good enough to be profitably funded with normal lending standards. The expansion of the borrower pool represents a boom that converts previously unfundable borrowers into now fundable borrowers. This boom may result in an immediate increase in lending volume followed by a bust, where the volume of lending collapses. Underlying the boom is a reliance on normal lending standards and underlying the bust is an endogenous switch to tight lending standards.

Our paper opens up several avenues for future research. For example, the government intervention that is analyzed effectively assumes that the negative shock that put the market on the path to the inefficient steady state is expected to never recur. But suppose it might recur. Is government intervention still optimal? Addressing this question requires a specification of the social cost of a government giveaway, e.g., a subsidized loan guarantee. With that we expect that it can be shown that if the likelihood that the shock recurs is low enough, then government intervention will be value enhancing. This is because with a low probability of a negative shock, the cost of an intervention can be amortized over a longer period of time.

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A Proofs and derivations

A.1 Within-period lending game

A.2 Steady state equilibria: Proof of Proposition 1

The three pairs (x, z) mentioned in Proposition 1 are solutions to (8) and (9) if $\lambda > \overline{x}$, $x^s < \overline{x}$, and $\frac{\lambda-\overline{x}}{\lambda-\lambda\overline{x}}\left(1+\delta\kappa^{-1}\right) < \overline{z}$. The first two of these hold by Assumption 2 and the third is a straight consequence of the second.

We claim that the three pairs indeed constitute equilibria, with $\theta = 1$, $\varphi^a = \varphi^r = 1$ and with R pinned down by Proposition 3. To prove this, first note that the law of motion (6) as well as the bank's maximization problem (4) are satisfied due to (8) and (9). The zero profit condition (5) pins down the interest rate (see our proof to Proposition 3). Finally, in any steady state the average borrower strictly prefers a loan today, that is,

$$\lambda\left(R_H - R + u\right) + (1-\lambda)(1-z)u + (1-\lambda)zJ^r - J^a > 0,$$

and since $R \leq R_H$ (which holds since $x^s \geq \underline{x}$ with $\underline{x}$ as in (10) due to Assumption 2) we have that $\theta = 1$ and $\varphi^a = \varphi^r = 1$.

A.3 Proof of Corollary 1

The flow of projects being funded in the pooling steady state is κ, compared to $\kappa x^s + \kappa(1-x^s)(1-\overline{z})$ in the screening steady state. The credit spread result follows directly from Proposition 3 and the fact that $R(x)$ is strictly decreasing in x. The equilibrium default rate is given by

$$\frac{\kappa(1-x)(1-z)}{\kappa(1-x)(1-z)+\kappa x} = \left(1+\frac{x}{(1-x)(1-z)}\right)^{-1}$$

which can further be simplified to

$$(1-\lambda)\left(1+\frac{\lambda z\delta\kappa^{-1}}{(1+\delta\kappa^{-1})(1-z)}\right)^{-1}.$$

Thus, when $\delta = 0$, the equilibrium default rate is always equal to $1-\lambda$, irrespective of the steady state.

A.4 Proof of Proposition 2

Let $x_0 \in [\underline{x}, \overline{x})$ ($x_0 \in (\overline{x}, \lambda]$). In that case, $z = \overline{z}$ ($z = 0$) is the optimal bank strategy (see (4)), and therefore the law of motion of x, (6), necessarily describes the unique equilibrium dynamics of x. By Assumption 6, $\theta_t = 1$ and therefore also $\varphi^a_t = 1 = \varphi^r_t$ (due to (7)).

The case $x_0 = \overline{x}$ is straightforward as $\overline{x}$ is a steady state.

Finally, if $x < \underline{x}$, $R(x) = \infty$, which is why the only possible equilibrium involves $\theta_t = 0$. In that region, therefore, the pool improves according to $\dot{x}_t = \delta(\lambda - x_t)$, until x hits $\underline{x}$, in which case the law of motion switches to be the same as the one for an initial quality $x_0 \in [\underline{x}, \overline{x})$.

A.5 Proof of Proposition 3

The zero profit condition (5) implies that

$$\Pi(R) = \kappa_H(R-1) + \kappa_L(1-z)(R_L-1) - (\kappa_H + \kappa_L)\tilde{c}z = 0.$$

Reformulating this we obtain

$$\kappa x(R-1)/(R_L-1) + \kappa(1-x)(1-z) + \kappa cz = 0$$

$$R = 1 + (1-R_L)\frac{cz + (1-x)(1-z)}{x}$$

which proves Proposition 3.

A.6 Proof of Proposition 4

Define $\theta(x)$ as in (12) and define $\hat{x}$ implicitly as the unique value of $x < \lambda$ with $\theta(x) = 1$. Such a value exists since $\theta(x)$ is strictly increasing and continuous in x with $\theta(0) = -\delta\kappa^{-1} < 0$ and $\lim_{x\to\lambda}\theta(x) = \infty$.

Assume $\hat{x} > \overline{x}$. Conjecture for any $x_0 \in [\overline{x}, \hat{x})$ that the equilibrium is one with $\theta_t = \theta(x_t)$. To verify the conjecture, we need to show that average borrowers are indifferent between taking a loan and waiting. Assuming $u \to 0$ in (1a), this is equivalent to

$$J_t^a = \lambda(R_H - R(x_t))$$

with

$$rJ_t^a = \dot{J}_t^a - \delta J_t^a.$$

Putting the two together, we obtain (13),

$$-\lambda R'(x)\dot{x} = (r+\delta)\lambda\left(R_H - R(x)\right).$$

The law of motion for x with $\theta < 1$ is $\dot{x}_t = (\kappa\theta + \delta)(\lambda - x)$, which, together with (13) yields (12) and therefore confirms that average borrowers are, by construction, precisely indifferent.

A.7 Proof of Corollary 3

By Assumption 2, $c \geq 1-\lambda$. Therefore, welfare in the screening steady state is bounded above,[26]

$$W^s = x^s r^\Delta - (1-\bar{z})\,(1-x^s) - c\bar{z} \leq x^s\left(r^\Delta + 1 - \bar{z}\right) - (1-\bar{z}) - (1-\lambda)\bar{z} = x^s\left(r^\Delta + 1 - \bar{z}\right) - (1-\lambda\bar{z})$$

Welfare in the pooling steady state is $W^p = \lambda(r^\Delta + 1) - 1$. Observe that W^s increases in x^s, so W^s can only ever be above W^p if x^s is as large as possible. Clearly, given the formula for x^s, x^s is largest as a function of δ if $\delta = \infty$ where $x^s = \lambda$. In that case, we find

$$W^s \leq x^s\left(r^\Delta + 1 - \bar{z}\right) - (1-\lambda\bar{z}) < \lambda\left(r^\Delta + 1 - \bar{z}\right) - (1-\lambda\bar{z}) = \lambda\left(r^\Delta + 1\right) - 1 = W^p$$

Therefore, welfare of the pooling steady state always dominates that of the screening steady state.

A.8 Proof of Proposition 5

We prove Proposition 5 in two steps. First, we determine the efficient screening policy $z^*(x)$ conditional on banks operating. Then we determine the optimal behavior for banks to operate $\theta^*(x)$.

A.8.1 Optimal screening policy $z^*(x)$

To do so, let $V(x,z)$ denote the present value of welfare if the current state of the market is x and the screening policy is z from hereafter, that is,

$$V(x,z) \equiv \frac{rx + \alpha^z x^z}{r + \alpha^z}(R_H - 1) + (1-z)\left(1 - \frac{rx + \alpha^z x^z}{r + \alpha^z}\right)(R_L - 1) - \tilde{c}z. \tag{26}$$

where $\alpha^z \equiv \kappa + \delta - \lambda\kappa z$ and $x^z \equiv \lambda - \lambda\frac{(1-\lambda)z}{(1-\lambda z)+\delta\kappa^{-1}}$. Also, denote by

$$v(x,z) \equiv r\left\{x(R_H - 1) + (1-z)(1-x)(R_L - 1) - \tilde{c}z\right\} \tag{27}$$

the flow value of policy z at state x. Finally, we call

$$d(x,z) \equiv \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{28}$$

the derivative of x at state x under policy z (see the law of motion in (6)). Observe that

$$rV(x,z) = v(x,z) + V_x(x,z)d(x,z) \tag{29}$$

as well as

$$d(x^s, \bar{z}) = 0 \;\; d(x^p, 0) = 0. \tag{30}$$

[26]We define all welfare expressions here as multiples of κ, for expositional clarity. κ multiplies both W^s and W^p equally.

We first prove the following helpful lemma.

Lemma 1. *We have:*

1. If $\lambda\kappa\rho \geq r+\kappa+\delta$, pooling is strictly optimal for any state x, i.e. $z^*(x)=0$.
2. If $\lambda\kappa\rho < r+\kappa+\delta$, $V(x,z)$ has negative cross-partials, $V_{xz}<0$.
3. If $\lambda\kappa\rho < r+\kappa+\delta$ and $V(x,0)>V(x,z_1)$ for some $z_1>0$, then also $V(x,0)>V(x,z_2)$ for any $z_2\in(0,z_1)$.

Proof. Assume $\lambda\kappa\rho \geq r+\kappa+\delta$. Suppose pooling were not strictly optimal for every state x. First, if $d(x,z^*(x))$ is ever non-negative for some $x<\lambda$, there must be a steady state at some $x^0\in[0,\lambda)$ with some $z^*(x^0)>0$. This cannot be optimal since

$$V(x^0,z^0) < V(x^0,0)$$

is equivalent to (after a few lines of algebra)

$$-\left(r+(1-\lambda)\alpha^p - rx^0\right)\left(\rho\kappa\lambda-(r+\kappa+\delta)\right) < (r+\alpha^s)(r+\alpha^p)c$$

which is true since the left hand side is negative. Second, assume $d(x,z^*(x))$ is positive everywhere. Then, $x^p=\lambda$ is still the unique steady state. Let $\mathbf{V}(x)$ the optimal value function. It has to hold that

$$r\mathbf{V}(x) = v(x,z^*(x)) + \mathbf{V}'(x)d(x,z^*(x)). \tag{31}$$

Rearranging,

$$\mathbf{V}'(x) = \frac{r\mathbf{V}(x)-v(x,z^*(x))}{d(x,z^*(x))} \equiv \mathbf{F}(\mathbf{V}(x),x).$$

Compare this to the ODE describing the value of pooling,

$$V_x(x,0) = \frac{rV(x,0)-v(x,0)}{d(x,0)} = \mathbf{F}^0(V(x,0),x)$$

Observe that $\mathbf{F}(V,x) > \mathbf{F}^0(V,x)$ for any x for which $z^*(x)>0$.[27] Since $\mathbf{V}(x^p)=V(x^p,0)$, it must be that $\mathbf{V}(x)<V(x,0)$ for x sufficiently small. This contradicts our assumption that $\mathbf{V}(x)$ is the optimal value function. Thus, pooling is optimal for every state.

Assume $\lambda\kappa\rho < r+\kappa+\delta$. Simple algebra based on (26) implies that

$$V_x = \frac{r}{r+\alpha^z}(R_H-1) + (1-z)\frac{r}{r+\alpha^z}(1-R_L) > 0$$

and

$$V_{xz} = r\frac{\lambda\kappa\rho-(r+\kappa+\delta)}{(r+\alpha^z)^2(1-R_L)} < 0.$$

[27]Note that $\mathbf{V}'(x)>0$ by a simple envelope argument.

$\square$

With this result in mind, we assume in the following that $\lambda\kappa\rho < r + \kappa + \delta$ and characterize $z^*(x)$.

Lemma 2. *Assume $\lambda\kappa\rho < r + \kappa + \delta$. The efficient screening policy $z^*(x)$ is to screen if $x < \overline{x}^*$ and to pool if $x > \overline{x}^*$, where*

$$V(\overline{x}^*, 0) = V(\overline{x}^*, \overline{z}) \tag{32}$$

as long as the solution to that equation is greater or equal to x^s. Otherwise, $\overline{x}^*$ is determined by

$$v_z(\overline{x}^*, 0) + V_x(\overline{x}^*, 0)d_z(\overline{x}^*, 0) = 0. \tag{33}$$

Proof. First, notice that $\overline{x}^*$ is indeed well-defined, in that if the solution to (32) is x^s, then (33) is also solved by x^s. Assume

$$V(x^s, 0) = V(x^s, \overline{z}).$$

Combining (29) and (30), we can rewrite $V(x^s, 0)$ and $V(x^s, \overline{z})$ and obtain

$$v(x^s, 0) + V_x(x^s, 0)d(x^s, 0) = v(x^s, \overline{z}) + V_x(x^s, \overline{z})d(x^s, \overline{z}).$$

Since $d(x^s, \overline{z}) = 0$, this can be combined into

$$v(x^s, \overline{z}) - v(x^s, 0) + V_x(x^s, 0)\left(d(x^s, \overline{z}) - d(x^s, 0)\right) = 0 \tag{34}$$

which is equivalent to (33) as v and d are linear in z. Moreover, going these steps backwards, if $\overline{x}^* < x^s$, then (34) holds with inequality and therefore

$$V(x^s, 0) > V(x^s, \overline{z}). \tag{35}$$

Now we proceed to our main argument, a proof by contradiction. We distinguish four possible cases.

Case 1: There exists $x > \overline{x}^*$ with $x \geq x^s$ where screening is optimal. If true, this would require there to be at least one point $x^0 \in [\overline{x}^*, \lambda)$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in (0, \overline{z}]$ such that $d(x^0, z^0) = 0$) over pooling. In math,

$$V(x^0, z^0) > V(x^0, 0).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, 0)$ and $V(x^s, z^0) > V(x^s, 0)$, which, by point 3 in Lemma 1, is contradicting either (32) or (35).

Case 2: There exists $x < \overline{x}^*$ with $x \geq x^s$ where pooling is optimal. If true, this would require there to be at least one point $x^0 \in (x^s, \overline{x}^*]$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in [0, \bar{z})$ such that $d(x^0, z^0) = 0$) over screening. In math,

$$V(x^0, z^0) > V(x^0, \bar{z}).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\bar{x}^*, z^0) > V(\bar{x}^*, \bar{z})$, which by point 3 in Lemma 1, contradicts (32).

Case 3: There exists $x > \bar{x}^*$ with $x \leq x^s$ where screening is optimal. If true, this would require there to be at least one point $x^0 \in [\bar{x}^*, x^s]$ where the planner strictly prefers to screen with some intensity $z^0 > 0$ in the current instant while pooling is chosen thereafter. That is,

$$v(x^0, z^0) + V_x(x^0, 0)d(x^0, z^0) > v(x^0, 0) + V_x(x^0, 0)d(x^0, 0).$$

Due to linearity of this equation, it also has to hold with $z^0 = \bar{z}$, and therefore also expressed as derivative,

$$v_z(x^0, 0) + V_x(x^0, 0)d_z(x^0, 0) > 0. \tag{36}$$

Since this is a linear equation in x^0, to be consistent with (33), it must be that (36) in fact holds for any $x^0 > \bar{x}^*$, including $x^0 = x^p = \lambda$. In that case, however, (36) simplifies to $v_z(x^p, 0) + V_x(x^p, 0)d_z(x^p, 0) > 0$, which is false, since $V_x(x, 0) > 0$, $d_z(x, 0) < 0$ and $v_z(x^p, 0) = -\kappa(1 - R_L)(c - (1 - \lambda)) < 0$ by Assumption 2.

Case 4: There exists $x < \bar{x}^* \leq x^s$ where pooling is optimal. Let $\mathbf{V}(x)$ be our conjectured value function left of $\bar{x}^*$. By design, $\mathbf{V}(x)$ solves

$$r\mathbf{V}(x) = v(x, \bar{z}) + \mathbf{V}'(x)d(x, \bar{z})$$

where $d(x, \bar{z}) = \alpha^{\bar{z}}(x^s - x)$ and $\mathbf{V}'(x)$ solves

$$(r + \alpha^{\bar{z}})\mathbf{V}'(x) = v_x(x, \bar{z}) + \mathbf{V}''(x)d(x, \bar{z}).$$

This ODE can be solved explicitly, giving[28]

$$\mathbf{V}'(x) = r(1 - R_L)\left(\frac{\rho}{r + \alpha^p} - \frac{\rho - \bar{z}}{r + \alpha^{\bar{z}}}\right)\left(\frac{x^s - x}{x^s - \bar{x}^*}\right)^{-\beta} + r(1 - R_L)\frac{\rho - \bar{z}}{r + \alpha^{\bar{z}}}$$

where $\beta = 1 + \frac{r}{\alpha^{\bar{z}}}$. The coefficient on the first term is positive, since we assumed $\rho\lambda\kappa < r + \kappa + \delta$. Thus, $\mathbf{V}'(x)$ is bounded above by

$$\mathbf{V}'(x) \leq \mathbf{V}'(\bar{x}^*) = r(1 - R_L)\frac{\rho}{r + \alpha^p}. \tag{37}$$

[28]Note that $v_x(x, \bar{z})$ is a constant in x.

Could it ever be that the planner prefers pooling in this region? If so, we would have an $x < \overline{x}^*$ with

$$v_z(x,0) + \mathbf{V}'(x)d_z(x,0) < 0$$

which due to (37) and the fact that $d_z(x,0) = -\kappa\lambda(1-x) < 0$ implies that

$$v_z(x,0) + \mathbf{V}'(\overline{x}^*)d_z(x,0) < 0.$$

Using the expressions in (27) and (28) we then see that this cannot hold as the left hand side is zero at $\overline{x}^*$ (by definition), and has a negative slope throughout,

$$v_{xz} + \mathbf{V}'(\overline{x}^*)d_{xz} = r(1-R_L)\left[-1 + \frac{\rho\lambda\kappa}{r+\alpha^p}\right] < 0$$

where again we used $\rho\lambda\kappa < r + \kappa + \delta$. This is a contradiction: there cannot be an $x < \overline{x}^*$ where pooling is optimal. □

A.8.2 Optimal bank operation policy $\theta^*(x)$

Next we focus on the optimal policy $\theta^*(x)$ for banks to operate. We prove the following result.

Lemma 3. *If it is strictly optimal to have banks operate at $\overline{x}^*$, the optimal policy describing when banks operate is bang-bang, that is,*

$$\theta^*(x) = \begin{cases} 0 & x < \underline{x}^* \\ 1 & x > \underline{x}^* \end{cases} \tag{38}$$

The threshold $\underline{x}^*$ is the supremum of all $x \in [0, \lambda]$ that solve

$$v(x, z^*(x)) + \mathbf{V}'(x)\left(\kappa\left(\lambda - x\right) - \kappa(1-x)z^*(x)\lambda\right) < 0 \tag{39}$$

where $\mathbf{V}(x)$ is the value function associated with the optimal screening policy $z^*(x)$.

Proof. Let $\underline{x}^*$ be defined as in (39) and let $\mathbf{V}(x)$ be the value function conditional on banks operating with screening policy $z^*(x)$. If it is optimal for the planner to follow the bang-bang policy (38), then its value function for $x \geq \underline{x}^*$ is given by $\mathbf{V}(x)$, whereas for $x < \underline{x}^*$ the value function solves

$$r\mathbf{V}(x) = \mathbf{V}'(x)\delta(\lambda - x)$$

which can be solved to express the marginal value in state x as

$$\mathbf{V}'(x) = \mathbf{V}'(\underline{x}^*)\left(\frac{\lambda - x}{\lambda - \underline{x}^*}\right)^{-1-r/\delta}.$$

Observe that this is increasing in x. To prove that the bang-bang policy (38) is indeed optimal, we need to prove that

$$\max_{z\in[0,\bar{z}]} v(x,z) + \mathbf{V}'(x)d(x,z,1) \leq \max_{z\in[0,\bar{z}]} \mathbf{V}'(x)d(x,z,0) \tag{40}$$

for $x < \underline{x}^*$, where

$$d(x,z,\theta) \equiv \theta\kappa(1-x)(1-z)\lambda - \theta\kappa x(1-\lambda) + \delta(\lambda - x)$$

is the speed at which the pool improves given (x,z,θ). Simplifying (40), we obtain

$$\max_{z\in[0,\bar{z}]} v(x,z) + \mathbf{V}'(x)\left[\kappa\lambda(1-z) - \kappa x(1-z\lambda)\right] \leq 0.$$

The left hand side of this inequality has a negative cross-partial in (x,z), since $v_{xz} < 0$ and $\mathbf{V}'(x)(1-x) \propto (\lambda-x)^{-r/\delta}\frac{1-x}{\lambda-x}$ increases in x. Thus, given that $z=\bar{z}$ is optimal for $x=\underline{x}^*$, it is also optimal for any $x<\underline{x}^*$.

The problem then reduces from (40) to showing that for $x<\underline{x}^*$

$$F(\lambda-x) \equiv v(x,\bar{z}) + \mathbf{V}'(x)\left[\kappa\lambda(1-\bar{z}) - \kappa x(1-\bar{z}\lambda)\right] < 0. \tag{41}$$

To see this, we first show that $F(y)$ is quasi-concave (only has a single local maximum) and therefore can at most have two roots. $F(y)$ is of the form

$$F(y) = -F_0 y + F_1 y^{-\alpha-1}(y-y_0) + const$$

where $\alpha = r/\delta > 0$, $F_0 = r(R_H-1) + r(1-\bar{z})(1-R_L) > 0$, $F_1 = \kappa(1-\lambda\bar{z})\mathbf{V}'(\underline{x}^*)(\lambda-\underline{x}^*)^{1+\alpha} > 0$, $y_0 = \lambda - x^s > 0$. F can only ever have a single local maximum as long as these parameters are positive:

$$F'(y) = 0 \quad \Leftrightarrow \quad y^{-\alpha-2}\left[(1+\alpha)y_0 - \alpha y\right] = F_1/F_0$$

The left hand side of this equation is strictly decreasing for $y \in (0,(1+\alpha)y_0/\alpha)$ with range $(0,\infty)$ and thus admits a unique solution for any $F_1/F_0 > 0$. This establishes that $F(y)$ is quasi-concave.

Since $F(y)$ is quasi-concave, it admits at most two roots, $y_1 < y_2$, in between which $F(y)$ is positive, and negative outside of $[y_1,y_2]$. Root y_2 must correspond to $\lambda-\underline{x}^*$: if y_1 were to correspond to $\lambda-\underline{x}^*$, $\underline{x}^*$ would not be the supremum of x with $F(\lambda-x)<0$ since for any $\epsilon>0$ small enough, $F(\lambda-(\underline{x}^*-\epsilon))>0$. But if $y_2=\lambda-\underline{x}^*$, then $F(\lambda-x)<0$ for any $x<\underline{x}^*$, which proves (41). □

A.9 Proof of Proposition 6

There exists $\bar{x}^C \geq \bar{x}$ such that banks screen if $x < \bar{x}^C$ and $\bar{V}_t < x_t + (1 - x_t)(1 - \bar{z})$. Banks prefer to lend after valuation at rate $z > 0$ iff

$$\frac{x_t r_t + (1 - x_t)(1 - z) r_L}{1 - (1 - x_t) z} - \frac{\tilde{c} z}{1 - (1 - x_t) z} > x_t r_t + (1 - x_t) r_L$$

which simplifies to

$$0 > (r_t - r_L) x_t^2 - (r_t - r_L) x + \tilde{c}.$$

The roots are

$$x_t^C = \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right] \tag{42}$$

where we denote the larger one by

$$\overline{x}_t^C = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right].$$

As $\tilde{c} \to 0$, the two roots (42) converge to 0 and 1, meaning tight standards everywhere.

Proof that $\bar{x}_t^C \geq \bar{x}$. We first prove a useful property. The interest rate at $\overline{x}$ in the unconstrained equilibrium is given by

$$r(\overline{x}) = (-r_L)\frac{1 - \overline{x}}{\overline{x}}.$$

With this interest rate we find a cutoff $\overline{x}^C$ of

$$\overline{x}^C(r = r(\overline{x})) = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{c}}{r(\overline{x}) - r_L}}\right] = 1 - c = \overline{x}. \tag{43}$$

Next we prove that $\bar{x}_t^C \geq \bar{x}$ more generally. We proceed by contradiction. Suppose there existed an $x_0 < \overline{x}$ and a continuous unbounded increasing function $\overline{V}_t$ starting with $\overline{V}_0 < \kappa x_0 + \kappa(1 - x_0(1 - \overline{z})$ such that pooling is optimal, that is, $x_0 > \overline{x}_t^C$. Observe that since $\tilde{r}_t$ declines monotonically along the transition and x_t increases, this also implies that $x_t > \overline{x}_t^C$ at any future time t. Thus, pooling remains optimal.

Let T be the time at which banks become unconstrained. It has to be the case that $x_T > \overline{x}$. If not, we would have a contradiction since $\overline{x}_T^C < x_T$ per the discussion above, but at the same time

$$\overline{x}_T^C = \overline{x}^C(r = r(x_T)) > \overline{x}^C(r = r(\overline{x})) = \overline{x}$$

following from (43) and the fact that $r(\cdot)$ is monotone (Proposition 11). Thus, $x_T > \overline{x}$.

Now compare the path x_t that ultimately reaches x_T to a different path that reaches x_T. This second path has unconstrained banks and begins an ϵ to the right of $\overline{x}$. On its way to the pooling steady state, it crosses x_T. Observe that this second path has a slower decline in the interest rate r_t as average borrowers are not indifferent along the path, $0 > \dot{r}_t > \dot{\tilde{r}}_t$. Moreover, the transition along the second path is faster as banks are unconstrained. Together, this implies that the initial interest rate $\tilde{r}_0$ along the first path must have been greater than the initial interest rate $r(\overline{x})$ along the second path. But this means that

$$\overline{x}^C(r = \tilde{r}_0) > \overline{x}^C(r = r(\overline{x})) = \overline{x}$$

which is a contradiction to our assumptions $x_0 < \overline{x}$ and $x_0 > \overline{x}^C(r = \tilde{r}_0) = \overline{x}^C_0$.

The case $\kappa x_t + \kappa(1 - \bar{z})(1 - x_t) < \overline{V}_t < \kappa$. Consider first $x_t > \bar{x}$. If banks were to impose tight lending standards, $z_t = \bar{z}$, then they would be unconstrained and the interest rate equals the interest rate in the competitive equilibrium. At the competitive interest rate, banks prefer tight lending standards only if $x_t \leq \bar{x}$. Thus for $x_t > \bar{x}$, it can only be that $z_t < \bar{z}$ and we need only consider whether normal lending standards dominate intermediate ones. The largest z_t that does not make banks unconstrained is $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$.

Given our derivation in Section 6 and the fact that $\bar{z}$ did not enter (25), pooling is preferable if and only if $x_t > \overline{x}^C_t$. For $x_t < \overline{x}^C_t$, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$ is the optimum. For $x_t \leq \overline{x}$, banks can screen without binding constraints. Given $x_t < x^C_t$, this is their optimal policy.

B Model with non-constant pool size

For this section, we assume that the pool size is not constant. We demonstrate that this economy gives rise to the exact same steady states and the exact same welfare predictions.[29]

B.1 Equilibria

Without a constant pool size, there are two state variables: m_H, the number of type-H borrowers in the pool and m_L, the number of type-L borrowers in the pool. The laws of motion of the state variables are given by

$$\dot{m}_H = \delta\lambda - \delta m_H - \kappa m_H \tag{44}$$

$$\dot{m}_L = \delta(1 - \lambda) - \delta m_L - \kappa(1 - z_t)m_L \tag{45}$$

The first term in both laws of motion stems from the constant inflow of $\delta\lambda$ type-H borrowers and $\delta(1 - \lambda)$ type-L borrowers. The second term captures the constant exit probability of borrowers in the pool. The final term is the flow rate of borrowers who receive a loan.

[29] For simplicity, we focus on the case of active banks, $\theta_t = 1$, throughout this section.

Observe that the first equation is independent of z_t. We can thus treat m_H as if it was at its steady state forever,

$$m_H = m_H^* = \frac{\delta\lambda}{\delta + \kappa} \tag{46}$$

We continue to denote the share of type-H borrowers by $x_t \equiv m_H/(m_H + m_L)$. Given (46), the law of motion of x_t can then be shown to be given by

$$\frac{\dot{x}}{x/\lambda} = \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{47}$$

The right hand side of (47) is identical to the one of (6) after substituting out κ_{Ht} and κ_{Lt} and setting $\theta_t = 1$. Thus, the only difference between the constant-pool-size model and the one in this section is that the speed in this one is altered by a factor x/λ, on the left hand side of (47). In particular, all results on steady states and their properties in Sections 3 carry over one-for-one to the model in this section.[30]

B.2 Welfare

The social planning problem now becomes

$$\max_{z_t \in [0,\bar{z}]} \int_0^\infty e^{-\rho t}\kappa \left\{ m_H^* r_H + (1-z_t) m_L r_L - \tilde{c} z_t (m_L + m_H) \right\} dt$$

subject to the law of motion for m_L, (45). One can show that it has the exact same properties as the planning problem in Section 4. Relative to the privately optimal threshold $\overline{x} = 1 - c$, which corresponds to

$$\overline{m}_L = \frac{\lambda}{1 + \kappa\delta^{-1}} \frac{c}{1-c}$$

there exists a socially optimal threshold $\overline{x}^* \equiv \frac{m_H^*}{m_H^* + \overline{m}_L^*}$ where $\overline{m}_L^*$ is determined by

$$\underbrace{(1 - \bar{z} + c\bar{z}) \frac{\rho \overline{m}_L^* + \alpha^s m_L^s}{\rho + \alpha^s} + c\bar{z} m_H^*}_{\text{Average social cost from lending to type-}L\text{ when screening}} \quad = \quad \underbrace{\frac{\rho \overline{m}_L^* + \alpha^p m_L^p}{\rho + \alpha^p}}_{\text{Average social cost from lending to type-}L\text{ when pooling}} \tag{48}$$

Here, we define the transition speeds for m_L under pooling and screening by $\alpha^p \equiv \kappa + \delta$ and $\alpha^s \equiv \kappa(1 - \bar{z}) + \delta$. The associated steady state values for m_L are given by $m_L^p = \frac{\delta(1-\lambda)}{\delta + \kappa}$ and $m_L^s = \frac{\delta(1-\lambda)}{\delta + \kappa(1-\bar{z})}$. Similar to Section 4, one can show here, too, that the social planner marginally prefers more pooling, that is,

$$\overline{m}_L^* > \overline{m}_L$$

The reason is identical to that in Section 4.

[30] What becomes harder to analyze with a non-constant pool size is slow thawing, since $\theta_t < 1$ affects both type-H and type-L borrowers, i.e. m_H is no longer constant at m_H^*.

An especially simple welfare result is the comparison of steady state welfares across steady states. Here, the question is whether it is the case that welfare in the pooling steady state exceeds that in the screening steady state. In the context of this model, this is satisfied if

$$(1 - \bar{z} + c\bar{z})m_L^s + c\bar{z}m_H^* > m_L^p$$

After some algebra, this simplifies to

$$1 + \delta^{-1}\kappa > (1 - \lambda)c^{-1} + \lambda\delta^{-1}\kappa\bar{z}$$

which is necessarily the case given our Assumption 2: $c > 1 - \lambda$. Thus, Corollary 3 also carries over to this model.