Monetary Policy with Opinionated Markets

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Abstract

Central banks (the Fed) and markets (the market) often disagree about the path of interest rates. We develop a model that explains this disagreement and study its implications for monetary policy and asset prices. We assume that the Fed and the market *disagree about expected aggregate demand.* Moreover, agents learn from data but not from each other—they are *opinionated* and information is fully symmetric. We then show that disagreements about future demand, together with learning, translate into disagreements about future interest rates. Moreover, these disagreements shape optimal monetary policy, especially when they are entrenched. The market perceives monetary policy "mistakes" and the Fed partially accommodates the market's view to mitigate the financial market fallout from perceived "mistakes." We also show that differences in the speed at which the Fed and the market react to the data—*heterogeneous data sensitivity*—matters for asset prices and interest rates. With heterogeneous data sensitivity, every macroeconomic shock has an embedded monetary policy "mistake" shock. When the Fed is more (less) data sensitive, the anticipation of these mistakes dampen (amplify) the impact of macroeconomic shocks on asset prices.

JEL Codes: E00, E12, E21, E32, E43, E44, G11, G12

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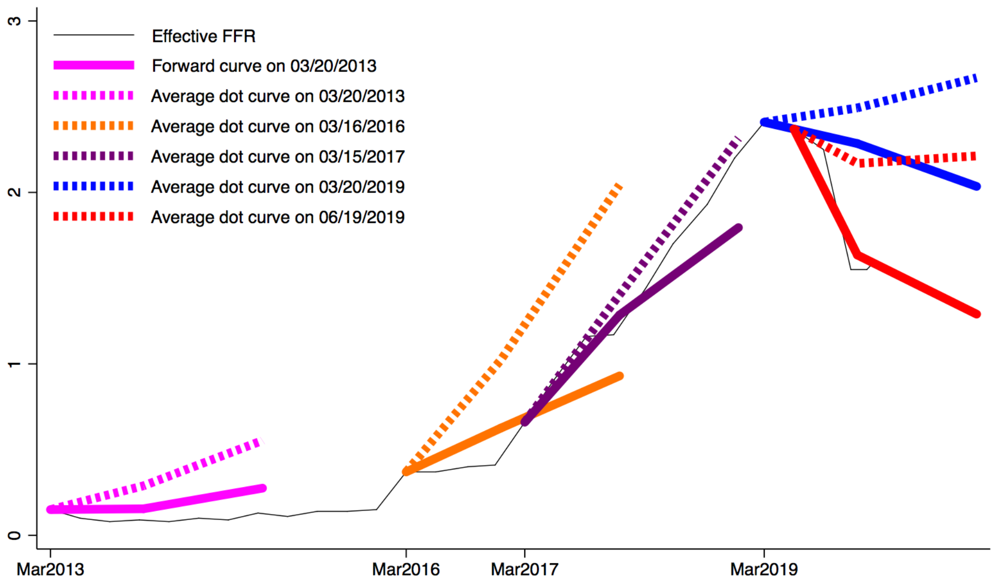

Figure 1: Each colored series plots, for select FOMC meetings, the FOMC average prediction for future fed funds rates (dotted line) and the forward fed funds rates—extracted from futures prices (solid line). The thin black line plots the effective fed funds rate.

1. Introduction

Figure 1 plots the evolution of the fed funds rate over time (thin black line), along with predicted paths. Dotted lines plot the average FOMC prediction and solid lines plot the forward rates that reflect the financial market's prediction. Each color-matched pair of lines plots data from the same FOMC meeting. The figure shows large disagreements between the FOMC predictions and the forward rates, especially around policy-inflection episodes. Ubide (2015) observes similar disagreements in other countries where central banks publish their expected interest rate paths (e.g., Sweden, Norway, and New Zealand).[1]

These disagreements about interest rates are difficult to explain with conventional macroeconomic models. The literature typically focuses on the Fed's superior information about its policy rule or economic activity (and its willingness to signal this information). However, Figure 1 shows that financial markets expect a different interest rate than the Fed *even after the Fed announces the interest rates it plans to set.* Using the Blue Chip Financial Forecasts, we document that this type of *confident disagreement* is a general feature of market participants'

[1] There are many adjustments one could make to Figure 1, while preserving its main qualitative features. The most significant one is to remove the embedded risk premium from the forward curve. We chose not to do so because there is a wide range of estimates for this premium. The most recent estimates by the Fed researchers suggest that this adjustment is of the order of one basis point per month (positive in the pre GFC period and negative after that), which is not nearly enough to eliminate the disagreements (see, e.g., Diercks et al. (2019)). Moreover, we have also found large disagreements between the Fed's Greenbook projections and the professional forecaster' consensus (in both the Blue Chip Financial Forecasts and the Survey of Professional Forecasters).

beliefs (see Section 2). In particular, forecasters from major financial institutions seem to have dogmatic and heterogeneous beliefs that they change only gradually—despite the fact that they see all other forecasts (with a delay of one month). We conjecture that there is no reason to think of the market's overall disagreements with the Fed to be any different than the disagreements among its major participants. In fact, market participants often have their own opinions and do not necessarily consider the Fed to have superior information about the state of the economy. To illustrate how opinionated the market can be, consider the FOMC meeting in December 2007—the run-up to the financial crisis—in which the Fed cut interest rates by 25 basis points. The market was expecting a larger interest rate cut so this was a "hawkish" policy surprise that led to a decline in stock prices. According to journal coverage, some market participants were quite pessimistic that deteriorating financial conditions would adversely affect the economy, and they thought the Fed did not realize the scope of the problem.[2]

In this paper, we build a model in which the market and the Fed have differences of opinion. We use the model to explain the disagreements for the path of interest rates, and study its implications for monetary policy and asset markets. Our key assumption is that the Fed and the market *disagree about expected aggregate demand.* Moreover, agents learn from data but not from each other—they are *opinionated* and information is fully symmetric. We show that disagreements about demand, together with learning, translate into disagreements about future interest rates. Moreover, these disagreements shape optimal monetary policy, especially when they are entrenched. We also show that differences in the speed at which the Fed and the market react to the data—*heterogeneous data sensitivity*—have important implications for how asset prices and interest rates react to macroeconomic shocks.

Our model is set in discrete time. The supply side features a single factor (capital) with nominal rigidities. The demand side features a representative household (the market) that makes consumption-saving and portfolio allocation decisions. In each period, the economy is subject to a shock that affects current spending without changing current productive capacity—we refer to this as an *aggregate demand shock.* The Fed sets the risk-free interest rate in an attempt to insulate the economy from aggregate demand shocks. However, the Fed sets the interest rate *under uncertainty about aggregate demand* for the current period. This assumption implies that the Fed cannot fully stabilize aggregate demand shocks. Instead, the Fed ensures asset prices and output are equal to their potential (efficient) levels "on average" *according to the Fed's belief.*

In this environment, the Fed's and the market's beliefs for aggregate demand affect current output. The Fed's belief matters because the Fed sets the interest rate under uncertainty about current aggregate demand. The market's belief matters because it "sets" current asset prices

[2] The day after the FOMC meeting the Wall Street Journal wrote: "Some on Wall Street yesterday criticized the Fed's actions so far as inadequate. 'From talking to clients and traders, there is in their view no question the Fed has fallen way behind the curve,' said David Greenlaw, economist at Morgan Stanley. 'There's a growing sense the Fed doesn't get it.' Markets believe a weakening economy will force the Fed to cut rates even more than they expected before yesterday, Mr. Greenlaw said."

and output under uncertainty about future output. Our main assumption is that the market and the Fed can have *belief disagreements* about the evolution of aggregate demand. Agents' beliefs are *dogmatic* in the sense that they know each other's beliefs and *agree to disagree. Agents also learn over time*: they update their beliefs as they observe realizations of aggregate demand.

These assumptions ensure that, when there is disagreement, the market expects the Fed to make *monetary policy "mistakes"* (we occasionally use quotes to remind the reader that these are mistakes under the market's belief, not under the Fed's belief or the objective belief). The anticipation of these mistakes affects current asset prices and output. Therefore, to stabilize current output, *the Fed needs to account for belief disagreements.*

To fix ideas, suppose that a persistent negative aggregate demand shock took place in the recent past so that aggregate demand is lower than average. Suppose agents disagree on the size of the persistent component of the aggregate demand shock: specifically, assume the Fed is less pessimistic than the market. In this case, the Fed will expect higher aggregate demand than the market both in the current period and in future periods. Knowing this, the market thinks the Fed will set interest rates too high. In particular, the market expects asset prices and output in future periods to be below the efficient level. The anticipation of these effects also puts downward pressure on asset prices and output in the *current* period. Consequently, to stabilize *current* asset prices and output, the Fed is forced to set a relatively low interest rate. Even though the Fed is optimistic, it sets an interest rate that is somewhere between the levels implied by its own (more optimistic) belief and the market's (more pessimistic) belief. Put differently, *disagreements with the market "constrain" the Fed's interest rate policy.*

Our first result formalizes this logic and shows that the Fed's optimal interest rate reflects a *weighted average of the Fed's belief and the market's belief.* Moreover, the relative weight on the market's belief is greater when the agents are more confident in their initial beliefs (and therefore learn more slowly).

We then explain the observed differences between the market's and the Fed's expectation for future interest rates—"the forward curve" and "the dot curve". Since agents learn from the same data, they expect disagreements to decline over time. In particular, for sufficiently distant horizons the forward curve reflects the market's *current* belief whereas the dot curve reflects the Fed's *current* belief. Intuitively, the market thinks *the Fed* will learn from data and come to *the market's* belief. Hence, the market thinks the Fed will set future interest rates that are more closely aligned with the market's current belief. Conversely, the Fed thinks the market will learn from data and come to the Fed's belief. Thus, the Fed believes it will be less constrained in future periods and will be able to set interest rates reflecting its current belief. Therefore, belief disagreements and learning provide a natural explanation for the patterns in Figure 1.

When agents have heterogeneous beliefs, they might also have *heterogeneous data sensitivity*: that is, they might learn from the data at different speeds (and in fact, this might be why they disagree). We show that this dimension of heterogeneity has clear implications for asset prices, interest rates, and risk-premia. Specifically, when the Fed's and the market's beliefs react to the data differently, demand shocks affect financial markets both directly, and indirectly because of an embedded *monetary policy "mistake"*. Put differently, *with heterogeneous data sensitivity, every macroeconomic shock has an embedded monetary policy "mistake" shock.*

To illustrate these effects, suppose the Fed becomes more data sensitive than the market (holding the market's sensitivity constant). In this case, a positive shock increases the Fed's optimism relatively more than the market's optimism, which means that the market expects the Fed to respond to the shock by setting interest rates "too high" in the future. Expectations of high interest rates dampen the direct effect of the shock on current asset prices. As a consequence, the shock translates into an amplified effect on the dot and forward curves (as well as on the gap between them) and a dampened effect on asset prices. Conversely, if the Fed becomes less data sensitive than the market (holding the market's sensitivity constant), then demand shocks have a smaller effect on the dot and forward curves but an amplified effect on asset prices.

Our main model isolates disagreements in an environment with symmetric information, while (as we mentioned) the bulk of the literature on monetary policy with imperfect information focuses on the Fed's superior information and its signaling motive. We extend our model to incorporate the signaling channel and show that it does not change our main insights, except for extreme cases of infinitely superior information by the Fed or infinite underconfidence in initial beliefs. Perhaps more interestingly, we show that the interaction of the signaling channel with the disagreement environment adds an extra layer of complexity to monetary policy: the Fed now needs to consider that it may face a disagreeable market. In particular, the Fed's attempt to signal the state of the economy through interest rate policy may backfire.

The rest of the paper is organized as follows. After discussing the related literature, we start in Section 2 by documenting facts about interest rate disagreements among forecasters that motivate our modeling ingredients. Section 3 introduces our environment and describes the equilibrium conditions. Section 4 provides benchmark results without disagreement. Section 5 analyzes the case with heterogeneous beliefs but common confidence (or data sensitivity). This section shows that disagreements affect optimal monetary policy and (together with learning) explain the gap between the forward curve and the dot curve. Section 6 analyzes the more general case that features heterogeneous confidence as well as heterogeneous beliefs. This section shows that heterogeneous data sensitivity changes the effect of macroeconomic shocks on asset prices and interest rates. Section 7 considers an extension in which the Fed has superior information. Section 8 provides final remarks, and is followed by several appendices.

Literature review. Our paper is related to an extensive literature on the signaling effects of monetary policy. The overarching idea is that the Fed has better information about its own belief or fundamentals than the market has, and both agents know this. The earlier literature focuses on how to design a transparent communication strategy that minimizes abrupt surprises (see, e.g., Poole et al. (2000); Blinder et al. (2008)). The more recent literature emphasizes that policy announcements reveal information about fundamentals and focuses on "cleansing" monetary policy shocks from this information effect (see, e.g., Campbell et al. (2012); Ramey (2016); Nakamura and Steinsson (2018); Andrade et al. (2019); Jarocinski and Karadi (2020)). In order to isolate our insights, our main model removes all of the—clearly realistic—signaling mechanisms. Therefore, we do not assume the Fed has superior information and instead model a situation in which both agents know what the other knows but process data differently.

An interesting strand of this literature documents that "high-frequency monetary policy surprises"—commonly used for identification purposes—are actually predictable from information *publicly available* before the announcement. Miranda-Agrippino (2016) documents this fact and interprets it as the market and the Fed having heterogeneous beliefs, and Miranda-Agrippino and Ricco (2018) develop a model in which belief heterogeneity emerges from imperfect information and noisy signaling. More recently, Sastry (2019) shows monetary policy surprises are predictable from public sentiment indicators, but he finds limited evidence of the signaling effect of these surprises once he controls for their predictable component (from sentiment indicators). Instead, his evidence suggests the Fed's belief responds more to public sentiment than the market's belief; and the market agrees to disagree with the Fed at the moment of the surprise but then gradually learns over time that the Fed was indeed better informed about these predictable shocks. This evidence is consistent with the key ingredients of our model, disagreements and learning from data, although we don't take a stand on who has a better track record.

Our paper is also related to an empirical literature documenting the large impact of monetary policy shocks on asset prices and the term structure of interest rates (e.g., Bernanke and Kuttner (2005); Gürkaynak et al. (2005a,b); Hanson and Stein (2015); Nakamura and Steinsson (2018); Goodhead and Kolb (2018)). Our model features some of the asset price behaviors identified in this literature. However, our focus is not so much on monetary surprises initiated by the Fed, but on the market's anticipation of the Fed's response to macroeconomic shocks. In this sense, our paper is connected with the extensive empirical literature documenting the response of asset prices to macroeconomic announcements. Specifically, this literature documents that these responses are time-varying and depend on stages of the business cycle and the expected impact on future interest rates (e.g., Gilbert et al. (2010); Goldberg and Grisse (2013)). Our model both embeds the anticipation of policy responses documented in this literature and shows the mechanisms by which belief disagreements influence these responses.[3]

Our policy analysis contributes to a growing literature that analyzes optimal macroeconomic policy without rational expectations (see Woodford (2013) for a review). This literature typically assumes the planner is rational but agents are boundedly rational due to frictions such as learning

[3]There is a complementary empirical finance literature that focuses on the impact of uncertainty about future macroeconomic policy on the current disagreement among investors, and on how these features can affect the term structure of interest rates and the risk premia (see, e.g., Giacoletti et al. (2020)).

(e.g., Evans and Honkapohja (2001); Eusepi and Preston (2011)), level-k thinking (e.g., García-Schmidt and Woodford (2019); Angeletos and Sastry (2018)), or cognitive discounting (Gabaix (forthcoming)).[4] The focus is on designing policies that address or are robust to agents' bounded rationality. Our approach has two key differences. First, we do not take a stand on who has rational beliefs: in fact, the market thinks it has correct beliefs and the Fed has incorrect beliefs—the opposite of the typical assumption. Second, our agents are not boundedly rational in the usual sense: both the market and the Fed have *dogmatic* beliefs about exogenous states and fully understand how those states map into endogenous outcomes. These assumptions lead to a different policy analysis and results. In our setting, the Fed's main non-standard concern is to mitigate the macroeconomic impact of the monetary policy "mistakes" perceived by the market.

Our normative analysis is also related to a literature that evaluates whether expanding standard monetary policy rules to incorporate financial markets' indicators is useful (e.g., Smets (1997); Bernanke and Gertler (1999); Bullard et al. (2002)). The nature of these models is very different from ours and they do not consider disagreements. An exception is Stein and Sunderam (2018), who assume the Fed has private information and perceives a (financial stability) benefit to reduce the volatility of bond prices (beyond the volatility implied by macroeconomic stability). In their setup, the Fed changes the short-term interest rate gradually to keep the long-term interest rates more stable, but this attempt backfires because the market correctly infers the implied relationship between short and long-term rates. They conclude that the Fed should ignore the bond market. That is, they argue the Fed should ignore the impact of its information revelation on the bond market. We focus on a problem that is orthogonal to theirs, as our model captures situations in which the market does *not* consider the Fed to have private information and the Fed's primary goal is macroeconomic stability. In these situations, the Fed *should* pay attention to the bond market (when the implied rate differs from its own belief), since not doing so would increase the volatility of asset prices and aggregate demand.[5]

Our analysis of the Blue Chip data is related to a growing literature that uses survey data to document belief distortions about macroeconomic outcomes. Much of the recent literature focuses on whether agents over-or-underreact to data (e.g., Coibion and Gorodnichenko (2015); Bordalo et al. (2018); Broer and Kohlhas (2018); Angeletos et al. (2020); Ma et al. (2020)). In contrast, we focus on agents' disagreements and their reaction to each other's beliefs. We provide evidence for confident disagreement: forecasters' beliefs are persistent and largely insensitive to the consensus belief. We also show that, as in our model, beliefs about the interest rate correlate with beliefs about aggregate demand (proxied by the GDP price index or the real GDP).

[4] A closely related literature assumes agents are also rational but lack common knowledge of each other's beliefs, and illustrates how the resulting coordination problems can lead to aggregate behavior that resembles some forms of bounded rationality (e.g., Woodford (2001); Angeletos and La'O (2010); Morris and Shin (2014); Angeletos and Lian (2018); Angeletos and Huo (2018)).

[5] This result is related to Vissing-Jorgensen (2019). She argues that market beliefs *do* influence actual monetary policy decisions, and goes on to postulate that FOMC members know this and use their speeches to influence markets. Our object of interest and model are different from hers.

Finally, this paper is related to our previous work on risk-centric macroeconomics (Caballero and Simsek (forthcoming, 2019)). There we also focus on shocks that affect asset prices and on a Fed that reacts to these shocks in order to limit their transmission to aggregate demand. The distinctive feature of this paper is our focus on disagreements between investors and the Fed, whereas our previous work analyzes disagreements among investors and the speculation that these disagreements generate.

2. Interest rate disagreements: Some facts from forecasters

Our model is built on the observation that disagreements on expected interest rates are driven by *disagreements about expected aggregate demand*. Moreover, we assume *confident disagreement:* that is, agents have dogmatic beliefs and do not consider the other agent to have superior information. In this section, we present evidence for these modeling ingredients from disagreements among professional forecasters. We focus on disagreements among forecasters to exploit the high quality and *forecaster*-level data on beliefs, which we do not have for the Fed. Our presumption is that the beliefs' traits observed across forecasters (major financial institutions) should also carry over to the Fed.

Specifically, we measure beliefs from Blue Chip Financial Forecasts (Blue Chip). Blue Chip is a monthly survey of several major financial institutions. Forecasters report predictions about interest rates and other outcomes for up to five quarters including the current quarter. We focus on the Fed funds rate (reported as the quarterly average)—which captures beliefs about the policy interest rate; and the GDP price index as well as the real GDP (reported as the annualized quarterly growth rate)—which proxy for beliefs about aggregate demand. We analyze predictions for the third quarter (beyond the current quarter) but the results are similar for other horizons. Our sample runs from January 2001 until February 2020.

Figure 2 illustrates the consensus (average) prediction together with the predictions from two major institutions: Goldman Sachs and Bear Stearns (until its failure in 2008). The panels show that higher interest rate predictions are typically associated with higher aggregate demand predictions (proxied by the GDP price index). In the early 2000s, Goldman Sachs and Bear Stearns were both more pessimistic about aggregate demand and predicted lower interest rates than the consensus. In the mid 2000s, both institutions turned more optimistic about demand and predicted higher interest rates. In the run-up to the financial crisis of 2008, Goldman Sachs became more pessimistic about demand and predicted lower interest rates; whereas Bear Stearns remained optimistic and predicted higher interest rates (until it eventually failed in early 2008). After the crisis, Goldman Sachs remained pessimistic and predicted low interest rates during the zero lower bound and the lift-off episodes (until recent years).

Importantly, Figure 2 also highlights that relative predictions are quite persistent. This persistence is difficult to reconcile with dispersed information: forecasters see each other's prediction as well as the consensus prediction (with a delay of one month) and yet they largely stay with

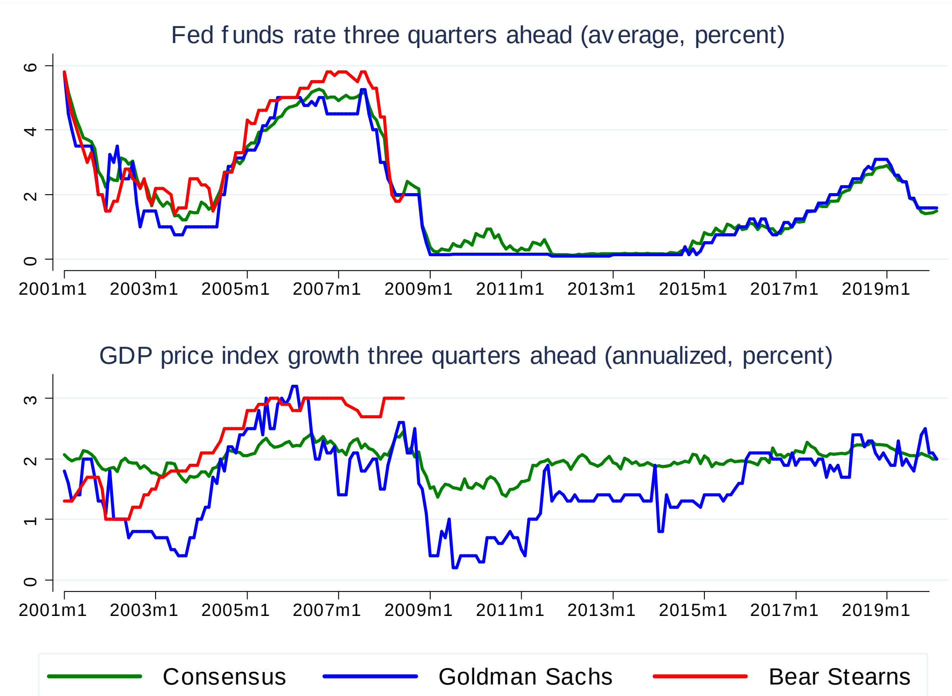

Figure 2: Select Blue Chip predictions for the Fed funds rate (top panel) and the GDP price index (bottom panel).

their own prediction. Rather, the persistence of predictions suggests *confident disagreement*: as in our model, forecasters seem to have *dogmatic beliefs* that they change only gradually.

We use regression analysis to show these observations more systematically. We first establish that higher interest rate predictions are associated with higher aggregate demand predictions. Specifically, we regress the Fed funds rate prediction on the GDP price index or the real GDP prediction, controlling for month and forecaster fixed effects. The first three columns of Table 1 illustrate that the coefficients are positive and significant. The coefficient for the GDP price index is larger and more significant—this is expected since nominal prices provide a more accurate proxy for aggregate demand than real output (which might also be driven by aggregate supply).

We next establish that interest rate predictions are persistent in a way that suggests confident disagreement rather than dispersed private information. With confident disagreement, predictions are correlated with their own past values. With dispersed private information, predictions instead are correlated with the consensus prediction's past values—assuming the consensus aggregates the dispersed information. We therefore run a "horse race" where we regress the Fed funds rate prediction on its one-month lag and the consensus prediction's one-month lag, controlling for forecaster fixed effects.[6] The fourth column of Table 1 shows that confident disagreement wins this horse race: the lagged own prediction has a much larger impact than the

[6]We exclude month fixed effects from this regression, since including them would absorb the effect of the consensus prediction.

Table 1: Correlates of interest rate predictions

	Fed funds rate (FFR) prediction						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GDP price index prediction	0.11**		0.11**				0.04**
	(0.02)		(0.02)				(0.01)
Real GDP prediction		0.03*	0.03+				0.01+
		(0.02)	(0.02)				(0.01)
FFR prediction last month				0.69**	0.69**	0.69**	0.68**
				(0.03)	(0.03)	(0.02)	(0.02)
FFR consensus last month				0.29**	-0.17**		
				(0.03)	(0.06)		
FFR futures last month					0.47**		
					(0.06)		
Time FE	Yes	Yes	Yes	No	No	Yes	Yes
Forecaster FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2 (adjusted, within)	0.03	0.00	0.03	0.96	0.97	0.48	0.49
Forecasters	110	111	110	108	108	108	107
Months	230	230	230	229	226	229	229
Observations	10,365	10,645	10,363	10,370	10,244	10,370	10,052

Note: The sample is an unbalanced panel of monthly Blue Chip forecasts between 2001-2020. Predictions and futures are for 3 quarters ahead. FFR is the quarterly average (percent) and the GDP price index and the real GDP are annualized quarterly growth rates (percent). Estimation is via OLS. Standard errors are in parentheses and clustered by forecaster and month. +, *, and ** indicate significance at 0.1, 0.05, and 0.01 levels, respectively.

lagged consensus prediction (and adding more lags does not change this result).

In the horse-race regression, the lagged consensus also has a significant effect. While this might be driven by dispersed private information, it might also reflect other public signals that correlate with the consensus. To investigate, we control for the one-month lag of the Fed funds futures rate for three quarters ahead (the same quarter as the predictions).[7] The fifth column shows the lagged futures rate has a larger effect than the lagged consensus. Moreover, once we control for the futures, the coefficient on the consensus changes signs (most likely due to collinearity). In contrast, the coefficient on the lagged own prediction is robust to including lagged futures (fifth column), month fixed effects—that capture forecasters' common reaction to all signals (sixth column), and the current own prediction for the GDP price index and the real GDP (the last column).

In sum, Table 1 shows that the results illustrated in Figure 2 hold more systematically. Interest rate forecasts correlate with aggregate demand forecasts. Forecasts also feature confident disagreement: forecasters seem to have dogmatic beliefs that they change only gradually. We next turn to our theoretical analysis, where we equip the Fed and the market with beliefs that satisfy these features and investigate the implications for monetary policy and asset prices.

[7] We obtain the futures rate for the quarter by averaging the implied futures rates for the months in the quarter.

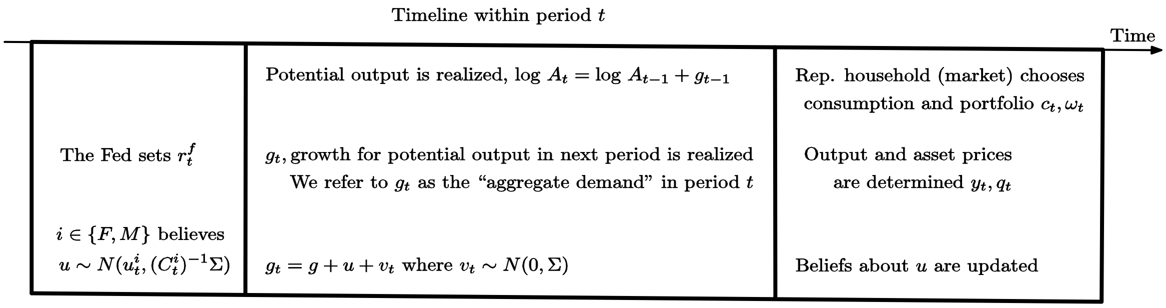

Figure 3: The timeline of events and the summary of the model.

3. Environment, equilibrium, and beliefs

In this section we introduce the basic environment, define and characterize the equilibrium, and describe the evolution of beliefs. We start with a brief overview of our model. The economy is set in discrete time with periods $t \in \{0, 1, ...\}$. There is a single consumption good and a single (and exogenous) factor of production—capital. There is no investment and depreciation and capital is normalized to one unit. Potential output is given by A_t. Actual output, y_t, can deviate from this level due to aggregate demand shocks and nominal rigidities.

The Fed sets the risk-free interest rate with the objective of keeping output close to its potential level. There is also a representative household (the market) that makes consumption and portfolio allocation decisions. We assume the Fed sets the interest rate for each period *before* observing the aggregate demand shock for that period. This implies that the Fed's expectation about aggregate demand affects the interest rate policy. Our key assumption is that the market and the Fed can have *heterogeneous beliefs* regarding the evolution of aggregate demand shocks. Our goal is to understand how disagreements between the market and the Fed affect equilibrium outcomes.

Figure 3 illustrates the timeline of events within a period. In the first phase, the Fed sets the risk-free interest rate. In the second phase, a shock that determines aggregate demand within the period is realized. Beliefs about aggregate demand are updated. In the third phase, the market chooses optimal allocations, markets clear, and the equilibrium levels of output and asset prices are determined. Throughout, we denote the Fed and the market with the superscript $i \in \{F, M\}$. We use $E_t^i[\cdot]$ and $var_t^i[\cdot]$ to denote agent i's expectation and variance in period t before the realization of shocks (in the first phase), and we use $\overline{E}_t^i[\cdot]$ and $\overline{var}_t^i[\cdot]$ to denote the corresponding beliefs after the realization of shocks (in the last phase).

Capital utilization and potential output. The rate of capital utilization in period t is endogenous and denoted by $\eta_t > 0$. When utilized at this rate, capital produces

$$y_t = \eta_t A_t. \tag{1}$$

We assume the Fed targets a particular level of utilization, normalized to one, $\eta_t = 1$. Hence $y_t = A_t$ captures potential output. Actual output can be above or below this level due to aggregate demand fluctuations. For analytical tractability, in the main text we assume capital utilization is costless. In particular, we do not model the welfare loss from utilizing capital above potential ($\eta_t > 1$). In Appendix B, we show that our analysis and results naturally extend to a setting in which capital utilization is costly and $\eta_t = 1$ corresponds to the efficient level of utilization (under an appropriate normalization).[8]

Aggregate demand shocks. Potential output follows the process

$$\log A_{t+1} = \log A_t + g_t. \tag{2}$$

Here, g_t denotes the growth rate of potential output (productivity) between periods t and $t+1$, which is realized in period t. In particular, by the time the economy reaches period t, there is no uncertainty about the potential output of the economy in the current period: $\log A_t = \log A_{t-1} + g_{t-1}$ and g_{t-1} is already determined. However, there is uncertainty about the potential output for the next period, g_t. As we will see, g_t has a one-to-one effect on aggregate spending in period t (see Eq. (17) and the subsequent discussion). Hence, for the rest of the paper, we refer to g_t as the *aggregate demand shock* in period t.[9]

Uncertainty and belief structure. We assume aggregate demand follows

$$\begin{aligned} g_t &= g + u + v_t \text{ for each } t, \\ \text{where } v_t &\sim N(0, \Sigma) \text{ for each } t \text{ and independent across } t. \end{aligned} \tag{3}$$

The term g captures the baseline level of aggregate demand. The random variable v_t captures transitory shocks to aggregate demand. These shocks are i.i.d. across periods with a Normal distribution. The term u is an unknown parameter that captures a one-time persistent shock to aggregate demand. It is realized at the beginning of the model but it is not observed by agents (see Figure 3).

Agents have heterogeneous prior beliefs about the unknown component of aggregate demand.

[8] Specifically, we assume that a higher utilization rate leads to faster depreciation of capital. This setup is slightly less tractable and requires a linear approximation. With this approximation, we obtain an equilibrium characterization (see Appendix B.2) that is very similar to the characterization that we present at the end of this section (see Section 3.2). Consequently, our qualitative results also apply in the extended model with costly utilization.

[9] Put differently, we focus on "news shocks" as a source of aggregate demand fluctuations (see Lorenzoni (2011) for a review). In Caballero and Simsek (forthcoming), we illustrate how changes in actual news about the future (g_t) generates similar macroeconomic effects as changes in perceived news ("noise"), actual or perceived risk, or risk aversion. There, we refer to these shocks as "risk premium shocks" since we focus on fluctuations that originate in financial markets and that are reflected in the valuations of risky financial assets. In this paper, we broaden the interpretation (and thus the terminology) to include aggregate demand fluctuations that do not necessarily originate in financial markets, e.g., changes in consumer sentiment.

Specifically, at the beginning of period 0, each agent $i \in \{F, M\}$ believes the unknown component is distributed according to a Normal distribution,

$$u \sim N\left(u_0^i, \left(C_0^i\right)^{-1} \Sigma\right). \tag{4}$$

Here, u_0^i denotes the perceived mean and $\left(C_0^i\right)^{-1} \Sigma$ denotes the perceived variance for agent i. The parameter C_0^i is a measure of the agent's *confidence* in its initial belief. As we will see, this parameter governs the rate at which the agent's belief reacts to data: specifically, it is an inverse measure of *data sensitivity*.

The key assumption is that the beliefs, $\left(u_0^F, C_0^F\right)$ and $\left(u_0^M, C_0^M\right)$, are not necessarily equal: that is, the market and the Fed can have different expectations or different confidence. We assume agents know each other's beliefs but "agree to disagree" (see Section 7 for a version of the model with heterogeneous information).

While agents disagree with one another, they both learn from the data. Specifically, in each period, the realization of aggregate demand provides the agents with a noisy signal about the persistent shock, $g_t - g = u + v_t$. Agents combine this data with their prior beliefs in (4) to form their posterior beliefs according to Bayes' rule. Hence, agents are "rational" given their initial beliefs. Nonetheless, we view agents' heterogeneous initial beliefs as resulting from a possibly misspecified learning model that we leave unmodeled for simplicity. One possibility is that agents receive public signals that are informative about u, but they interpret these signals differently, e.g., the Fed puts more weight on one signal whereas the market puts more weight on another signal.

Assets. There are two types of assets. There is a risk-free asset in zero net supply. We denote the one-period-ahead log return on the risk-free asset with r_t^f. There is also a "market portfolio" that represents claims to output. We denote the ex-dividend price of the market portfolio with $Q_t A_t$, so that Q_t corresponds to the ex-dividend price per unit of potential output.

We define the log price per potential output as $q_t \equiv \log Q_t$, and the one-period-ahead log return of the market portfolio as

$$r_{t,t+1} = \log \frac{y_{t+1} + Q_{t+1} A_{t+1}}{Q_t A_t}. \tag{5}$$

Note that the market return depends on the *endogenous* output and asset prices that will obtain in the next period.

Preferences and portfolio choice. The representative household (the market) makes decisions after observing each period's shocks and the interest rate (see Figure 3). The market maximizes expected time-separable log utility *according to its own belief*,

$$\overline{E}_t^M \left[\sum_{\tilde{t}=t}^{\infty} e^{-\rho \tilde{t}} \log c_{\tilde{t}} \right]. \tag{6}$$

Here, $e^{-\rho}$ denotes the one-period discount factor and the overline notation captures the timing within the period (after the shock and the interest rate decision). In every period, the market observes its wealth, $y_t + Q_t A_t$, and chooses how much to consume, c_t, and what fraction of its unspent wealth to allocate to the market portfolio, ω_t (with the residual fraction invested in the risk-free asset). We state the market's problem formally in Appendix A.1.1.

Equilibrium in asset markets. Asset markets are in equilibrium when the market allocates all of its wealth to the market portfolio,

$$\omega_t = 1. \tag{7}$$

Nominal rigidities and equilibrium in goods markets. The supply side of our model features nominal rigidities similar to the standard New Keynesian model. We relegate the details to Appendix A.1.2. There is a continuum of monopolistically competitive production firms that own the capital stock and produce intermediate goods (which are then converted into the final good). For simplicity, these production firms have set nominal prices that never change. The firms choose their capital utilization rate, $\eta_t > 0$, which leads to output $y_t = \eta_t A_t$. In the main text, we assume firms can increase their factor utilization for free. These features imply that output is determined by aggregate demand for goods (consumption):

$$y_t = c_t. \tag{8}$$

All output accrues to production firms in the form of earnings. Hence, the market portfolio is a claim on all production firms.

Monetary policy. Since goods prices do not change, the real interest rate is equal to the nominal interest rate, which is controlled by the monetary authority (the Fed). The Fed attempts to replicate log-potential output, which (by assumption) corresponds to $y_t = A_t$. However, the Fed might not be able to perfectly achieve its target because it sets the interest rate *before* observing the shocks within the period (see Figure 3). The Fed replicates the potential output *on average according to its own beliefs*. Specifically, the Fed sets r_t^f to ensure that

$$E_t^F [\log y_t] = \log A_t. \tag{9}$$

Targeting the average of the log-output (as opposed to the level) provides analytical tractability.

Definition 1. *An equilibrium is a collection of stochastic processes for allocations, $[A_t, y_t, c_t, \omega_t,]_{t=0}^{\infty}$, and prices, $Q_t, r_t^f, r_{t,t+1}$, such that potential output evolves according to* (2)*, the one-period-ahead return on the market portfolio is given by* (5)*, the market maximizes expected utility according to its own belief (cf. Eq.* (6) *and Appendix A.1.1), asset markets clear (cf. Eq.* (7)*), production firms maximize earnings (cf. Appendix A.1.2), goods markets clear (cf. Eq.* (8)*), and the Fed sets r_t^f to ensure output is equal to potential output on average according to its own belief [cf. Eq.* (9)*].*

We next present a generally applicable characterization of the equilibrium. In subsequent sections, we use this characterization to describe the equilibrium for different specifications of beliefs.

3.1. Equilibrium in the goods market

We start by establishing the equilibrium conditions in the goods market. Log-utility implies the representative household consumes a fraction of its lifetime income (see Appendix A.1.1 for a derivation):

$$c_t = \left(1 - e^{-\rho}\right)(y_t + Q_t A_t). \tag{10}$$

Combining this expression with Eq. (8), we obtain

$$y_t = \frac{1 - e^{-\rho}}{e^{-\rho}} Q_t A_t. \tag{11}$$

We refer to this equation as the *output-asset price relation.* Combining this relation with Eq. (9), we further obtain

$$E_t^F[q_t] = q^* \text{ where } q^* = \log \frac{e^{-\rho}}{1 - e^{-\rho}}. \tag{12}$$

We use $q_t \equiv \log Q_t$ to denote the log price per potential output. Output and asset prices are related through a wealth effect. Therefore, *monetary policy can be thought of as stabilizing asset prices.* The Fed targets the average log price per potential output, q^*, which is the level that ensures output is equal to its potential.

Using the output-price relation in Eq. (11), we can also write the payoff from the market portfolio as

$$y_{t+1} + Q_{t+1} A_{t+1} = e^{\rho} Q_{t+1} A_{t+1}.$$

Substituting this payoff into Eq. (5), we obtain the following expression for the equilibrium return on the market portfolio:

$$r_{t,t+1} = \log \frac{e^{\rho} Q_{t+1} A_{t+1}}{Q_t A_t} = \rho + g_t + q_{t+1} - q_t. \tag{13}$$

The one-period-ahead return on the market portfolio depends on the dividend yield, ρ, on the (known) growth rate, g_t, and on the log-change in asset prices between the current period and the next period, $q_{t+1} - q_t$.

3.2. Equilibrium in asset markets

We next establish the equilibrium conditions in asset markets. We derive the representative household's portfolio optimality condition in Appendix A.1.1. After substituting the equilibrium requirement, $\omega_t = 1$, portfolio optimality implies[10]

$$\overline{E}_t^M [r_{t,t+1}] = r_t^f + \frac{\overline{var}_t^M [r_{t,t+1}]}{2}. \tag{14}$$

In equilibrium, the discount rate (or the required return) on the market portfolio depends on the risk-free rate and the risk premium. As usual, the risk premium is proportional to the variance of the return on the market portfolio.

Combining Eqs. (13) and (14), we obtain an expression for the price of the market portfolio:

$$\begin{aligned} q_t &= \rho + g_t + \overline{E}_t^M [q_{t+1}] - \left(r_t^f + \frac{\overline{var}_t^M [r_{t,t+1}]}{2} \right) \\ &= \rho + g_t + E_{t+1}^M [q_{t+1}] - r_t^f - \frac{var_{t+1}^M [q_{t+1}]}{2}. \end{aligned} \tag{15}$$

Here, the second line uses Eq. (13) to calculate the variance of the one-period-ahead return. We also substitute agents' end-of-period expectations in period t with their beginning-of-period expectations in period $t+1$ (since no new information arrives between the last phase of period t and the first phase of period $t+1$—see Figure 3).

Eq. (15) illustrates that the equilibrium price increases one-to-one with the potential growth rate, g_t. Together with the output-asset price relation (cf. Eq. (11)), this implies aggregate spending increases one-to-one with g_t. This is why we refer to g_t as the aggregate demand in period t.

Next note that the Fed sets the interest rate to ensure $E_t^F [q_t] = q^*$ [cf. Eq. (12)]. Combining this observation with Eq. (15), we solve for the interest rate as

$$r_t^f = \rho + E_t^F [g_t] + E_t^F \left[E_{t+1}^M [q_{t+1}] - q^* \right] - \frac{E_t^F \left[var_{t+1}^M [q_{t+1}] \right]}{2}. \tag{16}$$

This expression illustrates that the Fed sets a higher interest rate when it expects greater aggregate demand, $E_t^F [g_t]$. More subtly, the Fed also sets a higher interest rate if it expects *the market* to be more optimistic about the one-period-ahead risk-adjusted asset price (higher $E_t^F \left[E_{t+1}^M [q_{t+1}] \right]$ or lower $E_t^F \left[var_{t+1}^M [q_{t+1}] \right]$). As illustrated by Eq. (15), the market's optimism about future asset prices increases asset prices in the *current* period, and the Fed increases the

[10] This expression also requires that the one-period-ahead market return follows a log-normal distribution, which applies in the equilibria we consider.

interest rate to offset this price increase. This mechanism plays an important role in our main results.

Substituting Eq. (16) into Eq. (15), we solve for the equilibrium price as

$$\begin{aligned} q_t &= q^* + g_t - E_t^F[g_t] \\ &\quad + \left(E_{t+1}^M[q_{t+1}] - E_t^F\left[E_{t+1}^M[q_{t+1}]\right]\right) - \frac{1}{2}\left(var_{t+1}^M[q_{t+1}] - E_t^F\left[var_{t+1}^M[q_{t+1}]\right]\right). \end{aligned} \tag{17}$$

This expression illustrates that the equilibrium price depends on surprises relative to *the Fed's* expectations. The first line concerns surprise shocks to aggregate demand, g_t. When aggregate demand is higher than the Fed expected when it set the interest rate, the price is higher than q^*. The second line concerns surprises to the market's expectation about risk-adjusted asset prices in the next period. When the market's expected price is higher (or its expected variance is lower) than the Fed expected it to be, then the equilibrium price is higher than q^*.

Eqs. (16) and (17), together with Eq. (11), provide a generally applicable equilibrium characterization. In subsequent sections, we solve these equations for various specifications of agents' beliefs.

3.3. Bayesian updating of beliefs

As Eqs. (16) and (17) suggest, the equilibrium depends on agents' conditional beliefs about aggregate demand, g_t. In our setting, these beliefs are non-trivial since agents learn over time. We next characterize agents' belief updating, which simplifies the subsequent analysis.

Recall that agent i starts with the belief, $u \sim N\left(u_0^i, \left(C_0^i\right)^{-1}\Sigma\right)$. In each period, she receives a signal about the shock, $g_t - g = u + v_t$, and updates its belief according to Bayes' rule. Therefore, at the beginning of each period t, the agent believes $u \sim N\left(u_t^i, \left(C_t^i\right)^{-1}\Sigma\right)$, where

$$u_t^i = \frac{C_0^i u_0^i + t\left(\overline{g}_{t-1} - g\right)}{C_0^i + t} \quad \text{where} \quad \overline{g}_{t-1} = \sum_{\tilde{t}=0}^{t-1} \frac{g_{\tilde{t}}}{t-1}.$$

Here, $\overline{g}_{t-1}$ denotes the average realization of aggregate demand up to (but excluding) the realization in period t. The agent's belief converges in probability to u. She eventually learns the unknown component of aggregate demand. The variance of agents' posterior belief is $\left(C_0^i + t\right)^{-1}\Sigma$, which illustrates that agents' confidence grows over time (that is, their parameter uncertainty declines over time).

Given the agent's belief about u, we can calculate its belief about aggregate demand, $g_t = g + u + v_t$. To simplify the notation, let us define $\mathbf{c}_{t_1,t_2}^i$ as the agent's confidence in period t_1 relative to a later period t_2:

$$\mathbf{c}_{t_1,t_2}^i = \frac{C_0^i + t_1}{C_0^i + t_2} \quad \text{for } t_1 \leq t_2. \tag{18}$$

Relative confidence satisfies the following identity, which we use in subsequent analysis:

$$\mathbf{c}^i_{t_1,t_2}\mathbf{c}^i_{t_2,t_3} = \mathbf{c}^i_{t_1,t_3} \text{ for } t_1 \leq t_2 \leq t_3. \tag{19}$$

With this notation, the agent believes $g_t \sim N\left(E^i_t[g_t], \Sigma^i_t\right)$, where

$$\begin{aligned} E^i_t[g_t] &= \mathbf{c}^i_{0,t}\left(g+u^i_0\right)+\left(1-\mathbf{c}^i_{0,t}\right)\overline{g}_{t-1} \qquad (20)\\ &= \mathbf{c}^i_{t-1,t}E^i_{t-1}[g_{t-1}]+\left(1-\mathbf{c}^i_{t-1,t}\right)g_{t-1} \\ \text{and } var^i_t[g_t] &= \Sigma^i_t \equiv \left(1+\frac{1}{C^i_0+t}\right)\Sigma \quad \text{ for } t\geq 0. \qquad (21)\end{aligned}$$

The agent's expectation for aggregate demand is a weighted average of its initial expectation, $E^i_0[g_0] = g + u^i_0$, and the average realization, $\overline{g}_{t-1}$. The second line writes the expectation as a weighted average of the most recent expectation and the most recent realization. The last line shows that the perceived variance about aggregate demand, Σ^i_t, reflects the uncertainty about the transitory shock and the parametric uncertainty about the persistent component.

Eqs. (16) and (17) suggest the equilibrium also depends on *higher order beliefs* about aggregate demand; in particular, expectations at time t about expectations that will obtain at time $t+1$. The following lemma characterizes these types of higher order expectations.

Lemma 1. *Consider expectations in period t_1 about expectations about aggregate demand in a subsequent period $t_2 \geq t_1$. For each agent $i \in \{F, M\}$ and $j \neq i$, we have:*

$$\begin{aligned} E^i_{t_1}\left[E^i_{t_2}[g_{t_2}]\right] &= E^i_{t_1}[g_{t_1}]. \qquad (22)\\ E^i_{t_1}\left[E^j_{t_2}[g_{t_2}]\right] &= \mathbf{c}^j_{t_1,t_2}E^j_{t_1}[g_{t_1}]+\left(1-\mathbf{c}^j_{t_1,t_2}\right)E^i_{t_1}[g_{t_1}]. \qquad (23)\end{aligned}$$

Eq. (22) says that agent i expects its own belief about aggregate demand in a future period to be the same as its current belief about aggregate demand. In contrast, Eq. (23) shows that agent i expects *the other agent's* belief about aggregate demand in a future period to be a *weighted average* of the other agent's current belief and its own belief. The weights depend on the other agent's relative confidence, $\mathbf{c}^j_{t_1,t_2}$. Intuitively, agent i expects agent j to learn from the data and to come toward agent i's own view. The expected speed of learning depends on the other agent's confidence. This implication of learning will be important for our results.

4. Benchmark with common beliefs

We start by analyzing a benchmark scenario with no disagreement between the Fed and the market. Specifically, suppose $u^F_0 = u^M_0 \equiv u_0$ and $C^F_0 = C^M_0 \equiv C_0$ so that agents start with the same initial belief $u \sim N\left(u_0, C_0^{-1}\Sigma\right)$ [cf. (4)].[11]

[11] Throughout, when agents have common confidence, we drop the superscripts and use $\mathbf{c}_{s,t}$ and Σ_t to denote the common levels of relative confidence and perceived variance [cf. (18) and (21)]. When they additionally have

In this context, we first solve for the equilibrium asset prices and interest rates. We then characterize the forward and dot curves and show that an increase in agents' optimism about the unknown component of demand, u_0, shifts both curves upward. In subsequent sections, we investigate how a similar shock affects the forward and dot curves when the Fed and the market disagree.

Equilibrium given common beliefs. Since there is no disagreement, we conjecture that asset prices change only because of surprise shocks to aggregate demand [cf. Eq. (17)]:

$$q_t = q^* + g_t - E_t[g_t] \text{ for each } t. \tag{24}$$

Suppose this conjecture holds for prices in period $t+1$. Then, we have

$$E_{t+1}[q_{t+1}] = q^* \text{ and } var_{t+1}[q_{t+1}] = var_{t+1}[g_{t+1}] = \Sigma_{t+1}. \tag{25}$$

Substituting these expressions into the equilibrium asset price Eq. (17), we see that Eq. (24) also holds for prices in period t. This verifies our conjecture for asset prices. Using Eq. (16), we solve for the corresponding interest rate as

$$r_t^f = \rho + E_t[g_t] - \frac{\Sigma_{t+1}}{2}. \tag{26}$$

With common beliefs, the market knows that the Fed will stabilize future asset prices on average around the efficient level, $E_{t+1}[q_{t+1}] = q^*$. This ensures that the interest rate depends on the expected aggregate demand, $E_t[g_t]$, and on the variance of aggregate demand for the next period, Σ_{t+1}. The variance matters because monetary policy is imperfect and cannot fully stabilize shocks to aggregate demand in the next period.

The forward and dot curves. We next fix a particular period, $t = 1$, and calculate the forward and the dot curves in this period: that is, the expected future interest rates according to the market's and the Fed's belief, respectively.

Using Eq. (26), we obtain

$$E_1^M\left[r_t^f\right] = E_1^F\left[r_t^f\right] = \rho + E_1[g_1] - \frac{\Sigma_{t+1}}{2} \text{ for } t \geq 1. \tag{27}$$

Here, we used Lemma 1 and Eq. (21). Hence, the forward and dot curves reflect agents' *current* belief about aggregate demand.

Finally, note that Eqs. $(26-27)$ together with the characterization of beliefs in Section 3.3 provide a closed-form solution for the interest rate and its expectations in terms of the initial beliefs, u_0^M, u_0^F, and the average past realization of aggregate demand, $\overline{g}_{t-1}$ (see Appendix A.2).

common beliefs, we use $E_t[g_t]$ and $var_t[g_t]$ to denote the common belief about aggregate demand.

Comparative statics of optimism and demand shocks. We end this section with two comparative static exercises that we will repeat once we add disagreement: an increase in initial optimism $\Delta u_0 > 0$, and an increase in the initial demand shock, $\Delta g_0 > 0$. In each case, we ask: How does the change affect the yield and the dot curve in period 1? How does it affect the current interest rate in period 1?

With common beliefs, the two comparative static exercises have essentially the same effects (this will not be the case once we add disagreement). The initial belief is given by [cf. (20)],

$$E_1 [g_1] = \mathbf{c}_{0,1} (g + u_0) + (1 - \mathbf{c}_{0,1}) g_0. \tag{28}$$

Therefore, either change induces an increase in optimism about aggregate demand in period 1, $\Delta E_1 [g_1] > 0$. Using Eq. (27), the induced increase in optimism shifts the forward and the dot curves upward by the same amount,

$$\frac{\Delta E_1^M \left[r_t^f \right]}{\Delta E_1 [g_1]} = \frac{\Delta E_1^F \left[r_t^f \right]}{\Delta E_1 [g_1]} = 1 \text{ for } t \geq 1. \tag{29}$$

Moreover, the current interest rate increases one-to-one with the induced increase in optimism,

$$\frac{\Delta r_1^f}{\Delta E_1 [g_1]} = 1. \tag{30}$$

5. Monetary policy with belief disagreements

We next introduce our main ingredient, belief disagreements about aggregate demand. In this section, we maintain the assumption that agents have the same level of confidence. In the next section, we consider the more general case with heterogeneous confidence. We start by presenting and deriving our main result, which describes how disagreements affect optimal monetary policy and generate differences in forward and dot curves. We then establish the comparative statics of optimism and demand shocks and contrast the results with the benchmark case with common beliefs.

Proposition 1. *Suppose $C_0^F = C_0^M = C_0$ and consider arbitrary u_0^F and u_0^M so the Fed and the market can disagree about aggregate demand.*

(i) The equilibrium price and the interest rate are given by

$$\begin{aligned} q_t &= q^* + g_t - E_t^F [g_t], & (31) \\ r_t^f &= \rho + (1 - \mathbf{c}_{t,t+1}) E_t^F [g_t] + \mathbf{c}_{t,t+1} E_t^M [g_t] - \frac{\Sigma_{t+1}}{2}. & (32) \end{aligned}$$

The optimal interest rate reflects in part the market's belief, with a weight that depends on relative confidence between the current and the next period, $\mathbf{c}_{t,t+1}$. For sufficiently high initial *confidence, the interest rate reflects* ***only the market's*** *belief,* $\lim_{C_0\to\infty} r_t^f = \rho + E_t^M [g_t] - \frac{\Sigma_{t+1}}{2}$.

(ii) The forward and dot curves in period 1 *are given by*

$$E_1^M \left[r_t^f\right] = \rho + E_1^M [g_1] - \frac{\Sigma_{t+1}}{2} - \mathbf{c}_{1,t} (1 - \mathbf{c}_{t,t+1}) \left(E_1^M [g_1] - E_1^F [g_1]\right), \tag{33}$$

$$E_1^F \left[r_t^f\right] = \rho + E_1^F [g_1] - \frac{\Sigma_{t+1}}{2} + \mathbf{c}_{1,t+1} \left(E_1^M [g_1] - E_1^F [g_1]\right). \tag{34}$$

Each curve reflects the ***corresponding agent's current belief*** *for aggregate demand, with an adjustment toward the other agent's belief that declines with horizon. For sufficiently long horizons, each curve reflects* ***only*** *the corresponding agent's belief,* $\lim_{t\to\infty} E_1^i \left[r_t^f\right] = \rho + E_1^i [g_1] - \frac{\Sigma}{2}$, *and the difference reflects the level of disagreement,* $\lim_{t\to\infty} E_1^M \left[r_t^f\right] - E_1^F \left[r_t^f\right] = E_1^M [g_1] - E_1^F [g_1]$.

The first part of the result characterizes the equilibrium asset price and the interest rate. The asset price is the same as in the benchmark case without disagreements [cf. (24)]. The interest rate is different and depends on a *weighted average of the Fed's and the market's beliefs about aggregate demand.* Unlike before, the Fed cannot set interest rates by focusing only on its own view of aggregate demand—it also needs to take into account the market's view and the extent of disagreement. In this sense, the Fed is "constrained" by disagreements with the market. Moreover, the weight on the market's belief is greater *when agents are more confident in their beliefs.* Hence, the more entrenched are belief disagreements, the more the Fed ignores *its own* beliefs. In the extreme, the interest rate *does not reflect the Fed's belief.*

The second part shows that, unlike the benchmark case, *the forward and dot curves trace out different expected interest rate paths* [Eq. (27)]. The forward curve reflects the market's belief for aggregate demand with an adjustment toward the Fed's belief—and vice versa for the dot curve. The adjustment terms decline with time horizon and eventually disappear. For sufficiently long horizons, each curve reflects the corresponding agent's current belief, and the difference between the curves reflects the extent of disagreement. Put differently, *belief disagreements manifest themselves as differences between the forward and dot curves.*

Sketch proof of Proposition 1, part (i). We next provide a sketch proof for the result, which is useful to provide intuition. To prove the first part, we conjecture that the asset price satisfies Eq. (31) for period $t+1$. We combine this conjecture with our general characterization from Section 3.2 and verify that the asset price also satisfies Eq. (31) in period t, with the equilibrium interest rate given by (32).

Our analysis in Section 3.2 shows that the equilibrium depends on the market's belief for the asset price, q_{t+1}. Using the conjectured price $q_{t+1} = q^* + g_{t+1} - E_{t+1}^F [g_{t+1}]$ (along with Lemma 1 for $t_2 = t_1$), we calculate:

$$\begin{aligned} E_{t+1}^M [q_{t+1}] &= q^* + E_{t+1}^M [g_{t+1}] - E_{t+1}^F [g_{t+1}] \\ var_t^M [q_{t+1}] &= var_{t+1}^M [g_{t+1}]. \end{aligned} \tag{35}$$

Unlike in the case with common beliefs, the market does not necessarily expect the asset price in the next period to be equal to q^* on average [cf. (25)]. Eq. (35) shows that the expected asset price depends on *expected future disagreements*. Intuitively, these disagreements capture the market's belief regarding how big a "mistake" the Fed will make in the next period. For instance, when $E_{t+1}^M [g_{t+1}] > E_{t+1}^F [g_{t+1}]$, the market thinks the Fed will be too pessimistic about future demand shocks and therefore will set low interest rates and stabilize asset prices at a high level (higher than q^*) on average.

Next note that Lemma 1 implies expected future disagreements satisfy

$$E_{t+1}^M [g_{t+1}] - E_{t+1}^F [g_{t+1}] = \mathbf{c}_{t,t+1} \left(E_t^M [g_t] - E_t^F [g_t] \right) \text{ for } t \geq 1. \tag{36}$$

Disagreements decline over time. Intuitively, agents are Bayesian and update their beliefs by observing *the same data*, $\overline{g}_{t-1}$, which reduces their disagreements. Moreover, disagreements are a *deterministic* function of time, because agents have the same level of confidence.

Combining Eqs. (35) and (36) with Eq. (17) from Section 3.2, we obtain Eq. (31) (see the appendix for details). The equilibrium price depends only on demand shocks, as in the benchmark case, because disagreements change deterministically and do not create additional uncertainty. We will see how this feature will change in the next section once we allow agents to have heterogeneous confidence. Moreover, by setting the interest rate appropriately, the Fed is still able to stabilize asset prices on average. Disagreements manifest themselves in the *interest rate* that the Fed must set to achieve this outcome.

Specifically, using Eq. (16) from Section 3.2, we calculate the equilibrium interest rates:

$$\begin{aligned} r_t^f &= \rho + E_t^F [g_t] + E_t^F \left[E_{t+1}^M [q_{t+1}] - q^* \right] - E_t^F \left[var_{t+1}^M [q_{t+1}] \right], \\ &= \rho + E_t^F [g_t] + E_{t+1}^M [g_{t+1}] - E_{t+1}^F [g_{t+1}] - \frac{\Sigma_{t+1}}{2}. \end{aligned} \tag{37}$$

Here, we have used Eq. (35) and dropped $E_t^F [\cdot]$ because the terms inside the expectation are deterministic. Intuitively, the equilibrium interest rate is affected by *expected future disagreements* [cf. Eq. (26)]. In particular, the market's expectation about the Fed's pessimism in the next period increases the interest rate in the current period. If the market thinks the Fed is too pessimistic, it also thinks the Fed will set low *future* rates. Therefore, the market expects high asset prices in the next period, $E_t^M [q_{t+1}]$, which exerts upward pressure on asset prices in the current period, q_t. Consequently, to stabilize asset prices around q^*, the Fed must increase the current interest rate, r_t^f.

Finally, substituting Eq. (36) into Eq. (37), we show that the interest rate is given by Eq. (32). Intuitively, the interest rate depends on a weighted average of current beliefs, because expected future disagreements depend on current disagreements. The weight on the market's belief depends on the relative confidence, $\mathbf{c}_{t,t+1}$, because this determines the extent to which disagreements in the next period $t+1$ will be smaller than in the current period t.

Sketch proof of part (ii). To prove the second part of the proposition, consider the forward curve. Taking the expectation of Eq. (32) according to the market's belief, we obtain

$$E_1^M\left[r_t^f\right] = \rho + (1 - \mathbf{c}_{t,t+1}) E_1^M\left[E_t^F[g_t]\right] + \mathbf{c}_{t,t+1} E_1^M[g_t] - \frac{\Sigma_{t+1}}{2}. \tag{38}$$

Substituting the higher order belief from Lemma 1, we prove Eq. (33). A symmetric argument establishes Eq. (34). The rest of the proof is straightforward and relegated to the appendix.

For intuition, note that as t increases, the higher order belief in Eq. (38), $E_1^M\left[E_t^F[g_t]\right]$, monotonically converges to $E_1^M[g_1]$ [cf. Lemma 1]. The market expects the Fed to learn over time and to converge to the market's belief. Therefore, the market expects future interest rates to be determined by its current belief, $E_1^F[g_1]$.

Finally, it is worth noting that the expressions in Proposition 1 together with the characterization of beliefs in Section 3.3 provide a closed-form solution for the interest rate as well as its expectations in terms of the initial beliefs, u_0^M, u_0^F, and the average past realization of aggregate demand, $\overline{g}_{t-1}$ (see Appendix A.3).

Comparative statics of optimism. To gain further intuition for the mechanisms that "constrain" the Fed, we consider the analogue of the comparative statics exercises we analyzed for the benchmark case with common beliefs. We start by considering an increase in initial optimism. In this case, since the Fed and the market do not necessarily agree, we increase the Fed's and the market's optimism one at a time.

First suppose the Fed becomes more optimistic without a change in the market's belief, $\Delta u_0^F > \Delta u_0^M = 0$. This increases the Fed's optimism in period 1 by $\Delta E_1^F[g_1] = \mathbf{c}_{0,1}\Delta u_0^F$ [cf. (20)]. Using Eqs. (33) and (34), the impact on the dot and forward curves is given by

$$\frac{\Delta E_1^F\left[r_t^f\right]}{\Delta E_1^F[g_1]} = 1 - \mathbf{c}_{1,t+1} \geq \frac{\Delta E_1^M\left[r_t^f\right]}{\Delta E_1^F[g_1]} = \mathbf{c}_{1,t}(1 - \mathbf{c}_{t,t+1}) \quad \text{for } t \geq 1. \tag{39}$$

As before, the Fed-optimism shock shifts both curves upward, but with greater effects on the dot curve [cf. Eq. (29)]. This gap arises because the Fed expects the market to learn. Hence, over longer horizons, the Fed expects to be able to set interest rates that reflect its increased optimism (whereas the market expects the *Fed* will learn instead). The top panels of Figure 4 illustrate the effects of a Fed-optimism shock on the forward and dot curves for a particular parameterization.

To calculate how much the current interest rate (in period 1) changes, note that Eq. (39) implies

$$\frac{\Delta r_1^f}{\Delta E_1^F[g_1]} = 1 - \mathbf{c}_{1,2}. \tag{40}$$

Unlike before, the interest rate in period 1 increases less than one-to-one with the size of the shock [cf. (39)]. Since disagreements are somewhat persistent, the market's expected interest rates

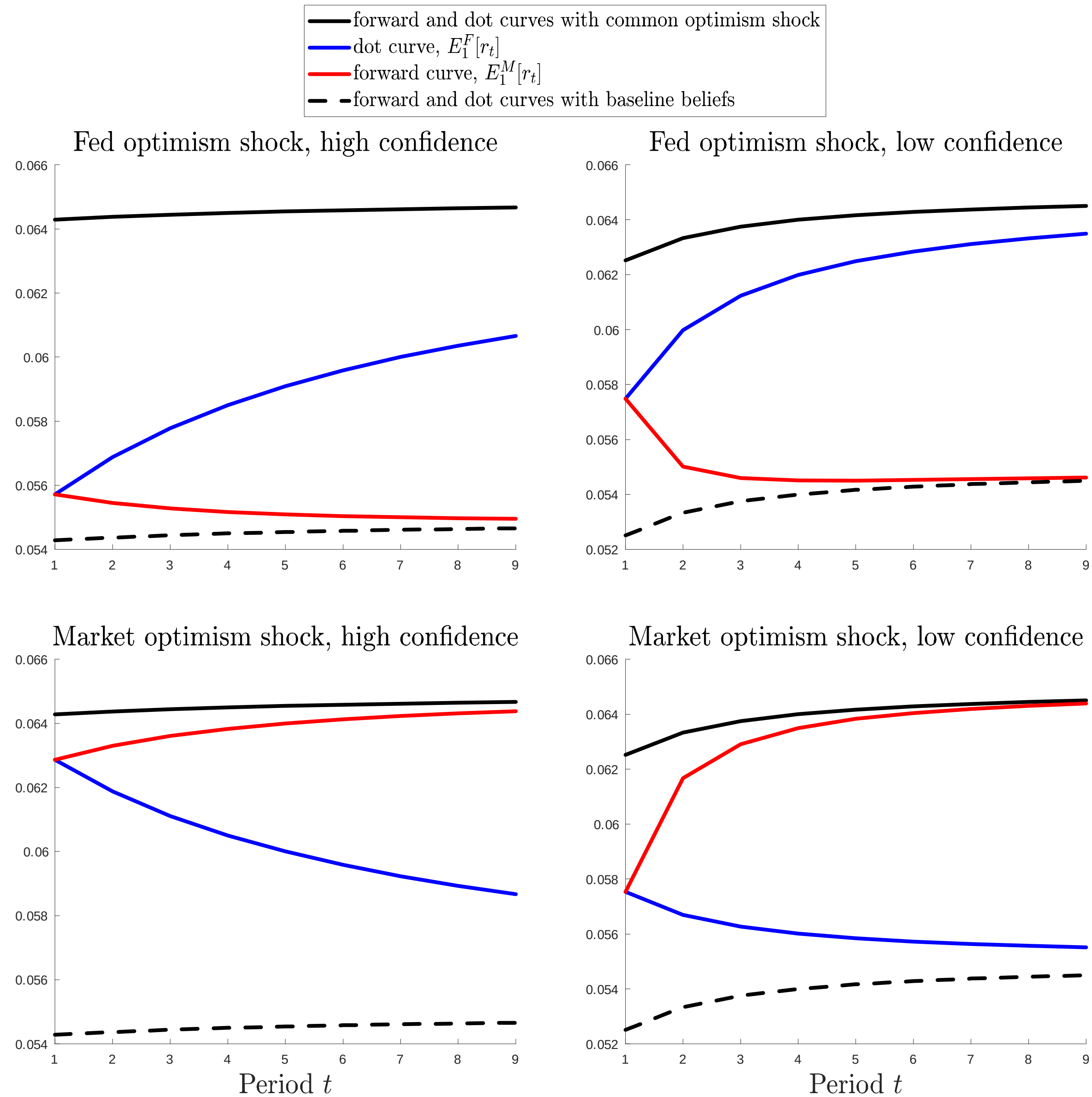

Figure 4: Top panels illustrate the effects of a Fed optimism shock without a change in market optimism. Bottom panels illustrate the market optimism shock. Left and right panels correspond to higher and lower levels of initial confidence, C_0. We normalize the shock so that the induced change in optimism for aggregate demand in period 1 is the same in all cases, $\Delta E_1[g_1] = \mathbf{c}_{0,1}\Delta u_0 = 0.01$.

beyond period 1 also increase (see Figure 4). Moreover, the market considers these increases a "mistake." These mistakes decrease expected future asset prices, which in turn exert downward pressure on current asset prices. Hence, even though the Fed is optimistic in period 1, it only needs to increase the current interest rate slightly to achieve its target asset price. Consistent with this intuition, the Fed increases the interest rate less when $\mathbf{c}_{1,2}$ is greater because in that case disagreements are more persistent. The top right panel of Figure 4 illustrates the effects of reducing initial confidence, C_0. This reduces $\mathbf{c}_{1,2} = \frac{C_0+1}{C_0+2}$ and enables the Fed to set a higher interest rate.

Next suppose the market becomes more optimistic without a change in the Fed's belief so that $\Delta u_0^M > \Delta u_0^F = 0$. This increases the market's optimism in period 1 by $\Delta E_1^M [g_1] = \mathbf{c}_{0,1} \Delta u_0^M$ [cf. (20)]. Using Eqs. (33) and (34), the impact on the forward and dot curves is given by

$$\frac{\Delta E_1^M \left[r_t^f\right]}{\Delta E_1^M [g_1]} = 1 - \mathbf{c}_{1,t} + \mathbf{c}_{1,t+1} \geq \frac{\Delta E_1^F \left[r_t^f\right]}{\Delta E_1^M [g_1]} = \mathbf{c}_{1,t+1} \quad \text{for } t \geq 1. \tag{41}$$

An increase in the market's optimism also shifts both curves upward, but with larger effects on the forward curve (see the bottom panels of Figure 4).

Using Eq. (41), we also calculate the effect on the current interest rate (in period 1) as

$$\frac{\Delta r_1^f}{\Delta E_1^M [g_1]} = \mathbf{c}_{1,2}. \tag{42}$$

The Fed raises the interest rate in response to the market's optimism about aggregate demand even though its own belief did not change. Since disagreements are persistent, the market expects the Fed to be too pessimistic and to set interest rates too low in future periods. In particular, although expected future interest rates increase, the market thinks these increases are insufficient (see Figure 4). These "mistakes" increase expected future asset prices, which in turn exert upward pressure on current asset prices. Therefore, the Fed is forced to increase the interest rate to achieve its target asset price. Consistent with this intuition, the Fed increases the interest rate more when $\mathbf{c}_{1,2}$ is greater because in that case disagreements are more persistent (see the bottom right panel of Figure 4).

As the final exercise of this section, consider an increase in the initial demand shock, $\Delta g_0 > 0$. With common confidence, both agents' optimism increases by the same amount, $\Delta E_1^i [g_1] \equiv \Delta E_1 [g_1] = (1 - \mathbf{c}_{0,1}) \Delta g_0$ [cf. Eq. (28)]. Therefore, Eqs. (29) and (30) from the common-belief benchmark continue to hold. We revisit this conclusion in the next section where we allow for heterogeneous confidence.

6. Monetary policy with heterogeneous data sensitivity

We next consider the general case in which agents have heterogeneous confidence levels, C_0^F and C_0^M, as well as heterogeneous initial beliefs, u_0^F and u_0^M. In this case, the Fed and the market have *heterogeneous data sensitivity.* Specifically, $C_0^i > C_0^j$ implies $\mathbf{c}_{s,t}^i > \mathbf{c}_{s,t}^j$ for each $t > s$: higher *initial* confidence implies a higher *relative* confidence at all times [cf. (18)]. In view of Eq. (20), this implies that agents put heterogeneous weights on new realizations of aggregate demand. In particular, the agent with smaller confidence puts greater weight on new data.

Our main result in this section shows that this type of heterogeneity in data sensitivity changes the relationship between asset prices and macroeconomic shocks. We start by presenting this result. We then illustrate the comparative statics of demand shocks and contrast the effects with the case with common data sensitivity. Overall, our analysis in this section shows that, if the Fed becomes more data sensitive (while keeping the market's data sensitivity unchanged), then demand shocks have a greater effect on the dot and forward curves (as well as on the gap between them), but a dampened effect on asset prices. The opposite happens if the Fed becomes less data sensitive.

Proposition 2. *Consider arbitrary $C_0^F, C_0^M, u_0^F, u_0^M$ so that the Fed and the market can have heterogeneous data sensitivity as well as heterogeneous initial beliefs about aggregate demand.*

The equilibrium price is given by

$$q_t = q^* + \mathbf{D_t}\left(g_t - E_t^F[g_t]\right). \tag{43}$$

Here, $\{\mathbf{D_t}\}_t$ is a deterministic sequence that captures ***the price impact of demand shocks.*** *It is the unique solution to the difference equation:*

$$\mathbf{D_t} = 1 + \left(\mathbf{c}_{t,t+1}^F - \mathbf{c}_{t,t+1}^M\right)\mathbf{D}_{t+1} \text{ with } \lim_{t\to\infty}\mathbf{D_t} = 1. \tag{44}$$

The solution satisfies $\mathbf{D_t} > 0$: above-average shocks create positive price impact. If the Fed is more data sensitive than the market, $C_0^F < C_0^M$ (resp. less data sensitive than the market $C_0^F > C_0^M$), then the price impact is dampened $\mathbf{D_t} < 1$ (resp. amplified $\mathbf{D_t} > 1$) relative to the case with common data sensitivity.

The equilibrium interest rate is given by

$$r_t^f = \rho + \left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)E_t^F[g_t] + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M E_t^M[g_t] - \frac{\mathbf{D}_{t+1}^2\Sigma_{t+1}^M}{2}. \tag{45}$$

The forward and dot curves in period 1 are given by

$$E_1^M\left[r_t^f\right] = \rho + E_1^M[g_1] - \frac{\mathbf{D}_{t+1}^2\Sigma_{t+1}^M}{2} - \mathbf{c}_{1,t}^F\left(1 - \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)\left(E_1^M[g_1] - E_1^F[g_1]\right), \tag{46}$$

$$E_1^F\left[r_t^f\right] = \rho + E_1^F[g_1] - \frac{\mathbf{D}_{t+1}^2\Sigma_{t+1}^M}{2} + \mathbf{D}_{t+1}\mathbf{c}_{1,t+1}^M\left(E_1^M[g_1] - E_1^F[g_1]\right). \tag{47}$$

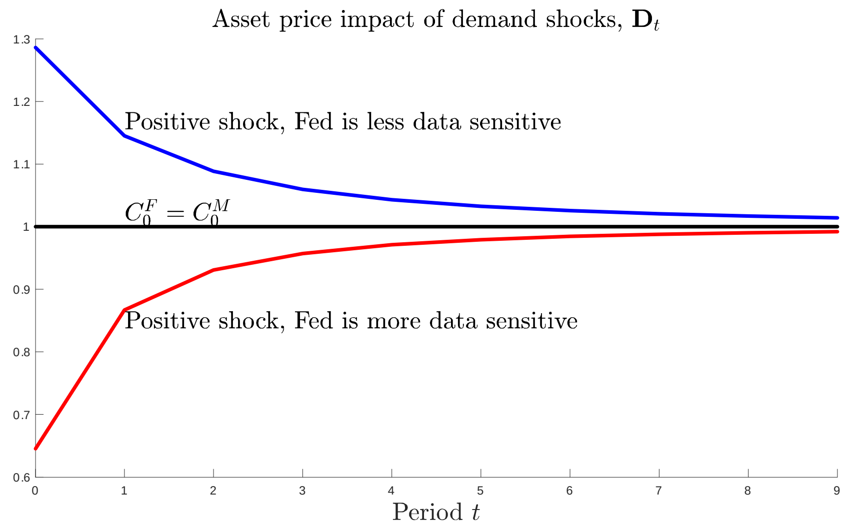

Figure 5: The equilibrium path for the price impact of demand shocks, $\{\mathbf{D}_t\}$, with heterogeneous data sensitivity.

The first part characterizes the equilibrium asset price. Common data sensitivity corresponds to the special case with $\mathbf{D_t} = 1$ [cf. Eq. (31)]. Hence, Eq. (43) shows that *heterogeneous data sensitivity changes the asset price impact of demand shocks.* If the Fed is more (less) data sensitive than the market, then demand shocks have a dampened (amplified) asset price impact. Figure 5 illustrates the equilibrium sequence, $\{\mathbf{D_t}\}$, for a particular parameterization and heterogeneous data sensitivity.

The rest of the result characterize the equilibrium interest rate as well as the forward and the dot curves. These expressions are similar to their counterparts with common data sensitivity [cf. Eqs. $(32-34)$]. The main difference concerns the variance term, $\mathbf{D}_{t+1}^2 \Sigma_{t+1}^M$, in the expressions for the interest rate. This term reflects the asset price variance induced by the demand variance. It illustrates that *heterogeneous data sensitivity affects asset price volatility and the risk premium*; and the equilibrium interest rate reflects this effect.

Sketch proof of Proposition 1. We next provide a sketch proof for the result, which is useful to understand why heterogeneous data sensitivity affects asset prices. The limit condition for the price impact holds, $\lim_{t\to\infty} \mathbf{D_t} = 1$, because in the long run disagreements disappear (due to learning) and the equilibrium approximates the benchmark case with common beliefs. We conjecture that the asset price satisfies Eq. (43) for period $t+1$. We then establish the same condition for period t (along with the other equilibrium conditions) proving the result by backward induction.

The key observation is that Eq. (43) and Eq. (20) together imply:

$$\begin{aligned} E_{t+1}^{M}\left[q_{t+1}\right] &= q^{*}+\mathbf{D}_{t+1}\left(E_{t+1}^{M}\left[g_{t+1}\right]-E_{t+1}^{F}\left[g_{t+1}\right]\right) \\ &= q^{*}+\mathbf{D}_{t+1}\left(\mathbf{c}_{t,t+1}^{M}E_{t}^{M}\left[g_{t}\right]-\mathbf{c}_{t,t+1}^{F}E_{t}^{F}\left[g_{t}\right]+\left[\begin{array}{c}\left(1-\mathbf{c}_{t,t+1}^{M}\right)\\ -\left(1-\mathbf{c}_{t,t+1}^{F}\right)\end{array}\right]g_{t}\right). \end{aligned} \tag{48}$$

Unlike the case with common data sensitivity, disagreement in the next period is stochastic and depends on the realization of current aggregate demand, g_t. Therefore, the market's expected price in the next period is also no longer deterministic [cf. Eqs. $(35-36)$]. For instance, when the Fed is more data sensitive, $1-\mathbf{c}_{t,t+1}^{F}>1-\mathbf{c}_{t,t+1}^{M}$, an increase in aggregate demand in the current period, g_t, leads to a *decrease* in the expected asset price in the next period. How does this happen? An increase in demand affects the extent of disagreement: it increases the Fed's optimism compared to the market, because the Fed reacts to the data more than the market. As this happens, the market anticipates a greater increase in interest rates—which it perceives to be a "mistake"—which leads to a lower asset price.

Next note that Eq. (17) describes the equilibrium price as a function of the surprises in the market's expected price, $E_{t+1}^{M}\left[q_{t+1}\right]-E_{t}^{F}\left[E_{t+1}^{M}\left[q_{t+1}\right]\right]$. Using Eq. (48), we obtain,

$$E_{t+1}^{M}\left[q_{t+1}\right]-E_{t}^{F}\left[E_{t+1}^{M}\left[q_{t+1}\right]\right]=\mathbf{D}_{t+1}\left[\begin{array}{c}\left(1-\mathbf{c}_{t,t+1}^{M}\right)\\ -\left(1-\mathbf{c}_{t,t+1}^{F}\right)\end{array}\right]\left(g_{t}-E_{t}^{F}\left[g_{t}\right]\right). \tag{49}$$

The surprise in the expected price depends on the surprise in the demand shock, $g_t-E_t^F\left[g_t\right]$, because this determines the extent of disagreement and thus the magnitude of the monetary policy "mistake' perceived by the market.

Eq. (43) implies the variance of the price is still deterministic. Then, substituting Eq. (49) into Eq. (17) from Section 3.2, we obtain

$$q_{t}=q^{*}+\left(1+\mathbf{D}_{t+1}\left[\begin{array}{c}\left(1-\mathbf{c}_{t,t+1}^{M}\right)\\ -\left(1-\mathbf{c}_{t,t+1}^{F}\right)\end{array}\right]\right)\left(g_{t}-E_{t}^{F}\left[g_{t}\right]\right).$$

Combining this with the difference equation (44), the price satisfies Eq. (43) also in period t.

It remains to show that the difference equation (44) has a unique solution $\{\mathbf{D_t}\}$. We establish this in Appendix A.4. We also show that the solution satisfies the properties in the proposition. In particular, $\mathbf{D_t}<1$ if and only if $C_0^F<C_0^M$. For intuition, suppose the Fed is more data sensitive than the market, $1-\mathbf{c}_{t,t+1}^{F}>1-\mathbf{c}_{t,t+1}^{M}$. In this case, the market expects the Fed to react to shocks more than (what the market thinks) is appropriate, which dampens the direct price impact of shocks [cf. (49)]. Conversely, when the Fed is less data sensitive, $1-\mathbf{c}_{t,t+1}^{F}<1-\mathbf{c}_{t,t+1}^{M}$, the market expects the Fed to underreact to shocks, which amplifies the direct price impact of shocks.

Finally, combining Eq. (49) with Eq. (16) from Section 3.2, we show that the interest rate satisfies Eq. (45). We also derive the forward and the dot curves by following similar steps as in the previous section (see Appendix A.4 for details).

Comparative statics of demand shocks. Next consider the comparative statics exercises for the expected interest rates that we analyzed earlier. Comparing Eqs. $(46-47)$ with Eqs. $(33-34)$ suggests that an increase in the Fed's or the market's optimism, $\Delta u_0^F > 0$ or $\Delta u_0^M > 0$, has similar effects on the forward and dot curves as in the case with common data sensitivity [cf. Figure 4].

In contrast, an increase in the initial demand shock, $\Delta g_0 > 0$, has distinct effects when agents have heterogeneous data sensitivity. Recall from Section 5 that, with common data sensitivity, these shocks shift the forward and the yield curve by the same amount. Intuitively, demand shocks *per se* do not create disagreement when agents have the same data sensitivity. With heterogeneous data sensitivity, this is no longer the case: *demand shocks create disagreement.* Therefore, demand shocks can induce a gap between the forward and the yield curves when there was none to begin with.

To see this, consider agents' beliefs in the first period, which are determined by the following analogue of Eq. (28):

$$E_1^i[g_1] = \mathbf{c}_{0,1}^i \left(g + u_0^i\right) + \left(1 - \mathbf{c}_{0,1}^i\right) g_0 \text{ for } i \in \{F, M\}. \tag{50}$$

Suppose the economy features, $u_0^F = u_0^M = 0$, and experiences the initial realization, $g_0 = g$, so that agents have the same beliefs in the next period, $E_1^F[g_1] = E_1^M[g_1] = g$. This ensures that the forward and dot curves in the next period are also the same [cf. Eqs. $(46-47)$.

Relative to this initial state, suppose the economy experiences a positive demand shock, $g_0 = g + \Delta g_0$ with $\Delta g_0 > 0$. First consider the case in which the Fed is more data sensitive, $1 - \mathbf{c}_{0,1}^F > 1 - \mathbf{c}_{0,1}^M$. In this case, Eq. (50) illustrates that a greater shock induces both agents to become more optimistic, but with a greater effect on the Fed, $\Delta E_1^F[g_1] > \Delta E_1^M[g_1] > 0$. Therefore, the curves shift as if there is a common-optimism shock, as in Section 4, combined with a Fed-optimism shock, as in Section 5. In particular, the dot curve increases more than the forward curve—especially for more distant horizons. The left panel of Figure 6 illustrates this case. Conversely, when the Fed is less data sensitive, $1 - \mathbf{c}_{0,1}^F < 1 - \mathbf{c}_{0,1}^M$, a greater demand shock has a larger effect on the market's optimism than the Fed's optimism, $\Delta E_1^M[g_1] > \Delta E_1^F[g_1] > 0$. Consequently, the forward curve increases more than the dot curve—especially for more distant horizons—as illustrated by the right panel of Figure 6.

This analysis also provides a complementary intuition for why heterogeneous data sensitivity changes the asset-price impact of demand shocks [cf. Figure 5]. When the Fed is more data sensitive than the market, a positive demand shock increases the dot curve more than the forward curve [cf. the left panel of Figure 6]. This generates a dampened increase in risky asset prices, because the market considers the Fed's additional reaction a mistake. Conversely, when the Fed

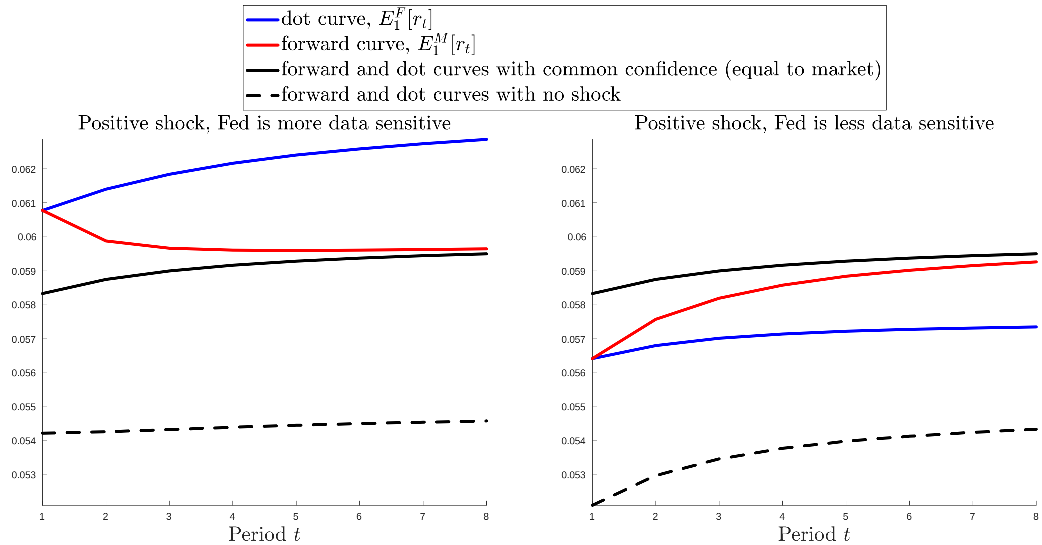

Figure 6: The comparative statics of an increase in the initial demand shock, $g_0 = g + \Delta g_0$ with $\Delta g_0 = 0.01$. The black solid lines illustrate the effects for a baseline case in which the Fed and the market have common data sensitivity. The left panel (resp. the right panel) illustrates the effects when the Fed has greater (resp. smaller) data sensitivity than in the baseline, while the market has the same data sensitivity as in the baseline.

is less data sensitive than the market, the shock increases the dot curve less than the forward curve [cf. the right panel of Figure 6]; and this induces an amplified effect on risky asset prices since the market considers the Fed's muted reaction a mistake.

7. Monetary policy signaling with disagreements

Our key ingredient is that the market and the Fed have belief disagreements. So far, we captured this ingredient by assuming that agents have no information asymmetry: they know each other's beliefs and "agree-to-disagree." While this assumption simplifies the analysis, it has the extreme feature that the market does not think the Fed has any information advantage. A large literature assumes the Fed has superior information and emphasizes the role of monetary policy in *signaling* this information to the market. In this section, we consider a version of our model in which the Fed has an information advantage. We show that our results still apply as long as agents have some heterogeneous prior beliefs (in addition to heterogeneous information).

We also analyze how belief disagreements affect the signaling mechanism. In an environment with belief disagreements, the market can also naturally disagree with the Fed's signal. Moreover, the Fed is unlikely to know ex ante whether the market will agree or disagree. This possibility leads to novel types of monetary policy shocks. If the market has disagreed with the signal, then an interest rate hike driven by a positive Fed signal has no impact on the market's optimism and reduces risky asset prices. If instead the market has agreed, then a similar interest rate *hike* increases the market's optimism and *increases* risky asset prices—as emphasized in the signaling literature. Hence, the asset price response to a monetary policy shock is informative about whether the market mostly agrees or mostly disagrees with the Fed's signal.[12]

7.1. Signaling with disagreements

Consider the same model with the only difference that in period 0 (and period 0 only) the Fed also receives a private signal about u (before the interest rate decision),

$$x^F = u + \varepsilon^F \text{, where } \varepsilon^F \sim N\left(0, I^{-1}\Sigma\right) \text{ is independent from } u. \tag{51}$$

Here, the market agrees with the Fed about the distribution of x^F conditional on u: that is, the market agrees that the Fed has an informative signal (the parameter I captures how informative is that signal). As before, agents start with prior beliefs, $N\left(u_0^i, C_0^{-1}\Sigma\right)$, and they know each others' prior beliefs. For simplicity, we focus on the case in which agents have common confidence, C_0. The rest of the model is unchanged.

In this setting, the Fed's *posterior belief* after observing the signal is given by $u \sim N\left(\overline{u}_0^F, \overline{C}_0^{-1}\Sigma\right)$, where

$$\overline{u}_0^F = \frac{C_0}{C_0+I}u_0^F + \frac{I}{C_0+I}x^F \text{ and } \overline{C}_0 = C_0 + I. \tag{52}$$

We conjecture an equilibrium in which the interest rate is a one-to-one (and increasing) function of the Fed's signal, x^F. Therefore, the equilibrium interest rate fully reveals the Fed's signal.

In the conjectured equilibrium, the market's posterior belief (conditional on observing the interest rate) is given by $u \sim N\left(\overline{u}_0^M, \overline{C}_0^{-1}\Sigma\right)$, where

$$\overline{u}_0^M = \frac{C_0}{C_0+I}u_0^M + \frac{I}{C_0+I}x^F. \tag{53}$$

Given posterior beliefs $\overline{u}_0^F, \overline{u}_0^M$, our earlier analysis applies. In particular, the equilibrium interest rate is given by

$$\begin{aligned} r_0^f &= \rho + g + \left(1 - \overline{\mathbf{c}}_{0,1}\right)\overline{u}_0^F + \overline{\mathbf{c}}_{0,1}\overline{u}_0^M - \frac{\Sigma_1}{2} \\ &= \rho + g + \frac{I}{C_0+I}x^F + \frac{C_0}{C_0+I}\left(\left(1-\overline{\mathbf{c}}_{0,1}\right)u_0^F + \overline{\mathbf{c}}_{0,1}u_0^M\right) - \frac{\Sigma_1}{2} \end{aligned} \tag{54}$$

[12] See Jarocinski and Karadi (2020) for evidence that not all central bank announcements are interpreted equally by the markets. They use high frequency data around these announcements to show that a surprise policy tightening can lead an increase or a decrease in stock prices, with very different macroeconomic effects. This evidence is consistent with the extension in this section. Stock prices decline when the market interprets the policy action as a hawkish Fed-belief shock (a disagreement shock, in our language). In contrast, stock prices increase when the market agrees with the Fed and interprets the policy action as as a positive signal on the state of the economy.

where $\overline{\mathbf{c}}_{0,1} = \frac{\overline{C}_0}{\overline{C}_0+1} = \frac{C_0+I}{C_0+I+1}$ [cf. (18)]. Here, the second line substitutes Eqs. $(52-53)$.

Hence, the equilibrium rate reflects the Fed's information, x^F, as well as belief disagreements. In extreme cases in which either $I \to \infty$ (the Fed is very informative) or $C_0 \to 0$ (agents are very underconfident in initial beliefs), the signal reflects only the Fed's information. In these cases, the Fed sets an interest rate that fully reflects its posterior belief. The market observes this decision and updates its belief—as emphasized by the signaling literature (e.g., Nakamura and Steinsson (2018)). Aside from the extreme cases, our earlier results still hold. The Fed's information advantage mitigates but does not overturn our mechanisms.[13]

7.2. Signaling with uncertainty and monetary policy shocks

In this setup, since the market starts with a different initial belief than the Fed, the market might also disagree about the informativeness of the Fed's signal. Importantly, the Fed might not know ex ante whether the market will agree or not. This possibility allows for novel types of monetary policy shocks.

To illustrate, suppose the market has one of two types, MA and MD, with ex-ante probabilities, α and $1-\alpha$, respectively. If the market is type MA ("agreeable"), then it agrees that the Fed has an informative signal as before. If instead the market is type MD ("disagreeable"), then it believes the Fed has no superior information: that is, it believes x^F is independent from u. In this case, the market also observes a signal that it believes is distributed according to,

$$x^{MD} = u + \varepsilon^{MD}, \text{ where } \varepsilon^{MD} \sim N\left(0, I^{-1}\Sigma\right) \text{ is independent from } u.$$

Suppose the Fed always observes x^{MD} but it believes it is independent from u. Therefore, the market of type MD and the Fed "agree to disagree" about the informativeness of their signals.[14] The Fed sets r_0^f under uncertainty about the market's type. The rest of the model is unchanged.

In this case, the equilibrium asset price is still determined by Eq. (15) after appropriately adjusting the expectations. Specifically, the asset price in period 0 depends on the market's posterior belief in period 0. If the market has type MA, then its posterior belief is the same as before, which we denote by $\overline{u}_0^{MA}$ [cf. (53)]. In contrast, if the market has type MD, then its posterior belief is $u \sim N\left(\overline{u}_0^{MD}, \overline{C}_0^{-1}\Sigma\right)$, where

$$\overline{u}_0^{MD} = \frac{C_0}{C_0+I}u_0^M + \frac{I}{C_0+I}x^{MD}.$$

[13] In this context, an interesting question is whether the Fed might have an incentive to adjust the interest rate to "distort" the market's posterior belief. In fact, given the equilibrium relationship in (54), if the Fed were to set rates higher than the equilibrium rate, then the market would infer greater x^F and would become more optimistic. In our setting, the Fed does not have an incentive to distort the market's belief—even if the market has a different belief than the Fed. This is because the Fed's objective is to stabilize output and it can equivalently achieve its objective (on average) by adjusting the interest rate. See Stein and Sunderam (2018) for a related model in which the Fed distors the market's belief, because it has an additional objective to reduce interest rate volatility.

[14] Giving the market its own signal does not play an important role beyond simplifying the expressions.

Note that $\overline{u}_0^{MA} \neq \overline{u}_0^{MD}$ (except for the non-generic case $x^{MD} = x^F$ that we ignore). Therefore, the asset price in period 0 reveals the market's type. Hence, starting period 1 onward, the equilibrium is the previous sections where the Fed has initial beliefs $\overline{u}_0^F$ and the market has initial beliefs either $\overline{u}_0^{MA}$ or $\overline{u}_0^{MD}$ depending on its type. In particular, the asset price in the next period is given by Eq. (31) regardless of the market's type (since r_1^F adjusts appropriately to ensure this outcome).

Next note that the interest rate in period 0 is still determined by Eq. (16) after accounting for the Fed's ex-ante expectations about the market's type. Following similar steps as before, and using $q_1 = q^* + g_1 - E_1^F[g_1]$, we calculate:

$$r_0^f = \rho + g + \frac{I}{C_0 + I}\left(\begin{array}{c} (1 - \overline{\mathbf{c}}_{0,1}(1-\alpha))\, x^F \\ + \overline{\mathbf{c}}_{0,1}(1-\alpha)\, x^{MD} \end{array}\right) + \frac{C_0}{C_0 + I}\left(\begin{array}{c} (1 - \overline{\mathbf{c}}_{0,1})\, u_0^F \\ + \overline{\mathbf{c}}_{0,1} u_0^M \end{array}\right) - \frac{\Sigma_1}{2}. \tag{55}$$

This expression illustrates that the Fed puts some weight on the disagreeable market's signal even though it thinks the signal is uninformative. This is for the same reason as in our baseline analysis. Unlike in our baseline analysis, the equilibrium asset price also depends on the realization of the market's type. Specifically, using (15), we obtain

$$q_0 = q^* + g_0 - E_0^F[g_0] + \begin{cases} \overline{\mathbf{c}}_{0,1}\alpha\left(x^{MD} - x^F\right) & \text{if type } MD \\ \overline{\mathbf{c}}_{0,1}(1-\alpha)\left(x^F - x^{MD}\right) & \text{if type } MA \end{cases}.$$

If the market is disagreeable, then it does not "buy" the Fed's signal. In this case, if the Fed's signal is relatively optimistic ($x^F > x^{MD}$), the interest rate is ex-post too high and the asset price declines. These episodes resemble monetary policy shocks of the following kind: the Fed increases interest rates in response to a positive signal, but the market does not consider the signal to be informative so asset prices fall.

Conversely, if the market is agreeable, then it "buys too much" into the Fed's signal. In this case, if the Fed's signal is relatively optimistic ($x^F > x^{MD}$), the interest rate is ex-post too low and the asset price rallies. These episodes resemble monetary policy shocks of a different kind: the Fed increases the interest rate in response to a positive signal and this makes the market so optimistic that asset prices actually increase.

8. Final remarks

We provide a model for analyzing monetary policy, expected interest rates, and asset prices when the Fed and the market do not see eye-to-eye. The model explains the large disagreements between the FOMC predictions and the forward rates. In the model, rstar (the natural interest rate) reflects the extent of disagreement as well as how entrenched each agent's views are. The larger the disagreement and the more entrenched the views, the more the Fed needs to weight the market's views in order to stabilize asset markets and aggregate demand. Despite this accommodation, in this environment the market sees every macroeconomic shock as the combination of the shock itself and an expected "policy mistake." This anticipation of policy-response mistakes changes the relationship between macroeconomic shocks and asset prices. When the Fed's belief is more sensitive to data, perceived monetary policy mistakes dampen the direct effect of shocks. Conversely, when the Fed's belief is less sensitive to data, the perceived monetary policy mistakes amplify shocks. These regimes also influence macroeconomic and asset price volatility, and the risk premium.

There are a number of ways to extend our model. For example, we adopted a stylized view where the Fed controls aggregate demand by supporting wealth, and the policy rate matters only through this channel. That is, in our model the Fed is willing to overweight the market's views when setting interest rates because it sees no cost in interest rate swings as long as they yield asset prices consistent with potential output. Of course, one could imagine contexts in which interest rates affect aggregate demand through other channels. The flavor of our main results would still hold in this case, but the Fed would put more weight on its own view for the policy interest rate, at the cost of introducing more volatility in risky asset markets.

We took the Fed's beliefs and confidence as primitives. In future research we will explore whether it is dynamically optimal for the Fed to choose its *effective* confidence, C_0^F, in order to control, for example, the market's risk premium. Similarly, effective confidence could be affected by constraints such as an effective lower bound or a gradualist policy framework.

Finally, in our version with disagreements and signaling, we assigned a constant probability to the Fed meeting an agreeable or disagreeable response from the market. In practice, it is highly likely that this probability is a function of the degree of ex-ante disagreement. If so, the Fed could become extremely reluctant to attempt signaling during periods of large ex-ante disagreement, which would represent yet another form of market's constraint on the Fed.

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A. Appendix: Omitted derivations

This appendix presents the derivations and proofs omitted from the main text.

A.1. Omitted derivations in Section 3

A.1.1. The portfolio problem and optimality conditions

The representative household's portfolio problem (in some period t) can be written as,

$$V_t(a_t) = \max_{\{c_{\tilde{t}}, \omega_{\tilde{t}}\}_{\tilde{t}=t}^{\infty}} \overline{E}_t^M \left[\sum_{\tilde{t}=t}^{\infty} e^{-\rho \tilde{t}} \log c_{\tilde{t}} \right] \tag{A.1}$$
$$a_{\tilde{t}+1} = (a_{\tilde{t}} - c_{\tilde{t}}) \left(\omega_{\tilde{t}} \exp\left(r_{\tilde{t},\tilde{t}+1}\right) + (1 - \omega_{\tilde{t}}) \exp\left(r_{\tilde{t}}^f\right) \right) \text{ for each } \tilde{t} \geq t.$$

Here, we used a_t to denote investors' total (cum-dividend) wealth at the end of period t. In every period, the representative household chooses how much to save, $a_t - c_t$, and what fraction of its savings to allocate to the market portfolio, ω_t. Since there is a representative household, asset market equilibrium implies

$$a_t = y_t + Q_t A_t. \tag{A.2}$$

As before, the equilibrium also implies $\omega_t = 1$ [cf. (7)].

The Bellman equation corresponding to problem $(A.1)$ is given by:

$$V_t(a_t) = \max_{c_t, \omega_t} \log(c_t) + e^{-\rho} \overline{E}_t^M \left[V_{t+1} \left((a_t - c_t) \left(\omega_t \exp(r_{t,t+1}) + (1 - \omega_t) \exp\left(r_t^f\right) \right) \right) \right]. \tag{A.3}$$

We conjecture a solution in which the value function takes the form

$$V_t(a_t) = \frac{\log(a_t)}{1 - e^{-\rho}} + v_t.$$

The first term in the value function captures the effect of holding greater wealth, which scales the investor's consumption proportionally at all times and states. The second term, v_t, is the normalized value function when the investor holds one unit of wealth ($a_t = 1$). This functional form implies,

$$\frac{\partial V_t^i}{\partial a_t} = \frac{1}{1 - e^{-\rho}} \frac{1}{a_t}. \tag{A.4}$$

First consider the optimality condition for c_t in the Bellman equation. Using Eq. $(A.4)$, we obtain:

$$\frac{1}{c_t} = \frac{e^{-\rho}}{1 - e^{-\rho}} \frac{1}{a_t - c_t}, \text{ which implies } c_t = \left(1 - e^{-\rho}\right) a_t. \tag{A.5}$$

Substituting the asset market clearing condition $(A.2)$, Eq. $(A.5)$ implies Eq. (10) from the main text.

Next consider the optimality condition for ω_t in the Bellman equation Using Eq. $(A.4)$, we obtain:

$$\overline{E}_t^M \left[\frac{\exp(r_{t,t+1}) - \exp\left(r_t^f\right)}{\omega_t \exp(r_{t,t+1}) + (1 - \omega_t) \exp\left(r_t^f\right)} \right] = 0.$$

In equilibrium, we also have $\omega_t = 1$ [cf. (7)] so this condition can be further simplified as:

$$\overline{E}_t^M \left[\exp\left(r_t^f - r_{t,t+1}\right)\right] = 1. \tag{A.6}$$

When $r_{t,t+1}$ is normally distributed, which is the case in our analysis, we have that $\exp\left(r_t^f - r_{t,t+1}\right)$ is log-normally distributed. Therefore, $\overline{E}_t^M \left[\exp\left(r_t^f - r_{t,t+1}\right)\right] = \exp\left(r_t^f - \overline{E}_t^M \left[r_{t,t+1}\right] + \frac{\overline{var}_t^M[r_{t,t+1}]}{2}\right)$. Substituting this into Eq. $(A.6)$, we obtain Eq. (14) in the main text. This completes the characterization of the representative household's optimality conditions.

A.1.2. New Keynesian microfoundation for nominal rigidities

The supply side of our model features nominal rigidities similar to the standard New Keynesian setting. There is a continuum of measure one of monopolistically competitive production firms denoted by ν. These firms own the capital stock (in equal proportion) and produce differentiated goods, $y_t(\nu)$, subject to the technology,

$$y_t(\nu) = \eta_t(\nu) A_t. \tag{A.7}$$

Here, $\eta_t(\nu) > 0$ denotes the firm's choice of capital utilization. We assume utilization is free (see Appendix B for a version with costly utilization). The production firms sell their output to a competitive sector that produces the final output according to the CES technology,

$$y_t = \left(\int_0^1 y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)},$$

for some $\varepsilon > 1$. Thus, the demand for the firms' goods implies,

$$y_t(\nu) \leq p_t(\nu)^{-\varepsilon} y_t, \text{ where } p_t(\nu) = P_t(\nu)/P_t. \tag{A.8}$$

Here, $p_t(\nu)$ denotes the firm's relative price, which depends on its nominal price, $P_t(\nu)$, as well as the ideal nominal price index, $P_t = \left(\int P_t(\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$.

Firms have preset prices that are equal to one another, $P_t(\nu) = P$. This also implies the relative price of a firm is fixed and equal to one, $p_t(\nu) = 1$. The firm chooses factor utilization $\eta_t(\nu) > 0$, to maximize its earnings, $y_t(\nu)$, subject to the supply constraint in $(A.7)$ and the demand constraint, $(A.8)$. The solution is given by $\eta_t(\nu) = \frac{y_t}{A_t}$: that is, each firm's production is determined by aggregate demand. Intuitively, since capital utilization is free, the marginal cost of production is zero and firms produce according to aggregate demand. Therefore, aggregate output is also determined by aggregate demand, which verifies (8) in the main text.

Proof of Lemma 1. Note that the identities trivially hold when $t_2 = t_1$. Suppose $t_2 > t_1$. Consider $i, j \in \{F, M\}$ where we allow j to be the same as i. Consider the second line of Eq. (20) for agent j. By repeatedly applying the equation and using Eq. (18), we obtain:

$$E_{t_2}^j [g_{t_2}] = \mathbf{c}_{t_1,t_2}^j E_{t_1}^j [g_{t_1}] + \left(1 - \mathbf{c}_{t_1,t_2}^j\right) \overline{g}_{t_1,t_2-1},$$

where $\overline{g}_{t_1,t_2-1} = \sum_{\tilde{t}=t_1}^{t_2-1} g_{\tilde{t}}/n$ denotes the average realization between periods t_1 and $t_2 - 1$. Considering agent i's expectation of this expression in period t_1, we obtain:

$$
\begin{aligned}
E_{t_1}^i\left[E_{t_2}^j\left[g_{t_2}\right]\right] &= \mathbf{c}_{t_1,t_2}^j E_{t_1}^j\left[g_{t_1}\right] + \left(1-\mathbf{c}_{t_1,t_2}^j\right) E_{t_1}^i\left[\overline{g}_{t_1,t_2-1}\right] \\
&= \mathbf{c}_{t_1,t_2}^j E_{t_1}^j\left[g_{t_1}\right] + \left(1-\mathbf{c}_{t_1,t_2}^j\right) E_{t_1}^i\left[g_{t_1}\right]. \qquad \text{(A.9)}
\end{aligned}
$$

Here, the first line uses the observation that $E_{t_1}^j\left[g_{t_1}\right]$ is known in period t_1. The second line observes that, for each $\tilde{t} \in \{t_1, .., t_2-1\}$, we have

$$
E_{t_1}^i\left[g_{\tilde{t}}\right] = E_{t_1}^i\left[u+v_{\tilde{t}}\right] = E_{t_1}^i\left[u+v_{t_1}\right] = E_{t_1}^i\left[g_{t_1}\right].
$$

Here, the equality in the middle follows since $v_{\tilde{t}}$ and v_{t_1} both have zero mean. Applying Eq. $(A.9)$ for $j=i$ implies Eq. (22). Applying it for $j \neq i$ implies Eq. (23) and completes the proof. □

A.2. Omitted derivations in Section 4

Most of the derivations are presented in the main text. Here, we provide a closed-form solution for the interest rate as well as its expectations in period 1.

First consider the interest rate. Combining Eq. (26) with the characterization of beliefs in Section 3.3 (for the special case with common initial beliefs), we obtain:

$$
\begin{aligned}
r_t^f &= \mathbf{r}_t^f\left(\overline{g}_{t-1}\right) + \mathbf{c}_{0,t} u_0 \text{ for } t \geq 1, \qquad \text{(A.10)} \\
\text{where } \mathbf{r}_t^f\left(\overline{g}_{t-1}\right) &= \rho + \mathbf{c}_{0,t} g + \left(1-\mathbf{c}_{0,t}\right) \overline{g}_{t-1} - \frac{\Sigma_{t+1}}{2}. \qquad \text{(A.11)}
\end{aligned}
$$

Here, the term $\mathbf{r}_t^f\left(\overline{g}_{t-1}\right)$ captures how agents' learning affects the interest rate. In particular, $\mathbf{r}_t^f\left(\overline{g}_{t-1}\right)$ is an increasing function. Intuitively, a greater past realization of aggregate demand, $\overline{g}_{t-1}$, increases optimism. Thus, the Fed needs to set a higher interest rate to stabilize asset prices on average. The term, $\mathbf{c}_{0,t} u_0$, captures how optimism about the persistent shock affects the interest rate. An increase in initial optimism, u_0, increases the interest rate in all periods but with a decreasing effect over time due to agents' learning.

Next consider the expected path of interest rates in period 1. Combining Eqs. (27) and (28), we obtain

$$
E_1^M\left[r_t^f\right] = E_1^F\left[r_t^f\right] = \rho + \mathbf{c}_{0,1}\left(g+u_0\right) + \left(1-\mathbf{c}_{0,1}\right) g_0 - \frac{\Sigma_{t+1}}{2} \text{ for } t \geq 1. \qquad \text{(A.12)}
$$

As we discussed in the main text, an increase in initial optimism, u_0, or initial news, g_0, shifts the forward and the yield curve upward by an equal amount.

A.3. Omitted derivations in Section 5

Proof of Proposition 1, part (i). Most of the proof is presented in the main text. Here, we complete the derivation of Eq. (31). Note that Eq. (17) implies

$$
\begin{aligned}
q_t &= q^* + g_t - E_t^F\left[g_t\right] \\
&\quad + \left(E_{t+1}^M\left[q_{t+1}\right] - E_t^F\left[E_{t+1}^M\left[q_{t+1}\right]\right]\right) - \frac{1}{2}\left(var_{t+1}^M\left[q_{t+1}\right] - E_t^F\left[var_{t+1}^M\left[q_{t+1}\right]\right]\right).
\end{aligned}
$$

Note also that Eqs. (35) and (36) imply $E_{t+1}^{M}[q_{t+1}]$ and $var_{t+1}^{M}[q_{t+1}]$ are both deterministic. Therefore $E_t^F\left[E_{t+1}^{M}[q_{t+1}]\right] = E_{t+1}^{M}[q_{t+1}]$ and $E_t^F\left[var_{t+1}^{M}[q_{t+1}]\right] = var_{t+1}^{M}[q_{t+1}]$. This establishes Eq. (31) and completes the proof of the first part. □

Proof of Proposition 1, part (ii). The derivation of the forward curve is presented in the main text. Here, we derive the dot curve, and we establish the limit results as $t \to \infty$. Taking the expectation of Eq. (32) according to the Fed's belief, we obtain,

$$\begin{aligned} E_1^F\left[r_t^f\right] &= \rho + (1-\mathbf{c}_{t,t+1}) E_1^F[g_t] + \mathbf{c}_{t,t+1} E_1^F\left[E_1^M[g_t]\right] - \frac{\Sigma_{t+1}}{2} \\ &= \rho + E_1^F[g_1] - \frac{\Sigma_{t+1}}{2} + \mathbf{c}_{1,t+1}\left(E_1^M[g_1] - E_1^F[g_1]\right). \end{aligned}$$

Here, the second line uses Lemma 1 and Eq. (19). This proves Eq. (34).

To derive the limit results, note that $\mathbf{c}_{1,t+1} = \frac{C_0+1}{C_0+t+1}$. Thus, $\mathbf{c}_{1,t+1}$ is decreasing in horizon t with $\lim_{t\to\infty} \mathbf{c}_{1,t+1} = 0$. This implies $\lim\limits_{t\to\infty} E_1^F\left[r_t^f\right] = \rho + E_1^F[g_1] - \frac{\Sigma_{t+1}}{2}$. Likewise, note that

$$(1-\mathbf{c}_{t,t+1})\,\mathbf{c}_{1,t} = \left(1 - \frac{C_0+t}{C_0+t+1}\right)\frac{C_0+1}{C_0+t} = \frac{1}{C_0+t+1}\frac{C_0+1}{C_0+t}.$$

Thus, $(1-\mathbf{c}_{t,t+1})\,\mathbf{c}_{1,t}$ is decreasing in horizon t with $\lim_{t\to\infty}(1-\mathbf{c}_{t,t+1})\,\mathbf{c}_{1,t} = 0$. This implies $\lim\limits_{t\to\infty} E_1^M\left[r_t^f\right] = \rho + E_1^F[g_1] - \frac{\Sigma_{t+1}}{2}$, completing the proof. □

Closed-form solutions. We next present a closed-form solution for the interest rate as well as its expectations in period 1.

First consider the interest rate. Combining Eq. (32) with the beliefs in (20), we obtain:

$$\begin{aligned} r_t^f &= \rho + (1-\mathbf{c}_{t,t+1}) E_t^F[g_t] + \mathbf{c}_{t,t+1} E_t^M[g_t] - \frac{\Sigma_{t+1}}{2} \\ &= \mathbf{r}_t^f(\overline{g}_{t-1}) + \mathbf{c}_{0,t}\left((1-\mathbf{c}_{t,t+1})\,u_0^F + \mathbf{c}_{t,t+1} u_0^M\right). \end{aligned} \tag{A.13}$$

Comparing this expression with Eq. $(A.10)$ illustrates that the equilibrium interest rate is similar to the case with common beliefs. The difference is that the interest rate depends on a weighted average of the Fed's and the market's initial belief for the unknown component of demand (see the main text for the intuition).

Next consider the expected path of interest rates in period 1. Note that agents' beliefs in period 1 is determined by the following analogue of Eq. (28):

$$E_1^i[g_1] = \mathbf{c}_{0,1}\left(g + u_0^i\right) + (1-\mathbf{c}_{0,1})\,g_0 \text{ for } i \in \{F, M\}. \tag{A.14}$$

Eq. $(A.14)$. Substituting these expressions into Eqs. (33) and (34), we obtain the following closed form solution for each $t \geq 1$:

$$E_1^M\left[r_t^f\right] = \rho + \mathbf{c}_{0,1}\left(g + u_0^M\right) + (1-\mathbf{c}_{0,1})\,g_0 - \frac{\Sigma_{t+1}}{2} - \mathbf{c}_{0,t}(1-\mathbf{c}_{t,t+1})\left(u_0^M - u_0^F\right), \tag{A.15}$$

$$E_1^F\left[r_t^f\right] = \rho + \mathbf{c}_{0,1}\left(g + u_0^F\right) + (1-\mathbf{c}_{0,1})\,g_0 - \frac{\Sigma_{t+1}}{2} + \mathbf{c}_{0,t+1}\left(u_0^M - u_0^F\right). \tag{A.16}$$

Comparative statics. The comparative statics of optimism follow from Proposition 1. In particular, Eqs. $(39-40)$ are implied by Eqs. (33) and (34) (after using the identity (19) for relative confidence).

The comparative statics of demand shocks follow from Eq. $(A.14)$, which implies $\Delta E_1^i [g_1] \equiv \Delta E_1 [g_1] = (1 - \mathbf{c}_{0,1}) \Delta g_0$. Combining this with the closed-form solution in Eqs. $(A.15 - A.16)$, we establish that Eqs. (29) and (30) from the common-belief benchmark continue to hold.

A.4. Omitted derivations in Section 6

Proof of Proposition 2. Most of the proof is presented in the main text. We characterize the solution to the difference equation (44), $\{\mathbf{D_t}\}$, and establish its properties. We then characterize the equilibrium interest rate as well as the forward curve and the dot curve.

To solve the difference equation, we first rewrite it as:

$$\mathbf{D_t} = 1 + \mathbf{C}_t \mathbf{D}_{t+1} \text{ with } \lim_{t\to\infty} \mathbf{D}_t = 1, \tag{A.17}$$

where we define

$$\mathbf{C}_t = \mathbf{c}^F_{t,t+1} - \mathbf{c}^M_{t,t+1}. \tag{A.18}$$

As a first step, we claim:

$$|\mathbf{C}_t| \leq |\mathbf{C}_0| < 1 \text{ for each } t. \tag{A.19}$$

To prove this claim, we observe that $\mathbf{c}^i_{t,t+1} = \frac{C_0^i + t}{C_0^i + t + 1}$ implies:

$$\mathbf{C}_t = \frac{C_0^F - C_0^M}{\left(C_0^F + t + 1\right)\left(C_0^M + t + 1\right)}.$$

This in turn proves the claim since,

$$|\mathbf{C}_t| \leq \frac{\left|C_0^F - C_0^M\right|}{\left(C_0^F + 1\right)\left(C_0^M + 1\right)} = |\mathbf{C}_0| = \left|\mathbf{c}^F_{0,1} - \mathbf{c}^M_{0,1}\right| < 1.$$

Here, the last inequality follows since $\mathbf{c}^F_{0,1}, \mathbf{c}^M_{0,1} \in (0,1)$.

We next iterate Eq. $(A.17)$ forward to obtain:

$$\mathbf{D_t} = 1 + \mathbf{C}_t + \mathbf{C}_t\mathbf{C}_{t+1} + \mathbf{C}_t\mathbf{C}_{t+1}\mathbf{C}_{t+2} + ... + \mathbf{C}_t\mathbf{C}_{t+1}...\mathbf{C}_{t+n} + \mathbf{C}_t\mathbf{C}_{t+1}...\mathbf{D}_{t+n}.$$

Taking the limit of the expression as $n \to \infty$ and using $\lim_{n\to\infty} \mathbf{D}_{t+n} = 1$ and $|\mathbf{C}_t| \leq |\mathbf{C}_0| < 1$ [cf. $(A.19)$], we obtain:

$$\mathbf{D_t} = 1 + \sum_{\tilde{t}=t}^{\infty} \mathbf{C}_t\mathbf{C}_{t+1}...\mathbf{C}_{\tilde{t}}. \tag{A.20}$$

Note also that $|\mathbf{C}_t| \leq |\mathbf{C}_0| < 1$ implies $|\mathbf{D_t}| \leq \frac{1}{1-|\mathbf{C}_0|}$. Thus, there exists a unique and finite solution characterized by $(A.20)$.

Next consider the properties of the solution. We first show that $\mathbf{D}_t > 0$ for each t. We have $\mathbf{D}_T > 0$ for each sufficiently large T (since $\lim_{t\to\infty} \mathbf{D}_t = 1$). Suppose this is true for some $t+1$. Then, Eq. $(A.17)$ implies:

$$\mathbf{D_t} = 1 + \mathbf{C}_t\mathbf{D}_{t+1} > 1 + \mathbf{C}_t > 0,$$

where the second equality follows since $|\mathbf{C}_t| < 1$ [cf. $(A.19)$]. By induction, this proves $\mathbf{D}_t > 0$ for each t.

We next prove that the solution satisfies the property,

$$\mathbf{D_t} < 1 \text{ if and only if } C_0^F < C_0^M. \tag{A.21}$$

First consider the case $C_0^F < C_0^M$. This implies $\mathbf{c}_{t,t+1}^F < \mathbf{c}_{t,t+1}^M$ for each t, which in turn implies $\mathbf{C}_t < 0$ for each t. We claim $\mathbf{D}_t \in (0,1)$ for each t. To show this, first note that for an arbitrary $\varepsilon > 0$ we have $\mathbf{D}_T \in (0, 1+\varepsilon)$ for sufficiently large T (since $\lim_{t\to\infty} \mathbf{D}_t = 1$). Let $\varepsilon = 1/|\mathbf{C}_0| - 1$ and note that it is strictly positive [cf. $(A.19)$]. Suppose $\mathbf{D}_{t+1} \in (0, 1+\varepsilon)$ for some $t+1$. Then, Eq. $(A.17)$ implies:

$$\mathbf{D_t} = 1 + \mathbf{C}_t \mathbf{D}_{t+1} < 1 + |\mathbf{C}_t| (1+\varepsilon) \leq 1 + |\mathbf{C}_0| (1+\varepsilon) = 1.$$

Here, the last inequality uses Eq. $(A.19)$. Thus, for any $\mathbf{D}_{t+1} \in (0, 1+\varepsilon)$, we obtain $\mathbf{D_t} \in (0,1) \subset (0, 1+\varepsilon)$. By induction, this proves $\mathbf{D_t} \in (0,1)$ for each t.

Next consider the case $C_0^F > C_0^M$. This implies $\mathbf{c}_{t,t+1}^F > \mathbf{c}_{t,t+1}^M$ for each t, which in turn implies $\mathbf{C}_t > 0$ for each t. In this case, the closed-form solution in $(A.20)$ implies $\mathbf{D}_t > 1$ for each t. This establishes $(A.21)$ and proves that the difference equation in (44) has a unique solution with the properties described in the proposition.

Next consider the equilibrium interest rate. Using Eq. (48), we obtain

$$E_t^F \left[E_{t+1}^M [q_{t+1}] - q^* \right] = \mathbf{c}_{t,t+1}^M \mathbf{D}_{t+1} \left(E_t^M [g_t] - E_t^F [g_t] \right).$$

Note also that $var_{t+1}^M [q_{t+1}] = \mathbf{D}_{t+1}^2 \Sigma_{t+1}$, which is deterministic. Substituting these observations into Eq. (16) from Section 3.2, we solve for the equilibrium interest rate as:

$$r_t^f = \rho + E_t^F [g_t] - \frac{\mathbf{D}_{t+1}^2 \Sigma_{t+1}}{2} + \mathbf{D}_{t+1} \mathbf{c}_{t,t+1}^M \left(E_t^M [g_t] - E_t^F [g_t] \right).$$

Rewriting this expression proves Eq. (45).

Next consider the forward and the dot curves. Taking the expectation of Eq. (45) according to the market's belief, we obtain,

$$\begin{aligned}
E_1^M \left[r_t^f \right] &= \rho + \left(1 - \mathbf{D}_{t+1} \mathbf{c}_{t,t+1}^M\right) E_1^M \left[E_t^F [g_t] \right] + \mathbf{D}_{t+1} \mathbf{c}_{t,t+1}^M E_1^M [g_1] - \frac{\mathbf{D}_{t+1}^2 \Sigma_{t+1}^M}{2} \\
&= \rho + \left(1 - \mathbf{D}_{t+1} \mathbf{c}_{t,t+1}^M\right) \left(\begin{array}{c} \mathbf{c}_{1,t}^F E_1^F [g_1] \\ + \left(1 - \mathbf{c}_{1,t}^F\right) E_1^M [g_1] \end{array} \right) + \mathbf{D}_{t+1} \mathbf{c}_{t,t+1}^M E_1^M [g_1] - \frac{\mathbf{D}_{t+1}^2 \Sigma_{t+1}^M}{2} \\
&= \rho + E_1^M [g_1] - \frac{\mathbf{D}_{t+1}^2 \Sigma_{t+1}^M}{2} + \mathbf{c}_{1,t}^F \left(1 - \mathbf{D}_{t+1} \mathbf{c}_{t,t+1}^M\right) \left(E_1^F [g_1] - E_1^M [g_1] \right).
\end{aligned}$$

Here, the second line uses Lemma 1 to evaluate the higher order belief. The last line establishes Eq. (46).

Likewise, taking the expectation of Eq. (45) according to the Fed's belief, we obtain,

$$\begin{aligned}
E_1^F\left[r_t^f\right] &= \rho + \left(1-\mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)E_1^F\left[g_1\right] + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M E_1^F\left[E_t^M\left[g_t\right]\right] - \frac{\mathbf{D}_{t+1}^2\Sigma_{t+1}^M}{2} \\
&= \rho + \left(1-\mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M\right)E_1^F\left[g_1\right] + \mathbf{D}_{t+1}\mathbf{c}_{t,t+1}^M \left(\begin{array}{c} \mathbf{c}_{1,t}^M E_1^M\left[g_1\right] \\ +\left(1-\mathbf{c}_{1,t}^M\right)E_1^F\left[g_1\right] \end{array}\right) - \frac{\mathbf{D}_{t+1}^2\Sigma_{t+1}^M}{2} \\
&= \rho + E_1^F\left[g_1\right] - \frac{\mathbf{D}_{t+1}^2\Sigma_{t+1}^M}{2} + \mathbf{D}_{t+1}\mathbf{c}_{1,t+1}^M\left(E_1^M\left[g_1\right]-E_1^F\left[g_1\right]\right).
\end{aligned}$$

This establishes (47) and completes the proof of the proposition. □

B. Appendix: Model with costly capital utilization

Our baseline model assumes capital utilization is free and the Fed targets a particular level of capital utilization, $\eta_t = 1$. While this simplifies the analysis, it leaves the welfare losses from utilizing capital above the target level ($\eta_t > 1$) unmodeled. In this appendix, we analyze a model in which capital utilization is costly. We show that (under appropriate normalization) $\eta_t = 1$ corresponds to the efficient level of utilization. We also show that (up to a linear approximation) the characterization of equilibrium is very similar to the characterization of the baseline model we presented in Sections 3.1 and 3.2. In particular, we obtain qualitative analogues of Eqs. (16) and (17) that characterize, respectively, the equilibrium interest rate and the equilibrium price. Since our results rely on these equations, it follows that our results qualitatively hold also in this version of the model with costly capital utilization.

B.1. Environment and equilibrium with costly utilization

Throughout, we focus on the components that are different than the baseline setting described in Section 3.

Capital utilization. In this version of the model, output is given by [cf. (1)],

$$y_t = \eta_t A_t k_t. \tag{B.1}$$

In the main text, we assumed there is no investment or depreciation and we normalized capital to one, $k_t = 1$. To introduce depreciation, we now explicitly keep track of capital. To parallel the main text, we also refer to $A_t k_t$ as *potential output*. We also denote the ex-dividend price of the market portfolio with $Q_t A_t k_t$. Hence, Q_t denotes the price of the market portfolio per potential output as in the main text.

The main difference is that capital depreciates according to an endogenous rate, $\delta(\eta_t)$. As before, productivity grows at the exogenous rate, g_t. Therefore, potential output evolves according to the following analogue of (2):

$$\log(A_{t+1}k_{t+1}) = \log(A_t k_t) + g_t - \delta(\eta_t). \tag{B.2}$$

Efficient level of utilization and output. As a benchmark, consider the efficient level of capital utilization and output. We analyze this frictionless benchmark in Section B.3 at the end of this appendix. Specifically, we consider a scenario with no nominal rigidities and no other distortions (in particular, monopoly distortions are corrected by appropriate subsidies). We show that firms' optimal choice of factor utilization satisfies:

$$\delta'(\eta_t) Q_t = 1. \tag{B.3}$$

Hence, efficient utilization ensures that the marginal depreciation rate is equal to the marginal product of capital.

To simplify this expression further, note that most of the analysis in Section 3.1 continues to apply in this case. In particular, the output-asset price relation in Eq. (11). Substituting $y_t = \eta_t A_t k_t$, this implies:

$$\eta_t = \frac{1 - e^{-\rho}}{e^{-\rho}} Q_t. \tag{B.4}$$

Combining Eqs. $(B.3)$ and $(B.4)$, we find that the efficient level of utilization solves:

$$\delta'(\eta^*)\,\eta^* = \frac{1-e^{-\rho}}{e^{-\rho}}. \tag{B.5}$$

We assume the depreciation function satisfies the following.

Assumption 1^{U}. $\delta(\eta)$ is strictly increasing and convex over $\mathbb{R}_+$. In addition, $\delta'(1) = \frac{1-e^{-\rho}}{e^{-\rho}}$.

The first part of the assumption ensures that there is a unique solution to $(B.5)$. The second part normalizes the efficient utilization to one, $\eta^* = 1$.

Nominal rigidities. Set against this benchmark, we consider firms that are subject to nominal rigidities similar to the standard New Keynesian setting. We relegate the details to Section B.3 at the end of this appendix. There is a continuum of monopolistically competitive production firms that own the capital stock and produce intermediate goods (which are then converted into the final good). For simplicity, these production firms have pre-set nominal prices that never change. The firms choose their capital utilization rate, $\eta_t > 0$, subject to the individual-firm analogues of Eqs. $(B.1)$ and $(B.2)$. These features ensure that output is determined by aggregate demand as long as the firm's price is above its marginal cost. Specifically, we obtain the following analogue of Eq. (8) in the main text,

$$y_t = c_t \text{ as long as } \frac{\varepsilon}{\varepsilon-1} \geq \delta'(\eta_t)\,Q_t. \tag{B.6}$$

Comparing Eqs. $(B.3)$ and $(B.6)$ illustrates that output is determined by aggregate demand in a neighborhood of the efficient benchmark. For simplicity, we consider the limit, $\varepsilon \to 1$, so that output is always determined by aggregate demand: that is, we recover Eq. (8) in the main text.

Monetary policy. The Fed attempts to ensure utilization and output are equal to their efficient levels on average. With Assumption 1^{U}, the efficient outcome is $\eta_t = 1$ and $y_t = A_t k_t$. Therefore, the Fed sets r_t^f to ensure output equals its potential on average as in the main text [cf. (9)]:

$$E_t^F[\log y_t] = \log A_t k_t.$$

The rest of the model is the same as in Section 3. The equilibrium is defined analogously to Definition 1.

B.2. Characterization of equilibrium with costly utilization

We next provide a partial characterization of the equilibrium. We show that (up to a linear approximation) Eqs. (16) and (17) from the main text remain qualitatively unchanged.

Equilibrium in the goods market. As we already noted, the output-asset price relation (11) also holds in this case. As before, this implies we can think of monetary policy as stabilizing the price of the market portfolio per potential output [cf. Eq. (12)]:

$$E_t^F[q_t] = q^* \text{ where } q^* = \log\frac{e^{-\rho}}{1-e^{-\rho}}. \tag{B.7}$$

Here, q^* is the level of the price that ensures factor utilization (and output) is at its efficient level, $\eta^* = 1$ [cf. $(B.4)$].

As before, we can write the payoff from the market portfolio that includes both output and the current asset price level as

$$y_{t+1} + Q_{t+1}A_{t+1}k_{t+1} = e^{\rho}Q_{t+1}A_{t+1}k_{t+1}.$$

Substituting this into Eq. (5), we obtain the following expression for the equilibrium return on the market portfolio:

$$\begin{aligned} r_{t,t+1} = \log \frac{e^{\rho}Q_{t+1}A_{t+1}k_{t+1}}{Q_t A_t k_t} &= \rho + g_t - \delta(\eta_t) + q_{t+1} - q_t \\ &= \rho + g_t + q_{t+1} - q_t - \delta\left(\frac{1-e^{-\rho}}{e^{-\rho}}\exp(q_t)\right) \end{aligned} \tag{B.8}$$

Here, the first line uses Eq. $(B.2)$ and the second line substitutes η_t in terms of the price using Eq. $(B.4)$.

Eq. $(B.8)$ illustrates that the one-period-ahead return in this case is slightly different than in the main text [Eq. (13)]. In particular, an increase in the price per unit of potential output leads not only to a direct reduction in the one-period-ahead return but also an indirect reduction as it increases capital depreciation (due to greater utilization). This introduces a nonlinearity that complicates the analysis.

For analytical tractability (and to ensure continuity with the main text), we proceed by considering a linear approximation. Specifically, linearizing Eq. $(B.8)$ around the efficient price level $q_t = q^*$, we obtain:

$$\begin{aligned} r_{t,t+1} &\simeq \rho + g_t + q_{t+1} - q_t - \delta^* - D(q_t - q^*), \\ \text{where } \delta^* &= \delta(\eta^*) \text{ and } D = \delta'(\eta^*). \end{aligned} \tag{B.9}$$

Here, the derived parameter δ^* denotes the depreciation rate associated with the efficient level of utilization ($\eta^* = 1$). The derived parameter $D > 0$ captures the marginal impact of capital utilization on depreciation around the efficient level. For the rest of the analysis, we assume investors choose their portfolios as if the return is given by the linear approximation in $(B.9)$.

Equilibrium in asset markets. The linear approximation ensures that the one-period-ahead market return follows a log-normal distribution. Consequently, the equilibrium return satisfies Eq. (14) from the main text:

$$\overline{E}_t^M[r_{t,t+1}] = r_t^f + \frac{\overline{var}_t^M[r_{t,t+1}]}{2}.$$

As before, the discount rate depends on the risk-free rate and the variance of the return on the market portfolio. Combining this expression with Eq. $(B.9)$, we also solve for the asset price. In particular, we obtain the following analogue of Eq. (15) from the main text:

$$(1+D)q_t - q^*D = \rho + g_t - \delta^* + E_{t+1}^M[q_{t+1}] - r_t^f - \frac{var_{t+1}^M[q_{t+1}]}{2}. \tag{B.10}$$

As before, the equilibrium price increases in the dividend yield (ρ), in the growth rate (g_t), in the expected price in the next period ($E_t^M[q_{t+1}]$); and decreasing in the risk-free interest rate (r_t^f) and in the variance of the price in the next period ($var_t^M[q_{t+1}]$). The only difference is that changes in these variables have a dampened effect on the asset price. This is because any increase in the asset price is also associated with an increase in depreciation (due to greater capital utilization) that reduces the expected return and mitigates the increase in the price.

To solve the equilibrium further, note that the Fed sets the interest rate to ensure $E_t^F[q_t] = q^*$ [cf. Eq. $(B.7)$]. Combining this observation with Eq. (15), we solve for the interest rate:

$$r_t^f = \rho + E_t^F[g_t] - \delta^* + E_t^F\left[E_{t+1}^M[q_{t+1}]\right] - q^* - \frac{E_t^F\left[var_{t+1}^M[q_{t+1}]\right]}{2}. \tag{B.11}$$

Substituting Eq. (16) into Eq. $(B.10)$, we also solve for the equilibrium price as:

$$q_t = q^* + \frac{1}{1+D}\left[\begin{array}{c} g_t - E_t^F[g_t] + \\ \left(E_{t+1}^M[q_{t+1}] - E_t^F\left[E_{t+1}^M[q_{t+1}]\right]\right) - \frac{1}{2}\left(var_{t+1}^M[q_{t+1}] - E_t^F\left[var_{t+1}^M[q_{t+1}]\right]\right)\end{array}\right]. \tag{B.12}$$

Comparing Eqs. $(B.11)$ and $(B.12)$ with their counterparts in the main text [cf. (16) and (17)] illustrates that the analysis remains largely unchanged. In particular, the expression for the risk-free rate is essentially the same as its counterpart in the main text (the only difference is the depreciation parameter, δ^*). The expression for the price is slightly different and features the dampening effect we discussed earlier. As before, the Fed attempts to neutralize shocks to asset valuations on average according to its belief. Any shock that it cannot neutralize affects the equilibrium price by a factor less than one $(1/(1+D))$.

Note also that all of our results in the main text rely on the partial characterization in Eqs. (16) and (17). Since Eqs. $(B.11)$ and $(B.12)$ are qualitatively the same, all of our results qualitatively generalize to this setting. For instance, it is easy to check that the equilibrium with common beliefs satisfies [cf. Eqs. (24) and (26) in Section 4]:

$$\begin{aligned} q_t &= q^* + \frac{1}{1+D}\left(g_t - E_t^F[g_t]\right) \\ r_t^f &= \rho + E_t^F[g_t] - \delta^* - \frac{\frac{1}{(1+D)^2} var_{t+1}^M[g_{t+1}]}{2}. \end{aligned}$$

In this case, the price reacts to changes in aggregate demand by less than one-to-one due to the dampening effect The interest rate is given by a similar expression as before with the difference that the variance of aggregate demand leads to a smaller risk premium due to the dampening effect.

Likewise, under Assumption D, the equilibrium with belief disagreements satisfies [cf. (31) and (37) in Section 5]:

$$\begin{aligned} q_t &= q^* + \frac{1}{1+D}\left(g_t - E_t^F[g_t]\right) \text{ for } t \geq 2, \\ \text{and } r_t^f &= \rho + E_t^F[g_t] + \frac{1}{1+D}\left(E_{t+1}^M[g_{t+1}] - E_{t+1}^F[g_{t+1}]\right) - \frac{\frac{1}{(1+D)^2} var_{t+1}^M[g_{t+1}]}{2} \quad \text{for } t \geq 2. \end{aligned}$$

As in the main text, the interest rate depends on expected future disagreements. The only difference is that future disagreements have a smaller impact on the interest rate due to the dampening effect. Therefore, our analysis in Section 5 remains qualitatively unchanged. As before, the interest rate is given by a weighted average of the Fed's and the market's beliefs. In this case, the weight on the market's belief is smaller and reflects the dampening effect (in addition to the confidence effect that we discussed in the main text).

B.3. New Keynesian microfoundation with costly utilization

In the rest of this appendix, we describe the firms' optimization problem in detail and derive Eqs. $(B.3)$ and $(B.6)$.

There is a continuum of measure one of monopolistically competitive production firms denoted by ν. These firms own the capital stock (in equal proportion) and produce differentiated goods, $y_t(\nu)$, subject to the technology,

$$y_t(\nu) = \eta_t(\nu) A_t k_t. \tag{B.13}$$

Here, $\eta_t(\nu) > 0$ denotes the firm's choice of capital utilization. Utilizing the capital at a higher rate leads to greater depreciation. Thus, potential output follows:

$$\log(A_{t+1}k_{t+1}(\nu)) = \log(A_t k_t(\nu)) + g_t - \delta(\eta_t(\nu)). \tag{B.14}$$

The firm maximizes its value subject to Eqs. $(B.13)$ and $(B.14)$ taking the price per capital (Q_t) as given. We will consider symmetric equilibria in which firms choose the same level of capital utilization and output so Eqs. $(B.13)$ and $(B.14)$ imply their aggregate counterparts in $(B.1)$ and $(B.2)$.

The production firm sells its output to a competitive sector that produces the final output according to the CES technology,

$$y_t = \left(\int_0^1 y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)},$$

for some $\varepsilon > 1$. Thus, the demand for the firms' goods implies,

$$y_t(\nu) \leq p_t(\nu)^{-\varepsilon} y_t, \text{ where } p_t(\nu) = P_t(\nu)/P_t. \tag{B.15}$$

Here, $p_t(\nu)$ denotes the firm's relative price, which depends on its nominal price, $P_t(\nu)$, as well as the ideal nominal price index, $P_t = \left(\int_0^1 P_t(\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$. We write the demand constraint as an inequality because the firm can in principle refuse to meet the demand for its goods.

Finally, we also assume there are linear subsidies designed to correct the inefficiencies that stem from the firm's monopoly power. In particular, the firm's revenue is given by $y_t(\nu) p_t(\nu)(1+\tau)$ where τ denotes a linear subsidy on output. These subsidies are financed by lump-sum taxes, that is, each firm pays $T = \tau \int_0^1 y_t(\nu) p_t(\nu) d\nu$. We set $1+\tau = \varepsilon/(\varepsilon-1)$ which will exactly neutralize the monopoly distortion.

We start by analyzing a frictionless benchmark without nominal rigidities. Specifically, suppose firms reset their price every period. Combining Eqs. $(B.13)$, $(B.14)$, and $(B.15)$, the firm's problem can be written as:

$$\begin{aligned} &\max_{p_t(\nu),\eta_t(\nu),y_t(\nu)} p_t(\nu) y_t(\nu)(1+\tau) - T + \exp(-\delta(\eta_t(\nu)))\exp(\delta(\eta_t)) Q_t A_t k_t, \\ &\text{s.t. } y_t(\nu) = \eta_t(\nu) A_t k_t \leq y_t p_t(\nu)^{-\varepsilon}. \end{aligned} \tag{B.16}$$

Here, the first term in the objective function captures the after-subsidy revenues. The second term captures the lump-sum tax. The last term captures the value of the firm's capital stock when it chooses the utilization rate, $\eta_t(\nu)$, and other firms choose the utilization rate, η_t. To understand this expression, note that $Q_t A_t$ is the price per unit of capital at the end of period t. This also incorporates depreciation that takes place between periods. Hence, $\exp(\delta(\eta_t)) Q_t A_t$ is the price per nondepreciated unit of capital at the beginning of the next period. Consequently, $\exp(-\delta(\eta_t(\nu)))\exp(\delta(\eta_t))Q_t A_t$ is the price per capital at the end of period t if the firm chooses the depreciation rate $\eta_t(\nu)$.

Note that the inequality constraint in (B.16) holds as equality (if not, the firm can always raise its price and increase profits). Using this observation, the optimality condition for $p_t(\nu)$ gives:

$$(1+\tau)\,p_t(\nu) = \frac{\varepsilon}{\varepsilon-1}\delta'(\eta_t(\nu))\exp(-\delta(\eta_t(\nu)))\exp(\delta(\eta_t))\,Q_t.$$

The firm ensures that its after-subsidy price is equal to a constant markup over marginal cost. The marginal cost depends on the marginal effect of factor utilization on depreciation, $\delta'(\eta_t(\nu))$. Substituting $1+\tau = \varepsilon/(\varepsilon-1)$, and using the observation that in a symmetric equilibrium all firms set the same price and allocation, $p_t(\nu) = p_t = 1, \eta_t(\nu) = \eta_t$, we obtain:

$$1 = \delta'(\eta_t)\,Q_t.$$

This proves Eq. (B.3) from Section B.2.

Set against this frictionless benchmark, the firms in our setting have preset nominal price are equal to one another, $P_t(\nu) = P$. This also implies the relative price of a firm is fixed and equal to one, $p_t(\nu) = 1$. Therefore, the firm solves the following analogue of problem (B.16):

$$\begin{aligned}&\max_{\eta_t(\nu),y_t(\nu)} y_t(\nu)(1+\tau) - T + \exp(-\delta(\eta_t(\nu)))\exp(\delta(\eta_t))\,Q_t A_t k_t, \\ &\text{s.t. } y_t(\nu) = \eta_t(\nu)\,A_t k_t \le y_t.\end{aligned} \tag{B.17}$$

Hence, the firm's only choice is whether to meet the demand at its fixed price. The firm will do so as long as its after-subsidy price $(1+\tau)$ is above the marginal cost, that is:

$$1+\tau \ge \delta'(\eta_t(\nu))\exp(-\delta(\eta_t(\nu)))\exp(\delta(\eta_t))\,Q_t.$$

Substituting $1+\tau = \varepsilon/(\varepsilon-1)$, and using the observation that in a symmetric equilibrium all firms set the same allocation, $\eta_t(\nu) = \eta_t$, we find that the firm meets the demand as long as:

$$\frac{\varepsilon}{\varepsilon-1} \ge \delta'(\eta_t)\,Q_t.$$

This proves Eq. (B.6) from Section B.2.

Comparing Eqs. (B.3) and (B.6) illustrates that in a neighborhood of the efficient benchmark output is determined by aggregate demand. For analytical tractability, we consider the limit in which markups are large, $\varepsilon \to 1$, in which case output is always determined by aggregate demand. In particular, we recover Eq. (8) from the main text.