# What do classroom spending decisions reveal about university objectives? 

James Thomas*

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#### Abstract

Every semester, a university decides which undergraduate courses to offer and how much to spend on instructors for these courses. These choices determine how efficiently resources from governments, donors, and families are used to benefit students; however, very little is known about how universities make these decisions. In this paper, I develop methods for understanding how universities make these classroom spending decisions. The methods include a test of whether a university's goal is to maximize student welfare and two methods for estimating parameters of a 'student driven' structure of university objectives. I apply my methods to administrative data from the University of Central Arkansas (UCA) and find that UCA has institutional preferences for decreasing enrollment in introductory business courses and increasing enrollments in introductory humanities and STEM courses. One counter-factual simulation shows a revenue neutral tax and subsidy policy which reduces the cost of offering introductory business courses and increases the cost of offering other introductory courses can induce UCA to offer courses that maximize student welfare. A second counter-factual simulation shows UCA could achieve the same student welfare at $38.5 \%$ of original costs in the absence of contractual constraints.


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## 1 Introduction

An important component of every university's mission is to educate undergraduates. To meet this component of their missions, universities decide which undergraduate courses to offer and how much to spend on instructors for these courses subject to budgetary and contractual constraints. Universities make these constrained decisions to maximize their objectives. These objectives illustrate what the university values and determines how the university responds to changes in constraints. This paper asks: Can these objectives be revealed by observing classroom spending decisions?

Objectives are fundamental to all economic analyses; however, very little evidence exists on the objectives of universities. ${ }^{1}$ This is surprising because universities are very important social institutions. There is abundant evidence that post-secondary outcomes have lasting effects on students in the labor market. ${ }^{2}$ Furthermore, there is growing evidence that institutional choices have important effects on post-secondary outcomes. ${ }^{3}$ Largely due to the value of undergraduate education in the labor market, large sums are spend on undergraduate education every year. In 2011, spending on post-secondary education comprised $2.7 \%$ of the United States gross domestic product (OECD, 2014). A better understanding of university objectives could be used to devise policies which lead universities to make decisions which benefits students and save money for taxpayers, families, and donors.

In this paper, I develop several tools for inferring university objectives from classroom spending decisions. I begin by developing a theoretical framework for analyzing classroom spending decisions. The framework casts these spending decisions as a sequential game between universities and students. In the first stage, universities observe constraints and the composition of the student body and decide which courses to offer and how much to spend on instructors for offered courses. In the second stage, students observe course offerings and spending on instruction and choose courses to maximize their utility. In a rational choice framework, the utility students achieve from these choices can be directly interpreted as a measure of welfare. This framework thus provides a link between classroom spending decisions and outcomes which may enter university objectives such as student course choices and welfare.

I use this theoretical framework to propose a method for statistically testing whether classroom spending decisions maximize student welfare. To develop this test, I derive the

[^1]first order conditions which define how much a welfare maximizing university would spend on instructors for offered courses. I then show that estimates of a course choice model and observed data on spending on instruction can be used to statistically test whether these first order conditions hold.

Next, I develop two methods for estimating parameters of a 'student driven' structure of university objectives. The student driven structure assumes universities receive payoffs from student welfare but permits the university to place different weights on the welfare of different students. This allows the university to value the welfare of certain students - such as upper class students or male students - relatively more than the welfare of other students. Furthermore, the structure also includes direct university preferences for class sizes. This allows the university to have institutional preferences for increasing enrollment in certain fields. ${ }^{4}$ This structure is student driven in the sense that university payoffs depend only on the academic experiences of students.

The first estimation method relates to a university's intensive margin decision of how much to spend on instructors for offered courses. First, I derive the first order conditions which define how much a student driven university would spend on instructors for offered courses. I then propose a variance minimization routine which solves for the parameter values which come closest to satisfying these first order conditions. These parameter estimates thus represent the values which best explain how much a university is observed spending on instructors for different courses.

The second estimation approach focuses on the university's extensive margin decision of which courses to offer. I propose a maximum likelihood estimator which solves for the parameter values which best explain why observed course offerings were preferred to all other feasible course offerings. The two alternative methods employ different empirical variation and have complementary strengths and weaknesses providing researchers with multiple tools for analysis.

I apply my inference methods using administrative data from the University of Central Arkansas. University of Central Arkansas (UCA) is a large public university in central Arkansas whose primary focus is teaching. UCA's teaching focus make analyzing the objectives underlying its classroom spending decisions especially interesting. The administrative data include information on all offered courses and information on the instructors teaching these courses between 1993 and 2013. Furthermore, the data include demographic information and full academic records for all students enrolled between 2004 and 2013. Importantly,

[^2]the data include instructor salaries and fraction of salaries paid for teaching. This allows me to connect the cost of offering courses to the effects of course offerings on students - a crucial link for inferring university objectives from observed classroom spending decisions.

The first stage of my empirical analysis is to estimate a multinomial choice model of students choosing courses. These estimates measure student preferences for course characteristics and estimate how much the desirability of a course increases when it is taught by a higher salaried instructor. To avoid issues of endogeneity and unobserved choice set heterogeneity my analysis focuses on choices of introductory courses. Estimates show introductory humanities courses are most popular with first year students while introductory business courses are most popular with sophomores, juniors and seniors. The estimates also show that students with higher ACT scores are relatively more attracted to introductory STEM courses. This corroborates existing literature which finds that initial preparation is an important determinant of whether a student pursues a STEM education (Arcidiacono, 2004; Stinebrickner and Stinebrickner, 2014). Finally, the estimates show that higher salaried instructors generally increase an introductory course's desirability but only to a small degree. This finding has important implications for universities: it implies that the vast amounts of resources spent hiring higher salaried instructors has relatively small effects on student course choices and student welfare.

The second stage of my analysis is to estimate parameters of a student driven objective structure taking student parameters as given. Preliminary analyses suggested the maximum likelihood estimation method was better suited for revealing UCA's objective parameters. I implement this method and find that UCA has institutional preferences for decreasing enrollment in introductory business courses and increasing enrollments in introductory humanities and STEM courses. This suggests UCA over invests in introductory STEM and humanities courses and under invests in introductory business courses relative to a university that is purely maximizing student welfare.

To place these estimates in context and to examine university behaviors under alternative constraints, I also develop tools for simulating classroom spending decisions under alternative objectives and constraints. I use these tools to examine two counter-factual simulations: First, I solve for counter-factual minimum costs which lead UCA to offer courses which maximize student welfare. The simulation suggests that a revenue neutral tax and subsidy policy which reduces the cost of offering introductory business courses and increases the cost of offering other introductory courses can induce UCA to offer courses which maximize student welfare at market costs. Second, I simulate course offerings and excess spending decisions which produce welfare efficiently in the absence of contractual constraints. This simulation shows UCA could achieve the same student welfare at $38.5 \%$ of original costs in
the absence of contractual constraints. While these scenarios may be undesirable for other reasons, it is useful to see how a revenue neutral policy could be used to benefit students and it is striking to see that students could receive the same benefit at drastically lower costs with changes in instructors and course composition.

Existing literature which models the behaviors of universities typically assumes a specific objective structure with little empirical justification for the choice. Epple, Romano, and Sieg (2006) examines admission, tuition, and financial aid behaviors of universities using a model which assumes a university's objective is to maximize its institutional quality. ${ }^{5}$ Hoxby (2012) seeks to understand why universities have endowments and assumes the university's objective is to maximize intellectual capital through instruction and research. Cyrenne and Grant (2009) investigate a model which assumes a university's objective is to maximize its reputation. There is clear disagreement on which objective structure is appropriate; furthermore, alternative objective structures may yield vastly different predictions. This highlights the need for additional analyses which reveal university objectives from observed behaviors.

Two papers which use observed behaviors to make inferences about university objectives are Bhattacharya, Kanaya, and Stevens (2014) and Turner (2014). Bhattacharya, Kanaya, and Stevens (2014) examines the admissions decisions of a selective British university and finds the university has lower admission thresholds for female and private school applicants. This suggests the university is interested in increasing the number of female and private school students in attendance. Turner (2014) examines the financial aid decisions of US colleges and finds schools are willing to pay an additional $\$ 284$ to have less privileged students attend their institution. This suggests the university is interested in increasing the number of less privileged students matriculating at their institution. My analysis uses completely different observed behaviors to infer objectives and thus complements these other works nicely.

The remainder of this paper proceeds as follows: Section 2 presents the theoretical framework for analyzing classroom spending decisions, Section 3 discusses what data are required for my analysis and describes the Arkansas Department of Higher Education administrative data, Section 4 presents the methods for inferring university objectives from classroom spending decisions, Section 5 describes my empirical analysis and presents estimates of student and university parameters, Section 6 describes the Marginal Improvement Algorithm and reports classroom spending decisions under counter-factual objectives and constraints, and Section 7 concludes.

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## 2 Theoretical Framework for Inferring University Objectives

In this section, I present a simple theoretical framework for inferring university objectives. Universities are complicated entities with functions ranging from preserving rare books to curing rare diseases. As such, inferring the objectives of an entire university is a daunting task. In this article, I focus on a university's decision of which undergraduate courses to offer and who to hire to teach these courses. As such, the theoretical framework developed in this section concerns course offering and hiring decisions of universities and the corresponding course choices of students.

### 2.1 Primitives

Index students by $i=1, \ldots, N$ and potential courses by $j=1, \ldots, J$. When specified, academic semesters are indexed by $t=1, \ldots T$. However, most of my analysis considers a static setting of one academic semester; therefore, semester subscripts are generally suppressed. In many settings, pre-requisites or other constraints imply that some students may not enroll in certain courses. To account for this heterogeneity, let $J_{i}$ denote the set of potential courses which would be open to student $i$ if they were offered.

The university chooses spending on instruction $c_{j}$ for every potential course $j \in J$ subject to a budget constraint. ${ }^{6}$ If spending exceeds a course specific minimum cost $m_{j}$, an instructor is hired and the course is offered; otherwise, the course is not offered. Let $d_{j}$ indicate whether course $j$ is offered and let $\mathbf{d}=\left[\begin{array}{lll}d_{1} & \cdots & d_{J}\end{array}\right]^{\prime}$ denote the full vector of course offering decisions. Formally:

$$
d_{j}=\mathbf{1}\left[c_{j} \geq m_{j}\right]
$$

Furthermore, suppose spending in excess of fixed cost may change unobserved instructor quality $I_{j}$ following:

$$
I_{j}= \begin{cases}\phi_{j}\left(c_{j}-m_{j}\right) & c_{j}>m_{j} \\ 0 & c_{j} \leq m_{j}\end{cases}
$$

where the quality of a baseline instructor in course $j$ is normalized to zero. Excess spending may increase instructor quality either because these funds are used to hire a more talented instructor or because increases in compensation motivate the same instructor to perform better. I assume the production function $\phi_{j}(\cdot)$ is differentiable but allow it to vary across

[^4]courses. For ease of exposition, define excess spending as
\[

e_{j}= $$
\begin{cases}c_{j}-m_{j} & c_{j}>m_{j} \\ 0 & c_{j} \leq m_{j}\end{cases}
$$
\]

and let $\mathbf{e}=\left[\begin{array}{lll}e_{1} & \cdots & e_{J}\end{array}\right]^{\prime}$ represent the full vector of excess spending decisions.

### 2.2 Student Utility

Suppose student utility from enrolling in course $j$ depends on student characteristics $X_{i}$, course characteristics $Z_{j}$ (possibly including expected class size), and unobserved instructor quality $I_{j}$ following a general additively separable structure:

$$
\begin{equation*}
U_{i j}=u_{i j}\left(Z_{j}(\mathbf{e}, \mathbf{d}), I_{j}, X_{i}\right)+\epsilon_{i j} \tag{1}
\end{equation*}
$$

where $\epsilon_{i j}$ is assumed to follow a Generalized Extreme Value (GEV) distribution (McFadden, 1978) and the deterministic utility function $u_{i j}(\cdot)$ is differentiable in $I_{j}$ and is allowed to vary across individuals and courses.

If student utility depends on expected class size, course offering and excess spending decisions affect the utility of each course indirectly through their effects on expected class sizes. ${ }^{7}$ To emphasize the importance of these general equilibrium effects, the dependence of $Z_{j}$ on decision vectors $\mathbf{e}$ and $\mathbf{d}$ is made explicit.

In this framework, student choice value is given by:

$$
\begin{equation*}
V_{i}(\mathbf{e}, \mathbf{d})=\max _{j \in J_{i}(\mathbf{d})}\left\{u_{i j}\left(Z_{j}(\mathbf{e}, \mathbf{d}), \phi_{j}\left(e_{j}\right), X_{i}\right)+\epsilon_{i j}\right\} \tag{2}
\end{equation*}
$$

where $J_{i}(\mathbf{d})$ denotes the set of courses which student $i$ is eligible for and are offered under offering vector $\mathbf{d}$.

### 2.3 Timing

University and student decisions proceed as follows:

1. The university observes all parameters, minimum costs $m_{j}$ for every potential course $j \in J$, observed student characteristics for every enrolled student $i$, and observed course

[^5]characteristics for every potential course $j \in J$.
2. The university observes the vector of courses $\mathbf{d}^{\text {con }}$ and associated excess spending levels $\mathbf{e}^{\text {con }}$ which they must offer and pay to honor contracts negotiated in prior periods
3. The university makes a two-tiered decision:
(a) The university decides which courses to offer by choosing the offering vector $\mathbf{d}$ (where ever offering vector must contain $\mathbf{d}^{\text {con }}$ ).
(b) The university chooses the excess spending vector $\mathbf{e}$ (where contracted courses must have excess spending levels $\left.\mathbf{e}^{\text {con }}\right)$.
4. Students observe d, e, observed student characteristics for every enrolled student $i$, observed course characteristics for every offered course, their own idiosyncratic preferences for offered courses, and their feasible set $J_{i}$, and choose one feasible offered course to maximize their utility. ${ }^{8}$

### 2.4 University's Problem

Denote the university's expected payoff from choosing offering vector $\mathbf{d}$ and excess spending vector $\mathbf{e}$ as $\mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}] .{ }^{9}$ The university's problem is to choose a spending vector $\mathbf{c}=$ $\left[\begin{array}{lll}c_{1} & \cdots & c_{J}\end{array}\right]^{\prime}$ to maximize the expected value of this objective function subject to a budget constraint and contractual constraints. Formally:

$$
\begin{aligned}
\mathbf{c}^{\star}= & \underset{\operatorname{argmax}_{\mathbf{c}}}{ }\{\mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]\} \\
& \text { s.t. } \sum_{j=1}^{J} c_{j} \leq E, c_{j} \geq 0, d_{j}=1 \text { iff } c_{j} \geq m_{j} \\
& \text { and } d_{j}=1 \text { and } c_{j}=e_{j}^{c o n}+m_{j} \text { if } d_{j}^{\text {con }}=1
\end{aligned}
$$

At a solution to the university's problem, all non-contracted courses where spending exceeds fixed cost must satisfy the following tangency conditions:

$$
\begin{gather*}
\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}=\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j^{\prime}}}  \tag{3}\\
\forall j, j^{\prime} \in J \text { s.t. } c_{j}>m_{j}, d_{j}^{c o n}=0 \text { and } c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{c o n}=0
\end{gather*}
$$

[^6]Intuitively, the tangency conditions described in Equation 3 impose that the marginal payoff of an additional dollar of spending must be equal across courses for which spending exceeds fixed cost. Importantly, the tangency condition does not apply to courses where the university only pays the fixed cost $f_{j}$ at a solution. These may be courses which are valuable to offer but are not particularly sensitive to excess spending.

## 3 Data for Inferring University Objectives

In this section, I discuss the data needed to implement the inference methods presented in this article. Following this, I introduce the Arkansas Department of Higher Education data used for my empirical analysis.

### 3.1 Necessary Data

Generally speaking, the inference methods presented in this article relate the costs of course offerings and hirings to the value of these decisions to students. As such, the methods require data which can be used to estimate both the costs of course offerings and hirings and the value of these decisions to students. Importantly, the methods require identifying institutional choices made freely rather than to honor contracts negotiated in prior periods. As such, the methods also require data which identify course offerings which were made freely.

For costs, two measures are required: first, researchers must observe how much an observed university spends on the instructor for every offered course; second, researchers must observe the minimum cost required to hire a minimally qualified instructor for every course. In the notation introduced in Section 2, the researcher must observe both the spending decision vector $\mathbf{c}$ and the minimum costs $m_{j}$ for every potential course $j \in J$.

Ideally, $\mathbf{c}$ is directly observed or can be constructed with limited assumptions. ${ }^{10}$ If $\mathbf{c}$ is available, $m_{j}$ can be approximated by classifying all offered courses into a discrete number of course types and computing the $k^{t h}$ percentile of the cost distribution for every course type where $k$ is small. In theory, researchers could use the minimum observed cost for each course type; however, the existence of outliers may make these minimums unrealistic. This estimation procedure is sensitive to the choice of $k$; therefore, inference methods which are sensitive to $m_{j}$ will also be sensitive to the choice of $k$.

[^7]To infer the value of course offerings and spending on instruction to students, researchers must estimate a multinomial choice model of students choosing courses. The parameters of a course choice model can be estimated with data on observed student characteristics, course choices, student choice sets, observed course characteristics, and spending on instruction.

In a general equilibrium setting in which class size affects utility estimating utility parameters is substantially more complicated than in standard multinomial choice models. Because class size is an equilibrium outcome, it is correlated with unobserved course specific attributes by construction. To see this more concretely, suppose course $j$ has the unobserved positive attribute that it meets at a convenient time and location. Because of this, enrollment in course $j$ will be unexpectedly high. Intuitively, a choice model which includes class size as a regressor will wrongfully attribute this unexpectedly high enrollment to positive effects of class size.

To estimate a general equilibrium model in which class size affects utility, one can adapt the Bayer and Timmins (2007) iterative IV method for estimating residential choice models in which the number of people in a locality affects its desirability. Broadly speaking, their approach iterates between predicting population sizes from observed attributes only and using these predicted sizes as instruments for actual population sizes. Bayer and Timmins (2007) note that it is important to have either variation in choice sets and/or observed heterogeneity in preferences across individuals to provide additional variation which reduces reliance on functional form assumptions. In most higher education contexts, both of these sources of variation exist: First, variation in choice sets exists because many courses have prerequisites and because students cannot retake courses which they have already completed. Second, observed student characteristics such as class year or SAT scores likely affect the relative desirability of different courses. As such, the Bayer and Timmins (2007) approach should adapt nicely to general equilibrium course choice models in which class size affects utility.

### 3.1.1 Non-offered courses

For some of the inference methods presented in this article it is necessary to observe characteristics of courses which are not offered in the analyzed semester. In many settings, researchers will only observe data on courses which are offered; in these cases, additional assumptions are needed to infer the characteristics of non-offered courses. Two approaches seem most feasible: First, researchers may assume deterministic choice utility and minimum costs depend on discrete observed characteristics with known support and that infinitely many potential courses exist at each point in the support. Second, researchers with access to panel data may assume that the entire set of feasible courses are offered in a set of academic
semesters.
To see the first approach more concretely, suppose courses are fully defined by academic department and course level. This implies all introductory chemistry courses have the same minimum cost and intrinsic popularity. In this case, estimates of a course choice model and minimum cost model can be used to forecast the utility and cost of a non-offered introductory chemistry course. Furthermore, this approach assumes that the university can always offer an additional introductory chemistry course.

The strength of the first approach is that it is computationally simpler and only requires one semester of data. One weakness of this approach is it ignores unobserved heterogeneity in the desirability and cost of courses. Specifically, this approach imposes that a non-offered introductory chemistry course is equivalent to offered courses with the same characteristics in terms of student utility and minimum cost. However, we may expect non-offered courses to be unobservably worse - from the perspective of the university - than observationally equivalent observed courses. Furthermore, the assumption that infinitely many potential courses exist at each point in the support is clearly unrealistic. While adding one introductory chemistry course is reasonable, adding twenty such courses would require major changes to faculty composition and facilities.

To illustrate the second approach, suppose a researcher has access to $T$ semesters of data on course offerings, spending on instruction, student characteristics, and course characteristics. In this approach, the researcher assumes that any course which was ever offered in any of the $T$ semesters could have been offered in any of the other $T$ semesters. As an example, if Chemistry 302 was offered in the Fall semester of 2007 it could have been offered in any other semester in the panel. As such, the set of non-offered courses in semester $t$ is given by all courses which are ever offered in any of the $T$ observed semesters but are not offered in semester $t$.

The strength of this second approach is that course fixed effects may be included to capture unobserved components of course utility. Furthermore, this approach places logical restrictions on the number of feasible courses to avoid unreasonable course offerings. The primary disadvantage of this approach is computational burden. This approach requires estimating multinomial choice models for every academic semester; if these models include course fixed effects estimating each one will be quite difficult. Another drawback of this approach is it yields a large number of non-offered courses in departments which change their course offerings often and few non-offered courses in departments where offerings are more stable. Finally, while this definition of non-offered courses is theoretically stronger than the definition of the first approach, there are still many practical issues which suggest
the true set of non-offered courses may be quite different from the constructed set. ${ }^{11}$

### 3.2 Arkansas Department of Higher Education Data

I will apply my methods for inferring university objectives using administrative data from the Arkansas Department of Higher Education. These data include information on all offered courses and information on the instructors teaching these courses for all four year and two year public institution of higher education in the state of Arkansas between 1993 and 2013. Furthermore, the data include demographic information and full academic records for all students who attended one of these institutions between 2004 and 2013.

The long panels of course and instructor information are crucial for estimating the minimum costs of offering different courses at each institution. Because so much data on spending on instruction is available, I can classify courses into finely defined course types for each institution and still use relatively low values for $k$ without worrying that estimates are driven by a small number of outliers. This provides reliable estimates of the minimum costs of offering each course.

The supplemental panel of student records is well suited for estimating multinomial course choice models. Students typically register for four to five courses each semester; as such, one semester of data typically provides more than enough student-course observations to precisely estimate utility parameters affecting student course choices.

## 4 Methods for Inferring University Objectives

In this section, I present various methods for making empirical inferences about university objectives. I begin by presenting a method for statistically testing whether classroom spending decisions maximize student welfare. To develop this test, I construct the first order conditions given by Equation (3) for the special case of a welfare maximizing university. I then show that estimates of a course choice model and observed data on spending on instruction can be used to statistically test whether these first order conditions hold.

Next, I develop two methods for estimating parameters of a 'student driven' structure of university objectives. The student driven structure assumes universities receive payoffs from student welfare but allows the university to place different weights on the welfare of different students. This allows the university to value the welfare of certain students-such as upper class students or male students - relatively more than the welfare of other students.

[^8]Furthermore, the structure also includes direct university preferences for class sizes. This allows the university to have institutional preferences for increasing enrollment in certain fields. ${ }^{12}$ This structure is student driven in the sense that university payoffs completely depend on the choice values and course choices of students.

To develop the first method for estimating student driven parameters, I construct the first order conditions given by Equation (3) for a student driven university as a function of student driven parameters. I then propose a variance minimization routine which solves for the parameter values which come closest to satisfying these first order conditions. These parameter estimates thus represent the values which best explain how much a university is observed spending on instructors for different courses. The second approach for estimating student driven parameters focuses on the university's decision of which courses to offer. I propose a maximum likelihood estimator which solves for the parameter values which best explain why observed course offerings were preferred to all other feasible course offerings. The two alternative methods employ different empirical variation and have complementary strengths and weaknesses providing researchers with multiple tools for analysis.

All methods apply to both the baseline setting in which students only value fixed course characteristics and instructor quality and to the general equilibrium setting in which students also value class size. In certain cases, I also discuss ways to use panel data to provide results which are more robust to functional form assumptions.

### 4.1 Are excess spending decisions consistent with utilitarian student welfare maximization?

In this subsection, I examine the special case of a university whose goal is to maximize student welfare giving equal weight to all students. Throughout the article, I refer to this baseline school as a utilitarian student welfare maximizing (U-SWM) university. I derive the tangency conditions described in Equation (3) for this university and show how observed data on spending on instruction and estimates of a student course choice model can be used to statistically test whether observed spending decisions satisfy the tangency conditions of this U-SWM university. This is equivalent to testing whether the incentives of an observed university are aligned with its students. I begin by describing the statistical test for the baseline setting in which students only value fixed course characteristics and instructor quality. Following this, I show how panel data can be used to reduce reliance on functional form assumptions about student utility. Finally, I describe the statistical test in the general equi-

[^9]librium setting in which students also value class size. I conclude by describing when these methods are appropriate and discussing the benefits and shortcomings of these methods.

### 4.1.1 Statistical test of U-SWM without class size effects

To make the university model described in Section 2 U-SWM, let the university's payoff from decision vectors $\mathbf{d}$ and $\mathbf{e}$ be the sum choice values over all students. The U-SWM problem is then given by:

$$
\begin{aligned}
\mathbf{c}^{\star}= & \operatorname{argmax}_{\mathbf{c}}\left\{\sum_{i=1}^{N} \mathbb{E}\left[V_{i}(\mathbf{e}, \mathbf{d})\right]\right\} \\
& \text { s.t. } \sum_{j=1}^{J} c_{j} \leq E, c_{j} \geq 0, \text { and } d_{j}=1 \text { iff } c_{j} \geq m_{j} \\
& \text { and } d_{j}=1 \text { and } c_{j}=e_{j}^{\text {con }}+m_{j} \text { if } d_{j}^{\text {con }}=1
\end{aligned}
$$

Because student preference shocks are assumed to follow a GEV distribution, a convenient property can be used to simplify the tangency conditions for a U-SWM university. For any $\theta_{j}$ which affects deterministic utility in course $j, \frac{d \mathbb{E}\left[V_{i}(\mathbf{e}, \mathbf{d})\right]}{d \theta_{j}}=\left(\frac{d u_{i j}}{d \theta_{j}}\right) P_{i j}(\mathbf{e}, \mathbf{d})$ where $P_{i j}(\mathbf{e}, \mathbf{d})$ is the probability individual $i$ chooses course $j$ given excess spending vector $\mathbf{e}$ and offering vector d. ${ }^{13}$ This property implies that the U-SWM version of the general tangency conditions given in Equation (3) is given by :

$$
\begin{gather*}
\sum_{i=1}^{N}\left(\frac{\partial u_{i j}}{\partial I_{j}}\right)\left(\frac{\partial \phi_{j}}{\partial e_{j}}\right) P_{i j}(\mathbf{e}, \mathbf{d})=\sum_{i=1}^{N}\left(\frac{\partial u_{i j^{\prime}}}{\partial I_{j^{\prime}}}\right)\left(\frac{\partial \phi_{j^{\prime}}}{\partial e_{j^{\prime}}}\right) P_{i j^{\prime}}(\mathbf{e}, \mathbf{d})  \tag{4}\\
\forall j, j^{\prime} \in J \text { s.t. } c_{j}>m_{j}, d_{j}^{\text {con }}=0 \text { and } c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{\text {con }}=0
\end{gather*}
$$

With functional form assumptions on the structure of $u_{i j}(\cdot)$ and $\phi_{j}(\cdot)$ and distribution of $\epsilon_{i j}$, researchers can estimate the parameters of a multinomial course choice model in which students choose one course to maximize utility defined in Equation 1. The parameters of this choice model can then be used to construct estimates of the tangency condition components $\left(\frac{\partial u_{i j}}{\partial I_{j}}\right),\left(\frac{\partial \phi_{j}}{\partial e_{j}}\right)$, and $P_{i j}(\mathbf{e}, \mathbf{d})$ for all students $i$ and offered courses $j$ s.t. $d_{j}=1$.

These estimates can be used to form test statistics which are the empirical analogs of the

[^10]tangency conditions:
\[

$$
\begin{equation*}
\hat{t}_{j j^{\prime}}=\left\{\sum_{i=1}^{N}\left(\frac{\partial \hat{u}_{i j}}{\partial I_{j}}\right)\left(\frac{\partial \hat{\phi}_{j}}{\partial e_{j}}\right) \hat{P}_{i j}(\mathbf{e}, \mathbf{d})\right\}-\left\{\sum_{i=1}^{N}\left(\frac{\partial \hat{u}_{i j^{\prime}}}{\partial I_{j^{\prime}}}\right)\left(\frac{\partial \hat{\phi}_{j^{\prime}}}{\partial e_{j^{\prime}}}\right) \hat{P}_{i j^{\prime}}(\mathbf{e}, \mathbf{d})\right\} \tag{5}
\end{equation*}
$$

\]

For the observed spending vector to be consistent with the goal of maximizing student welfare, $\hat{t}_{j j^{\prime}}$ must be statistically indistinguishable from zero for every course pair $j$ and $j^{\prime}$ for which $c_{j}>m_{j}, d_{j}^{c o n}=0, c_{j^{\prime}}>m_{j^{\prime}}$, and $d_{j^{\prime}}^{c o n}=0$. If $\hat{t}_{j j^{\prime}}$ is statistically positive (negative), it implies the welfare return on an additional dollar of spending is significantly higher (lower) in course $j$ relative to course $j^{\prime}$. This would be inconsistent with the goal of maximizing student welfare because welfare could be increased by marginally increasing (reducing) spending in course $j$ and reducing (increasing) spending in course $j^{\prime}$. Formally, the testing procedure is as follows:
$H_{0}$ : University spending is consistent with the goal of maximizing student welfare
$H_{a}$ : University spending is not consistent with the goal of maximizing student welfare

Testing procedure:

1. Identify the set of courses $\tilde{J}=\left\{j \in J \mid c_{j}>m_{j} \cap d_{j}^{\text {con }}=0\right\}$.
2. Use a bootstrap algorithm to estimate the distribution of the $\frac{\tilde{J}[\tilde{J}-1]}{2}$ dimensional random vector $\hat{\mathbf{t}}=\left[\begin{array}{lll}\hat{t}_{12} & \ldots & \hat{t}_{\tilde{J}-1 \tilde{J}}\end{array}\right] \cdot{ }^{14}$
3. Test the joint hypothesis: $H_{0}: \hat{t}_{j j^{\prime}}=0$ for all pairs of offered courses $j, j^{\prime} \in \tilde{J}$.

To implement the first step of this procedure, researchers can use observed data on $c_{j}$ and estimates of $m_{j}$ obtained as described in Subsection 3.1. To reduce sensitivity to estimation error in $\hat{m}_{j}$, researchers may use a stricter set: $\tilde{J}_{\delta}=\left\{j \in J \mid c_{j}>m_{j}+\delta \cap d_{j}^{\text {con }}=0\right\}$ where $\delta>0$. Choosing a large $\delta$ guarantees that spending exceeds minimum costs implying the tangency conditions must bind. However, as $\delta$ increases, the set of courses shrinks which reduces power to reject the null hypothesis.

### 4.1.2 Statistical test of U-SWM with class size effects

When class size affects choice utility, excess spending in course $j$ has direct effects on choice utility for course $j$ but also has indirect effects on choice utility for all courses through

[^11]changes in class sizes. These general equilibrium effects make simplifying the general tangency conditions in Equation 3 somewhat more difficult. A general version of the GEV property used previously is helpful: For any $\theta$ affecting deterministic utility in any course, $\frac{d \mathbb{E}\left[V_{i}(\mathbf{e}, \mathbf{d})\right]}{d \theta}=\sum_{j \in J_{i}(\mathbf{d})}\left(\frac{d u_{i j}}{d \theta}\right) P_{i j}(\mathbf{e}, \mathbf{d})$. This yields the following general equilibrium U-SWM tangency conditions:
\[

$$
\begin{gather*}
\sum_{i=1}^{N} \sum_{k \in J_{i}(\mathbf{d})}\left(\frac{d u_{i k}}{d e_{j}}\right) P_{i k}(\mathbf{e}, \mathbf{d})=\sum_{i=1}^{N} \sum_{k \in J_{i}(\mathbf{d})}\left(\frac{d u_{i k}}{d e_{j^{\prime}}}\right) P_{i k}(\mathbf{e}, \mathbf{d})  \tag{6}\\
\forall j, j^{\prime} \in J \text { s.t. } c_{j}>m_{j}, d_{j}^{c o n}=0 \text { and } c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{c o n}=0
\end{gather*}
$$
\]

where

$$
\frac{d u_{i k}}{d e_{j}}= \begin{cases}\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}+\frac{\partial u_{i j}}{\partial \tilde{n}_{j}} \frac{d \tilde{n}_{j}}{d e_{j}} & k=j  \tag{7}\\ \frac{\partial u_{i k}}{\partial \tilde{n}_{k}} \frac{d \tilde{n}_{k}}{d e_{j}} & k \neq j\end{cases}
$$

As discussed in Subsection 3.1, Bayer and Timmins (2007) estimates of the parameters of a general equilibrium course choice model can be used to estimate the tangency condition components $P_{i k}(\mathbf{e}, \mathbf{d}), \frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}$, and $\frac{\partial u_{i k}}{\partial \tilde{n}_{k}}$; however, they cannot be used to directly construct the effects of spending on class sizes given by $\frac{d \tilde{n}_{k}(\mathbf{e}, \mathbf{d})}{d e_{j}}$. These are complicated effects because they depend on the effects of spending on course utility and these effects depend on the effects of spending on class sizes. In Methodological Appendix A, I show how this recursive relationship can be unraveled to yield a closed form expression when $\epsilon_{i j}$ follows a type 1 extreme value distribution.

As before, these estimates can be used to construct test statistics which are the empirical analogs of the general equilibrium tangency conditions:

$$
\hat{t}_{j j^{\prime}}=\sum_{i=1}^{N} \sum_{k \in J_{i}(\mathbf{d})}\left(\frac{d \hat{u}_{i k}}{d e_{j}}\right) \hat{P}_{i k}(\mathbf{e}, \mathbf{d})-\sum_{i=1}^{N}\left(\sum_{k \in J_{i}(\mathbf{d})}\left(\frac{d \hat{u}_{i k}}{d e_{j^{\prime}}}\right) \hat{P}_{i k}(\mathbf{e}, \mathbf{d})\right)
$$

These test statistics can then be used to test whether observed spending is consistent with the goal of utilitarian student welfare maximization following the same procedure described in Subsection 4.1.1.

### 4.1.3 Statistical test of U-SWM with panel data

One concern with the baseline test described in Subsection 4.1.1 is the results may be sensitive to the functional form of student utility. In this subsection, I develop a complementary panel data test which is more robust to functional form assumptions. The idea behind this test
is that the researcher can solve for the return on spending parameters which exactly satisfy the U-SWM first order conditions in a given semester. If the university is U-SWM in every semester then these implied returns should be statistically similar for the same course in different semesters. In this subsection only, academic semesters are indexed by $t=1, \ldots, T$.

Suppose student utility falls into a general class of models in which utility from enrolling in course $j$ in semester $t$ depends on student characteristics $X_{i t}$, course characteristics $Z_{j t}$, and excess spending on instruction $e_{j t}$ in the following additively separable manner:

$$
\begin{equation*}
U_{i j t}=\theta_{j} \ln e_{j t}+\psi_{i j t}\left(Z_{j t}, X_{i t}\right)+\epsilon_{i j t} \tag{8}
\end{equation*}
$$

where $\epsilon_{i j t}$ is assumed to follow a Generalized Extreme Value distribution (McFadden, 1978) and the deterministic utility function $\psi_{i j t}(\cdot)$ is allowed to vary across individuals, courses, and semesters. While this class is generally quite flexible it places some restrictions on how spending on instruction affects utility. Most notably, although returns on spending are allowed to vary across courses they are not allowed to vary across individuals. Additionally, this structure imposes that concavity is generated by the natural logarithm function.

For this class of utility structures, the tangency conditions describing a U-SWM university's solution are:

$$
\begin{gather*}
\frac{\sum_{i=1}^{N_{t}} \theta_{j} P_{i j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}{e_{j t}}=\frac{\sum_{i=1}^{N_{t}} \theta_{j^{\prime}} P_{i j^{\prime} t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}{e_{j^{\prime} t}}  \tag{9}\\
\forall j, j^{\prime} \in J \text { s.t. } c_{j}>m_{j}, d_{j}^{c o n}=0 \text { and } c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{c o n}=0
\end{gather*}
$$

Rearranging yields:

$$
\begin{gathered}
\frac{\theta_{j}}{\theta_{j^{\prime}}}=\left(\frac{e_{j t}}{e_{j^{\prime} t}}\right)\left(\frac{\sum_{i=1}^{N_{t}} P_{i j^{\prime} t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}{\sum_{i=1}^{N_{t}} P_{i j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}\right) \\
\forall j, j^{\prime} \in J \text { s.t. } c_{j}>m_{j}, d_{j}^{c o n}=0 \text { and } c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{c o n}=0
\end{gathered}
$$

Notice the left hand side of this expression is invariant across semesters. This implies that if the U-SWM first order conditions are satisfied in both semesters $t$ and $t^{\prime}$ then the following conditions must hold:

$$
\begin{gather*}
\left(\frac{e_{j t}}{e_{j^{\prime} t}}\right)\left(\frac{\tilde{n}_{j^{\prime} t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}{\tilde{n}_{j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}\right)=\left(\frac{e_{j t^{\prime}}}{e_{j^{\prime} t^{\prime}}}\right)\left(\frac{\tilde{n}_{j^{\prime} t^{\prime}}\left(\mathbf{e}_{\mathbf{t}^{\prime}}, \mathbf{d}_{\mathbf{t}^{\prime}}\right)}{\tilde{n}_{j t^{\prime}}\left(\mathbf{e}_{\mathbf{t}^{\prime}}, \mathbf{d}_{\mathbf{t}^{\prime}}\right)}\right)  \tag{10}\\
\forall j, j^{\prime} \in J \text { S.t. } \min \left\{c_{j t}, c_{j t^{\prime}}\right\}>m_{j}, d_{j}^{c o n}=0 \text { and } \min \left\{c_{j^{\prime} t}, c_{j^{\prime} t^{\prime}}\right\}>m_{j^{\prime}}, d_{j^{\prime}}^{c o n}=0
\end{gather*}
$$

where $\tilde{n}_{j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)=\sum_{i=1}^{N} P_{i j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)$ is expected enrollment in course $j$ in semester $t$ given
decision vectors $\mathbf{e}_{\mathbf{t}}$ and $\mathbf{d}_{\mathbf{t}}$. Intuitively, this panel condition states that a U-SWM university responds to changes in relative intrinsic popularity by spending more in courses which are becoming more popular.

The benefit of the panel condition given in Equation (10) is it depends on excess spending $e_{j t}$ and equilibrium choice probabilities $P_{i j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)$ but does not depend on other parameters. Because no specific parameters are required to construct (10), researchers may conduct this test using flexible reduced form utility structures which are robust to functional form assumptions. ${ }^{15}$

As before, estimates of equilibrium choice probabilities and observed data can be used to construct the empirical analogs of (10):

$$
\hat{t}_{j j^{\prime}, t t^{\prime}}=\left(\frac{e_{j t}}{e_{j^{\prime} t}}\right)\left(\frac{\hat{\tilde{n}}_{j^{\prime} t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}{\hat{\tilde{n}}_{j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)}\right)=\left(\frac{e_{j t^{\prime}}}{e_{j^{\prime} t^{\prime}}}\right)\left(\frac{\hat{\tilde{n}}_{j^{\prime} t^{\prime}}\left(\mathbf{e}_{\mathbf{t}^{\prime}}, \mathbf{d}_{\mathbf{t}^{\prime}}\right)}{\hat{\tilde{n}}_{j t^{\prime}}\left(\mathbf{e}_{\mathbf{t}^{\prime}}, \mathbf{d}_{\mathbf{t}^{\prime}}\right)}\right)
$$

These test statistics can then be used to test whether observed spending is consistent with the goal of utilitarian student welfare maximization following the same procedure described in Subsection 4.1.1.

### 4.1.4 Discussion of tangency condition inference methods

These tangency condition tests of whether an observed university's behavior is consistent with utilitarian student welfare maximization can be conducted as long as the set of courses $\tilde{J}_{\delta}=\left\{j \in J \mid c_{j}>m_{j}+\delta \cap d_{j}^{c o n}=0\right\}$ is sufficiently large. Because of the way $m_{j}$ is estimated, $\tilde{J}_{\delta}=\left\{j \in J \mid c_{j}>m_{j}+\delta \cap d_{j}^{\text {con }}=0\right\}$ will only be small if non-contract courses are clustered in the lower regions of distributions of $c_{j}$ within course type. Researchers can readily evaluate course type specific distributions of $c_{j}$ and assess whether spending in noncontract courses is disperse enough to construct a sufficiently large $\tilde{J}_{\delta}$.

The tangency condition inference methods have several strengths: First, they provide a clear statistical test of whether observed behavior is consistent with utilitarian student welfare maximization. The methods test a specific structure of university objectives-rather than imposing a structure and estimating parameters assuming that structure is trueand they handle sampling error in estimates of student choice parameters appropriately. Furthermore, the trio of a baseline test, general equilibrium test, and panel data test offers researchers several inference tools which apply to a variety of settings and can be used to assess the robustness of results.

[^12]While these inference methods are desirable for their clarity and rigor, the tradeoff is they only offer narrow inferences about university objectives. Specifically, the tests can only reject or fail to reject that observed spending is consistent with utilitarian student welfare maximization. If the null hypothesis of U-SWM is rejected, these methods do not offer a preferable alternative. In subsequent sections, I introduce and discuss complementary inference methods which provide alternative objective structures for universities which are not U-SWM.

### 4.2 What university objectives best explain observed spending decisions

In the preceding subsection, I demonstrate how estimates of student choice parameters and minimum course costs can be used to test whether observed spending decisions are consistent with a specific structure of university objectives. While these tests are a useful place to start, they can only reject or fail to reject that spending is consistent with this structure. If the structure is rejected, it would be useful to find alternative structures which better explain observed spending.

In this subsection, I present methods for estimating the parameters of a more general 'student driven' structure of university objectives. The student driven structure assumes universities receive payoffs from student welfare but allows the university to place different weights on the welfare of different students. This allows the university to value the welfare of certain students - such as upper class students or male students - relatively more than the welfare of other students. Furthermore, the structure also includes direct university preferences for class sizes. This allows the university to have institutional preferences for increasing enrollment in certain fields. This structure is student driven in the sense that university payoffs completely depend on the choice values and course choices of students.

With this student driven structure, university payoffs are a weighted sum of expected student choice values and a function of expected class sizes. Formally

$$
\mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]=\left\{\sum_{i=1}^{N} \omega_{i} \mathbb{E}\left[V_{i} \mid \mathbf{e}, \mathbf{d}\right]\right\}+\Upsilon(\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d}))
$$

where $\omega_{i}$ are welfare weights, $\Upsilon(\cdot)$ is an unspecified function, and

$$
\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d})=\left[\begin{array}{lll}
\tilde{n}_{1}(\mathbf{e}, \mathbf{d}) & \cdots & \tilde{n}_{J}(\mathbf{e}, \mathbf{d})
\end{array}\right]^{\prime}
$$

represents a vector of expected class sizes given decision vectors e and $\mathbf{d}$. The student driven
university's problem is then given by:

$$
\begin{aligned}
\mathbf{c}^{\star}= & \operatorname{argmax}_{\mathbf{c}}\left\{\left(\sum_{i=1}^{N} \omega_{i} \mathbb{E}\left[V_{i} \mid \mathbf{e}, \mathbf{d}\right]\right)+\Upsilon(\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d}))\right\} \\
& \text { s.t. } \sum_{j=1}^{J} c_{j} \leq E, c_{j} \geq 0, \text { and } d_{j}=1 \text { iff } c_{j} \geq m_{j} \\
& \text { and } d_{j}=1 \text { and } c_{j}=e_{j}^{\text {con }}+m_{j} \text { if } d_{j}^{\text {con }}=1
\end{aligned}
$$

With this structure, the university's tangency conditions are given by:

$$
\begin{align*}
& \frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}=\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j^{\prime}}}  \tag{11}\\
& \forall j, j^{\prime} \in J \quad \text { s.t. } \quad c_{j}>m_{j}, d_{j}^{c o n}=0 \text { and } c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{c o n}=0
\end{align*}
$$

In the baseline setting where students do not value class sizes, $\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}$ is given by:

$$
\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}=\left\{\sum_{i=1}^{N} \omega_{i}\left(\left(\frac{\partial u_{i j}}{\partial I_{j}}\right)\left(\frac{\partial \phi_{j}}{\partial e_{j}}\right) P_{i j}(\mathbf{e}, \mathbf{d})\right)\right\}+\sum_{k=1}^{J}\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{k}}\right)\left(\frac{\partial \tilde{n_{k}}}{\partial e_{j}}\right)
$$

With functional form assumptions on the structure of $u_{i j}(\cdot)$ and $\phi_{j}(\cdot)$ and distribution of $\epsilon_{i j}$, researchers can use estimates of a multinomial course choice model to construct $\left(\frac{\partial u_{i j}}{\partial I_{j}}\right)$, $\left(\frac{\partial \phi_{j}}{\partial e_{j}}\right), P_{i j}(\mathbf{e}, \mathbf{d})$, and $\left(\frac{\partial \tilde{n}_{k}}{\partial e_{j}}\right)$ for all students $i$ and offered courses $j$ s.t. $d_{j}=1$.

In a general equilibrium setting where class size affects choice utility, $\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}$ is given by:

$$
\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}=\left\{\sum_{i=1}^{N} \omega_{i}\left(\sum_{k \in J_{i}(\mathbf{d})}\left(\frac{d u_{i k}}{d e_{j}}\right) P_{i k}\right)\right\}+\sum_{k=1}^{J}\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{k}}\right)\left(\frac{d \tilde{n_{k}}}{d e_{j}}\right)
$$

With functional form assumptions on the structure of $u_{i j}(\cdot)$ and $\phi_{j}(\cdot)$ and distribution of $\epsilon_{i j}$, researchers can construct $\frac{d u_{i k}}{d e_{j}}$ and $\frac{d \tilde{n_{k}}}{d e_{j}}$ with Bayer and Timmins (2007) estimates of a general equilibrium sorting model.

To identify the student driven preference parameters which best explain observed spending, I propose solving for the parameter values $\omega_{i}$ and $\frac{\partial \Upsilon}{\partial \tilde{n}_{k}}$ which come closest to satisfying these tangency conditions at observed spending levels. For a single academic semester, the university's excess spending tangency conditions state that the marginal returns $\frac{d E[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}$ must be equal for all courses where spending exceeds fixed costs. To solve for the parameter values which come closest to satisfying this condition at observed spending levels, I propose solving

$$
\left\{\omega^{\star}, \Upsilon^{\star}\right\}=\operatorname{argmin}_{(\omega, \Upsilon)}\left\{\operatorname{Var}_{j \text { s.t. } c_{j}>m_{j}, d_{j}^{c o n}=0}\left(\frac{d \mathbb{E}[\Pi \mid \omega, \Upsilon, \tilde{\mathbf{e}}, \tilde{\mathbf{d}}]}{d e_{j}}\right)\right\}
$$

where $\tilde{\mathbf{e}}$ and $\tilde{\mathbf{d}}$ represent observed excess spending. If the tangency conditions are satisfied for all pairs of courses $j$ and $j^{\prime}$ such that $c_{j}>m_{j}, d_{j}^{\text {con }}=0$ and $c_{j^{\prime}}>m_{j^{\prime}}, d_{j^{\prime}}^{\text {con }}=0$ then this objective variance is exactly zero. As such, the parameter values $\omega$ and $\Upsilon(\cdot)$ which minimize this variance are those which make the student driven structure of university objectives most consistent with observed data.

As in Subsection 4.1, this inference method requires identifying the set of courses $\tilde{J}=$ $\left\{j \in J \mid c_{j}>m_{j}, d_{j}^{c o n}=0\right\}$. As before, researchers may use a stricter set:

$$
\tilde{J}_{\delta}=\left\{j \in J \mid c_{j}>m_{j}+\delta, d_{j}^{c o n}=0\right\}
$$

where $\delta>0$ to reduce sensitivity to error in estimates of $m_{j}$. Choosing a large $\delta$ guarantees that spending exceeds minimum costs implying the tangency conditions must bind. However, as $\delta$ increases, the set of courses shrinks which reduces precision in estimates of $\omega$ and $\Upsilon$.

### 4.2.1 Identification and estimation

To implement the estimation algorithm described in this subsection it is necessary to specify a functional form for $\Upsilon(\cdot)$ and to aggregate students into types indexed by $\tau$. I consider two structures for $\Upsilon(\cdot)$ which both require aggregating courses into types indexed by $g$ : First, a simple linear structure:

$$
\Upsilon(\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d}))=\sum_{j=1}^{J} \gamma_{g(j)} \tilde{n}_{j}(\mathbf{e}, \mathbf{d})
$$

Second, a logarithmic structure which generates diminishing marginal returns to enrollments:

$$
\Upsilon(\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d}))=\sum_{j=1}^{J} \gamma_{g(j)} \ln \left[\tilde{n}_{j}(\mathbf{e}, \mathbf{d})\right]
$$

With the simple linear structure and aggregation of students, university objectives are given by:

$$
\mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]=\left\{\sum_{i=1}^{N} \omega_{\tau(i)} \mathbb{E}\left[V_{i} \mid \mathbf{e}, \mathbf{d}\right]\right\}+\sum_{j=1}^{J} \gamma_{g(j)} \tilde{n}_{j}(\mathbf{e}, \mathbf{d})
$$

In Methodological Appendix B, I show that the relative values of $\omega_{\tau}$ and $\gamma_{g}$ are overidentified as long as the number of student types plus the number of course types is less
than the total number of courses such that $c_{j}>m_{j}$ and $d_{j}^{\text {con }}=0$. Intuitively, identification of $\omega_{\tau}$ comes from students of type $\tau$ concentrating in classes where the university is under (over) investing relative to a U-SWM university. This implies that the university puts less (more) weight on the welfare of type $\tau$ students relative to the general student population. Identification of $\gamma_{g}$ comes from residual differences in marginal returns on spending which cannot be explained by student weights. For example, if course groups $g$ and group $g^{\prime}$ have identical student compositions but the marginal returns on spending are higher (lower) in group $g$ it must be that the university values enrollment in group $g$ courses less (more) than enrollment in group $g^{\prime}$ courses.

The fact that $\omega_{\tau}$ and $\gamma_{g}$ are over-identified provides a useful over-identification test of the student driven structure and other functional form assumptions. If all functional form assumptions are correct, the objective variance:

$$
\operatorname{Var}_{j \text { s.t. } c_{j}>m_{j}, d_{j}^{c o n}=0}\left(\frac{d \mathbb{E}[\Pi \mid \omega, \Upsilon, \tilde{\mathbf{e}}, \tilde{\mathbf{d}}]}{d e_{j}}\right)
$$

should be exactly zero at true parameter values $\omega_{\tau}$ and $\gamma_{g}$. If the objective variance is statistically positive at estimates of $\omega_{\tau}$ and $\gamma_{g}$, we can reject the joint structure of student utility and university objectives. As with any over-identification test, failure to reject the null hypothesis does not validate the model structure.

### 4.2.2 Discussion of marginal return variance minimizing inference methods

As with the tangency condition tests, these marginal return variance minimizing methods can only be used if the set of courses $\tilde{J}_{\delta}=\left\{j \in J \mid c_{j}>m_{j}+\delta, d_{j}^{c o n}=0\right\}$ is sufficiently large. Additionally, these variance minimizing methods are only appropriate when the student driven structure for university objectives is correct.

The strength of the variance minimizing methods described in this subsection is that they provide estimates of preference parameters for an objective structure which is more general than utilitarian student welfare maximization. Specifically, they quantify how the university weights the welfare of different students and how the university values enrollments in different courses. This paints a more interesting picture of university objectives than a binary test which can only reject or fail to reject a very specific objective structure.

While these variance minimizing methods provide interesting results they only apply if the student driven structure is correct and they depend on strong assumptions about the structures of $\omega$ and $\Upsilon(\cdot)$. While there is scope for assessing the validity of these assumptions before and after applying these methods, these tests are not guaranteed to reject the
assumptions when they are false. Misspecifications of these structures may lead to spurious conclusions about university objectives.

### 4.3 What university preferences best explain observed course offerings?

In the preceding subsection, I presented an algorithm for estimating parameters of a student driven objective structure by solving for parameter values which best explain observed excess spending decisions in non-contract courses. One shortcoming of this method is it relies on variation from non-contract courses with positive excess spending. This will be a small set of courses if salaries for non-contract instructors are at the bottom of the pay distribution as is often the case.

In this subsection, I propose complementary methods which estimate the parameters of a student driven objective structure which best explain observed course offering and excess spending decisions. Intuitively, the methods involve choosing the parameter values which minimize the number of alternative offering vectors which yield a higher payoff than the observed set of offered courses.

To illustrate these methods more concretely, note that it is often possible to combine the student driven tangency conditions given by 11 with the budget constraint to solve for the optimal excess spending vector for each offering vector. ${ }^{16}$ Denote these optimal excess spending vectors as: $\mathbf{e}(\mathbf{d})^{\star}$. The student driven university's problem can then be restated to focus on extensive margin decisions:

$$
\mathbf{d}^{\star}=\operatorname{argmax}_{\mathbf{d}}\left\{\sum_{i=1}^{N} \mathbb{E}\left[\Pi \mid \mathbf{e}(\mathbf{d})^{\star}, \mathbf{d} ; \omega, \Upsilon\right]\right\} \text { s.t. } \sum_{j=1}^{J} d_{j} m_{j} \leq E
$$

If the student driven structure is correct, the observed offering vector $\hat{\mathbf{d}}$ is the optimal vector $\mathbf{d}^{\star}$ and the observed excess spending vector $\hat{\mathbf{e}}$ is the optimal spending vector given the optimal offering vector $\mathbf{e}\left(\mathbf{d}^{\star}\right)^{\star}$ and the true parameter values $\omega$ and $\Upsilon$.

This implies that the observed decision vectors $\hat{\mathbf{d}}$ and $\hat{\mathbf{e}}$ must provide the highest university payoff at the true parameter values $\omega$ and $\Upsilon$ when the student driven structure is correct. Formally,

$$
\begin{aligned}
\mathbb{E}[\Pi \mid \hat{\mathbf{e}}, \hat{\mathbf{d}} ; \omega, \Upsilon] \geq \mathbb{E}\left[\Pi \mid \mathbf{e}(\mathbf{d})^{\star}, \mathbf{d} ; \omega, \Upsilon\right] \\
\forall \text { d s.t. } \sum_{j=1}^{J} d_{j} f_{j} \leq E \text { and } d_{j}=1 \text { if } d_{j}^{c o n}=1
\end{aligned}
$$

[^13]at the true parameter values $\omega$ and $\Upsilon$.
This suggests one method for estimating $\omega$ and $\Upsilon$ : Choose the values of $\omega$ and $\Upsilon$ which minimize the number of alternative feasible offering vectors which yield a higher payoff than the observed decision vectors. Formally,
\[

$$
\begin{equation*}
\{\hat{\omega}, \hat{\Upsilon}\}=\operatorname{argmin}_{(\omega, \Upsilon)}\left\{\sum_{\mathbf{d} \in \mathbf{D}(E)} \mathbf{1}\left\{\mathbb{E}\left[\Pi \mid \mathbf{e}(\mathbf{d})^{\star}, \mathbf{d} ; \omega, \Upsilon\right]>\mathbb{E}[\Pi \mid \hat{\mathbf{e}}, \hat{\mathbf{d}} ; \omega, \Upsilon]\right\}\right\} \tag{12}
\end{equation*}
$$

\]

where $\mathbf{D}(E)=\left\{\mathbf{d} \mid \sum_{j=1}^{J} d_{j} f_{j} \leq E\right.$ and $d_{j}=1$ if $\left.d_{j}^{\text {con }}=1\right\}$ is the set of feasible offering vectors given endowment $E$. However, because the objective function in Equation (12) is discontinuous, this minimization problem will generally not yield a unique solution.

To provide a unique solution, I propose treating expected payoffs as quantities which are measured with error and maximizing the probability that the observed vectors are the best choice. Formally,
$\{\hat{\omega}, \hat{\Upsilon}\}=\operatorname{argmax}_{(\omega, \Upsilon)}\left\{\operatorname{Pr}\left(\mathbb{E}[\Pi \mid \hat{\mathbf{e}}, \hat{\mathbf{d}} ; \omega, \Upsilon]+\zeta_{\mathbf{d}^{\star}} \geq \mathbb{E}\left[\Pi \mid \mathbf{e}(\mathbf{d})^{\star}, \mathbf{d} ; \omega, \Upsilon\right]+\zeta_{\mathbf{d}} \forall \mathbf{d} \in \mathbf{D}(E)\right)\right\}$
where $\zeta_{\mathbf{d}}$ is drawn from a type 1 extreme value distribution. This simplifies to:

$$
\{\hat{\omega}, \hat{\Upsilon}\}=\operatorname{argmax}_{(\omega, \Upsilon)}\left\{\frac{\exp (\mathbb{E}[\Pi \mid \hat{\mathbf{e}}, \hat{\mathbf{d}} ; \omega, \Upsilon])}{\sum_{\mathbf{d} \in \mathbf{D}(E)} \exp \left(\mathbb{E}\left[\Pi \mid \mathbf{e}(\mathbf{d})^{\star}, \mathbf{d} ; \omega, \Upsilon\right]\right)}\right\}
$$

Intuitively, estimates of $\omega$ and $\Upsilon$ obtained in this manner represent student driven preference parameters which best explain why observed course offerings and excess spending levels were preferred to all other possible decision vectors.

### 4.3.1 Identification and estimation

As in Subsection 4.2.1, this algorithm requires specifying a functional form for $\Upsilon(\cdot)$. As before, I consider both a simple linear structure:

$$
\Upsilon(\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d}))=\sum_{j=1}^{J} \gamma_{j} \tilde{n}_{j}(\mathbf{e}, \mathbf{d})
$$

and a logarithmic structure which generates diminishing marginal returns to enrollments:

$$
\Upsilon(\tilde{\mathbf{n}}(\mathbf{e}, \mathbf{d}))=\sum_{j=1}^{J} \gamma_{j} \ln \left[\tilde{n}_{j}(\mathbf{e}, \mathbf{d})\right]
$$

In Methodological Appendix D, I show that the inclusion of type 1 extreme value errors guarantee that $\gamma_{j}$ and $\omega_{i}$ are over-identified under very general circumstances; however, estimating individual specific welfare weights and course specific preferences is impractical. As such, I suggest grouping students into types $\tau$ and grouping courses into groups $g$ as before. Intuitively, identification of $\omega_{\tau}$ comes from differences across student types in preferences for the observed vector $\hat{\mathbf{d}}$-if type $\tau$ students prefer $\hat{\mathbf{d}}$ to most alternative offering vectors but type $\tau^{\prime}$ students do not particularly like $\hat{\mathbf{d}}$ this suggests the university values the welfare of type $\tau$ students more than the welfare of type $\tau^{\prime}$ students. Identification of $\gamma_{g}$ comes from the composition of courses offered in $\hat{\mathbf{d}}$ relative to the composition of courses in alternative offering vectors. If $\hat{\mathbf{d}}$ and $\mathbf{d}$ are equivalent from the perspective of students but $\hat{\mathbf{d}}$ offers more type $g$ courses this implies the university values enrollment in type $g$ courses more than other courses.

One challenge with implementing the estimation algorithm described in this subsection is that the set of feasible vectors $\mathbf{D}(E)$ may be so large that summing over all $\mathbf{d} \in \mathbf{D}(E)$ is impractical. To address this, researchers can use a subset of $\mathbf{D}(E)$ rather than the full set. This is akin to choosing the parameter values which make the observed offering vector yield a higher payoff than a subset of alternative feasible vectors. This provides a computationally feasible algorithm at the cost of modest efficiency losses.

As before, the fact that $\omega_{\tau}$ and $\gamma_{g}$ are over-identified provides a useful over-identification test of the student driven objective structure and other functional form assumptions. If all functional form assumptions are correct, the observed offering vector $\hat{\mathbf{d}}$ should yield a higher payoff than all alternative offering vectors at the true parameter values $\omega_{\tau}$ and $\gamma_{g}$. If there are alternatives which are statistically preferred to $\hat{\mathbf{d}}$ at estimates of $\omega_{\tau}$ and $\gamma_{g}$ we can jointly reject the student driven objective structure and other functional form assumptions. Once again, failure to reject the null hypothesis does not validate the model structure.

### 4.3.2 Discussion of best offering vector methods

The strength of the best offering methods described in this subsection is that they provide estimates of preference parameters for an objective structure which is more general than utilitarian student welfare maximization. Specifically, they quantify how the university weights the welfare of different students and how the university values enrollments in different courses. Additionally, this estimation method uses variation from all non-contract courses rather than the restricted set of non-contract courses with positive excess spending used for the method described in Subsection 4.2.

One disadvantage of these methods is they only apply if the student driven structure is correct and they depend on strong assumptions about the structures of $\omega$ and $\Upsilon(\cdot)$. While
there is scope for assessing the validity of these assumptions, these tests are not guaranteed to reject the assumptions when they are false. Misspecifications of these structures may lead to spurious conclusions about university objectives.

Another disadvantage of these methods is that they depend on the characteristics of non-offered courses. As discussed in Subsection 3.1.1, researchers rarely observe non-offered courses; this implies that strong assumptions about non-offered courses are required to implement this algorithm. Researchers can somewhat address this issue by using a subset of $\mathbf{D}(E)$ which only contains offering vectors which are deemed reasonable. Furthermore, these methods are very sensitive to estimates of minimum costs $m_{j}$ because these estimates determine which offering vectors are feasible and how much residual money is left for excess spending. Any error in estimates of $m_{j}$ may lead to spurious conclusions about university preference parameters.

## 5 Empirical Application

In this section, I describe estimation details and present parameter estimates for a model of course choice. As discussed in Subsection 3.1, estimates of this course choice model are central to the theoretical framework presented in Section 2 and are crucial elements of all inference methods presented in Section 4. Results of these inference methods are forthcoming.

### 5.1 Estimation Details

In the sequential game between a university and students described in Section 2, university decisions about which courses to offer and how much to spend on instructors hinge on how the university expects students to respond to these decisions. As such, it is crucial to obtain credible estimates of how student course choices depend on the set of offered courses and spending on instructors for these courses.

To obtain these estimates, I use a multinomial nested logit model of student course choices where nests are defined by academic fields. ${ }^{17}$ The nesting structure relaxes the independence of irrelevant alternatives assumption by allowing for correlation in unobserved preferences for courses of the same field. To avoid issues of endogeneity and unobserved heterogeneity in choice sets, I focus on introductory course choices only and assume that all introductory courses are in the choice sets of all enrolled students. As such, the estimation method should be viewed as a conditional nested logit in which students choose which introductory courses

[^14]to take conditional on already choosing to take some introductory course.
In this preliminary analysis, I consider the baseline setting in which class sizes do not affect the desirability of a course. Results for the general equilibrium setting in which class size affects course desirability are forthcoming.

### 5.1.1 Student utility, choice probabilities, and expected welfare

I assume the deterministic utility of introductory course $j$ for student $i$ depends on observed student characteristics $X_{i}$ and excess spending on instruction in course $j e_{j}$ as:

$$
u_{i j}\left(e_{j}\right)=X_{i} \beta_{f(j)}+\theta_{f(j)} \ln \left(e_{j}+1\right)
$$

where $f(j)$ indicates the academic field of introductory course $j$. The logarithmic structure is included to make the marginal utility of excess spending diminish as spending increases. ${ }^{18}$ $X_{i}$ includes gender and ACT scores to capture heterogeneous preferences for academic fields by gender and initial preparation. Furthermore, $X_{i}$ includes cohort dummy variables to allow for changes in relative preferences for introductory courses of different fields over the course of college.

I assume stochastic utility is given by deterministic utility with an additively separable error:

$$
U_{i j}\left(e_{j}\right)=u_{i j}+\epsilon_{i j}
$$

where $\epsilon_{i j}$ follows a nested logit structure where nests are defined by academic fields.
With this structure, the probability student $i$ chooses introductory course $j$ conditional on choosing an introductory course is given by:

$$
P_{i j}(\mathbf{e}, \mathbf{d})=\frac{\exp \left(\frac{u_{i j}}{\rho}\right)\left[\sum_{j^{\prime} \in f(j)} \exp \left(\frac{u_{i^{\prime}}}{\rho}\right)\right]^{\rho-1}}{\sum_{f=1}^{F}\left[\sum_{j^{\prime} \in f} \exp \left(\frac{u_{i j^{\prime}}}{\rho}\right)\right]^{\rho}}
$$

where $\rho$ is the nesting parameter. ${ }^{19}$ Furthermore, with this structure, student $i$ 's expected welfare from her choice of which introductory course to take is given by:

$$
\mathbb{E}\left[V_{i} \mid \mathbf{e}, \mathbf{d}\right]=\ln \left\{\sum_{f=1}^{F}\left[\sum_{j^{\prime} \in f} \exp \left(\frac{u_{i j^{\prime}}}{\rho}\right)\right]^{\rho}\right\}+\gamma
$$

[^15]where $\gamma$ is the Euler-Mascheroni constant. Importantly, choice probabilities and expected welfare both depend on the university's choice of which courses to offer $\mathbf{d}$ and how much to spend in excess of minimum costs to increase instructor quality e.

### 5.1.2 Maximum Likelihood Estimation

Let $C_{i t}$ represent the number of introductory courses taken by individual $i$ in semester $t$ and index these courses by $c$. Let $y_{i t c j}$ indicate whether individual $i$ chooses introductory course $j$ for choice $c$ in academic semester $t$. The conditional likelihood that student $i$ chooses her observed introductory course for choice $c$ in academic semester $t$ is given by:

$$
\mathscr{L}_{i t c}=\prod_{j \in \mathbf{d}_{\mathbf{t}}} P_{i j}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)^{y_{i t c j}}
$$

amounts of resources spent Taking products over courses within semesters, students, and semesters, the conditional likelihood of observing the observed introductory course choices is given by: ${ }^{20}$

$$
\mathscr{L}(\mathbf{y} ; \beta, \theta, \rho)=\prod_{t=1}^{T} \prod_{i=1}^{N_{t}} \prod_{c=1}^{C_{i t}} \prod_{j \in \mathbf{d}_{\mathbf{t}}} P_{i j}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}} ; \beta, \theta, \rho\right)^{y_{i t c j}}
$$

The log conditional likelihood is then given by:

$$
\ln \mathscr{L}(\mathbf{y} ; \beta, \theta)=\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{c=1}^{C_{i t}} \sum_{j \in \mathbf{d}_{t}} y_{i t c j} \ln P_{i j}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}} ; \beta, \theta, \rho\right)
$$

Estimates of student utility parameters $\beta, \theta$, and $\rho$ are obtained by numerically solving for the parameter values which maximize this log conditional likelihood.

### 5.2 Estimates of student utility parameters

Table 2 compiles estimates of the multinomial logit choice model. The estimates imply that a first year male student with average ACT scores is most attracted to introductory humanities and arts courses followed by STEM, occupational, social science and business. First year female students with average scores are also most attracted to introductory humanities and arts courses followed by social science, occupational, STEM, and business. While introductory business courses are unpopular with freshmen, they are quite popular with more

[^16]advanced students. With the exception of female seniors, sophomores, juniors, and seniors with average ACT scores favor introductory business courses to all other courses. Comparatively, while introductory humanities and arts courses are popular with Freshmen, they are rarely taken by more advanced students - sophomores, juniors and seniors with average ACT scores are least interested in taking introductory humanities and arts courses.

The estimates also imply that students with higher ACT scores are relatively more likely to enroll in introductory STEM courses and slightly less likely to enroll in introductory occupational courses. For example, while a first year male student with average ACT scores prefers taking introductory humanities and arts courses, a first year male student whose ACT scores are 1.5 standard deviations above the mean is approximately indifferent between introductory STEM and humanities courses. The finding that students with higher ACT scores are relatively more likely to enroll in introductory STEM courses is consistent with existing literature which finds that initial preparation is an important determinant of whether a student pursues a STEM education (Arcidiacono, 2004; Stinebrickner and Stinebrickner, 2014).

The return on excess spending results show that excess spending on instruction has a positive and significant effect on course desirability for all fields except business. However, the magnitudes of these estimates suggest the effects are quite small relative to non-spending preferences for fields. Social science courses appear most sensitive to excess spending but even these coefficients are small. The distribution of $\ln \left(e_{j}+1\right)$ has mean 7.71 and standard deviation 2.10. This implies first year male students with average ACT scores are approximately indifferent between an introductory social science course with $\ln \left(e_{j}+1\right)$ that is 3.67 standard deviations above zero and a introductory social science course with zero excess spending. Put simply, even in the field most sensitive to excess spending it takes large increases in spending to overcome latent differences in preferences.

This finding has important implications for universities. It implies that the vast amounts of resources spent hiring instructors who cost more than minimally qualified teachers has relatively small effects on student course choices and student welfare. To see this more concretely, consider the comparison of three simple hypothetical universities given in Table 3. As a baseline, suppose a university is currently offering one course from each field and hiring minimally qualified instructors to teach these courses. This costs $\$ 14,420$ and yields 61,829 units of welfare. Now suppose the university has surplus funds and wishes to either increase student welfare or increase enrollment in social science courses. In alternative A, the university spends its additional funds hiring another minimally qualified instructor to teach one additional social science course. Under this alternative, total cost is $\$ 17,124$, social science enrollment is $28.3 \%$ of total enrollment, and student welfare is 66,282 units.

In alternative B, the university spends the same amount of funds hiring a more qualified instructor to teach its one social science course. Under this alternative, total cost is still $\$ 17,124$, but social science enrollment is only $24.2 \%$ of the total, and student welfare is only 64,509. This implies that even for social science - where excess spending has the largest effects on utility - it is more efficient to increase student welfare or change student course choices by offering additional courses rather than hiring more qualified instructors.

This result provides an interesting complement to existing literature which examines the effects of instructor qualifications on student learning. Figlio, Shapiro, and Soter (2013) use data from Northwestern University and find students learn relatively more from non-tenure track instructors-who generally have lower salaries - than tenure track salaries. Additionally, Bettinger and Long (2010) use data from public four year colleges in Ohio and find that non-tenure track instructors make students more likely to take subsequent courses in a field. Together, my result and these results imply that the vast amounts of resources universities spend hiring more qualified instructors have small (or possibly negative) effects on the academic experiences of students.

### 5.3 Estimates of university parameters

Table 4 compiles estimates of university preference parameters obtained using the maximum likelihood estimation procedure discussed in Subsection 4.3. ${ }^{21}$ Intuitively, this estimation algorithm solves for parameter values which best explain why observed introductory course offerings were preferred to alternative feasible offering vectors. To be feasible, an alternative offering vector must contain all contracted introductory courses and must satisfy the budget constraint. ${ }^{22}$

For clarity, the results in Table 4 arise from a simple version of the student driven structure given by:

$$
\mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]=\sum_{i=1}^{N} \mathbb{E}\left[V_{i} \mid \mathbf{e}, \mathbf{d}\right]+\sum_{j=1}^{J} \gamma_{g(j)} \tilde{n}_{j}(\mathbf{e}, \mathbf{d})
$$

In this version of the student driven structure, the university gives equal weight to the welfare of all enrolled students but still possesses separate institutional preferences for class

[^17]enrollments.
The results in table 3 imply University of Central Arkansas (UCA) values enrollment in humanities and arts courses more than all other fields. After humanities and arts, UCA values STEM enrollment, social science enrollment, occupational enrollment and business enrollment. To directly see UCA's relative preference for humanities and arts enrollment, note that descriptive statistics in Table 1 show humanities courses comprise $31.6 \%-34.5 \%$ of introductory course spending but only $30.7 \%-32.2 \%$ of introductory course enrollment. This outsize investment reflects UCA's desire to increase enrollment in introductory humanities courses. Comparatively, business courses comprise only 5.7\%-7.3\% of introductory course spending but make up $8.1 \%-9.7 \%$ of introductory course enrollment. This under investment is consistent with an objective to draw students away from introductory business courses and into other fields.

Table 3 also includes measures assessing goodness of fit for the estimated student driven model. As discussed in Subsection 4.3.1, model fit can be evaluated by calculating the fraction of alternative feasible offering vectors which are preferred to the chosen vector in the estimated model. If a substantial fraction of alternative feasible offering vectors yield larger university payoffs than the chosen vector then the model is explaining university decisions poorly. The results of this analysis show that at most $0.5 \%$ of alternative feasible offerings are preferred to the chosen vector in any given semester. In two of the four semesters, all alternative feasible offerings yield lower university payoffs than the chosen vector. This indicates that the fitted model is explaining university choices remarkably well.

## 6 Counter-factual Simulations

In Section 5, I presented estimates of university preference parameters which show University of Central Arkansas (UCA) has a relative preference for increasing enrollments in STEM. To place estimates of university parameter values in context and to examine university behaviors under alternative constraints, I develop tools for simulating classroom spending decisions under alternative objectives and constraints. I use these tools to examine two counter-factual simulations: First, I solve for counter-factual minimum costs which lead UCA to offer courses which maximize student welfare. The simulation suggests that a revenue neutral tax and subsidy policy which increases the cost of offering introductory humanities, STEM, social science, and occupational courses and decreases the cost of offering business courses leads UCA to offer the same courses which maximize student welfare at market costs. Second, I simulate course offerings and excess spending decisions which produce welfare efficiently in the absence of contractual constraints. This simulation shows UCA could achieve the same
student welfare at $38.5 \%$ of original costs in the absence of contractual constraints. I begin this section by presenting an algorithm for approximating the classroom spending decisions of a university. Following this, I present results of counter-factual simulations.

### 6.1 Marginal Improvement Algorithm

This subsection presents a Marginal Improvement Algorithm (Chade and Smith, 2006) for approximating the course offerings and excess spending decisions of a university with a known objective function. Broadly speaking, the algorithm iteratively adds single courses which best complement previously selected courses until the budget constraint is satisfied or no marginal improving courses exist.

To illustrate these methods more concretely, note that it is often possible to combine tangency conditions given by 3 with the budget constraint to solve for the optimal excess spending vector for each offering vector. ${ }^{23}$ Denote these optimal excess spending vectors as: $\mathbf{e}(\mathbf{d})^{\star}$. The university's problem can then be restated to focus on extensive margin decisions: ${ }^{24}$

$$
\mathbf{d}^{\star}=\operatorname{argmax}_{\mathbf{d}}\left\{\mathbb{E}\left[\Pi \mid \mathbf{e}(\mathbf{d})^{\star}, \mathbf{d}\right]\right\} \forall \text { d s.t. } \sum_{j=1}^{J} d_{j} f_{j} \leq E
$$

Because the offering vector $\mathbf{d}$ is discrete, Lagrange methods cannot be used to characterize properties of the extensive margin solution. Furthermore, because the number of feasible $\mathbf{d}$ is typically very large, directly solving the problem is impractical. To solve for $\mathbf{d}^{\star}$, the Marginal Improvement Algorithm starts by selecting the single course offering which delivers the greatest expected payoff to the university. ${ }^{25}$ This involves computing the university's payoff for every $J$ potential course. Following this, the algorithm selects the best course to offer alongside this first course. This entails calculating the university's payoff for all remaining $J-1$ potential courses. The algorithm continues adding marginally improving course until marginal effects turn negative or until the constraint $\sum_{j=1}^{J} d_{j} m_{j} \leq E$ binds. Technical details on this algorithm are provided in Methodological Appendix E.

[^18]
### 6.2 Counter-factual costs which yield welfare maximizing course offerings

Table 5 contains simulation results which show that a revenue neutral tax and subsidy policy which increases the cost of offering introductory humanities, STEM, social science, and occupational courses and decreases the cost of offering business courses leads UCA to offer the same courses which maximize student welfare at market costs.

Column 1 reports optimal course offerings for a welfare maximizing university with a 1.9 million dollar endowment and no contractual constraints facing minimum costs estimated from data. The results suggest introductory business and occupational courses yield the best value to students while introductory STEM and humanities courses are less desirable. Column 2 reports offerings for an unconstrained university with the same endowment and minimum costs but with a student driven objective structure with parameter values reported in Table 3. These results show that UCA's preference for enrollment in humanities and STEM courses relative to occupational and business courses lead to many more humanities and STEM course offerings and fewer occupational and business courses when compared to the offerings which maximize student welfare.

Column 3 shows that the welfare maximizing offerings presented in column 1 will be chosen by a student driven university with approximately the same endowment under counterfactual minimum costs. The counter-factual costs represent a $87 \%$ increase in the minimum cost of introductory humanities courses, a $45 \%$ increase for STEM courses, a $32 \%$ increase for social science courses, a $14 \%$ increase in occupational courses, and a $21 \%$ decrease in the minimum cost of introductory business courses. This demonstrates that a tax and subsidy policy which modifies the relative minimum costs of offering different courses can induce a student driven university to offer courses which maximize student welfare. Furthermore, this policy would be approximately revenue neutral at the university's optimal course offerings. While such a policy may be impractical or undesirable for other reasons it is interesting to see what counter-factual minimum costs would price out institutional preferences for enrollments in different fields.

### 6.3 Welfare maximizing classroom spending decisions at University of Central Arkansas

Table 6 compares the UCA's observed course offerings in the Fall semester of 2007 to the course offerings which produce welfare efficiently in a scenario with no contractual constraints. The differences are quite striking. The counter-factual welfare maximizing UCA yields the same student welfare at $38.5 \%$ of original costs. These savings are primarily due
to reductions in spending on instruction. As discussed in Section 5, the student utility parameter estimates suggest spending on instruction has relatively small effects on course desirability. The Marginal Improvement Algorithm results show that the university can save vast sums of money at little cost to students by hiring only minimally qualified instructors.

The results also show that UCA could alter its introductory course composition to better serve students. The observed introductory course offering contains many more STEM and humanities courses and many fewer social science business courses than the counter-factual welfare maximizing course offerings. Importantly, although the counter-factual scenario is very different from observed course offerings the total number of courses offered is similar in both scenarios. This suggests the welfare maximizing offerings would not require large changes in facilities which would introduce costs not included in my analysis. While such a vastly different university may be undesirable for other reasons, it is striking to see that students could receive the same benefit with drastically lower costs and interesting to note what alternative classroom spending decisions achieve these savings.

## 7 Conclusion

In 1973, Daniel Bell described the university as "the axial institution of post-industrial society" (Bell, 1974). This is more true today than it was over four decades ago. Despite this, very little is known about how universities make decision. A better understanding of university objectives could lead to policies which benefit students and reduce financial burdens on taxpayers, families, and donors.

In this paper, I develop tools for inferring university objectives from decisions of which courses to offer and how much to spend on instructors for these courses. The methods include a statistical test of whether classroom spending decisions maximize student welfare and two methods for estimating parameters of a student driven structure of university objectives. I apply these methods to administrative data from University of Central Arkansas (UCA) and find UCA has institutional preferences for decreasing enrollment in introductory business courses and increasing enrollments in introductory humanities and STEM courses.

In addition to discussing methods for inferring the objectives of an observed university, I also present a method for simulating the classroom spending decisions of a university with alternative objectives or facing counter-factual constraints. I use this method to run two simulations: First, I show that a revenue neutral tax and subsidy policy which reduces the cost of offering introductory business courses and increases the cost of offering other introductory courses can induce UCA to offer courses which maximize student welfare. Second, I show that UCA could achieve the same student welfare at $38.5 \%$ of original costs in the
absence of contractual constraints. The savings primarily result from hiring less expensive instructors but are also generated by offering more introductory business courses and fewer introductory STEM and humanities courses. While these scenarios may be undesirable for other reasons, it is useful to see how a revenue neutral policy could be used to benefit students and it is striking to see that students could receive the same benefit with drastically lower costs with changes in instructors and course composition.

Given the paucity of empirical evidence on university objectives there is substantial room for additional work. My analysis focuses on inferring objectives from classroom decisions that are made conditional on existing employee contracts. A dynamic approach may reveal objectives from these long term contract decisions. Such an approach may also incorporate production of reputation into the university's objective function. Furthermore, there is a need for research which delves deeper and identifies more structural university objective parameters. Bhattacharya, Kanaya, and Stevens (2014) and Turner (2014) find evidence that universities prefer student compositions which over-represent students with certain characteristics; however, the authors have limited scope for exploring why the university prefers these students. Similarly, my analysis reveals a university's objective to draw students out of introductory business courses and into introductory STEM and humanities courses; however, it does not explain why STEM and humanities courses are preferred. A deep understanding of what motivates university decisions is an important step towards developing effective higher education policies.

## Methodological Appendix A: Expressions for effects of spending on choice probabilities and class sizes in general equilibrium

The effect of increasing spending in course $j$ on the expected number of students who choose course $j$ is recursively defined by:

$$
\begin{aligned}
\frac{d \tilde{n}_{j}}{d e_{j}} & =\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left[\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}+\frac{\partial u_{i j}}{\partial \tilde{n}_{j}} \frac{d \tilde{n}_{j}}{d e_{j}}\right] \\
& =\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left(\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}\right)+\left(\frac{d \tilde{n}_{j}}{d e_{j}}\right) \sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left(\frac{\partial u_{i j}}{\partial \tilde{n}_{j}}\right)
\end{aligned}
$$

where dependence on the spending vector $\mathbf{e}$ is suppressed. Rearranging yields:

$$
\begin{equation*}
\frac{d \tilde{n}_{j}}{d e_{j}}=\frac{\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left(\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}\right)}{1-\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left(\frac{\partial u_{i j}}{\partial n_{j}}\right)} \tag{13}
\end{equation*}
$$

This provides a closed form solution for the own-course effects of spending on expected class size which can be substituted into (7). Furthermore, the effects of increasing spending in course $j$ on the expected number of students who choose all other courses can be related to these own-course effects of spending on expected class size as follows:

$$
\frac{d \tilde{n}_{k}}{d e_{j}}=-\sum_{i=1}^{N} P_{i j} P_{i k}\left[\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}+\frac{\partial u_{i j}}{\partial \tilde{n}_{j}} \frac{d \tilde{n}_{j}}{d e_{j}}\right] \quad j \neq k
$$

Substituting for $\frac{d \tilde{n}_{j}}{d e_{j}}$ using (13) yields:

$$
\begin{equation*}
\frac{d \tilde{n}_{k}}{d e_{j}}=-\sum_{i=1}^{N} P_{i j} P_{i k}\left[\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}+\frac{\partial u_{i j}}{\partial \tilde{n}_{j}}\left\{\frac{\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left(\frac{\partial u_{i j}}{\partial I_{j}} \frac{\partial \phi_{j}}{\partial e_{j}}\right)}{1-\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\left(\frac{\partial u_{i j}}{\partial n_{j}}\right)}\right\}\right] \quad j \neq k \tag{14}
\end{equation*}
$$

## Methodological Appendix B: Identification of $\omega_{\tau}$ and $\gamma_{g}$ from variance minimization

Outline
Restrict to a small set of courses such that the system is exactly identified. Write out the linear system and explicitly solve for its unique solution. Note that this implies the system is over-identified.

## Methodological Appendix C: Optimal Excess Spending Decisions

In this appendix, I present methods for computing $\mathbf{e}(\mathbf{d})^{\star}$ for several alternative settings and utility structures. In most cases, it is infeasible to solve for $\mathbf{e}(\mathbf{d})^{\star}$ explicitly; however, it is often possible to define $\mathbf{e}(\mathbf{d})^{\star}$ implicitly and solve for a fixed point of these implicit definitions using a iterative algorithm.

## Example 1: Welfare Maximizing University - no effects of class size

This example solves for $\mathbf{e}(\mathbf{d})^{\star}$ for a welfare maximizing university in the baseline setting where class size does not effect course utility. For simplicity, let $J$ represent the set of courses offered under offering vector d. Suppose choice utility is given by:

$$
U_{i j}=\theta_{j} \ln \left(e_{j}+1\right)+\psi_{i j}\left(Z_{j}, X_{i}\right)+\epsilon_{i j}
$$

With this structure, a welfare maximizing university's tangency conditions are given by:

$$
\begin{equation*}
\frac{\theta_{j} \tilde{\mathbf{n}}_{j}(\mathbf{e})}{e_{j}+1}=\frac{\theta_{j^{\prime}} \tilde{\mathbf{n}}_{j^{\prime}}(\mathbf{e})}{e_{j^{\prime}}+1} \quad \forall j, j^{\prime} \in J \tag{15}
\end{equation*}
$$

and the binding budget constraint is given by:

$$
\sum_{j=1}^{J}\left(m_{j}+e_{j}\right)=E
$$

$\mathbf{e}(\mathbf{d})^{\star}$ can then be implicitly defined as:

$$
\begin{equation*}
e_{j}(\mathbf{d})^{\star}=\frac{\left[E+J-\sum_{j=1}^{J} m_{j}\right] \theta_{j} \tilde{\mathbf{n}}_{j}(\mathbf{e})}{\sum_{j^{\prime}=1}^{J} \theta_{j^{\prime}} \tilde{\mathbf{n}}_{j^{\prime}}(\mathbf{e})}-1 \tag{16}
\end{equation*}
$$

The following iterative algorithm can then be used to solve for a fixed point of this implicit definition:

1. Set initial excess spending values to be uniform across offered courses: $e_{j}^{1}=\frac{E-\sum_{j=1}^{J} m_{j}}{J}$
2. Compute expected class sizes given initial excess spending values: $\tilde{\mathbf{n}}_{j}^{1}=\tilde{\mathbf{n}}_{j}\left(\mathbf{e}^{1}\right)$
3. Use Equation (16) to compute new excess spending values: $e_{j}^{2}$
4. Repeat until sequential values of $\mathbf{e}$ become arbitrarily close.

## Example 2: Student Driven University - no effects of class size

This example solves for $\mathbf{e}(\mathbf{d})^{\star}$ for a student driven university in the baseline setting where class size does not effect course utility. As before, let $J$ represent the set of courses offered under offering vector $\mathbf{d}$ and suppose choice utility is given by:

$$
U_{i j}=\theta_{j} \ln \left(e_{j}+1\right)+\psi_{i j}\left(Z_{j}, X_{i}\right)+\epsilon_{i j}
$$

where $\epsilon_{i j}$ follows a type 1 extreme value distribution. With this structure, a student driven university's tangency conditions are given by:

$$
\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}=\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j^{\prime}}}
$$

where

$$
\begin{aligned}
\frac{d \mathbb{E}[\Pi \mid \mathbf{e}, \mathbf{d}]}{d e_{j}}= & \left\{\sum_{i=1}^{N} \omega_{i}\left(\frac{\theta_{j}}{e_{j}+1}\right) P_{i j}(\mathbf{e}, \mathbf{d})\right\}+\sum_{k=1}^{J}\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{k}}\right)\left(\frac{\partial \tilde{n}_{k}}{\partial e_{j}}\right) \\
= & \left(\frac{\theta_{j}}{e_{j}+1}\right)\left[\left\{\sum_{i=1}^{N} \omega_{i} P_{i j}(\mathbf{e}, \mathbf{d})\right\}+\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{j}}\right)\left(\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\right)\right] \\
& -\sum_{k \neq j}\left(\frac{\theta_{k}}{e_{k}+1}\right)\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{k}}\right)\left(\sum_{i=1}^{N} P_{i k} P_{i j}\right)
\end{aligned}
$$

and the binding budget constraint is given by:

$$
\sum_{j=1}^{J}\left(m_{j}+e_{j}\right)=E
$$

To simplify notation, I make the following substitutions:

$$
\begin{aligned}
& \alpha_{j}(\mathbf{e})=\left\{\sum_{i=1}^{N} \omega_{i} P_{i j}(\mathbf{e}, \mathbf{d})\right\}+\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{j}}\right)\left(\sum_{i=1}^{N} P_{i j}\left(1-P_{i j}\right)\right) \\
& \kappa_{j}(\mathbf{e})=\left(e_{j}+1\right)\left[\sum_{k \neq j}\left(\frac{\theta_{k}}{e_{k}+1}\right)\left(\frac{\partial \Upsilon}{\partial \tilde{n}_{k}}\right)\left(\sum_{i=1}^{N} P_{i k} P_{i j}\right)\right]
\end{aligned}
$$

This simplifies the first order conditions to:

$$
\frac{\theta_{j} \alpha_{j}(\mathbf{e})-\kappa_{j}(\mathbf{e})}{e_{j}+1}=\frac{\theta_{j^{\prime}} \alpha_{j^{\prime}}(\mathbf{e})-\kappa_{j^{\prime}}(\mathbf{e})}{e_{j^{\prime}}+1}
$$

$\mathbf{e}(\mathbf{d})^{\star}$ can then be implicitly defined as:

$$
e_{j}(\mathbf{d})^{\star}=\frac{\left(\theta_{j} \alpha_{j}(\mathbf{e})-\kappa_{j}(\mathbf{e})\right)\left(E+J-\sum_{j=1}^{J} m_{j}\right)}{\sum_{j^{\prime}=1}^{J}\left(\theta_{j^{\prime}} \alpha_{j^{\prime}}(\mathbf{e})-\kappa_{j^{\prime}}(\mathbf{e})\right)}-1
$$

# Methodological Appendix D: Identification of $\omega_{\tau}$ and $\gamma_{g}$ from maximum likelihood 

Outline
Type 1 extreme value assumption makes likelihood globally concave. Discuss normalizations.

## Methodological Appendix E: Marginal Improvement Algorithm for U-SWM Course Offerings

In this appendix, I describe a Marginal Improvement Algorithm (MIA) for solving for the optimal course offerings and excess spending decisions of a university. The algorithm requires that the university's objective as a function of offering vector $\mathbf{d}$ and excess spending vector e is known. Furthermore, the algorithm requires that the set of feasible courses $J$ and the budget endowment $E$ are observed.

Let $\Pi(\mathbf{d}, \mathbf{e})$ represent the university's objective given offering vector $\mathbf{d}$ and excess spending vector $\mathbf{e}$ and let $\mathbf{e}(\mathbf{d})^{\star}$ represent the university's optimal excess spending vector given offering vector $\mathbf{d}$. Algorithms for deriving $\mathbf{e}(\mathbf{d})^{\star}$ for various structures of student utility and university objectives are presented in Methodological Appendix D. Finally, let $\mathbf{v}_{\mathbf{j}}$ represent the elementary $J \times 1$ vector which contains 1 in entry $j$ and zeros in all other entries. The MIA proceeds as follows:

1. Solve for the best single course to offer alongside contracted courses:

$$
j_{1}^{\star}=\operatorname{argmax}_{j \in J}\left\{\Pi\left(\mathbf{v}_{\mathbf{j}}, \mathbf{e}\left(\mathbf{v}_{\mathbf{j}}\right)^{\star}\right)\right\} \quad \text { s.t. } m_{j} \leq E
$$

2. Solve for the best course to offer alongside $j_{1}^{\star}$ and contracted courses:

$$
j_{2}^{\star}=\operatorname{argmax}_{j \in J \backslash j_{1}^{\star}}\left\{\Pi\left(\mathbf{v}_{\mathbf{j}}+\mathbf{v}_{\mathbf{j}_{\mathbf{1}}^{\star}}, \mathbf{e}\left(\mathbf{v}_{\mathbf{j}}+\mathbf{v}_{\mathbf{j}_{\mathbf{1}}}\right)^{\star}\right)\right\} \quad \text { s.t. } m_{j}+m_{j_{1}^{\star}} \leq E
$$

3. In general, solve for the best $k+1$ courses to offer alongside previously chosen $k$ courses
and contracted courses:

$$
\begin{aligned}
j_{k+1}^{\star}= & \operatorname{argmax}_{j \in J \backslash \cup_{k^{\prime}=1}^{k} j_{k^{\prime}}^{\star}}\left\{\Pi\left(\mathbf{v}_{\mathbf{j}}+\sum_{k^{\prime}=1}^{k} \mathbf{v}_{\mathbf{j}_{\mathbf{k}^{\prime}}} \mathbf{e}\left(\mathbf{v}_{\mathbf{j}}+\sum_{k^{\prime}=1}^{k} \mathbf{v}_{\mathbf{j}_{\mathbf{k}^{\prime}}^{\star}}\right)^{\star}\right)\right\} \\
& \text { s.t. } m_{j}+\sum_{k^{\prime}=1}^{k} m_{j_{k^{\prime}}^{\star}} \leq E
\end{aligned}
$$

The algorithm terminates when either the best additional course decreases the university's objective or when no additional courses can be added without violating the budget constraint. Formally, the algorithm terminates if:

$$
\Pi\left(\sum_{k^{\prime}=1}^{k+1} \mathbf{v}_{\mathbf{j}_{\mathbf{k}^{\prime}}} \mathbf{e}\left(\sum_{k^{\prime}=1}^{k+1} \mathbf{v}_{\mathbf{j}_{\mathbf{k}^{\prime}}^{\star}}\right)^{\star}\right)<\Pi\left(\sum_{k^{\prime}=1}^{k} \mathbf{v}_{\mathbf{j}_{\mathbf{k}^{\prime}}^{\star}}, \mathbf{e}\left(\sum_{k^{\prime}=1}^{k} \mathbf{v}_{\mathbf{j}_{\mathbf{k}^{\prime}}^{*}}\right)^{\star}\right)
$$

or if

$$
\min _{j \in J \backslash \backslash \cup_{k^{\prime}=1}^{k} j_{k^{\prime}}^{\star}}\left\{m_{j}+\sum_{k^{\prime}=1}^{k} m_{j_{k^{\prime}}^{\star}}\right\}>E
$$

## References

Ahn, T., Arcidiacono, P., Hopson, A., Thomas, J., 2015. "Equilibrium Grade Inflation with Implications for Female Interest in STEM Majors." Working paper.

Altonji, J. G., Blom, E., Meghir, C., 2012. "Heterogeneity in Human Capital Investments: High School Curriculum, College Major, and Careers." Annual Review of Economics 4, 185223.

Arcidiacono, P., 2004. "Ability sorting and the returns to college major." Journal of Econometrics 121, 343-375.

Bayer, P., Timmins, C., 2007. "Estimating Equilibrium Models Of Sorting Across Locations." The Economic Journal 117(518), 353-374.

Bettinger, E. P., Long, B. T., 2010. "Does cheaper mean better? The impact of using adjunct instructors on student outcomes." The Review of Economics and Statistics 92(3), 598-613.

Bhattacharya, D., Kanaya, S., Stevens, M., 2014. "Are University Admissions Academically Fair?" Working paper.

Brodhead, R. H., 2012. "Brodhead: Advocating for the Humanities." Keynote address to annual meeting of the National Humanities Alliance. March 19, 2012.

Chade, H., Smith, L., 2006. "Simultaneous search." Econometrica 74(5), 1293-1307.
Cyrenne, P., Grant, H., 2009. "University decision making and prestige: An empirical study." Economics of Education Review 28, 237-248.

Epple, D., Romano, R., Sieg, H., 2006. "Admission, tuition, and financial aid policies in the market for higher education." Econometrica 74(4), 885-928.

Figlio, D. N., Schapiro, M. O., Soter, K. B., 2013. "Are Tenure Track Professors Better Teachers?" NBER Working Paper 19406.

Hoxby, C., 2012. "Endowment management based on a positive model of the university." NBER Working Paper 18626.

McFadden, D., 1978. "Modeling the choice of residential location." In A. Karlqvist, L. Lundqvist, F. Snickars, and J. Weibull, eds., Spatial Interaction Theory and Planning Models, North-Holland, Amsterdam, 75-96.

OECD, 2014. "Education at a Glance 2014: OECD Indicators." OECD Publishing.
Oreopoulos, P., Petronijevic, U., 2013. "Making College Worth It: A Review of the Returns to Higher Education." Future of Children 23(1), 41-65.

Stinebrickner, T., Stinebrickner, R., 2014. "A Major in Science? Initial Beliefs and Final Outcomes for College Major and Dropout." Review of Economic Studies 81, 426-472.

Turner, L. J., 2014. "The Road to Pell is Paved with Good Intentions: The Economic Incidence of Federal Student Grant Aid." Working paper.

# Table 1: Course Offerings, Spending, and Course Choices 

|  | Fall, 2007 <br> Introductory course offerings | Spring, 2008 | Fall, 2008 | Spring, 2009 |
| :--- | :---: | :---: | :---: | :---: |
| STEM | 279 | 242 |  |  |
| Social Sciences | 272 | 245 | 249 | 226 |
| Humanities | 433 | 407 | 257 | 258 |
| Occupational | 125 | 114 | 418 | 368 |
| Business | 65 | 57 | 107 | 99 |
|  |  | 61 | 58 |  |

Median spending per introductory course

| STEM | $\$ 8,410$ | $\$ 9,098$ | $\$ 8,632$ | $\$ 8,597$ |
| :--- | :--- | :--- | :--- | :--- |
| Social Sciences | $\$ 6,868$ | $\$ 6,354$ | $\$ 6,516$ | $\$ 6,292$ |
| Humanities | $\$ 6,547$ | $\$ 5,996$ | $\$ 5,802$ | $\$ 5,718$ |
| Occupational | $\$ 7,266$ | $\$ 6,824$ | $\$ 6,324$ | $\$ 5,531$ |
| Business | $\$ 9,480$ | $\$ 7,642$ | $\$ 7,391$ | $\$ 7,279$ |


| Share of total spending on introductory courses by field |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| STEM | $26.3 \%$ | $27.9 \%$ | $27.6 \%$ | $28.5 \%$ |
| Social Sciences | $21.7 \%$ | $21.3 \%$ | $22.2 \%$ | $24.5 \%$ |
| Humanities | $34.1 \%$ | $33.8 \%$ | $34.5 \%$ | $31.6 \%$ |
| Occupational | $10.6 \%$ | $11.3 \%$ | $9.1 \%$ | $8.4 \%$ |
| Business | $7.3 \%$ | $5.7 \%$ | $6.6 \%$ | $7.0 \%$ |

Share of total introductory student-course observations by field

STEM
Social Sciences
Humanities
Occupational
Business

Total Cost
Total Courses
22.2\%
20.9\%
21.8\%
21.7\%
26.8\%
27.0\%
27.5\%
28.2\%
31.6\%
32.1\%
32.2\%
30.7\%
$11.3 \%$
11.3\%
9.4\%
9.6\%
8.1\%
8.6\%
9.2\%
9.7\%
\$9,784,463
1174
\$8,369,421
1065
\$7,930,221
1092
\$7,109,921
1009

Statistics are for University of Central Arkansas. Students include all full time degree seeking undergraduates. Courses include all introductory undergraduate courses. All cost statistics are measured in 2012 dollars.

Table 2: Nested Logit Coefficient Estimates

|  | STEM | Social Science | Humanities | Occupational | Business <br> omitted |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Intercept | 0.2754 | 0.0980 | 0.4986 | 0.2188 |  |
|  | 0.0090 | 0.0091 | 0.0096 | 0.0143 |  |
| Female | 0.5140 | 0.8093 | 0.4560 | 0.6694 | omitted |
|  | 0.0112 | 0.0115 | 0.0128 | 0.0187 |  |
| ACT Z-score | 0.1518 | 0.0005 | -0.0049 | -0.0677 | omitted |
|  | 0.0062 | 0.0057 | 0.0094 | 0.0146 |  |
| Missing ACT | -0.2220 | -0.0550 | -0.1696 | 0.0017 | omitted |
|  | 0.0200 | 0.0181 | 0.0187 | 0.0253 |  |
| Sophomore | -1.7746 | -1.6281 | -2.3829 | -1.9234 | omitted |
|  | 0.0191 | 0.0173 | 0.0176 | 0.0274 |  |
| Junior | -2.2214 | -2.2204 | -3.2888 | -2.0030 | omitted |
|  | 0.0251 | 0.0233 | 0.0257 | 0.0317 |  |
| Senior | -0.9330 | -1.1566 | -2.3669 | -0.7185 | omitted |
|  | 0.0321 | 0.0313 | 0.0358 | 0.0397 |  |
| Marginal utility of | 0.0218 | 0.0520 | 0.0432 | 0.0067 | -0.0021 |
| log spending | 0.0010 | 0.0011 | 0.0012 | 0.0018 | 0.0019 |

Nesting parameter estimate: .9032; standard errors in italics.

Results are for a multinomial nested logit model of students choosing introductory courses. Nests are defined by academic fields. Data are from Fall and Spring academic semesters of 2007-08 and 2008-09 at University of Central Arkansas.

## Table 3: Adding Courses vs Spending on Instruction

|  | Baseline | "More courses" <br> Alternative | "More spending" <br> Alternative |
| :--- | :---: | :---: | :---: |
| Introductory course offerings |  |  |  |
| STEM | 1 | 1 | 1 |
| Social Science | 1 | 2 | 1 |
| Hum and Arts | 1 | 1 | 1 |
| Occupational | 1 | 1 | 1 |
| Business | 1 | 1 | 1 |
| Spending per course |  |  |  |
| STEM | $\$ 2,819$ | $\$ 2,819$ | $\$ 2,819$ |
| Social Science | $\$ 2,704$ | $\$ 2,704$ | $\$ 5,408$ |
| Hum and Arts | $\$ 2,976$ | $\$ 2,976$ | $\$ 2,976$ |
| Occupational | $\$ 2,663$ | $\$ 2,663$ | $\$ 3,258$ |
| Business | $\$ 3,258$ | $\$ 3,258$ |  |
|  |  |  | $\$ 17,124$ |
| Total Cost | $\$ 14,420$ | $\$ 17,124$ |  |
|  |  |  | $15.3 \%$ |
| Share of total student-course |  |  |  |
| STEM | $16.7 \%$ | $24.2 \%$ |  |
| Social Science | $16.7 \%$ | $14.0 \%$ |  |
| Hum and Arts | $17.6 \%$ | $28.3 \%$ | $17.0 \%$ |
| Occupational | $15.4 \%$ | $13.2 \%$ | $29.5 \%$ |
| Business | $18.6 \%$ | $16.0 \%$ | 64,509 |
| Student Welfare | $31.8 \%$ | $28.1 \%$ |  |

Enrollment shares and total welfare are calculated using estimates of the multinomial nested logit course choice model. Student sample is Fall, 2007 students at University of Central Arkansas

## Table 4: Parameter Estimates and Model Fit

|  | Preferences for enrollments |
| :--- | :---: |
| STEM | 0.778 |
|  | 0.038 |
| Social Science | 0.651 |
|  | 0.031 |
| Humanities and Arts | 1.229 |
|  | 0.058 |
| Occupational | 0.472 |
|  | 0.029 |
| Business | omitted |
|  |  |
| Share of alternatives preferred to chosen option |  |
| Fall 2007 | $0.5 \%$ |
| Spring 2008 | $0.1 \%$ |
| Fall 2008 | $0.0 \%$ |
| Spring 2009 | $0.0 \%$ |
|  |  |
| Alternatives per semester | 1000 |
| Number of semesters | 4 |
|  |  |
| Standard errors in italics. Parameters estimated by maximizing likelihood |  |
| that chosen course offerings are preferred to randomly drawn alternative |  |
| feasible course offerings. All alternative feasible offerings include |  |
| contracted courses. Standard errors calculated assuming estimated |  |
| student parameters are true values. |  |

# Table 5: Counterfactual costs which yield Welfare Maximizing Course Offerings 

Welfare Maximizing at market costs

(1)

Student Driven at market costs

(2)

Student Driven at tax/subsidy costs
(3)

Minimum costs
Market costs
STEM
Social Science
Humanities
Occupational
Business
\$2,819.0
\$2,704.0
\$2,976.0
\$2,663.0
\$3,258.0
Market costs
$\$ 2,819.0$
$\$ 2,704.0$
$\$ 2,976.0$
$\$ 2,663.0$
$\$ 3,258.0$
After tax/subsidy
$\$ 4,081.6$
$\$ 3,563.9$
$\$ 5,560.0$
$\$ 3,033.3$
$\$ 2,571.4$

## Simulated Optimal Course Offerings

STEM 34

34
160
34
Social Science
Humanities
Occupational
Business
86
78
86
423
21

Cost of Offerings
21
10
171
171
0
338
\$1,947,463
\$1,947,430
\$1,949,873
All simulations assume no contractual constraints and do not allow for spending in excess of minimum costs. Optimal course offerings are simulated using Marginal Improvement Algorithm taking estimates of university and student parameters as given. Costs of offerings are approximately 20\% of UCA's endowment in Fall, 2007

## Table 6: Observed and Welfare Maximizing UCA

|  | Observed | Welfare Maximizing |
| :--- | :---: | :---: |
| Introductory course offerings |  |  |
| STEM | 279 | 38 |
| Social Sciences | 272 | 518 |
| Humanities | 433 | 54 |
| Occupational | 125 | 110 |
| Business | 65 | 521 |
|  |  |  |
| Median spending per course | $\$ 8,410$ | $\$ 2,819$ |
| STEM | $\$ 6,868$ | $\$ 2,704$ |
| Social Sciences | $\$ 6,547$ | $\$ 2,976$ |
| Humanities | $\$ 7,266$ | $\$ 2,663$ |
| Occupational | $\$ 9,480$ | $\$ 3,258$ |
| Business |  |  |
|  |  | $3.0 \%$ |
| Share of students choosing each field | $22.2 \%$ | $38.7 \%$ |
| STEM | $26.8 \%$ | $4.5 \%$ |
| Social Sciences | $31.6 \%$ | $7.9 \%$ |
| Humanities | $11.3 \%$ | $45.9 \%$ |
| Occupational | $8.1 \%$ |  |
| Business |  | 0.385 |
| Relative Cost | 1 | 1.010 |
| Relative Welfare | 1174 | 1241 |
| Total Courses |  |  |

Welfare maximizing course offerings are obtained using Marginal Improvement Algorithm. Enrollment shares and total welfare are estimated using estimates of the multinomial nested logit course choice model. Student sample is Fall, 2007 students at University of Central Arkansas


[^0]:    *E-mail: james.r.thomas@duke.edu. I am deeply indebted to my advisor Peter Arcidiacono and commitee members V. Joseph Hotz, Robert Garlick, Hugh Macartney, and Arnaud Maurel without whom this work would not be possible. This project has also benefitted from useful feedback from participants in the Duke Labor Lunch Seminar. All remaining errors are my own.

[^1]:    ${ }^{1}$ Two notable exceptions are Bhattacharya, Kanaya, and Stevens (2014) and Turner (2014).
    ${ }^{2}$ See Altonji, Blom, and Meghir (2012) and Oreopoulos and Petronijevic (2013) for reviews.
    ${ }^{3}$ Ahn, Arcidiacono, Hopson, and Thomas (2015) and Stinebrickner and Stinebrickner (2014) argue that grading policies affect specialization decisions of students. Figlio, Shapiro, and Soter (forthcoming) and Bettinger and Long (2010; 2015) provide mixed results about the effects of instructor characteristics on specialization decisions of students.

[^2]:    ${ }^{4}$ As an example of institutional preferences for increasing enrollment in certain fields, Duke University President Richard Brodhead has made numerous public appearances to advocate for specialization in the humanities (Brodhead, 2015).

[^3]:    ${ }^{5}$ The authors define institutional quality is a function of mean student ability, expenditure per student, and mean income of students.

[^4]:    ${ }^{6}$ In many settings, universities may be contractually obligated to offer certain courses with certain instructors. For example, instructors with tenure must be paid to instruct in every semester. For these courses, the university does not truly choose spending on instruction.

[^5]:    ${ }^{7}$ Using expected class size is equivalent to assuming each student's idiosyncratic preferences are private information. Bayer and Timmins (2007) use an alternative justification for integrating out idiosyncratic prefences which is to assume there is a continuum of individuals with different unobserved preferences for each vector of observed characteristics.

[^6]:    ${ }^{8}$ In reality, students choose multiple courses in a semester. I abstract from this complication to focus on university objectives. Students do not observed the idiosyncratic preferences of other students.
    ${ }^{9}$ Expectations are taken over idiosyncratic shocks to student course preferences which are not observed by the university.

[^7]:    ${ }^{10}$ In many cases, instructors will receive one salary to teach multiple courses and possibly perform other functions. In these cases, assumptions are required to determine what share of an instructors total salary is for teaching each course.

[^8]:    ${ }^{11}$ For example, the instructor who taught a course in another semester may be teaching another course, on leave, or otherwise absent in the analyzed semester.

[^9]:    ${ }^{12}$ As an example of institutional preferences for increasing enrollment in certain fields, Duke University President Richard Brodhead has made numerous public appearances to advocate for the humanities (Duke Today, 2015).

[^10]:    ${ }^{13} P_{i j}(\mathbf{e}, \mathbf{d})=0$ if course $j$ is not in student $i$ 's choice set. This occurs either because course $j$ is not offered by the university or because student $i$ is not eligible to enroll in course $j$.

[^11]:    ${ }^{14}$ In theory, one could derive the true asymptotic distribution of $\hat{\mathbf{t}}$ since this random vector is a function of a maximum likelihood estimator which is asymptotically multivariate normal. However, the function is extremely complicated even for very simple utility structures which makes this derivation impractical.

[^12]:    ${ }^{15}$ Estimates of $P_{i j t}\left(\mathbf{e}_{\mathbf{t}}, \mathbf{d}_{\mathbf{t}}\right)$ must approximate how the university believes individual $i$ will choose courses. As such, researchers should avoid using student data which is not observed by the university or utility structures which are impractical.

[^13]:    ${ }^{16}$ See Methodological Appendix C for an illustration.

[^14]:    ${ }^{17}$ See McFadden (1978). Academic fields are: STEM, social science, humanities and arts, occupational, and business.

[^15]:    ${ }^{18} \mathrm{I}$ add 1 to excess spending to make the marginal utility of excess spending finite over the entire support of excess spending: $e_{j} \in[0, \infty)$.
    ${ }^{19} \rho \in(0,1] .1-\rho$ can be viewed as an indication of the correlation in unobserved preferences within the same academic field (McFadden, 1978).

[^16]:    ${ }^{20}$ This framework approximates choices by students within academic semesters as $C_{i t}$ independent choices. In reality, students are choosing the best bundle of $C_{i t}$ courses from all feasible bundles. I abstract from this complication to focus on university objectives. For an alternative framework which models the choices of course bundles see Ahn, Arcidiacono, Hopson, and Thomas (2015).

[^17]:    ${ }^{21}$ These are preliminary estimates which assume $\mathbf{e}(\mathbf{d})^{\star}=\mathbf{0}$ for all $\mathbf{d}$. Results which use the true $\mathbf{e}(\mathbf{d})^{\star}$ are in progress; however, as discussed in Subsection 5.2, estimates of student parameters imply these optimal excess spending levels will always be close to zero. As such, these preliminary estimates should be similar to the final estimates.
    ${ }^{22}$ As discussed in Subsection 4.3.1, the set of alternative feasible offering vectors is typically unfeasibly large making it necessary to draw a sample alternative feasible offering vectors. I sample 1000 feasible offering vectors for each academic semester.

[^18]:    ${ }^{23}$ See Methodological Appendix C for an illustration.
    ${ }^{24}$ I describe the algorithm for a setting in which the university has no contractual constraints. To incorporate contractual constraints, the algorithm should start with the set of courses which must be offered by contract.
    ${ }^{25} \mathrm{~A}$ variation of the algorithm selects courses which yield the greatest marginal improvement per minimum cost. This variation outperforms the standard version in settings where variation in minimum costs is large relative to marginal utilities of excess spending.

