Optimal Income Taxation with Endogenous Prices*

Alexey Kushnir Robertas Zubrickas

February, 2018

Abstract

We consider a Mirrleesian model of optimal income taxation with endogenous product prices. Given endogenous prices, any redistribution of income in the economy affects social welfare not only directly, but also through its influence on the level of product prices. To correct for this price externality, the optimal income tax schedule includes a new Pigouvian term. For competitive markets with increasing market supply, the Pigouvian term is positive for normal goods, negative for inferior goods, increasing for luxury goods, and decreasing for necessity goods. Using a calibrated model of the U.S. housing market, we quantify the price effect showing that it increases optimal marginal income tax by 4-5% for most income levels. We also analyze the Pigouvian term for oligopolistic markets, where the price effect on optimal income taxation persists even with the introduction of commodity and profit taxation. Our simulations of the U.S. housing market also show that optimal marginal income tax should be lower for more concentrated markets.

Keywords: Optimal income taxation, endogenous pricing, externality, competitive markets, oligopolistic markets, housing market.

JEL Classification: H21, H23, D43.

*Kushnir: Carnegie Mellon University, Tepper School of Business, Pittsburgh, USA, akushnir@andrew.cmu.edu; Zubrickas: University of Bath, Department of Economics, Bath, UK, r.zubrickas@bath.ac.uk. We are grateful to Simon Board, Craig Brett, David Childers, Emanuel Hansen, Louis Kaplow, Dirk Niepelt, Nicola Pavoni, Markus Reisinger, Florian Scheuer, Chris Sleet, Ron Siegel, Ali Shourideh, Dennis Epple, and Luca Rigotti and other people who generously provided feedback and offered insights at various stages of this project. Thanks to their precious help the paper has improved immensely.

1 Introduction

In the taxation literature, product markets are commonly assumed to be perfectly competitive (see Mirrlees, 2010). While this assumption is convenient for analysis as it implies fixed product prices, it is far from an accurate description of most markets.[1] According to recent studies (e.g., Azar et al., 2017; Grullon et. al., 2017), one third of U.S. industries are highly concentrated and over 75% of U.S. industries are now operating in markets that are more concentrated than 20 years ago.[2] Similar market structures are observed in Europe, where the market share of the top three food retailers ranges from 30 to 50% for most countries (Europe, 2012).

What are the implications for optimal income taxation if markets are not perfectly competitive? What is the size of the optimal income tax change due to price variability? The goal of this paper is to answer these questions and, thus, to bridge a gap in the public economics literature by incorporating endogenous product prices into the analysis of optimal income taxation.

We consider the standard Mirrlees (1971) framework in the presence of imperfectly competitive product markets.[3] There is a continuum of agents who care about the consumption of two goods: a numeraire good, which is produced with a constant returns to scale technology and perfectly elastic supply function, and a "main" good, which is produced with a decreasing returns to scale technology and strictly increasing supply function. As a consequence, the price of the numeraire good remains constant and the price of the main good is endogenously determined by the intersection of market supply and market demand—the market equilibrium condition. The objective of the public authority is to design an optimal income tax schedule that maximizes the sum of total agent utilities and weighted firm profits subject to three constraints: *the resource constraint,* which demands that the public authority raises a fixed level of public funds; *the incentive compatibility constraint,* which requires agents to reveal their productivity types; and *the market equilibrium condition,* which determines the price level.

Any change in income distribution creates an externality through its effect on market demand and, hence, the equilibrium price level. Thus, when determining optimal marginal income taxes, we obtain an additional term—referred to as the Pigouvian term—the role of which is to correct for the price externality. When the firm profit weight is smaller than the marginal social costs of raising public funds, we show that the Pigouvioan term is positive for normal

[1] See Atkinson (2012, p. 775) for an extensive discussion of how the existing taxation literature fails to take into account the underlying market structure and endogenous prices. Important exceptions are Myles (1987, 1989) and Auerbach and Hines (2001) who study *commodity taxation* in imperfectly competitive markets.

[2] See also articles in popular press by Theo Francis and Ryan Knutson, 2015, "Wave of Megadeals Tests Antitrust Limits in U.S." *The Wall Street Journal*, October 18 and The Editorial Board, 2015, "How Mergers Damage the Economy" *The New York Times*, October 31.

[3] We call markets *perfectly competitive* if firms are price takers and market supply is perfectly elastic. Otherwise, we call markets imperfectly competitive. They include *competitive* markets, where firms are price takers and aggregate supply has a positive slope, and *oligopolistic* markets, where each firm takes into account its influence on the equilibrium price.

goods and negative for inferior goods. Intuitively, when the main good is normal, the public authority wants to discourage agents from working too hard. Though this policy leads to a lower income level it also reduces the price of the main good. In addition, the Pigouvian term is increasing for luxury goods and decreasing for necessity goods. More strikingly, the presence of the Pigouvian term implies that lump sum taxation is not the first best, i.e., it is not optimal when agents' productivity is perfectly observable. Another implication of endogenous prices is that the famous end-point result of zero marginal tax for the highest income bracket ceases to hold (see Sadka, 1976 and Seade, 1977).

To estimate the price effect on optimal income tax rates we use the U.S. housing market as an example. This market is particularly suitable for this purpose because housing costs account for almost a third of average household expenditures in the U.S. Using a calibrated model of the U.S. housing market due to Albouy et al. (2016), we estimate the optimal income tax schedule in the presence of endogenous prices. We find that the price effect leads to $4-5\%$ increase in optimal marginal income tax at most income levels. The change in optimal marginal income tax is also decreasing in income, which is consistent with housing being a necessity good. Compared to an equilibrium without the price effect, agents also provide less effort, which is in line with our theoretical predictions.

We also extend our results to markets with various forms of oligopolistic competition. In markets where firms are engaged in imperfect competition, two opposite effects—the price externality effect and the anticompetitive effect—influence the optimal income tax schedule. Under the price externality effect, as in competitive markets, higher income leads to greater market demand and, therefore, higher prices for normal goods. Hence, the public authority wants to increase optimal income tax to compensate for the price externality. The anticompetitive effect arises because of inefficiently small quantities of goods traded in oligopolistic market equilibrium. To offset this inefficiency and to increase the quantities of goods traded, the public authority wants to stimulate labor income by decreasing the marginal income tax level. Decreasing marginal income tax has an effect similar to a corrective subsidy in the case of commodity taxation and oligopolistic markets (see Auerbach and Hines, 2001). For the U.S. housing example, we show that both effects are increasing in absolute value with additional market power. The anticompetitive effect, however, dominates the price externality effect leading to lower optimal marginal income taxes for more concentrated markets.

We also study whether commodity and profit taxation can alleviate the price externality and the anticompetitive effect of oligopolistic markets. In competitive markets (where the anticompetitive effect is not present), the optimal commodity tax exactly offsets the price externality, thereby eliminating the Pigouvian term in the optimal marginal tax formula. This is in contrast to Atkinson and Stiglitz (1976) who showed that commodity taxation in not needed in the presence of the optimal income tax schedule. Their result, however, holds only when firm profits are fully taxed, which is contrary to what we assume in our main model. If we consider 100% profit taxation, the Pigouvian term in the optimal marginal tax formula vanishes, which is in line with the production efficiency theorem of Diamond and Mirrlees (1971) stating that agents' productive efforts should not be distorted in the presence of the optimal income tax code. In oligopolistic markets (where the anticompetitive effect is present), however, we show that the Pigouvian term persists despite commodity taxation or 100% profit taxation. This result is parallel to Auerbach and Hines (2001) who show that it is too costly for the public authority to fully eliminate the price externality using commodity taxation in imperfectly competitive markets.

We now briefly outline the main contribution of the paper to the literature. The detailed literature review is postponed until Section 6. Compared to the growing literature on optimal income taxation with endogenous wages in labor markets (e.g., Rothschild and Scheuer, 2013; Sachs, Tsynvisky, and Werquin, 2016), we consider endogenous prices in product markets. In contrast to endogenous wages that are agent- or occupation-specific (see, e.g., Ales, Kurnaz, and Sleet, 2015), product prices studied in this paper are the same for all agents in the economy. Although agents generally cannot work simultaneously for several sectors of the economy (as assumed in the occupational choice papers), agents can consume several products. Hence, one could think about externalities studied in this paper as "vertical" because they affect all agents in the economy. The externalities in the endogenous wages literature are "horizontal" because they are different across agents or occupations.[4] In competitive markets, both wages and product prices are determined by the marginal product of the production function in equilibrium. The previous literature, however, bundles all consumption goods in one product and normalizes product prices to one (see, e.g., Scheuer and Werning, 2017). In contrast, we normalize wages to one and consider two levels of product prices in the economy.

Further, the previous papers assume that each agent cannot influence wage schedule, i.e. agents are price-takers. This assumption is a reasonable approximation for labor markets. In contrast, many product markets are oligopolistic. Hence, there is no counterpart to our analysis of oligopolistic markets in the literature on optimal income taxation. However, the literature does include a few important studies analyzing *commodity taxation* in imperfectly competitive markets (Myles 1987, 1989; Auerbach and Hines, 2001). Our paper can be viewed as one extednding their analysis to optimal income taxation settings.

In addition, most public economics studies that include analysis of labor markets assume a constant return to scale production technology (see, e.g., Sachs, Tsyvinsky, and Werquin, 2016).[5] In contrast, our results rely on a non-constant return to scale technology to allow for

[4]Rothschild and Stiglitz (2016) allow agents to engage in two activities simultaneously, where one activity is traditional and the other one is rent-seeking. The externality in rent-seeking activity is still agent specific. Rothschild and Stiglitz (2014) generalize their setting to several activities.

[5]Some notable exceptions are Scheuer and Werning (2016, 2017) and Rothschild and Scheuer (2014).

an upward-sloping aggregate supply and market equilibrium effects. Even if the production function has increasing or decreasing returns to scale, a common assumption in the literature is that firm profits belongs to the public authority or are taxed at 100%. In this case, production technology type is a factor in determining optimal income taxation schedule. In contrast, our analysis regards firm profits as a part of the objective function.[6]

The remainder of the paper is organized as follows. Section 2 introduces the model. In Section 3, we consider competitive markets, analyze properties of the optimal marginal income tax schedule, and provide simulation results estimating the size of income tax change due to price variability. Section 4 extends our analysis to oligopolistic markets. We study commodity and profit taxation in Section 5. Section 6 provides a detailed literature review and Section 7 concludes. The omitted proofs are postponed to Appendix A. Appendix B contains a labor market that supports our main model. Appendix C contains additional simulation results.

2 Model

In the economy, there is a continuum of agents indexed by their productivity type n that is distributed according to probability density function $f(n) > 0$ with support $[\underline{n}, \overline{n}]$. Agent n's labor income is given by $z = n\ell$, where ℓ is the number of hours worked. The labor cost is represented by an increasing and convex function $c(\ell)$.

An agent's disposable income after tax $T(z)$ equals $y = z - T(z)$, which is spent on the consumption of two goods: a numeraire good and good X (refered to as the main good in the introduction). The numeraire good is produced with a homogeneous of degree one technology and traded in a competitive market that results in a fixed price normalized to 1 and zero firm profits. Good X is produced with a decreasing returns to scale technology yielding positive profits. Let p and $\Pi(p)$ denote the price and the profit of firms producing good X.[7] We assume that these profits are spent solely on the consumption of the numeraire good.

All agents have identical preferences for consumption represented by an indirect utility function $v(p, y)$, which is concave and increasing in y. Agent's net utility is defined by

$$U(p, y, \ell) = v(p, y) - c(\ell).$$

The social welfare function is given by the aggregation of agents' net utilities together with the

[6] Ales and Sleet (2016) consider a public authority that can tax firm profits only at level below 100%. The main results of Scheuer and Werning (2017) also hold when firm profits are not fully taxed.

[7] We do not model explicitly why firms that produce the numeraire good do not switch to a more profitable production of good X. One could think about firms having a lack of technology or patents or facing other barriers to entry. The high degree of persistent performance differences even among similar firms is a well-documented phenomenon (see, for example, Syverson (2011) for a recent survey).

firm profits weighted by $\omega \geq 0$

$$W = \int H(U(p, y(n), \ell(n)))f(n)dn + \omega\Pi(p),$$

where H is an increasing and concave function. The public authority wants to maximize the social welfare function W subject to three constraints. The first one is resource constraint

$$\int T(z(n))f(n)dn = \int (n\ell(n) - y(n))\, f(n)dn \geq R, \tag{1}$$

which ensures that the public authority covers its own expenses $R \geq 0$ that are spent solely on the numeraire good. The second one is incentive compatibility constraint

$$U\left(p, n\ell(n) - T(n\ell(n)), \ell(n)\right) \geq U\left(p, m\ell(m) - T(m\ell(m)), m\ell(m)/n\right) \tag{2}$$

for all $n, m \in [\underline{n}, \overline{n}]$. Constraint (2) ensures that an agent with productivity n does not want to seek the income of an agent with different productivity.

The third constraint is a *market equilibrium condition* that determines price p. This condition varies across market structures. In *competitive* markets (Section 3), where we assume that firms are price-takers and market supply has a positive slope, the market equilibrium condition requires the market supply equal to the market demand for good X. In *oligopolistic markets* (Section 4), where each firm takes into account its influence on the level of product price, the market equilibrium condition is determined by the firm profit maximization condition.

Overall, the only difference between our framework and the model of Mirrlees (1971) is that we do not assume fixed prices in the economy. Instead, the price of one of the goods is endogenously determined by the market equilibrium condition. The effect of this condition on optimal income taxation is the main subject of our subsequent analysis.

3 Competitive Market

In this section, we analyze the problem of optimal income taxation in competitive markets, where the price of good X is determined by the following market equilibrium condition

$$S(p) = \int x(p, y(n))f(n)dn. \tag{3}$$

On the left-hand side we have market supply $S(p)$ and on the right side the market demand for good X, where $x(p, y)$ is the Walrasian demand function of an agent with disposable income y. We determine the Walrasian demand function by using the Roy's identity $x(p, y) = -v_p(p, y)/v_y(p, y)$. We consider a non-decreasing supply function $S'(p) \geq 0$ and zero fixed costs so that firm profits coincide with total producers' surplus $\Pi(p) = \int_0^p S(\tilde{p})d\tilde{p}$. We also assume that the demand for good X satisfies the law of demand $x_p < 0$ and has the same curvature with respect to income, either $x_{yy} \geq 0$ or $x_{yy} < 0$, for any income level. In Appendix B, we show how our model can be supported with a labor market where agents supply their effort for a fixed wage and the firms spend its profits solely on the numeraire good. We also explain why condition (3) ensures that the product and labor markets clear in the economy.

To highlight the main ideas we start analyzing complete information case with observable agent productivity types. We then proceed to the analysis of the incomplete information case.

3.1 Complete Information

In this subsection, we assume that agent productivity types are observable. Hence, the public authority can tailor taxes to each individual, i.e., $T(z, n)$. Observing that $T(z, n) = n\ell(n) - y(n)$, the public authority's problem is to find price p, the distribution of disposable income $y(n)$, and the distribution of individual labor supply $\ell(n)$ to maximize[8]

$$\max_{p,\, y(n),\, \ell(n)} \int H(U(p, y(n), \ell(n)))f(n)dn + \omega\Pi(p) \text{ subject to (1) and (3).}$$

The Lagrangian of the public authority's problem is given by

$$\mathcal{L} = \int (H(U(p, y(n), \ell(n))) + \omega\Pi(p) + \lambda(n\ell(n) - y(n) - R) + \gamma(S(p) - x(p, y(n))))f(n)dn,$$

where λ and γ are multipliers corresponding to constraints (1) and (3), respectively. The first-order conditions are equal then

$$y(n) : H'v_y - \lambda - \gamma x_y = 0, \tag{4}$$

$$\ell(n) : -H'c_\ell + \lambda n = 0, \tag{5}$$

$$p : \int (H'v_p + \omega\Pi'(p) + \gamma(S'(p) - x_p))f(n)dn = 0. \tag{6}$$

The optimal marginal income tax $t(z, n) = T_z(z, n)$ is found then from the individual utility maximization problem $\max_z U(p, z - T(z, n), z/n)$ that implies $t = 1 - c_\ell/(nv_y)$. Using the first-order conditions (4) and (5) we obtain the following result.

Theorem 1. *In competitive markets with endogenous prices and complete information, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{\gamma}{\lambda}x_y. \tag{7}$$

[8]We maximize over p becuase the price is determined implicitly by equation (3). If we had an explicit function how price p depends on fundamentals we could use it to substitute into indirect utility and maximize only over $(y(n), l(n))$.

Let us understand when the optimal marginal income tax is different from zero. First, we notice that when resource constraint (1) is binding we have $\lambda > 0$. Then, taking into account Roy's identity $x = -v_p/v_y$ and $\Pi'(p) = S(p)$, conditions (4) and (6) imply

$$\gamma = \frac{S(p)(\lambda - \omega)}{S'(p) - \int (x_p + x_y x) f dn}. \tag{8}$$

Therefore, if the profit weight differs from the marginal social costs of raising public funds ($\omega \neq \lambda$) the marginal income tax is different from zero, i.e., lump-sum taxation is not optimal.

Corollary 1. *In competitive markets with endogenous prices and complete information, lump-sum taxation is not optimal if the firm profit weight is different from marginal social costs of raising public funds.*

Corrolary 1 states that the effort-distorting taxation is superior to lump-sum taxation. Intuitively, the distribution of income affects social welfare not only directly, but also through its influence on the level of product prices. Thus, the public authority finds it optimal to correct for the price externality by distorting agent's first-best effort level.

The externality interpretation of the effort-distorting taxation can be formalized by linking the Pigouvian term $\gamma x_y/\lambda$ to the equilibrium price change due to local change in income distribution. For this purpose, consider a small increase in income from $y(n')$ to $y(n') + \phi$ for all types $n' \in [n - \delta/2, n + \delta/2]$ for some n. Using the Implicit Function Theorem the market equilibrium condition (3) implies

$$\frac{dp}{d\phi} = \frac{x_y(p, y(n)) f(n) \delta}{S'(p) - \int x_p(p, y(n)) f(n) dn}. \tag{9}$$

Since market supply has a positive slope $S'(p) > 0$ and individual demand has a negative slope $x_p < 0$ we obtain that $dp/d\phi \sim x_y(p, y(n))$. At the same time, equation (8) implies that multiplier γ is positive when $\omega < \lambda$. To see this, note that the Slutsky equation implies that $x_p + x_y x = h_p$, where h is the compensated (Hicksian) demand function. Then, the law of compensated demand $h_p \leq 0$ together with $S'(p) > 0$ implies $\gamma > 0$. Taking into account that $\lambda > 0$ we also obtain that the Pigouvian term proportional to $x_y(p, y(n))$. Overall, we obtain the following result.

Proposition 1. *In competitive markets with endogenous prices when the social weight on firm profits is smaller than the marginal social costs of raising public funds ($\omega < \lambda$), the Pigouvian term is proportional to the price change resulting from a local change in income distribution.*

We also observe that when $\omega < \lambda$, the sign and the monotonicity properties of the Pigouvian term are determined by x_y. Hence, we obtain a positive Pigouvian term for normal goods and negative for inferior goods. Furthermore, our assumption that the demand has the same curvature with respect to income (either $x_{yy} \geq 0$ or $x_{yy} < 0$ for any income level) also implies that x_y is increasing for luxury goods and decreasing for necessity goods (see the proof of Proposition 2).

Proposition 2. *In competitive markets with endogenous prices when the social weight on firm profits is smaller than the marginal social costs of raising public funds ($\omega < \lambda$), the Pigouvian term is positive for normal goods and negative for inferior goods, increasing for luxury goods and decreasing for necessity goods.*

3.2 Incomplete Information

In this subsection, we consider agents' productivity being their private information. In this case, the public authority cannot condition tax schedule on agent type. Observing that $T(z) = n\ell(n) - y(n)$, the public authority's problem is now to find price p, the distribution of disposable income $y(n)$, and the distribution of individual labor supply $\ell(n)$ to maximize

$$\max_{p,\, y(n),\, \ell(n)} \int H(U(p, y(n), \ell(n)))f(n)dn + \omega\Pi(p) \text{ subject to (1), (2), and (3).}$$

Let $\mu(n)$ denote the multiplier on the incentive-compatibility constraint, E^c the elasticity of compensated labor supply, and E^u the elasticity of uncompensated labor supply (see the proof of Theorem 2 for details). Using the derivations similar to the complete information case and the standard Mirrleesian model, we obtain

Theorem 2. *In competitive markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{1+E^u}{E^c}\frac{v_y\mu}{\lambda n f} + \frac{\gamma}{\lambda}x_y. \tag{10}$$

The optimal income tax formula (10) has two terms. The first one is the standard Mirrleesian term that balances work incentives with the public authority's redistributive and budgetary objectives. The second term is the Pigouvian term that corrects for the price externality, as we discussed in the complete information case.

The immediate implication of Theorem 2 is that the seminal end-point results of Sadka (1976) and Seade (1977) no longer hold with price endogeneity. In the standard model with fixed prices the optimal tax formula has only the incentive term and the transversality condition implies that the optimal marginal tax for the most productive agents is zero. Intuitively, if the marginal tax were positive, the public authority could instead impose zero tax on any income in excess of the current income of the most productive agents. Then, these agents would respond by exerting an additional effort to achieve the previously not feasible level of utility.

Since the amount of tax collected would not change while the most productive agents would obtain a higher level of utility, the overall welfare would increase. With endogenous prices, however, the aforementioned tax relief would change the income distribution in the economy with implications for product prices. Therefore, the public authority cannot increase the utility of the most productive agents without influencing the rest of the agents.

3.3 Price Effect Estimate

To estimate the size of the effect of endogenous prices on optimal income taxation, we turn to the U.S. housing market. This market suits particularly well to quantify the results obtained in the previous section because housing expenditures account for a significant part of the total household expenditures. In particularly, Davis and Ortalo-Magné (2011) estimates that housing expenditures account for 37% of total expenditures at the bottom and for 17% of total expenditures at the top quartiles of the U.S. income distribution. Consumer Expendiure Survey (2015) also reports that the average share of U.S. household expenditure on housing equals to 32.9%.

To model the demand side of the housing market, we use the model of Albouy et al. (2016) who estimate a non-homothetic constant elasticity of substitution (NH-CES) utility function for the U.S. housing market. If we denote the consumption of housing by x and of other goods by g, the NH-CES utility function can be written as

$$u(x,g) = \left(\frac{\eta x^{\frac{\sigma-1}{\sigma}} + \theta_1}{\theta_2 - (1-\eta) g^{\frac{\sigma-1}{\sigma}}} \right)^{\frac{\sigma}{\beta(\sigma-1)}}, \tag{11}$$

where σ is a substitution parameter, β is a non-homotheticity parameter, η is a distribution parameter, $\theta_1 = \frac{1-\sigma-\beta\eta}{\beta\sigma}$, and $\theta_2 = \frac{1-\sigma-\beta(\eta-1)}{\beta\sigma}$.[9] This function becomes a standard CES function (Arrow et al., 1961) when $\beta \to 0$ and Cobb-Douglas when $\sigma \to 1$. Albouy et al. (2016) calibrate the model and show that this utility function fits well the patterns of housing consumption in the U.S. by passing tests imposed by rationality and household mobility. The parameters that fit the data are $\sigma = 2/3$, $\eta = 1/8$, $\beta = 4/3$, which yields the following utility specification[10]

$$u(x,g) = \left(\frac{27 - 14 g^{-1/2}}{2x^{-1/2} + 3} \right)^{3/2}. \tag{12}$$

Following Kanbur and Tuomala (2013) (see also Saez et al. (2012)), we assume that the labor supply elasticity is equal to 1/3, which implies the cost function $c(\ell) = \ell^4/4$.

Regarding the supply side of the housing market, we consider the standard constant price

[9] We omit household size and neutral shifts in quality of life parameters as in Albouy et al. (2016).

[10] Albouy et al (2016) mentioned to us in private communication they are working on updating these estimates. As soon as we obtain their new estimates, we update our results as well.

elasticity function $S(p) = sp^{\varepsilon}$, where s is a scale parameter and ε is the price elasticity of supply.[11] We calibrate scale parameter $s = 0.0021$ to match the average share of U.S. household expenditure on housing equal to 32.9%. The estimates of the price elasticity of supply ε vary significantly across countries and even across cities. In particular, Saiz (2010) show that ε highly depends on geographical and regulatory constraints within U.S. metropolitan areas. Following his estimates for the average U.S. metropolitan area, we consider $\varepsilon = 1.75$. In Appendix C, we also present the results for inelastic supply $\varepsilon = 0$ that better describes the housing supply in major UK cities,[12] and the results for a quite elastic supply $\varepsilon = 3$ that is closer to the estimates used in Green et al. (2005) and Epple and Romer (1991).

For the distribution of agent abilities $F(n)$, we consider the lognormal function $\ln(m, \sigma)$ that matches well the empirical pattern of U.S. income. Following Kanbur and Tuomala (2013), we consider mean $m = e^{-1}$ and standard deviation $\sigma = 0.7$. In Appendix C, we also present results for the Champernowne distribution that fits better an upper tail of income distribution. Finally, we set the level of public expenditures at $R = 0$, the weight on producer surplus at $\omega = 0$, and assume that social welfare function is linear $H(u) = u$.

Figure 1 presents our main simulation results. The top left figure shows the optimal marginal tax for the economy with endogenous prices (solid line) and for the economy with fixed prices (dashed line). For the latter economy, we take the price of housing equal to the equilibrium price of housing in the economy with endogenous prices. The simulations show that the change in marginal tax $t_{endogeneous} - t_{fixed}$ (the top right figure) is positive for all income levels. Since housing is a normal good this confirms our theoretical predictions of Proposition 2. The change in marginal tax rate also decreases in income that corresponds to housing being a necessity good. Compared to the economy with fixed prices, agents also supply less labor (the middle right figure), which is also in line with our theoretical predictions. The existence of price externalities prompts the public authority to impose higher marginal taxes to subdue the negative effect of higher market demand on housing prices. While agents pay smaller absolute taxes (the middle left figure), the total change in disposable income and agent utility are still negative for most income levels (the low left and right figures).

4 Oligopolistic Competition

In this section, we relax the assumption that firms are price-takers and consider instead markets with a varying degree of oligopolistic competition. In particular, we consider $M \geq 1$ firms that have the same convex cost function $K(X_i)$ of producing X_i units of good X. Let us denote the inverse aggregate demand function by $p(X)$, where $X = \int x(p, y(n))f(n)dn$.[13] We can

[11] This functional form corresponds to a Cobb-Douglas production function (see, e.g., Epple et al. (2010)).
[12] See "How to solve Britain's housing crisis, " *The Economist*, 2017, August 5.
[13] The assumption $x_p < 0$ ensures that the inverse aggregate demand function $p(X)$ is well defined.

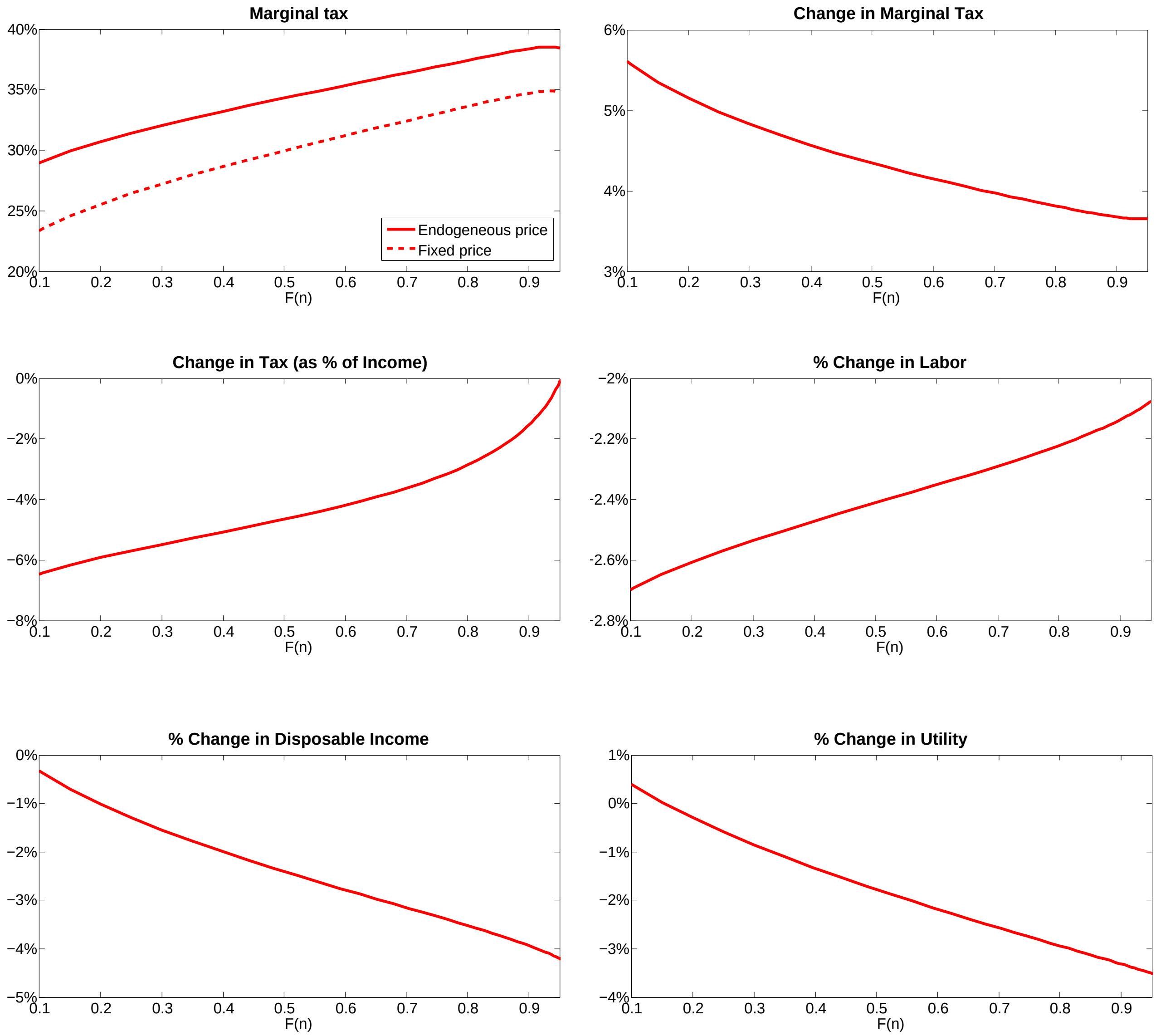

Figure 1. The graphs show the optimal marginal taxes, change in the optimal marginal tax, % change in labor supply, and % change in agent utility between the optimal tax regimes in economies with endogeneous and fixed price.

then write firm i's profit as $X_i p(X) - K(X_i)$, where the market clearing condition ensures $\sum_{i=1}^{m} X_i = X$. When firm i maximizes its profits it forms a belief, or a conjectural variation, about the other firms response to the unit change in its output level

$$\frac{d(\sum_{j\neq i} X_j)}{dX_i} = \theta, \text{ where } -1 \leq \theta \leq M - 1. \tag{13}$$

The first order condition for profit maximization can be expressed by

$$p(X) - K'(X_i) + (1+\theta)X_i\ p'(X) = 0. \tag{14}$$

The conjectural variation model was introduced by Bowley (1924), and it captures a wide variety of types of oligopolistic competition. For instance, competitive equilibrium corresponds to $\theta = -1$ when firms expect the rest of industry to absorb exactly its output expansion, conjectural variation $\theta = 0$ represents the Cournot-Nash model when each firm considers the output of the other firms unchanged, and the collusive behavior of firms maximizing their joint profits leads to $\theta = M - 1$.[14]

For subsequent analysis, it is convenient to express the market equilibrium condition in terms of market price rather than quantity. In what follows, we also limit attention to symmetric equilibria $X_i = X/M$ and zero weight on firm profits $\omega = 0$. Hence, the market equilibrium condition reduces to

$$J(X, X_p, p) \equiv p - K'\left(\tfrac{X}{M}\right) + (1+\theta)\frac{X}{MX_p} = 0, \tag{15}$$

where $X_p = \int x_p(p, y(n))f(n)dn$. The next theorem establishes the formula of the optimal marginal income tax for oligopolistic markets. Its derivation is similar to that of Theorem 2 and, therefore, relegated to the appendix.

Theorem 3. *In oligopolistic markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = \frac{1+E^u}{E^c}\frac{v_y\mu}{\lambda n f} + \frac{\gamma}{\lambda}(-J_1 x_y - J_2 x_{py}), \tag{16}$$

where J_i is the partial derivative with respect to ith argument.

Similar to Theorem 2, the tax formula consists of the standard Mirrleesian term and the Pigouvian term. However, the Pigouvian term has now two terms reflecting the fact that the equilibrium price under oligopolistic competition depends not only on market demand but also on its slope. Note that in a competitive market when $\theta = -1$ the second part of the Pigouvian term disappears rendering the tax formula as in (10).[15] Similarly, the result of Corollary 1 that lump sum taxation is not first best extend to oligopolistic markets.

As for competitive case, we can now demonstrate that the absolute value of the Pigouvian term remains proportional to the price externality resulting from a change in income distribution. Specifically, consider a small local increase in income level from $y(n')$ to $y(n') + \phi$ for all types $n' \in [n - \delta/2, n + \delta/2]$ for some n. The market equilibrium condition (15) then implies

$$\left|\frac{dp}{d\phi}\right| = \left|\frac{(J_1 x_y + J_2 x_{py})f(n)\delta}{dJ/dp}\right|, \tag{17}$$

[14] For an excellent overview of various conjectural variational parameters see Perry (1982).

[15] Note that $-J_1$ does not depend on n and, thus, can be included in Lagrange multiplier γ.

where $dJ/dp = J_1 X_p + J_2 X_{pp} + J_3$ and $X_{pp} = \int x_{pp}(p, y(n)) f(n) dn$. Similar to Proposition 1, we establish the following result for the absolute value of the Pigouvian term.

Proposition 3. *In oligopolistic markets with endogenous prices, the absolute value of the Pigouvian term is proportional to the price change resulting from a local change in income distribution.*

To understand the sign of the Pigouvian term we need to determine the sign of multiplier γ, which we do in Proposition A1 in Appendix A. To explain its result, let us consider a situation where the public authority can propose a small unit rebate to consumers of good X. If the equilibrium price increase in response to the rebate is smaller than the rebate itself we call this situation "no tax overshifting" (see Seade (1985) and Myles (1987)). Proposition A1 shows that with no tax overshifting, the sign of the Pigouvian term coincides with the sign of the price change resulting from a local change in income distribution.

Price Effect Estimate: Oligopolistic Competition

We now turn to the numerical simulations of U.S. housing market for oligopolistic markets. In particular, we use again the housing model of Albouy et al. (2016) to analyze the optimal marginal income tax schedule for two values of conjectural variation parameter corresponding to competitive and Cournot-Nash models. Though we present our results below only for two values of θ, our simulations confirm that they hold for any $\theta \in [-1, 0]$.[16]

Let us consider an economy with $M = 2$ firms and two forms of oligopolistic competition: competitive market with $\theta = -1$ and Cournot-Nash model with $\theta = 0$. In competitive markets the firm maximization problem reduces to $p - K'(\frac{X}{M}) = 0$, where X equals market supply $S(p)$ in market equilibrium. To make sure that our analysis reduces to competitive market sturcture of Section 3 where market supply $S(p) = sp^{\varepsilon}$ with $s = 0.0021$ and $\varepsilon = 1.75$, we assume that firm's marginal costs satisfy

$$K'(X_i) = \left(\frac{X_i M}{s}\right)^{\frac{1}{\varepsilon}}.$$

For our simulations, we again consider Albouy et al. (2016) utility (11) calibarated for the U.S. housing market. In Appendix C, we show how one obtains indirect utility $v(p, y)$, demand $x(p, y)$, and demand slope $x_p(p, y)$ for this utility specification.

Figure 2 presents our main simulation results. The left figure shows the optimal marginal income tax schedules for competitive market $\theta = -1$ with endogeneous prices (thick solid red line) and fixed price (thick dashed red line) and for Cournot-Nash model $\theta = 0$ with endogeneous price (thin solid blue line) and fixed price (thin dashed blue line). As for previous

[16] Our simulations break down for θ close to $M - 1$ reflecting to the fact the profit maximization problem of monopoly is not well defined for Albouy et al. (2016) utility specification.

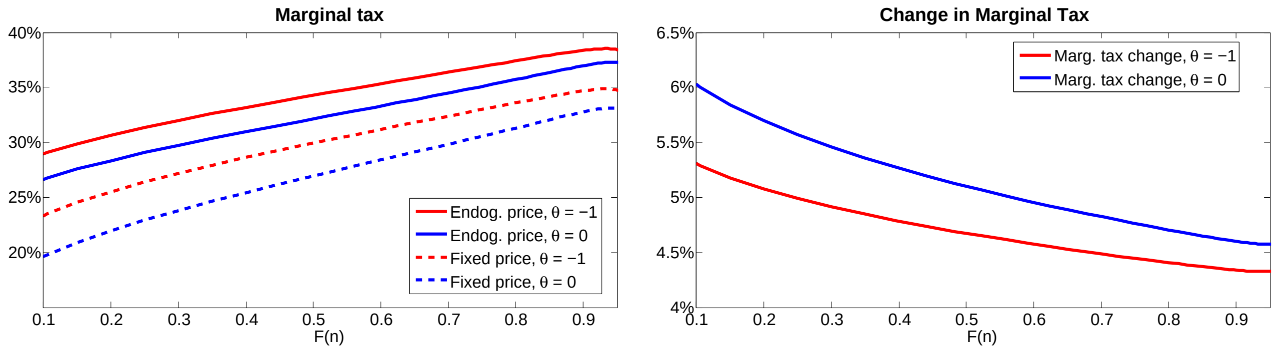

Figure 2. The graphs illustrate the optimal marginal income taxation for various income percentiles for competitive market $\theta = -1$ and Cournot-Nash model with 2 firms and $\theta = 0$. The left figure shows the optimal marginal income tax level for competitive market with endogeneous prices (thick solid red line) and fixed price (thick dashed red line) and for Cournot-Nash model with endogeneous price (thin solid blue line) and fixed price (thin dashed blue line). The right figure presents the change in the optimal marginal income tax for competitive market (thick red line) and Cournot-Nash model (thin blue line).

simulations, the markets with fixed and endogenous prices have the same prices. The right figure presents the change in marginal tax $t_{endogeneous} - t_{fixed}$ for competitive market (thick red line) and Cournot-Nash model (thin blue line).

The graphs illustrate that there are two opposite effects that influence the optimal income tax schedule in oligopolistic markets. The first one is the price externality effect that larger income leads to larger market demand and, hence, higher prices for normal goods. Hence, the public authority wants to increase optimal income tax to compensate for the price externality. Both solid lines lie above the corresponding dashed lines on the left figure. In addition, the size of the price effect is increasing in θ, which is illustrated on the right figure.

The second effect is anticompetitive effect of oligopolistic markets that leads to inefficient low quantities traded in the market. To offset this effect and to increase the amount of goods traded in equilibrium, the public authority wants to stimulate labor income by decreasing the marginal income tax level. In particular, the tax schedule for Cournot-Nash model (dashed blue line) is below the tax schedule for competitive market (dashed blue line). A smaller marginal income tax for more concentrated markets is similar to a corrective subsidy in the case of commodity taxation (see Auerbach and Hines, 2001).

Since for endogenous prices the tax schedule for Cournot-Nash market (blue solid line on the left figure) lie below the tax schedule for competitive market (red solid line on the same figure) the anticompetitive effect dominates the price effect for U.S. housing market. This leads to optimal marginal income tax schedule being smaller for markets with larger values conjectural variation parameter.

5 Commodity and Profit Taxation

The main result of this paper is the emergence of an additional term in the optimal income tax formula that is aimed at correcting for price externality arising from changes in income distribution. In this section, we study the question how robust our main result is in the presence of other forms of taxation – commodity and profit, in particular. Arguably, and similarly to the Pigouvian term, commodity and profit taxation could also play a role in offsetting price externalities arising from changes in income distribution.

Before we present our main findings, we make the following technical assumption.

Assumption 1. *Individual demand function $x(p, y)$ satisfy that $x_{py} \neq \rho x_y + \nu$, where $\rho, \nu \in \mathbb{R}$.*

We require the cross derivative x_{py} of the demand function be not a linear transformation of its partial derivative x_y or, in other words, the demand and its price slope follow different dynamics with respect to income.

First, we consider the case of commodity taxation that takes the form of an excise tax imposed on good X. Let the tax be paid by producers and resultant revenues be used to fund public expenditures. We show that

Theorem 4. *With optimal commodity taxation the Pigouvian term of optimal income taxation is (i) zero if the market is competitive and (ii) non-zero if the market is oligopolistic.*

As shown earlier, in competitive markets the price externality is proportional to the change in demand x_y resulting from a change in income. We observe that the additional amount paid in commodity tax that arises from additional income is also proportional to the change in demand x_y. Thus, we argue that it is possible to design the excise tax such that it fully corrects for the price externality of income taxation, thus, rendering the Pigouvian term obsolete. Intuitively, our finding suggests that the optimal solution to the price externality problem is not to restrict workers from earning too much but make them pay for the externality they create (and use the tax revenue for redistributive or budgetary purposes).

However, commodity taxation can only partially correct for price externality in oligopolistic markets because price externality arises not only from changes in demand but also from oligopolistic competition. This implies that in the economy with oligopolistic markets income taxation continues to play a role in correcting for price externality. From a practical perspective, there is also room for a non-zero Pigouvian term even in competitive markets with commodity taxation. In particular, when extended to multiple goods with variable marginal costs of production our analysis suggests differentiated commodity taxation with different goods taxed in accordance with the extent of price externality. If such differentiation is infeasible, the solution to price externalities is the extension of income taxation with the Pigouvian term that aggregates all price externalities over all goods

Next, we consider the problem of optimal income taxation in the presence of profit taxation. In our model, the optimal profit tax is 100%, which follows from the linearity of profits in the maximization problem and is in line with the related theoretical literature on taxation (Diamond and Mirrlees (1971); Atkinson and Stiglitz (1976)). In our exposition of the effects of profit taxation, we abstract from commodity taxation which is, as argued by Atkinson and Stiglitz (1976), superfluous under 100% profit taxation. The next result shows that with profit taxation we obtain the same conclusions as in Theorem 4.

Theorem 5. *With 100% profit taxation the Pigouvian term of optimal income taxation is (i) zero if the market is competitive and (ii) non-zero if the market is oligopolistic.*

In competitive markets 100% profit taxation eliminates the Pigouvian term from optimal income taxation. This finding can be linked with the production efficiency result of Diamond and Mirrlees (1971). Their result says that optimal taxation should not distort aggregate production efficiency, which is not the case with an effort-distorting Pigouvian term. With 100% profit taxation, it is welfare enhancing for the public authority to maximize total produce and redistribute it in the population. Importantly, however, with oligopolistic markets the Pigouvian term does not disappear even with 100% profit taxation. There is no production efficiency under oligopolistic pricing which can imply that better work incentives will translate into disproportionately higher prices rather than more goods produced. In this case, it also remains optimal to restrain work incentives by the means of additional income taxation.[17]

6 Literature Review

The results reported in the present paper are connected to several strands of the literature. First, our analysis is closely related to the study of optimal income taxation in the presence of endogenous wages in labor markets. Stiglitz (1982) is one of the first to consider a setting in which workers are not perfect substitutes in production.[18] In this case, the general equilibrium effects imply that the optimal tax policy should subsidize high-talent workers and tax low-talent workers. Extending this analysis to a setting where workers have two-dimensional skill characteristics and an occupational choice, Rothschild and Scheuer (2013) found that the ability of workers to select their occupation involves a more progressive tax schedule than in a model without occupational choice. Ales, Kurnaz, and Sleet (2015) use a related multi-task assignment model with finite one-dimensional agent types to study a change in optimal income policy in response to a technical change. In a study of the continuum of one-dimensional agent types and general constant returns to scale production functions, Sachs, Tsyvinskiy, and

[17]The inefficiency of oligopolistic pricing has also an effect on optimal commodity taxation as discussed in Myles (1987) and Auerbach and Hines (2001).

[18]See also Feldstein (1973), Allen (1982), and Stern (1982).

Werquin (2016) show that the optimal labor supply in an equilibrium is determined by a complicated integral equation. Using this equation, they analyze the incidence of a tax reform with the actual U.S. tax code as a starting point.

The endogenous wages in the papers just cited arise because of a non-linear production technology where an agent's efforts on various tasks complement each other. The agent's effort on one task then influences the equilibrium wage on the other tasks (see, e.g., Sachs, Tsyvinskiy, and Werquin, 2016). To simplify analysis these papers assume a constant returns to scale production technology. In contrast, we consider a non-constant return to scale production technology to allow for an upward-sloping aggregate supply function with firms earning positive profits in equilibrium. In addition, our analysis of the price effect on optimal income taxation is more tractable because the prices are determined only by aggregate demand compared to more complicated type-specific or occupation-specific wages (see, e.g., Ales, Kurnaz, and Sleet, 2015). As a result, we were able to obtain an analytical expression for the Pigouvian term for each agent and analyze its properties for a wide spectrum of individual demand functions, including for normal and inferior goods.

The literature analyzing endogenous wages in labor markets also studies the impact of externalities on the optimal income tax schedule.[19] Rothschild and Scheuer (2016) study the corrective role of income taxation in a setting where agents can engage in a rent-seeking activity with private returns being different from social returns. Lockwood, Nathanson, and Weyl (2017) integrate tax considerations into an assignment model of agents to professions in which high-paying professions have negative externalities and low-paying professions have positive externalities. They estimate that the welfare gains from an optimal income taxation policy targeted to compensate these externalities are small. Rothschild and Scheuer (2014) also develops a unifying framework to analyze externalities and general equilibrium effects on optimal income taxation. Compared to this literature, the price effect studied in the present paper belongs to the category of general equilibrium effects without agents imposing any direct externalities.

Our analysis of the price effect is also related to the production efficiency theorem of Diamond and Mirrlees (1971) stating that the optimal taxation entails efficient production. Hence, optimal taxation does not explicitly depend on production technology.[20] This results holds, however, only under the condition that firm profits are either owned by the public authority or fully taxed. In the present paper, we analyze how production should be distorted in the optimum if firm profits cannot be fully taxed.

Only a few papers consider models in which firms can earn positive profits that are not fully taxed by the public authority. For example, Ales and Sleet (2016) incorporate a firm-

[19]Stantcheva (2014) also studies how adverse selection in labour markets influences optimal income taxation.

[20]See Scheuer and Werning (2016) for an insightful connection between the seminal model of Diamond and Mirrlees (1971) on commodity taxation and Mirrlees (1971) on income taxation.

CEO assignment model in Mirrleesian framework to study the taxation of top CEO incomes. They consider a public authority maximizing a weighted sum of tax revenue and firm profits. Though 100% profit taxation is optimal in their setting, the authors argue convincingly that the public authority should not take all the firm profits when firm entry decisions are also considered in the model. Similarly, Scheuer and Werning (2017) consider an assignment model to study the taxation of superstars.[21] They consider a supermodular production technology allowing for positive firm profits. One of their results establishes that the optimal income taxation schedule, as a function of agent preferences and agent skill distribution, does not directly depend on the production technology even when firm profit are not fully taxed. Both papers do not consider, however, variable product prices. In particular, there is only one consumption good with a price that can be normalized to one. In contrast, we consider the prices of two goods, of which only one price can be normalized to one. The price for the other good is endogenously determined in equilibrium. We showed that the optimal income tax schedule does depend on the production technology in the presence of price externalities.

Our paper also highlights the importance of the full profit taxation assumption for the interaction between income and commodity taxation. When all firm profits are fully taxed Atkinson and Stiglitz (1976) established that commodity taxation is unnecessary given the optimal income tax schedule (see also Mirrlees, 1976). On the other hand, the commodity taxation plays a crucial role into compensating for price externality in the presence of optimal income taxation when profits are not fully taxed.

Our paper is also related to a small body of literature analyzing commodity taxation in imperfectly competitive markets (Myles, 1987, 1989; Auerbach and Hines, 2001). Similar to Myles (1987, 1989), we show that it is too costly for the public authority to completely eliminate price externality in the presence of imperfect competition. We also obtain a decrease in optimal marginal income tax due to the price effect, which is parallel to providing a subsidy to correct for price externality as shown in Auerbach and Hines (2001). Our results can be seen as an extension of the analysis of this literature to optimal income taxation settings.

Our paper is also closely related to an important strand of literature focused on the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Oswald (1983), Ireland (2001), Kanbur and Tuomala (2013).[22] These papers are typically motivated by empirical observations according to which people care not only about their absolute level of consumption, but also how it compares with that of others. In our paper, relativity concerns arise endogenously because income distribution influence product prices through the market equilibrium condition.

[21] See also Pikety, Saez, and Stantcheva (2014) and Shourideh (2014) for the analyses of optimal taxation of top labor and capital incomes.

[22] A similar idea that individuals may seek a more equal income distribution in order to improve the terms of trade is explored by Zubrickas (2012).

7 Conclusion

In this paper, we note that a change in income distribution affects social welfare not only directly through changes in income level, but also through changes in product prices. This price effect leads to an additional Pigouvian term in the optimal income tax formula. For competitive markets, the Pigouvian term is positive for normal goods, negative for inferior goods, increasing for luxury goods, and decreasing for necessity goods. Using the U.S. housing market as an example, we show that the price effect increases optimal marginal income tax by 4-5% for most income levels.

For oligopolistic markets, the anticompetitive effect that leads to inefficiently low quantities traded in equilibrium. To offset this effect, the public authority wants to stimulate labor income by decreasing marginal income tax level. For U.S. housing market, the anticompetitive effect on income taxes dominates the price effect leading to a smaller optimal marginal income tax level for more concentrated markets. We also show that the Pigouvian term in oligopolistic markets is robust to the introduction of commodity and profit taxation.

Our study of the price effect is only the first step of incorporating an underlying market structure into the analysis of optimal income taxation. The housing spending accounts only for a third U.S. household expenditures. Hence, the overall price effect can be significantly larger if one takes into the price response of the other industries. We leave the analysis of the price effect in the economy with several industries as an exciting question for future research.

The price effect on optimal income taxation exists only if competitive firms have non-negative profits and the public authority cannot fully tax them. This highlights an important interaction between income and profit taxation. It is hard to analyze this interaction within our framework because the optimal profit taxation is either 0% or 100% in our model. However, Ales and Sleet (2016) and Scheuer and Werning (2017) outline possible models with firm entry and exit where the intermediate level of profit taxation is optimal. The interaction of the optimal income taxation and profit taxation is an important open question that we leave for future research.[23]

In addition, it is hard to analyze optimal tax policies in healthcare (Grossman, 1972) and other industries without considering dynamic models. The previous literature extended Mirrleesian model to dynamic settings (e.g., Farhi et al., 2012). Incorporating the price effect into these settings is an important open question that we also leave for future research.

Moreover, the equilibrium price response is an important consideration beyond income taxation policies. The welfare assessment of subsidies, welfare benefits, pensions, minimum wages, etc., would be biased unless the public authority takes into account the price effect studied in this paper.

[23]We thank Chris Sleet and Ali Shourideh for pointing this out to us.

Appendix A – Proofs

Proof of Theorem 1. The statement follows from the argument presented in the main text.

Proof of Corollary 1. The statement follows from the argument presented in the main text.

Proof of Proposition 1. The statement follows from the argument presented in the main text.

Proof of Proposition 2. The statement of the proposition about normal and inferior goods follows from $x_y > 0$ for normal goods and $x_y < 0$ for inferior goods.

We now show that the assumption that x_{yy} does not change its sign implies $x_{yy} \geq 0$ for luxury goods. From contrary, let us assume that $x_{yy} \geq 0$ for all income levels. Then

Consider $x_{yy}(p, y)$ at $y = 0$, which can be expressed as the limit

$$x_{yy}(p, 0) = \lim_{h \to 0} \frac{x(p, 2h) - 2x(p, h) + x(p, 0)}{h^2}$$

or, observing that $x(p, 0) = 0$,

$$x_{yy}(p, 0) = \lim_{h \to 0} \frac{x(p, 2h) - 2x(p, h)}{h^2} \tag{A.1}$$

By the definition of the luxury good, i.e., $p(x(p, y))/y$ is increasing in y, we have that

$$\frac{x(p, 2h)}{2h} \geq \frac{x(p, h)}{h}$$

or $x(p, 2h) \geq 2x(p, h)$. Hence, the limit in A.1 is non-negative, which implies that $x_{yy}(p, 0) \geq 0$. Then, by the assumption that x_{yy} does not change its sign we have $x_{yy} \geq 0$ for all y, which proves the proposition for luxury goods. Using the analogous argument, one can show that the proposition also holds for necessity goods.

Proof of Theorem 2. The public authority's objective is to find the tax schedule $T(z)$ that maximizes the social welfare function W subject to the resource and incentive compatibility constraints and also accounting for the price effects of tax policy stemming from the market equilibrium condition. Observing that $T(z) = n\ell(n) - y(n)$ and using the market equilibrium condition as a constraint, we can express the public authority's problem as finding the price p, levels of disposable income $y(n)$, and levels of individual labor supply $\ell(n)$ that maximize

$$\max_{p,\, y(n),\, \ell(n)} \int H(U(p, y(n), \ell(n))) f(n) dn + \omega \Pi(p) \text{ subject to (1), (2), and (3).}$$

It is analytically more convenient to reformulate this maximization problem so that the choice variables are price p, labor supply $\ell(n)$, and agent utility $u(n) = U(p, y(n), \ell(n))$ (see Mirrlees (1976)). From the latter expression we can invert disposable income $y(n)$, which we express as a function $y = q(p, u, \ell)$. Using the new set of independent variables, we rewrite the market equilibrium condition as

$$S(p) = \int x(p, q(p, u, \ell)) f(n) dn \tag{A.2}$$

and the resource constraint as

$$\int \left(n\ell(n) - q(p, u, \ell)\right) f(n) dn = R. \tag{A.3}$$

Incentive compatibility constraints (2) can be written as $u(n) = \max_m U(p, y(m), m\ell(m)/n)$. The envelope theorem then implies that[24]

$$u'(n) = \frac{dU}{dn} = \frac{\ell(n) c_\ell(\ell(n))}{n}. \tag{A.4}$$

Given the new formulation, the public authority chooses functions $u(n)$, $\ell(n)$, and price p to maximize

$$\max_{p,\, u(n),\, \ell(n)} \int H(u(n)) f(n) dn + \omega \Pi(p) \text{ subject to (A.2), (A.3), and (A.4).}$$

The Lagrangian of the public authority's problem is given by

$$\begin{aligned} \mathcal{L} = \int & [(H(u(n)) + \omega\Pi(p) + \lambda(n\ell(n) - q(p, u(n), \ell(n)) - R) \\ & + \gamma(S(p) - x(p, q(p, u(n), \ell(n)))))f(n) + \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n)]dn, \end{aligned}$$

where γ, λ, and $\mu(n)$ are multipliers corresponding to constraints (A.2), (A.3), and (A.4), respectively. After integrating $\int \mu(n) u'(n) dn$ by parts, the first-order conditions can be written as

$$\begin{aligned} u(n) &: [H'(u) - (\lambda + \gamma x_y) q_u] f(n) - \mu'(n) = 0, \\ \ell(n) &: [\lambda n - (\lambda + \gamma x_y) q_\ell] f(n) - \mu(n)(c_\ell(\ell) + \ell c_{\ell\ell}(\ell))/n = 0, \\ p &: \int [\omega \Pi'(p) - \lambda q_p + \gamma(S'(p) - x_p - x_y q_p)] f(n) dn = 0, \end{aligned}$$

along with the transversality conditions $\mu(\underline{n}) = \mu(\overline{n}) = 0$. By implicit differentiation, we obtain the derivatives $q_u = 1/v_y,\ q_\ell = c_\ell/v_y,\ q_p = -v_p/v_y = x$, where the last expression follows from

[24]The second-order condition also requires $u(n)$ be a non-decreasing function.

the Roy's identity. The first-order conditions then reduce to

$$u(n): \left(H'(u) - \frac{\lambda + \gamma x_y}{v_y}\right) f - \mu'(n) = 0, \tag{A.5}$$

$$\ell(n): \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \gamma x_y)\right) f - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.6}$$

$$p: \omega S(p) - \lambda S(p) + \gamma \left(S'(p) - \int (x_p + x_y x) f dn\right) = 0. \tag{A.7}$$

In deriving (A.7), we also use the market equilibrium condition (A.2) and that $\Pi'(p) = S(p)$.

To find the expression for the optimal marginal income tax $t(z) = T'(z)$ we note that individual maximization problem $u(n) = \max_m U(p, y(m), z(m)/n)$ implies that $nv_y y'(n) - c_\ell z'(n) = 0$. Given that $y = z - T(z)$ the optimal marginal tax rate must satisfy $t(z) = 1 - c_\ell/(nv_y)$. Rearranging the condition (A.6) we obtain

$$\frac{t}{1-t} = \left(1 + \frac{\ell c_{\ell\ell}}{c_\ell}\right) \frac{v_y \mu}{\lambda n f} + \frac{\gamma}{\lambda} x_y, \tag{A.8}$$

with the multiplier μ found from (A.5)

$$\mu(n) = \int_n^{\overline{n}} \left(\frac{\lambda + \gamma x_y}{v_y} - H'(u)\right) f(n) dn.$$

Next, we demonstrate that $1 + \ell c_{\ell\ell}/c_\ell = (1 + E^u)/E^c$, where E^c is the elasticity of compensated labor supply and E^u is the elasticity of uncompensated labor supply E^u. Observing that $y'(n) = z'(n)(1-t)$, we can express the individual utility maximization condition as

$$v_y(p, y)\; n(1-t) - c_\ell(\ell) = 0 \tag{A.9}$$

or, using $y = w\ell + \overline{y}$, where $w = n(1-t)$ is a net wage rate and $\overline{y}$ is non-labor income,

$$v_y(p, w\ell + \overline{y})\; w - c_\ell(\ell) = 0. \tag{A.10}$$

Implicitly differentiating (A.10) we obtain

$$\frac{\partial \ell}{\partial w} = -\frac{v_{yy} w\ell + v_y}{v_{yy} w^2 - c_{\ell\ell}} = \frac{v_{yy} w\ell + v_y}{c_{\ell\ell} - v_{yy} w^2}.$$

Then, the elasticity of uncompensated labor supply $E^u = \partial \ell/\partial w\; (w/\ell)$ is equal to

$$E^u = \frac{v_{yy} w\ell + v_y}{c_{\ell\ell} - v_{yy} w^2}\; \frac{w}{\ell} = \frac{v_{yy}(c_\ell/v_y)^2 + c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2},$$

where we use $w = c_\ell/v_y$ from (A.10).

To obtain the elasticity of compensated labor supply, E^c, we employ the Slutsky equation $E^c = E^u - E^m$, where $E^m = w(\partial \ell / \partial \overline{y})$ is the income effect parameter:

$$E^m = w \, \frac{\partial \ell}{\partial \overline{y}} = \frac{v_{yy}(c_\ell / v_y)^2}{c_{\ell\ell} - v_{yy}(c_\ell / v_y)^2}.$$

Thus, the elasticity of compensated labor supply, E^c, is given by

$$E^c = E^u - E^m = \frac{v_{yy}(c_\ell/v_y)^2 + c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2} - \frac{v_{yy}(c_\ell/v_y)^2}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2} = \frac{c_\ell/\ell}{c_{\ell\ell} - v_{yy}(c_\ell/v_y)^2}.$$

Thus, we obtain that $1 + \ell c_{\ell\ell}/c_\ell = (1 + E^u)/E^c$, which completes the proof of the theorem.

Proof of Theorem 3. In oligopolistic markets, the public authority's problem is

$$\max \int H(u(n)) f(n) dn$$

$$s.t. \begin{cases} \frac{du}{dn} = \frac{\ell c_\ell}{n} & (\mu(n),\ \text{incentice compatibility}) \\ \int \left[n\ell(n) - y(n) \right] f(n) dn \geq R & (\lambda,\ \text{resource constraint}) \\ J(X, X_p, p) = 0 & (\gamma,\ \text{oligopolistic market equilibrium}) \\ X - \int x(p, y(n)) f(n) dn = 0 & (\alpha_0,\ \text{market demand}) \\ X_p - \int x_p(p, y(n)) f(n) dn = 0 & (\alpha_1,\ \text{market demand slope}) \end{cases}$$

with Largrange multipliers introduced next to their corresponding constraints. The Lagrangian of this problem is given by

$$\mathcal{L} = \int [(H(u(n)) + \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n) + \lambda(n\ell(n) - y(n) - R) + \gamma J(X, X_p, p)$$
$$+ \alpha_0(X - x(p, y)) + \alpha_1(X_p - x_p(p, y)))f(n)]dn$$

Using $y = q(u, \ell, p)$ with $q_u = 1/v_y,\ q_\ell = c_\ell/v_y$, and $q_p = x$, we obtain the first-order conditions as

$$u(n):\left(H'(u)-\frac{\lambda+\alpha_0 x_y+\alpha_1 x_{py}}{v_y}\right)f-\mu'(n)=0, \tag{A.11}$$

$$\ell(n):\left(\lambda n-\frac{c_\ell}{v_y}(\lambda+\alpha_0 x_y+\alpha_1 x_{py})\right)f-\mu(n)(c_\ell+\ell c_{\ell\ell})/n=0, \tag{A.12}$$

$$p:-\lambda X+\gamma J_3-\int(\alpha_0(x_p+x_y x)+\alpha_1(x_{pp}+x_{py}x))f(n)dn=0, \tag{A.13}$$

$$X:\alpha_0+\gamma J_1=0, \tag{A.14}$$

$$X_p:\alpha_1+\gamma J_2=0. \tag{A.15}$$

where J_1, J_2, and J_3 are partial derivatives of J with respect to its first, second, and third arguments respectively. Using equations (A.14) and (A.15) we obtain

$$u(n):\left(H'(u)-\frac{\lambda-\gamma J_1 x_y-\gamma J_2 x_{py}}{v_y}\right)f-\mu'(n)=0, \tag{A.16}$$

$$\ell(n):\left(\lambda n-\frac{c_\ell}{v_y}(\lambda-\gamma J_1 x_y-\gamma J_2 x_{py})\right)f-\mu(n)(c_\ell+\ell c_{\ell\ell})/n=0, \tag{A.17}$$

$$p:-\lambda X+\gamma\left(J_1 X_p+J_2 X_{pp}+J_3+\int(J_1 x_y+J_2 x_{py})xf(n)dn\right)=0, \tag{A.18}$$

where we denote $X_{pp}=\int x_{pp}(p,y(n))f(n)dn$. Following the same steps when deriving the optimal income taxes in Theorem 2, we obtain the marginal income tax formula given in (16). Lastly, equation (A.16) gives the expression for multiplier

$$\mu(n)=\int_n^{\overline{n}}\left(\frac{\lambda+\gamma(-J_1 x_y-J_2 x_{py})}{v_y}-H'(u)\right)f(n)dn.$$

Proposition A1. *Suppose that in response to a small unit rebate r given for good X its equilibrium price p increases by less than the rebate itself. The Pigouvian term for agent type n is positive if and only if the change in price in response to an increase in income levels in the neighborhood of $y(n)$ is positive.*

Proof. Let us denote $dJ/dp=J_1X_p+J_2X_{pp}+J_3$. We first show that the rebate condition implies that the sign of γ coincides with the sign of dJ/dp. We can rewrite the first-order condition (A.18) as

$$\frac{dJ}{dp}\left(1+\frac{\int(J_1 x_y+J_2 x_{py})xf(n)dn}{dJ/dp}\right)=\frac{\lambda}{\gamma}X(p). \tag{A.19}$$

Since $\lambda X(p)$ is positive the sign of γ coincides with the sign of the leftside, which we denote by

A. Now consider a small unit rebate r, which increases an agent's income by $rx(p,y)$. Applying the Implicit Function Theorem to the equilibrium condition $J(X, X_p, p) = 0$, we obtain that the effect of rebate on the equilibrium price is given by

$$\left.\frac{dp}{dr}\right|_{r=0} = -\frac{\int (J_1 x_y + J_2 x_{py}) x f(n) dn}{dJ/dp}. \tag{A.20}$$

Taking into account the proposition hypothesis that $\left.\frac{dp}{dr}\right|_{r=0} < 1$, we conclude from (A.41) and (A.20) that the sign of γ coincides with the sign of dJ/dp.

We now consider a change in income level from $y(n')$ to $y(n') + \phi$ for all types $n' \in [n - \delta/2, n + \delta/2]$ for some n. The resultant change in price is given by

$$\frac{dp}{d\phi} = -\frac{(J_1 x_y + J_2 x_{py}) f(n)\delta}{dJ/dp} = \frac{-\gamma(J_1 x_y + J_2 x_{py}) f(n)}{\gamma dJ/dp}\delta. \tag{A.21}$$

Since the sign of γ coincides with sign of (dJ/dp), which implies a positive denominator, the numerator – Pigouvian term for agent with productivity n – is positive if and only if $dp/d\phi > 0$.

Proof of Theorem 4. Let b be an excise tax imposed on good X. Then, the equilibrium condition for market price changes to

$$J(X, X_p, p) - b = 0, \tag{A.22}$$

where $J(X, X_p, p) = p - K'\left(\frac{X}{M}\right) + (1+\theta)\frac{X}{MX_p}$ as defined in (15). The firm profits are given by $\Pi(X, p, b) = (p - b)X - MK(\frac{X}{M})$. We impose one more constraint ensuring that firms receive non-negative profit

$$\Pi(X, p, b) \geq 0. \tag{A.23}$$

The Lagrangian of the public authority's problem is given by

$$\begin{aligned}\mathcal{L} = \int &[(H(u(n)) + \lambda(n\ell(n) + bX - y(n) - R) + \gamma(J(X, X_p, p) - b) \\ &+ \alpha_0(X - x(p,y)) + \alpha_1(X_p - x_p(p,y)))f(n) \\ &+ \alpha_2 \Pi(X, p, b) + \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n)]dn,\end{aligned}$$

where multiplier α_2 corresponds to condition (A.23), with the other multipliers defined earlier.

The first-order conditions are

$$u(n) : \left(H'(u) - \frac{\lambda + \alpha_0 x_y + \alpha_1 x_{py}}{v_y} \right) f - \mu'(n) = 0, \tag{A.24}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda + \alpha_0 x_y + \alpha_1 x_{py}) \right) f - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.25}$$

$$p : -\lambda X + \gamma J_3 + \alpha_2 \Pi_2 - \int (\alpha_0(x_p + x_y x) + \alpha_1(x_{pp} + x_{py} x)) f(n) dn = 0, \tag{A.26}$$

$$b : \lambda X - \gamma + \alpha_2 \Pi_3 = 0, \tag{A.27}$$

$$X : \alpha_0 + \lambda b + \gamma J_1 + \alpha_2 \Pi_1 = 0, \tag{A.28}$$

$$X_p : \alpha_1 + \gamma J_2 = 0, \tag{A.29}$$

where J_i and Π_i denote partial derivatives with respect to their argument i, respectively. Taking into account that $J_3 = 1$ and $\Pi_2 = -\Pi_3$, we use the last three conditions to transform the first three conditions as

$$u(n) : \left(H'(u) - \frac{\lambda - (\gamma J_1 + \lambda b + \alpha_2 \Pi_1) x_y - \gamma J_2 x_{py}}{v_y} \right) f - \mu'(n) = 0, \tag{A.30}$$

$$\ell(n) : \left(\lambda n - \frac{c_\ell}{v_y}(\lambda - (\gamma J_1 + \lambda b + \alpha_2 \Pi_1) x_y - \gamma J_2 x_{py}) \right) f - \mu(n)(c_\ell + \ell c_{\ell\ell})/n = 0, \tag{A.31}$$

$$p : \int ((\gamma J_1 + \lambda b + \alpha_2 \Pi_1)(x_p + x_y x) + \gamma J_2 (x_{pp} + x_{py} x)) f(n) dn = 0. \tag{A.32}$$

Condition (A.32) implies that the optimal excise tax equals

$$b = -\frac{\alpha_2 \Pi_1}{\lambda} - \frac{\gamma \int (J_2(x_{pp} + x_{py} x) + J_1(x_p + x_y x)) f(n) dn}{\lambda \int (x_p + x_y x) f(n) dn}. \tag{A.33}$$

First, we observe that $\gamma \neq 0$. Otherwise, condition (A.33) implies that $b \leq 0$ because of $\Pi_1 \geq 0$ and $\alpha_2 \geq 0$. But $b \leq 0$ implies that $\alpha_2 = 0$ because of positive profits $\Pi > 0$, which in turn implies that $\alpha_2 = 0$. Then the first-order condition (A.27) is violated. We arrive at a contradiction.

Condition (A.31) implies that the Pigouvian term equals to

$$\frac{(-\gamma J_1 - \lambda b - \alpha_2 \Pi_1) x_y - \gamma J_2 x_{py}}{\lambda}. \tag{A.34}$$

Condition (A.33) then implies that the Pigouvian term can be written as

$$\frac{\gamma J_2}{\lambda} \left(\frac{\int (x_{pp} + x_{py} x) f(n) dn}{\int (x_p + x_y x) f(n) dn} - x_{py} \right). \tag{A.35}$$

We notice that $J_2 = 0$ for competitive markets ($\theta = -1$), which yields a zero Pigouvian term proving part (i) of the theorem. But for $\theta > -1$ we have that the derivative $J_2 \neq 0$. Furthermore, the expression in the brackets cannot be equal to zero for all n because x_{py} is a non-constant function as implied by Assumption 1. Hence, the Pigouvian term does not reduce to 0 in oligopolistic markets with commodity taxation.

Proof of Theorem 5. With 100% profit taxation, the Lagrangian of the public authority's problem is given by

$$\begin{aligned}\mathcal{L} = \int [&(H(u(n)) + \lambda(n\ell(n) + pX - MK(\tfrac{X}{M}) - y(n) - R) + \gamma J(X, X_p, p) \\ &+ \alpha_0(X - x(p, y)) + \alpha_1(X_p - x_p(p, y)))f(n) \\ &+ \mu(n)(u'(n) - \ell(n)c_\ell(\ell(n))/n)]dn.\end{aligned} \tag{A.36}$$

The first-order conditions with respect to p and X are

$$p : \gamma J_3 - \int (\alpha_0(x_p + x_y x) + \alpha_1(x_{pp} + x_{py} x))f(n)dn = 0, \tag{A.37}$$

$$X : \alpha_0 + \lambda(p - K'(\tfrac{X}{M})) + \gamma J_1 = 0, \tag{A.38}$$

whereas those with respect to $u(n)$, $\ell(n)$, and $X_p(p)$ remain as in (A.24), (A.25), and (A.29), respectively. The Pigouvian term remains the same as in Theorem 3 :

$$\frac{-\gamma(J_1 x_y + J_2 x_{py})}{\lambda}. \tag{A.39}$$

Observing that $p - K'(\frac{X}{M}) = -(1+\theta)\frac{X}{MX_p}$ from (15), we obtain from (A.38) that the multiplier α_0 is equal to

$$\alpha_0 = \frac{\lambda(1+\theta)X(p)}{MX_p(p)} - \gamma J_1. \tag{A.40}$$

Thus, we can express the first-order condition (A.37) as

$$\gamma \int (J_3 + J_1(x_p + x_y x) + J_2(x_{pp} + x_{py} x))f(n)dn = \frac{\lambda(1+\theta)X}{MX_p} \int (x_p + x_y x)f(n)dn. \tag{A.41}$$

If $\theta = -1$ (competitive market) condition (A.41) implies that $\gamma = 0$ and, thus, a zero Pigouvian term, which proves part (i) of the theorem. With $\theta > -1$ (oligopolistic market), we respectively obtain $\gamma \neq 0$. In addition, conditions (A.29) and (A.40) imply that at least one of the multipliers α_0 or α_1 must be non-zero. Since by Assumption 1 we have that x_{py} is not a linear transformation of x_y, the Pigouvian term must be non-zero, which proves the theorem.

Appendix B – General Equilibrium Model

In this section, we show that our model of competitive markets can be supported with a labor market and a consumer's utility maximization problem. In particular, we consider two competitive industries: one producing the numeraire good G and the other producing good X. We label these industries as G and X respectively. We assume that agents can earn wage w for effective labor hours supplied (i.e., $n\ell(n)$) in both industries, the price for the numeraire good is normalized to $p_g = 1$, and the price for good X equals to p.

Industry G has a homogeneous of degree one production technology $F_g(L_g) \equiv L_g$, where L_g is the amount of labor used in production of good G. Since the price of numeraire good is normalized to 1, the profit maximization condition implies that $w = p_g = 1$, *zero profits*, and *any level of the equilibrium labor demand* L_g^d in industry G.

Industry X has a production technology with decreasing returns to scale $F_x(L_x)$, e.g., $F_x(L_x) = AL_x^a$, where L_x is the amount of labor supplied and $0 < A$, $0 < a < 1$ are constants. Hence, firm profit maximization problem

$$\max_{L_x} p \cdot F_x(L_x) - wL_x$$

The solution to this maximization problem leads to the equilibrium labor demand L_x^d in industry X. Taking into account that $w = 1$ for the production function mentioned we have $L_x^d(p) = (aAp)^{\frac{1}{1-a}}$. The equilibrium market supply of good X then equals $S_x(p) = F(L_x^d(p))$. We also assume that firms spend all their profits $\Pi_x(p) = pS_x(p) - L_x^d(p)$ on the numeraire good G. The government also spends all its revenue R on numeraire good G.

On the demand side of the economy, we assume that agent's preferences can be summarized by utility function $u(x, g) - c(\ell)$, where (x, g) is the amount of good X and the numeraire G consumed by the agent. Utility u is a continuous function representing locally non-satiated preferences.

Consider an agent with productivity n who works ℓ hours. Taking into account that equilibrium wage $w = p_g = 1$ and tax schedule $T(n\ell)$ her income equals $n\ell - T(n\ell)$. Hence, agent's maximization problem is

$$\begin{aligned} &\max_{x,\, g,\, \ell} u(x, g) - c(\ell) \\ &s.t. \;\; p \cdot x + g \leq n\ell - T(n\ell) \end{aligned} \tag{B.1}$$

The solution to the above problem is labor supply $\ell^*(n, p)$ and consumption bundle $(x^*(n, p), g^*(n, p))$. Overall, aggregate labor supply and consumer demand equal

$$L^s(p) = \int n\ell^*(n, p)f(n)dn, \quad X(p) = \int x^*(n, p)f(n)dn, \quad G(p) = \int g^*(n, p)f(n)dn.$$

The economy must satisfy three market clearing conditions:

$$S_x(p) = X(p) \tag{B.2}$$

$$S_g(p) = G(p) + \Pi_x(p) + R \tag{B.3}$$

$$L^s(p) = L^d_x(p) + L^d_g(p), \tag{B.4}$$

where the market clearing condition (B.3) for numeraire G demands that the market supply equals the market demand for G plus profits of industry X and government spending R.

Let us show that condition (B.2) is the only one that we should consider in optimal income taxation problem. Since any level $L^d_g(p)$ satisfies the maximization problem of firms producing numeraire good G (see above), we are free to choose $L^d_g(p) = L^s(p) - L^d_x(p)$ to clear the labor market. Taking into account that $S_g(p) = L^d_g(p)$ and $\Pi_x(p) = pS_x(p) - L^d_x(p)$ condition (B.3) can be equivalently rewritten as

$$L^d_g(p) + L^d_x(p) = G(p) + pS_x(p) + R$$

Given conditions (B.2) and (B.4) this is equivalent to

$$\int n\ell^*(n,p)f(n)dn = G(p) + pX(p) + R.$$

This condition follows from the budget constraint of agent's maximization problem (B.1) when the government spending constraint is satisfied as equality $\int T(n\ell^*(n,p))f(n)dn = R$ (as we assume). Overall, the only independent market clearing condition that we should take into account in the optimization problem is (B.2) – the market clearing condition for good X.

Appendix C U.S. Housing Market

In this appendix, we present some additional simulation results for the U.S. housing market that we used in the main text to quantitatively estimate the size of price effect on optimal income taxation. Our calibrations solve the following problem

$$\max_{p,\,y(n),\,l(n)} \int \left(v(p, y(n)) - c(\ell(n))\right) f(n)dn \tag{C.1}$$

$$s.t. \begin{cases} \int \left(n\ell(n) - y(n)\right) f(n)dn = R. \\ v(p, y(n)) - c(\ell(n)) \geq v(p, y(n-1)) - c((n-1)\ell(n-1)/n) \\ S(p) = \int x(p, y(n))f(n)dn \end{cases}$$

where we consider only adjacent and downward binding incentive compatibility constraints because agent's utility function satisfies the single-crossing condition (see Mirrlees (1976)). As described in the main text, to obtain the indirect utility function we consider a non-homothetic constant elasticity of substitution (NH-CES) utility function of Albouy et al. (2016)

$$u(x, g) = \left(\frac{27 - 14g^{-1/2}}{2x^{-1/2} + 3}\right)^{3/2}.$$

The corresponding expenditure function equals to

$$e(p, u) = \frac{4(p^{1/3} + 7^{2/3}u^{4/9})^3}{9(-9 + u^{2/3})^2}.$$

Hence, the indirect utility function $v(p, y)$ is implicitly determined by the following equation

$$y = \frac{4(p^{1/3} + 7^{2/3}v(p, y)^{4/9})^3}{9(-9 + v(p, y)^{2/3})^2}.$$

We use this equation together with the resource constraint, incentive compatibility constraint, and the market equilibrium constraint in maximization problem (C.1). Following Kanbur and Tuomala (2013), we also assume that agent costs equal $c(\ell) = \ell^4/4$.

Section 3 in the main text presents our simulation results for competitive market with supply function $S = sp^{\varepsilon}$, where $\varepsilon = 1.75$ corresponds to the price elasticity of the average U.S. metropolitan area (Saiz (2010)) and $s = 0.0021$ is a constant calibrated to match the average share of U.S. housing expenditure of 32.9% in equilibrium (see Albouy et al. (2016)). Since the housing supply elasticity differs widely for various countries and regions, we reestimate the change in optimal income tax for a median citizen for additional elasticity parameters. Table 1 presents our results for perfectly inelastic supply function $\varepsilon = 0$, which describes better a short-run housing supply or the housing supply in major UK cities (The Economist, 2017,

	$\varepsilon = 0$	$\varepsilon = 1.75$	$\varepsilon = 3$
Δt	17%	4.5%	3%

Table 1: The change in optimal marginal income tax Δt between endogeneous and fixed price regimes for a median citizen for various elasticities of housing supply.

Aug. 5), and a more elastic supply function with $\varepsilon = 3$, which is closer to the estimates of the price elasticity of U.S. housing supply obtained by Green et al. (2005) and Epple and Romer (1991).[25] The results suggest that the price effect could be significantly higher for markets with more inelastic supply functions.

In addition to extending our results to various forms of housing supply elasticities, we consider an alternative distribution of agent abilities. The lognormal distribution does not match well the upper tail of the income distribution (at least for the U.S. data). As an alternative, we consider the Champernowne distribution that could be a better approximation of the income distribution for some countries and approaches a Pareto distribution for the upper tail of income. The probability density function of the Champernowne distribution is

$$f(n) = \frac{m^{\theta} n^{\theta-1}}{(m^{\theta} + n^{\theta})^2}$$

where θ is a shape parameter and m is a scale parameter. We consider parameters $\theta = 3$ and $m = 2$ that fit well the U.S. data (see Kanbur et al., 2013).

Figure 3 presents our results. Qualitatively the results are quite similar to the simulation results using the lognormal distribution. The change in marginal tax between endogeneous and fixed price economies is positive for all agents, equals around 6% for a median citizen, and decreasing with agent abilities (as well as agent's income). The absolute level of tax as percentage of income did not change compared to negative change for the case of lognormal distribution. The change in labor supply, disposable income, and agent utility exhibit similar patterns to the case of the lognormal distribution.

[25]For both elasticities we calibarated parameter s to match the average share of U.S. housing expenditure of 32.9% in equilibrium ($s = 0.07$ for $\varepsilon = 0$, $s = 0.0021$ for $\varepsilon = 1.75$, and $s = 0.00015$ for $\varepsilon = 3$).

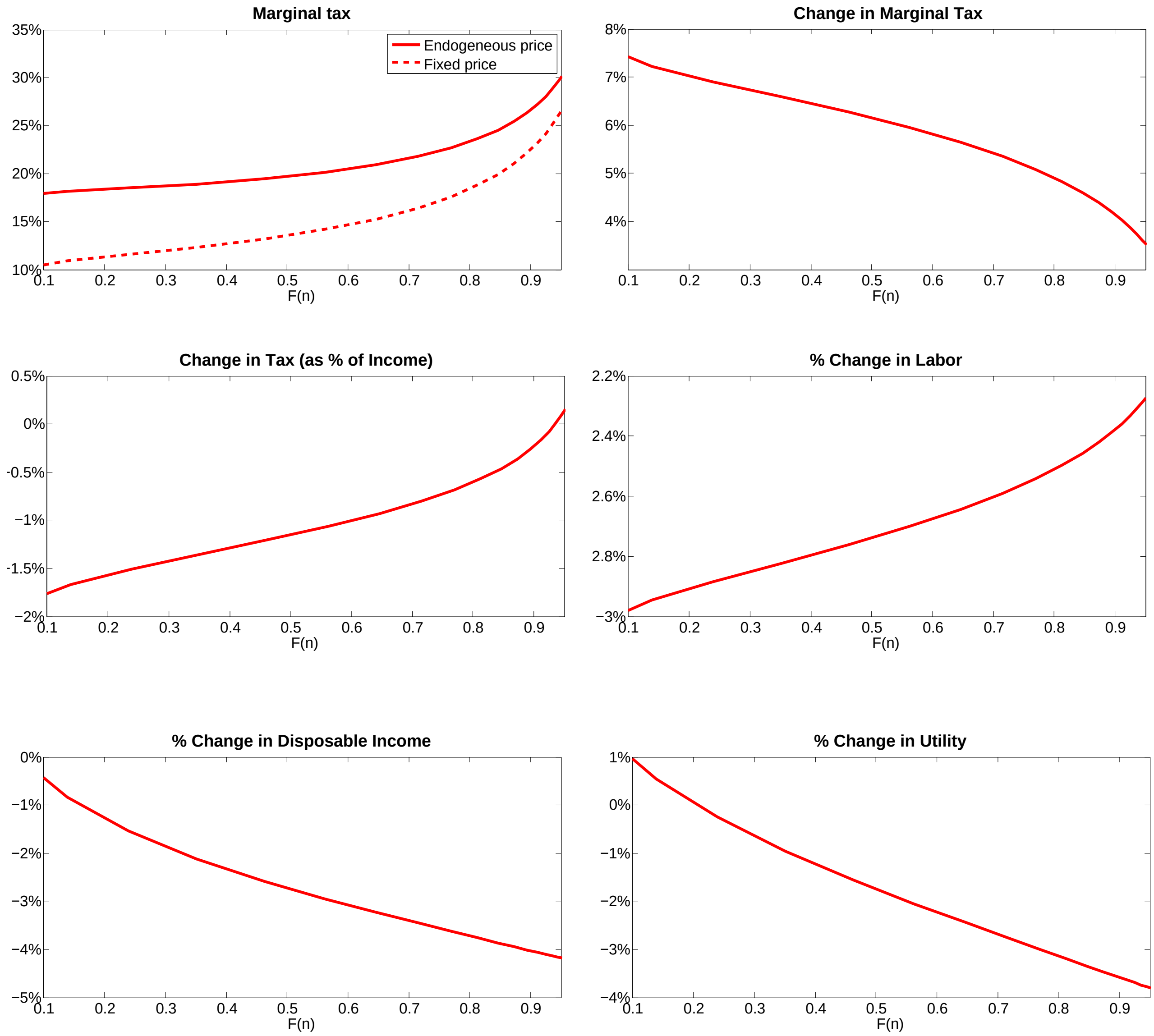

Figure 3. The figure presents the results of our simulations for Champernowne distribution of agent abilities. The graphs show the optimal marginal taxes, change in the optimal marginal tax, % change in labor supply, and % change in agent utility between the optimal tax regimes in economies with endogeneous and fixed price.

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