

# Spatial Equilibrium, Search Frictions and Efficient Regulation in the Taxi Industry

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## JOB MARKET PAPER

### Abstract

This paper analyzes the dynamic spatial equilibrium of taxicabs and shows how common taxi regulations lead to substantial inefficiencies. Taxis compete for passengers by driving to different locations around the city. Search costs ensure that optimal search behavior will still result in equilibrium frictions in the form of waiting times and spatial mismatch. Medallion limit regulations and fixed fare structures exacerbate these frictions by preventing markets from clearing on prices, leaving empty taxis in some areas, and excess demand in other areas. To analyze the role of regulation on frictions and efficiency, I pose a dynamic model of search and matching between taxis and passengers under regulation. Using a comprehensive dataset of New York City yellow medallion taxis, I use this model to compute the equilibrium spatial distribution of vacant taxis and estimate intraday demand. My estimates show that search frictions reduce welfare by \$422 million per year, or 62%. Counterfactual analysis reveals that existing regulations attain only 11% of the efficiency implied by a social planner's solution, while the adoption of optimized two-part tariff pricing would lead to 89% efficiency, or a welfare gain on the order of \$2.1B per year. The addition of directed matching technology to an optimized regime would increase welfare even further, by approximately \$2.4B per year.

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# 1 Introduction

Since the pioneering work of Diamond (1981, 1982a,b), Mortensen (1982a,b) and Pissarides (1984, 1985), the search and matching literature has focused on understanding the role of search frictions in impeding the efficient clearing of markets. Search and matching literature examines many markets where central or standardized exchange is not possible, including labor markets (e.g., Shimer and Wright (2005)), marriage markets (e.g., Mortensen (1988)), and financial markets (e.g., Duffie, Gârleanu, and Pedersen (2002, 2005)). In each case, market outcomes differ from the canonical, Walrasian ideal of perfectly frictionless trade and competition, as prices and quantities fail to reflect the intersection of supply and demand. As illustrated by these examples, search markets comprise some of the most important areas of the economy, so it is essential to understand the nature of frictions in different environments and their impact on market function and welfare. In this paper, I study spatial search and matching in the taxicab industry. This industry offers a uniquely well-suited environment to study the search process at a very granular level as well as the ability of regulation to mitigate the cost of search frictions.

The taxicab industry is a critical component of the transportation infrastructure in large urban areas. Approximately 700 million passengers are transported annually in the United States, generating \$16.0 billion in revenues.<sup>1</sup> Urban taxi markets are distinguished from most other public transit options by a lack of centralized control; taxis do not service established routes or coordinate search behavior. Instead, individual drivers decide where to provide service. The supply of taxis in any area is determined by the aggregation of these choices. Vacant taxis and customers do not know each others' locations, so in expectation, empty cabs and waiting customers will coexist while search is conducted. This paper analyzes these frictions and the extent to which they are driven by the search behavior of vacant taxis. How and where drivers decide to search for passengers directly affects the availability of taxi service across the city. From a driver's perspective, location decisions have important economic tradeoffs. Taxis are drawn to search in the most concentrated pedestrian areas, but competitive pressure (i.e., the threat of losing fares to other taxis) incentivizes drivers to spread search among a broader set of locations. The confluence of these incentives gives rise to equilibrium search behavior that can leave some areas with little to no service while in other areas, empty taxis will wait in long queues for passengers.

A second defining characteristic of the U.S. taxi market is that both prices and quan-

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<sup>1</sup>See technical report Brennan (2014) for more information.

tities are typically regulated by municipal authorities. Although the scope and magnitude of regulations vary across cities, nearly all local regulators implement the same pair of instruments: two-part tariff pricing and entry restrictions. The two-part tariff price structure consists of a one-time fixed fare and a distance-based fare (e.g., \$2.50 per-ride plus \$2.50 per-mile, as in New York City). Entry restrictions strictly limit the number of taxis that are licensed to provide service (e.g., 13,500 total permits to operate a taxi service in a city). Although local taxi regulations might be aimed at mitigating several potential externalities, fixed prices and quantities prevent traditional market mechanisms from efficiently rationing supply and demand, so taxi drivers will not be fully incentivized to focus their search in the highest-demand areas. Fixed prices directly influence the spatial search behavior of taxis by differentially rewarding search for, say, a long ride versus a short ride. Thus the type of two-part tariff used - a high distance fee and low fixed fee, or vice-versa, will lead to different equilibrium spatial distributions of service. In this way, the spatial sorting of taxis is endogenously determined by fare regulations. The overall level of service is further dependent on entry regulations. If local authorities ignore the spatial effects of regulation, there is a risk of adding unnecessary and wasteful costs to the search process between supply and demand.

In this paper, I model taxi drivers' location choices as a dynamic spatial oligopoly game in which vacant drivers choose where to provide service given the locations of their competition and profitability of different search locations. I show how regulations impact taxi drivers' search behavior, which in turn influences the equilibrium spatial distribution of taxi service. To empirically analyze this model, I use data from the New York City Taxi and Limousine Commission (TLC), which provides trip details including the time, location and fare paid for all 350 million taxi rides in New York from January, 2011 to December, 2012.<sup>2</sup> Using TLC data together with a model of taxi search and matching, I estimate the spatial, inter-temporal distribution of supply and demand in equilibrium. Importantly, the data only reveal matches made between taxis and customers as a consequence of search activity, but do not show underlying supply or demand; I do not observe a measure of vacant taxis or the number of customers who want a ride. Because these objects are necessary to measure search frictions and welfare in the market, I present an identification strategy using the spatial equilibrium model together with a microfounded matching specification.

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<sup>2</sup>New York's taxi industry is approximately 25% of the U.S. taxi market, the largest in the United States. Source: my own calculation based on 2013 data from the NYC Taxi and Limousine Commission and Brennan (2014)

Identification is obtained by first mapping the observed spatial distribution of matches in each period across the day to corresponding time-location levels of supply and demand via an aggregate matching function and then finding both supply and demand via the structural restrictions imposed by the spatial equilibrium model. The most common methods for estimating dynamic oligopoly games involve solving for equilibrium policies for each type of agent at each point in time. These methods are infeasible here due to the extremely high-dimensional state space. Instead, I model and solve for an equilibrium where taxis play against the *distribution* of competitors instead of individuals.

I use this model to evaluate welfare in this market as well as the welfare effects associated with several potential policy alternatives. Baseline estimates of welfare indicate that the New York taxi industry generates \$257 million in annual consumer surplus and \$1.6 billion in taxi profits. Approximately \$422 million is lost annually as a result of search frictions due to both unmatched supply and demand as well as matches occurring for some customers who have lower valuations than others. By simulating market outcomes over a grid of potential two-part tariffs, I am able to solve for an optimal fare structure and show that one with no fixed fee, based on distance pricing alone, leads to \$289 million in annual consumer welfare, a gain of 12.5%. With this pricing result, I derive a social planner’s solution to the taxi allocation problem when the planner is constrained by current medallion levels. Results suggest that a social planner could generate \$2.6 billion in consumer welfare, a gain of 821%, through a combination of lower prices, which reduce the rents accruing to medallion stakeholders, and spatial reallocation. Finally, I measure the value added by eliminating matching frictions, a reflection of technology used by popular new taxi market entrants such as Uber and Lyft; I show that a combined policy of optimal tariffs, marginal-cost pricing, and on-demand matching technology would generate \$2.6 in consumer welfare each year in the New York market, slightly exceeding the welfare induced by a social planner in the presence of search and matching frictions.

## Related Literature

This paper integrates ideas from the search and matching literature, spatial economics and empirical studies of industry dynamics. My model is built around the aggregate matching function concept of canonical search-theoretic models.<sup>3</sup> Traditionally, matching functions

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<sup>3</sup>A starting point for the implementation of these models in studying labor markets can be found in surveys of search-theoretic literature by Mortensen (1986) and Mortensen and Pissarides (1999) and Shimer and Wright (2005). Blanchard and Diamond (1991) introduces the aggregate matching function concept.

directly built in frictions as a functional form assumption, but some recent literature has derived matching functions under explicit microfoundations. My model builds on the taxi search and matching framework of Lagos (2000), which studies endogenous search frictions using a stylized environment of taxi search and competition. I extend the Lagos model to link spatial search with regulation, and to reflect several important aspects of the real-world search and matching process. Specifically, I add non-stationary dynamics, a more realistic and flexible spatial structure, stochastic and price-sensitive demand, fuel costs, and regulation in the form of two-part tariff pricing and fixed quantities. Further, to facilitate estimation of this model, I also allow for drivers' location choices to be influenced by unobservable factors (e.g., it might be easier to continue travel in the same direction the cab is facing than to turn around). Because Lagos's aggregate matching function assumes frictionless matching *within* a given location, a type of friction I explicitly want to consider, I replace this function with a specification from Burdett, Shi, and Wright (2001). There, authors derive a matching function directly from a single-good, random-matching competitive equilibrium. The micro-foundations are well-suited to the taxi industry and my approach in particular, as they generate endogenous frictions within single location as a function of the number of buyers and sellers.

I integrate the search and matching framework with a dynamic oligopoly model in the tradition of Hopenhayn (1992) and Ericson and Pakes (1995), which characterize Markov-perfect equilibria in entry, exit and investment choices given some uncertainty in the evolution of the states of firms and their competitors. Instead of these choices, however, I model taxi drivers' problem as a series of location choices made throughout the day whenever cabs fail to find passengers. Given the non-stationary environment, this problem generates a particularly high-dimensional state space, so I use numerical approximate dynamic programming methods to mitigate computational burden. My approach exploits the large number of agents (taxi drivers) in the system by assuming that state transitions are perfectly forecastable by drivers. This assumption not only permits solving for equilibrium search policies of taxis, but also reflects a more realistic behavioral model: here, an agent's strategic location game involves a play against the expected distribution of competitors throughout the day, rather than all possible realizations of competitors' states. This approach has precedent in literature on non-stationary firm dynamics (e.g., Weintraub, Benkard, Jeziorski, and Van Roy (2008), Melitz and James (2007)) as well in auction models with many bidders (e.g., Hong and Shum (2010)).

This paper also contributes to literature on spatial economics by modeling the spa-

tial dispersion of firms as an equilibrium outcome of the search process. While there is no unifying theory of spatial economics, there is a long history of literature tracing the interactions between space and economic activity.<sup>4</sup> My model underscores some common themes in the spatial equilibrium literature. Classic migration models of Rosen (1979), Roback (1988), and more recent work (e.g. Diamond (2012), Allen and Arkolakis (2014)), highlight equilibrium tradeoffs between city characteristics, wages and migration. My spatial model reveals a conceptually similar no-arbitrage equilibrium that demonstrates tradeoffs between competition, profitability of locations, and the movement of vacant taxis over space.

Finally, this paper contributes to our understanding of the taxi industry by analyzing driver search behavior and the effects of regulation. To my knowledge, this is the first empirical analysis of the spatial search dynamics of taxis. The most closely related study is Frechette, Lizzeri, and Salz (2015) [hereafter, FLS], which models the labor supply dynamics of taxis to ask how customer waiting times and welfare are impacted by medallion regulations and matching technology. As with my paper, FLS study the effect of regulations on search frictions and welfare. The key difference is that they focus on the labor supply decision rather than the spatial location decision.<sup>5</sup> As a result they model the dynamic equilibrium effects of regulation on aggregate (i.e., city-wide) levels of service and demand, whereas I model the dynamic equilibrium effects of regulation on spatial distributions of supply and demand. My identification relies on a fixed labor supply, however, which is only valid when medallion limits are binding in the busiest hours. Though these approaches differ substantially, they lead to similar predictions when comparing similar counterfactuals.

A diverse set of literature exists to address whether taxi regulation is necessary at all. Among this literature, both the theoretical and empirical findings offer mixed evidence. These studies point to successful regulation's function to reduce transaction costs (Gallick and Sisk (1987)), prevent localized monopolies (Cairns and Liston-Heyes (1996)), correct for negative externalities (Schrieber (1975)), and establish efficient quantities of vacant

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<sup>4</sup>For a history of spatial economics, see Ohta and Thisse (1993) For a review of modern spatial economics, see Fujita, Krugman, and Venables (1999). Important early literature capturing these effects appear in *The Isolated State* (von Thünen (1966)), Von Thünen's 1826 treatise showing how spatial sorting and agglomeration patterns develop purely through competition and market-determined land prices, as well as Hotelling (1929), a workhorse model that predicts the spatial locations of competitive firms as an equilibrium outcome.

<sup>5</sup>There is an additional body of literature on taxi drivers' labor supply choices, including Camerer, Babcock, Loewenstein, and Thaler (1997), Farber (2005), Farber (2008) and Crawford and Meng (2011). These studies investigate the labor-leisure tradeoff for drivers, and ask how taxi drivers' labor supply is influenced by daily wage targets and other factors.

cabs (Flath (2006)). Other authors assert that regulation has lead to restricted quantities and higher prices (Winston and Shirley (1998)) and that low sunk and fixed costs in this industry are sufficient to support competition (Häckner and Nyberg (1995)). My paper contributes to this debate by showing where regulation can help taxi markets by incentivizing more efficient spatial allocations of supply, and also showing where regulations contribute to high prices and cartel-like profits which accrue to a small class of stakeholders.

This paper is organized as follows. Section 2 details taxi industry characteristics relating to search, regulation and spatial sorting, as well as a description of the data. Section 3 presents the behavioral model of taxi search and matching. Section 4 discusses the empirical strategy for computing equilibrium and estimating parameters with the data. Results are presented in section 5, with an analysis of counterfactual policies in section 6. Section 7 concludes.

## 2 The Taxi Industry

### 2.1 Industry Characteristics

#### Fragmented Firms

In the U.S., taxi service is highly fragmented. The market share of the largest firm is less than 1%, and the largest four firms make up less than 3% of the overall market. There are many individual firms, some consisting of a single owner-operator. While there is increased concentration at the local level in which taxis operate, most non-owner drivers lease taxis from owners. The typical lease arrangement has drivers paying a fixed leasing cost, paying for their own gas and insurance, and collecting all residual revenue as profit. Given these arrangements, drivers do not centrally coordinate their search behavior. Instead, each driver independently searches for passengers, competing with other drivers for the same demand.

It is important to note that some cities permit taxis to operate both a street-hail service, where passengers are acquired via driving and searching, as well as a for-hire-vehicle (FHV) service, which pre-arrange rides with customers via telephone or internet. In other cities, including New York City, these two services are treated as separate markets by regulation. Yellow medallion taxis are only permitted to operate a street-hail service, and as such may not pre-arrange rides with customers. New York’s FHVs are separately licensed for pre-arranged rides, and are likewise not permitted to operate a street-hail service.

In recent years, several firms including Uber, Lyft, Curb and Sidecar have entered the taxi industry, all of which take advantage of mobile technology to match customers with cabs, and thereby greatly reduce frictions associated with taxi search and availability. The precipitous expansion and success of these firms is suggestive of the enormous benefits associated with reduced search costs compared with traditional taxi markets. At the moment, these companies also enjoy relatively little regulation, though this environment is changing rapidly as local regulators revise taxi laws to address the new platforms. My analysis, which focuses on the traditional, street-hail taxi markets that still dominate service in the largest cities, will reveal the extent of spatial frictions induced by regulation as well as those induced by a random versus directed search mechanism (i.e. sorting customers by willingness-to-pay).

## Regulation

The U.S. taxi market is highly regulated by local municipalities. The most common regulatory scheme is the combination of a fixed two-part tariff fare pricing structure and entry restrictions. The two-part tariff fare structure should be familiar to taxi customers; it consists of a one-time fixed fee and a distance-based fee. The other most common form of regulation is entry restriction. Most U.S. cities limit the number of legal taxis by requiring them to hold a permit or medallion, the supply of which are capped.<sup>6,7</sup> Entry restrictions are often controversial; critics argue that they are a product of regulatory capture, and serve to enrich medallion owners by limiting competition. Proponents of regulation highlight several market failures that arise in an unregulated environment: congestion externalities, localized market power in remote locations, and potentially high bargaining costs.<sup>8</sup>

Across the U.S. and elsewhere around the world, taxi regulations are at odds with the new wave of technology-centered entrants in the taxi industry. Often these companies implement different pricing regimes and much looser entry restrictions than those imposed by regulatory authorities, leading to a variety legal disputes as stakeholders in the traditional taxi business suffer losses and as policy makers argue that safety and worker-rights

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<sup>6</sup>For a survey of entry restrictions across forty U.S. cities, see Schaller (2007).

<sup>7</sup>These licenses are also tradable, and the mere fact that they tend to have positive value, sometimes in excess of one million dollars, implies that this quantity cap is binding and below that of an unrestricted equilibrium.

<sup>8</sup>See the discussion of regulation papers above.



regulations have been undermined.<sup>9</sup> These ongoing policy questions highlight the need for an analysis of the effects of these new entrants, a central question of this paper.

## The Spatial Dimension

The spatial availability of taxis is of evident concern to municipal regulators around the country: policies of various types have been enacted in different cities to control the spatial dimension of service. For example, in the wake of criticism over the availability of taxis in certain areas, New York City issued licenses for 6,000 additional medallion taxis in 2013 with special restrictions on the spatial areas they may service.<sup>10</sup> Specifically, these green-painted “Boro Taxis” are only permitted to pick up passengers in the boroughs outside of Manhattan.<sup>11</sup> Though the city’s traditional “yellow taxis” have always been able to operate in these areas, it’s apparent that service was scarce enough relative to demand that city regulators intervened by creating the Boro Taxi service. This intervention highlights the potential discord between regulated prices and the location choices made by taxi drivers. This paper aims to characterize both the location incentives faced by taxis and equilibrium spatial distribution of supply, and to analyze the role of regulations on market outcomes.

## 2.2 Data Overview

New York City is the largest taxi market in the United States, with 236 million passenger trips in 2014, or about 34% of all U.S. service. In 2009, The Taxi and Limousine Commission of New York City (TLC) initiated the Taxi Passenger Enhancement Project, which mandated the use of upgraded metering and information technology in all New York medallion cabs. The technology includes the automated data collection of taxi trip and fare information. I use TLC trip data from all New York City medallion cab rides given from August 1, 2012 to September 30, 2012. An observation consists of information related to a single cab ride. Data include the exact time and date of pickup and drop-offs, GPS coordinates of pickup and drop-off, trip distance, and trip time length for approximately 28 million rides.<sup>12</sup> New York cabs typically operate in two separate shifts of 9-12 hours

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<sup>9</sup>See, e.g., [forbes.com/sites/ellenhuet/2015/06/19/could-a-legal-ruling-instantly-wipe-out-uber-not-so-fast/](http://forbes.com/sites/ellenhuet/2015/06/19/could-a-legal-ruling-instantly-wipe-out-uber-not-so-fast/).

<sup>10</sup>See, e.g., [cityroom.blogs.nytimes.com/2013/11/14/new-york-today-cabs-of-a-different-color/](http://cityroom.blogs.nytimes.com/2013/11/14/new-york-today-cabs-of-a-different-color/)

<sup>11</sup>A map of the Boro taxi service area is available at [www.nyc.gov/html/tlc/images/features/map\\_service\\_area.png](http://www.nyc.gov/html/tlc/images/features/map_service_area.png)

<sup>12</sup>Using this information together with geocoded coordinates, we might learn for example that cab medallion 1602 (a sample cab medallion, as the TLC data are anonymized) picks up a passenger at the

Table 1: Taxi Trip and Fare Summary Statistics

Rate Type	Variable	Obs.	10%ile	Mean	90%ile	S.D.
Standard Fares	Total Fare (\$)	24,075,211	4.50	9.51	16.00	5.57
	Dist. Fare (imputed, \$)	24,044,354	1.22	4.55	9.50	3.36
	Flag Fare (imputed, \$)	24,044,354	2.5	2.82	3.50	0.37
	Distance (mi.)	24,075,239	0.78	2.26	4.17	2.10
	Trip Time (min.)	24,075,239	4.0	11.1	20.2	7.3
JFK Fares	Total Fare (\$)	381,270	45	48.35	52	3.52
	Distance (mi.)	381,270	5.00	16.60	20.67	4.72
	Trip Time (min.)	381,270	24.0	40.1	60.1	17.0

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to individual taxi trips taken in New York City between August 1, 2012 to September 30, 2012 for two fare types. The first is the standard metered fare (TLC rate code 1), in which standard fares apply, representing 98.1% of the data. The second is a trip to or from JFK airport (TLC rate code 2). Total Fare and Distance data are reported for each ride in the dataset. The two main fare components are a distance-based fare and a flag-drop fare. I predict these constituent parts of total fare using the prevailing fare structure on the day of travel and the distance travelled. Flag fare calculations include the presence of time-of-day surcharges. Any remaining fare is due to a charge for idling time. Units are reported in parentheses.

each, with a mandatory shift change between 4-5pm. I focus on the “day-shift” period of 6am until 4pm, after which I assume all drivers stop working.

A unique feature of New York taxi regulation is that medallion cabs may only be hailed from the street, and are not authorized to conduct pre-arranged pick-ups, which are the exclusive domain of licensed livery cars. As a result, the TLC data only record rides originating from street-hails. This provides an ideal setting for analyzing taxi search behavior, since all observed rides are obtained through search. Table (1) provides summary facts for this data set.

Most of the time, New York taxis operate in Manhattan. When not providing rides within Manhattan, the most common origins and destinations are to New York’s two city airports, LaGuardia (LGA) and John F. Kennedy (JFK). Instead of conducting a search for passengers, taxis form queues and wait in line for next available passengers. The costly waiting times and travel distances is offset by larger fares, however. Table (2) below

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corner of Bowery and Canal at 2:17pm of August 3rd, 2012, and then drives that passenger for 2.9 miles and drops her off at Park Ave and W. 42nd St. at 2:39pm, with a fare of \$9.63, flat tax of \$0.50, and no time-of-day surcharge or tolls, for a total cost of \$10.13. Cab 1602 does not show up again in the data until his next passenger is contacted

Table 2: Taxi Trips and Revenues by Area

Time	Place	Obs.	Mean Fare	Trip Share	Rev. Share
All Times	Intra-Manhattan Trips	23,360,688	\$8.94	83%	65%
	Airport Trips	1,579,264	\$33.51	6%	16%
	Other Trips	3,233,128	\$18.60	11%	19%
Weekdays, Day-shift	Intra-Manhattan Trips	7,238,867	\$9.12	87%	70%
	Airport Trips	536,108	\$33.24	6%	19%
	Other Trips	509,293	\$18.44	6%	11%

Taxi trip and fare data come from New York Taxi and Limousine Commission (TLC). This table provides statistics related to the locations of taxi trips taken in New York City between August 1, 2012 to September 30, 2012. Intra-Manhattan denotes trips which begin and end within Manhattan, Airport Trips are trips with either an origin or destination at either LaGuardia or JFK airport, and Other Trips captures all other origins and destinations within New York City. Statistics are reported for all times as well as the day-shift period of a weekday, from 6am until 4pm. The latter category is the focus of my analysis.

provides statistics related to the frequency and revenue share of trips between Manhattan, the two city airports and elsewhere.

### 3 Model

I begin with a city of  $L$  locations connected by a network of routes and a discrete, finite time horizon  $t = \{1, \dots, T\}$ , where  $t$  can be thought of as intra-day times such as minutes. At time  $t = 1$  the work day begins; at  $t = T$  it ends. A location can be thought of as a spatial area within the city.<sup>13</sup> In the empirical analysis below, I explicitly define locations by dividing a map into areas. Vacant taxi drivers search for passengers within a location. When taxis find passengers, they drive them from some origin location to a destination location.

The distance between each location is given by  $\delta_{ij}$  where  $i \in \{1, \dots, L\}$  represents the trip origin and  $j \in \{1, \dots, L\}$  represents the destination. Similarly, the mean travel time between each location is given by  $\tau_{ij}$ . In each location  $i$  at time  $t$ , the number of people wish to move to a new location (taxi customers) is random and given by  $u_i^t$  where  $u_i^t$  is drawn from a Poisson distribution with parameter  $\lambda_i$ . I assume that taxi drivers know the

<sup>13</sup>e.g., a series of blocks bounded by busy thoroughfares, different neighborhoods, etc.

distribution of demand at every location and time. Taxis only receive fare revenue from passengers, and earning revenue net of fuel costs is assumed to be the single objective faced by working drivers.

There are two types of locations, *searching locations* and *airports*. Searching locations comprise most a city; they are locations in which cabs drive around in search for passengers, with no guarantee that they will find one. Airport locations require taxi queueing. Taxis wait in queue for a guaranteed passenger ride once they reach the front of the queue.<sup>14</sup> The following subsections detail the dynamics of searching and matching in these two location types.

### 3.1 Searching Locations

At the start of each five-minute period, taxis search for passengers. The number of taxis in each location at the start of the period is given by the sum of vacant taxis who have to location  $i$  to search, and previously employed taxis who have dropped off a passenger in location  $i$ , denoted as  $v_i$ . Matches can only occur among cabs and customers within the same location. I assume that once a cab meets a mover, a match is made and the cab is obliged to go wherever the passenger wants within the city. A cab may not refuse a ride after contact is made.<sup>15</sup> The meetings are random and the number of matches made in location  $i$ , time  $t$  is given by  $m_i^t$ . The exact specification of the matching function is explained in detail below.

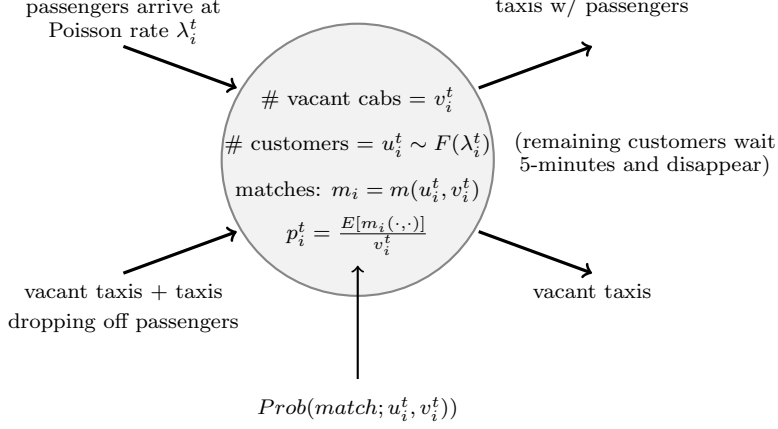
A taxi driver's behavior is dependent on the probability that he will match with a customer. Within a location,  $m(v_i, u_i)$  matches are randomly assigned between supply and demand. As such, the ex-ante probability that a cab will find a mover in location  $i$  at time  $t$  in a given period is  $p_i^t = \frac{\mathbb{E}_{u_i^t}[m(u_i^t, v_i^t) | \lambda_i^t]}{v_i^t}$ . The matching between taxis and passengers within a location is illustrated in Figure 1.

**The Spatial Matching Function** To model how many customers are observed by all vacant cabs in a location, I begin with the following observation: in a location with very a small area, say a single intersection of streets, all taxis will see all customers, so that matches will be given by a simple *min* function, i.e.  $m_i = \min\{u_i, v_i\}$ , as used in Lagos (2000). In larger areas, say a collection of city blocks, the matching function should capture

<sup>14</sup> Airport rides comprise roughly 5% of all taxi trips, and 16% of revenues.

<sup>15</sup> In New York, the TLC prohibits refusals, c.f. [www.nyc.gov/html/tlc/html/rules/rules.shtml](http://www.nyc.gov/html/tlc/html/rules/rules.shtml)

Figure 1: Flow of demand, matches and vacancies in location  $i$



This illustration depicts the sources of taxi arrivals and departures in location  $i$  and time  $t$ . At the beginning of a period, all taxis conducting search in location  $i$  are either dropping off passengers or vacant and searching from previous periods. Matches are then made according to the matching function  $m(u_i, v_i)$ . At the end of the period, newly employed taxis leave for various destinations and vacant taxis continue searching.

the possibility that some taxis within a larger area will not necessarily encounter any customers within the same area. This means that even though taxi drivers may choose to search in a neighborhood with some demand for taxi rides, it's possible that some drivers and customers never meet. This type of *intra-location* search friction is a consequence of limited visibility, mobility and information on the part of taxi drivers and customers.

To model intra-location search frictions, I use a simple aggregate matching function, given by equation (1). This function is derived from a simple urn-ball matching problem first formulated in Butters (1977), where  $u$  balls are randomly placed in  $v$  urns, and a “match” only occurs for the first ball placed in any urn. An equilibrium matching function based on this problem is derived in Burdett, Shi, and Wright (2001), where  $u$  represents buyers and  $v$  represents firms selling a single unit of some good. This basic premise has appealing analogs to the taxi market. To motivate its use in this study, I make the following assumptions about search activity within a period: (1) customers and drivers can only match if they are in the same neighborhood, (2) taxis are spread across different street blocks, (3) customers, without knowing exactly where vacant taxis are, but choose a street

block to search on, (4) each taxi can provide only one ride. It is therefore possible that some customers search on the same block and only the ones who first see cabs are matched, while elsewhere within the neighborhood, some cabs encounter no passengers.

In a location  $i$  with  $u$  customers and  $v$  taxis, aggregate matches are given by

$$m(v_i, u_i) = v_i \left( 1 - \left( 1 - \frac{1}{\alpha v_i} \right)^{u_i} \right). \quad (1)$$

This function exhibits several important properties for purposes of this study. First, if locations are defined to be very small, granular areas such as a single street intersection, we would expect levels of inputs  $u$  and  $v$  to also shrink, and for any available cab and customer to find each other with little effort. For small  $u$  and  $v$ , this matching function becomes frictionless: it is bounded below by  $\min\{u, v\}$ . Second, this function exhibits constant returns to scale for larger values of inputs. If locations are drawn large enough, so as to encompass a large number of vacant taxis and searching customers, the number of expected matches will be proportional to the number of taxis and customers. This ensures that intra-location frictions are invariant to how I choose to draw locations, as long as locations are drawn large enough.<sup>16</sup> I denote  $\alpha$  as an efficiency parameter; all else equal, larger values of alpha generate fewer matches.

This matching function is also invertible, a critical property for identification of its inputs (i.e., supply and demand) from data which reflect its output (i.e., observed matches). Once I solve for the equilibrium distribution of supply, I invert the matching function to find the distribution of customer arrival-rates that rationalize the observed spatial pattern of matches. There is a one-to-one relationship between the matching function and its demand inputs, which allows for an inversion necessary to identify demand inputs as a function of matching data, as discussed further in section 4.3.

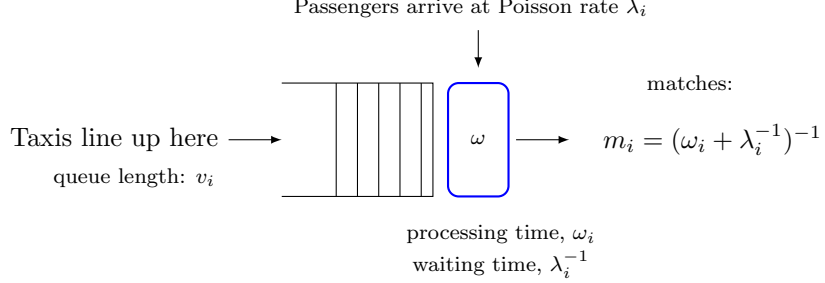
### 3.2 Queueing Locations

At airports, taxis pull into a queue and wait for passengers to match with cabs at the front of the queue. Demand in each location  $i$  is governed by a Poisson arrival rate  $\lambda_i$ , and there is an additional processing time  $\omega_i$  which reflects how, once a unit of demand has arrived, it takes a very short time until that passenger is in the front taxi and the taxi has exited the cab-stand to make way for the next taxi. The expected total time for each taxi to be

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<sup>16</sup>For example, when  $u = v$ , the match rate for both supply and demand will be approximately 65% for  $u, v$  around 10, 63.4% for  $u, v$  around 100, and converging towards around 63.2% as  $u, v \rightarrow \infty$ .

Figure 2: Flow of demand, matches and vacancies in queueing location  $i$



matched with a passenger once he is at the front of the line is therefore given by  $\omega_i + \lambda_i^{-1}$ . Thus, the flow of matches out of queueing location  $i$  is given by  $m_i = (\omega_i + \lambda_i^{-1})^{-1}$ . The units of  $\omega_i$  are fractional-periods, the same as  $\lambda_i$ , so that  $m_i$  is in units of matches per period. The expected waiting time experienced by a taxi at the back of a queue of length  $v_i$  is given by  $v_i \cdot (\omega_i + \lambda_i^{-1})$ . Figure 2 illustrates this process.

This figure depicts the sources of taxi arrivals and departures at an airport location  $i$ . All taxis searching for a customer at the airport are vacant, and must arrive to a queue. The queue waiting time endogenously depends on how many taxis are in it, and on the arrival rate of passengers at the airport. After waiting, taxis are matched with customers with probability 1, after which time they leave for various destinations. To understand how queues fit into observed data on matches, note that all taxi customers at the airport eventually get a ride, and all queued taxi drivers eventually get a passenger. Thus matches are taken to be equal to the minimum between the number of customers and the number of taxis who can service the queue in one period, given by  $\omega_i^{-1}$  for airport location  $i$ .

### 3.3 Revenue and Costs

Taxis earn revenue from giving rides. At the end of each ride, the taxi driver is paid according to the fare structure. The fare structure is defined as follows:  $b$  is the one-time flag-drop fare and  $\pi$  is the distance-based fare, with the distance  $\delta_{ij}$  denoting the distance between  $i$  and  $j$ . Thus the fare revenue earned by providing a ride from  $i$  to  $j$  is  $b + \pi\delta_{ij}$ .

At the same time, taxis spend time and money in pursuit of passengers. When a driver departs from location  $i$  to location  $j$ , with or without a passenger, he will arrive  $\tau_{ij}$  periods later. Since there is a finite amount of time in a day, the cost of time is the opportunity cost of earning other revenue while in-transit; taxis may spend time to drive to more profitable locations. Taxis also pay a cost of fuel when traveling, given by  $c_{ij}$ .

Thus the net revenue of any passenger ride is given by

$$\Pi_{ij} = b + \pi\delta_{ij} - c_{ij}. \quad (2)$$

This profit function sums the total fare revenue earned net of fuel costs in providing a trip from location  $i$  to  $j$ .

### 3.4 Taxi Drivers' States, Actions and Payoffs

Taxi drivers compete with each other for rides, and the total number of taxis in the city is fixed by regulation. One more vacant taxi on the road implies a lower probability that any taxi driver finds a passenger at any moment. This competition is also spatially relevant: an additional vacant taxi searching in a neighborhood imposes lower match probabilities for other taxis in the same neighborhood than it does for taxis on the other side of town.

More formally, a taxi driver's behavior is dependent on the state of the world,  $\mathcal{S}$ .  $\mathcal{S}$  describes, for any driver, his own location and a measure of taxis across all locations including those in transit between locations. A driver's own location at time  $t$  is given by  $\ell_a^t \in \{1, \dots, L\}$ , where  $a$  indexes the driver. The industry state at time  $t$  is a count of vacant taxis  $v_i^t$  in each location  $i$ , and a count of employed taxis  $v_{e,k}^t$  actively *in-transit* between locations, where all "in-between" states are indexed by  $k \in \{1, \dots, K\}$ .<sup>17</sup> Thus the relevant taxi-specific state at time  $t$  can be described by

$$\mathcal{S}_a^t = \{\ell_a^t, \{v_i^t\}_{i \in \{1, \dots, L\}}, \{v_{e,k}^t\}_{k \in \{1, \dots, K\}}\}. \quad (3)$$

Denote  $\mathcal{S} = \{\mathcal{S}_a^t\}_{\forall a, t}$  so that  $\mathcal{S}$  reflects the entire spatial *and* intertemporal distribution of vacant and employed taxis. At the beginning of each five-minute period, taxi drivers

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<sup>17</sup>Though at any moment employed taxis are not directly competing with vacant taxis for passengers, accounting for the number and location employed taxis is an important component of the state variable because the eventual arrival of employed taxis and subsequent transition to vacancy is payoff-relevant for the dynamic decision-making problem of currently vacant taxis.



make a conjecture about the current-period state and transition probabilities into the next period. Given this conjecture, they assign value  $V_i$  to each location  $i \in \{1, \dots, L\}$ .

I define the drivers' ex-ante (i.e., before observing any shocks and before any uncertainty in passenger arrivals is resolved) value as

$$V_i^t(\mathcal{S}) = \mathbb{E}_{p_i|\lambda_i, \mathcal{S}^t} \left[ p_i(u_i^t, v_i^t) \overbrace{\left( \sum_j M_{ij}^t \cdot (\Pi_{ij} + V_j^{t+\tau_{ij}}) \right)}^{\text{Exp. Value of Fare}} + \underbrace{(1 - p_i(u_i^t, v_i^t)) \cdot \mathbb{E}_{\varepsilon_{a,j}^{t+1}} \left[ \max_{j \in A(i)} \left\{ V_j^{t+\tau_{ij}} + \mathbb{I}_{[j=i]} \gamma - c_{ij} + \varepsilon_{a,j} \right\} \right]}_{\text{Exp. Value of Vacancy}} \middle| \lambda_i^t \right]. \quad (4)$$

This expression can be decomposed as follows: Drivers in  $i$  expect to contact a passenger with probability  $p_i$ . I assume matches are *randomly determined* within a location, so  $p_i$  is computed as the expected number of matches in location  $i$  divided by the total number of taxis searching in that same location,

$$p_i(\lambda_i^t, v_i^t) = \frac{\mathbb{E}_{u_i^t}[m_i(u_i^t, v_i^t) | \lambda_i^t]}{v_i^t}, \quad (5)$$

where the number of customers  $u_i^t$  is drawn from a  $\text{Poisson}(\lambda_i^t)$  distribution and the number of taxis  $v_i^t$  is an element of the spatial distribution of taxis  $\mathcal{S}^t$ . Conditional on a passenger contact, drivers expect to receive mean profits, given by the probability of a trip to each location  $j$  (denoted by the transition probability matrix  $M^t$ ), multiplied by the current profit associated with each possible ride plus the continuation value of being in location  $j$  in  $\tau_{ij}$  periods.<sup>18</sup>

$$\sum_j M_{ij}^t \cdot (\Pi_{ij} + V_j^{t+\tau_{ij}}) \quad (6)$$

A search for passengers occurs within the period. At the end of the period, any cabs which remain vacant can choose to relocate or stay put to begin a search for passengers in the next period. Relocation over longer distances requires more time. Vacant drivers choose to search next period in the adjacent location that maximizes the net present value

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<sup>18</sup>Note that  $M^t$  has superscript  $t$  because preferences of passengers change throughout the day.

of profits,

$$\mathbb{E}_{\varepsilon_{a,j}^{t+1}} \left[ \max_{j \in A(i)} \left\{ V_j^{t+\tau_{ij}} + \mathbb{I}_{[j=i]} \gamma - c_{ij} + \varepsilon_{a,j} \right\} \right], \quad (7)$$

where  $\varepsilon_{aj}$  is an idiosyncratic shock to the perceived value of search in each alternative location  $j$ . This shock accounts for unobservable reasons that individual drivers may assign a slightly greater value to one location over another. For example, traffic conditions and a taxi's direction of travel within a location may make it inconvenient to search anywhere but further along the road in the same direction.<sup>19</sup> The parameter  $\gamma$  represents an extra payoff associated with choosing to stay put: when taxis continue search where they are currently located, they will receive extra value associated with time and effort savings.<sup>20</sup> The set  $A(i)$  reflects the set of locations available to vacant taxis. I assume that  $A(i)$  is the entire set of locations in the city, except when the day is almost over, then  $A(i)$  reflects the locations that are attainable within the amount of time left in the day.

Thus  $V_i^t$  is the expected value of a search, evaluated prior to realizing a draw from the distribution of arriving customers (which affects the match probabilities  $p_i$  by influencing the number of matches experienced in each location), and before realizing the draw of valuation shocks  $\varepsilon_i$ . Taxi drivers have beliefs over the policy function of vacant cabs governing state transitions in each period, denoted  $\tilde{\sigma}^t$ . Because time ends at period  $T$ , all continuation values beyond period  $T$  are set to zero:  $V_i^t = 0 \forall t > T, \forall i$ .

### 3.5 Drivers' Choice Problem

To form a strategy, taxi drivers compare the expected present values to search in each alternative location and account for the costs associated with traveling, both in terms of fuel expense and the opportunity costs of time. The strategy dictates where a taxi will search next for passengers if he fails to find a passenger in the current period.

Equation (7) gives the continuation value associated with ending up vacant, from the perspective of the beginning of the period. At the end of each period, vacant drivers must

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<sup>19</sup>The terms  $\varepsilon_{aj}$  further ensure that equilibrium will be in pure strategies, as drivers' will have best responses even among otherwise identical locations. Further, it ensures that vacant taxis leaving one location will mix among several alternative locations rather than moving to the same location, a feature broadly corroborated by data.

<sup>20</sup>In other words, the total continuation value of staying put in time  $t$  is neither equal to  $V_i^t$  nor is it equal to the same value a full period later,  $V_i^{t+1}$  but rather somewhere in between.

decide where to search for passengers in the next period by choosing the location with the highest present value net of transportation costs. Drivers in location  $i$  move to location  $j^*$  according to:

$$j^* = \arg \max_j \{V_j^{t+\tau_{ij}} + \mathbb{I}_{[j=i]} \gamma - c_{ij} + \varepsilon_{ja}\}. \quad (8)$$

To compute the firm's strategies, I define the ex-ante choice-specific value function as  $W_i(j_a, \mathcal{S}^t, \lambda^t)$ , which represents the net present value of payoffs conditional on taking action  $j_a$  while in location  $i$ , before  $\varepsilon_{ai}^t$  is observed:

$$W_i^t(j_a, \mathcal{S}^t) = \mathbb{E}_{\mathcal{S}^{t+\tau_{ija}}} \left[ V_{ja}^{t+\tau_{ija}}(\mathcal{S}^{t+\tau_{ija}}, \cdot, \cdot) - c_{ija} \right]. \quad (9)$$

Defining  $W_i^t$  separately from  $V_i^t$  permits a simple expression of taxi drivers' conditional choice probabilities: the probability that a driver in  $i$  will choose  $j \in A(i)$  conditional on observing state  $\mathcal{S}^t$ , but before observing  $\varepsilon_{ia}^t$ , is given by the multinomial logit formula:

$$P_i[j_a | \mathcal{S}^t] = \frac{\exp(W(j_a, \mathcal{S}^t)/\sigma_\varepsilon)}{\sum_{k \in A(i)} \exp(W(j_k, \mathcal{S}^t)/\sigma_\varepsilon)}. \quad (10)$$

This expression defines drivers' policy functions  $\sigma_i^t$  in each location  $i$  and time  $t$  as the probability of optimal transition from an origin  $i$  to all destinations  $j$  conditional on future-period continuation values. The collection of these policy functions form a transition probability matrix from any origin to any destination, which I denote as  $\{\sigma_i^t\}$ . Note that only vacant taxis transition according to these policies. Employed taxis will transition according to passenger transition probabilities given by  $M_i^t$ .

### 3.6 Intraday timing

At time  $t = 1$ , taxis have an initial spatial distribution, which I denote as  $\mathcal{S}^0$  and take as exogenous. I choose an exogenous distribution in two steps. First, I begin with the time zero empirical distribution of matches, then solve for a candidate spatial equilibrium of taxis in each period. Next, I take the candidate equilibrium distribution of vacant taxis at 12pm and take this as a new time zero distribution of all taxis. So distributed, nature decides which taxis match with passengers and which ones are left vacant. The employed taxis leave the market while employed and accept a payoff  $\Pi_{ij} + V_j^{t+\tau_{ij}}$ . The vacant taxis realize a profit of 0 in  $t = 1$  and an average present value of his own state is given by

equation (7). Equation (8) defines which locations the vacant taxis will search in next period. In period  $t = 2$ , locations of vacant taxis are updated, the employed taxis who are still in-transit are noted, and nature again decides which vacant taxis find passengers.

### Timing

1. Taxis are distributed according to  $\mathcal{S}^1$ .
2. Draws are taken from arrival rates  $\lambda_1^1, \dots, \lambda_L^1$ , leading to  $u_1, \dots, u_L$  customers.
3. Nature randomly assigns  $m_i$  matches in each location according to matching function.
4. Employed taxis leave for destinations.
5. Remaining customers disappear.
6. Remaining vacant taxis choose a location to search in during the next period.
7. Vacant taxis arrive in new locations and some previously hired taxis arrive, forming new distribution  $\mathcal{S}^2$ .
8. Repeat until reaching  $\mathcal{S}^T$ .

Note that, regarding item 6, if a vacant taxi perceives some far-away location  $j_0$  as best, he may either choose to move directly to that far-away location over the course of several periods, in which case he is not available to give rides until arriving in that location, or else he may move *in the direction of* that far away location by moving to, say, an adjacent location  $j_1$  and continuing to search along the way to  $j_0$ . Which choice is made depends which destination  $j_0$ , or  $j_1$ , solves equation (8).

Regarding item 7, “some hired taxis arrive”: many hired taxis are in-transit for more than one period. Suppose hired taxis providing service from location  $i$  to  $j$  will take 3 periods to complete the trip. Then only the taxis who were 1 period away at time  $t - 1$  will arrive in  $j$  in period  $t$ .

### 3.7 Equilibrium

Taxi drivers’ policy functions depend on the current state, beliefs about the policies of competitors, as well as an information set which includes the fixed price schedule, the arrival rates of demand conditional on time and weather conditions, and the geography

of routes and distances. The current state is unobservable; taxi drivers do not see where other taxis are, but rather have beliefs about the distribution and policy functions of their competitors.<sup>21</sup> Beliefs over competitors' policies, conditional on the distribution of all vacant cabs, allow taxis to infer how the state will update in future periods. This implied transition of the current state as a function of the policies of competitors is denote as  $\tilde{Q}_i^t$ . A driver is assumed to have already "learned" the arrival rates of demand, so that any observed deviation from the expected number of people hailing a taxi in a given location is taken as a draw from the known Poisson distribution. The optimization problem facing taxis is the choice of *where* to locate when vacant. Since the time  $t$  state and transition beliefs summarize all relevant information about the competition, taxis condition only on the current-period, so that an optimal location choices at time  $t$  can be made using time  $t - 1$  information. This Markovian structure permits a definition of equilibrium as follows:

**Definition** *Equilibrium is a sequence of states  $\{S_i^t\}$ , transition beliefs  $\{\tilde{Q}_i^t\}$  and policy functions  $\{\sigma_i^t\}$  over each location  $i = \{1, \dots, L\}$ , and an initial state  $\{S_i^0\}_{\forall i}$  such that:*

- (a) *In each location  $i \in \{1, \dots, L\}$ , at the start of each period, matches are made according to equation (1) and are routed to new locations according to transition matrix  $\mathbf{M}^t$ . The aggregate movement generates the employed taxi transition kernel  $\nu(\mathbf{v}_e^{t+1}|\mathbf{v}_e^t, \mathbf{M}^t, \mathbf{m}^t)$  where  $\mathbf{v}_e^t$  is the distribution of employed taxis across locations in period  $t$  and  $\mathbf{m}^t$  is the distribution of matches across locations.*
- (b) *In each location  $i \in \{1, \dots, L\}$ , at the end of each period, each vacant taxi driver follows a policy function  $\sigma_{i,a}^t(S^t, \tilde{Q}_i^t)$  that (a) solves equation (8) and (b) derives expectations under the assumption that the state transition is determined by transition kernel  $\tilde{Q}_i^t$ . The aggregate movement generates the vacant taxi transition kernel  $\mu(\mathbf{v}_v^{t+1}|\mathbf{v}_v^t, \tilde{\sigma}^t, \mathcal{S}^t)$  where  $\mathbf{v}_v^t$  is the distribution of vacant taxis in period  $t$ .*
- (c) *State transitions are defined by the combined movement of vacant taxis and employed taxis, defined by  $Q(S^{t+1}|\tilde{S}^t) = \nu(\mathbf{v}_e^{t+1}|\mathbf{v}_e^t, \mathbf{M}^t, \mathbf{m}^t) + \mu(\mathbf{v}_v^{t+1}|\mathbf{v}_v^t, \tilde{S}^t)$ .*
- (d) *Agent's expectations are rational, so that transition beliefs are self-fulfilling given optimizing behavior:  $\tilde{Q}_i^t = Q_i^t$  for all  $i$  and  $t$ .*

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<sup>21</sup>Of course drivers will see other taxis while driving around, but since other taxis may be vacant or employed, and on- or off-duty, I assume drivers do not update beliefs based on this noisy measure of competition within a neighborhood.

Existence of this equilibrium is a direct consequence of the finite horizon and finite action-space (e.g., Maskin and Tirole (2001)). Uniqueness can be established by a simple backward induction argument, presented in Appendix A3.

## 4 Empirical Strategy

### 4.1 Discretizing time and space

The spatial equilibrium model is one of discrete times and locations. To connect these features with data, I fix a period length and define a set of locations on a map of New York. I focus on a period length of five minutes. Taxis form expectations of the state, location profitability, transition probabilities, demand arrivals and policy functions for every five-minute period of a weekday and month, so that, for example, on all weekdays in June from 1:00pm to 1:05pm, taxis will be playing the exact same strategy. In this sense,  $t$  is taken to be a {5-minute interval, Weekday Y/N, Month} combination.

I use data from 6am-4pm on Weekdays from August to September, 2012. This time range corresponds to a typical day-shift among New York taxi drivers and it represents a period in which almost most medallions are utilized. Figure (3) illustrates this. Note that levels shown will be smaller than the total medallion count, 13,237. Medallions will not appear when taxis are on duty but show no activity (e.g., any combination of unsuccessful search, taking a break, waiting in queue). Some amount of this discrepancy is also attributable to out-of-service cabs due to maintenance work, and lost observations from cleaning inaccurate data.<sup>22</sup> For estimation, I assume that 12,500 cabs are operating between Manhattan and the two airports. The August-September period of 2012 was selected because it straddles a change in regulated tariff prices, which I will use in part to estimate customers' demand elasticities.

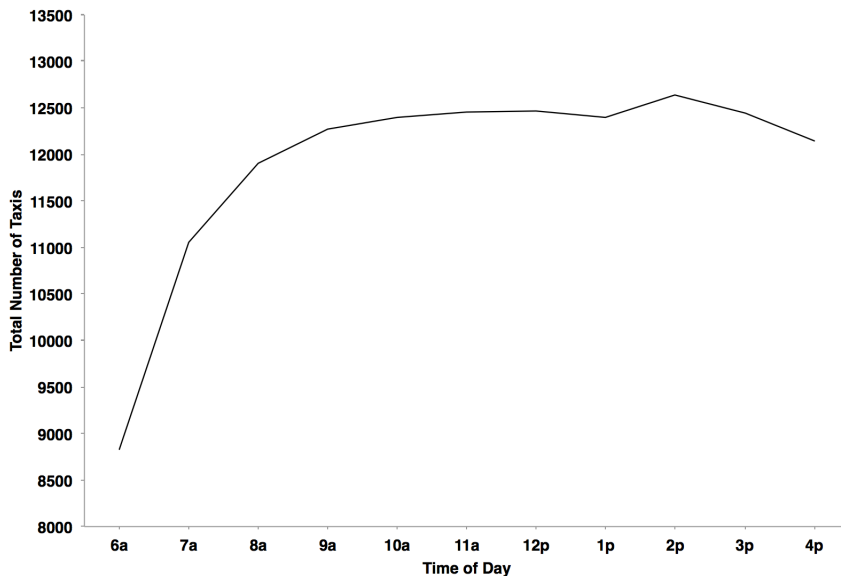
Locations are set by associating taxi rides' GPS points of origin with one of 48 locations.<sup>23</sup> These locations, shown in Figure (4), are created via clustering census tracts. While there is some arbitrariness involved in their exact specification, the number of locations used is a compromise between tradeoffs; more locations give a richer map of spatial densities and choice behavior, but impose greater requirements on both the dataset and computational resources. Further, approximately 94% of all taxi rides originate in Man-

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<sup>22</sup>See Appendix (A.1) for more details on data cleaning procedures

<sup>23</sup>This association is achieved via the point-in-polygon matching procedure outlined in Brophy (2013). Thanks to Tim Brophy for the code.

Figure 3: Total medallion activity by hour



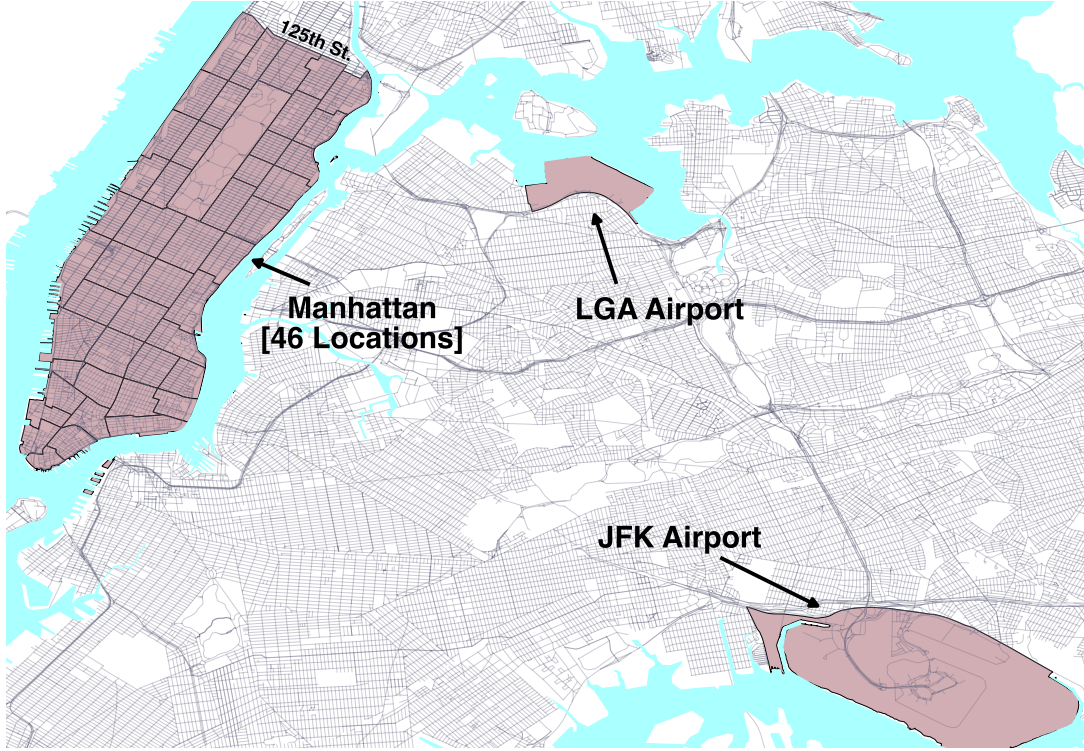
This figure is derived from TLC trip data. In each month, I count the number of unique medallions appearing in the dataset by hour. This figure depicts the mean of this number across all months of 2012.

hattan or one of New York’s two airports. Because of the sparsity of data in the other boroughs, I focus on the set of locations falling within Manhattan below 125th street and the two New York City airports, Lagueardia and J.F.K. Figure 4 depicts these locations on the map of New York.

Four important features of the model are identified directly off the data. First is  $M_{ij}^t$ , the transition probability of employed taxis in each period and location. In each period, I record the probability of transition from each origin to each destination conditional on a taxi matching with a passenger. The mean of these probabilities over each month form the transition probabilities  $M_{ij}^t$ . The second and third directly identified features are travel times and distances between locations. To find the travel time,  $\tau_{ij}$ , and distance,  $\delta_{ij}$ , between each origin and destination, I take the average time and distance of taxi rides between each possible pair of locations within a month.

Aggregating days over a month affords reasonable approximations of travel times and distances between each possible origin and destination, especially for origin-destination pairs less frequently traveled by taxis. However, allowing these features to change with

Figure 4: 48-location Map of New York City



Each divided section of Manhattan depicts a location  $i$ . Locations are created by aggregating census-tract boundaries, which broadly follow major thoroughfare divisions. The expected travel time and distance between these locations is computed separately *for each origin and destination pair* as the average of all observations within each origin and destination.

each month permits the equilibrium to change as roads and location features change over time. The fourth feature is the cost of fuel per mile  $c$ , taken as the average fuel price in New York City in 2012, divided by the average fuel economy in the New York taxi fleet, 29 mpg.<sup>24</sup> Using  $c$  I compute the cost of traveling between any origin and destination as  $c_{ij} = c \cdot \delta_{ij}$

After I record the distances between each origin and destination, I can derive  $\Pi_{ij}$ , the expected profits associated with each possible trip. Recall from equation (2) that

<sup>24</sup>Data come from the New York City Taxi and Limousine Commission 2012 Fact Book. The high fuel economy rating is due to the medallion taxi fleet being made up of approximately 60% hybrid vehicles



$\Pi_{ij} = b + \pi\delta_{ij} - c_{ij}$ , where the regulated fare structure is given by the set  $\{b, \pi\}$ .

The spatial equilibrium model identifies the equilibrium spatial distribution of taxis given taxis' knowledge of the arrival rates of passengers across space and time. TLC data do not contain any information regarding the arrival rates of passengers, so these parameters will be estimated.

## 4.2 Equilibrium Computation

Solving the full dynamic programming problem with large state spaces is burdened by the curse of dimensionality. Under the assumption that taxi drivers are symmetric, atomistic agents, whose actions do not measurably impact the payoffs of competitors, the model reduces to a single-agent problem. As such, there is only one policy function to solve for at any location and time period. However, the total number of taxis still contributes to the computational burden because it affects number of states over which continuation values must be computed.<sup>25</sup>

I implement an approximation method that takes advantage of the large number of agents in my empirical application: I assume that the 13,237 active taxis in my data form a continuum of agents. When there is a continuum of agents facing state transitions, any probability transition matrices become deterministic transition matrices.<sup>26</sup> In the taxi model, state transitions are composed of the combined transitions of vacant and employed taxis. Under the continuum assumption, these transitions are therefore also deterministic.

The advantage of this assumption is that instead of computing policy and value functions for all possible states, I only need to compute a single, deterministic equilibrium path for the state  $\{\mathcal{S}^t\}_{t=0}^T$ .<sup>27</sup>

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<sup>25</sup>Specifically, when there are  $N$  taxis,  $L$  locations, and  $T$  time periods, continuation values and (symmetric) policy functions must be computed for every point in the state space, which numbers  $T$  times  $N^L$ , or: number of states  $(T, N, L) = T \cdot \frac{N^L}{(N-L)!L!}$ . Storing and solving for policy functions and value functions in this setting would be infeasible.

<sup>26</sup>This insight appears in firm dynamics literature where continuums of firms are modeled, such as Hopenhayn (1992), as well as in literature which views this approach as an approximation, as in Weintraub et al. (2008).

<sup>27</sup>Using traditional computational methods in a non-stationary, finite-time environment, continuation values can be computed by evaluating the probability of transitioning into each possible state next period multiplied by the value function computed at each state. Policy functions are also computed from every possible state to determine transitions. By implementing the continuum assumption, continuation values in every period are computed by evaluating only one point in the state space: next period's *known, deterministic* state. Similarly, if the current period's state is known, the policy function need only be computed from that state.

The algorithm I use to compute equilibrium requires an initial, exogenous state  $\mathcal{S}^0$ . Using this, I solve for continuation values backwards from the last period  $T$  and use forward simulation to find optimal transition paths given continuation values. Given some state  $\mathcal{S}^t$ , remaining period continuation values  $\{V^{t+1}, V^{t+2}, \dots, V^T\}$ , I solve for optimal policies at time  $t$  which, given the deterministic transitions approximation, results in an equilibrium path from  $\mathcal{S}^t$  to  $\mathcal{S}^{t+1}$ . The job of the computational algorithm is to find equilibrium transition paths and value functions which are mutually consistent. This method reduces the total number of equilibrium continuation values and policy functions that I need to search for to  $T \cdot L$ , offering a drastic improvement in computational burden.

#### 4.2.1 The Taxi Equilibrium Algorithm

The algorithm that I implement takes as inputs all model primitives, parameters, and a time zero state, and returns the equilibrium state and policy functions for each location and each time period. Equilibrium states constitute a  $L \times T$  matrix (i.e., how many taxis are in each location in each period), and equilibrium policy functions constitute a  $L \times L \times T$  matrix (i.e., the probability of vacant taxi transition from any location  $i \in \{1, \dots, L\}$  to any location  $j \in \{1, \dots, L\}$  in each period). Broadly, the algorithm uses backwards iteration to solve for continuation values and forward simulation to generate transition paths. For computational efficiency, the algorithm moves in an alternating, asymmetric backwards and forwards sequence through the current time step  $t \in \{1, \dots, T\}$ , where backwards moves update continuation values and forwards moves update transition paths. The algorithm terminates when all transition paths and continuation values are self-filling and consistent with equilibrium. Below I provide an outline of the taxi equilibrium algorithm.

The Taxi Equilibrium Algorithm begins at time  $T$ . With a guess of the industry state  $S_0^T$ , I can determine a candidate continuation value  $V^T(S_0^T)$ . Next, the algorithm moves backwards to period  $T - 1$ , again guessing the state  $S_0^{T-1}$ . With this guess and a given value of the parameter vector  $\lambda$ , I compute expected number of matches and vacancies. The flow of matches between locations is then estimated (see Appendix (A.2) for detail). Then, I conduct a policy function iteration to determine the optimal policies for each taxi driver state (i.e., each location) given the candidate continuation values next period. The policy functions define the transition of vacant cabs. This information, together with the computed transitions of matched cabs, defines the transition kernel,  $Q(S'|S)$ . The algorithm updates the time  $T$  state via  $Q : S_0^{T-1} \rightarrow S_1^T$ , and then updates period  $T$

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**Algorithm 1** Taxi Equilibrium Algorithm

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```
1: Set parameters  $\{\lambda^t\}_{t \in \{1, \dots, T\}}$  (where  $\lambda^t = \{\lambda_i^t\}_{i \in \{1, \dots, L\}}$ ).
2: Set counter  $k = 0$ 
3: Guess  $\mathcal{S}_0^T$  and compute  $V^T(\mathcal{S}_0^T, \lambda^T)$ 
4: for  $\tau = T - 1$  to  $1$  do ▷ Backwards Iteration
5:   Guess  $\mathcal{S}_0^t$  and compute  $V^t(\mathcal{S}_0^t, \lambda^t)$ 
6:   for  $t = \tau$  to  $T - 1$  do ▷ Fwd. Iteration to  $T$  for each step back
7:     Compute matches  $m_i^t := E[m(\mathcal{S}_k^t | \lambda^t)]$ 
8:     Compute matched taxi transitions  $m_{ij}^t = M^t \cdot m_i^t$ 
9:     Compute unmatched cabs at the end of period  $\mu_{ij}^t$ 
10:    Find eq'm policy fcts.  $\sigma_k^t(V_k^{t+1})$  to determine vacant taxi transitions
11:     $\sigma_0^t$  and  $m_{ij}^t$  imply transition to  $\tilde{\mathcal{S}}^{t+1}$ 
12:    Update next period state  $\mathcal{S}_{k+1}^{t+1} \leftarrow \tilde{\mathcal{S}}^{t+1}$ 
13:    Update next period continuation values as  $V^{t+1}(\mathcal{S}_{k+1}^{t+1}, \lambda^t)$ 
14:     $k \leftarrow k + 1$ 
15:  end for
16: end for
17: Update  $\mathcal{S}_0^1 \leftarrow \mathcal{S}_0^{(T/2)}$  ▷ A more informed guess of  $\mathcal{S}_0^1$ 
18: repeat
19:   Fix  $\tau = 1$ 
20:   Iterate on steps 6 to 15
21: until  $|V_k^t - V_{k-1}^t| \leq \epsilon \quad \forall t$ 
```

---

continuation values to  $V^T(\mathcal{S}_1^T)$ . Next, the algorithm moves backwards to period  $T - 2$  and again guesses  $\mathcal{S}_0^{T-2}$  and now computes  $Q : \mathcal{S}_0^{T-2} \rightarrow \mathcal{S}_1^{T-1} \rightarrow \mathcal{S}_2^T$  as well as  $V^T(\mathcal{S}_2^T)$  and  $V^{T-1}(\mathcal{S}_1^{T-1})$ . This process repeats all the way back to time zero. Because the time zero state is never updated, it remains an exogenous input.

At every period, including time zero, the initial guess  $\mathcal{S}_0^t$  is derived by distributing taxis across locations according to the distribution of matches. Though this state is unobserved, the distribution of matches reasonably approximates the distribution of taxis. A more detailed description of this algorithm and initial state choice appears in Appendix (A.2).

### 4.3 Model Estimation

Estimation of this model requires solving for several thousand demand parameters as well as the variance parameter for the logit shocks, the matching-efficiency parameter, and a parameter reflecting the extra value to taxis of choosing to search in the same location

Table 3: Parameter List

Parameter	No. Elements	Description
$\lambda_i^t$	5,760	Demand arrival rates
$\sigma_\varepsilon$	1	Variance of $\varepsilon_i^t$ shocks
$\gamma$	1	Extra value to staying-put
$\alpha$	1	Matching efficiency

This table describes the full set of parameters to be estimated. Refer to section 3 for model details.

as last period. Table (3) outlines the set of parameters to be estimated. The next two subsections describe this process in more detail.

#### 4.3.1 Estimation and Identification of Demand Parameters

The method described in the above section finds equilibrium states, transitions and policies for taxis given knowledge of the arrival rates of passengers. These arrival rates, described by the set of Poisson parameters  $\{\lambda_i^t\}$  (hereafter denoted as  $\boldsymbol{\lambda}$ ), are the primary parameters to be estimated. In brief, the strategy is to estimate the distribution  $\boldsymbol{\lambda}$  by finding values of the parameter which, given the matching function, generate an equilibrium distribution of taxis that rationalize the observed number of taxi-passenger matches. In the above section I described a computational method to map  $\boldsymbol{\lambda}$  to an equilibrium distribution of taxis. The next step is to show that this mapping is invertible, so that given observed matches and the equilibrium distribution of taxi service, I can uniquely infer the distribution of demand given by  $\boldsymbol{\lambda}$ .

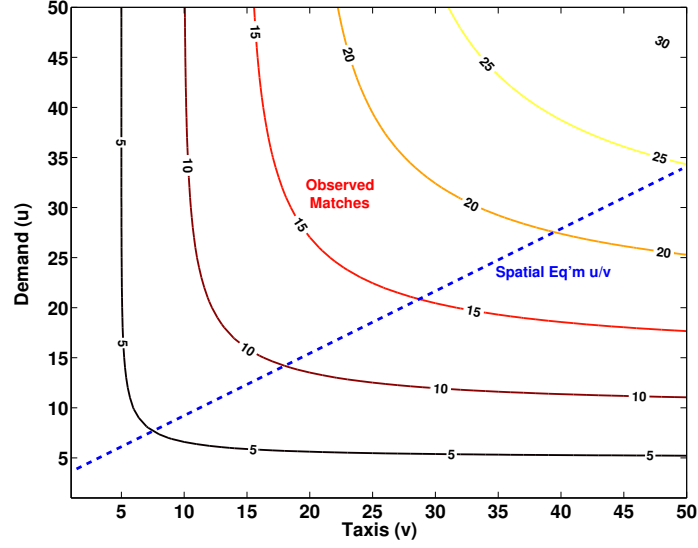
**Proposition 4.1.** *The expected number of matches  $E[m(v_i^t, \lambda_i^t)|v_i^t]$  is one-to-one in  $\lambda_i^t$ .*

*Proof.* See Appendix (A.3) □

This result shows that the number of matches given the equilibrium state can be inverted back into the demand parameter. Let  $\Gamma$  be an operator which maps the spatial distribution of demand  $\boldsymbol{\lambda}$  into the corresponding equilibrium distribution of taxis  $\boldsymbol{v}$ , and let  $\Psi$  map  $\{\boldsymbol{m}, \boldsymbol{v}\}$  into  $\boldsymbol{\lambda}$  according to the inversion described above. By using empirically observed expected matches  $\boldsymbol{m}_0$ , the parameters  $\boldsymbol{\lambda}$  can therefore be obtained by imposing the structural restriction  $\boldsymbol{\lambda} = \Psi(\Gamma(\boldsymbol{\lambda}), \boldsymbol{m}_0)$ .

This identification argument is illustrated in figure (5). It shows the matching function depicted as a series of iso-match contours as functions of supply and demand. The solution

Figure 5: Identification of Supply and Demand from Matches




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**Algorithm 2** NFP Algorithm

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- 1: Guess parameters  $\lambda_0 = \{\lambda^t\}_{t \in \{1, \dots, T\}}$ .
  - 2: Run Taxi Equilibrium Algorithm ( $\lambda_0$ ) to produce equilibrium distribution of supply  $\{v^t\}$
  - 3: Invert empirical matches given  $\{v^t\}$  to infer demand  $\tilde{\lambda}$
  - 4: Check that  $|\tilde{\lambda} - \lambda_0| < \epsilon$  for some tolerance level  $\epsilon$ .
  - 5: If yes, done. If no, update guess to  $\lambda_1$  and repeat.
- 

to the spatial equilibrium model can be represented as an equilibrium state  $\mathcal{S} = \{v_i^t\}_{\forall i, t}$  given the set of customer arrival rates  $\lambda$ . This defines a set of equilibrium *market tightness* measures (i.e. supply-to-demand ratios) for each location-time. The intersection of these equilibrium ratios with the iso-match contours, where the contour-level is the observed number of matches in the data, identifies the levels of both supply and demand.

Algorithm (2) shows how this fixed point is computed in the context of the taxi supply equilibrium algorithm. The termination of the NFP algorithm identifies the spatial, intertemporal distribution of Poisson parameters  $\lambda = \{\lambda_i^t\}_{\forall i, \forall t}$  and also implies the termination of the TEA algorithm, which identifies the equilibrium spatial, intertemporal distribution of vacant taxis,  $\mathcal{S}(\lambda) = \{v_i^t\}_{\forall i, \forall t}$ .

### 4.3.2 Estimation of Remaining Parameters

The two remaining parameters to be estimated are the scale parameter  $\sigma_\varepsilon$  and the same-location bonus value,  $\gamma$ . To identify these two parameters I use a standard MSM estimator designed to rationalize several moments in the data: drivers' total time spent employed and unemployed, total distance travelled while employed, and the probability of the next ride being given from location  $i$  conditional on the last dropoff being in location  $i$ . To link the MSM estimator with the NFP algorithm, I run the NFP over a two-dimensional grid of parameter values, and identify the point  $(\sigma_\varepsilon^*, \gamma^*)$  which minimizes the GMM criterion function. Details are given in Appendix (A.4).

## 4.4 Demand Curve Estimation

I assume that taxi drivers face a constant-elasticity demand curve of the form:

$$\ln(Q_{ij}^t) = \sum_{k=\{r_1, r_2, \dots, r_K\}} \mathbb{I}_{[k \leq d_{ij} < k']} \left( \alpha_{0,k} + \sum_s \mathbb{I}_{s,i} (\alpha_{1,k,s} \ln(P_{ij}) + \alpha_{2,k,s} X_i) \right) + \delta^t \quad (11)$$

This equation can be broken down as follows:  $Q_{ij}^t$  reflects the willingness-to-pay for taxi service from  $i$  to  $j$  given some price  $P_{ij}^t$ , other characteristics  $X_i$  of location  $i$ , and a time-of-day fixed effect  $\delta^t$ .  $\mathbb{I}_s$  is an indicator representing the number of public transit stations present in location  $i$ . The three  $s$ -groups are 0, 1-2, and 3+ transit stations.  $\mathbb{I}_{[k \leq d_{ij} < k']}$  is an indicator representing rides of different lengths. I chose  $\{r_0, r_1, r_2, r_3\} = \{0\text{mi.}, 2\text{mi.}, 4\text{mi.}, 6\text{mi.}\}$  so there will be four ride-length categories, rides under 2 miles in length, rides between 2 and 4 miles, rides between 4 and 6 miles, and for rides greater than 6 miles. Price elasticities  $\alpha_{1,k,s}$  are different for each  $s$ -type of location, and each ride-length category. Separating elasticities by ride-lengths reflects the idea that different types of customers demand different types of rides. Separating elasticities by transit-station access reflects the idea that the outside option is different in different locations. Control coefficients  $\alpha_{2,k}$  are also estimated separately for rides of different lengths. I cannot view information about individual customers, but allowing parameters to differ by ride length will permit some differentiation that is especially useful for considering spatial effects. Finally,  $X_i$  is a matrix of observable covariates including demographic information and location-characteristics.

In the previous section I describe the estimation of  $\lambda = \{\lambda_i^t\}_{\forall i, \forall t}$ . The elements of  $\lambda$  are the Poisson parameters describing the per-period arrival rates of customers in each location and time. Recall that the travel preferences of customers are observed in the data and given by the transition matrix  $M_{ij}^t$ . Thus, the arrival rate of passengers in  $i$  who wish to move to  $j$  at time  $t$  is given by:

$$\lambda_{ij}^t = \lambda_i^t \cdot M_{ij}^t. \quad (12)$$

Note that the origin-destination arrivals of passengers will therefore be Poisson distributed with parameter  $\lambda_{ij}^t$ , and therefore the mean arrivals per-period will be equal to  $\lambda_{ij}^t$ . The estimates  $\lambda_{ij}^t$  can then be regarded as the expectation of points on the demand curve,  $E^\lambda(Q_{ij}^t | P_{ij}, X_i, \delta^t)$ .<sup>28</sup>

I re-write equation (11) to relate these arrival rates to the price of taxi rides conditional on observable features of the market across time and locations. The demand specification is thus given by

$$\ln(\lambda_{ij}^t) = \sum_{k=\{r_1, r_2, \dots, r_K\}} \mathbb{I}_{[k \leq d_{ij} < k']} \left( \alpha_{0,k} + \sum_s \mathbb{I}_{s,i} (\alpha_{1,k,s} \ln(P_{ij}) + \alpha_{2,k,s} X_i) \right) + \delta^t + \epsilon_{i,j,t}, \quad (13)$$

where prices  $P_{ij}^t$  are computed directly from the fare structure (i.e., flag fare + dist. fare  $\cdot d_{ij}$ ). In this demand system, all customers of a given type have the same price elasticities, and variation comes from two sources: the variation in prices for trips between different origins and destinations, and the variation in prices before and after the September, 2012 fare change. I also condition on demographic information within each location.  $X_i$  contains location-specific data: log mean income, log mean commute times, log public transit travel times between each pair of locations, whether the origin is in midtown and whether the destination is an airport. “Midtown” here refers to the four locations which include or sit between Penn Station and Grand Central Station, and is an indicator because many customers arrive into these locations from trains, and the observed matches are much higher in this region.<sup>29</sup> Because airport customers are potentially very different from other taxi customers, and because most of the relevant elasticities and controls don’t apply (e.g.

<sup>28</sup>Note that this expectation is with respect to the arrival of passengers *conditional* on some exact set of circumstances. This is not the same as the expectation of the econometric model  $\ln(Q_{ij}^t) = f(P_{ij}, X_i, \delta^t) + \epsilon_{ij}^t$  in which  $\epsilon$  reflects a source of variation orthogonal to the covariates.

<sup>29</sup>This effect is evident in figure (7), a map of mean demand estimates in each location.

mean incomes of residents, ride-lengths of 0-2 miles or 2-4 miles, etc.), I specify airport demand slightly differently, according to the following specification:

$$\ln(\lambda_{ij}^t) = \beta_0 + \beta_1 \ln(P_{ij}) + \beta_2 d_{ij} + \epsilon_{i,j,t}, \quad (14)$$

where  $d_{ij}$  is the distance between the airport and the destination  $j$ .

To estimate parameters, I conduct an OLS procedure on the log-linear model as in equation (13) for all trips originating in Manhattan, and in equation (14) for trips originating at *each* airport. Recall that since prices are fixed within a location and time period, this specification does not suffer from simultaneity bias as would traditional non-instrumented demand models.

## 5 Empirical Results

This section presents estimation results of the spatial equilibrium model. To begin, Table (4) shows a summary of the estimated parameters and the corresponding equilibrium spatial supply of taxis. Panel A shows estimation results for the per-period Poisson arrival rates of customers  $\lambda_i^t$  across time and locations, as well as point estimates for three scalar parameters: the variance of unobservable shocks,  $\sigma_\varepsilon$ , the matching efficiency parameter  $\alpha$ , and the extra value to cabs for choosing to remain in the same location to search next period. Note that the parameter  $\gamma$  is computed and expressed as a fraction of value functions between period  $t$  and period  $t + 1$ . Thus the value 0.5 means there is a bonus equal to  $0.5 \cdot (V_i^t - V_i^{t+1})$ , implying the choice is worth an additional 1/2 of a period of search value over moving to any adjacent location.

Figure (6) shows the distribution of estimated levels of vacant taxis and customers across all locations and all times of day. The subsections thereafter discuss the dynamic feature of these estimates in more detail.

### 5.1 Equilibrium Supply and Demand Estimation

Denote the spatial equilibrium of taxis conditional on the fare structure and the intertemporal spatial distribution of demand as  $\mathcal{S}(\lambda, b, \pi)$ .  $\mathcal{S}(\lambda, b, \pi)$  is given by a  $T \times L$  matrix, as it specifies the equilibrium path of the state over time. Note that  $\lambda$  is also a  $T \times L$  matrix. I run the TEA algorithm, nested within the NFP Algorithm for each month of data under



Table 4: Results Summary

<b>Panel A: Parameter Estimates Summary</b>			
Estimated Parameter	Number of Estimates	Point Estimate	Mean/Min/Max
$\{\lambda_i^t\}$	5,760	See Fig. (6)	72.71 / 0.04 / 2346
$\sigma_\varepsilon$	1	12.5	n.a.
$\gamma$	1	0.5 periods	n.a.
$\alpha$	1	1.3	n.a.
...			
<b>Panel B: Equilibrium Summary</b>			
Estimated Object	Number of Elements	Computed Value	Mean/Min/Max
$\{S_i^t\}$	5,760	See Fig. (6)	110.39 / 0 / 896.08

This table presents a brief summary of estimation results and equilibrium solutions.

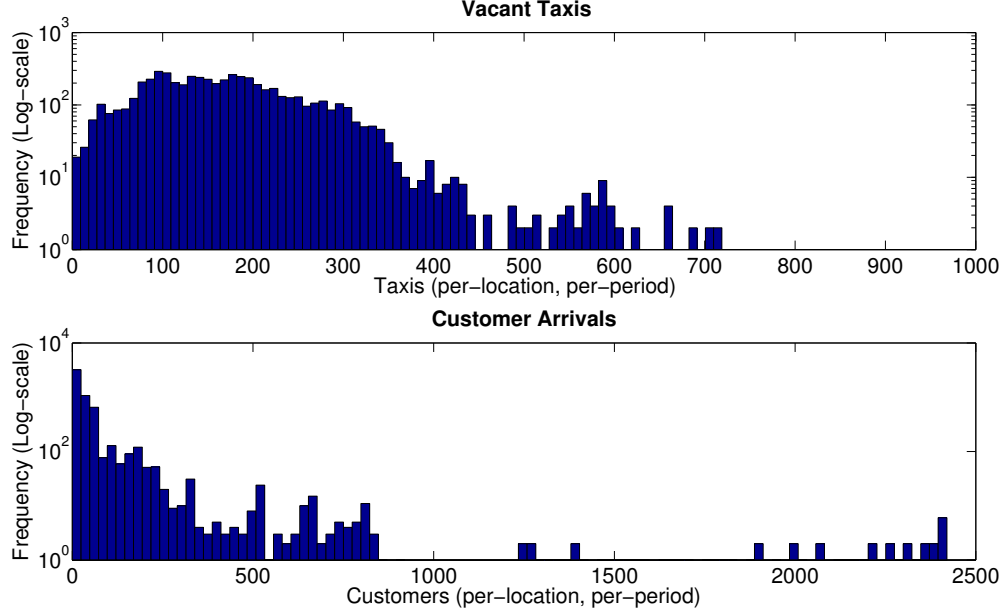
the assumption that the distribution of demand is the same within the same month during weekdays. When the outer NFP algorithm concludes, I have identified both  $\mathcal{S}$  and  $\lambda$ .

### 5.1.1 Supply and Demand Results

Figure (7) depicts supply and demand for taxi rides across all locations, averaged across all periods of the day. It depicts how both taxis and passenger demand is most highly concentrated in the central part of Manhattan. We see that, in general, the number of vacant taxis is sufficient to meet demand in the absence of search frictions. Because this view aggregates across time, I also present time-of-day results below for two selected locations to illustrate how matches are formed from supply and demand dynamics within a location.

Figures (8) and (9) show results for two busy locations, representing areas around West Village and Midtown, respectively. Both graphs depict the equilibrium supply of vacant taxis, estimated arrival rates of customers looking for a taxi, the equilibrium number of matches, and the model's fit against the observed number of matches in the data. Each series is shown from 6a-4p, in 5-minute increments. Figure (8) shows that there are periods of relative oversupply and undersupply (relative to demand) of taxis at different times of

Figure 6: Supply and Demand Estimates



This table shows the distribution of parameter estimates and equilibrium vacant taxis. Frequency is depicted on a log-scale to accommodate large differences in taxi and customer populations across space and time.

day. Figure (9) shows an oversupply of taxis at the same moment there is an undersupply shown in Figure (8). This illustrates spatial misallocation as an equilibrium outcome.<sup>30</sup>

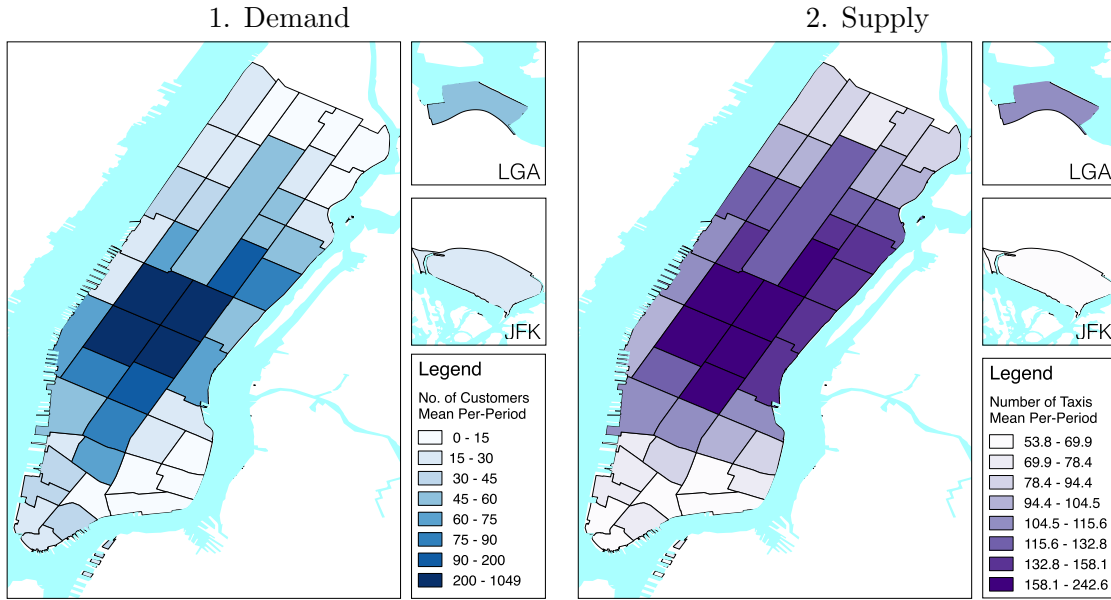
With these estimates in hand, I exploit price variation across different origin-destination pairs, as well as variation stemming from a regulated fare increase, to estimate how demand responds to price changes.

## 5.2 Demand Curve Estimation

Table (5) provides results of the demand estimation of equation (13). As outlined in section 4, an observation is a location-time within a weekday from 6a-4p. The dependent variable is the expected passenger arrivals ( $\lambda_i^t$ ) in each 5-minute time interval and price variation comes from differences in expected customer fares paid for rides originating in each location. Table (5) identifies price elasticities of passenger arrivals between 0.02 at the airport to 1.3 for inter-Manhattan trips longer than 4 miles. These estimates are used to define the

<sup>30</sup>Results for more locations are available in Appendix (A.5).

Figure 7: Map of Estimated Supply and Demand



This figure maps supply and demand estimates for September, 2012. The left panel shows the mean number of demand arrivals in each location from 6a-4p, and the right panel shows the mean number of vacant taxis in each location from 6a-4p.

demand curve in each location. Very low price elasticity at LaGuardia in part reflects both an abundance of business travel (for which transportation costs are often paid by one's firm) as well as a lack of good alternative travel options to Manhattan.<sup>31</sup>

### 5.3 Frictions and Welfare

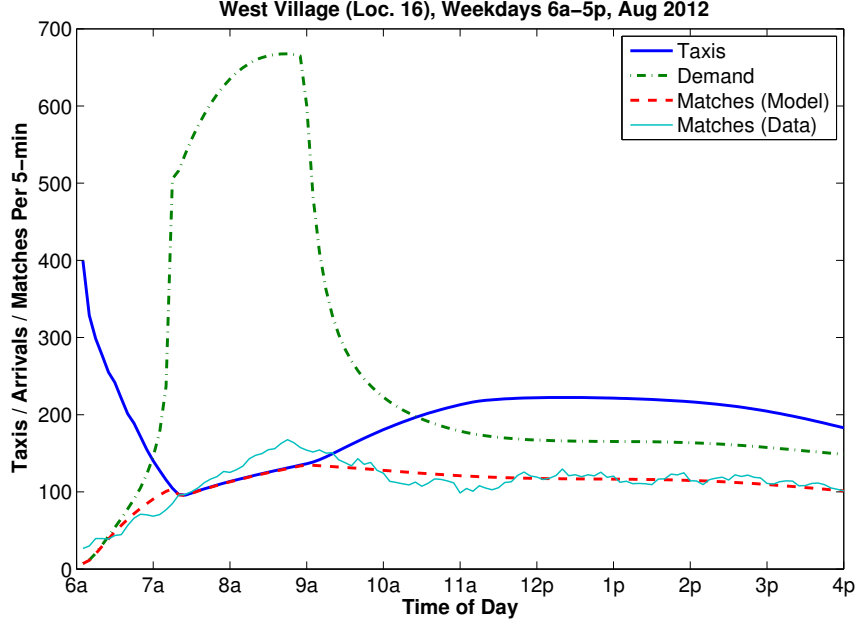
With demand functions estimated, I directly compute estimates of consumer and producer surplus by integrating under the demand function in each location-time and summing across all times and locations.<sup>32</sup>

Importantly, the log-linear demand specification is problematic for integration, as it implies that there will always be some small number of customers at any price. Absent any further extrapolation to estimate the lowest price for which demand equals zero, the integral of these demand curves grows arbitrarily large as the lower integrand approaches

<sup>31</sup>There is no subway service to LaGuardia as there is for JFK Airport.

<sup>32</sup>Explicit formulas used to compute these measures are provided in the Appendix.

Figure 8: Intra-Location Dynamics: Undersupply



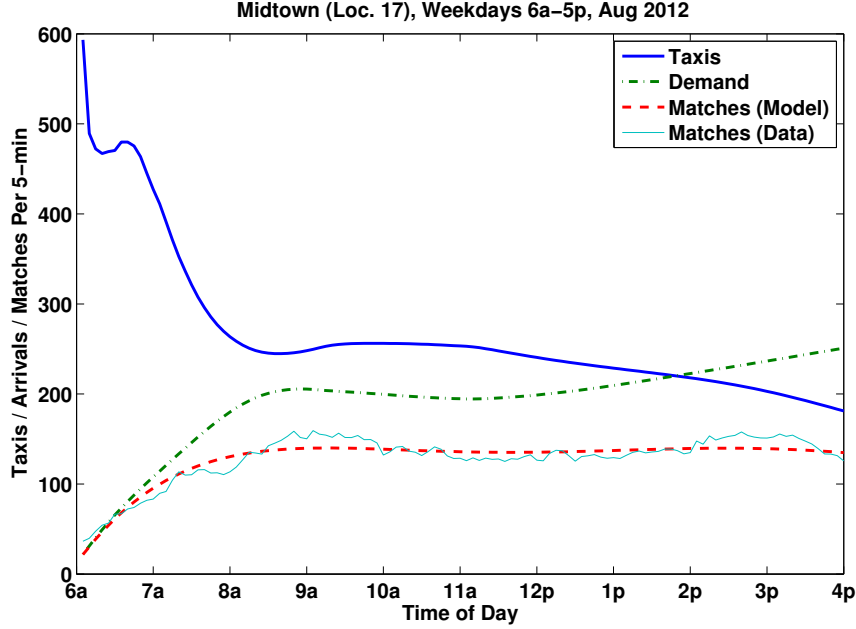
This figure shows the supply of taxis and the arrival of passengers from 6am to 4pm, compared with the expected number of matches. The model fit with observed matches is also shown. Each point depicts the over- or under-supply of taxis relative to demand in each 5-minute interval. This location reveals a substantial under-supply of taxis from 7am until 10am.

zero. I follow Hausman (1999) by computing a lower bound estimate of consumer surplus by computing the area beneath the supporting hyperplane tangent to the estimated elasticities at current prices.

Consumer surplus in matching markets is illustrated in figure (10). Panel I depicts frictions when customers are sorted by their valuations. The full triangle  $A \cup B$  reflects the entire available surplus in this market at price  $p_i^t$ , and the area  $B$  is the lost surplus due to not all demand being matched. This is a kind of “best-case scenario” of welfare loss due to frictions, because only low-valuation demand gets left out.

Panel II depicts frictions under random matching, the matching mechanism faced by taxis. Again,  $C \cup D$  reflects the total available surplus. Customers are matched randomly, so the lost surplus due to matching frictions is represented as area  $D$ . There is a combination of low-valuation and high-valuation customers being matched, and so realized surplus is

Figure 9: Intra-Location Dynamics: Oversupply



This figure shows the supply of taxis and the arrival of passengers from 6am to 4pm, compared with the expected number of matches. The model fit with observed matches is also shown. Each point depicts the over- or under-supply of taxis relative to demand in each 5-minute interval. This location reveals an over-supply of taxis in the morning and early afternoon, but under-supply during the afternoon rush-hour periods.

reduced by the fraction  $\frac{m_i^t}{Q_d^t}$  of total available surplus. Note that  $D > B$ , so realized surplus  $C$  is strictly lower than  $A$ .

Table (6) shows that there is a lower bound consumer welfare of \$281,100 per day for New York taxi service during weekdays from 6a–4p in September, 2012. Producer profits in each shift are \$1.76M, or \$140 per driver.<sup>33</sup> A crude approximation of annual welfare, then, is obtained by multiplying this number by 2.5 to estimate welfare from 4p–6a, and again by 365 days. This calculation yields \$257 million in consumer surplus and \$1.6 billion in producer profits. Using these welfare numbers as a baseline, I analyze the welfare effects of counterfactual prices and regulations in section 6.

Beyond the baseline welfare estimates, table (6) also depicts two more cases: the first of which is where matching frictions exist but customers are sorted by valuation, so that

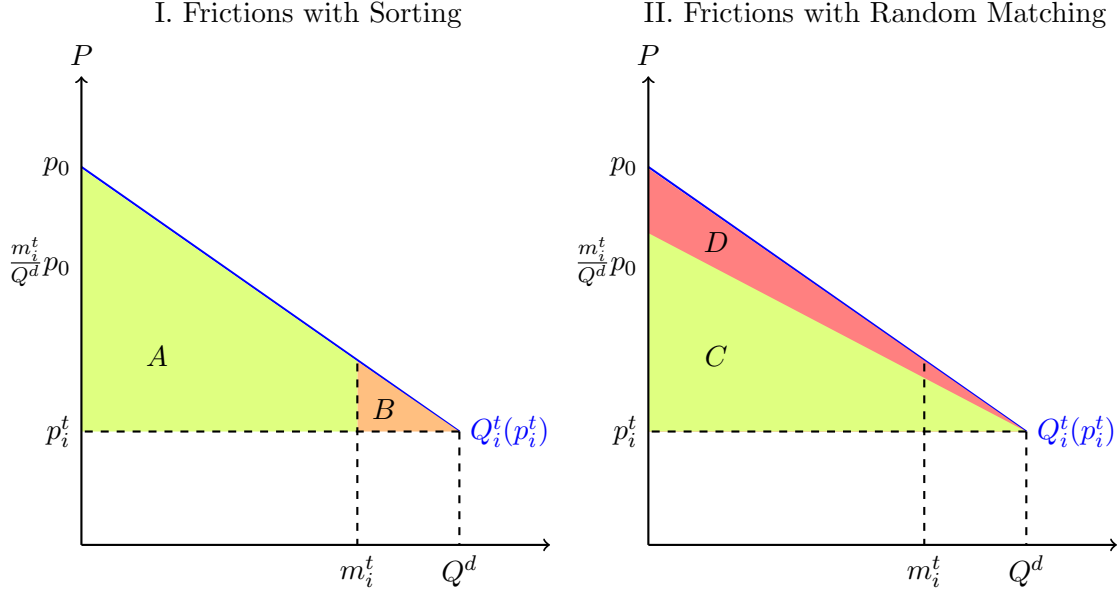
<sup>33</sup>I use the term “producer profits” to denote revenues net of fuel costs.

Table 5: Estimation results: Log Arrivals-per-location per-period

Variable	From Manh.				From LGA	From JFK
	0-2 Miles	2-4 Miles	4-6 Miles	6+ Miles		
<b>log price</b>						
Zero MTA stations	-1.865 (0.594)	-1.075 (0.426)	-2.41 (0.476)	-0.212 (0.632)	-0.978 (0.126)	
1-2 MTA stations	-0.987 (0.542)	-0.924 (0.666)	-1.611 (0.726)	-0.637 (0.534)		-0.927 (0.105)
3+ MTA stations	-0.645 (0.417)	-1.101 (0.799)	-1.527 (0.886)	-1.636 (0.672)		
<b>log transit time</b>						
Zero MTA stations	-0.138 (0.181)	-0.490 (.312)	-1.156 (0.561)	-1.361 (0.570)		
1-2 MTA stations	-0.372 (0.228)	-0.448 (0.339)	-2.268 (0.713)	-1.246 (0.465)		
3+ MTA stations	-0.701 (0.204)	-0.785 (0.353)	-1.183 (0.501)	-0.548 (0.376)		
<b>log commute time</b>						
Zero MTA stations	-7.060 (1.000)	-4.973 (0.808)	-3.338 (1.422)	-4.323 (1.115)		
1-2 MTA stations	-2.840 (1.628)	-2.469 (1.393)	-0.491 (1.411)	-2.263 (0.921)		
3+ MTA stations	-3.399 (0.956)	-1.339 (0.887)	-2.445 (1.690)	-2.999 (0.677)		
<b>log location income</b>						
Zero MTA stations	2.392 (0.422)	1.438 (0.488)	1.094 (0.422)	1.140 (0.337)		
1-2 MTA stations	1.073 (0.204)	0.627 (0.208)	0.779 (0.220)	0.791 (0.142)		
3+ MTA stations	1.380 (0.319)	0.590 (0.245)	0.652 (0.381)	0.869 (0.160)		
<b>origin in midtown</b>	2.258 (0.266)	1.396 (0.228)	0.557 (0.377)	2.302 (0.258)		
<b>destination is airport</b>	-0.148 (0.027)			3.181 (0.347)		
<b>distance to destination</b>	-0.148 (0.027)				0.200 (0.015)	-0.033 (0.010)
<b>Intercept</b>	0.659 (6.108)	5.018 (5.892)	11.901 (7.582)	5.937 (5.003)	1.003 (0.349)	2.707 (0.297)
Time Fixed Effects	Y	Y	Y	Y	N	N
N	293760	323520	220800	221760	11520	11520
R <sup>2</sup>	.6122	.4071	.3891	.5601	.0217	.0390

Demand data come from model estimates. Dependent variable is  $\log(\text{demand})$ , where demand is the arrival rate of customers per five minutes. Observations are mean customer arrivals and prices for an origin-destination pair, time-of-day, month and year. For example, one observation is the estimated mean customer arrivals in location 23 (Penn Station area) with destination location 28 (Central Park), from 2:00p-2:05p on any weekday in August, 2012. Specification includes time-of-day fixed effects. Remaining data come from the following sources: prices (TLC data), transit times (Google Maps), transit stations (NYC OpenData), location commute times and incomes (Census Bureau). Standard errors are clustered at the level of origin-location.

Figure 10: Consumer Surplus with Frictions



This figure depicts sources of welfare loss generated under different types of frictions. Panel I depicts frictions when customers are sorted by their valuations. Panel II depicts frictions under random matching.

only those with the highest willingness to pay are matched with taxis. This case generates \$447.9 thousand in consumer welfare, or \$409 million annually. The second case is where matching was perfectly frictionless; in this case the New York taxi market would attain \$743.6 thousand in welfare per shift, or \$679 million annually. These two cases highlight the welfare cost associated with search frictions: the current market attains only 62% of the frictionless ideal. Decomposing this lost welfare, about one third can be attributed to the lack of customer sorting, and the remaining two thirds is due to lost trades from market participants not finding each other.

Table 6: Estimated Results: Daily, Single-Shift Welfare Measures

Welfare Measure	Consumer Surplus (lower bound)	Producer Profits
Current Market: Frictions w/o sorting ( $C$ )	\$281.1K	\$1.76M
Sorting Market: Frictions w/ sorting ( $A$ )	\$447.9K	\$1.76M
Ideal Frictionless: ( $A \cup B$ ) or ( $C \cup D$ )	\$743.6K	\$3.16M

This table depicts welfare measures with references to Figure (10). Consumer surplus is computed via the Hausman (1999) approach, which provides a lower-bound measure. Producer profits derive directly from estimated matches multiplied by prices for each origin, destination and time-of-day. Profits reflect daily, single-shift revenues net of fuel costs. These profit measures do not include drivers’ or medallion-holders’ opportunity costs, as such they may be thought of as net-revenues.

## 6 Policy Experiments

In this section, I examine the effect of demand shocks, supply shocks and changes in regulated prices on equilibrium frictions and welfare. I hold fixed the basic institutional details of the New York City taxi market: two-part tariff pricing, medallion limits, and medallion rental prices. Throughout, I will take each of these as policy variables, and study the change in each while holding some others fixed. I also compute a social planner’s solution under these constraints. Doing this will allow me to disentangle the costs and consequences of each regulatory constraint and the cost of frictions associated with each. I take the New York City locations and distances as primitive, so specific policy results should be interpreted as relevant only to this market.

### 6.1 Supply Shocks: Effect of Green Cabs Initiative on Manhattan and Airport Taxis

Approximately 6% of taxi trips involve service provided outside of Manhattan or above 125th street. In 2013, a new class of taxis known as Boro Taxis or “green cabs” began operating in Manhattan above 110th street and in the outer boroughs. A natural first question to ask is what is the effect on Manhattan of the introduction of green cabs. I



model this policy as relating to a shift of search activity among all remaining yellow taxis, approximately 1,000 cabs, into the interior Manhattan and airport locations.

Next, I consider the effects of increasing the number of cabs permitted on the roads. Though such a policy would unambiguously reduce individual cab driver revenues in the short run, cities will expand the number of cab medallions to match service to growing populations.<sup>34</sup> I examine how an increase of 1,000 cabs changes consumer welfare and producer profits, and, using the spatial model, examine which neighborhoods benefit the most.

I assume that all 12,500 medallions are on the road, and, in the event of an increase, all new medallions would also take to the road.<sup>35</sup> Table (7) displays the results of this experiment. It shows that consumer surplus increases by 4%, overall firm profits increase by 3.4% (as more rides are now being given than before), while individual driver profits decrease by 4.1%, as the total profits are now split among more drivers.

Table 7: Estimated Welfare Change: Increase Total Cabs by 1,000

<b>Total No. Cabs</b>	<b>Consumer Surplus (lower bound)</b>	<b>Producer Profits</b>	<b>Producer Profits (per-driver)</b>
12,500	\$281.1K	\$1.76M	\$140.4
13,500	\$292.5K	\$1.82M	\$134.6
Change	+4.0%	+3.4%	-4.1%

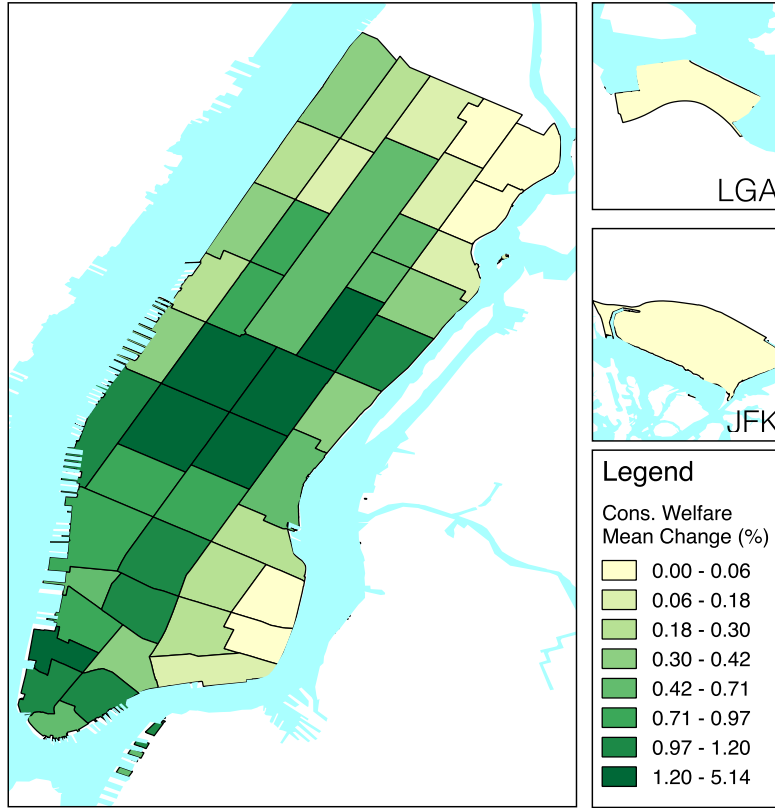
This table shows the daily, single-shift welfare measures in the present market with 13,237 total cabs, compared to that of a counterfactual market in which there are 14,237 total cabs. Both cases take prices to be post-September, 2012 (i.e., \$2.50 plus \$2.50/mile). Note that estimated taxi profits do not include revenues attributable to waiting-time fees or tips.

Figure (11) shows the spatial distribution of welfare changes as a result of an increase of 1,000 taxis. It can be seen in this figure that consumers benefit the most in central, high-demand areas. Note that prices have not changed, but the number of rides serviced has increased, so more customer rides are being given. For taxis, the increase in passengers in these central areas offsets the negative effects of an increase in the number of competitors.

<sup>34</sup>Recent discussion by regulators of a proposed increase in New York can be found at [www.nyc.gov/html/tlc/downloads/pdf/statement\\_of\\_findings\\_11\\_04\\_2013.pdf](http://www.nyc.gov/html/tlc/downloads/pdf/statement_of_findings_11_04_2013.pdf)

<sup>35</sup>There were 13,237 licensed cabs during this time. Because I eliminate the outer-boroughs from my analysis, and because some medallion taxis will be taken off of the road for maintenance, I model the number of taxis operating within the 48 locations as 12,500, consistent with the 96% trip share shown on Table (2) as well as figure (3).

Figure 11: Consumer welfare effect of a 1,000 taxi increase



This figure maps the welfare effects of a hypothetical increase in the medallion limits by 1,000 taxis, given prices as of September 2012. It shows the mean percent change - across all time periods - in consumer surplus for each location. Consumers at both airports are equally well off under both regimes because the model specifies that airport customers always find a taxi.

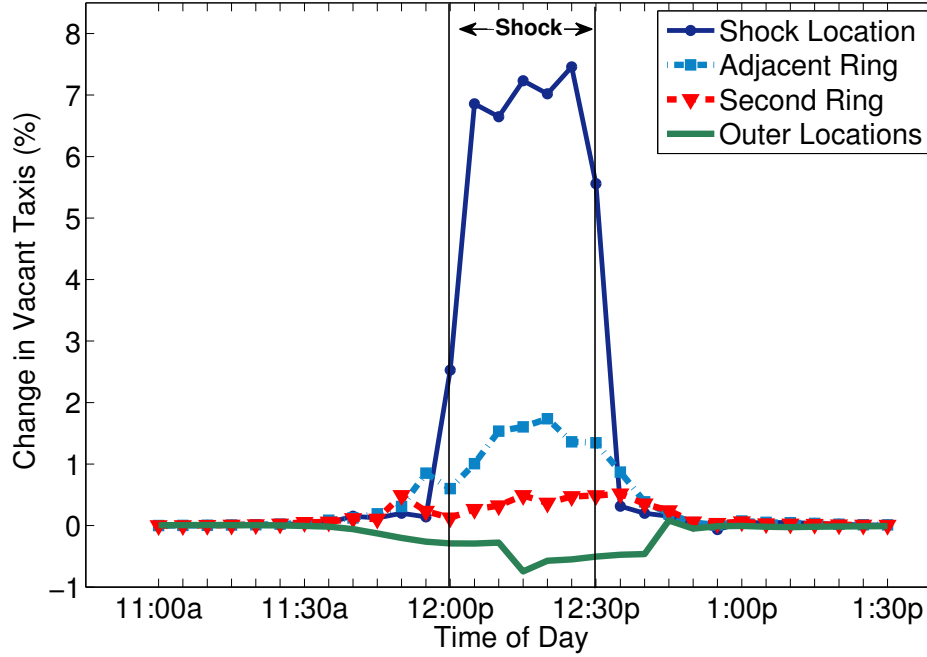
Conversely, in some peripheral locations to the north-east and south-east, consumers end up with similar levels of service before and after, but an increase in the number of taxis dilutes the profit share for individual drivers.<sup>36</sup>

## 6.2 Demand Shocks: effect of a transient, local demand shock

Next, I consider the effect of an expected, transient shock to demand in a single location. This shock mimics the surge in demand for taxis at the end of an public event (e.g. a

<sup>36</sup>Note that the spatial distribution of per-capita profits for taxis only illustrates profits that are attributable to different locations *on average*, as a typical driver spends the day in many locations.

Figure 12: Change in taxi supply due to a local demand shock



This figure depicts the percentage change in the supply of vacant taxicabs resulting from a demand shock in the Union Square neighborhood of five times normal demand from 12:00pm to 12:30pm. The adjacent ring refers to all neighboring locations. The second ring refers to all locations which neighbor the adjacent ring. Outer locations refer to all other locations.

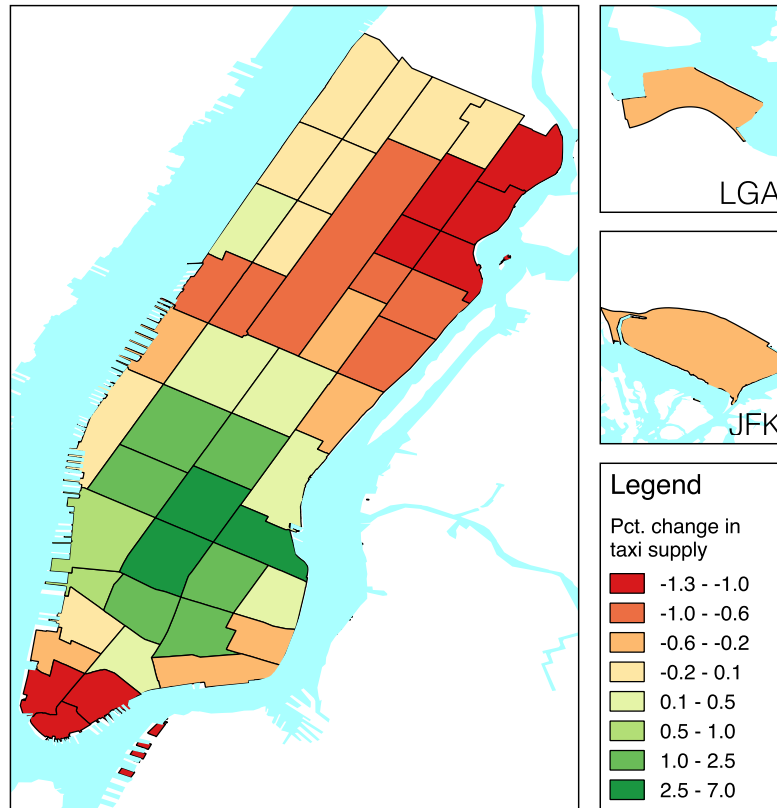
parade, ball game, concert, etc), lasting for 30 minutes. In particular, I am interested in the effects on the taxi supply over several spatial zones: within the shock location, adjacent locations, nearby non-adjacent locations, and in non-nearby locations.

To conduct this experiment, I begin by predicting demand associated with the current fare structure, \$2.50 plus \$2.50/mi. The shock is modeled as a percentage increase in demand over predicted demand within a single location in each relevant period. I simulate the demand shock occurring in the Union Square area beginning at 12:00p and lasting until 12:30p.<sup>37</sup> This area is relatively central, and has several adjacent locations in which I can analyze the effects of the shock.

Figure (12) depicts the spatial taxi supply effects resulting from the local demand

<sup>37</sup>This location is the area bounded by 14th-27th St. and 3rd-8th Ave.

Figure 13: Mapping the change in taxi supply due to a local demand shock



This map depicts the percentage change in the supply of vacant taxicabs at 12:15pm, resulting from a demand shock in the Union Square neighborhood of five times normal demand from 12:00pm to 12:30pm.

shock. It shows that the supply of taxis does respond to the increase in demand from just before the surge to just after, quickly equilibrating back to original levels. In the location of the shock, extra vacant taxis flow into Union Square, increasing more than 7% from original levels. There is a corresponding increase in empty cabs by almost 2% in the adjacent neighborhoods, and an increase of around 0.3% in the second ring of adjacent neighborhoods. Given that the total supply of taxis is fixed, these supply increases draw down the supply elsewhere in the city, as farther locations experience an average drop of up to 0.8% around 12:15pm.

Figure (13) depicts the spatial extend of the counterfactual supply shifts at the peak of the demand shock, from 12:00pm to 12:05pm. It shows that the increase in the supply

of taxi service is "financed" by a deficit in both upper Manhattan, particularly the Upper East Side, as well as from lower Manhattan. The shock location in Union Square has the highest increase, while nearby locations experience an increase in supply due to proximity. This spatial spillover effect stems from both an increase in taxi activity from providing extra service from the shock location to nearby locations (e.g., more pick-ups in Union Square lead to increased drop-offs nearby), but also from the increased value of search in adjacent locations (e.g., any pick-up in a nearby location will more likely result in a drop-off in the higher-valued shock location).

These results suggest a role for policy to mitigate the negative externality generated away from Union Square: the TLC could in this example tax rides in high-demand areas (i.e., a "surge" price that is not collected by cabs) and subsidize taxi trips in affected outlying areas to prevent service shortfalls in locations away from the demand shock. Alternatively, or in tandem with a tax/subsidy policy, the TLC could permit more taxis to be on the road during high-demand times, such that equilibrium levels of service are not reduced anywhere. Importantly, this discussion only applies to the case in which the demand spike was fully expected and internalized into the optimal search patterns of taxis. In contrast, an unexpected shock to demand would not cause taxis to change their search behavior precisely because their decisions are formed off the expectations of demand in each period.

### 6.3 Pricing Effects: The 2012 Fare Hike

Another natural question to ask of this model, which is estimated before and after the time of a regulated fare increase, is what was the effect of that price change? On September 4, 2012, the fare structure increased from \$2.50 plus \$2.00/mile, to \$2.50 plus \$2.50/mile, and the flat-fare charged for rides between JFK and Manhattan increased from \$45 to \$52. Average fares increased by about 17%. The effect of this change is obtained by first inputting the original fare structure (\$2.50, \$2.00, \$45) into the demand estimation, which estimates the arrival rates of passengers across all locations and times-of-day. Using these estimates, I then solve for the equilibrium distribution of taxi service using the spatial model. Finally, I use the resulting supply and demand estimates for each location and time to predict the number of matches made in equilibrium and the total surplus generated from these matches. Results are shown in Table (8).

This table shows that consumer surplus decreased by 4.7% as a result of the fare change,

Table 8: Estimated Welfare Change: 2012 fare increase

Price Input	Consumer Surplus (lower bound)	Producer Profits
8/12 Prices	\$294.9K	\$1.76M
9/12 Prices	\$281.1K	\$1.79M
Change	-4.7 %	+1.7%

This table shows the change in daily, single-shift welfare measures from before and after the September, 2012 fare change. On September 4, 2012, the fare structure increased from \$2.50 plus \$2.00/mile, to \$2.50 plus \$2.50/mile, and the flat-fare charged for rides between JFK and Manhattan increased from \$45 to \$52.

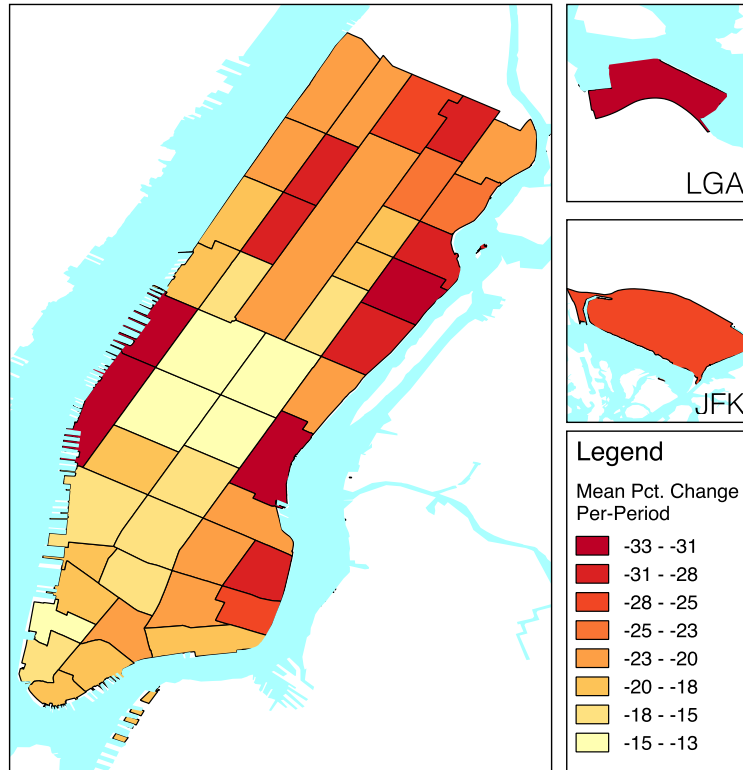
while producer profits increased by 1.7%. Note that these welfare measures are aggregates of time- and location-specific measures. The spatial model reveals a very detailed picture about the spatial extent of the consumer welfare losses. Figure (14) shows the change in welfare in each location as a result of the fare change. This figure shows that more peripheral locations experienced greater consumer surplus losses, an intuitive pattern given that consumers in these locations tend to demand longer-distance rides, so that an increase in the distance-fare will have a greater impact on the total price paid. The loss in customers in these locations also impacts drivers, mitigating the increased profitability of individual rides in these locations by reducing the probability of finding a match. As a result, drivers shift search to less-affected areas.

## 6.4 Optimal Tariff Pricing under Fixed Medallion Limits

Why should the two-part tariff be used in the taxi market? In contrast to the ubiquity of fixed-plus-distance fare structure, there are essentially no fixed costs to providing a taxi ride.<sup>38</sup> Fixed costs may plausibly arise from the time spent initially stating one’s destination, or from the transaction time at the end of a ride, but these costs are likely to be quite small relative to the time taken to drive a single mile. Thus from the perspective of this model, fixed costs of taxi rides are assumed to be zero. Given this assumption, and given the existence of a fixed “flag-drop” fare, the service price for a very short trip might greatly exceed marginal cost. It is therefore tempting to conclude that charging a fixed fee here is inefficient, but this question is slightly more complicated; The fixed fee also

<sup>38</sup>The two-part tariff structure can be seen in historical photos of taximeters going back to the 1940’s in the US, and I have found earlier photos advertising *strictly* fixed-fare rides in some cities.

Figure 14: Consumer welfare effect of September, 2012 fare increase



This figure maps the welfare effects of the September, 2012 fare hike. The intensity of color in each location depicts the degree of percentage loss in consumer welfare after the change.

influences taxis' search behavior and therefore the spatial distribution of service. Existence of this fee may incentivize vacant cabs to distribute themselves spatially into more efficient allocations than they would under a pure distance-based fee. What type of fare structure is optimal? The spatial equilibrium model can be used to answer this question.

This section proceeds in two steps. First, I discuss the optimal tariff in the current regime; that is, one in which medallion limits are held fixed at 13,500 and taxi drivers pay a portion of their revenues towards renting the taxi medallion for a shift. Second, I discuss some implications of existing medallion rental costs and question the extent to which these high rental costs reflect economic rents accruing to medallion owners. Finally, I discuss how high rental prices impose a price wedge which represents an additional market distortion, and I further derive tariff prices that would support current taxi driver earnings in the

absence of rental fees.

#### 6.4.1 Optimal Tariffs in the Current Regime

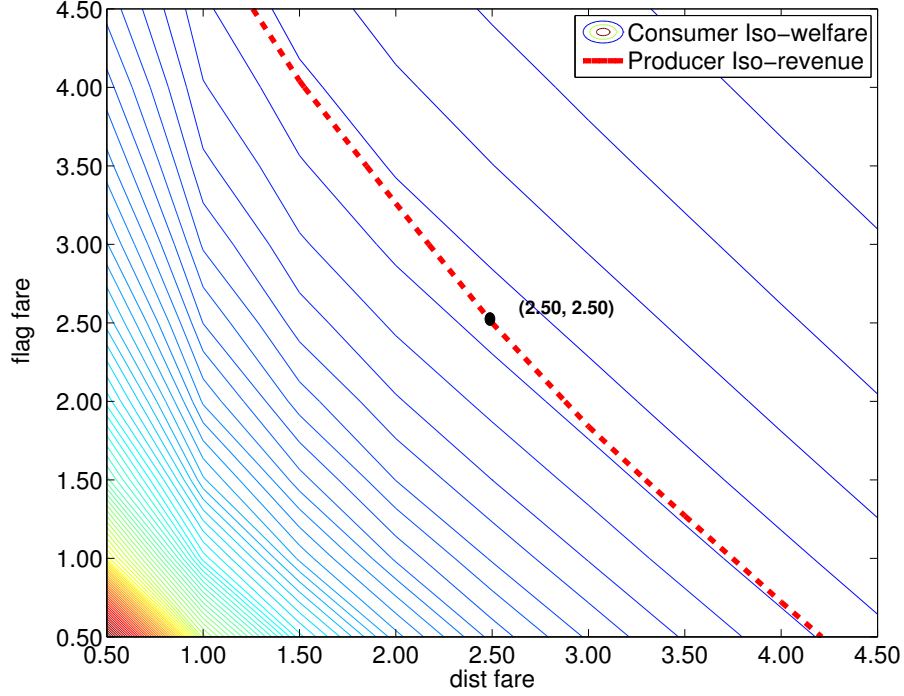
The first question of this section is to ask which combination of distance-based and fixed fees are optimal from a consumer welfare perspective. To answer this, I hold fixed the medallion constraint and compute counterfactual equilibria across a grid of one hundred possible two-part tariffs, recording the effects on consumer welfare and producer profits. To ensure an equivalent comparison, I fix taxi revenues at their prevailing level in September, 2012 at a fare structure of \$2.50 plus \$2.50/mi. and search for the alternative fare structure that yields the highest consumer surplus subject to the constraint that firm profits remaining fixed.

Figure (15) depicts a surface of consumers' iso-welfare curves across a grid of counterfactual fare structures. It also depicts the iso-profit curve for taxis against the same grid: holding fixed profits earned under September, 2012 fares. It shows that, subject to firms being equally well-off as they are currently, consumer welfare is maximized when the fare structure is \$4.20 per mile and zero fixed fee. Though this result is intuitive, it is far from obvious given the complex and subtle underlying shifts in equilibrium search behavior. Consumers in this model receive no fixed benefit to finding a cab, just as cabs do not face any fixed cost of providing an additional ride. The fixed fee introduces a wedge between the cost of providing a very short ride and its corresponding price.

Moving to the optimal tariff of \$4.20 per mile and zero fixed fee generates \$289 million in annual welfare, a 12.5% gain over current levels. Note that taxi profits remain fixed by construction. Changing the tariff structure to a distance-only fee will have spatial consequences, however. As shown in Figure (14), putting additional weight on distance increases prices in locations whose customers demand longer average travel distances. An optimized tariff based only off distance can be viewed as a simple, consumer welfare-enhancing improvement in the fare structure, and one that should be acceptable to drivers. The spatial view tells us that it nevertheless cannot be regarded as a Pareto improvement, as some customers who demand longer rides would pay more under these prices.



Figure 15: Welfare of counterfactual fare-structures



This figure depicts consumers' equilibrium iso-welfare curves, from low-levels (blue) to high levels (red) evaluated within the space of counterfactual two-part tariffs. In aggregate, consumers are equally well-off with fare-structures along a given curve. Also depicted is a single equilibrium iso-profit curve on the firm-side, evaluated at *existing* profits as of September, 2012. Taxis will be indifferent to fare-structures that sit along this curve.

#### 6.4.2 Optimal Tariffs in a Zero-Profit Regime

In the last 15 years, the price for a New York taxi medallion has increased by approximately 500%, peaking around \$1.3 Million in 2012.<sup>39</sup> These sales prices reflect the lack of free entry, and, correspondingly, prices which surpass marginal costs.<sup>40</sup> Binding medallion

<sup>39</sup>Medallion sales data come from the NY Taxi and Limousine Commission. Though prices have since declined significantly in the wake of competitive pressure from new services, they remain above \$500,000 as of 2015.

<sup>40</sup>Positive medallion values directly imply a binding entry limit; the right to drive a taxi in this market is valued much higher than the wages earned by a driver. Free entry would allow taxis to enter the market until this value was driven to zero.

restrictions limit supply below that of a competitive market, leading to cartel-like rents that accrue directly to medallion owners through high leasing prices paid by taxi drivers to rent medallion-equipped cars. Though rental rates in part fund the capital costs of providing cars, typical day-shift rates clearly exceed the cost of car ownership. Specifically, the New York TLC regulates lease prices to be no greater than \$105 for a 12-hour day-shift and between \$115-129 per 12-hour evening shift, so that medallion owners will earn more than \$220 per day.<sup>41</sup>

In this section, I ask how medallion rental costs pass through to consumer prices. There are two components to this question: first, what portion of current taxi fares go towards paying medallion owners? This question regards the wedge between prices paid by customers and revenues earned by drivers. Second, if there were no medallion rental cost, and all fare dollars were retained by taxi drivers, how low could fares be and still support taxi drivers' wages at today's levels? The second question addresses the fact that lower prices to consumers would increase demand and thereby increase the match rate for taxis, allowing for prices which are even lower than those implied by the answer to the first question.

To address both questions, I will make the following assumptions: I first assume that medallion holders charge the full \$105 rental rate. Second, because my model does not predict revenues from waiting-time fees or tips, I assume that these extra revenue sources are a fixed percentage of the base fare, and I compute this percentage by dividing the mean empirically observable total fare (a value which includes revenue from waiting fees and tips) by the model's predicted base fare (i.e., the flag fare plus distance fare). This calculation produces a value of 1.9992, or almost exactly a doubling of revenue from these additional sources. Third, I assume that in the absence of outside medallion owners, drivers must pay depreciation costs of ownership on top of fuel prices, an additional cost which may influence spatial choices. The Internal Revenue Service provides an estimate of \$0.23.<sup>42</sup> Finally, I assume that there are no additional costs of ownership (e.g., financing costs of a vehicle, storage costs, etc).

What portion of current taxi fares pass through to medallion owners? I note that driver incomes average \$275 per day in my sample.<sup>43</sup> A rental price of \$105 is 38.1% of daily

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<sup>41</sup>For hybrid cars, the lease cap is \$108 for the day-shift, and \$118-132 for the evening-shift. TLC lease cap rules can be accessed at: [www.nyc.gov/html/tlc/downloads/pdf/fleet\\_drivers\\_rights\\_poster.pdf](http://www.nyc.gov/html/tlc/downloads/pdf/fleet_drivers_rights_poster.pdf)

<sup>42</sup>See additional details at [www.irs.gov/uac/IRS-Announces-2012-Standard-Mileage-Rates,-Most-Rates-Are-the-Same-as-in-July](http://www.irs.gov/uac/IRS-Announces-2012-Standard-Mileage-Rates,-Most-Rates-Are-the-Same-as-in-July)

<sup>43</sup>Cash tips are generally not reported in this data, but credit card tips are. I assume that the average

revenues. This directly implies that this fraction of the fare can be thought of as a transfer to medallion owners: \$0.95 fixed fee plus \$0.95/mi. In September, 2012 the average fare amount (not counting tips, taxes or tolls) was \$12.39. Thus, an average of \$4.72 per trip is paid by customers to medallion owners. But these measures only reflect a transfer from consumers to medallion owners, not the true economic costs of rent seeking. The economic costs come from the price wedge; markets do not fully clear when some customers are priced out at current fare levels. The next counterfactual considers the case in which medallion owners were not allowed to collect rent payments. This is alternatively thought of as a regime in which we tax the entire rental cost and subsidize customer taxi rides.

To estimate what prices could be supported if a zero-profit condition were imposed on medallion owners, I search for fare prices that would allow drivers to earn the same take-home wages if the leasing costs were zero, but capital costs (i.e., depreciation of the taxi) were charged to drivers. Note that drivers currently take home the difference between total revenues and leasing costs, fuel costs, and credit card costs.<sup>44</sup> At this wage level, drivers are willing to supply labor at existing levels.

In Figure (16), I compute equilibrium for a variety of possible fare-structures and find the point at which revenues are equal to the current mean daily revenues \$275 *minus* medallion leasing costs of \$105, so that take-home wages are equal to the same levels as they are currently, \$170. It shows that for fixed taxi driver revenues, consumer surplus remains maximized when the flag-fare is set to zero, and the fare is computed only off of distance. If medallion rents are set to zero but drivers pay for depreciation and other costs, then a fare of \$1.00 per mile is sufficient to generate the same income. This result implies that up to a 64% fare reduction would be possible in the absence of rent-seeking, much more than the naive estimate of 38% as described above.

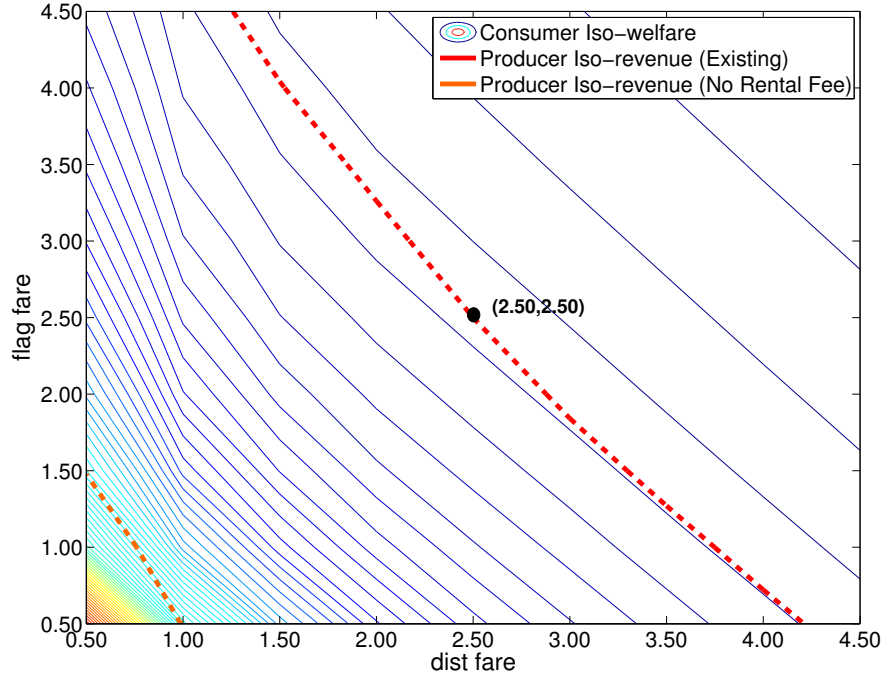
Table (9) shows the aggregate welfare benefit induced by this change. There are immense benefits to reducing prices by such a large amount. Consumer welfare increases by 717% and matches increase by 163%. Taxi driver revenues are unchanged by construction, but medallion owners lose all of their surplus, as would be expected under a free-entry policy.

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cash tip is equal to the average credit card tip in the day shift, about 21%, and compute total daily revenues accordingly.

<sup>44</sup>Credit card costs are a flat-rate of \$11 per-shift, as per [www.nyc.gov/html/tlc/downloads/pdf/fleet\\_drivers\\_rights\\_poster.pdf](http://www.nyc.gov/html/tlc/downloads/pdf/fleet_drivers_rights_poster.pdf)

Figure 16: Welfare under zero-profit condition



This figure depicts consumers' equilibrium iso-welfare curves, from low-levels (blue) to high levels (red) evaluated within the space of counterfactual two-part tariffs. In aggregate, consumers are equally well-off with fare-structures along a given curve. Also depicted is the equilibrium iso-profit curves of taxis, evaluated at two points: current revenues and equivalent revenues when leasing costs are zero. Taxis will be indifferent to fare-structures that sit along these curves.

## 6.5 Reducing Frictions through Matching Technology

In 2011, Uber began operating in New York City, and has quickly grown to be a substantial force in the taxi market. An agreement with the New York TLC in 2013 permitted medallion taxis to use Uber technology to facilitate matching between taxis and passengers.<sup>45</sup> The rapid rise of ride-sharing services has sparked intense debate over taxi regulation in New York and elsewhere, as these new services often clash with existing regulations and incumbent stakeholders of medallion rights. This section contributes to this discussion by considering the value of improved matching technologies. I also estimate the cost of excess

<sup>45</sup>See [newsroom.uber.com/2012/12/another-big-win-e-hail-coming-to-nyc/](http://newsroom.uber.com/2012/12/another-big-win-e-hail-coming-to-nyc/)

Table 9: Estimated Welfare Change: All rents transferred to lower prices

<b>Tariff</b>	<b>Consumer Surplus (lower bound)</b>	<b>Matches</b>
Baseline, 9/12 Prices	\$281.1K	123K
(\$1/mi., \$0 flag)	\$2,297.0K	324K
Change	+717%	+163%

This table shows the change in daily, single-shift welfare measures from adopting a policy in which medallion owners may not earn rents (above and beyond costs of depreciation and credit card processing). Instead, these lost profits are transferred to consumers through lower prices. The prevailing price is one in which taxi drivers earn the same net revenues before and after the policy change.

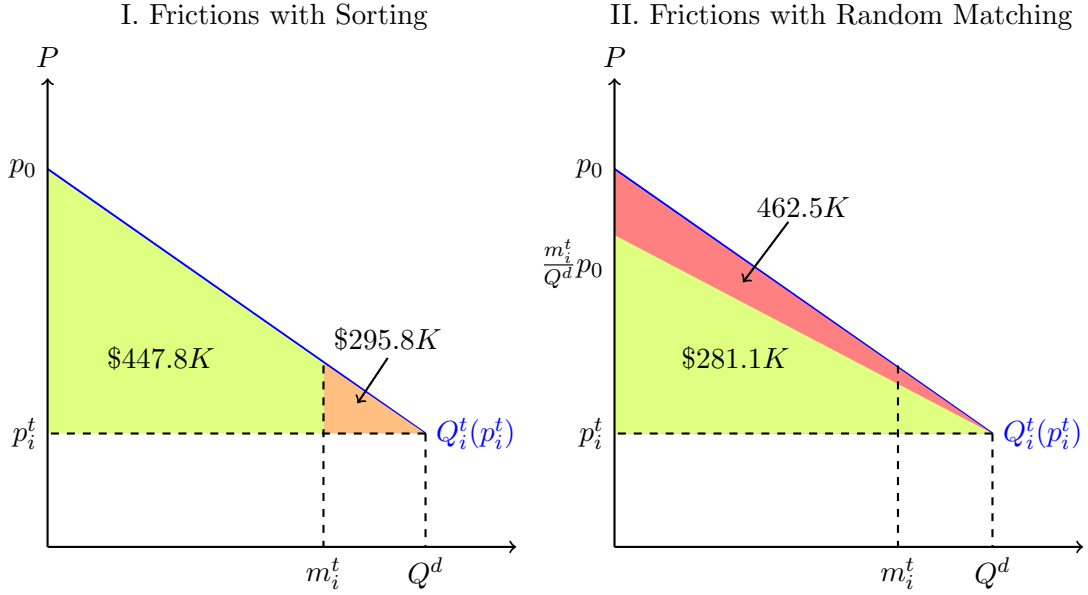
rents accruing to medallion owners, which could otherwise be diverted into price cuts and driver profits.

I first exploit the separate estimates of passenger demand and equilibrium cab-passenger matches to derive the costs of the search process inherent in the street-hail taxi market. This analysis estimates both the cost of allocative inefficiency stemming from random matching (where customers are not differentiated by their valuation, or willingness-to-pay, for taxis), and the foregone surplus of unserved passengers who fail to find a match.

Figure (10) illustrates how surplus is computed in the current taxi market as the area labeled  $C$ . Suppose there is a hypothetical world in which all customers who demanded a taxi at a particular price were matched with one. This scenario is made possible by a combination of a real-time bilateral matching technology, and a supply of taxis sufficient to service all customers. The new generation of ride-sharing services offer technologies to address both of these requirements. The first is real-time matching between customers and cabs using a mobile application, and the second is variable pricing to clear the market. Though I can not identify market clearing prices with this model, I can estimate the cost of matching frictions at any price in the absence of supply constraints. The area  $A \cup B \cup C$  of figure (10) records this ideal level of surplus.

Figure (17) displays the results of table (6) in graphical form. It shows that at least \$462,500 per shift is lost in consumer surplus, and approximately \$1.4M in total revenues due to unserved customers. Under the simple approximation that service in all shifts are equally valued by cabs and customers, and over each day of the year, together this represents a lower-bound estimate of annual costs of about \$1.7 billion. This number is a lower bound due to the lower bound measure on consumer surplus. Of these losses, about

Figure 17: Consumer Surplus with Valuations



This figure depicts two sources of welfare loss generated under random matching with frictions. Given values are welfare measures associated with a single 6a-4p weekday shift.

\$166.7 million per shift is the loss to consumers is due to random matching alone, which represents 36% of the total consumer welfare cost of unserved matches. implying that even when there are not enough taxis to serve all customers, the ability to simply sort by customers' valuations would generate non-trivial efficiency gains.

### 6.5.1 Discussion on Free-entry

The above sections provide some insight into the value of a free-entry policy. A free-entry policy would eliminate rents by driving lease prices down until they equal the marginal cost of vehicle ownership. Such a policy would enable these rents to be absorbed instead by market participants and would generate additional value by eliminating the implicit price wedge. I do not predict how many taxis would be on the road under free entry.<sup>46</sup> Using the counterfactuals above, though, I can predict some general effects of free entry: Since the current medallion supply is binding, for at least most of the day, free entry would likely

<sup>46</sup>Estimating these costs in the context of the spatial equilibrium model is a topic of future work.

lead to an increase in supply, the benefits of which are discussed in section 6.3, and a fall in price towards marginal cost, as discussed in the above subsection.

The success of modern web-based taxi hailing services is partially attributable to the enormous value added by mixing free entry with matching technology, which together accounts for nearly all sources of welfare-enhancing policies discussed in this paper. Notably, however, these companies still price service according to two-part tariffs, which is demonstrated in sections 6.1 and 6.3 to be inefficient.<sup>47</sup>

## 6.6 Combining *mc*-pricing, Optimal Tariffs and Matching Technology

Several of the above subsections highlight potential improvements to taxi regulation. Adjusting the two-part tariff along drivers' iso-profit curves to be solely distance-based generates welfare by inducing better spatial allocations of supply and demand. A policy which lowers the overall price level by imposing a zero-profit condition on medallion owners (implementation specifics aside) will directly generate large welfare gains. Improved matching technology can further be used to nearly eliminate search frictions by ensuring that all passengers find a taxi. I model a matching technology which perfectly matches taxis with customers who demand a ride within the *same* location.

Using the spatial equilibrium model and demand system estimates, I simulate the effects of a regulatory regime that enacts all three policies simultaneously. Table (10) displays the consumer welfare gains associated with a switch to these policies. It shows that consumer welfare increases to \$2.6 million per shift and matches increase to about 324,000 per shift. Note that these gains surpass the constrained social planner's optimal configuration for cabs, given the addition of frictionless matching. This result underscores the importance of improved matching on consumer welfare in this market. Given the benefits associated with these changes, the results provide insight into the success of Uber, Lyft and other new firms which operate with an on-demand matching technology combined with essentially free-entry.

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<sup>47</sup>For details on the fare structures of the two most popular ride-sharing services, see [uberestimate.com/prices/](http://uberestimate.com/prices/), and [www.lyft.com/help/article/1515660](http://www.lyft.com/help/article/1515660)

Table 10: Estimated Welfare Change: optimal tariffs, no owner rents, on-demand matching

Policy	Consumer Surplus (lower bound)	Matches
Baseline, 9/12 Prices	\$281.1K	123K
Combined Policy Change	\$2,599.0K	324K
Change	+824%	+163%

This table shows the change in daily, single-shift welfare measures from adopting (1) an optimal tariff policy, (2) a zero-rental rate policy and reducing prices so that taxis earn equal amounts before and after, and (3) a technology which perfectly matches customers with cabs who are in the same location.

## 6.7 A Social Planner's Spatial Distribution

Can competition generate an efficient spatial equilibrium? I can determine the efficient spatial distribution of taxis by considering the problem of a constrained social planner whose goal is arrange the supply of cabs so as to maximize consumer surplus such that total supply remains fixed at 13,237. I assume that the planner can only move taxis around in space by influencing their search behavior. Taxis remain constrained by travel time and by search frictions, but are endowed with preferences which perfectly align with maximizing consumer welfare across the city. This exercise will reveal that the distribution of taxis attained in competitive equilibrium is inefficient in the sense that taxis are systematically providing service to some locations that value service less than others. For a given level of demand, the difference in welfare attained between the competitive regime and the constrained planner's regime reflects the cost of spatial frictions induced by fixed prices.

To compute welfare under the constrained planner's regime, I implement the following procedure: First, I assume that the marginal cost of taxi service is equal to \$1.00 per mile, and set the tariff exactly equal to equal this, a basic efficiency condition.<sup>48</sup> Second, I endow taxi drivers with the planner's incentives. Recall that in the equilibrium model, taxis' policy functions depend on the (private) value of search in each location, represented by value functions. To solve the social planner's problem, I replace period-profits in the value function specification with consumers' demand curves in each location and time, evaluated at the expected number of matches.<sup>49</sup> These points on the demand curve represent the

<sup>48</sup>This marginal cost is only an approximation, based on the finding above that 2012 levels of service are supportable under these prices if rental rates were zero.

<sup>49</sup>As above, I use the linear, lower-bound estimate of the demand curve.



Table 11: Estimated Welfare Change: Constrained Social Planner’s Solution

<b>Tariff</b>	<b>Consumer Surplus (lower bound)</b>	<b>Matches</b>
Baseline, 9/12 Prices	\$281.1K	123K
Social Planner	\$2,589.0K	460K
Change	+821%	+274%

This table shows the change in daily, single-shift welfare measures from enacting the social planner’s solution where medallion levels are held fixed. The social planner induces taxi drivers to solve the optimal location choice problem so as to maximize consumer surplus instead of profits.

marginal surplus associated with adding a single cab in each location and time. Finally, I solve the problem by running the modified TEA algorithm, which, as before, begins with an initial distribution of taxis. When the algorithm runs, it identifies policy functions that maximize consumer welfare by moving taxis around until there is no policy which attains a higher surplus.

Overall results are shown in Table (11). Consumer surplus attained is equal to \$2.6 million and 460,000 matches per shift, or about \$2.4 billion per year assuming equal surplus across the entire day and each day of the year. Compared with the prevailing surplus in 2012, this reflects a 821% increase in consumer welfare and a 274% increase in matches. Note, however, that most of these gains are attributable to lower prices. Table (9) shows the outcome of a \$0 flag + \$1/mi. tariff where taxis choose locations to maximize profits, which leads to a singles-shift surplus of \$2.3 million and 257,000 matches per shift. Viewed from this alternative baseline, the social planner is able to improve surplus by about \$292 thousand (about \$266M per year) through spatial reallocation alone.<sup>50</sup>

## 7 Conclusion

Supply and demand in the taxi market is uniquely shaped by space. Regulation influences how taxis and their customers search for one another and how often they find each other. This paper models a dynamic spatial equilibrium in the search and matching process be-

<sup>50</sup>The constrained planner’s solution could be implemented as a dynamic and spatially differentiated tax and subsidy policy; there is a price tax or subsidy  $s_{it}^*$  for each origin-destination pair that generates competitive incentives equivalent to the planner’s incentives, though this would likely pose far too complex of a price schedule for taxis to fully internalize.

tween taxis and passengers, highlighting a unique mapping between passenger-cab matches and equilibrium supply and demand. Using trip data from New York yellow taxis, I estimate this model to recover the unobservable spatial and inter-temporal distribution of demand during the day-shift. Using variation in trip prices, I specify and estimate demand curves for each time-of-day and in each of 48 New York locations. With identified demand curves, a spatial equilibrium model of taxi supply permits predictions of welfare outcomes under alternative regulatory and technological circumstances.

I show that welfare attained in the New York market is \$257 million per year, a reduction in by \$422 million per year compared to an environment with frictionless matching. I identify several welfare-enhancing policy options: First, an improved fare structure would be strictly distance-based. It leads to welfare gains through reducing search frictions and a better spatial allocation of taxis relative to demand. Second, restricted quantity controls generate rents for medallion owners, but these profits impose significant costs on consumers in the form of higher prices. I show that a marginal-cost pricing policy combined with a distance-only tariff generates 89% of the surplus achieved by a social planner's solution. Third, improved matching technologies, as seen in popular ride-sharing services such as Uber and Lyft, generate substantial gains by nearly eliminating search frictions. Finally, a policy which combines a directed matching technology, optimal tariff structure, and marginal-cost pricing would increase consumer welfare by approximately nine times current levels, slightly more than the social planner's outcome when constrained under current matching technology.

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## A Appendix

### A.1 Data Cleaning

Taxi trip and fare data are subject to some errors from usage or technology flaws. A quick analysis of GPS points reveals that some taxi trips appear to originate or conclude in highly unlikely locations (e.g., the state of Maine) or even impossible locations (e.g., the ocean). I first drop any apparently erroneous observations. Next, I drop observations outside of the locations of interest, Manhattan and the two airports. This section describes how data are cleaned and provides some related statistics.

#### Data Cleaning Routine

1. Begin with merged trip and fare data from August, 2012 to September, 2012.
2. Drop observations outside of USA boundaries.
3. Drop observations outside of the New York area.
4. Drop duplicates in terms of taxi driver ID and date-time of pickup.
  - Most of these appear to be erroneous.
5. Drop observations outside of Manhattan (bounded above by 125th st.), LaGuardia Airport and JFK Airport.
  - JFK and LGA airport areas are defined by their corresponding census tract.
6. Drop observations which cannot be mapped to any of the 48 locations summarized in Figure (4).
  - E.g., GPS point falls inside of manhattan boundaries, but in impossible location such as the East River.

Table (12) shows the incidence of each cleaning criterion.

### A.2 Equilibrium Algorithm Details

Recall that  $S_0^t$  is a state vector of the number of vacant taxis in each location at time  $t$ . The initial guess of the state in each period,  $S_0^t$ , is assigned by allocating the total number

Table 12: Data Cleaning Summary

Procedure	Criterion Applied	Obs. Change
Drop Errors	1. Initial Data	28,927,944
	2. obs. outside USA	-749,623
	3. obs. outside NYC	-5,298
	4. drop duplicates	-57
Drop Unusable Data	5. keep manhattan + airports	-3,622,803
	6. un-mapped data	-117,249

**Final Data Set:** 24,432,914 observations

This table summarizes the data cleaning routine for TLC data from 8/1/2012-9/30/2012.

of taxis (12,500) according to the empirical distribution of matches. As the TEA algorithm runs, each vector  $\mathcal{S}_0^t$  for  $t \geq 2$  is updated as  $t - 1$  transitions are computed given the  $t - 1$  initial state and value functions for  $t, t + 1, \dots, T$ . Only one term,  $\mathcal{S}_0^1$  remains exogenously chosen.

The TEA algorithm begins at time  $t = T$  and runs backwards to time  $t = 1$ . At each step backwards, the algorithm updates policy functions and resulting transitions forward from the current step  $t$  to the final step  $T$ . Once reaching time  $t = 1$ , the exogenous guess  $\mathcal{S}_0^1$  is updated to equal the equilibrium distribution of taxis at the middle of the day,  $t = 60$  (i.e., 12pm). Following this update, the TEA algorithm always begins at  $t = 1$ , with the exogenous state guess fixed, and computes transitions through the end of the day, and at each period forward updates the value functions. The algorithm repeatedly cycles forward through the day, updating transitions and value functions, terminating only when transitions and value functions are mutually consistent.

### A.3 Proofs

**Proof of Proposition 4.1** The expected number of matches  $E[m(v, \lambda)|v]$  is one-to-one in  $\lambda$ .

*Proof.* First decompose  $E[m|v, \lambda]$  as follows:

$$E[m|v, \lambda] = v \sum_{k=0}^{\infty} \left( 1 - \left( 1 - \frac{1}{\alpha v} \right)^k \right) f_{\lambda}(k)$$



Let  $\rho = \left(1 - \frac{1}{\alpha v}\right)$ . Then with some algebra we can write:

$$\begin{aligned}
E[m|v, \lambda] &= v - v \sum_{k=0}^{\infty} \rho^k f_{\lambda}(k) \\
&= v - v \sum_{k=0}^{\infty} \frac{\rho^k \lambda^k e^{-\lambda}}{k!} \\
&= v - v \frac{e^{-\lambda}}{e^{-\rho\lambda}} \sum_{k=0}^{\infty} \frac{(\rho\lambda)^k e^{-\rho\lambda}}{k!} \\
&= v - v \cdot e^{-\lambda(1-\rho)} \\
&= v(1 - e^{-\frac{\lambda}{\alpha v}})
\end{aligned}$$

Where the second step follows from the pmf of the Poisson distribution. It is straightforward to show from this step that  $E[m(v, \lambda)|v]$  is strictly increasing in  $\lambda$ . Since a strictly increasing function is one-to-one,  $\{m(\cdot, \cdot), v\} \leftrightarrow \lambda$  is one-to-one.  $\square$

### Proof of Equilibrium Uniqueness

*Proof.* To show that there is a unique equilibrium to the dynamic oligopoly game, I need to show that players' best response curves intersect only once at every period in time. First note that since any taxi driver plays against the expected distribution of his competitors, and since all transitions are taken to be deterministic with respect to the taxi drivers' decision problem, this game can be viewed as a two-player game: the agent versus his competition. Further, all agents are atomistic, so that no single individual's action has any influence on competitors' strategies. Thus I only need to show that a drivers' best response function (i.e., which location to search in given the distribution of competitors) intersects his competitors' action (i.e., the aggregated policy function) at one point. To do this I will show that a taxi's best response function is strictly decreasing in the level of competition. Specifically, I want to show that for a taxi in location  $i$ , his best response curves (given by policy function  $\sigma_i(j_a|\mathcal{S}^t)$ ) are strictly decreasing in  $v_i^t$ , the payoff-relevant component of the state (recall  $\mathcal{S}^t$  also includes the in-transit status of all employed and vacant taxis who are not actively searching in period  $t$ ).

Recall that this policy function is given by:

$$\sigma_i(j_a|\mathcal{S}^t) = \frac{\exp(W_i^t(j_a, \mathcal{S}^t)/\sigma_\varepsilon)}{\sum_k \exp(W_i^t(j_k, \mathcal{S}^t)/\sigma_\varepsilon)}.$$

It is straightforward to see that  $\frac{\delta \sigma_i(j_a)}{\delta v_{ja}^t} < 0 \Leftrightarrow \frac{\delta W_i(j_a)}{\delta v_{ja}^t} < 0 \Leftrightarrow \frac{\delta V_{ja}}{\delta v_{ja}^t} < 0$ . I will therefore show that value functions are strictly decreasing functions of the state variable. To do this, begin at the last period,  $t = T$ . In this period, continuation values are all equal to zero, so that the only profit may be earned from successful search within period  $T$ . The value function for location  $i$  in the terminal period may be rewritten as:

$$V_i^T(\mathcal{S}) = \mathbb{E}_{p_i} [p_i(u_i^T, v_i^T) | \lambda_i^T] \left( \sum_j M_{ij}^T \cdot \Pi_{ij} \right).$$

The state of competition is summarized by the number of vacant cabs in location  $i$ ,  $v_i^T$ . In Appendix A.3, I prove that  $\mathbb{E}[m_i^t | v_i^t, \lambda_i^t] = v \cdot (1 - e^{-\frac{\lambda}{\alpha v}})$ . Thus,  $p_i(v | \lambda) = (1 - e^{-\frac{\lambda}{\alpha v}})$ , a strictly downward sloping function of  $v$ . Since the expected profits associated with a fare is independent of the state, this directly implies that  $V_i^T(\mathcal{S})$  is downward sloping in the relevant component of the state,  $v_i^T$ .

Now let's turn our attention to a generic period  $t$ . The goal is to show that  $V_i^t$  is decreasing in  $v_i^t$  as long as continuation values are also decreasing. Since this is true in period  $T$ , by induction this will prove that  $V_i^t$  is decreasing in  $v_i^t$  for all  $t$ . To proceed, suppose  $V_j^t(\mathcal{S})$  is decreasing in  $v_j^k$  for all  $j$  and for all  $k > t$ . For clarity I display equation (4) below:

$$\begin{aligned} V_i^t(\mathcal{S}) = \mathbb{E}_{p_i | \lambda_i, \mathcal{S}^t} \left[ p_i(u_i^t, v_i^t) \overbrace{\left( \sum_j M_{ij}^t \cdot (\Pi_{ij} + V_j^{t+\tau_{ij}}) \right)}^{\text{Exp. Value of Fare}} + \right. \\ \left. (1 - p_i(u_i^t, v_i^t)) \cdot \underbrace{\mathbb{E}_{\varepsilon_{a,j}^{t+1}} \left[ \max_{j \in A(i)} \left\{ V_j^{t+\tau_{ij}} + \mathbb{I}_{[j=i]} \gamma - c_{ij} + \varepsilon_{a,j} \right\} \right] \middle| \lambda_i^t}_{\text{Exp. Value of Vacancy}} \right]. \quad (15) \end{aligned}$$

Since we have assumed that  $V_j^{t+\tau_{ij}}(\mathcal{S})$  is decreasing, it is easy to see that if the expected

value of a fare is greater than the expected value of a vacancy over the next period, then any decrease in the probability of finding a fare will decrease  $V_i^t$ . The assumption that taxis can opt to not search in any location ensures this condition will always hold; taxis may always choose to exercise the option to remain vacant and thereby forgo the opportunity to profit in any period.<sup>51</sup> Thus an increase in  $v_i^t$  decreases the probability of matching,  $p_i$ , which thereby decreases  $V_i^t$ .

I showed above that value functions are decreasing in the level of competition. This directly implies that policy functions are decreasing functions of the competitor's state. Since competitors' policies are independent of an atomistic taxi's actions, best response curves will intersect only once. Therefore equilibrium is unique.

□

#### A.4 Estimation and Simulated Moments

I identify  $\sigma_\varepsilon$ ,  $\gamma$ , and  $\alpha$  by simulating individual taxi trip data and comparing simulation moments with their empirical counterparts. The moments are as follows: (1) Mean total travel time of employed cabs, (2) Mean total vacancy time, (3) Mean total distance of employed cabs, and (4) the probability that a driver's next match is in the same location as his most recent drop-off. Note that these moments will depend at least in part on these two parameters;  $\sigma_\varepsilon$  reflects how much of a taxi driver's location choice depends on observable features within the model. A high value of  $\sigma_\varepsilon$  should lead to behavior that appears random from the perspective of the model, including longer vacancy periods, whereas a low value implies that the model is capturing incentives well, and thus behavior should conform to the model's valuation of locations.  $\gamma$  reflects the value of saved time when drivers choose to stay in the same location next period as they ended up in last period. A low gamma means that drivers are, all else equal, indifferent between searching in the current location and adjacent ones. Thus, a lower probability of staying-put is expected. A high gamma likewise implies a higher probability of staying put. Table (13) displays each simulation moment compared with its observed value.

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<sup>51</sup>For clarity, this option value is not written. Under this assumption, value functions are more precisely written with an additional max operator in front, i.e.  $V_i^t(\mathcal{S}) = \max([\text{Exp. Value of Vacancy}], \tilde{V}_i^t(\mathcal{S}))$ , where  $\tilde{V}$  is given by equation (15), allowing drivers to choose vacancy with probability one instead of facing a probability of getting a ride. In simulations, taxis very rarely exercise this option.

Table 13: Data and Simulation Moments

<b>Moment</b>	<b>Data Average</b>	<b>Simulation Average</b>
Total Vacant Wait Time (min.)	83.56	126.06
Total Vacant Distance Travelled (mi.)	39.71	34.97
Total Employed Time (min.)	203.35	34.41
Pr(pickup in $i$   drop-off in $i$ )	.4483	.4386

This table summarizes the data and simulation moments used to estimate remaining model parameters.

## A.5 Aggregated Estimation Results

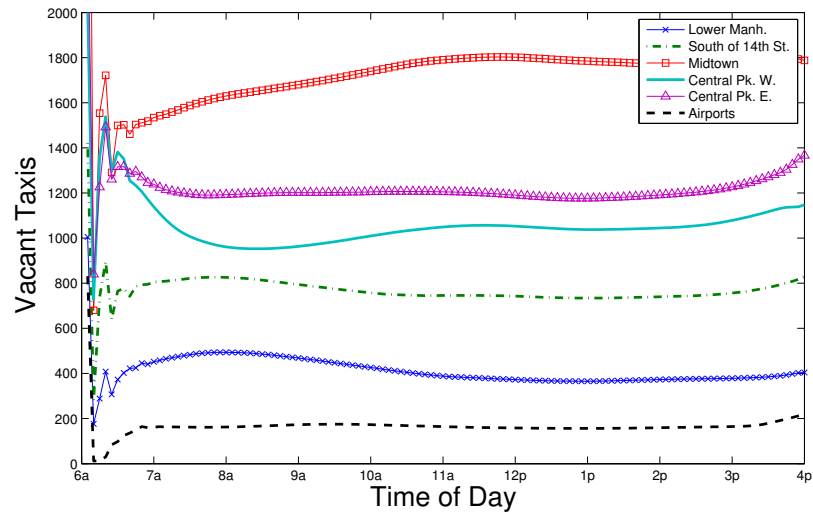
I present results that aggregate results over all 48 locations to six locations.<sup>52</sup> For a map of corresponding locations, see Figure (21) below.

The results above demonstrate that the while taxi supply maintains some coverage across all locations throughout the day, there are intra-day trends in spatial availability. The first-period drop in supply reflects the beginning of the model, as all taxis are modeled as vacant in the initial period. Spatial mismatch is immediately evident, in that Midtown service very thin while the Uptown areas (Central Park East and West) are highly over-supplied relative to demand. One contributing factor to this relative oversupply is that mean fares are about 15% higher in these areas than in Midtown.

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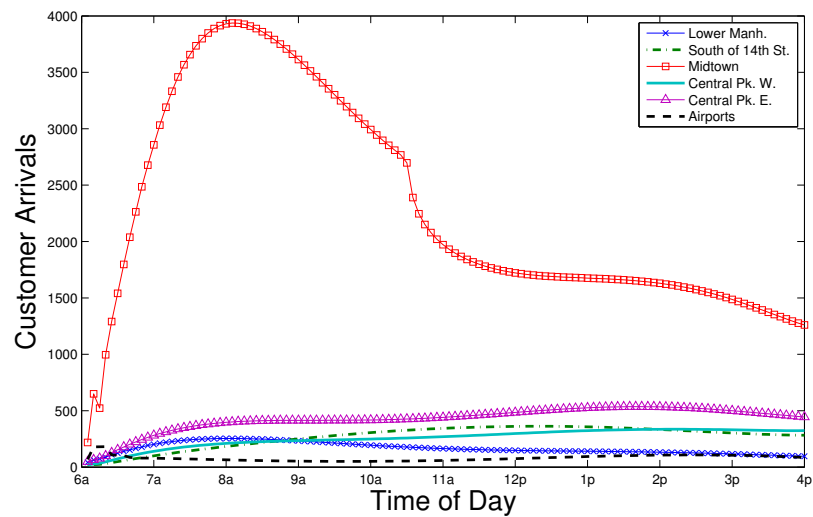
<sup>52</sup>Detailed, disaggregated results for all 5,760 demand estimates and the corresponding distribution of taxis are available upon request.

Figure 18: Equilibrium Vacant Taxis: Weekdays 6a-4p, 9/2012 (Six-Loc. Aggregates)



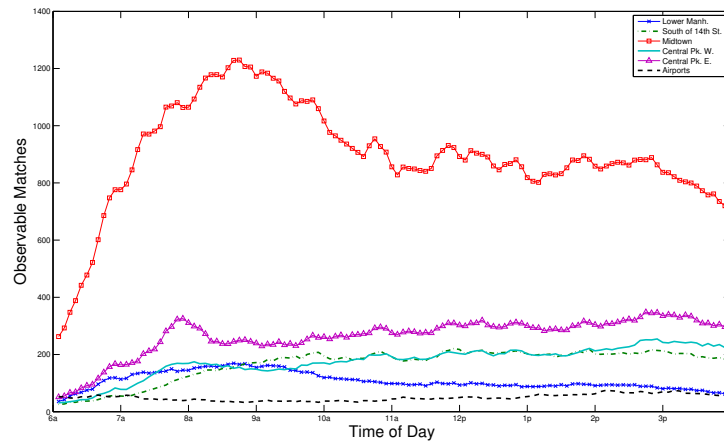
This figure depicts the equilibrium spatial distribution of taxis. Results across 48 locations are aggregated to six locations. Results are depicted for the weekday taxi drivers' day shift, from 6a-4p, under demand estimated over the month of September, 2011.

Figure 19:  $\lambda$  estimates: Weekdays 6a-4p, 9/2012 (Six-Loc. Aggregates)



This figure depicts the estimated arrival rates of demand for taxi service. Results across 48 locations are aggregated to six locations. Estimates shown are for the month of September, 2011.

Figure 20: Empirical Matches: Weekdays 6a-4p, 9/2012 (Six-Loc. Aggregates)



This figure depicts the empirical distribution of matches during September, 2011, where each of the 48 locations are aggregated to six locations. Demand estimates are identified off of these data given the taxis' spatial equilibrium.

Figure 21: Reference Map for Six-Location Results (Note: JFK not shown)

