The Sources of Capital Misallocation*

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Abstract

We develop a methodology to analyze capital misallocation (dispersion in static marginal products) measure the contributions of technological/informational frictions and a rich class of firm-specific factors. An application to Chinese manufacturing firms reveals that adjustment costs and uncertainty, while significant, generate only a modest amount of MPK dispersion, which stems largely from other factors. For large US firms, adjustment costs are relatively more salient, though firm-specific factors still account for the bulk of observed misallocation. We also find that heterogeneity in technologies/markups account for a limited fraction of observed misallocation in China, but a potentially large share for US firms.

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1 Introduction

A large and growing body of work analyzes the 'misallocation' of productive resources across firms, i.e., dispersion in static marginal products, and the resulting adverse effects on aggregate productivity and output.[1] A number of recent studies examine the role of specific factors hindering period-by-period marginal product equalization. Examples of such factors include adjustment costs, imperfect information, financial frictions, as well as firm-specific 'distortions' stemming from economic policies or other institutional features. The importance of disentangling the role of these forces is self-evident. For one, a central question, particularly from a policy standpoint, is whether misallocation stems largely from efficient sources e.g., technological factors like adjustment costs or heterogeneity in production technologies, or inefficient ones, such as policy-induced distortions or markups. Similarly, understanding the exact nature of distortions – e.g., the extent to which they are correlated with firm characteristics – is essential to analyze their implications beyond static misallocation, for example, on firm entry and exit decisions and investments that influence future productivity.[2]

In this paper, we develop and implement a tractable methodology to distinguish various sources of capital misallocation using observable data on revenues and inputs. Our analysis proceeds in two steps. First, we augment a standard general equilibrium model of firm dynamics with a number of forces that contribute to *ex-post* dispersion in static marginal products, specifically (i) capital adjustment costs, (ii) informational frictions, in the form of imperfect knowledge about firm-level fundamentals and (iii) a class of firm-specific factors, meant to capture all other forces influencing investment. This includes, but is not limited to, unobserved heterogeneity in markups and/or production technologies, financial frictions, or institutional/policy-related distortions. In this first part of our analysis, rather than take a stand on the exact nature of these factors, we adopt a flexible specification that allows for time-variation and correlation with firm characteristics. The environment is an extension of the canonical Hsieh and Klenow (2009) framework to include dynamic considerations in firms' investment decisions. The main contribution of this part is an empirical strategy designed to precisely measure the contribution of each of these forces using widely available firm-level data.

In the second part of our analysis, we analyze various candidates for the firm-specific factors in (iii) above. First, we extend our methodology to investigate the role of unobserved heterogeneity in markups and production technologies in generating observed misallocation. Next, we analyze the effect of policies that affect or restrict the size of firms. We also describe a model of financial/liquidity considerations. We show that the latter two forces are, in a sense, isomorphic

[1]Throughout the paper we use the term misallocation to refer to dispersion in static marginal products, whether stemming from distortionary factors or efficient ones, for example, adjustment costs.

[2]See, e.g., Restuccia and Rogerson (2017) for an in-depth discussion of these margins.

to a broader set of firm-specific factors and so are difficult to disentangle using production-side data alone. In other words, additional information (e.g., firm-level financial data) would be required to separately quantify their impact.

Our key innovation is to explore the sources of marginal product dispersion within a unified framework and thus provide a more robust decomposition of the observed misallocation in the data. In contrast, we show that focusing on particular sources one-by-one while abstracting from others – a common approach in the literature – is potentially problematic. To the extent the data reflect the combined influence of a number of factors, examining them in isolation runs the risk of reaching biased conclusions of their severity and contribution to observed misallocation. Our strategy for disentangling these forces is based on a simple insight: although each moment is a complicated function of multiple factors, making any single moment insufficient to identify a particular factor, combining the information in a wider set of moments – specifically, elements of the covariance matrix of capital and revenues – can be extremely helpful in disentangling these factors. Indeed, we show that allowing these forces to act in tandem is essential to reconcile a broad set of moments in firm-level investment dynamics.

To understand the measurement difficulty, consider, as an example, convex adjustment costs. When they are the only force present, a single moment, for example, the variability of investment, has an intuitive, one-to-one mapping with their magnitude – the greater the adjustment cost, the lower is investment volatility. However, suppose that there are other factors that also dampen investment volatility (e.g., firm-specific distortions or implicit 'taxes' correlated with fundamentals). In this case, using this single moment in isolation to make inferences regarding adjustment costs leads to an upward bias. As a second example, consider the effects of firm-level uncertainty, which reduces the contemporaneous correlation between investment and fundamentals. However, a low measured correlation could also be the result of other firm-specific factors (e.g., distortions or markups) that are uncorrelated with fundamentals. Again, using this moment in isolation runs the risk of incorrectly measuring the quality of information.

Our empirical strategy overcomes this difficulty by jointly examining a set of carefully chosen moments. We formalize this identification strategy using a salient special case of our model – when firm-level fundamentals follow a random walk. The tractability of this case allows us to derive analytical expressions for the moments and prove that they uniquely identify the underlying structural parameters that determine the contribution of each factor. Specifically, four moments, namely, (1) the variance of investment, (2) the autocorrelation of investment, (3) the correlation of investment with past fundamentals, and (4) the covariance of the marginal (revenue) product of capital ($mrpk$) with fundamentals together identify adjustment costs, uncertainty and the magnitude and correlation structure of other firm-specific factors.

The intuition behind this result is easiest to see in a simple pairwise analysis – this set of moments comprises pairs that have opposing effects on a corresponding pair of structural parameters. As an example, consider the challenge described earlier of disentangling adjustment costs from other idiosyncratic factors that dampen the firm's incentives to respond to changing fundamentals. Both of these forces depress the volatility of investment. However, they have opposing effects on the autocorrelation of investment - convex adjustment costs create incentives to smooth investment over time and so tend to make investment more serially correlated. A distortion that directly reduces the response to fundamentals, on the other hand, increases the relative importance of transitory factors in investment, reducing the autocorrelation. Holding all else fixed, these two moments allow us to separate the two forces. Similar arguments can be developed for the remaining factors as well. In our quantitative work, where we depart from the polar random walk case, we demonstrate numerically that the same logic carries through.

This logic also emerges in the second part of our analysis, where we dig deeper into factors other than adjustment/information frictions. First, we use moments of labor and materials usage to investigate the role of unobserved firm-specific variation in markups and technologies. Under some conditions, the former is pinned down by the dispersion in materials' share of revenues. For the latter, we show how the observed covariance between the marginal products of capital and labor can be used to derive an upper bound on the potential for heterogeneity in capital intensities. Intuitively, holding overall returns to scale fixed, a high production elasticity of capital implies a low labor elasticity, so this type of heterogeneity in technologies is a source of negative covariance between revenue-capital and revenue-labor ratios. Therefore, the observed correlation between these objects disciplines the potential for misallocation from this channel. Next, we show how policies that affect or restrict firm size and/or financial/liquidity costs can show up as firm-specific factors that are correlated with fundamentals.

We apply our methodology to data on manufacturing firms in China over the period 1998-2009. These data, taken from the Annual Surveys of Industrial Production, represent a census of all state and non-state manufacturing firms above a certain size threshold. Our results show evidence of economically significant adjustment and informational frictions. However, they account for only a relatively modest fraction of observed misallocation among Chinese firms (about 1% and 10% of overall dispersion in the marginal product of capital, respectively). Losses in aggregate total factor productivity (TFP) from these two sources (relative to the undistorted first-best) are 1% and 8%. These findings suggest that a substantial portion of observed misallocation in China is due to other firm-specific factors, both correlated with fundamentals (and therefore, vary over time with the fortunes of the firm) and ones that are essentially permanent. These lead to TFP losses of 38% and 36%, respectively.[3]

[3]Our estimation also allows for distortions that are transitory and uncorrelated with firm characteristics. However, our estimation finds them to be negligible.

We also apply the methodology to data on publicly traded firms in the US. Although the two sets of firms are not directly comparable, the US numbers serve as a useful benchmark to put our results for China in context.[4] As one would expect, the overall degree of misallocation is considerably smaller for publicly traded US firms. More interestingly, a larger share (about 11%) of observed *mrpk* dispersion is accounted for by adjustment costs. Uncertainty and other correlated factors play a smaller role than among Chinese firms, reducing aggregate TFP by 1% and 3%, respectively. However, even for these firms, other firm-specific fixed factors, although considerably smaller in absolute magnitude than in China, remain quite significant as drivers of *mrpk* dispersion. Our estimates suggest eliminating them could increase TFP by as much as 13%. In sum, the US results underscore the importance of factors other than technological and informational frictions in determining the allocation of capital.

How much of these firm-specific factors can be accounted for by variation in markups or technologies? Our results reveal a modest scope for these forms of heterogeneity in China – together, they account for at most 27% of *mrpk* dispersion. In contrast, for US publicly traded firms, they can explain as much as 90%. These findings suggest that unobserved heterogeneity is a promising explanation for the observed 'misallocation' in the US, but that the predominant drivers among Chinese firms lie elsewhere e.g., additional market frictions or institutional/policy-related distortions. Our analysis shows that size-dependent policies and certain forms of financial market imperfections are possible candidates.

Before concluding, we show that these patterns – specifically, the relative contributions of the various forces to observed misallocation – are robust to a number of variations of our baseline setup. For example, they are largely unchanged when we allow for non-convex adjustment costs or when labor is also assumed to be subject to the same frictions and distortions as capital. In the latter instance, since both inputs are affected by each of the forces, the *absolute* importance of all factors – i.e., the impact on aggregate TFP and output – is much higher. For example, adjustment costs and uncertainty in China are estimated to lead to TFP drops of 36% and 32%, respectively, and correlated and permanent factors 144% and 90%. We interpret these estimates as an upper bound, with reality likely falling somewhere in between this and the baseline version with frictionless labor. We also address several potential measurement-related issues, including allowing for sectoral heterogeneity.

The paper is organized as follows. Section 2 describes our model of production and frictional investment. Section 3 spells out our approach to identifying these frictions using the analytically tractable random walk case, while Section 4 details our numerical analysis and

[4]We also report results for Chinese publicly traded firms as well as Colombian and Mexican manufacturing firms. The results regarding the role of various factors in driving misallocation are quite similar to our baseline findings for Chinese manufacturers.

presents our quantitative results. Section 5 further investigates the potential sources of firm-specific idiosyncratic factors. Section 6 explores a number of variants on our baseline approach. We summarize our findings and discuss directions for future research in Section 7. Details of derivations and data work are provided in the Appendix.

Related literature. Our paper relates to several branches of literature. We bear a direct connection to the growing body of work focused on measuring and quantifying the effects of resource misallocation.[5] Following the seminal contributions of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), recent attention has shifted toward analyzing the roles of specific factors in generating misallocation. Important contributions include work by Asker et al. (2014) on adjustment costs, Buera et al. (2011), Moll (2014), Gopinath et al. (2017) and Midrigan and Xu (2014) on financial frictions, David et al. (2016) on uncertainty and Peters (2016) on markup dispersion. Several recent papers study subsets of these factors in combination. For example, Gopinath et al. (2017) show that the interactions of capital adjustment costs and size-dependent financial frictions are important in determining the recent dynamics of misallocation in Spain. Kehrig and Vincent (2017) combine financial and adjustment frictions to investigate misallocation within firms, while Song and Wu (2015) estimate a model with adjustment costs, permanent distortions and heterogeneity in markups/technologies.

Our primary contribution is to develop a unified framework that encompasses many of these factors and devise an empirical strategy based on observable firm-level data to disentangle them. We augment a standard adjustment cost model with information frictions and a flexible class of additional, potentially distortionary, factors. Our modeling of these factors as implicit taxes that can be correlated with fundamentals follows the approach taken by, e.g., Restuccia and Rogerson (2008), Guner et al. (2008), Bartelsman et al. (2013), Buera et al. (2013), Buera and Fattal-Jaef (2016) and Hsieh and Klenow (2014). An analytically tractable special case of our model allows us to prove identification in an intuitive and transparent fashion. Our findings underscore the importance of studying such a broad set of forces in tandem. This breadth is partly what distinguishes us from the work of Song and Wu (2015), who abstract from time-variation in firm-level distortions (as well as in firm-specific markups/technologies), ruling out, by assumption, any role for so-called 'correlated' or size-dependent distortions.[6] Many papers in the literature – e.g., Restuccia and Rogerson (2008), Bartelsman et al. (2013), Hsieh and Klenow (2014) and Bento and Restuccia (2016) – emphasize the need to distinguish such factors from those that are orthogonal to fundamentals. This message is reinforced by our quantitative findings, which reveal a significant role for correlated factors (in addition to

[5] Restuccia and Rogerson (2017) and Hopenhayn (2014) provide recent overviews of this line of work.

[6] We also differ from Song and Wu (2015) in our explicit modeling (and measurement) of information frictions and in our approach to quantifying heterogeneity in markups/technologies.

uncorrelated, permanent ones), particularly in developing countries such as China.

Our methodology and findings also have relevance beyond the misallocation context, perhaps most notably for studies of adjustment and informational frictions. A large literature has examined the implications of adjustment costs, examples of which include Cooper and Haltiwanger (2006), Khan and Thomas (2008) and Bloom (2009). Our analysis shows that accounting for other firm-specific factors acting on firms' investment decisions is potentially crucial in order to accurately estimate the severity of these frictions and reconcile a broader set of micro-level moments. A similar point applies to recent work on quantifying firm-level uncertainty, for example, Bloom (2009), Bachmann and Elstner (2015) and Jurado et al. (2015).

2 The Model

We consider a discrete time, infinite-horizon economy, populated by a representative household. The household inelastically supplies a fixed quantity of labor N and has preferences over consumption of a final good. The household discounts time at rate β. The household side of the economy is deliberately kept simple as it plays a limited role in our study. Throughout the analysis, we focus on a stationary equilibrium in which all aggregate variables remain constant.

Production. A continuum of firms of fixed measure one, indexed by i, produce intermediate goods using capital and labor according to

$$Y_{it} = K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2}, \quad \hat{\alpha}_1 + \hat{\alpha}_2 \leq 1 \; . \tag{1}$$

These intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left(\int \hat{A}_{it} Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} ,$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between intermediate goods and $\hat{A}_{it}$ represents an idiosyncratic demand shifter. This is the only source of fundamental uncertainty in the economy (i.e., we abstract from aggregate risk).

Market structure and revenue. The final good is produced frictionlessly by a representative competitive firm. This yields a standard demand function for intermediate good i:

$$Y_{it} = P_{it}^{-\theta} \hat{A}_{it}^{\theta} Y_t \quad \Rightarrow \quad P_{it} = \left(\frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\theta}} \hat{A}_{it} \; ,$$

where P_{it} denotes the relative price of good i in terms of the final good, which serves as numeraire. Revenues for firm i at time t are

$$P_{it}Y_{it} = Y_t^{\frac{1}{\theta}} \hat{A}_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} \ ,$$

where

$$\alpha_j = \left(1 - \frac{1}{\theta}\right) \hat{\alpha}_j, \quad j = 1, 2 \ .$$

This framework accommodates two alternative interpretations of the idiosyncratic component $\hat{A}_{it}$: as a firm-specific shifter of either demand or productive efficiency, and so we simply refer to $\hat{A}_{it}$ as a firm-specific fundamental.

Input choices. In our baseline analysis, we assume that firms hire labor period-by-period under full information at a competitive wage W_t.[7] At the end of each period, firms choose investment in new capital, which becomes available for production in the following period. Investment is subject to quadratic adjustment costs, given by

$$\Phi\left(K_{it+1}, K_{it}\right) = \frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^2 K_{it} \ , \tag{2}$$

where $\hat{\xi}$ parameterizes the severity of the adjustment cost and δ is the rate of depreciation.[8]

Investment decisions are likely to be affected by a number of additional factors (other than productivity/demand and the level of installed capital). These could originate, for example, from distortionary government policies – e.g., taxes, size restrictions or regulations, or other features of the institutional environment – from other market frictions that are not explicitly modeled – e.g., financial frictions – or from un-modeled heterogeneity in markups/production technologies. For now, we do not take a stand on the precise nature of these additional factors. To capture them, we follow, e.g., Hsieh and Klenow (2009), and introduce a class of idiosyncratic 'wedges' that appear in the firm's optimization problem as proportional taxes on the flow cost of capital. We denote these wedges by T_{it+1}^K and, in a slight abuse of terminology, refer to them as 'distortions' or wedges throughout the paper, even though they may partly reflect sources that are efficient (for example, production function heterogeneity). In Section 5, we demonstrate how progress can be made in further disentangling some of these sources.

The firm's problem in a stationary equilibrium can be represented in recursive form as (we

[7]We relax this assumption in Section 6.2.

[8]We generalize this specification to include non-convex costs in Section 6.1. Our quantitative results change little.

suppress the time subscript on all aggregate variables)

$$\mathcal{V}(K_{it},\mathcal{I}_{it}) = \max_{N_{it},K_{it+1}} \quad \mathbb{E}_{it}\left[Y^{\frac{1}{\theta}}\hat{A}_{it}K_{it}^{\alpha_1}N_{it}^{\alpha_2} - WN_{it} - T_{it+1}^{K}K_{it+1}\left(1-\beta\left(1-\delta\right)\right) - \Phi\left(K_{it+1},K_{it}\right)\right] \\ + \quad \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] ,$$

where $\mathbb{E}_{it}[\cdot]$ denotes the firm's expectations conditional on $\mathcal{I}_{it}$, the information set of the firm at the time of making its period t investment choice. We describe this set explicitly below. The term $1-\beta(1-\delta)$ is the user cost per unit of capital.

After maximizing over N_{it}, this becomes

$$\mathcal{V}(K_{it},\mathcal{I}_{it}) = \max_{K_{it+1}} \quad \mathbb{E}_{it}\left[GA_{it}K_{it}^{\alpha} - T_{it+1}^{K}K_{it+1}\left(1-\beta\left(1-\delta\right)\right) - \Phi\left(K_{it+1},K_{it}\right)\right] \\ + \quad \mathbb{E}_{it}\beta\left[\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] , \tag{3}$$

where $G \equiv (1-\alpha_2)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}$, $A_{it} \equiv \hat{A}_{it}^{\frac{1}{1-\alpha_2}}$ and $\alpha \equiv \frac{\alpha_1}{1-\alpha_2}$ is the curvature of operating profits (revenues net of wages).[9]

Equilibrium. We can now define a *stationary equilibrium* in this economy as (i) a set of value and policy functions for the firm, $\mathcal{V}(K_{it},\mathcal{I}_{it})$, $N_{it}(K_{it},\mathcal{I}_{it})$ and $K_{it+1}(K_{it},\mathcal{I}_{it})$, (ii) a wage W and (iii) a joint distribution over $(K_{it},\mathcal{I}_{it})$ such that (a) taking as given wages and the law of motion for $\mathcal{I}_{it}$, the value and policy functions solve the firm's optimization problem, (b) the labor market clears and (c) the joint distribution remains constant through time.

Characterization. We solve the model using perturbation methods. In particular, we log-linearize the firm's optimality conditions and laws of motion around the undistorted non-stochastic steady state, where $A_{it} = \bar{A}$ and $T_{it}^{K} = 1$. Appendix A.1.1 derives the following log-linearized Euler equation:[10]

$$k_{it+1}\left((1+\beta)\xi + 1 - \alpha\right) = \mathbb{E}_{it}\left[a_{it+1} + \tau_{it+1}\right] + \beta\xi\mathbb{E}_{it}\left[k_{it+2}\right] + \xi k_{it} , \tag{4}$$

where ξ is a composite parameter that captures the degree of adjustment costs and τ_{it+1} summarizes the effect of T_{it+1}^{K} on the firm's investment decision.

[9] Allowing for labor market distortions that manifest themselves in firm-specific wages has no effect on our identification strategy or our results about the sources of *mrpk* dispersion – see Appendix A.2. In Section 6.2, we subject the firm's labor choice to the same frictions – whether due to adjustment costs, informational frictions or distortionary factors – as its capital investment decision and show that this setup leads to a very similar specification with suitably re-defined fundamentals and curvature.

[10] We use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $x_{it} = \log X_{it}$.

Stochastic processes. We assume that A_{it} follows an AR(1) process in logs with normally distributed i.i.d. innovations, i.e.,

$$a_{it} = \rho a_{it-1} + \mu_{it}, \quad \mu_{it} \sim \mathcal{N}\left(0, \sigma_\mu^2\right) , \tag{5}$$

where the parameter ρ is the persistence of firm-level fundamentals and σ_μ^2 the variance of the innovations.

For the distortion, τ_{it}, we adopt a specification that allows for a rich correlation structure, both over time as well as with firm fundamentals. Specifically, τ_{it} is assumed to have the following representation:

$$\tau_{it} = \gamma a_{it} + \varepsilon_{it} + \chi_i, \quad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \chi_i \sim \mathcal{N}\left(0, \sigma_\chi^2\right) , \tag{6}$$

where the parameter γ controls the extent to which τ_{it} co-moves with fundamentals. If $\gamma < 0$, the distortion discourages (encourages) investment by firms with stronger (weaker) fundamentals – arguably, the empirically relevant case. The opposite is true if $\gamma > 0$. The uncorrelated component of τ_{it} has an element, ε_{it}, that is i.i.d. over time and a permanent term, denoted χ_i. Thus, the severity of these factors is summarized by 3 parameters: $(\gamma, \sigma_\varepsilon^2, \sigma_\chi^2)$.

Information. Next, we spell out $\mathcal{I}_{it}$, the information set of the firm at the time of choosing period t investment, i.e., K_{it+1}. This includes the entire history of its fundamental shock realizations through period t, i.e., $\{a_{it-s}\}_{s=0}^{\infty}$. Given the AR(1) structure of uncertainty, this history can be summarized by the most recent observation, namely a_{it}. The firm also observes a noisy signal of the following period's innovation in fundamentals:

$$s_{it+1} = \mu_{it+1} + e_{it+1}, \qquad e_{it+1} \sim \mathcal{N}\left(0, \sigma_e^2\right) ,$$

where e_{it+1} is an i.i.d., mean-zero and normally distributed noise term. This is in essence an idiosyncratic 'news shock,' since it contains information about future fundamentals. Finally, firms also perfectly observe the uncorrelated transitory component of distortions, ε_{it+1} (as well as the fixed component, χ_i) at the time of choosing period t investment. They do not see the correlated component but are aware of its structure, i.e., they know γ.

Thus, the firm's information set is given by $\mathcal{I}_{it} = (a_{it}, s_{it+1}, \varepsilon_{it+1}, \chi_i)$. Direct application of

Bayes' rule yields the conditional expectation of the fundamental a_{it+1}:

$$\begin{aligned} a_{it+1}|\mathcal{I}_{it} &\sim N\left(\mathbb{E}_{it}\left[a_{it+1}\right],\mathbb{V}\right) \quad \text{where} \\ \mathbb{E}_{it}\left[a_{it+1}\right] &= \rho a_{it}+\frac{\mathbb{V}}{\sigma_e^2}s_{it+1}, \quad \mathbb{V}=\left(\frac{1}{\sigma_\mu^2}+\frac{1}{\sigma_e^2}\right)^{-1} . \end{aligned}$$

There is a one-to-one mapping between the posterior variance $\mathbb{V}$ and the noisiness of the signal, σ_e^2 (given the volatility of fundamentals, σ_μ^2). In the absence of any learning (or 'news'), i.e., when σ_e^2 approaches infinity, $\mathbb{V}=\sigma_\mu^2$, that is, all uncertainty regarding the realization of the fundamental shock μ_{it+1} remains unresolved at the time of investment. In this case, we have a standard one period time-to-build structure with $\mathbb{E}_{it}\left[a_{it+1}\right]=\rho a_{it}$. At the other extreme, when σ_e^2 approaches zero, $\mathbb{V}=0$ and the firm becomes perfectly informed about μ_{it+1} so that $\mathbb{E}_{it}\left[a_{it+1}\right]=a_{it+1}$. It turns out to be more convenient to work directly with the posterior variance, $\mathbb{V}$, and so, for the remainder of the analysis, we will use $\mathbb{V}$ as our measure of uncertainty.

Optimal investment. Appendix A.1.1 derives the log-linearized version of the firm's optimal investment policy:

$$k_{it+1}=\psi_1 k_{it}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_3\varepsilon_{it+1}+\psi_4\chi_i \tag{7}$$

where

$$\begin{aligned} \xi\left(\beta\psi_1^2+1\right) &= \psi_1\left((1+\beta)\xi+1-\alpha\right) \\ \psi_2=\frac{\psi_1}{\xi\left(1-\beta\rho\psi_1\right)}, \qquad \psi_3 &= \frac{\psi_1}{\xi}, \qquad \psi_4=\frac{1-\psi_1}{1-\alpha} . \end{aligned} \tag{8}$$

The coefficients ψ_1–ψ_4 depend only on production (and preference) parameters, including the adjustment cost, and are independent of assumptions about information and distortions. The coefficient ψ_1 is increasing and ψ_2-ψ_4 decreasing in the severity of adjustment costs, ξ. If there are no adjustment costs (i.e., $\xi=0$) , $\psi_1=0$ and $\psi_2=\psi_3=\psi_4=\frac{1}{1-\alpha}$. At the other extreme, as ξ tends to infinity, ψ_1 approaches one and ψ_2-ψ_4 go to zero. Intuitively, as adjustment costs become large, the firm's choice of capital becomes more autocorrelated and less responsive to fundamentals and distortions. Our empirical strategy essentially relies on identifying the coefficients in the policy function, ψ_1 and $\psi_2\left(1+\gamma\right)$, from observable moments. Expression (8) shows that, for given values of α and β, we can use the estimate of ψ_1 to compute ξ. Next, we can use that value, along with the estimate of $\psi_2\left(1+\gamma\right)$ to compute γ.

Aggregation. We now turn to the aggregate economy, and in particular, measures of aggregate output and TFP. In Appendix A.1.2, we show that aggregate output can be expressed as

$$\log Y \equiv y = a + \hat{\alpha}_1 k + \hat{\alpha}_2 n \ ,$$

where k and n denote the (logs of the) aggregate stock of capital and labor inputs, respectively, and aggregate TFP, denoted by a, is given by

$$a = a^* - \frac{(\theta\hat{\alpha}_1 + \hat{\alpha}_2)\,\hat{\alpha}_1}{2}\sigma^2_{mrpk} \qquad \frac{da}{d\sigma^2_{mrpk}} = -\frac{(\theta\hat{\alpha}_1 + \hat{\alpha}_2)\,\hat{\alpha}_1}{2} \ , \tag{9}$$

where a^* is the level of TFP in the absence of all frictions (i.e., where static marginal products are equalized) and σ^2_{mrpk} is the cross-sectional dispersion in (the log of) the marginal product of capital ($mrpk_{it} = p_{it}y_{it} - k_{it}$). Thus, aggregate TFP monotonically decreases in the extent of capital misallocation, which in this log-normal world is summarized by σ^2_{mrpk}. The effect of σ^2_{mrpk} on aggregate TFP depends on the elasticity of substitution, θ, and the relative shares of capital and labor in production. The higher is θ, that is, the closer we are to perfect substitutability, the more severe the losses from mis-allocated resources. Similarly, fixing the degree of overall returns to scale in production, for a larger capital share, $\hat{\alpha}_1$, a given degree of misallocation has larger effects on aggregate outcomes.[11]

In our framework, a number of forces – adjustment costs, information frictions, and distortions – will lead to *mrpk* dispersion. Once we quantify their contributions to σ^2_{mrpk}, equation (9) allows us to directly map those contributions to their aggregate implications.

Measuring the contribution of each factor is a challenging task, since all the data moments confound all the factors (i.e., each moment reflects the influence of more than one factor). As a result, there is no one-to-one mapping between moments and parameters – to accurately identify the contribution of any factor, we need to explicitly control for the others. In the following section, we overcome this challenge by exploiting the fact that these forces have different implications for different moments.[12]

[11] Aggregate output effects are larger than TFP losses by a factor $\frac{1}{1-\hat{\alpha}_1}$. This is because misallocation also reduces the incentives for capital accumulation and therefore, the steady-state capital stock.

[12] Asker et al. (2014) make a similar observation – they find that a one period time-to-build model (but no adjustment costs) produces very similar patterns in σ^2_{mrpk} across countries compared to a model with a rich structure of adjustment costs. But, the implications for other moments (e.g. the variability of investment) are quite different – see columns (3) and (5) of Table 9 in that paper, along with the accompanying discussion and footnote 37.

3 Identification

In this section, we lay out our strategy to identify to overcome a primary challenge of our framework – namely, we provide a methodology to tease out the role of adjustment costs, informational frictions and other factors using observable moments from firm-level data on revenues and investment. We use a tractable special case – when firm-level shocks follow a random walk, i.e., $\rho = 1$ – to derive analytic expressions for key moments, allowing us to prove our identification result formally and make clear the underlying intuition. When we return to our general model in the following section, we will demonstrate numerically that this intuition extends to the case with $\rho < 1$.

We assume that the preference and technology parameters – the discount factor, β, the curvature of the profit function, α, and the depreciation rate, δ – are known to the econometrician (e.g., calibrated using aggregate data). The remaining parameters of interest are the costs of capital adjustment, ξ, the quality of firm-level information (summarized by $\mathbb{V}$), and the severity of distortions, parameterized by γ, σ_ε^2 and σ_χ^2.

Our methodology uses a set of carefully chosen elements from the covariance matrix of firm-level capital and fundamentals (since α is assumed known, the latter can be directly measured using data on revenues and capital). Note that $\rho = 1$ implies non-stationarity in levels and so we work with moments of (log) changes. This means that we cannot identify σ_χ^2, the variance of the fixed component.[13] Here, we focus on the four remaining parameters, namely ξ, γ, $\mathbb{V}$ and σ_ε^2. Our main result is to show that these are exactly identified by the following four moments: (1) the autocorrelation of investment, denoted $\rho_{k,k_{-1}}$, (2) the variance of investment, σ_k^2, (3) the correlation of period t investment with the innovations in fundamentals in period $t-1$, denoted $\rho_{k,a_{-1}}$ and (4) the coefficient from a regression of $\Delta mrpk_{it}$ on Δa_{it}, which we denote $\lambda_{mrpk,a}$.

Several of these moments have been used in the literature to quantify the various factors in isolation. For example, $\rho_{k,k_{-1}}$ and σ_k^2 are standard targets in the literature on adjustment costs – see, e.g., Cooper and Haltiwanger (2006) and Asker et al. (2014). The lagged responsiveness to fundamentals, $\rho_{k,a_{-1}}$, is used by Klenow and Willis (2007) in a price setting model to quantify information frictions. The covariance of $mrpk$ with fundamentals – which we proxy with $\lambda_{mrpk,a}$ – is often interpreted as indicative of correlated distortions, e.g., Bartelsman et al. (2013) and Buera and Fattal-Jaef (2016). We will use the tractability of the random walk case to shed light on the necessity of analyzing these moments/factors in tandem (and the potential biases from doing so in isolation).

Our main result is stated formally in the following proposition:

[13] For our numerical analysis in Section 4, we use a stationary model (i.e., with $\rho < 1$) and use σ_{mrpk}^2, a moment computed using levels of capital and fundamentals, to pin down σ_χ^2.

Proposition 1. *The parameters ξ, γ, $\mathbb{V}$ and σ_ε^2 are uniquely identified by the moments $\rho_{k,k_{-1}}$, σ_k^2, $\rho_{k,a_{-1}}$ and $\lambda_{mrpk,a}$.*

3.1 Intuition

The proof of Proposition 1 (in Appendix A.3) involves tedious, if straightforward, algebra. Here, we provide a more heuristic argument that highlights the intuition behind the result. Specifically, we analyze the parameters of interest in pairs and show that they can be uniquely identified by a pair of moments, holding the other parameters fixed. To be clear, this is a local identification argument – our goal here is simply to provide intuition about how the different moments can be combined to disentangle the different forces. The identification result in Proposition 1 is a global one and shows that the four moments uniquely pin down all four parameters.

Adjustment costs and correlated distortions. We begin with adjustment costs, parameterized by ξ and correlated distortions, γ. The relevant moment pair is the variance and autocorrelation of investment, σ_k^2 and $\rho_{k,k_{-1}}$. Both of these moments are commonly used to estimate quadratic adjustment costs – for example, Asker et al. (2014) target the former and Cooper and Haltiwanger (2006) (among other moments), the latter. In our setting, these moments are given by:

$$\sigma_k^2 = \left(\frac{\psi_2^2}{1-\psi_1^2}\right)(1+\gamma)^2\sigma_\mu^2 + \frac{2\psi_3^2}{1+\psi_1}\sigma_\varepsilon^2 \tag{10}$$

$$\rho_{k,k_{-1}} = \psi_1 - \psi_3^2\frac{\sigma_\varepsilon^2}{\sigma_k^2}\ , \tag{11}$$

where the ψ's are defined in equation (8). Our argument rests on the fact that the two forces have similar effects on the variability of investment, but opposing effects on the autocorrelation. To see this, recall that ψ_1 is increasing and ψ_2 and ψ_3 decreasing in the size of adjustment costs, but all three are independent of γ. Then, holding all other parameters fixed, σ_k^2 is decreasing in both the severity of adjustment costs (higher ξ) and correlated factors (more negative γ).[14] The autocorrelation, $\rho_{k,k_{-1}}$, on the other hand, increases with ξ but decreases as γ becomes more negative (through its effect on σ_k^2). Intuitively, while both factors dampen the volatility of investment, they do so for different reasons – adjustment costs make it optimal to smooth investment over time (increasing its autocorrelation) while correlated factors reduce sensitivity to the serially correlated fundamental (reducing the autocorrelation of investment).

[14] The latter is true only for $\gamma > -1$, which is the empirically relevant region.

The top left panel of Figure 1 shows how these properties help identify the two parameters. The panel plots a pair of 'isomoment' curves: each curve traces out combinations of the two parameters that give rise to a given value of the relevant moment, holding the other parameters fixed. Take the σ_k^2 curve: it slopes upward because higher ξ and lower γ have similar effects on σ_k^2 – if γ is relatively small (in absolute value), adjustment costs must be high in order to maintain a given level of σ_k^2. Conversely, a low ξ is consistent with a given value of σ_k^2 only if γ is very negative. An analogous argument applies to the $\rho_{k,k_{-1}}$ isomoment curve: since higher ξ and more negative γ have opposite effects on $\rho_{k,k_{-1}}$, the curve slopes downward. As a result, the two curves cross only once, yielding the unique combination of the parameters that is consistent with both moments. By plotting curves corresponding to the empirical values of these moments, we can uniquely pin down the pair (ξ, γ) (holding all other parameters fixed).

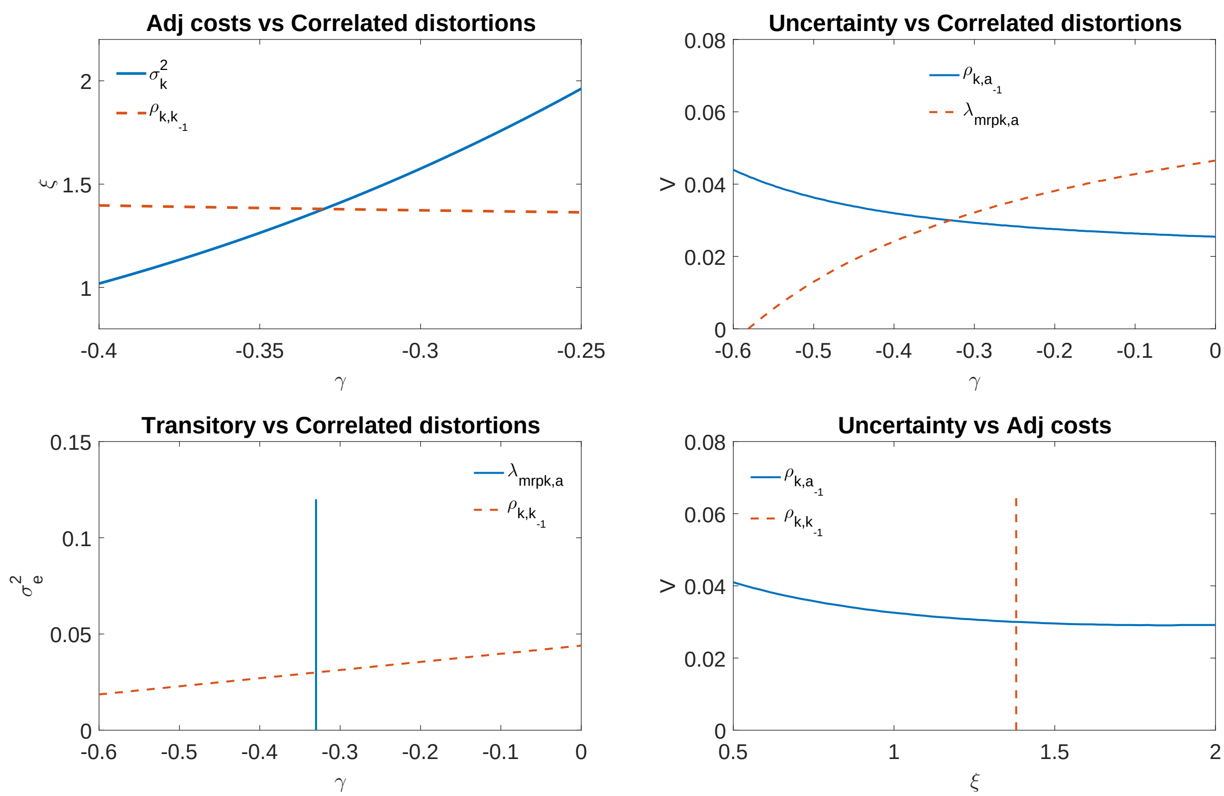


Figure 1: Pairwise Identification - Isomoment Curves

The graph also illustrates the potential bias introduced when examining these forces in isolation. For example, estimating adjustment costs while ignoring correlated distortions (i.e., imposing $\gamma = 0$) puts the estimate on the very right-hand side of the horizontal axis. The estimate for ξ can be read off the vertical height of the isomoment curve corresponding to the targeted moment. Because the σ_k^2 curve is upward sloping, targeting this moment alone leads to an overestimate of adjustment costs (at the very right of the horizontal axis, the curve is above the point of intersection, which corresponds to the true value of the parameters).[15] Targeting $\rho_{k,k_{-1}}$ alone leads to a bias in the opposite direction – since the $\rho_{k,k_{-1}}$ curve is downward sloping, imposing $\gamma = 0$ yields an underestimate of adjustment costs.

The remaining panels in Figure 1 repeat this analysis for other combinations of parameters. Each relies on the same logic as shown in the top left panel.

Uncertainty and correlated distortions. To disentangle information frictions from correlated factors (the top right panel), we use the correlation of investment with past innovations in fundamentals, $\rho_{k,a_{-1}}$, and the regression coefficient $\lambda_{mrpk,a}$. These moments can be written as:

$$\rho_{k,a_{-1}} = \left[\frac{\mathbb{V}}{\sigma_\mu^2}\left(1-\psi_1\right)+\psi_1\right]\frac{\sigma_\mu\psi_2\left(1+\gamma\right)}{\sigma_k} \tag{12}$$

$$\lambda_{mrpk,a} = 1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right). \tag{13}$$

A higher $\mathbb{V}$ implies a higher correlation of investment with lagged fundamental innovations. Intuitively, the more uncertain is the firm, the greater the tendency for its actions to reflect fundamentals with a 1-period lag. In contrast, a higher (more negative) γ increases the relative importance of transitory factors in the firm's investment decision, reducing its correlation with fundamentals. Therefore, to maintain a given level of $\rho_{k,a_{-1}}$, a decrease in $\mathbb{V}$ must be accompanied by a less negative γ, i.e., the isomoment curve slopes downward. On the other hand, higher uncertainty and a more negative gamma both cause *mrpk* to covary more positively with contemporaneous fundamentals, a, leading to an upward sloping $\lambda_{mrpk,a}$ curve. Together, these two curves pin down $\mathbb{V}$ and γ, holding other parameters fixed.

As before, the graph also reveals the direction of bias when estimating these factors in isolation. Assuming full information ($\mathbb{V} = 0$) and using $\lambda_{mrpk,a}$ to discipline the strength of correlated distortions – e.g. as in Bartelsman et al. (2013) and Buera and Fattal-Jaef (2016) – overstates their importance. Using the lagged responsiveness to fundamentals to discipline information frictions while abstracting from correlated factors understates uncertainty.

Transitory and correlated distortions. To disentangle correlated from uncorrelated transitory factors, consider $\lambda_{mrpk,a}$ and $\rho_{k,k_{-1}}$. The former is increasing in the severity of correlated distortions, but independent of transitory ones, implying a vertical isomoment curve. The latter is decreasing in both types of distortions – a more negative γ and higher σ_ε^2 both increase the importance of the transitory determinants of investment, yielding an upward sloping isomoment

[15] This approach would also predict a counter-factually high level of the autocorrelation of investment.

curve.

Uncertainty and adjustment costs. Finally, the bottom right panel shows the intuition for disentangling uncertainty from adjustment costs. An increase in the severity of either of these factors contributes to sluggishness in the response of actions to fundamentals, i.e., raises the correlation of investment with past fundamental shocks $\rho_{k,a_{-1}}$. However, the autocorrelation of investment $\rho_{k,k_{-1}}$ is independent of uncertainty and determined only by adjustment costs (and other factors). Thus, holding those other factors fixed, the autocorrelation of investment in combination with the correlation of investment with lagged shocks jointly pin down the magnitude of adjustment frictions and the extent of uncertainty.

4 Quantitative Analysis

The analytical results in the previous section showed a tight relationship between the moments $\left(\rho_{k,a_{-1}}, \rho_{k,k_{-1}}, \sigma_k^2, \lambda_{mrpk,a}\right)$ and the parameters $(\mathbb{V}, \xi, \sigma_\varepsilon^2, \gamma)$ for the special case of $\rho = 1$. In the first part of this section, we use this insight to develop an empirical strategy for the more general case where fundamentals follow a stationary AR(1) process and apply it to data on Chinese manufacturing firms. This allows us to quantify the severity of the various forces and their impact on misallocation and economic aggregates. For purposes of comparison, we also provide results for publicly traded firms in the US.[16] In the second piece of this section, we extend our methodology to explore some specific candidates behind our general specification of alternative factors.

4.1 Parameterization

We begin by assigning values to the more standard preference and production parameters of our model. We assume a period length of one year and accordingly set the discount factor $\beta = 0.95$. We keep the elasticity of substitution θ common across countries and set its value to 6, roughly in the middle of the range of values in the literature. We assume constant returns to scale in production, but allow the parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ to vary across countries. In the US, we set these to standard values of 0.33 and 0.67, respectively, which implies $\alpha = 0.62$.[17] A number

[16]The two sets of firms are not directly comparable due to their differing coverage (for example, the Chinese data include many more small firms). To address this concern, in Appendix E, we repeat the analysis on the set of Chinese publicly traded firms. We find patterns that are quite similar to those for Chinese manufacturing firms, suggesting that the cross-country differences in the importance of different factors are a robust feature of the data. This conclusion is further supported by results for two additional developing countries, Colombia and Mexico, also presented in Appendix E.

[17]This value is very close to the estimate of 0.59 in Cooper and Haltiwanger (2006). We also directly estimated α following the indirect inference approach in, e.g., Cooper et al. (2015). Specifically, we choose

of recent papers, for example, Bai et al. (2006), have found that capital share's of value-added is about one-half in China and so we set $\hat{\alpha}_1 = \hat{\alpha}_2 = 0.5$ in that country. These values imply an α equal to 0.71 in China.[18]

Next, we turn to the parameters governing the process for fundamentals, a_{it}: the persistence, ρ, and the variance of the innovations, σ_μ^2. Under our assumptions, the fundamental is directly given by (up to an additive constant) $a_{it} = va_{it} - \alpha k_{it}$ where va_{it} denotes the log of value-added. Controlling for industry-year fixed effects to isolate the firm-specific idiosyncratic component of fundamentals, we use a standard autoregression to estimate the parameters ρ and σ_μ^2.[19]

To pin down the remaining parameters – the adjustment cost, ξ, the quality of firm information, $\mathbb{V}$, and the size of other factors, summarized by γ and σ_ε^2 – we follow a strategy informed by the results in the previous section. Specifically, we target the correlation of investment growth with lagged shocks to fundamentals ($\rho_{\iota,a_{-1}}$), the autocorrelation of investment growth ($\rho_{\iota,\iota_{-1}}$), the variance of investment growth (σ_ι^2) and the correlation of the marginal product of capital with fundamentals ($\rho_{mrpk,a}$).[20] Finally, to infer σ_χ^2, the fixed component of distortions in equation (6), we match the overall dispersion in the marginal product of capital, σ_{mrpk}^2, which is clearly increasing in σ_χ^2. Thus, by construction, our parameterized model will match the observed misallocation in the data, allowing us to decompose the contribution of each factor. We summarize our empirical approach in Table 1.

4.2 Data

The data on Chinese manufacturing firms are from the Annual Surveys of Industrial Production conducted by the National Bureau of Statistics. The surveys include all industrial firms (both state-owned and non-state owned) with sales above 5 million RMB (about $600,000).[21] We use data spanning the period 1998-2009. The original data come as a repeated cross-section.

target the coefficient from an OLS regression of value-added on capital and match it to that from an identical regression performed on model simulated data. This procedure also yields $\alpha = 0.62$.

[18]The curvature of the profit function, α, plays a key role in determining the TFP/output implications of a given degree of σ_{mrpk}^2, but does not significantly affect the estimates of the contributions of the various factors, the main focus of this paper. For example, using the same capital share for both countries yields a very similar decomposition of observed misallocation. See also Section 6.2, where introducing labor distortions leads to a higher α, as well as Section 6.4, where we allow for sectoral heterogeneity in α.

[19]It is straightforward to extend our analysis to allow for a firm-specific fixed component in the stochastic process for the fundamental. We performed this exercise on the US data and arrived at very similar results.

[20]We work with the growth rate of investment to partly cleanse the data of firm-level fixed-effects, which have been shown to be a significant component in cross-sectional variation in investment (in the analytical cases studied earlier, we used the level of investment, i.e., the growth rate of capital). See Morck et al. (1990) for a more detailed discussion of this issue. However, in Appendix D.1, we show that our results are largely unchanged if we use the autocorrelation and variance of investment (in levels, rather than growth rates).

[21]Industrial firms correspond to Chinese Industrial Classification codes 0610-1220, 1311-4392 and 4411-4620, which includes mining, manufacturing and utilities.

Table 1: Parameterization - Summary

Parameter	Description	Target/Value
Preferences/production		
θ	Elasticity of substitution	6
β	Discount rate	0.95
$\hat{\alpha}_1$	Capital share	0.33 US/0.50 China
$\hat{\alpha}_2$	Labor share	0.67 US/0.50 China
Fundamentals/frictions		
ρ	Persistence of fundamentals	$\rho_{a,a_{-1}}$
σ_μ^2	Shocks to fundamentals	σ_a^2
$\mathbb{V}$	Signal precision	$\rho_{\iota,a_{-1}}$
ξ	Adjustment costs	$\rho_{\iota,\iota_{-1}}$
γ	Correlated factors	$\rho_{mrpk,a}$
σ_ε^2	Transitory factors	σ_ι^2
σ_χ^2	Permanent factors	σ_{mrpk}^2

A panel is constructed following almost directly the method outlined in Brandt et al. (2014), which also contains an excellent overview of the data for the interested reader. The Chinese data have been used multiple times and are by now familiar in the misallocation literature – for example, Hsieh and Klenow (2009) – although our use of the panel dimension is rather new. The data on US publicly traded firms comes from Compustat North America. We use data covering the same period as for the Chinese firms.

We measure the firm's capital stock, k_{it}, in each period as the value of fixed assets in China and of property, plant and equipment (PP&E) in the US, and investment as the change in the capital stock relative to the preceding period.[22] We construct the fundamental as $a_{it} = va_{it} - \alpha k_{it}$, where we compute value-added from revenues using a share of intermediates of 0.5. Ignoring constant terms that do not affect our calculations, we measure the marginal product of capital as $mrpk_{it} = va_{it} - k_{it}$. First differencing k_{it} and a_{it} gives investment and changes in fundamentals between periods. To isolate the firm-specific variation in our data series, we extract a time-by-industry fixed-effect from each and use the residual as the component that is idiosyncratic to the firm. In both countries, industries are classified at the 4-digit level. This is equivalent to deviating each firm from the unweighted average within its industry in each time period and serves to eliminate any aggregate components, as well as render our calculations to

[22]Our baseline measure of the capital stock uses the book value of assets. In Section 6.4 (details in Appendix D.2), we construct the capital stock using a perpetual inventory method on the US data, following the approach, for example, in Eberly et al. (2012) and re-estimate the model parameters. Although the point estimates are somewhat different, the overall patterns in terms of the role of various factors is unchanged.

be within-industry, which is a standard approach in the literature. After eliminating duplicates and problematic observations (for example, firms reporting in foreign currencies), outliers, observations with missing data etc., our final sample consists of 797,047 firm-year observations in China and 34,260 in the US. Appendix B provides further details on how we build our sample and construct the moments, as well as summary statistics from one year of our data, 2009.[23]

Table 2 reports the target moments for both countries. The first two columns show the fundamental processes, which have similar persistence but higher volatility in China. The remaining columns show that investment growth in China is more correlated with past shocks, is more volatile and less autocorrelated, that there is a higher correlation between firm fundamentals and *mrpk*, and that the overall dispersion in the *mrpk* is substantially higher than among publicly traded US firms. This variation will lead us to find significant differences in the severity of investment frictions and distortions across the two sets of firms.

Table 2: Target Moments

	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
China	0.91	0.15	0.29	−0.36	0.76	0.14	0.92
US	0.93	0.08	0.13	−0.30	0.55	0.06	0.45

4.3 Identification

Before turning to the estimation results, we revisit the issue of identification. Although we no longer have analytical expressions for the mapping between moments and parameters, we use a numerical experiment to show that the intuition developed in Section 3 for the random walk case applies here as well. In that section, we used a pairwise analysis to demonstrate how various moments combine to help disentangle the various sources of observed misallocation. Here, we repeat that analysis by plotting numeric isomoment curves in Figure 2, using the moments and parameter values for US firms (from Tables 2 and 3, respectively). The graph reveals the same broad patterns as Figure 1, indicating that the logic of that special case goes through here as well.[24]

[23] We have also examined the moments year-by-year. They are reasonably stable over time.

[24] The differences in the precise shape of some of the curves in the two figures come partly from the departure from the random walk case and also from the fact that they use slightly different moments (Figure 2 works with changes in investment and $\rho_{mrpk,a}$ while Figure 1 used changes in k and $\lambda_{mrpk,a}$).

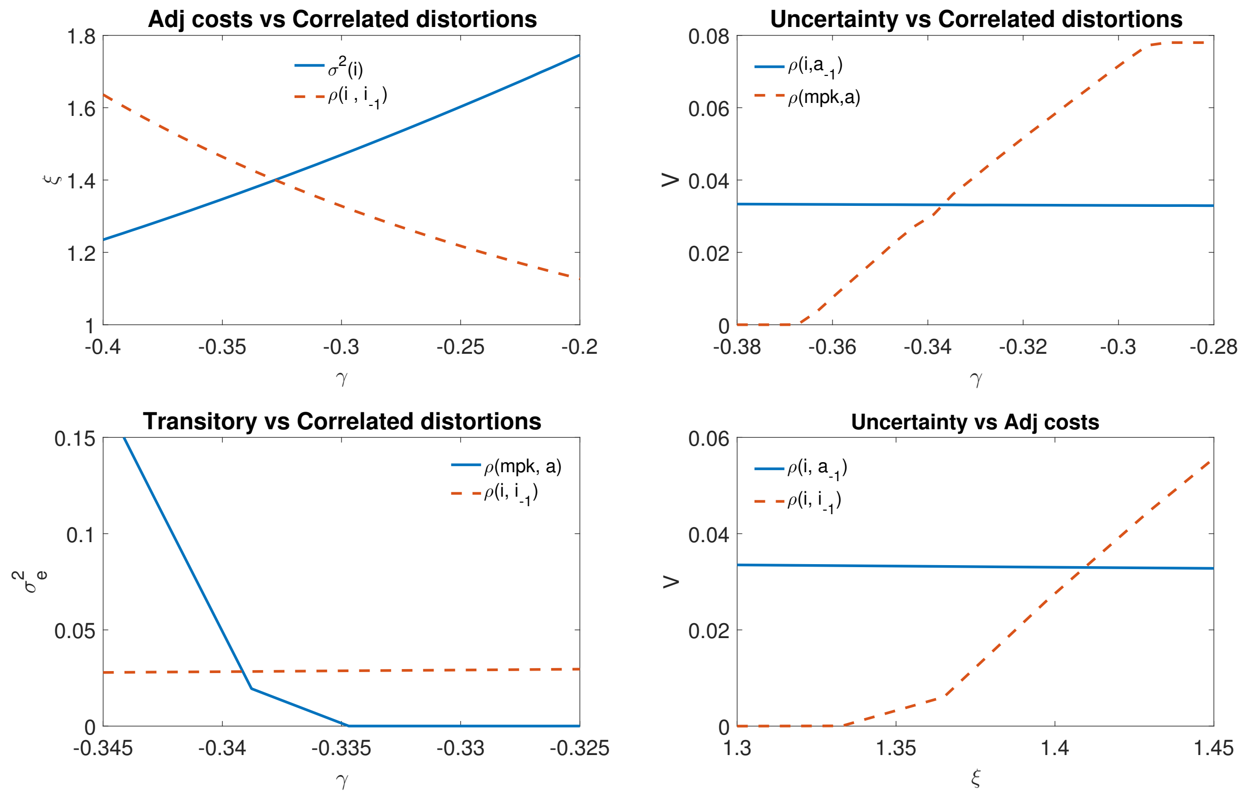


Figure 2: Isomoment Curves - Quantitative Model

4.4 The Sources of Misallocation

Table 3 contains our baseline results. In the top panel we display the parameter estimates. In the second panel, we report the contribution of each factor to dispersion in the *mrpk*, which we denote $\Delta\sigma^2_{mrpk}$.[25] These are calculated under the assumption that only the factor of interest is operational, i.e., in the absence of the others, so that the contribution of each one is measured relative to the undistorted first-best.[26] The third panel expresses this contribution as a percentage of the total *mrpk* dispersion measured in the data, denoted $\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$. Because of interactions between the factors, there is no *a priori* reason to expect these relative contributions to sum to one. In practice, however, we find that the total is reasonably close to one, allowing us to interpret this exercise as a decomposition of total observed misallocation. In the bottom panel of the table, we compute the implied losses in aggregate TFP, again relative to the

[25]For adjustment costs, we do not have an analytic mapping between the severity of these costs and σ^2_{mrpk}, but this is a straightforward calculation to make numerically; for each of the other factors, we can compute their contributions to misallocation analytically.

[26]An alternative would be to calculate the contribution of each factor holding the others constant at their estimated values. It turns out that the interactions between the factors are small at the estimated parameter values, so the two approaches yield similar results. Table 9 in Appendix C shows that the effects of each factor on *mrpk* dispersion in the US are close under either approach. Interaction effects are even smaller in China.

undistorted first-best level, i.e., $\Delta a = a^* - a$. Once we have the contribution of each factor to $mrpk$ dispersion, computing these values is simply an application of expression (9).

Table 3: Contributions to Misallocation

			Other Factors		
	Adjustment Costs	Uncertainty	Correlated	Transitory	Permanent
Parameters	ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
China	0.13	0.10	−0.70	0.00	0.41
US	1.38	0.03	−0.33	0.03	0.29
$\Delta\sigma^2_{mrpk}$					
China	0.01	0.10	0.44	0.00	0.41
US	0.05	0.03	0.06	0.03	0.29
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$					
China	1.3%	10.3%	47.4%	0.0%	44.4%
US	10.8%	7.3%	14.4%	6.3%	64.7%
Δa					
China	0.01	0.08	0.38	0.00	0.36
US	0.02	0.01	0.03	0.01	0.13

Adjustment costs. Our results show evidence of economically significant adjustment frictions. For example, for US firms, the estimate of $\xi = 1.38$ in Table 3 implies a value of 0.2 for the primitive parameter $\hat{\xi}$ in the adjustment cost function (2).[27] Though differences in datasets and methods complicate direct comparisons with earlier estimates in the literature, this is within the range, albeit towards the lower end, of those estimates. For example, Asker et al. (2014) report an estimate of 8.8 for their convex adjustment cost parameter for US manufacturing firms. To interpret this difference, consider a firm that doubles its capital stock in a year. Our estimate for $\hat{\xi}$ implies that such a firm would incur adjustment costs equal to about 11% of the value of this investment, whereas the corresponding figure using the Asker et al. (2014) estimate would be 60%. Our estimates are closer, and slightly higher than Cooper and Haltiwanger (2006), who find $\hat{\xi} = 0.05$ for US manufacturing firms and Bloom (2009), who finds a value of zero using Compustat data.[28]

Apart from the differences across these studies in time period and the set of firms, these estimates also vary for the reasons highlighted in Section 3. There, we discussed the poten-

[27]The mapping between ξ and $\hat{\xi}$ is derived in equation (21) in Appendix A.1.1. We use an annual depreciation rate of $\delta = 0.10$.

[28]These are estimates of the quadratic component alongside additional components in the cost function, e.g., fixed costs. Section 6.1 shows that our estimate of $\hat{\xi}$ changes little in the presence of a fixed component.

tial bias in estimating adjustment costs in isolation, i.e., without controlling for other factors correlated with fundamentals. The papers mentioned abstract from these factors, though they target different moments. For example, Asker et al. (2014) match the overall variability of investment (among other moments), but do not try to match the autocorrelation, while Cooper and Haltiwanger (2006) do the reverse. As the arguments in Section 3 showed, in the presence of correlated factors, the former strategy would tend to overstate the true extent of adjustment frictions, while the latter would understate it. This lies at the heart of the difference in estimates.[29] To show this more clearly, we also estimated a version of our model in which we abstract from the other forces and parameterize the adjustment cost to match a single moment in the data. If we target the volatility of investment growth, σ_ι^2, this procedure yields a considerably larger estimate of ξ of 2.3 in the US, about 60% higher than the baseline value. However, the implied autocorrelation of investment growth from this approach is much higher than that observed in the data, -0.17 vs a true value of -0.30, exactly the pattern predicted by the theory. A strategy targeting only the autocorrelation leads to the opposite conclusion (a lower estimated value for ξ), but at the cost of a counterfactually high variability of investment. This exercise partly explains the range of estimates of adjustment costs in the literature – when adjustment costs are estimated without explicitly controlling for other factors, the results can be quite sensitive to the particular moments chosen.[30] Indeed, our results suggest that explicitly accounting for these additional factors is essential in order to reconcile a broad set of moments in firm-level investment dynamics.

The estimated value of ξ is significantly lower in China compared to the US. Intuitively, investment in China is both more volatile and less serially correlated than for US firms. Together with the other moments, this implies a lower degree of adjustment frictions. Importantly, as was the case with US firms, one would reach very different conclusions from examining a model with only adjustment costs. For example, a strategy of estimating such a model by targeting σ_ι^2 in China yields an estimate for ξ of about 1.5, roughly 10 times larger than the one in Table 3.

In both countries, however, the estimated adjustment costs do not contribute significantly to observed misallocation. This is particularly so in China – if this were the only friction, *mrpk* dispersion would be 0.01, which is about 1% of the observed σ_{mrpk}^2. As we would expect from the higher estimate of ξ, the contribution of adjustment costs in the US is higher, though still

[29] Another potential source of differences is the fact that we work with investment growth rates rather than levels, unlike those papers. It turns out that this makes only a small difference for the results. In Appendix D.1 we re-estimate our model targeting the variance and autocorrelation of investment in levels (instead of the corresponding moments in growth rates). The estimate of ξ changes only slightly.

[30] For example, Table IV in Bloom (2009) highlights the wide variation in these estimates, ranging from zero to as high as 20.

modest (by themselves, adjustment costs lead to *mrpk* dispersion of 0.05, about 11% of the observed σ^2_{mrpk}). The corresponding losses in aggregate TFP are about 1% and 2% in the two countries, respectively.

This does not mean the adjustment costs are irrelevant for understanding firm-level investment dynamics. To see this, consider the implications of setting adjustment costs to zero in the US while holding the other parameters at their estimated values: the variance of investment growth spikes to 1.68 (compared to 0.06 in the data) and the autocorrelation drops to -0.62 (data: -0.30). However, σ^2_{mrpk} falls only modestly, from 0.45 to 0.41. Re-estimating the model without adjustment costs (and dropping the autocorrelation as a target) also leads to a counterfactually low autocorrelation (-0.43).[31] In other words, while adjustment frictions are an important determinant of investment dynamics, they do not generate significant dispersion in static marginal products.

Uncertainty. Table 3 shows that firms in both countries make investment decisions under considerable uncertainty, with the information friction more severe for Chinese firms. As a share of the prior uncertainty, σ^2_μ, residual uncertainty, $\frac{\mathbb{V}}{\sigma^2_\mu}$, is 0.42 in the US and 0.63 in China.[32] In an environment where imperfect information is the only friction, we have $\sigma^2_{mrpk} = \mathbb{V}$, so the contribution of uncertainty alone to observed misallocation can be directly read off the second column in Table 3 – namely 0.10 in China and 0.03 in the US. These represent about 10% and 7% of total *mrpk* dispersion in the two countries, respectively. The implications for aggregate TFP are substantial in China – losses are about 8% – and are lower in the US, about 1%. Note, however, that imposing a one period time-to-build assumption where firms install capital in advance without any additional information about innovations in fundamentals, i.e. setting $\mathbb{V} = \sigma^2_\mu$, would overstate the role of uncertainty (and bias the estimates of adjustment costs and other parameters). Indeed, doing so yields estimates of $\mathbb{V}$ that are about 55% higher in China and a factor of 2.5 times higher in the US.

'Distortions'. The last three columns of Table 3 show that other, potentially distortionary, factors play a significant role in generating the observed *mrpk* dispersion in both countries.

[31]The estimates for the other parameters change as well: notably, γ becomes more negative in order to match σ^2_ι.

[32]Our values for $\frac{\mathbb{V}}{\sigma^2_\mu}$ are similar to those in David et al. (2016), who find 0.41 and 0.63 for publicly traded firms in the US and China, respectively. The absolute values of $\mathbb{V}$ are different but are not directly comparable – David et al. (2016) focus on longer time horizons (they analyze 3-year time intervals). This might lead one to conclude that ignoring other factors – as David et al. (2016) do – leads to negligible bias in the estimate of uncertainty. But, this is not a general result and rests on the fact that adjustment costs and uncorrelated distortions are estimated to be modest. Then, as Figure 2 shows, the sensitivity of actions to signals turns out to be a very good indicator of uncertainty. If, on the other hand, adjustment costs and/or uncorrelated factors were much larger, the bias from estimating $\mathbb{V}$ alone can be quite significant.

Turning first to the correlated component, the negative values of γ suggest that they act to disincentivize investment by more productive firms and especially so in China. The contribution of these distortions to *mrpk* dispersion is given by $\gamma^2\sigma_a^2$, which amounts to 0.44 in China, or 47% of total misallocation. The associated aggregate consequences are also quite sizable – TFP losses from these sources are 38%. In contrast, the estimate of γ in the US is significantly less negative than in China, suggesting that these types of correlated factors are less of an issue for firms in the US, both in an absolute sense – the *mrpk* dispersion from these factors in the US is 0.06, less than one-seventh that in China – and in relative terms – they account for only 14% of total observed *mrpk* dispersion in the US. The corresponding TFP effects are also considerably smaller for the US - losses from correlated sources are only about 3%.

Next, we consider the role of distortions that are uncorrelated with firm fundamentals. Table 3 shows that purely transitory factors (measured by σ_ε^2) are negligible in both countries, but permanent firm-specific factors (measured by σ_χ^2) play a prominent role. Their contribution to *mrpk* dispersion, which is also given by σ_χ^2, amounts to 0.41 in China and 0.29 in the US. Thus, their absolute magnitude in the US is considerably below that in China, but in relative terms, these factors seem to account for a substantial portion of measured misallocation in both countries. The aggregate consequences of these types of distortions are also significant, with TFP losses of 36% in China and about 13% in the US.

In sum, the estimation results point to the presence of substantial distortions to investment, especially in China, where they disproportionately disincentivize investment by more productive firms. What patterns in the data lead us to this conclusion? The *mrpk* in both countries shows significant dispersion and a high correlation with fundamentals, indicating a dampened response of investment to fundamentals. In principle, this pattern could emerge from adjustment costs, imperfect information or correlated distortions. However, the autocorrelation of investment growth, $\rho_{\iota,\iota_{-1}}$, in the data is relatively low, which bounds the severity of adjustment frictions. Similarly, the response of investment to past shocks, $\rho_{\iota,a_{-1}}$, is also modest, limiting the role of the informational friction. Hence, the estimation assigns a substantial role to correlated distortions, particularly in China, as well as fixed distortions, in order to generate the observed patterns in the *mrpk*.[33] Section 6 shows that this result is robust to a number of modifications to our baseline setup, e.g. allowing for non-convex adjustment costs, a frictional labor choice and additive measurement error. Further, we have applied the methodology to data on Colombian and Mexican firms (in addition to the set of publicly traded firms in China) - the results resemble those for Chinese manufacturing firms, in that they point to a substantial role for correlated factors, as well as fixed ones (details are in Appendix E).

[33] A high value for $\rho_{mrpk,a}$ also limits the scope for uncorrelated transitory distortions as an important driver of investment decisions.

5 Firm-Specific Factors: Some Candidates

In this section, we dig deeper into the firm-specific factors contributing to observed misallocation. Specifically, we extend our baseline framework and empirical methodology to investigate three potential sources – heterogeneity in markups and production technologies, size-dependent policies and financial considerations.

5.1 Heterogeneity in Markups and Technologies

In our baseline setup, all firms within an industry (1) had homogeneous production technologies and (2) were monopolistically competitive facing CES demand curves and therefore, have identical markups. As a result, unobserved firm-level heterogeneity in technologies and/or markups would show up in our estimates for firm-specific factors. Here, we explore this possibility using a modified version of our baseline model which allows for such heterogeneity. This requires more assumptions as well as additional data, but it allows to provide an upper bound on the contribution of these elements to observed misallocation.

We begin by generalizing the production function from Section 2 to include intermediate inputs and to allow for (potentially time-varying) heterogeneity in capital intensities. Specifically, the output of firm i is now given by

$$Y_{it} = K_{it}^{\hat{\alpha}_{it}} N_{it}^{\hat{\zeta}-\hat{\alpha}_{it}} M_{it}^{1-\hat{\zeta}} \ ,$$

where M_{it} denotes intermediate or materials input. In what follows, we abstract from adjustment/information frictions in firms' input decisions. This is largely in the interest of simplicity, but it can also be justified by the relatively modest role played by these dynamic considerations in our baseline estimates.[34] Capital and labor choices are each subject to a factor-specific 'distortion' (in addition to the markup), denoted T_{it}^{K} and T_{it}^{N}, respectively. The choice of intermediates is distorted by the firm-specific markup. Under this structure, we can put to use the powerful methodology pioneered by De Loecker and Warzynski (2012) to measure markups at the firm level without taking a stand on the nature of competition/demand.[35]

[34]It is possible to extend the identification methodology from Section 3 to explicitly include heterogeneity in $\hat{\alpha}$ and markups. Although this would require more assumptions (e.g., on the correlation structure of markups/technologies with fundamentals and over time) and make the intuition more complicated, the basic insights should still go through.

[35]The method is also robust to the presence of distortions in the market for intermediate inputs, so long as they are reflected in the price that the firm pays. In other words, even if firms pay idiosyncratic prices for intermediate inputs, the method accurately identifies markup dispersion.

The contribution of markup dispersion. Identification of markup dispersion makes use of the following optimality condition from the firm's cost minimization problem:

$$P_t^M = MC_{it}\left(1-\hat{\zeta}\right)\frac{Y_{it}}{M_{it}} \quad \Rightarrow \quad \frac{P_t^M M_{it}}{P_{it}Y_{it}} = (1-\hat{\zeta})\frac{MC_{it}}{P_{it}}\ , \tag{14}$$

where P_t^M is the price of materials and MC_{it} is the marginal cost of the firm. This condition states that, at the optimum, the firm sets the materials share in gross output equal to the inverse of the markup, $\frac{MC_{it}}{P_{it}}$, multiplied by the materials elasticity $1-\hat{\zeta}$.

Expression (14) suggests a simple way to estimate the cross-sectional dispersion in markups. The left-hand side is the materials' share of revenue – the dispersion in this object (in logs) maps one-for-one into (log) markup dispersion across firms.[36] The results of applying this procedure are reported in the first rows of the two panels in Table 4. The variance of the share of materials in revenue is about 0.09 in the US Compustat data and 0.05 in China, accounting for about 28% of σ^2_{mrpk} among the US firms, but only about 4% of σ^2_{mrpk} among Chinese manufacturing firms. Thus, markup heterogeneity composes a significant fraction of observed misallocation among US publicly traded firms but seems to be an almost negligible force in China.

The contribution of heterogeneity in technology. Cost minimization also implies that the average revenue products of capital and labor are given by:[37]

$$\log\left(\frac{P_{it}Y_{it}}{K_{it}}\right) = \log\frac{P_{it}}{MC_{it}} - \log\hat{\alpha}_{it} + \tau_{it}^K + \text{Constant} \tag{15}$$

$$\log\left(\frac{P_{it}Y_{it}}{N_{it}}\right) = \log\frac{P_{it}}{MC_{it}} - \log(\hat{\zeta}-\hat{\alpha}_{it}) + \tau_{it}^N + \text{Constant} \tag{16}$$

$$\approx \log\frac{P_{it}}{MC_{it}} + \frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\log\hat{\alpha}_{it} + \tau_{it}^N + \text{Constant}\ , \tag{17}$$

where τ_{it}^K and τ_{it}^N are the logs of the capital and labor wedges T_{it}^K and T_{it}^N, respectively, and $\bar{\alpha}$ is the average capital elasticity across firms.[38] Observed average revenue products are combinations of the firm-specific production elasticities as well as markups and distortionary factors. Importantly, the expressions reveal that the capital elasticity, $\hat{\alpha}_{it}$ has opposing effects on the average products of capital and labor. Specifically, firms with a high $\hat{\alpha}_{it}$ will, *ceteris paribus*, tend to have a low average product of capital and a high average product of labor. This property enables us to use the observed covariance of the average products to bound the extent of

[36] Note that this assumes no heterogeneity in the materials elasticity, $1-\hat{\zeta}$. To the extent there is heterogeneity in $1-\hat{\zeta}$ that is uncorrelated (or negatively correlated) with markups, the strategy would overestimate markup variation.

[37] See Appendix A.4 for details.

[38] The third equation is derived by log-linearizing (16) around $\hat{\alpha}_{it} = \bar{\alpha}$.

variation in $\hat{\alpha}_{it}$. Let

$$arpk_{it} \equiv \log\left(\frac{P_{it}Y_{it}}{K_{it}}\right) - \log\left(\frac{P_{it}}{MC_{it}}\right)$$
$$arpn_{it} \equiv \log\left(\frac{P_{it}Y_{it}}{N_{it}}\right) - \log\left(\frac{P_{it}}{MC_{it}}\right)$$

denote the markup-adjusted average revenue products of capital and labor. Appendix A.4 proves the following result:

Proposition 2. *Suppose* $\log\hat{\alpha}_{it}$ *is uncorrelated with the distortions* τ_{it}^{K} *and* τ_{it}^{N}*. Then, the cross-sectional dispersion in* $\log\hat{\alpha}_{it}$ *satisfies*

$$\sigma^2(\log\hat{\alpha}_{it}) \leq \frac{\sigma^2_{arpk}\sigma^2_{arpn} - cov(arpk, arpn)^2}{2\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}cov(arpk, arpn) + \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2\sigma^2_{arpk} + \sigma^2_{arpn}}. \tag{18}$$

The bound in (18) is obtained by setting the correlation between the distortionary factors τ_{it}^{K} and τ_{it}^{N} to 1. Given the observed second moments of $(arpk_{it}, arpn_{it})$, this maximizes the potential for variation in $\hat{\alpha}_{it}$, which, as noted earlier, is a source of negative correlation between $arpk_{it}$ and $arpn_{it}$. The expression for the bound reveals the main insight: the more positive the covariance between $(arpk_{it}, arpn_{it})$, the lower is the scope for heterogeneity in $\hat{\alpha}_{it}$.

To compute this bound for the two countries, we set $\hat{\zeta}$, the share of materials in gross output, to 0.5. The results, along with the moments, are reported in Table 4. Heterogeneous technologies can potentially account for a substantial portion of σ^2_{mrpk} in the US - as much as 62% - and a more modest, though still significant, fraction in China, about 23%.[39] The last row of Table 4 shows that in total, unobserved heterogeneity in markups and technologies can potentially explain as much as 90% of measured misallocation in the US and at most about 27% in China.

Hsieh and Klenow (2009) perform an alternative experiment to bound the role of technological heterogeneity: they attribute all the variation in firm-level capital-labor ratios to heterogeneity in $\hat{\alpha}_{it}$. In our setting, this amounts to assuming that $\tau_{it}^{K} = \tau_{it}^{N}$, which implies:

$$k_{it} - n_{it} = arpn_{it} - arpk_{it} \approx \frac{\hat{\zeta}}{\hat{\zeta}-\bar{\alpha}}\log\hat{\alpha}_{it} \quad \Rightarrow \quad \sigma^2(k_{it} - n_{it}) = \left(\frac{\hat{\zeta}}{\hat{\zeta}-\bar{\alpha}}\right)^2\sigma^2(\log\hat{\alpha}_{it}).$$

[39] There is some evidence that the share of intermediates may be higher in China than the US, see, e.g., Table 1 in Brandt et al. (2014). We re-computed the bound with $\hat{\zeta} = 0.25$ and obtained very similar results. We also verified the accuracy of the approximation by working directly with (16) instead of the log-linearized version in (17). This yielded bounds that were slightly lower for both countries: 53% and 17% of σ^2_{mrpk} in the US and China, respectively.

Table 4: Heterogeneous Markups and Technologies

	China		US	
Moments				
$\sigma^2\left(\log \frac{P_{it}Y_{it}}{P_t^M M_{it}}\right)$	0.05		0.09	
$\text{cov}\left(arpk_{it}, arpn_{it}\right)$	0.41		0.12	
$\sigma^2\left(arpk_{it}\right)$	1.37		0.41	
$\sigma^2\left(arpn_{it}\right)$	0.76		0.25	
Estimated $\Delta\sigma^2_{mrpk}$				
Dispersion in Markups	0.05	(3.8%)	0.09	(28.3%)
Dispersion in $\log\hat{\alpha}_{it}$	0.30	(23.1%)	0.19	(62.2%)
Total	0.35	(26.9%)	0.28	(90.5%)

Notes: The values in parentheses in the bottom panel are the contributions to $mrpk$ dispersion expressed as a fraction of total σ^2_{mrpk}.

This procedure yields estimates for $\sigma^2\left(\log\hat{\alpha}_{it}\right)$ that are quite close to those in Table 4: 0.27 (compared to 0.30) for China and 0.16 (compared to 0.19) in the US.

5.2 Size-Dependent Policies

Our baseline results showed a significant role for factors correlated with firm-level fundamentals, especially in developing countries such as China. Here, we discuss how policies that affect or restrict the size of firms have very similar effects. A number of papers have pointed out the prevalence of distortionary size-dependent policies across a range of countries, for example, Guner et al. (2008). Many of these policies take the form of restrictions (or additional costs) associated with acquiring capital and/or other inputs. To be clear, our goal is not to explore the role of a particular policy in China or the US. Rather, we show how policies that are common in a number of countries can generate patterns that are, in a sense, isomorphic to factors correlated with fundamentals.

To analyze the effects of such policies, we generalize our baseline specification of firm-specific factors in equation (6) to also allow for factors that vary systematically with chosen level of capital. Formally,

$$\tau_{it} = \gamma_k k_{it} + \gamma a_{it} + \varepsilon_{it} + \chi_i \ ,$$

where the parameter γ_k indexes the severity of these additional factors. The empirically relevant case is when $\gamma_k < 0$, which implicitly penalizes larger firms. This specification captures the essence of the policies discussed above in a tractable way (e.g., it allows to continue to use perturbation methods). With this formulation, the log-linearized Euler equation takes the form

$$k_{it+1}\left((1+\beta)\xi+1-\alpha-\gamma_k\right)=(1+\gamma)\,\mathbb{E}_{it}\left[a_{it+1}\right]+\varepsilon_{it+1}+\chi_i+\beta\xi\mathbb{E}_{it}\left[k_{it+2}\right]+\xi k_{it}\ . \tag{19}$$

Expression (19) is identical to expression to (4), but with $\alpha+\gamma_k$ taking the place of α. It is straightforward to derive the firm's investment policy function and verify that the same adjustment goes through, i.e., expressions (7) and (8) hold, with α everywhere replaced by $\alpha+\gamma_k$. Intuitively, the size-dependent component, γ_k, changes the effective degree of curvature in the firm's investment problem – although the curvature of the profit function remains α, the firm acts as if it is $\alpha+\gamma_k$. If $\gamma_k<0$, the distortion dampens the responsiveness of investment to shocks. If $\gamma_k>0$, the responsiveness of investment is amplified.

Importantly, these effects are broadly similar to those coming from γ: indeed, if γ_k were the only factor distorting investment choices, the implied law of motion for k_{it} is identical (up to a first-order) to one with only productivity-dependent factors, where $\gamma=\frac{\gamma_k}{1-\alpha-\gamma_k}$. The implication of this isomorphism is that we cannot distinguish the two factors using observed series of capital and revenues alone. This challenge also applies to the case when other factors are present, though the mapping between the two is more complicated (and affects the other parameters as well). We detail this mapping in Appendix A.5.

What about the contribution to misallocation? Table 5 reports the results for Chinese firms for two different values of γ_k, namely -0.18 and -0.36 (these values imply effective curvatures $\alpha+\gamma_k$ equal to one-quarter and one-half of the true α, respectively). The table shows two key results – first, a more negative γ_k reduces the estimated γ (i.e., makes it less negative), suggesting that our baseline estimate of correlated factors could be picking up such size-dependent policies. The total contribution of both types of correlated distortions remains quite stable, ranging between 40% and 47%. Second, the estimates of adjustment costs remain quite modest over this wide range of curvature.

5.3 Financial Frictions

In this section, we lay out an extension of our model that subjects firms to liquidity costs and show that they can be mapped to the size-dependent distortions analyzed in the previous subsection. We assume that firms face a liquidity cost $\Upsilon\left(K_{it+1},B_{it+1}\right)$, where B_{it+1} denotes holdings of liquid assets, which earn an exogenous rate of return $R<\frac{1}{\beta}$. The cost is increasing (decreasing) in K_{it+1} (B_{it+1}). This specification captures the idea that firms need costly liquidity in order to operate (e.g., to meet working capital needs). Using a continuous penalty function rather than an occasionally binding constraint allows us to continue using perturbation methods. Note also that this differs from the standard borrowing constraint used widely in the literature

Table 5: Size vs Productivity-Dependent Factors

	Correlated Factors			
	Size-Dependent γ_k	Prod.-Dependent γ	Total	Adj. Costs ξ
$\alpha+\gamma_k=0.71$ (baseline)				
Parameters	0.00	-0.70		0.13
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	0.0%	47.4%	47.4%	1.3%
$\alpha+\gamma_k=0.54$				
Parameters	-0.18	-0.51		0.21
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	14.2%	25.4%	39.6%	2.3%
$\alpha+\gamma_k=0.36$				
Parameters	-0.36	-0.33		0.29
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	29.6%	10.2%	39.8%	3.2%

on financial frictions. Our firms are not constrained in terms of their ability to raise funds. This implies that self-financing, which often significantly weakens the long-run bite of borrowing constraints, plays no role here.[40]

We use the following flexible functional form for the liquidity cost:

$$\Upsilon\left(K_{it+1},B_{it+1}\right)=\hat{\nu}\frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2}}\ ,$$

where $\hat{\nu}$, ω_1 and ω_2 are all positive parameters. The marginal liquidity cost of capital, after optimizing over the choice of B_{it+1} is given by (derivations in Appendix A.6)

$$\Upsilon_{1,t+1}\equiv\frac{d\Upsilon\left(K_{it+1},B_{it+1}\right)}{dK_{it+1}}=\nu\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}K_{it+1}^{\omega}\ , \tag{20}$$

where ν and ω are composite parameters. The former is always positive, while the latter is of indeterminate sign. If ω is positive (negative), the marginal cost of liquidity is increasing (decreasing) in K_{it+1}.

The log-linearized Euler equation takes the same form as (19), with

$$\gamma_k=-\omega\left(\frac{\bar{\Upsilon}_1}{\bar{\Upsilon}_1+\kappa}\right)\ ,$$

[40]See, for example, Midrigan and Xu (2014) and Moll (2014). Gopinath et al. (2017) show that a richer variant of the standard collateral constraint can have important implications during a period of transition, even if it generates only modest amounts of misallocation in the long-run.

where $\bar{\Upsilon}_1$ is the marginal cost of liquidity in the deterministic steady state and $\kappa = 1 - \beta(1 - \delta) + \hat{\xi}\delta(1 - \beta(1 - \frac{\delta}{2}))$. Intuitively, the fraction $\frac{\bar{\Upsilon}_1}{\bar{\Upsilon}_1 + \kappa}$ is the share of liquidity in the marginal user cost of capital in the steady state. The result shows that liquidity considerations manifest themselves as a size-dependent factor of the form described in Section 5.2. The sign depends on the sign of ω: if $\omega > 0$, then $\gamma_k < 0$, so costly liquidity dampens incentives to adjust capital in response to fundamentals (since the liquidity cost is convex). The opposite happens if $\omega < 0$.

Thus, liquidity considerations are a promising candidate for correlated and/or size-dependent factors. Cross-country differences in liquidity requirements (summarized by the parameters ν and ω) and/or costs (i.e., $1 - \beta R$) will translate into variation in the severity of our measures of correlated firm-specific factors. However, our results here also highlight the difficulty in separating them from other factors using production-side data alone. One would need additional data, e.g., on firm-level liquidity holdings, to disentangle the role of liquidity from other forces.

In sum, our findings in sections 5.1-5.3 provide some guidance on the factors beyond adjustment and information frictions that influence investment decisions. For US publicly traded firms, observed dispersion in capital-output ratios could be driven to a large extent by unobserved heterogeneity in production technologies and therefore, may not be a sign of misallocated capital. On the other hand, the scope for this type of heterogeneity appears limited among Chinese manufacturing firms, suggesting a greater role for inefficient factors like size-dependent policies or financial imperfections.

6 Robustness and Extensions

In this section, we explore a number of variants on our baseline approach. We generalize our specification of adjustment costs to include a non-convex component. We also use this exercise to assess the accuracy of the log-linearized solution, since this case requires nonlinear solution techniques. We consider the implications of a frictional labor choice. We explore a number of measurement concerns, including the potential for measurement error. Our main conclusions about the relative contribution of various factors to observed misallocation is robust across these exercises.

6.1 Non-Convex Adjustment Costs

Our baseline model only allowed for convex adjustment costs. This allowed us to use perturbation techniques which yielded both analytical tractability for our identification arguments as well as computational efficiency. However, it raises two questions: one, is the log-linearization a sufficiently good approximation for the true non-linear solution? And two, are the results robust to allowing for non-convex adjustment costs? Here, we address both of these concerns by extending our baseline setup to include non-convex costs and solving the model without linearization. Specifically, the adjustment cost function now takes the form:

$$\Phi\left(K_{it+1}, K_{it}\right)=\frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right)^2 K_{it}+\hat{\xi}_f \mathbb{I}\left\{I_{it} \neq 0\right\} \pi\left(A_{it}, K_{it}\right) \ ,$$

where $I_{it}=K_{it+1}-(1-\delta) K_{it}$ denotes period t investment and $\mathbb{I}\{\cdot\}$ the indicator function. Capital adjustment costs are composed of two components: the first is the quadratic cost, the same as before. The second is a fixed component that must be incurred if the firm undertakes any non-zero investment. This component is parameterized by $\hat{\xi}_f$ and scales with profits (so that it does not become negligible for large firms), a common formulation in the literature, see, e.g., Asker et al. (2014).

Because of the fixed component, we cannot use perturbation methods to solve this version of the model. Therefore, we do so non-linearly using a standard value function iteration and re-estimate the parameters. We now have an additional parameter, $\hat{\xi}_f$. To pin this down, we add a new moment: the share of 'non-adjusters,' i.e., firms that make very small adjustments to their capital stock in a particular period. Specifically, we match the share of firms with net investment rates of less than 5% in absolute value, which is 14% of firms in China and 27% of firms in the US.

We report the results in Table 6. The estimated value for $\hat{\xi}_f$ is quite small in both countries. The value in the US, which implies a cost of about 0.2% of annual profits, is somewhat lower than previous estimates in the literature, underscoring the importance of controlling for other factors when estimating adjustment frictions.[41] The remaining parameters and their relative contributions to σ^2_{mrpk} are quite close to their values in the baseline analysis. These results demonstrate that (1) non-convex costs play a limited role in leading to *mrpk* dispersion, (2) abstracting from them does not significantly bias our estimates of other parameters and their contributions to misallocation and (3) the perturbation approach used for our baseline results is quite accurate.

6.2 Frictional Labor

Our baseline analysis makes the rather stark assumption of no adjustment or information frictions in labor choice, making it a static decision with full information. Although this is not an uncommon assumption in the literature, it may not be an apt description of labor markets

[41] For example, Bloom (2009) estimates a fixed adjustment cost of 1% of annual sales for US Compustat firms. Asker et al. (2014) and Cooper and Haltiwanger (2006) work with data on US manufacturing firms and estimate this parameter at 9% of annual sales and 4% of the capital stock, respectively.

Table 6: Non-Convex Adjustment Costs

Parameters	$\hat{\xi}$	(ξ)	$\hat{\xi}_f$	$\mathbb{V}$	γ	σ_ε^2	σ_χ^2
China	0.034	(0.23)	0.000	0.09	−0.635	0.00	0.45
US	0.135	(0.92)	0.002	0.03	−0.320	0.02	0.29
$\frac{\Delta\sigma_{mrpk}^2}{\sigma_{mrpk}^2}$							
China	4.3%		0.0%	10.0%	38.1%	0.0%	48.8%
US	11.1%		0.4%	7.5%	13.5%	4.4%	64.4%

Notes: The second column (in parentheses) reports the value of the normalized adjustment cost parameter, ξ, for purposes of comparison to Table 3. The mapping between ξ and $\hat{\xi}$ is given in expression (21).

in developing economies such as China. In this section, we extend our analysis to depart from this assumption. In Appendix A.7.1, we show that when labor is subject to the same forces as capital – adjustment and informational frictions and other factors – the firm's intertemporal investment problem takes the same form as in expression (3), but where the degree of curvature is equal to $\alpha = \alpha_1 + \alpha_2$ (and with slightly modified versions of the G and A_{it} terms). Thus, the qualitative analysis of the model is unchanged, although the quantitative results will differ since we now have $\alpha = \alpha_1 + \alpha_2 = 0.83$. Table 7 reports results from this specification for the Chinese firms. The top panel of the table displays the target moments recomputed under this scenario. A comparison to the baseline moments in Table 2 shows that under the assumption of frictional labor, the correlation of investment with lagged shocks increases, as does the correlation of the *mrpk* with fundamentals. The second panel reports the associated parameter estimates. They imply a higher level of adjustment costs, greater uncertainty and more severe correlated distortions. As a result, a lower level of the permanent component of uncorrelated distortions, σ_χ^2, is needed to match the dispersion in the *mrpk*.

The bottom panel of Table 7 reports the contribution of each factor to total misallocation and computes the implications for aggregate TFP. There is a noticeable increase in the impact of adjustment costs from the baseline case – now, they account for almost 13% of *mrpk* dispersion in China (compared to 1% above). There is also a slight increase in the impact of uncertainty (from 10% to 11%). Further, the effects on aggregate productivity are much larger than in the baseline scenario – here, these forces distort both inputs into production. Adjustment costs and imperfect information now lead to TFP losses of about 36% and 32%, respectively. Thus, this version of our model illustrates the potential for large aggregate consequences of adjustment/information frictions. However, despite the increased impact of these forces (in both relative and absolute terms), the results also confirm a key finding from before, namely, the important role of other correlated and permanent factors. Indeed, these factors compose about 80% of the measured *mrpk* dispersion, leading to TFP gaps relative to the first-best of about 144% and 90%, respectively.

Table 7: Frictional Labor - China

Moments	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
	0.92	0.16	0.33	-0.36	0.81	0.14	0.94
Parameters			ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
			0.78	0.11	-0.68	0.04	0.30
Aggregate Effects							
$\Delta\sigma^2_{mrpk}$			0.12	0.11	0.48	0.04	0.30
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$			12.8%	11.3%	51.2%	4.0%	32.2%
Δa			0.36	0.32	1.44	0.11	0.90

6.3 Measurement Error

Measurement error is an important and challenging concern, not just for our analysis but for the misallocation literature more broadly. In an important recent contribution, Bils et al. (2017) propose a method to identify additive measurement error. Here, we apply their methodology to our data. The Bils et al. (2017) approach essentially involves estimating the following regression:

$$\Delta rev_{it} = \Phi mrpk_{it} + \Psi \Delta k_{it} - \Psi \left(1 - \lambda\right) mrpk_{it} \cdot \Delta k_{it} + D_{jt} + \epsilon_{it} \ ,$$

where Δrev_{it} and Δk_{it} denote changes in (log) revenues and capital respectively, D_{jt} is a full set of industry-year fixed effects and $mrpk_{it}$ is (the log of the) marginal revenue product of capital. The key object is the coefficient on the interaction term. Bils et al. (2017) show that, under certain assumptions, λ equals the ratio of the true dispersion in the $mrpk$ to its measured counterpart (and inversely, $1 - \lambda$ is the contribution of measurement error to the observed σ^2_{mrpk}). Intuitively, to the extent measured $mrpk$ deviations are due to additive measurement error, revenues of firms with high observed $mrpk$ will display a lower elasticity with respect to capital.

Estimating this regression in our data yields estimates for λ of 0.92 in China and 0.88 in the US. These values suggest that, in both countries, only about 10% of the observed σ^2_{mrpk} can be accounted for by additive measurement error. Of course, it must be pointed out that this method is silent about other forms of measurement error (e.g., multiplicative).[42]

[42] There are a few approaches in the literature to deal with multiplicative measurement error, e.g. Collard-Wexler and De Loecker (2016) and Song and Wu (2015) make some progress on this dimension after imposing additional structure.

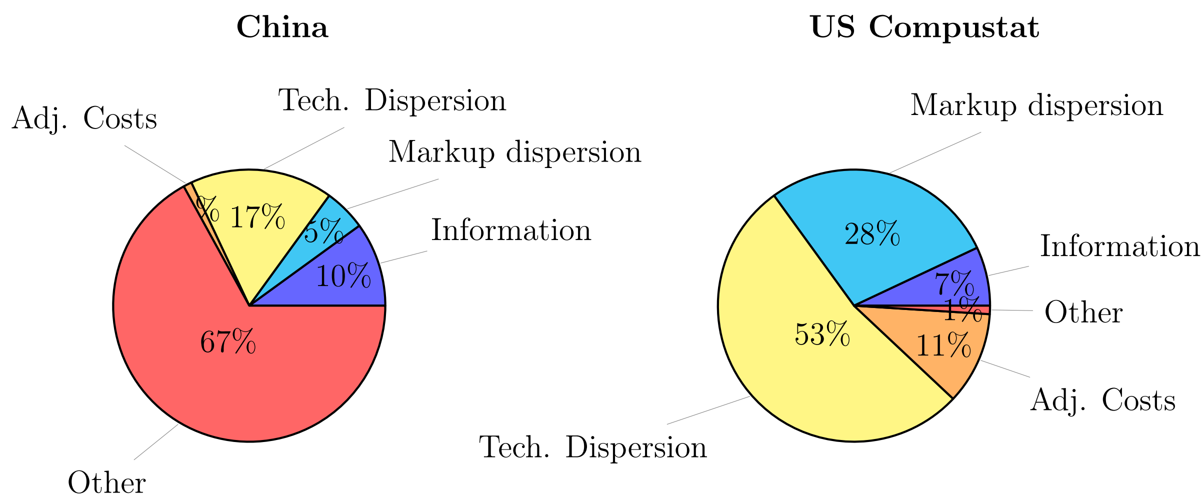
6.4 Additional Measurement Concerns

In this subsection, we address two other measurement-related issues. The first stems from our use of book values for capital. Although this is a common approach in the misallocation literature, e.g., Hsieh and Klenow (2009) and Gopinath et al. (2017), other papers use the perpetual inventory method along with data on investment good price deflators to construct an alternative measure for capital. To address this concern, we compute firm-level capital stocks for US firms, where data on the relevant price indices are readily available, using the approach outlined in Eberly et al. (2012). The results from re-estimating the model using these measures, presented in Appendix D.2, are broadly in line with our baseline findings. They point to a somewhat larger role for adjustment costs (the autocorrelation of investment growth is higher under this method and the variance lower, leading to a higher estimate of ξ), which account for about 27% of total σ^2_{mrpk} (compared to 11% under our baseline approach). The contribution of uncertainty is essentially unchanged at about 6%. Importantly, other firm-specific factors continue to play a key role in generating the observed $mrpk$ dispersion.

The second concern relates to sectoral heterogeneity in the structural parameters. We have estimated our model separately for US firms for the 9 major sectors of the industrial classification (e.g., manufacturing, construction, services, etc.). Specifically, we allowed for sector-specific parameters in production (we infer sector-specific α's using sectoral labor shares obtained from the BEA), adjustment frictions, uncertainty, as well as other factors. The details of this procedure are outlined in Appendix D.3 and the results are presented in Table 13 in that appendix. Although there is some variation across sectors, the overall patterns in the role of various factors (bottom panel of that table) are similar to those from our baseline analysis. The contribution of adjustment costs to observed misallocation is generally quite modest – the highest contribution is about 20% of σ^2_{mrpk} in Manufacturing and the lowest is 2% in Finance, Insurance and Real Estate. Uncertainty accounts for 5-10% across sectors, leaving the bulk of observed misallocation within each sector to be accounted for by other factors.

7 Conclusion

In this paper, we have laid out a model of investment featuring multiple factors that interfere with static marginal product equalization, along with an empirical strategy to disentangle them using widely available firm-level production data. Figure 3 summarizes our main results on the sources of misallocation in China (left panel) and the US (right panel). They suggest that much of the observed misallocation stems not from technological and informational frictions but, rather, from other firm-specific factors, in particular, ones that are correlated with firm productivity/size, and ones that are permanent. They also show that misspecification of demand and production technologies can potentially account for a significant portion of observed misallocation in a developed country like the US, but less so in China. There, size-dependent policies or certain forms of financial imperfections may be more fruitful avenues to pursue. Crucially, analyzing these factors in isolation would have led to very different conclusions, highlighting the value of using a unified framework and empirical approach like the one here.



Notes: The numbers for the contribution of technological dispersion denote the upper bound as calculated in footnote 39.

Figure 3: The Sources of Misallocation

There are several promising directions for future work. Our findings suggest that misallocation of productive resources, particularly in countries like China, are largely driven by factors that systematically disincentivize investment by larger/more productive firms or are uncorrelated and permanent to the firm. They provide a guide for future research linking these factors, for example, to specific policies and/or features of the institutional environment. A straightforward first step would be to analyze subsamples of firms – e.g., small vs. large, state-owned vs. private in China, etc. Applying our methodology on a more disaggregated level (for example, as we do across US sectors in Appendix D.3) might also be helpful in identifying segments of developing economies that are more 'distorted' than others and the underlying sources. Buera et al. (2013) show how irreversibility in government policy can result in fixed distortions at the firm-level. Our results show that further progress in separating the effects of specific policies and/or frictions is likely to require additional data (e.g., financial data). It also seems reasonable to conjecture that observed misallocation is the combined effect of a number of policies, so the main message of this paper – the need to use a broad set of data moments to discipline the effects of individual factors – is relevant for this line of work as well.

Our findings have implications beyond static marginal product dispersion. Midrigan and

Xu (2014) show that the same factors behind static misallocation can have larger effects on aggregate outcomes by influencing entry and exit decisions. Similarly, a number of recent papers examine the impact of distortions on the life-cycle of the firm and the distribution of productivity itself, e.g., Hsieh and Klenow (2014), Bento and Restuccia (2016) and Da-Rocha et al. (2017). An important insight from these papers is that the exact nature of the underlying distortions (e.g., their correlation with firm fundamentals) is key to understanding their dynamic implications. An ambitious and important next step would be to use an empirical strategy like the one in this paper to analyze richer environments featuring some of these elements.

References

Asker, J., A. Collard-Wexler, and J. De Loecker (2014): "Dynamic Inputs and Resource (Mis)Allocation," *Journal of Political Economy*, 122, 1013–1063.

Bachmann, R. and S. Elstner (2015): "Firm Optimism and Pessimism," *European Economic Review*, 79, 297–325.

Bai, C.-E., C.-T. Hsieh, and Y. Qian (2006): "The Return to Capital in China," *Brookings Papers on Economic Activity*, 2006, 61–101.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013): "Cross Country Differences in Productivity: The Role of Allocative Efficiency," *American Economic Review*, 103, 305–334.

Bento, P. and D. Restuccia (2016): "Misallocation, Establishment Size, and Productivity," Tech. rep., National Bureau of Economic Research.

Bils, M., P. J. Klenow, and C. Ruane (2017): "Misallocation or Mismeasurement?" *Working Paper*.

Bloom, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

Brandt, L., J. Van Biesebroeck, and Y. Zhang (2014): "Challenges of Working with the Chinese NBS Firm-Level Data," *China Economic Review*, 30, 339–352.

Buera, F. J. and R. N. Fattal-Jaef (2016): "The Dynamics of Development: Innovation and Reallocation," *working paper*.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101, 1964–2002.

Buera, F. J., B. Moll, and Y. Shin (2013): "Well-Intended Policies," *Review of Economic Dynamics*, 16, 216–230.

Collard-Wexler, A. and J. De Loecker (2016): "Production Function Estimation with Measurement Error in Inputs," Tech. rep., National Bureau of Economic Research.

Cooper, R., G. Gong, and P. Yan (2015): "Dynamic Labor Demand in China: Public and Private Objectives," *The RAND Journal of Economics*, 46, 577–610.

Cooper, R. W. and J. C. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *The Review of Economic Studies*, 73, 611–633.

Da-Rocha, J.-M., M. M. Tavares, and D. Restuccia (2017): "Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity," Tech. rep., National Bureau of Economic Research.

David, J. M., H. A. Hopenhayn, and V. Venkateswaran (2016): "Information, Misallocation, and Aggregate Productivity," *The Quarterly Journal of Economics*, 131, 943–1005.

De Loecker, J. and F. Warzynski (2012): "Markups and Firm-Level Export Status," *The American Economic Review*, 102, 2437–2471.

Eberly, J., S. Rebelo, and N. Vincent (2012): "What Explains the Lagged-Investment Effect?" *Journal of Monetary Economics*, 59, 370–380.

Eslava, M., J. Haltiwanger, A. Kugler, and M. Kugler (2004): "The Effects of Structural Reforms on Productivity and Profitability Enhancing Reallocation: Evidence from Colombia," *Journal of Development Economics*, 75, 333–371.

Gopinath, G., Ş. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017): "Capital Allocation and Productivity in South Europe," *The Quarterly Journal of Economics.*

Guner, N., G. Ventura, and Y. Xu (2008): "Macroeconomic Implications of Size-Dependent Policies," *Review of Economic Dynamics*, 11, 721–744.

Hopenhayn, H. A. (2014): "Firms, Misallocation, and Aggregate Productivity: A Review," *Annu. Rev. Econ.*, 6, 735–770.

Hsieh, C. and P. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

Hsieh, C.-T. and P. J. Klenow (2014): "The Life Cycle of Plants in India and Mexico," *The Quarterly Journal of Economics*, 129, 1035–1084.

İmrohoroğlu, A. and Ş. Tüzel (2014): "Firm-Level Productivity, Risk, and Return," *Management Science*, 60, 2073–2090.

Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring Uncertainty," *The American Economic Review*, 105, 1177–1216.

Kehrig, M. and N. Vincent (2017): "Do Firms Mitigate or Magnify Capital Misallocation? Evidence from Planet-Level Data," *Working Paper*.

Khan, A. and J. K. Thomas (2008): "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76, 395–436.

Klenow, P. J. and J. L. Willis (2007): "Sticky Information and Sticky Prices," *Journal of Monetary Economics*, 54, 79–99.

Midrigan, V. and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *The American Economic Review*, 104, 422–458.

Moll, B. (2014): "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *The American Economic Review*, 104, 3186–3221.

Morck, R., A. Shleifer, R. W. Vishny, M. Shapiro, and J. M. Poterba (1990): "The Stock Market and Investment: Is the Market a Sideshow?" *Brookings Papers on Economic Activity*, 1990, 157–215.

Peters, M. (2016): "Heterogeneous Mark-Ups, Growth and Endogenous Misallocation," *Working Paper*.

Restuccia, D. and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11, 707–720.

——— (2017): "The Causes and Costs of Misallocation," Tech. rep., National Bureau of Economic Research.

Song, Z. and G. L. Wu (2015): "Identifying Capital Misallocation," *Working Paper*.

Tybout, J. R. and M. D. Westbrook (1995): "Trade Liberalization and the Dimensions of Efficiency Change in Mexican Manufacturing Industries," *Journal of International Economics*, 39, 53–78.

Appendix: For Online Publication

A Derivations

A.1 Baseline Model

This appendix provides detailed derivations for our baseline analysis.

A.1.1 Model Solution

The first order condition and envelope conditions associated with (3) are, respectively,

$$\begin{aligned} T^K_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1},K_{it}\right) &= \beta\mathbb{E}_{it}\left[\mathcal{V}_1\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] \\ \mathcal{V}_1\left(K_{it},\mathcal{I}_{it}\right) &= \Pi_1\left(K_{it},A_{it}\right)-\Phi_2\left(K_{it+1},K_{it}\right) \end{aligned}$$

and combining yields the Euler equation

$$\mathbb{E}_{it}\left[\beta\Pi_1\left(K_{it+1},A_{it+1}\right)-\beta\Phi_2\left(K_{it+2},K_{it+1}\right)-T^K_{it+1}\left(1-\beta\left(1-\delta\right)\right)-\Phi_1\left(K_{it+1},K_{it}\right)\right]=0$$

where

$$\begin{aligned} \Pi_1\left(K_{it+1},A_{it+1}\right) &= \alpha G A_{it+1}K^{\alpha-1}_{it+1} \\ \Phi_1\left(K_{it+1},K_{it}\right) &= \hat{\xi}\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right) \\ \Phi_2\left(K_{it+1},K_{it}\right) &= -\hat{\xi}\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right)\frac{K_{it+1}}{K_{it}}+\frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right)^2 \\ &= \frac{\hat{\xi}}{2}(1-\delta)^2-\frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}}\right)^2 \end{aligned}$$

In the undistorted $\left(\bar{T}^K=1\right)$ non-stochastic steady state, these are equal to

$$\begin{aligned} \bar{\Phi}_1 &= \hat{\xi}\delta \\ \bar{\Phi}_2 &= \frac{\hat{\xi}}{2}(1-\delta)^2-\frac{\hat{\xi}}{2} \\ \bar{\Pi}_1 &= \alpha\bar{G}\bar{A}\bar{K}^{\alpha-1} \end{aligned}$$

Log-linearizing the Euler equation around this point yields

$$\mathbb{E}_{it}\left[\beta\bar{\Pi}_1\pi_{1,it+1}-\beta\bar{\Phi}_2\phi_{2,it+1}-\tau^K_{it+1}\left(1-\beta\left(1-\delta\right)\right)-\bar{\Phi}_1\phi_{1,it}\right]=0$$

where $\tau^K_{it+1} = \log T^K_{it+1}$ and

$$\begin{aligned}
\bar{\Pi}_1 \pi_{1,it+1} &\approx \alpha \bar{G} \bar{A} \bar{K}^{\alpha-1} \left(a_{it+1} + (\alpha - 1) k_{it+1} \right) \\
\bar{\Phi}_1 \phi_{1,it} &\approx \hat{\xi} \left(k_{it+1} - k_{it} \right) \\
\bar{\Phi}_2 \phi_{2,it+1} &\approx -\hat{\xi} \left(k_{it+2} - k_{it+1} \right)
\end{aligned}$$

Rearranging gives

$$k_{it+1} \left((1+\beta)\xi + 1 - \alpha \right) = \mathbb{E}_{it} \left[a_{it+1} + \tau_{it+1} \right] + \beta \xi \mathbb{E}_{it} \left[k_{it+2} \right] + \xi k_{it}$$

where

$$\xi = \frac{\hat{\xi}}{\beta \bar{\Pi}_1}, \qquad \tau_{it+1} = -\frac{1 - \beta (1-\delta)}{\beta \bar{\Pi}_1} \tau^K_{it+1}$$

which is expression (4) in the text. Using the steady state Euler equation,

$$\beta(\bar{\Pi}_1 + 1 - \delta) - \beta \bar{\Phi}_2 = 1 + \bar{\Phi}_1 \quad \Rightarrow \quad \alpha \beta \bar{G} \bar{A} \bar{K}^{\alpha-1} = 1 - \beta (1-\delta) + \hat{\xi} \delta \left(1 - \beta \left(1 - \frac{\delta}{2} \right) \right)$$

we have

$$\begin{aligned}
\xi &= \frac{\hat{\xi}}{1 - \beta (1-\delta) + \hat{\xi} \delta \left(1 - \beta \left(1 - \frac{\delta}{2} \right) \right)} \qquad (21) \\
\tau_{it+1} &= -\frac{1 - \beta (1-\delta)}{1 - \beta (1-\delta) + \hat{\xi} \delta \left(1 - \beta \left(1 - \frac{\delta}{2} \right) \right)} \tau^K_{it+1}
\end{aligned}$$

To derive the investment policy function, we conjecture that it takes the form in expression (7). Then,

$$\begin{aligned}
k_{it+2} &= \psi_1 k_{it+1} + \psi_2 (1+\gamma) \mathbb{E}_{it+1} a_{it+2} + \psi_3 \varepsilon_{it+2} + \psi_4 \chi_i \\
\mathbb{E}_{it} \left[k_{it+2} \right] &= \psi_1 k_{it+1} + \psi_2 (1+\gamma) \rho \mathbb{E}_{it} \left[a_{it+1} \right] + \psi_4 \chi_i \\
&= \psi_1 \left(\psi_1 k_{it} + \psi_2 (1+\gamma) \mathbb{E}_{it} \left[a_{it+1} \right] + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i \right) + \psi_2 (1+\gamma) \rho \mathbb{E}_{it} \left[a_{it+1} \right] + \psi_4 \chi_i \\
&= \psi_1^2 k_{it} + (\psi_1 + \rho) \psi_2 (1+\gamma) \mathbb{E}_{it} \left[a_{it+1} \right] + \psi_1 \psi_3 \varepsilon_{it+1} + \psi_4 (1 + \psi_1) \chi_i
\end{aligned}$$

where we have used $\mathbb{E}_{it} \left[\varepsilon_{it+2} \right] = 0$ and $\mathbb{E}_{it} \left[\mathbb{E}_{it+1} \left[a_{it+2} \right] \right] = \rho \mathbb{E}_{it} \left[a_{it+1} \right]$. Substituting and rearranging,

$$
\begin{aligned}
& \left(1+\beta\xi\psi_4\left(1+\psi_1\right)\right)\chi_i+\left(1+\beta\xi\psi_1\psi_3\right)\varepsilon_{it+1} \\
+ \quad & \left(1+\beta\xi\left(\psi_1+\rho\right)\psi_2\right)\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\xi\left(1+\beta\psi_1^2\right)k_{it} \\
= \quad & \left(\left(1+\beta\right)\xi+1-\alpha\right)\left(\psi_1 k_{it}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_3\varepsilon_{it+1}+\psi_4\chi_i\right)
\end{aligned}
$$

Finally, matching coefficients gives

$$
\begin{aligned}
\xi\left(\beta\psi_1^2+1\right) &= \psi_1\left((1+\beta)\xi+1-\alpha\right) \\
1+\beta\xi\left(\psi_1+\rho\right)\psi_2 &= \psi_2\left((1+\beta)\xi+1-\alpha\right)\Rightarrow\psi_2=\frac{1}{1-\alpha+\beta\xi\left(1-\psi_1-\rho\right)+\xi} \\
1+\beta\xi\psi_1\psi_3 &= \psi_3\left((1+\beta)\xi+1-\alpha\right)\Rightarrow\psi_3=\frac{1}{1-\alpha+\left(1-\psi_1\right)\beta\xi+\xi} \\
1+\beta\xi\psi_4\left(1+\psi_1\right) &= \psi_4\left((1+\beta)\xi+1-\alpha\right)\Rightarrow\psi_4=\frac{1}{1-\alpha+\xi\left(1-\beta\psi_1\right)}
\end{aligned}
$$

A few lines of algebra yields the expressions in (8).

A.1.2 Aggregation

To derive aggregate TFP and output, substitute the firm's optimality condition for labor

$$
N_{it}=\left(\frac{\alpha_2 Y^{\frac{1}{\theta}}}{W}\hat{A}_{it}K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}
$$

into the production function (1) to get

$$
Y_{it}=\left(\frac{\alpha_2 Y^{\frac{1}{\theta}}}{W}\right)^{\frac{\hat{\alpha}_2}{1-\alpha_2}}\hat{A}_{it}^{\frac{\hat{\alpha}_2}{1-\alpha_2}}K_{it}^{\frac{\hat{\alpha}_1}{1-\alpha_2}}
$$

and using the demand function, revenues are

$$
P_{it}Y_{it}=Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}A_{it}K_{it}^{\alpha}
$$

Labor market clearing implies

$$
\int N_{it}di=\int\left(\frac{\alpha_2 Y^{\frac{1}{\theta}}}{W}\right)^{\frac{1}{1-\alpha_2}}A_{it}K_{it}^{\alpha}di=N
$$

so that

$$\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} = \left(\frac{N}{\int A_{it}K_{it}^{\alpha}di}\frac{1}{Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}}\right)^{\alpha_2} \Rightarrow P_{it}Y_{it} = Y^{\frac{1}{\theta}}\frac{A_{it}K_{it}^{\alpha}}{\left(\int A_{it}K_{it}^{\alpha}di\right)^{\alpha_2}}N^{\alpha_2}$$

By definition,

$$MRPK_{it} = \alpha\frac{A_{it}K_{it}^{\alpha-1}}{\left(\int A_{it}K_{it}^{\alpha}di\right)^{\alpha_2}}Y^{\frac{1}{\theta}}N^{\alpha_2}$$

so that

$$K_{it} = \left(\frac{\alpha Y^{\frac{1}{\theta}}A_{it}}{MRPK_{it}}\right)^{\frac{1}{1-\alpha}}\left(\frac{N}{\int A_{it}K_{it}^{\alpha}di}\right)^{\frac{\alpha_2}{1-\alpha}}$$

and capital market clearing implies

$$K = \int K_{it}di = \left(\alpha Y^{\frac{1}{\theta}}\right)^{\frac{1}{1-\alpha}}\left(\frac{N}{\int A_{it}K_{it}^{\alpha}di}\right)^{\frac{\alpha_2}{1-\alpha}}\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di$$

The latter two equations give

$$K_{it}^{\alpha} = \left(\frac{A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}}{\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di}K\right)^{\alpha}$$

Substituting into the expression for $P_{it}Y_{it}$ and rearranging, we can derive

$$P_{it}Y_{it} = \frac{\frac{A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}}{\left(\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di\right)^{\alpha}}}{\left(\frac{\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}di}{\left(\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di\right)^{\alpha}}\right)^{\alpha_2}}Y^{\frac{1}{\theta}}K^{\alpha_1}N^{\alpha_2}$$

Using the fact that $Y = \int P_{it}Y_{it}di$, we can derive

$$Y = \int P_{it}Y_{it}di = Y^{\frac{1}{\theta}}AK^{\alpha_1}N^{\alpha_2}$$

where

$$A = \left(\frac{\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}di}{\left(\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di\right)^{\alpha}}\right)^{1-\alpha_2}$$

or in logs,

$$a=(1-\alpha_2)\left[\log\left(\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}\right)-\alpha\log\left(\int A_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}\right)\right]$$

The first term inside brackets is equal to

$$\frac{1}{1-\alpha}\overline{a}-\frac{\alpha}{1-\alpha}\overline{mrpk}+\frac{1}{2}\left(\frac{1}{1-\alpha}\right)^2\sigma_a^2+\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)^2\sigma_{mrpk}^2-\frac{\alpha}{(1-\alpha)^2}\sigma_{mrpk,a}$$

and the second,

$$\frac{\alpha}{1-\alpha}\overline{a}-\frac{\alpha}{1-\alpha}\overline{mrpk}+\frac{1}{2}\alpha\left(\frac{1}{1-\alpha}\right)^2\sigma_a^2+\frac{1}{2}\alpha\left(\frac{1}{1-\alpha}\right)^2\sigma_{mrpk}^2-\frac{\alpha}{(1-\alpha)^2}\sigma_{mrpk,a}$$

Combining,

$$a=(1-\alpha_2)\left[\overline{a}+\frac{1}{2}\frac{1}{1-\alpha}\sigma_a^2-\frac{1}{2}\frac{\alpha}{1-\alpha}\sigma_{mrpk}^2\right]$$

and

$$\begin{aligned}
y &= \frac{1}{\theta}y+(1-\alpha_2)\,\overline{a}+\frac{1}{2}\frac{1-\alpha_2}{1-\alpha}\sigma_a^2-\frac{1}{2}\alpha\frac{1-\alpha_2}{1-\alpha}\sigma_{mrpk}^2+\alpha_1 k+\alpha_2 n\\
&= \frac{\theta}{\theta-1}(1-\alpha_2)\,\overline{a}+\frac{\theta}{\theta-1}\frac{1}{2}\frac{1-\alpha_2}{1-\alpha}\sigma_a^2-\frac{\theta}{\theta-1}\frac{1}{2}\alpha\frac{1-\alpha_2}{1-\alpha}\sigma_{mrpk}^2+\hat{\alpha}_1 k+\hat{\alpha}_2 n\\
&= a+\hat{\alpha}_1 k+\hat{\alpha}_2 n
\end{aligned}$$

where, using $a_{it}=\frac{1}{1-\alpha_2}\hat{a}_{it}$, $\sigma_a^2=\left(\frac{1}{1-\alpha_2}\right)^2\sigma_{\hat{a}}^2$ and $\alpha=\frac{\alpha_1}{1-\alpha_2}$,

$$\begin{aligned}
a &= \frac{\theta}{\theta-1}\bar{\hat{a}}+\frac{1}{2}\frac{\theta}{\theta-1}\frac{1}{1-\alpha_1-\alpha_2}\sigma_{\hat{a}}^2-\frac{1}{2}(\theta\hat{\alpha}_1+\hat{\alpha}_2)\,\hat{\alpha}_1\sigma_{mrpk}^2\\
&= a^*-\frac{1}{2}(\theta\hat{\alpha}_1+\hat{\alpha}_2)\,\hat{\alpha}_1\sigma_{mrpk}^2
\end{aligned}$$

which is equation (9) in the text.

To compute the effect on output, notice that the aggregate production function is

$$y=\hat{\alpha}_1 k+\hat{\alpha}_2 n+a$$

so that

$$\begin{aligned}\frac{dy}{d\sigma^2_{mrpk}} &= \hat{\alpha}_1 \frac{dk}{da}\frac{da}{d\sigma^2_{mrpk}} + \frac{da}{d\sigma^2_{mrpk}} \\ &= \frac{da}{d\sigma^2_{mrpk}}\left(1+\hat{\alpha}_1\frac{dk}{da}\right)\end{aligned}$$

In the stationary equilibrium, the aggregate marginal product of capital must be a constant, denote it by $\bar{R}$, i.e., $\log\hat{\alpha}_1 + y - k = \bar{r}$ so that

$$k = \frac{1}{1-\hat{\alpha}_1}\left(\log\hat{\alpha}_1 + \hat{\alpha}_2 n + a - \bar{r}\right)$$

and

$$\frac{dk}{da} = \frac{1}{1-\hat{\alpha}_1}$$

Combining,

$$\frac{dy}{d\sigma^2_{mrpk}} = \frac{da}{d\sigma^2_{mrpk}}\left(1+\frac{\hat{\alpha}_1}{1-\hat{\alpha}_1}\right) = \frac{da}{d\sigma^2_{mrpk}}\frac{1}{1-\hat{\alpha}_1}$$

A.2 Firm-Specific Wages

In this appendix, we show that to the extent distortions to the labor choice are reflected in firm-specific wages, they change the interpretation of fundamentals but otherwise do not affect our analysis of capital misallocation. In particular, they do not contribute to measured *mrpk* dispersion and so our strategy for disentangling the various sources of capital misallocation and our estimates for their magnitudes go through unchanged.

We allow wages to vary at the firm level due to distortions, i.e., introduce $W_{it} \equiv WT^N_{it}$ into the firm's problem, which becomes

$$\begin{aligned}\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right) = \max_{N_{it},K_{it+1}} \quad & \mathbb{E}_{it}\left[Y^{\frac{1}{\theta}}\hat{A}_{it}K^{\alpha_1}_{it}N^{\alpha_2}_{it} - WT^N_{it}N_{it} - T^K_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right) - \Phi\left(K_{it+1},K_{it}\right)\right] \\ + \quad & \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right]\end{aligned}$$

The labor choice satisfies the first order condition

$$N_{it} = \left(\alpha_2\frac{Y^{\frac{1}{\theta}}\hat{A}_{it}K^{\alpha_1}_{it}}{WT^N_{it}}\right)^{\frac{1}{1-\alpha_2}}$$

Substituting, we can derive operating profits (revenues net of total wages) as

$$\begin{aligned}
P_{it}Y_{it} - WT_{it}^{N}N_{it} &= Y_t^{\frac{1}{\theta}}\hat{A}_{it}K_{it}^{\alpha_1}\left(\alpha_2 Y_t^{\frac{1}{\theta}}\frac{\hat{A}_{it}K_{it}^{\alpha_1}}{WT_{it}^{N}}\right)^{\frac{\alpha_2}{1-\alpha_2}} - WT_{it}^{N}\left(\alpha_2 Y_t^{\frac{1}{\theta}}\frac{\hat{A}_{it}K_{it}^{\alpha_1}}{WT_{it}^{N}}\right)^{\frac{1}{1-\alpha_2}} \\
&= (1-\alpha_2)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\frac{\hat{A}_{it}^{\frac{1}{1-\alpha_2}}}{(T_{it}^{N})^{\frac{\alpha_2}{1-\alpha_2}}}K_{it}^{\frac{\alpha_1}{1-\alpha_2}} \\
&= GA_{it}K_{it}^{\alpha}
\end{aligned}$$

which is the same form as in the baseline version, except now the fundamental A_{it} also incorporates the effect of the labor distortion:[43]

$$A_{it} \equiv \left(\frac{\hat{A}_{it}}{(T_{it}^{N})^{\alpha_2}}\right)^{\frac{1}{1-\alpha_2}}$$

With this re-interpretation, the firm's dynamic investment decision is still given by (3). To see that these labor taxes do not contribute to *mrpk* dispersion, assume that they are the only friction, i.e., the capital choice is made under full information with no adjustment costs or uncertainty. The capital choice is then static and given by

$$K_{it} = \left(\frac{\alpha G\hat{A}_{it}^{\frac{1}{1-\alpha_2}}}{(T_{it}^{N})^{\frac{\alpha_2}{1-\alpha_2}}}\right)^{\frac{1}{1-\alpha}}$$

Combining this with the expression for revenues, the measured *mrpk* is equal to

$$\begin{aligned}
mrpk_{it} &= \text{Const.} + p_{it} + y_{it} - k_{it} \\
&= \text{Const.} + \frac{-\alpha_2}{1-\alpha_2}\tau_{it}^{N} + \frac{1}{1-\alpha_2}\hat{a}_{it} + (\alpha-1)\frac{-\alpha_2}{1-\alpha_2}\frac{1}{1-\alpha}\tau_{it}^{N} + (\alpha-1)\frac{1}{1-\alpha_2}\frac{1}{1-\alpha}\hat{a}_{it} \\
&= \text{Const.}
\end{aligned}$$

So, T_{it}^{N} does lead to any measured dispersion in the *mrpk*.

A.3 Identification

In this appendix we derive analytic expressions for the four moments in the random walk case, i.e., when $\rho = 1$, and prove Proposition 1.

[43] Note that this is also the a_{it} we would measure from the data using the definition $a_{it} = va_{it} - \alpha k_{it}$.

Moments. From expression (7), we have the firm's investment policy function

$$k_{it+1} = \psi_1 k_{it} + \psi_2 (1+\gamma) \mathbb{E}_{it} [a_{it+1}] + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i$$

and substituting for the expectation,

$$k_{it+1} = \psi_1 k_{it} + \psi_2 (1+\gamma) (a_{it} + \phi (\mu_{it+1} + e_{it+1})) + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i$$

where $\phi = \frac{\mathbb{V}}{\sigma_e^2}$ so that $1 - \phi = \frac{\mathbb{V}}{\sigma_\mu^2}$. Then,

$$\Delta k_{it+1} = \psi_1 \Delta k_{it} + \psi_2 (1+\gamma) ((1-\phi) \mu_{it} + \phi \mu_{it+1} + \phi (e_{it+1} - e_{it})) + \psi_3 (\varepsilon_{it+1} - \varepsilon_{it})$$

We will use the fact that

$$\begin{aligned} \text{cov}(\Delta k_{it+1}, \mu_{it+1}) &= \psi_2 (1+\gamma) \phi \sigma_\mu^2 \\ \text{cov}(\Delta k_{it+1}, e_{it+1}) &= \psi_2 (1+\gamma) \phi \sigma_e^2 \\ \text{cov}(\Delta k_{it+1}, \varepsilon_{it+1}) &= \psi_3 \sigma_\varepsilon^2 \end{aligned}$$

Now,

$$\begin{aligned} \text{var}(\Delta k_{it+1}) &= \psi_1^2 \text{var}(\Delta k_{it}) + \psi_2^2 (1+\gamma)^2 (1-\phi)^2 \sigma_\mu^2 \\ &+ \psi_2^2 (1+\gamma)^2 \phi^2 \sigma_\mu^2 + 2\psi_2^2 (1+\gamma)^2 \phi^2 \sigma_e^2 + 2\psi_3^2 \sigma_\varepsilon^2 \\ &+ 2\psi_1 \psi_2 (1+\gamma)(1-\phi) \text{cov}(\Delta k_{it}, \mu_{it}) - 2\psi_1 \psi_2 (1+\gamma) \phi \text{cov}(\Delta k_{it}, e_{it}) \\ &- 2\psi_1 \psi_3 \text{cov}(\Delta k_{it}, \varepsilon_{it}) \end{aligned}$$

where substituting, rearranging and using the fact that the moments are stationary gives

$$\sigma_k^2 \equiv \text{var}(\Delta k_{it}) = \frac{(1+\gamma)^2 \psi_2^2 \sigma_\mu^2 + 2(1-\psi_1) \psi_3^2 \sigma_\varepsilon^2}{1 - \psi_1^2}$$

which can be rearranged to yield expression (10).

Next,

$$\begin{aligned} \text{cov}(\Delta k_{it+1}, \Delta k_{it}) &= \psi_1 \text{var}(\Delta k_{it}) + \psi_2 (1+\gamma)(1-\phi) \text{cov}(\Delta k_{it}, \mu_{it}) \\ &- \psi_2 (1+\gamma) \phi \text{cov}(\Delta k_{it}, e_{it}) - \psi_3 \text{cov}(\Delta k_{it}, \varepsilon_{it}) \\ &= \psi_1 \text{var}(\Delta k_{it}) - \psi_3 \text{cov}(\Delta k_{it}, \varepsilon_{it}) \\ &= \psi_1 \sigma_k^2 - \psi_3^2 \sigma_\varepsilon^2 \end{aligned}$$

so that

$$\rho_{k,k_{-1}} \equiv \text{corr}\left(\Delta k_{it}, \Delta k_{it-1}\right) = \psi_1 - \psi_3^2 \frac{\sigma_\varepsilon^2}{\sigma_k^2}$$

which is expression (11). Similarly,

$$\begin{aligned} \text{cov}\left(\Delta k_{it+1}, \Delta a_{it}\right) &= \text{cov}\left(\Delta k_{it+1}, \mu_{it}\right) \\ &= \psi_1 \text{cov}\left(\Delta k_{it}, \mu_{it}\right) + \psi_2 \left(1+\gamma\right)\left(1-\phi\right)\sigma_\mu^2 \\ &= \psi_1 \psi_2 \left(1+\gamma\right)\phi\sigma_\mu^2 + \psi_2 \left(1+\gamma\right)\left(1-\phi\right)\sigma_\mu^2 \\ &= \left(1-\phi\left(1-\psi_1\right)\right)\psi_2\left(1+\gamma\right)\sigma_\mu^2 \end{aligned}$$

and from here it is straightforward to derive

$$\rho_{k,a_{-1}} \equiv \text{corr}\left(\Delta k_{it}, \Delta a_{it-1}\right) = \left[\frac{\mathbb{V}}{\sigma_\mu^2}\left(1-\psi_1\right) + \psi_1\right] \frac{\sigma_\mu \psi_2 \left(1+\gamma\right)}{\sigma_k}$$

as in expression (12).

Finally,

$$mrpk_{it} = \text{Const} + p_{it} + y_{it} - k_{it} = \text{Const} + a_{it} + \alpha k_{it} - k_{it} = \text{Const} + a_{it} - \left(1-\alpha\right) k_{it}$$

so that

$$\Delta mrpk_{it} = \Delta a_{it} - \left(1-\alpha\right)\Delta k_{it} = \mu_{it} - \left(1-\alpha\right)\Delta k_{it}$$

which implies

$$\text{cov}\left(\Delta mrpk_{it}, \mu_{it}\right) = \left(1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right)\sigma_\mu^2$$

and

$$\begin{aligned} \lambda_{mrpk,a} \equiv \frac{\text{cov}\left(\Delta mrpk_{it}, \mu_{it}\right)}{\sigma_\mu^2} &= 1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi \\ &= 1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right) \end{aligned}$$

which is expression (13).

To see that the correlation $\rho_{mrpk,a}$ is decreasing in σ_ε^2, we derive

$$\begin{aligned}\text{var}\left(\Delta mrpk_{it}\right) &= \sigma_\mu^2 + \left(1-\alpha\right)^2\sigma_k^2 - 2\left(1-\alpha\right)\text{cov}\left(\Delta k_{it},\mu_{it}\right)\\
&= \sigma_\mu^2 + \left(1-\alpha\right)^2\left(\frac{\psi_2^2\left(1+\gamma\right)^2\sigma_\mu^2 + 2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2}{1-\psi_1^2}\right) - 2\left(1-\alpha\right)\psi_2\left(1+\gamma\right)\phi\sigma_\mu^2\\
&= \frac{1}{1-\psi_1^2}\left(\left(\left(1-\psi_1^2\right)\left(1-2\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right) + \left(1-\alpha\right)^2\left(1+\gamma\right)^2\psi_2^2\right)\sigma_\mu^2\right)\\
&+ \frac{1}{1-\psi_1^2}\left(2\left(1-\alpha\right)^2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2\right)\end{aligned}$$

so

$$\rho_{mrpk,a} = \frac{\left(1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right)\sigma_\mu\sqrt{1-\psi_1^2}}{\sqrt{\left(\left(1-\psi_1^2\right)\left(1-2\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right) + \left(1-\alpha\right)^2\left(1+\gamma\right)^2\psi_2^2\right)\sigma_\mu^2 + 2\left(1-\alpha\right)^2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2}}$$

Proof of Proposition 1. Write the variance of investment as

$$\sigma_k^2 = \psi_1^2\sigma_k^2 + \left(1+\gamma\right)^2\psi_2^2\sigma_\mu^2 + 2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2$$

To rewrite the last term as a function of an observable moment, use the autocovariance of investment,

$$\sigma_{k,k_{-1}} = \psi_1\sigma_k^2 - \psi_3^2\sigma_\varepsilon^2 \tag{22}$$

and substitution yields

$$\sigma_k^2 = \psi_1^2\sigma_k^2 + \left(1+\gamma\right)^2\psi_2^2\sigma_\mu^2 + 2\left(1-\psi_1\right)\left(\psi_1\sigma_k^2 - \sigma_{k,k_{-1}}\right) \tag{23}$$

To eliminate the second term, use the equation for $\lambda_{mrpk,a}$ to solve for

$$\left(1+\gamma\right)\psi_2\phi = \frac{1-\lambda_{mrpk,a}}{1-\alpha} = \tilde{\lambda} \tag{24}$$

where $\tilde{\lambda}$ is a decreasing function of $\lambda_{mrpk,a}$ that depends only on the known parameter α. Substituting into the expression for the covariance of investment with the lagged shock, $\sigma_{k,a_{-1}}$, and rearranging yields

$$\left(1+\gamma\right)\psi_2 = \frac{\sigma_{k,a_{-1}}}{\sigma_\mu^2} + \tilde{\lambda}\left(1-\psi_1\right) \tag{25}$$

which is an equation in ψ_1 and observable moments. Substituting into (23) gives

$$\sigma_k^2 = \psi_1^2\sigma_k^2 + \left(\frac{\sigma_{k,a_{-1}}}{\sigma_\mu^2} + \tilde{\lambda}\left(1-\psi_1\right)\right)^2\sigma_\mu^2 + 2\left(1-\psi_1\right)\left(\psi_1\sigma_k^2 - \sigma_{k,k_{-1}}\right)$$

and rearranging, we can derive

$$0=\left(\hat{\lambda}^2-1\right)(1-\psi_1)^2+2\left(\hat{\lambda}\rho_{k,a_{-1}}-\rho_{k,k_{-1}}\right)(1-\psi_1)+\rho_{k,a_{-1}}^2 \tag{26}$$

where

$$\hat{\lambda}=\frac{\sigma_\mu}{\sigma_k}\tilde{\lambda}=\frac{\sigma_\mu}{\sigma_k}\left(\frac{1-\lambda_{mrpk,a}}{1-\alpha}\right)$$

Equation (26) represents a quadratic equation in a single unknown, $1-\psi_1$, or equivalently, in ψ_1. The solution features two positive roots, one greater than one and one less. The smaller root corresponds to the true ψ_1 that represents the solution to the firm's investment policy. The value of ψ_1 pins down the adjustment cost parameter ξ as well as ψ_2 and ψ_3. We can then back out γ from (25), ϕ (and so $\mathbb{V}$) from (24) and finally, σ_ε^2 from (22).

□

A.4 Heterogeneity in Markups/Technologies

The firm's cost minimization problem is

$$\min_{K_{it},N_{it},M_{it}} R_tT_{it}^KK_{it}+W_tT_{it}^NN_{it}+P_t^MM_{it} \quad s.t. \quad Y_{it}\leq K_{it}^{\hat{\alpha}_{it}}N_{it}^{\hat{\zeta}-\hat{\alpha}_{it}}M_{it}^{1-\hat{\zeta}}$$

The first order condition on M_{it} gives

$$P_t^M=\left(1-\hat{\zeta}\right)\frac{Y_{it}}{M_{it}}MC_{it} \quad\Rightarrow\quad \frac{P_t^MM_{it}}{P_{it}Y_{it}}=\left(1-\hat{\zeta}\right)\frac{MC_{it}}{P_{it}}$$

where MC_{it} is the Lagrange multiplier on the constraint (i.e., the marginal cost). Rearranging gives expression (14). In logs,

$$\log\frac{P_{it}}{MC_{it}}=\log\left(1-\hat{\zeta}\right)+\log\frac{P_{it}Y_{it}}{P_t^MM_{it}} \quad\Rightarrow\quad \sigma^2\left(\log\frac{P_{it}}{MC_{it}}\right)=\sigma^2\left(\log\frac{P_{it}Y_{it}}{P_t^MM_{it}}\right)$$

Similarly, the optimality conditions for K_{it} and N_{it} yield:

$$\begin{aligned}\log\frac{P_{it}Y_{it}}{K_{it}} &= \log\frac{P_{it}}{MC_{it}}-\log\hat{\alpha}_{it}+\tau_{it}^K+\text{Constant}\\ \log\frac{P_{it}Y_{it}}{N_{it}} &= \log\frac{P_{it}}{MC_{it}}-\log\left(\hat{\zeta}-\hat{\alpha}_{it}\right)+\tau_{it}^N+\text{Constant}\end{aligned}$$

Log-linearizing around the average $\hat{\alpha}_{it}$, denote it $\bar{\alpha}$, and ignoring constants yields $\log\left(\hat{\zeta}-\hat{\alpha}_{it}\right)\approx -\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\log\hat{\alpha}_{it}$. Substituting gives expression (17).

Proof of Proposition 2. Assuming $\log \hat{\alpha}_{it}$ is uncorrelated with τ_{it}^K and τ_{it}^N,

$$\begin{aligned}
cov\left(arpk_{it}, arpn_{it}\right) &= -\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\sigma^2_{\log\hat{\alpha}} + \text{cov}\left(\tau_{it}^k, \tau_{it}^n\right) \\
\sigma^2_{arpk} &= \sigma^2_{\log\hat{\alpha}} + \sigma^2_{\tau^k} \\
\sigma^2_{arpn} &= \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2 \sigma^2_{\log\hat{\alpha}} + \sigma^2_{\tau^n}
\end{aligned}$$

From here, we can solve for the correlation of the distortions:

$$\rho\left(\tau_{it}^K, \tau_{it}^N\right) = \frac{\text{cov}\left(arpk_{it}, arpn_{it}\right) + \frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\sigma^2_{\log\hat{\alpha}}}{\sqrt{\sigma^2_{arpk} - \sigma^2_{\log\hat{\alpha}}}\sqrt{\sigma^2_{arpn} - \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2 \sigma^2_{\log\hat{\alpha}}}}$$

which is increasing in $\sigma^2_{\log\hat{\alpha}}$. An upper bound for $\sigma^2_{\log\hat{\alpha}}$, denoted $\bar{\sigma}^2_{\log\hat{\alpha}}$, is where $\rho\left(\tau_{it}^K, \tau_{it}^N\right) = 1$, and substituting and rearranging gives

$$\bar{\sigma}^2_{\hat{\alpha}} = \frac{\sigma^2_{arpk}\sigma^2_{arpn} - \text{cov}\left(arpk_{it}, arpn_{it}\right)^2}{2\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\text{cov}\left(arpk_{it}, arpn_{it}\right) + \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2 \sigma^2_{arpk} + \sigma^2_{arpn}}$$

□

A.5 Size-Dependent Policies

In this appendix, we detail the relationship between size and productivity-dependent factors. First, note that our empirical strategy can be thought of as essentially recovering the law of motion for k_{it} – in particular, the coefficients ψ_1, $\psi_2(1+\gamma)$, ψ_3 and ψ_4. Importantly, these estimates are invariant to assumptions about γ_k, which only affects the mapping from these coefficients to the underlying structural parameters. For example, suppose we assume $\gamma_k = 0$. Then, given our values for (α, β, δ), the estimated ψ_1 identifies the adjustment cost parameter ξ. Next, the value of ξ can be used to pin down ψ_2, allowing us to recover γ from the estimated $\psi_2(1+\gamma)$. This procedure can be applied for any given γ_k as well. Since the estimated ψ_1 and $\psi_2(1+\gamma)$ do not change, for any γ_k, the adjustment cost parameter becomes, from (8),

$$\xi = \psi_1 \frac{1-\alpha-\gamma_k}{\beta\psi_1^2 + 1 - \psi_1(1+\beta)} .$$

The next step is the same as before: the estimated ξ implies a value for ψ_2, which then allows us to back out γ from the estimated $\psi_2(1+\gamma)$. Table 5 applies this procedure for various values of γ_k to trace out a set of parameters that are observationally equivalent, i.e., that cannot be distinguished using only data on capital and revenues.

A.6 Financial Frictions

Including the liquidity cost, the firm's recursive problem can be written as

$$\begin{aligned}\mathcal{V}(K_{it},B_{it},\mathcal{I}_{it}) = \max_{B_{it+1},K_{it+1}} \quad & \mathbb{E}_{it}\left[\Pi(K_{it},A_{it}) + RB_{it} - B_{it+1} - T^K_{it+1}K_{it+1}(1-\beta(1-\delta))\right] \\ - \quad & \Phi(K_{it+1},K_{it}) - \Upsilon(K_{it+1},B_{it+1}) + \beta\mathbb{E}_{it}\left[\mathcal{V}(K_{it+1},B_{it+1},\mathcal{I}_{it+1})\right]\end{aligned}$$

The first order conditions are given by

$$\begin{aligned}\mathbb{E}_{it}\left[\beta\Pi_1(K_{it+1},A_{it+1}) - \beta\Phi_2(K_{it+2},K_{it+1})\right] &= T^K_{it+1}(1-\beta(1-\delta)) + \Phi_1(K_{it+1},K_{it}) + \Upsilon_1(K_{it+1},B_{it+1}) \\ -\Upsilon_2(K_{it+1},B_{it+1}) + \beta R &= 1\end{aligned}$$

Note that

$$\Upsilon_2(K_{it+1},B_{it+1}) = -\hat{\nu}\omega_2\frac{K^{\omega_1}_{it+1}}{B^{\omega_2+1}_{it+1}}, \qquad \Upsilon_1(K_{it+1},B_{it+1}) = \hat{\nu}\omega_1\frac{K^{\omega_1-1}_{it+1}}{B^{\omega_2}_{it+1}}$$

Using the FOC for B_{it+1}

$$\begin{aligned}1 &= \hat{\nu}\omega_2\frac{K^{\omega_1}_{it+1}}{B^{\omega_2+1}_{it+1}} + \beta R \quad \Rightarrow \quad B_{it+1} = \left(\frac{\hat{\nu}\omega_2}{1-\beta R}\right)^{\frac{1}{\omega_2+1}} K^{\frac{\omega_1}{\omega_2+1}}_{it+1} \\ \Upsilon_1(K_{it+1},B_{it+1}) &= \hat{\nu}\omega_1\frac{K^{\omega_1-1}_{it+1}}{B^{\omega_2}_{it+1}} = \hat{\nu}\omega_1\frac{K^{\omega_1-1}_{it+1}}{\left(\frac{\hat{\nu}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}} K^{\frac{\omega_2\omega_1}{\omega_2+1}}_{it+1}} = \left(\frac{\hat{\nu}}{\omega_2^{\omega_2}}\right)^{\frac{1}{\omega_2+1}} \omega_1(1-\beta R)^{\frac{\omega_2}{\omega_2+1}} K^{\frac{\omega_1-(\omega_2+1)}{\omega_2+1}}_{it+1} \\ &= \nu(1-\beta R)^{\frac{\omega_2}{\omega_2+1}} K^{\omega}_{it+1},\end{aligned}$$

where

$$\begin{aligned}\nu &\equiv \left(\frac{\hat{\nu}}{\omega_2^{\omega_2}}\right)^{\frac{1}{\omega_2+1}} \omega_1 \\ \omega &\equiv \frac{\omega_1 - (\omega_2+1)}{\omega_2+1}.\end{aligned}$$

Log-linearizing,

$$\begin{aligned}\bar{\Upsilon}_1 + \bar{\Upsilon}_1 \upsilon_{1t+1} &\approx \nu(1-\beta R)^{\frac{\omega_2}{\omega_2+1}} \bar{K}^{\omega} + \nu(1-\beta R)^{\frac{\omega_2}{\omega_2+1}} \bar{K}^{\omega}\omega k_{it+1} \\ \bar{\Upsilon}_1 \upsilon_{1t+1} &\approx \nu(1-\beta R)^{\frac{\omega_2}{\omega_2+1}} \bar{K}^{\omega}\omega k_{it+1}.\end{aligned}$$

Substituting into the FOC,

$$\begin{aligned}&\mathbb{E}_{it}\left[\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}\left(a_{it+1}+(\alpha-1)\,k_{it+1}\right)+\beta\hat{\xi}\left(k_{it+2}-k_{it+1}\right)-\tau^{K}_{it+1}\left(1-\beta\left(1-\delta\right)\right)\right]\\ =\;&\hat{\xi}\left(k_{it+1}-k_{it}\right)+\nu\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}\bar{K}^{\omega}\omega k_{it+1}\ ,\end{aligned}$$

or

$$k_{it+1}\left(\left(1+\beta\right)\xi+1-\alpha-\gamma_k\right)=\mathbb{E}_{it}\left[a_{it+1}+\tau_{it+1}\right]+\beta\xi\mathbb{E}_{it}\left[k_{it+2}\right]+\xi k_{it}\ ,$$

where

$$\begin{aligned}\gamma_k &= -\frac{\nu\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}\omega\bar{K}^{\omega}}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}=-\frac{\nu\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}\omega\bar{K}^{\omega}}{\nu\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}\bar{K}^{\omega}+1-\beta\left(1-\delta\right)+\hat{\xi}\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)}\\ &= -\frac{\omega\bar{\Upsilon}_1}{\bar{\Upsilon}_1+\kappa}\end{aligned}$$

where we have substituted in from the steady state Euler equations and $\kappa\equiv1-\beta\left(1-\delta\right)+\hat{\xi}\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)$.

A.7 Frictional Labor

In this appendix, we provide detailed derivations for the case of frictional labor.

A.7.1 Model Solution

When labor is chosen under the same frictions as capital, the firm's value function takes the form

$$\begin{aligned}\mathcal{V}\left(K_{it},N_{it},\mathcal{I}_{it}\right)=\max_{K_{it+1},N_{it+1}}\quad&\mathbb{E}_{it}\left[Y^{\frac{1}{\theta}}\hat{A}_{it}K_{it}^{\alpha_1}N_{it}^{\alpha_2}\right]\\ -\;&\mathbb{E}_{it}\left[T_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi\left(K_{it+1},K_{it}\right)\right]\\ -\;&\mathbb{E}_{it}\left[T_{it+1}WN_{it+1}\left(1-\beta\left(1-\delta\right)\right)+W\Phi\left(N_{it+1},N_{it}\right)\right]\\ +\;&\mathbb{E}_{it}\left[\beta\mathcal{V}\left(K_{it+1},N_{it+1},\mathcal{I}_{it+1}\right)\right]\end{aligned}\tag{27}$$

where the adjustment cost function $\Phi\left(\cdot\right)$ is as defined in expression (2). Because the firm makes a one-time payment to hire incremental labor, the cost of labor W is now to be interpreted as the present discounted value of wages. Capital and labor are both subject to the same adjustment frictions, the same distortions, denoted T_{it+1}, and are chosen under the same information set, though the cost of labor adjustment is denominated in labor units.

The first order and envelope conditions yield two Euler equations:

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1},K_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_1 Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{\alpha_1-1}N_{it+1}^{\alpha_2}-\beta\Phi_2\left(K_{it+2},K_{it+1}\right)\right]\\ \mathbb{E}_{it}\left[WT_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(N_{it+1},N_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_2 Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{\alpha_1}N_{it+1}^{\alpha_2-1}-\beta W\Phi_2\left(N_{it+2},N_{it+1}\right)\right]\end{aligned}$$

To show that this setup leads to an intertemporal investment problem that takes the same form as (3), we prove that there exists a constant η such that $N_{it+1}=\eta K_{it+1}$ which leads to the same solution as if the firm were choosing only capital facing a degree of curvature $\alpha=\alpha_1+\alpha_2$.

Under this conjecture, we can rewrite the firm's problem in (27) as

$$\begin{aligned}\tilde{\mathcal{V}}\left(K_{it},\mathcal{I}_{it}\right)=\max_{K_{it+1}} \quad & \mathbb{E}_{it}\left[\frac{\eta^{\alpha_2}}{1+W\eta}Y^{\frac{1}{\theta}}\hat{A}_{it}K_{it}^{\alpha_1+\alpha_2}-T_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right)\right]\\ + \quad & \mathbb{E}_{it}\left[-\Phi\left(K_{it+1},K_{it}\right)+\beta\tilde{\mathcal{V}}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right]\end{aligned}$$

Let $\{K_{it}^*\}$ be the solution to this problem. By definition, it must satisfy the following optimality condition

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1}^*,K_{it}^*\right)\right] &= \mathbb{E}_{it}\left[\beta\frac{\left(\alpha_1+\alpha_2\right)Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{1+W\eta}\right]\\ &- \mathbb{E}_{it}\left[\beta\Phi_2\left(K_{it+2}^*,K_{it+1}^*\right)\right]\end{aligned} \tag{28}$$

Now substitute the conjecture that $N_{it}^*=\eta K_{it}^*$ into the optimality condition for labor from the original problem and rearrange to get:

$$\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1}^*,K_{it}^*\right)\right]=\mathbb{E}_{it}\left[\beta\frac{\alpha_2 Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{W\eta}-\beta\Phi_2\left(K_{it+2}^*,K_{it+1}^*\right)\right] \tag{29}$$

If η satisfies

$$\frac{\alpha_1+\alpha_2}{1+W\eta}=\frac{\alpha_2}{W\eta} \qquad \Rightarrow \qquad W\eta=\frac{\alpha_2}{\alpha_1} \tag{30}$$

then (29) is identical to (28). In other words, under (30), the sequence $\{K_{it}^*,N_{it}^*\}$ satisfies the optimality condition for labor from the original problem. It is straightforward to verify that this also implies that $\{K_{it}^*,N_{it}^*\}$ satisfy the optimality condition for capital from the original problem:

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1}^*,K_{it}^*\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_1 Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}-\beta\Phi_2\left(K_{it+2}^*,K_{it+1}^*\right)\right]\\ &= \mathbb{E}_{it}\left[\beta\frac{\alpha_2 Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{W\eta}-\beta\Phi_2\left(K_{it+2}^*,K_{it+1}^*\right)\right]\end{aligned}$$

Thus, we can analyze this environment in an analogous fashion to the baseline setup, where the firm's intertemporal optimization problem takes the same form as expression (3), with $\alpha = \alpha_1 + \alpha_2$, $G = \frac{\eta^{\alpha_2} Y^{\frac{1}{\theta}}}{1+W\eta}$ and $A_{it} = \hat{A}_{it}$.

A.7.2 Aggregation

To derive aggregate output and TFP for this case, we use the fact that, as shown above, $N_{it} = \eta K_{it}$ where $\eta = \frac{\alpha_2}{\alpha_1 W}$. Substituting into the revenue function gives

$$P_{it}Y_{it} = Y^{\frac{1}{\theta}} \hat{A}_{it} \eta^{\alpha_2} K_{it}^{\alpha_1+\alpha_2} = Y^{\frac{1}{\theta}} \hat{A}_{it} \eta^{\alpha_2} K_{it}^{\alpha}$$

By definition,

$$MPRK_{it} = \alpha Y^{\frac{1}{\theta}} \hat{A}_{it} \eta^{\alpha_2} K_{it}^{\alpha-1}$$

so that

$$K_{it} = \left(\frac{\alpha Y^{\frac{1}{\theta}} \hat{A}_{it} \eta^{\alpha_2}}{MPRK_{it}} \right)^{\frac{1}{1-\alpha}}$$

so that

$$\begin{aligned} P_{it}Y_{it} &= Y^{\frac{1}{\theta}} \eta^{\alpha_2} \hat{A}_{it} \left(\frac{\alpha Y^{\frac{1}{\theta}} \eta^{\alpha_2} \hat{A}_{it}}{MRPK_{it}} \right)^{\frac{\alpha}{1-\alpha}} \\ &= \alpha^{\frac{\alpha}{1-\alpha}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha}} \eta^{\frac{\alpha_2}{1-\alpha}} \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}} \end{aligned}$$

and

$$Y = \int P_{it}Y_{it} di = \alpha^{\frac{\alpha}{1-\alpha}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha}} \eta^{\frac{\alpha_2}{1-\alpha}} \int \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}} di$$

or, rearranging,

$$Y = \alpha^{\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}} Y^{\frac{1}{\theta}\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}} \eta^{\frac{\hat{\alpha}_2}{1-\alpha}} \left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}} di \right)^{\frac{\theta}{\theta-1}}$$

Capital market clearing implies

$$K = \int K_{it} di = \alpha^{\frac{1}{1-\alpha}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha}} \eta^{\frac{\alpha_2}{1-\alpha}} \int \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di$$

so that

$$K^{\hat{\alpha}_1} N^{\hat{\alpha}_2} = \alpha^{\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}} Y_t^{\frac{1}{\theta}\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}} \eta^{\hat{\alpha}_2 + \frac{\alpha_2}{1-\alpha}(\hat{\alpha}_1+\hat{\alpha}_2)} \left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di \right)^{\hat{\alpha}_1+\hat{\alpha}_2}$$

Aggregate TFP is

$$A = \frac{Y}{K^{\hat{\alpha}_1} N^{\hat{\alpha}_2}} = \frac{\left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}} di\right)^{\frac{\theta}{\theta-1}}}{\left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di\right)^{\hat{\alpha}_1+\hat{\alpha}_2}}$$

Following similar steps as in the baseline case, we can derive

$$a = a^* - \frac{1}{2}\frac{\theta}{\theta-1}\frac{\alpha}{1-\alpha}\sigma^2_{mrpk}$$

Under constant returns to scale in production, this simplifies to

$$a = a^* - \frac{1}{2}\theta\sigma^2_{mrpk}$$

The output effects are the same as in the baseline case.

B Data

As described in the text, our Chinese data are from the Annual Surveys of Industrial Production conducted by the National Bureau of Statistics. The data span the period 1998-2009 and are built into a panel following quite closely the method outlined in Brandt et al. (2014). We measure the capital stock as the value of fixed assets and calculate investment as the change in the capital stock relative to the preceding period. We construct firm fundamentals, a_{it}, as the log of value-added less α multiplied by the log of the capital stock and (the log of) the marginal product of capital, $mrpk_{it}$ (up to an additive constant), as the log of value-added less the log of the capital stock. We compute value-added from revenues using a share of intermediates of 0.5 (our data does not include a direct measure of value-added in all years). We first difference the investment and fundamental series to compute investment growth and changes in fundamentals. To extract the firm-specific variation in our variables, we regress each on a year by time fixed-effect and work with the residual. Industries are defined at the 4-digit level. This eliminates the industry-wide component of each series common to all firms in an industry and time period (as well the aggregate component common across all firms) and leaves only the idiosyncratic variation. To estimate the parameters governing firm fundamentals, i.e., the persistence ρ and variance of the innovations σ^2_μ, we perform the autoregression implied by (5), again including industry by year controls. We eliminate duplicate observations (firms with multiple observations within a single year) and trim the 3% tails of each series. We additionally exclude observations with excessively high variability in investment (investment rates over 100%). Our final sample in China consists of 797,047 firm-year observations.

Our US data are from Compustat North America and also spans the period 1998-2009. We measure the capital stock using gross property, plant and equipment. We treat the data in exactly the same manner as just described for the set of Chinese firms. We additionally eliminate firms that are not incorporated in the US and/or do not report in US dollars. Our final sample in the US consists of 34,260 firm-year observations.

Table 8 reports a number of summary statistics from one year of our data, 2009: the number of firms (with available data on sales), the share of GDP they account for, and average sales and capital.

Table 8: Sample Statistics 2009

	No. of Firms	Share of GDP	Avg. Sales ($M)	Avg. Capital ($M)
China	303623	0.65	21.51	8.08
US	6177	0.45	2099.33	1811.35

For the analyses in Section 5.1, labor is measured as the number of employees in the US Compustat data and wage bill in the Chinese data. Expenditures on intermediate inputs are reported in the Chinese data. In the US, we construct a measure of intermediates following the method outlined in İmrohoroğlu and Tüzel (2014), i.e., as total expenses less labor expenses, where total expenses are calculated as sales less operating income (before depreciation and amortization, Compustat series OIBDP). From here we can calculate materials' share and the markup-adjusted revenue products of capital and labor. We isolate the firm-specific variation in these series following a similar procedure as described above, i.e., by extracting a full set of industry by time fixed effects and working with the residual. We trim the 1% tails of each series.

C Interactions Between Factors

In the main text (specifically, Table 3), we measured the contribution of each factor in isolation, i.e., setting all other forces to zero. The top panel of Table 9 reproduces those estimates (labeled 'In isolation') and compares them to the case where all the other factors are held fixed at their estimated levels (labeled 'Joint'). The table shows some evidence of interactions, but since adjustment and informational frictions are modest, the numbers are quite similar under both approaches.

Table 9: Interactions Between Factors - US

	Adj Costs	Uncertainty	Other Factors		
			Correlated	Transitory	Permanent
In isolation					
$\Delta\sigma^2_{mrpk}$	0.05	0.03	0.06	0.03	0.29
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	10.8%	7.3%	14.4%	6.3%	64.7%
Joint					
$\Delta\sigma^2_{mrpk}$	0.04	0.03	0.08	0.00	0.29
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	8.0%	5.7%	17.4%	0.3%	64.7%

D Robustness

D.1 Investment Moments

As discussed in footnotes 20 and 29, in this subsection, we re-estimate our model targeting the autocorrelation and variance of investment in levels, rather than growth rates. The values of these moments are 0.25 and 0.04, respectively, in the US and 0.04 and 0.08 in China. The other target moments are the same as in Table 2. Table 10 reports the results. A comparison to Table 3 shows that the parameter estimates are quite close to the baseline, as are the contributions to *mrpk* dispersion – adjustment costs and uncertainty account for between 15% and 20% of σ^2_{mrpk} in the two countries, correlated factors play a large role in China and less so in the US, while fixed factors are quite significant in both countries.

Table 10: Using Moments from Investment in Levels

	Adjustment Costs	Uncertainty	Other Factors		
			Correlated	Transitory	Permanent
Parameters	ξ	$\mathbb{V}$	γ	σ^2_{ε}	σ^2_{χ}
China	0.37	0.11	-0.72	0.02	0.38
US	1.77	0.04	-0.31	0.19	0.28
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$					
China	4.3%	11.9%	48.9%	2.5%	40.8%
US	12.1%	8.1%	13.2%	42.4%	62.8%

D.2 Measurement of Firm-Level Capital

Our baseline analysis uses reported book values of firm-level capital stocks. Here, we use the perpetual inventory method along with investment good price deflators to construct an alternative measure of capital for the US firms. To do this, we follow the approach in Eberly et al. (2012). Here, we briefly describe the procedure and refer the reader to that paper for more details. We use the book value of capital in the first year of our data as the starting value of the capital stock and use the recursion:

$$K_{it} = \left(K_{it-1} \frac{P_{Kt}}{P_{Kt-1}} + I_{it} \right) (1 - \delta_j)$$

to estimate the capital stock in the following years, where I_t is measured as expenditures on property, plant and equipment, P_K is the implicit price deflator for nonresidential investment, obtained from the 2013 Economic Report of the President, Table 7, and δ_j is a four-digit industry-specific estimate of the depreciation rate. We calculate the useful life of capital goods in industry j as $L_j = \frac{1}{N_j} \sum_{N_j} \frac{PPENT_{it-1} + DEPR_{it-1} + I_{it}}{DEPR_{it}}$ where N_j is the number of firms in industry j, $PPENT$ is property, plant and equipment net of depreciation and $DEPR$ is depreciation and amortization. The implied depreciation rate for industry j is $\delta_j = \frac{2}{L_j}$. We use the average value for each industry over the sample period.

Table 11 reports the estimation results. The parameters governing firm fundamentals, ρ and σ^2_μ, are quite close to the baseline values, as is the total amount of observed misallocation, σ^2_{mrpk}.[44] The autocorrelation of investment growth is somewhat higher and its volatility somewhat lower, which together lead to a higher estimate of the adjustment cost parameter, ξ. This is reflected in the higher contribution of these costs to *mrpk* dispersion, which is about 27% of the total (compared to 11% in the baseline). The estimated degree of uncertainty is close to the baseline value. Together, these two forces account for about 33% of the observed misallocation, compared to about 18% under our baseline calculations. Thus, our finding of a key role for other firm-specific factors continues to hold – these factors account for roughly two-thirds of σ^2_{mrpk}. The largest component shows up as a permanent factor that is orthogonal to firm fundamentals. The time-varying correlated and uncorrelated components contribute only modestly.

Similar to the exercise in Appendix D.1, we have also re-estimated the model using this alternative measure of firm-level capital stocks and targeting the autocorrelation and variability of investment in levels, rather than growth rates. The results are reported in Table 12. The estimates are broadly in line with those in Table 11 and are extremely close to the baseline

[44]Even in the last year of the sample, the correlation of the two capital stock measures exceeds 0.95.

Table 11: Perpetual Inventory Method for Capital - US firms

Moments	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
	0.94	0.07	0.15	-0.18	0.55	0.01	0.43
Parameters			ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
			5.80	0.02	-0.17	0.05	0.26
Aggregate Effects							
$\Delta\sigma^2_{mrpk}$			0.12	0.02	0.02	0.05	0.26
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$			27.5%	5.7%	4.3%	12.8%	59.9%
Δa			0.05	0.01	0.01	0.02	0.11

ones in Table 3. To see why, we have also computed the implied values of the autocorrelation and variance of investment using the parameter estimates from Table 11. This gives values of 0.69 and 0.02, respectively, compared to the empirical values of 0.57 and 0.02. Because the estimation in Table 11 already matches these (non-targeted) moments fairly closely, explicitly targeting them does not have a large effect.

Table 12: Perpetual Inventory Capital and Investment in Levels - US

	ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
Parameters	1.65	0.03	-0.32	0.00	0.28
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	12.0%	6.9%	14.0%	0.7%	64.3%

D.3 Sectoral Analysis

In this appendix, we repeat our analysis for US firms at a disaggregated sectoral level, allowing for sector-specific structural parameters.

We begin by computing sector-specific α's (curvature in the profit function) using data on value-added and compensation of labor by sector from the Bureau of Economic Analysis, Annual Industry Accounts.[45] To match the SIC (or NAICS) classifications in Compustat, we compute labor's share of value-added for the 9 major sectors of the industrial classification – Agriculture, Forestry and Fishing; Mining; Construction; Manufacturing; Transportation, Communications and Utilities; Wholesale Trade; Retail Trade; Finance, Insurance and Real Estate; Services.[46]

[45] The data are available at `https://www.bea.gov/industry/iedguide.htm`.

[46] Most of these correspond one-for-one with sectors reported by the BEA data. There, Transportation and Utilities are reported separately, as are several subcategories of services, which we aggregate. The only sector we were unable to include from the BEA data was Information, as it does not line up one-for-one with an SIC or NAICS category. The shares are calculated as the average over the most recent period available, 1998-2011

To translate these shares into a value of α, note that under our assumptions of monopolistic competition and constant returns to scale in production, labor's share of value-added is equal to $LS = \frac{\theta-1}{\theta}(1-\hat{\alpha}_1)$ where $1-\hat{\alpha}_1$ is the labor elasticity in the production function. Then, solving for $\hat{\alpha}_1$ and substituting into the definition of α, we have

$$\alpha = \frac{\alpha_1}{1-\alpha_2} = \frac{\frac{\theta-1}{\theta} - LS}{1-(1-\hat{\alpha}_1)\frac{\theta-1}{\theta}} = \frac{\frac{\theta-1}{\theta} - LS}{1-LS}$$

Implementing this procedure yields the values of α in the top panel of Table 13.[47]

Next, we re-compute our cross-sectional moments for each sector, using the values of α to estimate fundamentals. We continue to control for time and industry fixed-effects to extract the firm-specific components of the series (there are multiple four-digit industries within each sector). We report the target moments in the first panel of Table 13. We then estimate the model separately for each sector, allowing the structural parameters governing the various sources of misallocation to vary across sectors. The resulting parameter estimates are presented in the second panel of the table and the implied contribution of each factor to *mrpk* dispersion in the last two panels.

There is some heterogeneity across the sectors, both in the overall extent of misallocation as well as in the estimates for the underlying factors. For example, adjustment costs are largest in manufacturing, where they account for as much as 20% of the observed misallocation and are smallest in FIRE. But, overall, the main message from our baseline analysis continues to hold – adjustment and information frictions, although significant, do not create a lot of *mrpk* dispersion, leaving a substantial role for other firm-specific factors. While the results point to some heterogeneity in the correlation structure of these factors, the permanent component seems to play a key role across all sectors.

E Estimates for Other Countries/Firms

In this appendix, we apply our empirical methodology to two additional countries for which we have firm-level data - Colombia and Mexico - as well as to publicly traded firms in China.

The Colombian data come from the Annual Manufacturers Survey (AMS) and span the years 1982-1998. The AMS contains plant-level data and covers plants with more than 10 employees, or sales above a certain threshold (around \$35,000 in 1998, the last year of the data). We use data on output and capital, which includes buildings, structures, machinery

(which roughly lines up with the period of the firm-level data, 1998-2009).

[47] We have also calculated this value for the entire US economy by summing across all the sectors reported by the BEA. This gives an aggregate labor share of 0.56 and an implied α of 0.62, exactly our baseline value.

and equipment. The construction of these variables is described in detail in Eslava et al. (2004). Plants are classified into industries defined at a 4-digit level. The Mexican data are from the Annual Industrial Survey over the years 1984-1990, which covers plants of the 3200 largest manufacturing firms. They are also at the plant-level. We use data on output and capital, which includes machinery and equipment, the value of current construction, land, transportation equipment and other fixed capital assets. A detailed description is in Tybout and Westbrook (1995). Plants are again classified into industries defined at a 4-digit level. Data on publicly traded Chinese firms are from Compustat Global. Due to a lack of a sufficient time-series for most firms, we focus on single cross-section for 2015 (the moments use data going back to 2012). Similarly, due to the sparse representation of many industries, we focus on those with at least 20 firms. For all the datasets, we compute the target moments following the same methodology as outlined in the main text of the paper. Our final samples consist of 44,909 and 3,208 plant-year observations for Colombia and Mexico, respectively, and 1,055 firms in China.

Table 14 reports the moments and estimated parameter values for these sets of firms, as well as the share of *mrpk* dispersion arising from each factor and the effects on aggregate productivity. The results are quite similar to those for Chinese manufacturing firms in Table 3 in the main text. The contribution of adjustment costs and uncertainty to misallocation is rather limited, and that of uncorrelated transitory factors negligible - across these sets of firms, a large portion of misallocation stems from correlated and permanent firm-specific factors.

Table 13: Sector-Level Results

Moments	α	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
Agr., Forestry and Fishing	0.77	0.92	0.11	0.13	−0.37	0.92	0.03	0.61
Mining	0.76	0.91	0.10	0.16	−0.29	0.74	0.07	0.35
Construction	0.49	0.93	0.15	0.17	−0.28	0.71	0.07	0.69
Manufacturing	0.59	0.94	0.08	0.10	−0.32	0.50	0.05	0.43
Trans., Comm. and Utilities	0.67	0.94	0.04	0.13	−0.32	0.58	0.03	0.38
Wholesale Trade	0.65	0.94	0.08	0.18	−0.31	0.67	0.05	0.57
Retail Trade	0.61	0.96	0.02	0.20	−0.30	0.25	0.02	0.20
Finance, Insurance and Real Estate	0.78	0.90	0.09	0.28	−0.32	0.77	0.07	0.61
Services	0.38	0.95	0.10	0.03	−0.28	0.31	0.08	0.53
Parameters				ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
Agr., Forestry and Fishing				0.83	0.05	−0.78	0.01	0.09
Mining				0.49	0.04	−0.56	0.00	0.13
Construction				0.65	0.08	−0.50	0.00	0.32
Manufacturing				3.35	0.03	−0.17	0.18	0.28
Trans., Comm. and Utilities				0.55	0.02	−0.55	0.00	0.25
Wholesale Trade				0.55	0.04	−0.54	0.00	0.30
Retail Trade				1.97	0.01	−0.07	0.03	0.17
Finance, Insurance and Real Estate				0.18	0.06	−0.80	0.00	0.26
Services				0.81	0.04	−0.14	0.00	0.44
Contribution to misallocation: $\Delta\sigma^2_{mrpk}$								
Agr., Forestry and Fishing				0.07	0.05	0.45	0.01	0.09
Mining				0.06	0.04	0.19	0.00	0.13
Construction				0.04	0.08	0.26	0.00	0.32
Manufacturing				0.09	0.03	0.02	0.18	0.28
Trans., Comm. and Utilities				0.01	0.02	0.10	0.00	0.25
Wholesale Trade				0.02	0.04	0.20	0.00	0.30
Retail Trade				0.02	0.01	0.00	0.03	0.17
Finance, Insurance and Real Estate				0.01	0.06	0.31	0.00	0.26
Services				0.02	0.04	0.02	0.00	0.44
Share of misallocation: $\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$								
Agr., Forestry and Fishing				0.11	0.08	0.74	0.02	0.15
Mining				0.18	0.10	0.54	0.00	0.37
Construction				0.05	0.11	0.37	0.00	0.47
Manufacturing				0.21	0.07	0.05	0.41	0.63
Trans., Comm. and Utilities				0.03	0.05	0.26	0.01	0.65
Wholesale Trade				0.04	0.07	0.35	0.00	0.54
Retail Trade				0.08	0.05	0.01	0.14	0.85
Finance, Insurance and Real Estate				0.02	0.09	0.51	0.00	0.42
Services				0.05	0.07	0.04	0.00	0.83

Table 14: Additional Countries/Firms

Moments	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
Colombia	0.95	0.09	0.28	-0.35	0.61	0.07	0.98
Mexico	0.93	0.07	0.17	-0.39	0.69	0.02	0.79
China Compustat	0.96	0.04	0.30	-0.42	0.76	0.04	0.41
Parameters			ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
Colombia			0.54	0.05	-0.55	0.01	0.60
Mexico			0.13	0.04	-0.82	0.00	0.42
China Compustat			0.15	0.03	-0.69	0.00	0.18
$\Delta\sigma^2_{mrpk}$							
Colombia			0.02	0.05	0.30	0.01	0.60
Mexico			0.00	0.04	0.36	0.00	0.42
China Compustat			0.00	0.03	0.22	0.00	0.18
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$							
Colombia			2.5%	5.6%	30.9%	0.7%	61.3%
Mexico			0.5%	4.9%	44.9%	0.0%	52.8%
China Compustat			0.8%	6.3%	54.0%	0.2%	43.7%
Δa							
Colombia			0.01	0.02	0.13	0.00	0.26
Mexico			0.00	0.02	0.16	0.00	0.18
China Compustat			0.00	0.02	0.19	0.00	0.16