Investing in Children's Skills: An Equilibrium Analysis of Social Interactions and Parental Investments

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Abstract

This paper studies the effects of social interactions on the dynamics of children's skills. I build a dynamic equilibrium model of child development with two key ingredients: peer groups forming endogenously and parental investments responding to the child's social interactions. I estimate the model via simulated method of moments using a dataset of U.S. adolescents. Exploiting within school / across cohort variations in potential peers' compositions, I identify the degree of complementarity between parents and peers in skill formation. I find that the environment where children grow up permanently shapes their developmental trajectories through the effects of social interactions. Moving a child at age 12 to an environment where children have 1 percentile higher skills at age 16, on average, improves her skills rank at age 16 by 0.63 percentiles. The effects are in proportion to the exposure time throughout childhood, with an average effect of 0.48 percentiles if the child is moved at age 15. As model validation, I show that these results track the estimates of the exposure effects of neighborhoods in Chetty and Hendren (2016a). Decomposing the exposure effects, I find that *peers* alone account for more than half of the overall findings, while the *school* and the *neighborhood* quality account for the remainder. Finally, I evaluate the effects on child development of a policy that integrates a sizable fraction of low-skilled children into a high-income environment. I find: (i) the effects depend on the endogenous formation of new peer groups; (ii) the policy generates dynamic equilibrium effects on parental investments and social interactions, which, if ignored, would lead to policy predictions for children's skills of approximately seven times smaller.

JEL Classification: C51, J13, J24

Keywords: Skill Formation, Social Interactions, Child Development, Model Validation, Out-of-sample Prediction, Equilibrium Treatment Effects, Heterogeneous Treatment Effects

1 Introduction

This paper analyzes the effect of social interactions on skill formation in children. In particular, I build and estimate a model of child development, where children grow up in different *environments*, which are defined by: peers' composition, neighborhood quality and school quality. The dynamics of skills is governed by a technology of skill formation, which depends upon parental investments, the current child's skills and the environment-specific inputs. In this framework, I shed light on the importance of the dynamic effects of children's endogenous social interactions and the parental investment decisions in explaining developmental differences between different environments. A growing consensus in the literature emphasizes the importance of neighborhoods in shaping children's opportunities later in life (Chetty and Hendren, 2016a,b; Chetty et al., 2016a,b). However, despite extensive research, the mechanisms behind these results remain unexplained. This paper reconciles the previous findings of childhood exposure to neighborhood with the role of children's social interactions in child development.

This project advances the current literature of child development by building and estimating a dynamic equilibrium model of children's skill formation with two innovative empirically grounded features. First, within different environments, children endogenously select their peer groups based on their preferences for their peers' characteristics. Social interactions can exhibit the tendency of children to become friends with others who share similar characteristics: a phenomenon called homophily bias. Second, parental investments respond to changes in peer groups. Decisions regarding parental investments depend upon a child's current peers, as well as on expectations about future peer groups. Equilibrium effects arise from the socially determined aspects of parental investments. In this framework, parental investments not only directly affect a child's skills, but also affect the development of the child's peers through social interactions. Consequently, the individual return on investing in children is affected by the equilibrium parental engagement within each environment.

Skills are formed dynamically through a technology of skill formation, which defines the complementarities between parental investments and the other inputs of child development in producing a child's skills: the current endowment of skills, the skills of peers, the school quality and the neighborhood quality. In this framework, there are two main channels through which peers affect parental behavior. First, contemporaneous changes in current peers and parental investments are related to the *static* complementarity between the two inputs. Second, permanent changes in peer composition affect parental behavior through the *dynamic* complementarity in skill formation. In other words, a permanent change in peer composition affect the return of parental investments through the dynamic aspect of skill formation.

The model is estimated using data on U.S. adolescents from the National Longitudinal Study of Adolescent Health (Add Health). Add Health provides information about friendships within each school, which is key for analyzing the formation of peer groups. Moreover, information about child achievements and parental investments are available.

The identification of the model comes with two main challenges: (i) unobserved heterogeneity in how peer groups are endogenously formed; and (ii) children's skills and parental investments are unobserved. Ignoring these issues by using correlational relationships would cause the model's estimates and subsequent quantitative analysis to be biased.

The first challenge presents itself from the fact that peer groups may be formed based on additional unobserved heterogeneity, which can cause correlation between peer groups' realization and the residual unexplained variation in skill formation. To address this concern, I implement a standard instrumental variable (IV) approach in the literature. This identification strategy exploits random variations in cohort composition within school / across cohorts. The idea behind this identification strategy is simple: random changes in cohort compositions affect the opportunities for friendships between children. These shifts in the formation of peer groups affect the return of parental investments and the subsequent parental decisions.[1]

In addressing the second challenge, Cunha et al. (2010) illustrate that even the classical measurement error in measuring a child's skills can cause important biases in estimating the technology of children's skill formation. Following the approach in Cunha et al. (2010) and Agostinelli and Wiswall (2016), I implement a dynamic latent factor model, which allows me to identify the joint distribution of latent skills and investments by exploiting multiple measurements in the data.

I estimate the model via simulated method of moments (SMM). I find that parental investments and peers are substitute inputs in producing children's skills. At the same time, I find a strong dynamic complementarity between parental investments and future expected peers. As a result of these two findings, a permanent change in peer composition has two opposing effects on parental investments. On one hand, "better" peers generate contemporaneous substitution effects in investment decisions due to the high substitutability in the production function. On the other hand, higher expected future skills for peers produce an "income" effect through the dynamic complementarity of skill formation. Parents have the incentive to invest

[1] For previous use of similar source of identifying variation, see Hoxby (2000); Hanushek et al. (2003); Ammermueller and Pischke (2009); Lavy and Schlosser (2011); Lavy et al. (2012); Bifulco et al. (2011); Burke and Sass (2013); Card and Giuliano (2016); Carrell et al. (2016); Olivetti et al. (2016); Patacchini and Zenou (2016)

more in their children because a higher-skilled child benefits more from higher-skilled peers in the future.

Furthermore, my estimates suggest that the formation of peer groups displays an extensive degree of homophily bias. I show evidence of homophily bias with respect to a child's race and level of latent skills. A child who is in the lower quartile of the skill distribution and belongs to a minority group is four times more likely to befriend a same-race child than a different-race child. In addition, the same child is two times more likely to befriend a same-skill and same-race child than a same-race child in the upper quartile of skill distribution.

I first use the estimated model to analyze the extent to which growing up in different environments accounts for the variation in children's outcomes. I find sizable effects for children moving to better environments. The effects are in proportion to the exposure time. The earlier children are moved, the higher the effect. A child who is moved at age 12 to an environment where children have 1 percentile higher skills at age 16 exhibits, on average, an improvement in her skills rank at age 16 by 0.63 percentiles. The average effect is 0.48 percentiles if the child is moved at age 15. As model validation, I show that my findings track (out-of-sample) the quasi-experimental findings of childhood exposure effects of neighborhoods for the U.S. from Chetty and Hendren (2016a). In addition, my model allows me to decompose these effects. I find that *peers* account for more than half of the exposure effects.

The relative importance of peers for the exposure effects underlines the role of policies that change peers' composition and promote socioeconomic integration in environments, as a way to improve outcomes for disadvantaged children. I find that by moving the most disadvantaged children (in the lower quartile of skill distribution) from a low-income environment to a high-income environment generates important dynamic equilibrium effects, with heterogeneous treatment effects for both the moved and receiving children. I first consider a large-scale policy, i.e. a policy that moves a sizable fraction of disadvantaged children into a higher-income environment (approximately 5% of the population of the receiving cohort). I find that the policy increases the skills of the moved population of 16-year-old children, on average, by approximately 0.40 standard deviations. On average, I do not find any adverse effect for receiving children. On the contrary, when the fraction of moved population increases to 30%, I find that the policy generates *winners* and *losers*. First, I find that the policy increases the skills of the moved population of 16-year-old children on average by 0.22 standard deviations. In contrast, there is an adverse effect for receiving children, with the skills of 16-year-old children decreasing, on average, by 0.15 standard deviations. Additionally, I find that children who remained in the sending environment benefit from the outflow of the most disadvantaged companions, with an average increase in skills at age 16 of 0.17 standard deviations.

I find that large-scale changes in peers' composition generate important equilibrium feedback effects, and as a result amplify the policy effects. Ignoring equilibrium effects would lead to large biases in counterfactual policy predictions for children's final skills. In the case of the larger policy, I find that the policy predictions for the children's skills in the receiving environment would be approximately seven times smaller. Part of the bias is due to the dynamic-equilibrium feedback effects on parental investments. In fact, in the absence of dynamic-equilibrium feedback effects, the *static* complementarity between parents and peers dominates the dynamic effects of the policy.

I find that policy effects for receiving and remaining children reduce in magnitude as the fraction of moved children decreases. An increase of inflow of the most disadvantaged children from the low-income environment to the high-income environment increases the probability of the receiving children becoming friends with the new companions. For the same reason, an increase of the outflow of the moved population benefits children who remain in the sending environment. For children who were moved, the opposite is true. The higher the outflow of disadvantaged companions, the higher the chances that the moved children remain friends with each other in the new environment.

My structural model allows me to analyze the distributional policy effects. I find that large-scale changes in peers' composition exhibit heterogeneous treatment effects as a result of the endogenous formation of new peer groups. Children with lower skills (in the first quartile of the skills distribution in each subpopulation): (i) benefit the most in leaving disadvantaged social environments; (ii) benefit the most amongst the children who remained in the sending environment; (iii) are the ones who are more adversely affected in receiving the new peers. Furthermore, I find stronger policy effects for minorities, with detrimental effects in black and Hispanic children living in the receiving environment. This is explained by the fact that most of the moved children are minorities, and as a result, the minority children from the receiving environment are more likely to interact with the new companions because of the race effects in the endogenous formation of peer groups. In line with this result, previous empirical studies pointed out that peer effects seem to be stronger *intra*-race and for minorities (see Hoxby, 2000; Angrist and Lang, 2004; Imberman et al., 2012)

The paper will be presented as follows. In Section 2, I discuss the related literature. In Section 3, I present the data used for the empirical work and preliminary empirical results. In Section 4 and 5, I present the model. In Section 6, I describe the identification strategy. Section 7 contains a discussion of the structural estimation and results. Section 8 and Section 9 discuss the quantitative analysis and the model validation. Section 10 concludes.

2 Related Literature

This paper builds upon two important areas of the literature: child development and social interaction. There is extensive evidence in the literature on parental and public investment in children that highlights the important role of play inside and outside the household on the development of children's skills (see Todd and Wolpin, 2003, 2007; Del Boca et al., 2014). Cunha and Heckman (2008) and Cunha et al. (2010) estimate a dynamic latent factor model of cognitive and non-cognitive skill formation, allowing for unobservability of both inputs and outputs, endogeneity of inputs and unobserved child-specific heterogeneity. They find that investments made early in life are more effective in remediations for low-skilled children. Agostinelli and Wiswall (2016) follow the framework considered in Cunha et al. (2010) and develop a new identification strategy for the technology of skill formation with unknown total factor productivity and unknown return to scale. Their empirical results show a pattern of rapid skill development from age 5 to 14. They find that as children age, skill inequality increases. Estimates reveal that investments are more productive at early ages and in particular for disadvantaged children. This paper is the first work in the literature that sheds light upon the dynamic equilibrium effects of children's social interactions and the parental investment decisions in explaining developmental differences in children.

A wide set of previous work analyzed peer effects in various outcomes. Manski (1993) points out the challenges in identifying peer effects by considering three observable equivalent specifications in a model of social interactions: peer effects (endogenous effects), selection into peer groups (correlated effects) and common exogenous (contextual) effects. One part of the literature tried to overcome this challenge in identification of peer effects by exploiting exogenous variation in peer-group composition.[2] For example, in Abdulkadiroglu et al. (2014), the identification of peer effects at school is based on test-score discontinuity in admission criteria. In the context of college students, De Giorgi et al. (2010) and Sacerdote (2001), respectively, exploited random assignments of peers at college. Arcidiacono et al. (2012) develop a new algorithm for estimating peer effects using panel data and which controls for peer selection and unobserved heterogeneity using University of Maryland transcript data. Finally, Sacerdote (2001) highlights that the literature's findings on peer effects are quantitatively and statistically larger when considering non-linear models of peer effects. However, results are often context

[2]For a complete literature review on the identification of peer effects through experiments, see Sacerdote (2014)

specific, a potential limitation for policy analysis. This paper is exploiting quasi-experimental variation in cohort composition within school / across cohorts, to identify the degree of complementarity between parental investments and peers in the technology of skill formation. The set of policy-invariant parameters of the model are used to evaluate policies that have not previously been implemented.

A parallel literature started to consider identification and estimation of peer effects within micro-funded models of behavior and social interactions (see Brock and Durlauf, 2001a,b, 2007; Blume et al., 2011, 2015). Calvó-Armengol et al. (2009) estimate a model of adolescent effort choice within a social network. The authors define peers as the set of nominated children in Add Health data. However, the authors do not consider any model of network formation and peer selection, while they control for peer selection through network-specific fixed effects. This is equivalent to assuming that peers select themselves into groups but friendship formation within each group is independent of observable and unobservable characteristics of people in the group. They find that the level of an adolescent's connectivity (position within the network measured by Katz–Bonacich centrality) is an important predictor for their school performance. Finally, Fu and Mehat (2016) estimate a model of student achievements and a class-tracking regime with endogenous parental effort using ECLS-K. They find that accounting for endogenous parental responses to class-quality changes is key to evaluating class-tracking policies. These works explicitly focus on the contemporaneous peer effects on children's outcomes. My paper contributes in this literature by building a new structural model of child development and peer effects, and by highlighting the importance of dynamic peer effects in shaping the developmental trajectories of children.

Analysis of endogenous network formation became popular among both theoretical (see for example Jackson and Wolinsky, 1996; Bala and Goyal, 2000; Dutta et al., 2005; Mele, 2010) and econometric studies (see Christakis et al., 2010; Sheng, 2014; Auerbach, 2016; Graham, 2016, 2017). Carrell et al. (2013) estimates peer effects on academic performance at the United States Air Force Academy. Using an assignment algorithm designed to foster the academic achievement of the lowest-ability students, the authors find a negative treatment effect for the targeted group. The authors provide evidence that this finding is the result of endogenous peer-group formation, which displays the tendency of students to generate homogeneous subgroups. This result underlines the importance of accounting for endogenous peer-group formation once considering policies that manipulate peer composition.

Important progress has been made in Badev (2016), where the author develops a model of individual behavior and endogenous peer selection. He estimates the model using Add Health data on smoking decisions and friendship nominations. He finds that neglecting the endogeneity of the networks leads to important biases on policy evaluations. However, Badev (2016) focuses on the contemporaneous effects of peers, and through this paper, I will emphasize how the dynamic aspect of peer effects is key in understanding the role of social interactions in child development. Additionally, the empirical analysis in Badev (2016) looks at a specific outcome for adolescents (smoking decisions), while my empirical analysis will be based on a dynamic latent factor model. This allows me to consistently study peer effects on children's skills and to avoid relying on arbitrary variables as measures for children's outcomes.

Finally, my work sheds light on the mechanisms behind the recent research on the effects of neighborhood exposure on children. Chetty and Hendren (2016a) find sizable childhood exposure effects of neighborhood. Their results show that the return for children's outcomes of moving to a better neighborhood is in proportion to the amount of time spent in that neighborhood, with a rate of 4% decline for each additional year of exposure to the origin area. In a companion paper, Chetty and Hendren (2016b) find strong correlation between the childhood exposure effects and specific characteristics of neighborhoods, like racial segregation, income inequality, school quality, and social capital. My estimated model replicates (out-of-sample) the findings in Chetty and Hendren (2016a), and it decomposes the causal effects of neighborhoods in different policy-relevant mechanisms, like the effect of peer composition, school quality and neighborhood quality in child development.

3 Data and Empirical Evidence

3.1 The National Longitudinal Study of Adolescent to Adult Health (Add Health)

This paper uses the National Longitudinal Study of Adolescent to Adult Health (Add Health).[3] The Add Health original sample comprises students among 132 representative schools in the United States. There are 90,118 students, ranging between grade 7 and grade 12 in the 1994–1995 school year (Wave I). A subsample of students (20,745) is selected for having an additional home interview (in-home). The home interview includes new questions for the children and a questionnaire for one of their parents. The dataset includes specific information on family background, students' school grades and their scores in the Add Health Picture Vocabulary Test (AHPVT – a revised version of the Peabody Picture Vocabulary Test [PPVT]) , as well as information about children's peers.

[3] For additional information about the dataset, see Appendix A or visit http://www.cpc.unc.edu/projects/addhealth

A main source of information that makes the Add Health dataset particularly attractive for achieving the objective of this project is the friendship nomination. During the first two waves, children were asked, both during the in-home and in-school interviews, to nominate their best five male and best five female friends. This detailed information helps me to reconstruct the structure of friendship for every child in the sample by simply matching their identifier. Additionally, during the in-home interview, children are asked about their relationship with their parents. Respondents provide information regarding whether, during the last four weeks, they were involved in specific activities with their parents. The activities include: going shopping, sport activities, going to a movie/museum/concert or sport event, talking about personal problems or school, or working on a project for school. I use all the activities as measures of parental investments.

One important challenge in the empirical analysis of peer effects using Add Health comes from the fact that children are able to nominate up to five friends for each gender. This feature of the data can lead to a potential censoring and mismeasurement of peer groups (see for example Chandrasekhar and Lewis, 2016; Griffith, 2017). In the sample, I use for my empirical analysis approximately 11% of children showing a full list of 10 best friends within the school roster.[4] To address this concern, I construct the peer-group information for each child from both the individual child's list of friends as well as from the unilateral friendship nominations coming from the other children who are not nominated. In other words, if child i does not nominate child j as a friend, but child j nominates child i, then I consider them as friends in the data. In a case where child i's list of friends is binding, I am able to recover additional friends in his peer group, alleviating the truncation problem. Furthermore, through my empirical analysis of parental investments and peer effects, I implement an IV estimation analysis to deal with the endogeneity of the network formation. Given that my instrument is unrelated to the network structure of friends, this approach is also effective in dealing with mismeasurement of peer effects.[5]

[4]By gender, respectively, 28% of male and 32% of female children report a list of five best same-gender friends within the school roster

[5]A common instrument in the analysis of peer effects is constructed with exogenous characteristics of the friends of friends (see for example Bramoullé et al., 2009; Calvó-Armengol et al., 2009; Patacchini and Zenou, 2012). The validity of this instrument requires the correct measurement of the network structure, at least until the second degree of separation between links.

3.2 Descriptive Statistics

Table 1 reports descriptive statistics for the sample I use in the estimation of the model. The average age is 15.65. In terms of racial composition, 16% of the children are black and 17% are Hispanic, while the remaining 67% are white (or other races). On average, children report 4.48 friends out of the maximum number of 10 possible nominations.[6] The average PPVT raw score is 64.26, while the average grades for English, math, history and science vary from 2.72 to 2.86.[7,8] The average family income is $42,884 (in 1994), while the average number of years of schooling for the child's mother is 13.13.[9]

Table 1 also provides descriptive statistics on measures of a parent's (mother's) level of engagement, which I use to identify latent parental investments in my empirical analysis. The most frequent activities performed by children with their mothers in the four weeks preceding the in-home interview are: shopping (72%), talking about school work (63%) and school activities (54%). On average, from one-third to approximatively a half of children had a conversation about personal issues or an argument with their mothers (47% of them talked with their mothers about someone they are dating or about a party they attended, 39% talked about a personal problem and 33% had a serious argument). A quarter of children went with their mother at least once to a movie theater, museum, concert or sport events, while 38% went to a religious service. Finally, approximately 10% of children either played a sport or worked on a school project with their mother.

3.3 Empirical Findings on the Endogeneity of Network Formation

In this section, I provide some empirical evidence that friendships are not formed at random, but instead children display the tendency to become friends with others who are similar to themselves: a phenomenon called homophily bias. One important concern in any empirical analysis of peer effects is related to the endogeneity of the network formation (see Carrell et al., 2013). In particular, the tendency of children with a similar background (both observable and unobservable characteristics) to socialize together can be an important challenge for identi-

[6] Here, I report the average number of nominated friends. Later, in the empirical section, I will consider the total number of friends, which is constructed by also including the unilateral friendship nominations of other children. The average number of friends is approximately seven.

[7] The national GPA in 1994 for U.S. high schools was 2.44 in math, 2.50 in science and 2.63 in English. Source: the National Center for Education Statistics, The Nation's Report Card

[8] Only 34 out of 19,713 children achieved the maximum PPVT score. Only five children scored the minimum.

[9] Using the Current Population Survey, I find that mothers of children with similar age, as in Add Health, have an average family income for 1994 of $42,759. Their average number of years of education is 12.63.

fication. Furthermore, evaluation of many policies which can change cohort composition in specific social environments (e.g. school vouchers, housing vouchers, classroom tracking, etc.) is required to predict new policy-induced children's networks to account for social interactions and to predict the effects on the dynamics of children's skills.

Following the method in Currarini et al. (2010), I test for homophily bias in the formation of peer groups by looking at the homophily bias index (hereafter referred to as HBI) developed in Coleman (1958). The intuition behind the HBI is straightforward: the index captures the tendency of friendships to be biased towards other children of the same type, adjusting for the relative frequency of that specific type in the overall population. In detail, letting $f_{x,s}$ be the average fraction of friends who are of the same type x at school s, and $q_{x,s}$ being the total fraction of children of type x in a given school s, then we have:

$$HBI_{x,s} = \frac{f_{x,s}}{q_{x,s}} \tag{1}$$

A value of one for the HBI index in (1) suggests that friendships are formed at random, preserving the frequencies of school composition in peer-group composition. On the other hand, the homophily bias in network formation generates an HBI index greater than one: the fraction of same-type friends consistently exceeding the fraction of that type of children at school. Figure 1 shows graphically the HBI for race. The x-axis represents the values for the fraction of same-race children in the school ($q_{x,s}$). The y-axis displays the fraction of same-race friends ($f_{x,s}$). Each point in Figure 1 represents the average fraction of friends of the same race for individuals of a specific race in a specific school. Figure 1 shows that children of different races tend to form friendships with same-race children at a higher frequency than the frequencies of racial composition at school. Figure 2 is the analogue of the previous figure with respect to a child's skills. Each of the sub-figures in Figure 2 uses a different criterion for defining "same-skills children" relative to the standard deviation of the skills distribution. Each of the four specifications exhibits the tendency of children to become friends with other children with the same level of skills.[10]

3.4 Empirical Findings on Parental Investments and Peers' Skills

In this section, I show the extent to which changes in peers' skills induce changes in parental investment behavior. The results provide the support for the framework of my model of child

[10] The null hypothesis of random formation of peer groups with respect to race and skills is rejected in both cases at a 1% significance level.

development and peer effects described in the next section. In addition, these empirical findings are used in the structural estimation of my model as identifying moments. Consider the following empirical model of investment decision:

$$I_{i,s,t} = \beta_0 + \beta_1 \ln h_{i,s,t} + \beta_2 \ln \overline{H}_{i,s,t} + X_i' \beta_3 + \beta_s + u_{i,s,t}\,, \tag{2}$$

where $I_{i,s,t}$ is the parental investment (as a fraction of time) for parent of child i, in school s when she is t years old, which is recovered through a latent factor model (see Section 5.1) using data on parental engagement described in the previous section. The child's skills are defined as $h_{i,s,t}$, while $\overline{H}_{i,s,t}$ is the mean of her peers' skills. X_i is a vector of the child's and parents' exogenous characteristics and β_s is the school fixed effects. The coefficient β_2 represents the effect of peers' skills on investment decisions. The equation in (2) is similar to the investment decision function estimated in the previous literature, where I additionally include peers' skills as an explanatory variable (see Cunha et al., 2010; Agostinelli and Wiswall, 2016; Attanasio et al., 2017a,b,c).

I first estimate the model in (2) using school fixed effects. Column (1) of Table 2 shows the results.[11] The effect of peers on parental investment is negative and statistically significant at the 5% level. The estimate of -1.44 indicates that doubling peers' skills is associated with a decrease in investments of 1.44 percentage points. On the other hand, the effect of a child's skills on parental investment behavior is positive and statistically significant at the 1% level. The point estimate of +2.66 suggests that doubling a child's skills induces parental investments to increase by 2.66 percentage points. Overall, the school fixed effects estimates suggest that parental investments respond in opposite direction to changes in their own child's skills in comparison to changes in skills of their child's peers.

Following the analysis of endogenous network formation in the previous section, I address the endogeneity of peers' skills in (2), implementing a within-school instrumental variable (IV) estimator. I use variation in the racial compositions of different cohorts within the same school to analyze the effect of changes in peers' skills on parental investments, where cohorts are defined by children's ages (for previous use of a similar source of within-school/across-cohorts identifying variation for peer effects, see Hoxby, 2000; Hanushek et al., 2003; Ammermueller and Pischke, 2009; Lavy and Schlosser, 2011; Lavy et al., 2012; Bifulco et al., 2011; Burke and

[11] All results in Table 2 are adjusted for measurement error through the latent factor model explained in Section 5.1.

Sass, 2013; Card and Giuliano, 2016; Carrell et al., 2016; Olivetti et al., 2016; Patacchini and Zenou, 2016). The idea behind this identification strategy is simple: the differences in cohort composition define the choice set for the children's network formations, shifting the peer-group realizations and identifying the causal effect of peers' skills on investment decisions.[12]

Evidence of homophily bias in friendships in Section 3.3 underlines that the heterogeneous effects in the formation of peer groups in children depend on the individual characteristics of the child. Differences in the fraction of children from a minority group between cohorts would asymmetrically impact the friendship realizations that depend on race. Likewise, differences in the fraction of low-skilled children between cohorts would predict dissimilar changes in peers' skills for low-skilled versus high-skilled children. For this reason, I implement an IV specification which allows for different effects of cohort racial composition based on the individual child's own race and skills.

I construct two different instrumental variables for whether the child is part of a minority group or not. In the case of white children, the instrument corresponds to the interactions between the individual child's log-skills ($\ln h_{i,t}$) and the fraction of white children in that cohort. In the case of children from a minority group, the children's skills are interacted with the fraction of black children in that cohort.[13] Allowing for the heterogeneous effects of cohort compositions is important in terms of predicting power on the formation of peer groups, and consequently for the *relevance* of the instrumental variables.

The validity of the instruments relies on (i) the conditional independence and (ii) the exclusion restriction. The first condition requires that differences in racial composition between cohorts be uncorrelated with the unobserved heterogeneity in investment decisions. This assumption would be consistent with a sorting model of neighborhood/school choice through which parents decide where to permanently move according to their expectations about the school's composition. Random differences between the ex-ante expectations and the actual realization of the new cohort's composition would generate exogenous shifts in the set of potential peers. Additionally, conditional independence is valid under the assumption that the latent factor model fully captures the child-specific unobserved heterogeneity, which is a common assumption in studies that estimate the technologies of skill formation (see Cunha and Heckman, 2007, 2008; Cunha et al., 2010; Agostinelli and Wiswall, 2016).[14] The exclusion re-

[12] This identification strategy does not require that friendships are only formed within a unique cohort. It only requires that changes in cohort composition alter the peer-choice set between cohorts. Empirically, most of the friendship nominations are within the same cohort.

[13] The regression is estimated with a within-school IV estimator, and the instrumental variables are all transformed with a within-school transformation.

[14] An exception to this can be found in Cunha et al. (2010), where the authors consider additional model specific-

striction requires that the differences in racial composition between cohorts within the same school affect parental investment decisions only through the peer-effects channel, and not directly in any other way.

Figure 3 shows graphically the first-stage coefficients of the two instruments. In the background of the two figures is a histogram that shows the distribution of the two instruments (after controlling for the explanatory variables in (2)), revealing the identifying sources of the variation. Figures 3a-3b show the different predictions for peers' skills due to changes in compositional effects by race: an increase in the fraction of children with the same race within the same cohort predicts a decrease in the expected level of peers' skills in the case of a child from a minority group, while it predicts an increase in the expected level of peers' skills for a white child in that cohort. Specifically, for the average child, an increase of 20% of same-race children within the same cohort induces an increase of peers' skills of approximately 1.7% if the child is white, while it induces a decline of peers' skills of 2.2% if the child belongs to a minority group[15]. Results are stronger for the high-skilled children. For children in the 95^{th} percentile of the skills distribution, a 20% increase of same-race children within the same cohort induces an increase of peers' skills of approximately 6% if the child is white, while it induces a decline of peers' skills of 7% if the child belongs to a minority group. Moreover, I formally test the relevance of the two instrumental variables in Panel B of Table 2. I find that the two instruments are relevant, with an F-statistic of the test of joint significance equal to 11.78.[16]

Column (2) in Table 2 reports the IV estimates. Peer effects on parental investments are both statistically and quantitatively different from the estimate of the school fixed effects estimator. In fact, using shifts induced from within-school/across-cohort changes in cohort composition, I find that the causal effect of peers' skills on parental investment is positive. The estimate suggests that doubling the average of skills of a child's peers is associated with an increase of parental investment of 0.72 percentage points.

Anticipating the next discussions regarding the identification of the model of investment decisions and the formation of peer groups, there are three main findings from the above estimates which are extremely informative for identification of the model: (i) point estimates of

ations that allow the factors to be correlated with unobserved heterogeneity in investment decisions. However, in order to identify the model, the authors need additional exclusion restrictions. One possible exclusion restriction is that during the first period of development, skills are uncorrelated with the unobserved heterogeneity.

[15] The average for children's skills is approximately 1.1

[16] Stock and Yogo provide critical values to test weak IV conditions based on the F-stat of excluded instruments. Those critical values can be interpreted as a test, with a 5 % significance level, of the hypothesis that the maximum relative bias (with respect to the OLS estimates) is 10% or at least 15%. In this case, Stock and Yogo's critical values for the F-stat of the excluded instruments are 19.93 (10%) and 11.59 (15%)

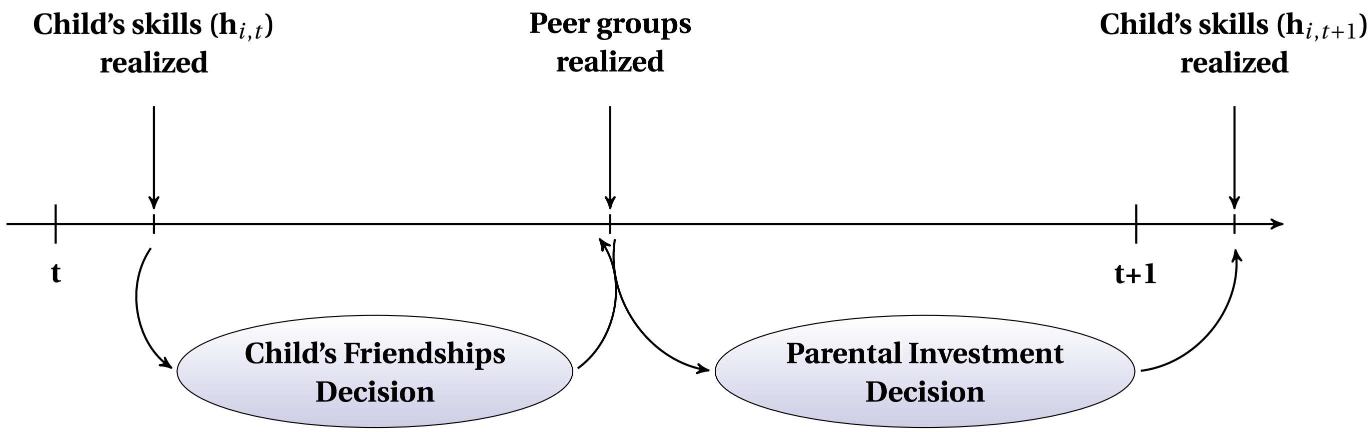
fixed effect estimator; (ii) the permanent nature of shift-induced changes in peers by the instrumental variables and the associated causal findings; (iii) The relative bias of the school fixed effects estimates relative to the IV estimates. Through the lens of the structural model, these three pieces of information from the empirical analysis will directly map into three specific features of the model of investment decisions and formation of peer groups: (i) *static* complementarity between parents and peers in producing skills; (ii) *dynamic* complementarity between parents and peers in producing skills; (iii) endogenous network formation and selection into peer groups on unobservables.

4 An Equilibrium Model of Parental Investments and Endogenous Network Formation

The social environment in which children live predicts their success later in life, defining important room for policy interventions in fostering children's skills. However, analysis of any policy which changes the composition of a specific social context (e.g. school vouchers, housing vouchers, classroom tracking, etc.) requires knowledge of the endogenous policy-induced changes in parental investments and peer groups in order to predict the overall policy effects. This is particularly relevant considering the empirical evidence on endogenous network formation and peer effects on parental investments shown in the previous section. For this reason, in this section, I develop the model that serves as the basis of my empirical and quantitative analysis.

This model represents a network economy populated by a finite number of families, each formed by one parent (mother) and one child. There are several environments $e \in \{1, \ldots, E\}$, each populated by N_e number of families. Children can form social networks only within each environment e. Children from different environments are isolated from each other and they cannot socially interact. The model has four periods ($T = 4$), each consisting of one year. The first period ($t = 1$) is when children are 13 years old, while the last period ($t = T$) is when children are 16 years old. Since I observe a negligible percentage of people changing school during the considered period (probably because children are enrolled in high school), I simplify the model by assuming that the parent cannot decide to change their environment during the model's period of study.

Parents and children solve different problems. Mothers altruistically invest time to foster children's skills. Children endogenously decide their peers according to their skills and other



exogenous characteristics, forming a potentially large social network within the environment. Potential skills spillovers for child development take place between peers within a children's social network. Peers' skills spillovers affect parental investment productivity. In equilibrium, parental investments form the within-environment children's skills distribution, determining the children's social interactions. This mechanism generates an equilibrium-feedback effect on parental behavior caused by peer effects through the formed social network.

4.1 The initial conditions

The initial period of the model ($t = 1$) is fixed when children are 13 years old. At the beginning of this period, I assume each family i draws the vector of individual initial conditions composed of the initial skills for both mother and child (m_i, $h_{i,1}$), its exogenous characteristics X_i, and the neighborhood quality d and school quality s. The peers' composition is defined by the set of children who share the same neighborhood quality and school quality. The combination of peers' composition, neighborhood quality and school quality defines the environments where children live. I do not allow mobility of a family between different environments during the period of consideration (when the child is between 13 and 16 years old). In Add Health, I observe a negligible percentage of people changing school neighborhood during the considered period. This finding is probably related to the fact that parents do not tend to move once their child is already enrolled in high school. Hence, I use this assumption in the model since it significantly simplifies the model. However, different realizations of initial conditions generate the sorting of people into different environments in terms of parents' skills, income and child's skills, which is a key mechanism to analyze children's social interactions and the consequences of social isolation in generating inequality in children's outcomes. Perhaps, in an economy where selection into environments is based on a family's skills and income, children raised in high-skilled and high-income families will tend to interact with peers also raised in high-skilled and high-income families. The characterization of these social-interaction patterns are due to the sorting into environments.

4.2 Skill Formation

At each period t, children's skills $(h_{i,t})$ evolve dynamically through a technology of skill formation. The children's skills in the next period are produced by the current stock of children's skills, parental investment, peer effects, school quality and neighborhood quality. The first two inputs are generally considered in the literature of child development (see for example Cunha and Heckman, 2007, 2008; Cunha et al., 2010; Heckman and Masterov, 2007; Del Boca et al., 2014, 2016). Here, peer effects $(\overline{H}_{i,t})$ are captured by the average of peers' skills:

$$\overline{H}_{i,t} = \frac{1}{\sum_{j \in N_e} L_{i,j,t}} \sum_{j \in N_e} L_{i,j,t} \cdot h_{j,t} \quad , \tag{3}$$

where $L_{i,j,t}$ is an indicator function, which equals one if child i and child j are friends, and zero otherwise. The formation of peer groups is endogenous in the model and is defined by the decision of children (see next section).

The choice of the average effect of peers' skills approach is in line with previous literature on peer effects and social networks, where the mean effect (unweighted or weighted average) is considered a first-order approximation of the peers' externality (see Brock and Durlauf, 2001a,b, 2007; Blume et al., 2011, 2015; Calvó-Armengol et al., 2009; Patacchini and Zenou, 2012; Patacchini et al., 2012). However, in this framework, peer effects can be potentially highly non-linear, depending on the technology specification.[17] I allow the dynamics of children's skills to be affected also by the parental investments $(I_{i,t})$, some individual specific neighborhood/school effects $(A_{i,d,s,t})$ and total factor productivity (TFP). The technology of skill formation which defines children's skills in the next period looks as follows:

$$h_{i,t+1} = {h_{i,t}}^{\alpha_1} \cdot \left[\alpha_2 \left(I_{i,t}\right)^{\alpha_3} + (1-\alpha_2)\left(\overline{H}_{i,t}\right)^{\alpha_3}\right]^{\frac{\alpha_4}{\alpha_3}} \cdot A_{i,d,s,t} \cdot exp(\xi_{i,t+1})\,, \tag{4}$$

where I assume that $(\alpha_1, \alpha_2, \alpha_4) \in (0,1)$ and $\alpha_4 \in (-\infty, 1]$. The stochastic component $\xi_{i,t+1}$ represents the production function shocks. It is unrealized at time t and it affects children's skill dynamics. It represents the variation of skills dynamics unexplained by the specified techno-

[17]See Sacerdote (2001) for the importance of non-linearity in empirical analysis of peer effects

logy in (4). I allow $\xi_{i,t+1}$ to be correlated with the unobservable heterogeneity in the formation of peer groups process $\nu_{i,j,t}$, which represents the unexplained variation in friendship realizations between children from the model in (5). Specifically, I define the shock for the formation of peer groups to be $\nu_{i,j,t} = \widetilde{\nu}_{i,j,t} + \zeta_{i,j,t}$, where $\zeta_{i,j,t} \sim \mathrm{N}(0, \sigma^2_{\zeta})$, and it is potentially correlated with the production shock $\xi_{i,t+1}$. The correlation between production function shock $\xi_{i,t+1}$ and friendship shock $\nu_{i,j,t}$ effectively allow the possibility of selection into peer groups on unobservables.

The specification in (4) allows parents and peers to vary from being perfect complements to being perfect substitute inputs. Additionally, technology in (4) is consistent with the idea of dynamic complementarity of skills evolution, where higher skills today induce higher skills tomorrow (see Cunha and Heckman, 2007). Equation (4) allows me to have a flexible specification for the analysis of peer effects in children's skills accumulation. The level of *static* complementarity/substitutability between parents and peers is defined by α_3, while the *dynamic* complementarity between investment and future peers comes from the self-productivity of skills to beget skills, i.e. the complementarity of future peers with future skills.

4.3 The Child's Problem

At the beginning of every period t, each child i decides to become friends with another child j, independently of their parents. I define the process of children's network formation as a function of the children's skills, their exogenous characteristics (X_i, X_j) and a vector of environment-specific characteristics (O_e) such as race composition and population size[18] The network-formation process takes place only within the same environment, generating social isolation between children of different areas. Empirically, this is consistent with the fact that I observe friendships only within the same school. At the same time, within the same environment, the children's meeting process is "frictionless", meaning that each child meets the other children in that social context. However, friendships are endogenously formed by the joint decision of children.

Following a similar specification as in Christakis et al. (2010), Goldsmith-Pinkham and Imbens (2013) or Graham (2017), child i's utility to become friends with child j at time t is:

$$u^C_{i,j,t} = \delta_1 + \delta_2 \ln h_{j,t} + \delta_3 X_j + \delta_4 \mathbb{1}(X_i = X_j) + \delta_5 \left(\ln h_{i,t} - \ln h_{j,t}\right)^2 + \delta_6 O_e + \delta_7 t - \nu_{i,j,t}, \quad (5)$$

where $\nu_{i,j,t}$ is a utility shock for the formation of peer groups and δs are the parameters

[18]Figure B-1 shows that the probability of forming a friendship is a function of the population size of the school.

associated with each variable affecting the friendship decision. This utility function has a similar representation to the one used in the demand for products in the literature of industrial organization, where individuals may have direct preferences over the attributes of the potential partners. This part is captured by δ_2 and δ_3. The difference from that literature comes from the other component of the utility function $\delta_4 \mathbb{1}(X_i = X_j) + \delta_5 \left(\ln h_{i,t} - \ln h_{j,t}\right)^2$, which captures the propensity of children to interact with children who are alike both in terms of skills and other individual characteristics. This phenomenon is called homophily bias in the network literature (see Jackson, 2008; Christakis and Fowler, 2009). A specific age (t) effect in the formation of peer groups is captured by δ_7. Hence, each child i solves the following problem for each potential future peer j at each period t:

$$V^C(h_{i,t}, X_i, h_{j,t}, X_j, O_e) = \max\left\{0,\ u^C_{i,j,t}\right\} \tag{6}$$

where I normalize to zero the value to have no friend.[19] Child i and child j become friends if both children find the friendship beneficial, i.e.:

$$L^*_{i,j,t} = \begin{cases} 1 & \text{if} \quad u^C_{i,j,t} > 0 \quad \& \quad u^C_{j,i,t} > 0 \\ 0 & \quad \text{otherwise} \end{cases} . \tag{7}$$

The model in (6) does not consider a potential decrease of marginal returns to additional friendships as the number of friends increases. Perhaps an alternative way of modeling friendship formation could consider children with a limited endowment of time who optimally allocate their time in interacting with other children. In this case, the social interactions would be limited by this time constraint and children would have to coordinate relative to their own time constraints. While the latter model seems more realistic, the lack of data on time allocation between peers as well as the additional computational burden in the model directed me to the model in (6). Model (6) represents a simple and flexible way of capturing the endogeneity of peer groups and the main driving forces affecting friendship formation. For convenience, let us define W as the set of variables of the utility function in equation (5):

$$W_{i,t} = \left[1,\ h_{j,t},\ X_j,\ \mathbb{1}(X_i = X_j),\ \left(\ln h_{i,t} - \ln h_{j,t}\right)^2,\ O_e, t\right].$$

The conditional probability for child i and child j to become friends, under the independence assumption between utility shocks is then:

[19]The underlying assumption is that the outside option is common for different types of children and for different meetings.

$$Pr(L^*_{i,j,t} = 1) \equiv P_{i,j,t}(h_{i,t}, X_i, h_{j,t}, X_j, O_e) = Pr(\nu_{i,j,t} \leq W'_{i,t}\delta) \cdot Pr(\nu_{j,i,t} \leq W'_{j,t}\delta) \quad (8)$$

where the probability of two children connecting together can be higher (lower) in a case where the two children have the same characteristics (δ_4). Also, if δ_5 is negative, a higher difference in skills will reduce the probability of the two children deciding to connect, while a positive coefficient will increase it. In other words, the sign of the coefficient will reflect a positive or negative assortative matching between children with respect to their skill development. All children compute the utility of forming a friendship with other children within the social network, and the entire social network graph is determined. The set of probabilities between different children as in (8) forms the probabilities of the possible networks in each environment e.

In this framework, children do not directly make investments in themselves (e.g. a study effort decision problem), which perhaps could depend on their own skills as well as the effort their peers make. This hypothetical modeling choice would consider a specific aspect of social interactions either emerging from a conformity effect or from strategic complementarities in skill formation between children (see Blume et al., 2015). In the next section, I will describe the technology of skill formation and how children's skills evolve over time. The dynamics of skills depend on both a child's own level of skills as well as the level of peers' skills, capturing the potential effects of studying effort, allowing for other more general peer effects and reducing the computational burden of the model.

4.4 The Parenting Problem

4.4.1 Preferences

I assume that each parent in family i at any period t has preferences over their own consumption ($c_{i,t}$) and over the skills of their children ($h_{i,t}$), while they do not receive any direct utility from time spent with their children ($I_{i,t}$). I additionally assume that preferences are stable over time. Parental investments are made dynamically to foster children's skills over time. This specification is in line with the recent literature in child development (see Cunha, 2013; Del Boca et al., 2014, 2016; Mullins, 2016; Gayle et al., 2015, 2016; Caucutt and Lochner, 2017). There is only one decision maker regarding parental investments, as I assume that the mother–father interactions occur at a prior stage . At any period, each parent is endowed with τ units of time and decides how to allocate this endowment between working ($\tau - I_{i,t}$) and parenting ($I_{i,t}$). Finally, I assume that the utility function for parents i at period t is as follows:

$$u^P(c_{i,t}, h_{i,t}) = \frac{c_{i,t}^{1-\gamma_1} - 1}{1-\gamma_1} + \gamma_2 \frac{h_{i,t}^{1-\gamma_3} - 1}{1-\gamma_3}, \tag{9}$$

where $\gamma_2 > 0$.[20] The specification in (9) underlines the main parent's trade-off: the benefit of higher children's skills at the cost of their own foregone consumption. Another model choice I could use would be to define parental investments in terms of the effort parents need to make in order to invest in their children's skills, and the associated utility cost of that effort. In this case, the trade-off would be between the altruistic benefit of fostering children's skills and the disutility of the required effort. The two specifications are isomorphic.[21] Finally, family income is defined by the mother's labor and non-labor income. I assume that both the mother's hourly wage ($w_{i,t}$) and non-labor income ($y_{i,t}$) are a function of her skills (m_i). The exact wage and non-labor income specifications are described in Section 5, where the relationship between a mother's skills and her non-labor income aims to capture the potential effect of her skills in assortative mating and family formation.

4.4.2 Terminal value

I assume that the parent' s problem ends when children reach 16 years of age. This assumption can be read as the fact that children leave the household at 16 years old or that after that age parental investments become unproductive. I think of the child's final skills at 16 as an initial condition of another developmental process which I am not modeling here, such as finishing high school, starting a job or going to college. Hence, I allow a possible change in parental preferences over the skills of the final childhood period. I am defining the terminal value for the parent i with respect to children's skills as follows:

$$V_4^P(h_{i,4}) = \gamma_4 \cdot \frac{h_{i,4}^{1-\gamma_5} - 1}{1-\gamma_5}, \tag{10}$$

where both γ_3 and γ_4 are free parameters that potentially differ from the altruistic parameter

[20] Preferences over consumption or a child's outcomes are logarithmic functions if, respectively, $\gamma_1 = 1$ or $\gamma_3 = 1$.

[21] In this specification, I abstract from the labor–leisure decision margin. The main reasons are related to the lack of data on Add Health about parent's leisure choices, which would allow me to identify the elasticity of leisure choice with respect to changes in peers' skills. Additionally, adding another endogenous variable to the model would increment its computational burden. For this reason, my policy counterfactual experiments will only focus on policy-induced change in peers' skills and the associated change in the return of parenting, while I will abstract from any welfare analysis and/or changes in family resources, which would need additional important predictions on the family time allocation between working, leisure and parenting

specified in (9).

4.4.3 The recursive representation of the parent's problem

The two endogenous-state variables of the problem are, respectively, the child's skills ($h_{i,t}$, individual-state variable) and peers' skills ($\overline{H}_{i,t}$, aggregate-state variable). The dynamics of the network state within each environment are taken as given from the parent' s perspective. Parents form expectations with respect to the next period's average skills of peers. Different types of peers in the next period affect the return of investment in their offspring today through the dynamics of the child's skills. Anticipating the discussion on the equilibrium in Section 4.5, the consistency condition in this economy is that the expectations about the next period's peers' skills will be consistent with the transition probabilities generated by the endogenous network formation from the child's problem (see Section 4.3).[22] The parent's problem can be represented as follows:

$$V_t^P(h_{i,t}, \overline{H}_{i,t}) = \max_{I_{i,t}\in[0,\tau]} u^P(c_{i,t}, h_{i,t}) + \beta E\left[V_{t+1}^P(h_{i,t+1}, \overline{H}_{i,t+1}) \,\middle|\, h_{i,t}\right] \tag{11}$$

$$s.t. \quad c_{i,t} = (\tau - I_{i,t}) \cdot w_{i,t} + y_{i,t}$$

where $\beta \in (0,1)$ is the discount factor, while the consumption $c_{i,t}$ is a function of earnings (through the labor supply $\tau - I_{i,t}$) and the non-labor income $y_{i,t}$. Parents are uncertain about the production shock as well as their child's peer group in the next period, which will affect their future investment productivity. The law of motion for the next period's child's skills (or technology) is defined in (4), while the law of motion for peer effects ($\overline{H}_{i,t+1}$) is as follows:

$$Pr\left(\overline{H}_{i,t+1} = \frac{1}{\sum_{j\in N_e} L_{i,j,t+1}} \sum_{j\in N_e} L_{i,j,t+1} \cdot h_{j,t+1}\right) = \prod_{i=1}^{N_e} \pi_{i,j,t+1}^{L_{i,j,t+1}} (1-\pi_{i,j,t+1})^{1-L_{i,j,t+1}} \tag{12}$$

where $L_{i,j,t}$ is an indicator function equal to one if child i and child j are friends, and zero otherwise. Given the conditional independence assumption about the formation of peer groups, the stochastic law of motion in (12) represents the probability distribution of N_e independently and differently distributed Bernoulli random variables (friendships), where $\pi_{i,j,t+1}$ is the relative probability of that friendship happening.

[22] The consistency condition between the individual behavior of parents and the aggregate distribution of skills in the network is the analogue of the consistency condition used to solve recursive competitive equilibrium in macroeconomic models with aggregate externalities.

4.5 Equilibrium of the Network Economy

In this section, I describe the equilibrium of the economy. For computational reasons, I restrict my attention only to the (short-memory) Markovian equilibrium, where the parent's and child's policy functions depend only on the current realization of state variables during each period. Nevertheless, a desirable property of the Markovian equilibrium is that, in this framework, it generates non-ergodic skill dynamics (i.e. the property of skill formation depending on the history of developmental inputs throughout childhood), a key mechanism in explaining diverging patterns in outcome inequality in children. In fact, as I will explain later in the policy analysis, moving children at age 13 to a different environment predicts *persistent* effects in the dynamics of children's skills. Skills beget skills through many mechanisms: self-production, better peers and higher investments.

Alternative classes of equilibrium concepts consist of longer-memory equilibria where parents' and children's behavior is explained both by the realization of the current states as well as by the equilibrium path history that led to that state. This would lead to even stronger dynamic equilibrium spillover effects of skills, because it would strengthen the role of social interactions in explaining children's developmental differences through the determinants of the equilibrium path of a child's development.

Parents and children have two different and separate problems. In particular, parents observe the current realization of their offspring's peer groups and then form expectations about the next period's peer groups when deciding on today's investment. Parents take as a given both the dynamics of network structure as well as the distribution of children's skills within the social network. At any point in time, children decide about their friends, generating the network of friendships. Then, parents decide how much time to invest in their offspring, forming expectations with respect to the next period's distribution of peers' skills. Given that the next period's distribution of peers' skills is an endogenous object in the model, the equilibrium characterization will take into account the consistency condition between parents' expectations and both skills and network equilibrium realizations.

Definition 1. *A Markovian equilibrium of the network economy is a set of functions* $\{I_t(\cdot),\, \mathbb{1}_t(\cdot)\}_{t=13}^{16}$ *such that:*

1. $\mathbb{1}^*(\cdot)$ *solves the child's problem in (6), for every period* t*,*

2. $I_t^*(\cdot)$ *solves the parent's problem in (11) , for every period* t*,*

3. *The probability for the formation of peer groups is consistent with the skills dynamics generated by the parental optimal behavior:*

$$\pi_{i,j,t+1} = P_{i,j,t+1}\,(h^*_{i,t+1}, X_i, h^*_{j,t+1}, X_j, O_e) \;\; \textit{for all } i, j, t$$

where

$$h^*_{i,t+1} =$$

$$h^{*\;\alpha_1}_{i,t} \cdot \left[\alpha_2\,(I^*_{i,t}(\cdot))^{\,\alpha_3} + (1-\alpha_2) \left(\frac{1}{\sum_{j\in N_e} L^*_{i,j,t}} \sum_{j\in N_e} L^*_{i,j,t} \cdot h^*_{j,t} \right)^{\alpha_3} \right]^{\frac{\alpha_4}{\alpha_3}} \cdot A_{i,d,s,t} \cdot exp(\xi_{i,t+1}),$$

for any production shock realization $\xi_{i,t+1}$.[23]

Definition 4.5 provides that both parents and children maximize their utility at each point in time. The last equilibrium condition, the consistency condition, closes the model. In fact, condition (3) implies that the endogenous stochastic network structure, which depends on the skill dynamics, is determined simultaneously in equilibrium from both the parents' and the children's optimal behavior.

Theorem 1. *In this economy, a Markovian equilibrium exists.*

See proof in Appendix D.

Theorem 1 formalizes the existence of equilibrium of the model and is the theoretical base of the algorithm used in my simulation-based estimation procedure.

A common feature of any model of social interactions and spillover effects is the potential existence of multiple equilibria. Multiplicity can arise from the presence of "strong" peer externalities. In this framework, this translates into a strong complementarity between parental investments and peers' skills, which is reflected directly in a low value for the CES complementarity parameter α_3. The possibility of multiple equilibria creates a challenge for the use of standard econometric methods through the presence of an indeterminacy condition in the map from the observed data to the structural parameters. In this case, the estimation procedure would require implementing additional steps to recover the parameters of the model.

[23]The same condition can be rewritten in terms of expected next period child's skills

$$\pi_{i,j,t+1} = \int P_{i,j,t+1}\,(h^*_{i,t+1}, X_i, h^*_{j,t+1}(\xi_{i,t+1}), X_j, O_e)\,dF(\xi_{i,t+1}) \;\; \text{for all } i, j, t \;,$$

where $F(\xi_{i,t+1})$ is the distribution of the production shocks.

Three possible solutions can be considered. First, a common approach in the literature is to assume that the data is generated from a specific equilibrium selection. Generally, the equilibrium selection rule considers the equilibrium with the highest welfare amongst all the possible equilibria (see for example Lazzati, 2015; Fu and Gregory, 2017).

A second approach consists of partially identifying the model. In this case, the econometrician does not need to make any assumptions about the equilibrium selection. A set of moment inequalities arises from the different equilibria and can be used to create bounds on the structural parameters of the model (using, for example, the moment inequalities estimator in Chernozhukov et al., 2007; Andrews and Soares, 2010; Pakes et al., 2015)

A third approach, which is the one I use here, is to determine (if possible) which specific parameter (or set of parameters) is responsible for the presence of multiple equilibria, and for what specific threshold value of that parameter multiplicity arises.

In my model, the key parameter which determines whether the model generates multiple equilibria is the CES parameter of complementarity between parental investments and peers' skills (α_3). A high level of complementarity between parents and peers can generate multiplicity: within each environment, the parental decisions of other parents affect the individual decisions of everybody else, creating possible extreme equilibria where no parents invest at all or all the parents invest the majority of their time in child development. This statement on how peer externalities affect parental investment decisions is a testable prediction. This means that the previous empirical results in Section 3 can be considered as a pre-test for multiplicity in this model. Specifically, the fact that by using within-school cross-sectional variation in peers' skills I find a negative effect in parental investment decisions suggests that the two inputs cannot be too complementary; to let the model reproduce that cross-sectional negative relationship between investments and peers' skills, the complementarity between parents and peers should be less than in the Cobb–Douglas case (so the CES parameter α_3 should be bigger than 0).[24] Given this low level of complementarity, I am going to implement a "guess and verify" method, in which I assume that the equilibrium is locally unique for values of $\alpha_3 \in (0,1]$, and will then computationally verify if the assumption is correct by implementing a monotone method for equilibria computations. This method requires calculation of the two possible extremal equilibria of this economy using the algorithm in Topkis (1979), and then simply comparing them. For the data-driven model's parametrization of $\alpha_3 \in (0,1]$, I find no evidence of multiple equi-

[24] A recent work of Datta et al. (2017) shows that in a similar environment, a macro growth model with externalities, the unique equilibrium is proved in a case where externality is not big, like in the case of constant return to scale in the production technologies.

libria.

5 Econometric Specification

5.1 The Latent Factor Models

In line with the recent literature on child development (see Cunha et al., 2010; Agostinelli and Wiswall, 2016; Attanasio et al., 2017a,b,c), I implement a dynamic latent factor model to map the key unobserved variables of the model into data. The factor model overcomes the main problem in the analysis of skill formation: mis-measurement of skills and the arbitrariness of test-score scales relative to the scale of skills. For both mothers' and children's skills, I follow the latent factor model implemented in Agostinelli and Wiswall (2016) as follows:

$$
\begin{aligned}
Z^h_{i,t,k} &= \mu^h_{t,k} + \lambda^h_{t,k} \cdot \ln h_{i,t} + \epsilon^h_{i,t,k} \\
Z^m_{i,k} &= \mu^m_{t,k} + \lambda^m_k \cdot \ln m_i + \epsilon^m_{i,k}
\end{aligned}
\tag{13}
$$

The index k is for indexing each of the multiple measurements (proxies) Z for each latent factor. Because the location and scale of skills can differ from the arbitrary location and scale of the proxies I use, I implement the factor model in (13) with free measurement parameters (μ and λ). Finally, the noises in (13) have a mean of zero in any period and for any measure.[25] I assume the common independence conditions about the measurement error to hold. These conditions include the independence of measurement noises with both a child's and mother's latent skills and between different measures of skills, as well as between different children and over time (for more details, see Appendix C).

Mapping data to the distribution of latent investment is challenging due to the nature of the proxies included in Add Health. Add Health asks children whether they have been engaged in specific activities with their mothers in the last four weeks. Examples of activities are "gone shopping," "played a sport," "gone to a movie, play, museum, concert, or sports event" or "had a talk about a personal problem." Each question requires a "yes" or "no" answer, generating a set of binary proxies for investments defined by $Z^I_{i,k} \in \{0,1\}$, where i and k indexes are relative to the child and the specific question. These measures can be considered indicators as to whether parents spent some time with their children or not. Hence, each measure of investment can

[25]Given the intercept $\mu_{t,m}$, the assumption of a mean of zero $\epsilon_{t,m}$ errors is without loss of generality.

be thought of as a Bernoulli random variable with probability $p_k(I_{i,t})$, a function of the latent investment. I adopt a similar approach as in Del Boca et al. (2014), and I consider a specific parametric distribution for $p_k(I_{i,t})$, which is a Beta distribution with parameters Beta(α + $Z^I_{i,t,k}$, $1 + \beta$ - $Z^I_{i,t,k}$).[26] I can now draw p_k from this distribution to recover, for each measure k, the latent distribution of parental investments $F_k(I_{i,t})$. Let $\widehat{p}_{i,t,k}$ be the draw from the parametric distribution for some observation i at time t, and I can impute the level of investment by inverting the probability function at $\widehat{p}_{i,t,k}$ (assuming the inverse exists):

$$I^k_{i,t} = p_k^{-1}(\widehat{p}_{i,t,k}). \tag{14}$$

where α and β define the location and scale for the latent investments.[27] To assure that imputed levels of investments are constrained between 0 and τ (the max time in the model), I map each specific probability into the fraction of time spent with children ($\frac{I_{i,t}}{\tau}$). Each probability of observing a measure of investment equal to one increases with respect to the fraction of parental investments (higher parental investments lead to a higher probability of observing, in data, children involved in activity "k" with their parents). Moreover, a desirable property for the probability function is that $\lim_{I_{i,t}\to 0} p_k(I_{i,t}) = 0$, $\lim_{I_{i,t}\to \tau} p_k(I_{i,t}) = 1$. This means that once the fraction of invested time goes to zero or to one, the probability of observing a parent involved in that specific activity goes to zero or to one. For all these reasons, I choose the following simple functional form, which respects all the required properties:

$$p_k(I_{i,t}) = \left(\frac{I_{i,t}}{\tau}\right)^{\lambda^I_{t,k}} \quad \lambda^I_{t,k} > 0, \tag{15}$$

where the parameter $\lambda^I_{t,k}$ is the loading factor for each activity m.

5.2 Parametric Assumptions of the Model

In this section, I illustrate the assumptions I am making in order to parametrically estimate the model. I consider three different types of neighborhood quality according to the income distribution in the data. For each school I have in Add Health, I compute the within-school mean family income and then assign each family to the low-, medium- or high-income neighborhood

[26] Del Boca et al. (2014) use the same approach to address the issue of measuring continuous skills with a discrete test score as well as the possibility of measurement error of the measured score. Additionally, this method overcomes the problem of a measurement floor and ceiling of test scores

[27] In contrast to the skills case, latent time investments have a well-known location and scale, which is in terms of time units or fractions of total time. I will measure this information by looking at the time allocation of parents on the American Time Use Survey (ATUS).

type according to the median school income.[28] This distinction is made based on the terciles of income-distribution (33^{th} percentiles, between the 33^{th} and 66^{th} percentile and above the 66^{th} percentile).[29] Families first draw their race (I allow race to be either black, Hispanic or other) and their neighborhood type from a joint distribution P(d,r), which I directly estimate from the data.[30] After race and neighborhood are drawn, I assume that the initial distribution of a family's log-skills (mother and child) are drawn from a conditional bivariate distribution:

$$(\ln h_{i,1}, \ln m_i) \sim N(\eta_{r,d}, \Sigma_{r,d}) \tag{16}$$

where I allow mean and variance of latent skills to vary by race and neighborhood. The specification allows me to have a flexible framework to capture potential sorting of families within specific environments. Taking into consideration this kind of assortative pattern is fundamental in order to properly identify the peer effects in child development.

I also make some parametric assumptions about the measurement error equations in (13). I assume that the measurement noises for each measure are mean-zero normally distributed $\epsilon_k \sim$ N(0,$\sigma^2_{\epsilon,k}$) for any measure k at any age t.

The TFP term in the technology of skill formation is composed of two parts: neighborhood-quality effects and school-quality effects. For each neighborhood type d, I assume there is a distribution of school-quality families drawn together with their neighborhood realization: $A_s \sim$ N($\eta_{s,d}$,$\sigma^2_{s,d}$). To generalize the school effects in skill dynamics, the parametric functional form for the technology of skill formation I bring to data is:

$$h_{i,t+1} = {h_{i,t}}^{\alpha_1} \cdot \left[\alpha_2 \left(I_{i,t}\right)^{\alpha_3} + (1-\alpha_2)\left(\overline{H}_{i,t}\right)^{\alpha_3} + \alpha_5\, A_s^{\alpha_3}\right]^{\frac{\alpha_4}{\alpha_3}} \cdot A_{d,t} \cdot exp(\xi_{i,t+1})\,, \tag{17}$$

where the share parameter of A_s (α_5) is normalized to 1 given that the variance of the latent school fixed effects is a free parameter estimated directly in the data ($\sigma^2_{s,d}$). Technology in (17) generalizes the school effects to have individual specific elasticities of skill production with respect to school quality. The environment-specific TFP term is assumed to be a function of the neighborhood quality and child's age as follow: $A_{d,t}$ = exp($\gamma_{0,tfp}$ + $\gamma_{1,tfp}$ · d + $\gamma_{2,tfp}$ · t). Additionally, I am assuming some parametric form for both shocks of skill production and preference for children's peer groups. Production shocks are assumed to be mean-zero normally distributed $\xi_{i,t+1} \sim$ N(0,σ^2_ξ), while a child's preference shocks for friendships, defined in eq. (5),

[28]The results do not change if I consider the median family income within the Census Tract where families live
[29]For further details on the descriptive statistics for each of the neighborhood types, see Table B-1.
[30]This empirical distribution represents the probability for each of the nine possible combinations of race and neighborhoods and is directly observed and estimable from the data

are distributed as a standard logistic . This implies that the probability that child i and child j become friends at any time t is:

$$P_{i,j,t} \equiv Pr(L^*_{i,j,t} = 1) = \frac{exp(W'_{i,t}\delta)}{1 + exp(W'_{i,t}\delta)} \cdot \frac{exp(W'_{j,t}\delta)}{1 + exp(W'_{j,t}\delta)} \tag{18}$$

where $W_{i,t}$ and $W_{j,t}$ are the set of variables affecting the decision and are defined in eq. (4.3), while δ are the common utility parameters.

Finally, I assume that both the mother's hourly wage $(w_{i,t})$ and non-labor income $(y_{i,t})$, at any period t, are defined as a function of the mother's skills in a linear fashion, as in the classic Mincer wage equation (see Mincer, 1958):

$$\begin{bmatrix} \ln w_{i,t} \\ \ln y_{i,t} \end{bmatrix} = \begin{bmatrix} \kappa_{1,0} \\ \kappa_{2,0} \end{bmatrix} + \ln m_i \cdot \begin{bmatrix} \kappa_{1,1} \\ \kappa_{2,1} \end{bmatrix} + \begin{bmatrix} \epsilon^w_{i,t} \\ \epsilon^y_{i,t} \end{bmatrix}$$

where $\epsilon^w_{i,t}$ and $\epsilon^y_{i,t}$ are measurement noises.

6 Identification of the Model

As explained above, both mother's and child's skills as well as parental investments are assumed to be unobserved. With this in mind, the goal is to identify peer effects on parental investment decisions and skill formation, dealing with the endogeneity of the peers network formation. In this section I describe how I approach this task. The identification of the wage and non-labor income process is standard: in both cases, once the scale and location for the latent mother's skills are identified (see Section 6.1), it is a simple linear errors-in-variables models. In the next sections, I focus more on the more challenging part of the model for identification. I am following the logical order from the model: (i) initial conditions; (ii) network formation process; (iii) static and dynamic complementarity between parents and peers.

6.1 Identification of the Initial Conditions

The main challenge in the identification of the family skills distribution comes from the fact that skills are unobserved and they have not a natural scale and location. Recalling that the initial conditions are assumed to be normally distributed,

$$(\ln h_{i,1}, \ln m_i) \sim N\left(\eta_{r,d}, \Sigma_{r,d}\right),$$

where $\eta_{r,d} = \begin{bmatrix} \eta^h_{r,d} \\ \eta^m_{r,d} \end{bmatrix}$ and $\Sigma_{r,d} = \begin{bmatrix} {\sigma^h_{r,d}}^2 & \sigma^{h,m}_{r,d} \\ \sigma^{h,m}_{r,d} & {\sigma^m_{r,d}}^2 \end{bmatrix}$ are, respectively, the vector of means and the variance–covariance matrix for skills for a specific neighborhood type (d) and race (r). I normalize the scale and location of skills for a subgroup of the population. Without loss of generality, I normalize the distribution of skills for white (r=w) families living in the lowest-income neighborhood ($d = 1$) as follows:

Normalization 1. *Initial period normalization:*

- $\eta_{w,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Sigma_{w,1} = \begin{bmatrix} 1 & \sigma^{h,m}_{w,1} \\ \sigma^{h,m}_{w,1} & 1 \end{bmatrix}$

Under normalization 1, I am able to identify the remaining measurement equation parameters and the initial joint distribution of skills for each race and each neighborhood type. The initial period loading factors (λ) and the location parameters (μ) are identified as follows:

$$[\lambda^h_{1,k}, \lambda^m_k] = \left[\frac{Cov(Z^h_{i,1,k}, Z^h_{i,1,j})}{Cov(Z^h_{i,1,j}, Z^h_{i,1,1})}, \frac{Cov(Z^m_{i,k}, Z^m_{i,j})}{Cov(Z^m_{i,j}, Z^m_{i,1})}\right] \text{ for } k \neq j \text{ and for } k, j \neq 1 \tag{19}$$

$$[\mu^h_{1,k}, \mu^m_{1,k}] = \left[E[Z^h_{i,1,k}|r = w, d = 1], \quad E[Z^m_{i,k}|r = w, d = 1]\right] \text{ for all } k. \tag{20}$$

Using both (19) and (20), it is possible to identify the means and the variance-covariance matrix of the latent skills for all the rest of race and neighborhood type combinations in the following way:

$$\eta_{r,d} = \left[\frac{E[Z^h_{i,1,k}|r,d] - \mu^h_{1,k}}{\lambda^h_{1,k}}, \frac{E[Z^m_{i,k}|r,d] - \mu^m_k}{\lambda^m_k}\right] \text{ for all } (r,d) \neq (w,1) \tag{21}$$

$$\Sigma_{r,d} = \begin{bmatrix} \dfrac{Cov(Z^h_{i,1,k}, Z^h_{i,1,j}\,|r,d)}{\lambda^h_{1,k}\,\lambda^h_{1,j}} & \dfrac{Cov(Z^h_{i,1,k}, Z^m_{i,j}\,|r,d)}{\lambda^h_{1,k}\,\lambda^m_j} \\ \dfrac{Cov(Z^h_{i,1,k}, Z^m_{i,j}\,|r,d)}{\lambda^h_{1,k}\,\lambda^m_j} & \dfrac{Cov(Z^m_{i,k}, Z^m_{i,j}\,|r,d)}{\lambda^m_k\,\lambda^m_j} \end{bmatrix} \text{ for all } (r,d). \tag{22}$$

Repeating the identification of mean and variance–covariance matrix for all the neighborhood types, and for each race, I am able to identify the distribution of initial conditions in the economy.[31] Finally, I identify the variance of the measurement errors for each measure:

$$\sigma^2_{\epsilon^m,k} = Var(Z^m_{i,k}) - (\lambda^m_k)^2 Var(\ln m_i)$$
$$\sigma^2_{\epsilon^h,t,k} = Var(Z^h_{i,t,k}) - (\lambda^h_{t,k})^2 Var(\ln h_{i,t}) \text{ for all } t \tag{23}$$

where each variable on the right-hand side of the equations in (23) is already identified. The repeated cross-sectional dimension of the Add Health data allows me to observe the same test score administered to children at different ages. Hence, following the approach in Agostinelli and Wiswall (2016), I assume the PPVT to be an age-invariant measure – that is, the latent skills load into the PPVT in the same manner through the ages of consideration (13 to 16).[32] As shown in Agostinelli and Wiswall (2016), this assumption is sufficient to identify an unknown TFP term and an unknown return to scale in the CES technology case.[33]

6.2 Identification of the Formation of Peer Groups

Using the measurement system defined in (13), I can rescale the observed measures for skills to be:

$$\widetilde{Z}^h_{i,t,k} \equiv \frac{Z^h_{i,t,k} - \mu^h_{t,k}}{\lambda^h_{t,k}} = \ln h_{i,t} + \frac{\epsilon^h_{i,t,k}}{\lambda^h_{t,k}} \tag{24}$$

where $\widetilde{Z}^h_{i,t,k}$ represents a monotone transformation of raw test scores that permit consideration of the data on a comparable scale as the latent skills. Each transformed measure is composed by two elements: the factor and the noise. For this reason, in order to use this information to identify the peer-group formations, we need to deal with the presence of this measurement error. In particular, I exploit the identified parametric distribution of the measurement errors to integrate out the noise from the data.

Following the notation in (4.3), let us define W to be the set of variables of the utility function

[31] Notice that the identifying assumption here is that the measurement parameters are common for all the children. This assumption is common in the literature of latent factor models.

[32] Agostinelli and Wiswall (2016) defines an age-invariant measure to be a repeated achievement test, the assessment of which would not depend on a child's age, but on their cognitive development

[33] It also allows me to identify the measurement parameters for all remaining proxies for skills through all the periods of the model

of child i to become friends with child j as in equation (5) and $\widetilde{W}$ to be the analogue measured in the data:

$$W_i = \Big[1,\ h_{j,t},\ r_j,\ \mathbb{1}(r_i = r_j),\ \big(h_{i,t} - h_{j,t}\big)^2,\ O_e,\ t\Big],$$

$$\widetilde{W}_i(\epsilon^h_{i,t,k}, \epsilon^h_{j,t,k}) = \left[1,\ (\widetilde{Z}^h_{j,t,k} - \frac{\epsilon^h_{j,t,k}}{\lambda^h_k}),\ r_j,\ \mathbb{1}(r_i = r_j),\ \left((\widetilde{Z}^h_{i,t,k} - \frac{\epsilon^h_{i,t,k}}{\lambda^h_k}) - (\widetilde{Z}^h_{j,t,k} - \frac{\epsilon^h_{j,t,k}}{\lambda^h_k})\right)^2,\ O_e,\ t\right]. \tag{25}$$

The conditional probability of child i and child j becoming friends, given their skills and race, is as follows:

$$\begin{aligned} P_{i,j,t} &= Pr(\nu_{i,j,t} \leq W_i'\delta) \cdot Pr(\nu_{j,i,t} \leq W_j'\delta) \\ &= \int Pr\left(\nu_{i,j,t} \leq \widetilde{W}_i(\epsilon^h_{i,t,k}, \epsilon^h_{j,t,k})'\delta\right) \cdot Pr\left(\nu_{j,i,t} \leq \widetilde{W}_j(\epsilon^h_{j,t,k}, \epsilon^h_{i,t,k})'\delta\right)\ d\Phi(\epsilon^h_{i,t,k}, \epsilon^h_{j,t,k}; \Sigma_{\epsilon,t}) \end{aligned} \tag{26}$$

where $\Phi(\cdot;\Sigma_{\epsilon,t})$ is the bivariate normal CDF of the measurement errors with zero mean and a variance–covariance matrix $\Sigma_{\epsilon,t} = \begin{bmatrix} \sigma^2_{\epsilon^h,t,k} & 0 \\ 0 & \sigma^2_{\epsilon^h,t,k} \end{bmatrix}$, which is already identified in (23).

Equation (26) shows that I can identify the parameters of the model of peer-group formation by mapping the noisy measures of children's skills available in data to information about the children's latent skills and their effects on children's friendship decisions.

6.3 Identifying Peer Effects in Child Development

Given the complexity of the above dynamic equilibrium model, it is important to develop an easy intuition behind the source of variation which will lead to identifying peer effects on child development. In particular, one helpful source of identifying variation comes from the endogenous response of parental investments to peer quality, allowing us to infer the degrees of both *static* and *dynamic* complementarity between parents and peers in producing a child's skills. These parent response elasticities are key for identification because they map directly into the deep structural parameters of the technology of skill formation and are the key margin of interest for policy predictions. The main object of interest for identification is the full joint distribution of inputs of development $\Psi(\Omega)=\left\{\left\{I^*_{i,t}, h_{i,t}, \overline{H}^*_{i,t}\right\}_i\right\}_t$, where Ω is the set of structural parameters left to identify, the peers' skills spillover determined in the equilibrium social network is defined as $\overline{H}^*_{i,t}$, and the star defines the equilibrium realization.

6.3.1 Static Complementarity

A useful set of moments to identify the static complementarity between parents and peers in skill formation is the partial correlations of parental investments and peers' skills. In fact, an intuitive prediction of my model is a positive (negative) cross-sectional relationship between parental investments and peers in the presence of a high degree of *static* complementarity (substitutability) between parents and peers in child development. Once peer quality increases, the return on investing in child development increases as well, making parents more productive in parenting. On the other hand, parents have an incentive to decrease their costly investments once an alternative substitute input (peers) increases. Hence, the *static* complementarity/substitutability between parents and peers is identified by the cross-sectional variation between investments and peers' skills.

A specific but helpful representation of these moments is given by the linear regression in (2), which provides useful information about the variation of investment decisions with respect to the type of peers. Specifically, the coefficient β_2, which represents the effect of peers' skills on investment decisions, is informative about the degree of *static* complementarity/substitutability (α_3) between parental investment and peers in the production of children's skills.

6.3.2 Dynamic Complementarity

The *dynamic* complementarity between parents and peers is identified by looking at the variation in parental investments induced by changed expectations for future peers' skills. For example, as a thought experiment, consider two children (and their relative families) who are alike in all dimensions including their peer groups, but only one of them is assigned *permanently* (treatment) to a different (better) peer group, while the other child (control) has no change in peer composition. Given the permanent nature of the experiment, the two parents now observe today's peers and have different expectations about tomorrow's peers. Again, the model's prediction is helpful in fixing ideas: an increased expectation of tomorrow's peers' skills would change the return on today's investments. Current investments would foster the child's skills of tomorrow, thus benefiting more from better future peers. Hence, the higher the dynamic complementarity, the higher the difference in parental investments induced by the policy between the control and the treatment of children: the treatment induces higher investments because of the effects of expectations of future peers.

My empirical identification strategy is based on the instrumental variable approach explained in Section 3.4, which represents the quasi-experimental analogue of the identifying variation considered in the above thought experiment. Specifically, in order to estimate the causal effects of permanent peers changing investment decisions, I use the random realization of different cohort composition of children within the same school to analyze how permanent changes in peers affect parental investments. The idea behind this identification strategy is simple. The cohort realization *permanently* defines the choice set for the children's network formation. Hence, different cohort compositions shift parents' expectations about future peer groups, allowing us to identify the causal effect of a permanent change in peer groups on investment decisions (β_2) and, consequently, the dynamic complementarity of investments and peers.[34]

7 Estimation

I implement a two-step estimation algorithm to alleviate the estimation burden of this model. In the first step, I estimate the initial conditions, the wage and the family income process, as well as the distribution of school quality for low/medium/high neighborhood types.[35,36] The second step of estimation is focused on the rest of the model: preferences, technology and parameters of the network formation. I use a simulation-based estimation technique which allows me to simulate the distributions of investments and skills over the entire childhood and replicate the statistics (M) I observe in Add Health data. In greater detail, I simulate the equilibrium dynamics of investments, skills and networks and use the realized simulated data set to compute the analogous statistics (M_S) observed in the data. Hence, the second-step estimator is a simulated method of moments (SMM) estimator:

[34] This identification strategy does not require that friendships are only formed within a unique cohort. It only requires that changes in cohort composition also change the peer-choice set between cohorts. Empirically, most of the friendship nominations are within the same cohort.

[35] The school quality distributions are identified estimating the school fixed-effects distribution from a value-added model of child development. This method is simple and alleviates the computational burden of estimating different school quality parameters in the second step of the estimation. See Appendix E for further details.

[36] Identification of latent skills requires at least three proxies for each latent variable. In the case of a mother's skills, Add Health provides only information about a mother's years of education. I use the NLSY79 sample to recover the loading factor and location parameter for this proxy.

$$\widehat{\Omega} = \arg\min_{\Omega} (M - M_S(\Omega))\, W\, (M - M_S(\Omega)) \tag{27}$$

where Ω is the set of structural parameters previously described and W represents the weighting matrix.[37] In the estimation procedure, I set the weighting matrix to be the inverse of the diagonal variance–covariance matrix of moments computed by bootstrapping the data. The selected moments include: (i) first- and second-order moments for the conditional distribution of skills by race and neighborhood type; (ii) the set of auxiliary coefficients from the two auxiliary regressions in (2), as well as auxiliary coefficients for the technology of skill formation (in an *indirect inference* fashion); (iii) the set of moments about network formation, including the homophily bias index for skills and race for each neighborhood type, which I described during the previous empirical analysis in Section 3.3.

7.1 Structural Estimates

7.1.1 Network formation parameters

Table 3 shows estimates for the formation of peer groups. The unconditional effects of age, race or skill level on the formation of peer groups are negligible (the unconditional numbers of friends by race and by skills are similar). Race and skills play a role through homophily bias in friendship formation, affecting the composition (not the quantity) of friends. Qualitatively, coefficients in Table 3 are consistent with the empirical evidence of homophily bias both in race and in skills. Specifically, I find that for any race, the fact that the other child is of the same race is highly predictive about the friendship realization between the two children. The coefficients associated with being of the same race are, respectively, 0.76, 0.70 and 0.60 for black, Hispanic and white children. The higher coefficient is for black children. The probability for the formation of peer groups depends on whether children have similar skills. In particular, the higher the difference in children's skills, the lower the probability of the two children becoming friends, with a coefficient of -0.038. These homophily bias effects get bigger in context with a lower fraction of minorities, which is shown by how the two coefficients of homophily bias

[37] The choice of the simulated method of moments with respect to a likelihood-based method is due to three reasons: first of all, the SMM approach overcomes the additional source of computational burden which arises from the multi-dimension integration problem associated with the maximum-likelihood estimator of this model. Secondly, because of the data structure, I observe each child only for two consecutive waves (with a temporal distance of one year) in Add Health, making the SMM a more flexible estimator in combining the information of skills dynamics from the dataset. Finally, the SMM does not require any assumption about the cross-sectional distribution of the children's skills over childhood.

interact with the total fraction of black and white children (0.042 and -0.063, respectively). I find that the correlation of unobservable heterogeneity in the formation of peer groups with the production shocks is -0.40, while its standard deviation is 0.11. I do not find relevant effects of the specific race or skills level in the probability level (only through homophily bias).

To better interpret the estimates, I plot the marginal probabilities of two children becoming friends over the spectrum of children's skills and for different races of children living in the poorest environment (see Figure 4). For a black low-skilled child (within the first quintile of skills distribution), the probability of becoming friends with white children is four times lower than with a same-race counterpart. At the same time, it is two times more likely for the same child to become friends with children having similar skills than for children in the top decile of skills distribution. For a white low-skilled child (within the first quintile of skills distribution), the racial gap is lower (around 2.5 times), while the effect of skills is similar.

7.1.2 Technology parameters

Table 4 shows estimates for the technology of skill formation. I find a high degree of *static* substitutability between parents and peers, with a complementarity parameter (α_3) of approximately 0.95 (and associated elasticity of substitution of $\frac{1}{1-0.95} = 20$). I find a degree of self-productivity (α_1), i.e. the ability of skills to beget skills, of 0.75. This means that a 1% increase in current skills would predict an average of 0.75% . This result is qualitatively in line with the previous research on the estimation of technology of skill formation (Cunha and Heckman, 2007, 2008; Cunha et al., 2010; Agostinelli and Wiswall, 2016). The magnitudes of the share parameters are meaningless due to the different scale between peers' skills (normalized at age 13 to have unit variance) and investments (in yearly hours). However, I find that the estimated value of 0.009 implies that to completely offset a change of one standard deviation in peers' skills, parental investments need to change by approximately four hours per week (for the mean parent). The return to scale of the combined parents–peers inputs (α_4) is 0.77, suggesting non-linear peer effects in skills dynamics even when parents and peers are approximately perfect substitutes (linear). I find that the total factor productivity is an increasing function of the neighborhood quality ($\gamma_{1,tfp}$=0.008) and of the age of children ($\gamma_{2,tfp}$=0.030). The coefficients for school quality represent each neighborhood-type mean and standard deviation of the school fixed effects. I find that average school quality is increased by neighborhood type (from -0.03 for low-income neighborhoods to 0.04 for high-income neighborhoods), while the standard deviation is decreased (from 0.26 for low-income neighborhoods to 0.18 for high-income neighborhoods). Finally, I find production shocks to be important in explaining the total variation in skills dynamics, with a coefficient for the standard deviation of shocks (σ_{ξ}) of 0.70.

7.1.3 Preferences Parameters

Panel A in Table 5 shows estimates for preference parameters. I find both utility for consumption and a child's skills to be relatively concave, with a higher degree of curvature for consumption relative to a child's skills. I find a relatively high degree of parental altruism towards a child's skills: parents care about their child's skills through ages 13 to 15 almost as much as their own consumption, while they care twice as much about the final continuation value for their child relative to their own current consumption. The last result underlines the importance of allowing for a different parameterization for the final period preference, which, as explained above, can capture a different developmental process for children, such as finishing high school, starting a job or going to college.

7.1.4 Wage-Income Process and Initial Conditions

Panel B in Table 5 shows estimates for the wage and non-labor income process. A mother's skills are very predictive of both, suggesting an elasticity of 0.44 and 1.03 for wages and non-labor. Table 6 shows estimates for the initial conditions by neighborhood type (low/medium/high family income). The normalized mean and variance for a mother's skills and a child's initial skills are for white families living in a low-income neighborhood (neighborhood 1). The other subpopulations' means and variances are relative to the normalized ones. I find that children from minorities start with a lower mean of initial skills relative to their reference white children. White children living in a higher family income neighborhood (neighborhoods 2 or 3) have higher mean initial skills relative to white children in lower-income neighborhoods. I find similar patterns in terms of mother's skills.

7.2 Sample Fit

Table B-2 and Table B-3 report the sample fit for the auxiliary regressions of investments and the dynamics of a child's skills. The model is able to replicate the empirical findings on parental investments and skill dynamics. Table B-2 also reports the 95% confidence interval to show that the simulated coefficients are not statistically different from the fitted coefficients. More importantly, Table B-2 shows that the model is able to replicate the switch in sign in the peers' skills, from the cross-sectional (negative) to the permanent (positive) variation in peers, through the different degrees of static versus dynamic complementarity of parental investments and peers' skills in skill formation.

Table B-3 shows that the estimated model is able to fit the auxiliary coefficients for the dynamic aspect of skill formation, suggesting that the estimated technology of skill formation provides the proper marginal productivity for the developmental inputs. The reported 95% confidence intervals suggest that the data and simulated coefficients are not statistically different.

In terms of neighborhood effects on child development, the estimated model is able to fit the differential patterns of skill formation between different neighborhoods and for different races. Figures B-2-B-7 show the sample fit for the mean and standard deviation of skills by age, race and for each of the three types of neighborhood I consider (low/medium/high income).

Figures B-8-B-9 show the sample fit for the homophily bias index for skills and race in different neighborhoods (low/medium/high income). Figure B-8 shows that the model tracks the findings on the skills homophily bias in friendship formation. The model replicates the fact that high-skilled children display a higher bias toward children with similar skills.

Figure B-9 shows that the model is able to replicate the findings of homophily bias by race in different neighborhoods. However, within the high-income neighborhood, while the data indicate a fall in the homophily bias index for Hispanic children relative to black children, the model indicates a common tendency of homophily bias between the two races.

8 The Exposure Effects of Environments on Child Development

In this section, I analyze the extent to which environments explain differences in children's final achievements. In order to calculate the treatment effect of better environments, I simulate the counterfactual dynamics of skills as the outcome of being moved to a different environment. I compute the treatment effect by simulating each child in each different environment and school and at any age I consider in the model.

Within the estimated model, I can now calculate the treatment effect of moving from any considered environment to any of the others. Specifically, consider the case where a child i is permanently moved from a environment e to a environment e' at age m. The individual treatment effect in this case is:

$$TE_i(e, e', m) = y_i^*(e, e', m) - y_i^*(e) \tag{28}$$

where $y_i^*(e)$ represents the baseline child percentile in the skill distribution at age 16, while $y_i^*(e, e', m)$ represents the child percentile in the skill distribution at age 16 if the child is moved from her original environment e to a new environment e' at age m. To simplify the analysis and to have a comparable setting with the previous literature, I first characterize environments by their mean percentiles of skills of permanently based children at age 16 ($\overline{y}_e$). Second, I consider a specific parametric relationship between a child's outcome and the associated mean percentiles of children's skills at age 16 in each environment ($\overline{y}_e$):

$$y_i^* = \psi_0 + \psi_{1,m}\overline{y}_e + \epsilon_i \tag{29}$$

where $\overline{y}_e$ represents the associated mean percentile of children's skills in environment e. In this case, $\psi_{1,m}$ represents the average treatment effect of being permanently moved at age m from a environment e to a new environment e' which has a mean of children's skills at age 16 that is one percentile higher.

Finally, I can compute the exposure effects of environments, which represent the effects on skill formation of an additional year in a environment with a mean of children's skills at age 16 that is one percentile higher. Following equation (29), the exposure effects are simply $\psi_{1,m}$ - $\psi_{1,m+1}$.

Figure 5 shows that environments have sizable effects on skill formation. Moving a child at age 12 into a environment with a mean of children's skills at age 16 that is one percentile higher causes an increase of her skills by approximately 0.65 percentiles. This effect declines by age. I find an exposure effect of 0.048, which means that the outcomes of moved children converge to the outcomes of receiving children at a rate of 4.8% per year of exposure. In other words, moving a child to a better environment at 14 rather than at 13 years old causes the child to lose almost 5% of the benefit of moving. The exposure effects imply approximately a 15% higher benefit from moving to the same environment at 12 rather than at 15 years old.[38]

8.1 Model Validation: Comparison with Exposure Effects in Chetty and Hendren (2016a)

The specification in (29) allows me to compare my results with those in Chetty and Hendren (2016a). The authors implement the same specification to analyze the childhood exposure

[38] My model starts at age 13 when children draw their skills after the realization of the environment where they live. In this exercise, when I move children at age 12, it means that I allow children to be in the environment before they draw their skills. In contrast, when I move them at age 13, I let them first draw their skills in the original environment and then move them into the new environment.

effects of environments for the United States. The authors consider individual income percentiles at age 24 as their variables of interest. Still, my results are comparable with those in Chetty and Hendren (2016a) under the reasonable assumption that expected individual income is defined by any monotone rank-preserving function of children's skills (higher skills imply average higher individual income).[39]

Figure 5 shows that my model tracks the findings in Chetty and Hendren (2016a), who find that an additional year of exposure to a environment with a mean income of one percentile higher for permanently moved children increases a child's income later in life by approximately 0.04 percentiles. The authors also find exposure effects to be stronger for families above the median income distribution. Table 7 shows the comparison of model predictions and their estimates with respect to family income. The model's predictions are in line with the heterogeneous exposure effects.

It is still an open question, however, as to which factors drive the exposure effects. In the next section, I decompose the overall exposure effects into three classes of environment-specific amenities: peers, school quality and environment quality.

8.2 Decomposition of Childhood Exposure Effects

One advantage of my structural model is that I can now decompose the previous findings of exposure effects with respect to each of the components which characterized an environment in my model: *peers, school quality* and environment quality.

With the estimated model, I can replicate the previous simulated experiment of moving children within different scenarios to isolate the effects of each of the previous inputs. I first move children and compute the treatment effect of moving to different environments at different ages due exclusively to the associated change of peers (keeping the previous level of both school and environment quality fixed). Secondly, I compute the effect of moving children when the associated treatment is composed by both changes in peers and the new school quality (keeping the previous level of environment quality fixed). Finally, I calculate the overall treatment effect of moving at different ages associated with the new peers, the new school and the new environment quality. In this way, I can ascertain the contribution of each component to the overall effects of childhood exposure to environments.

Table 8 shows the decomposition of the overall effect. Peers alone account for more than

[39] I could also estimate a relationship between individual income at age 24 and children's skills at age 16 and then use this estimated function to predict individual income for the simulation exercise. Any monotone relationship between these two variables would give me the same results in terms of rank effects.

half of the childhood exposure effects, while school and environment quality account for the rest. This means that more than half of the effect of an additional year in a environment with mean skills one percentile higher for permanently moved children is caused by the child's social interactions. Whether a child leaves a disadvantaged environment affects the quality of the child's social interactions (in terms of peers' skills). The dynamic complementarity of skill formation causes this effect to have a higher return earlier than later throughout adolescence. Each additional year of adolescence spent in an environment with mean skills one percentile lower for permanently moved children worsens the child's skills at age 16 by 0.027 percentiles, exclusively through social interactions. The exposure to the same social interactions from age 12 to age 15 would cause a reduction of skills by approximately 0.08 percentiles.

The overall exposure effects are bigger for disadvantaged children, and in this case, peer effects account for almost two-thirds. Children who have a low endowment of skills and are from lower-income families are more likely to live in low-income environments, where they have higher chances of interacting with low-skilled peers due to the homophily bias effects in peer-group formation. This channel explains why this group of children experiences the greatest benefit from leaving these disadvantaged environments. The exposure effect in this case is approximately 0.05, and peers alone account for up to 60% of this finding. For this specific subgroup of children from low-income families, each additional year of adolescence spent in an environment with mean skills of one percentile lower for permanently moved children worsens a child's skills by around 0.034 percentiles at age 16, exclusively through social interactions. Figure 6 shows graphically the same result for lower-skilled children.

These results suggest the importance of social interactions alone in explaining the differences in developmental trajectories in children from different environments. In the next section, I analyze the effects of policies that target disadvantaged children and change the peers' composition between different environments, for example, policy that promote socioeconomic integration between environments or schools.

9 Policy Analysis

The decomposition of the exposure effects of environments suggests that more than half of the effects come from peers and social interaction. This result calls for policies which focus on changes in peers' composition between different environments to overcome the negative effects of growing up in disadvantaged environments. I analyze how this kind of policy, if implemented on a large scale (i.e. when a sizable fraction of children are moved into a new environment), can generate important equilibrium effects in skill dynamics through the changes in social interactions.

9.1 Large Scale Changes in Peers' Composition

The estimated model reveals that skills dynamics depend on a peers' composition and associated children's social interactions. In this section, I analyze the quantitative equilibrium effects, created by policies that change cohort compositions into different social contexts, on the dynamics of skills. Specifically, I want to understand the implications of changes in cohort compositions within both *receiving* and *sending* environments. In order to answer this question, I perform a simulated counterfactual analysis where I move disadvantaged children (and their parents), i.e. children with low skill endowment at age 13 and living in a poor environment, into a high-income environment. This policy has diverse effects on three subgroups of people: (i) parents and children who are moved; (ii) parents and children who live in the receiving (high-income) environment; (iii) parents and children who remain in the low-income environment.

The simulated policy targets a specific group of disadvantaged children: children who are within the first quartile of skills distribution at age 13 in the low-income environment. More than 70% of these children are from a racial minority and their initial (age 13) skills are, on average, about a standard deviation below the population mean of log-skills at age 13. On the other hand, the racial composition of the receiving environment is different: the racial minority makes up only 18% of the population. On average, receiving children are 0.12 standard deviations above the population mean of children's log-skills at age 13. I consider the policy effects when the moved children group is about 5% and 30% of the receiving cohort. These sizable changes in cohort composition allows me to analyze the equilibrium effects of the policy. In the last part of this section, I will discuss more in detail how the effects change as a function of the fraction of moved children.

Table 9 shows the effects of the policy that moves the smaller fraction of children (5% of the receiving population). Panel A reports the effects of the policy on children's skills for both the moved and receiving children. For each group, I report the baseline as well as the counterfactual skill dynamics. To assess the importance of equilibrium effects, I also report the counterfactual results without any dynamic equilibrium effects (column called "No Equilibrium"). In this case, I just solve the parent's behavioral problem without considering any equilibrium feedback effects from the endogenous response of other parents' behavior after the policy change. The first finding is that this policy has small effects for the receiving children, with a minimal decrease in skills at age 16 of 3%. On the other hand, I find that moved children increase their skills at age 16, on average, by 55% (0.4 of a standard deviation of children's skills at age 16).[40]

Panel B in Table 10 shows the effects of the policy change on parental investment decisions. I find that parents of moved children increase their investments overall, fostering the effects of the policy on their own child's skills dynamics. On the other hand, parents of receiving children do not respond to changes in peer composition.

Table 10 displays the effects for the larger policy. Panel A in Table 10 reports the effects of the policy on children's skills also for the remained children. In this case, the policy creates winners and losers: an average increase in skills at age 16 of 31% and 23%, respectively, for moved and remaining children is associated with an average decline of 15% in receiving children. An alternative interpretation of the results is in terms of the standard deviation of skills distribution: I find that moved and remaining children increase their log-skills on average by 0.22 and 0.17, respectively, of a standard deviation of children's log-skills at age 16, while, on average, log-skills for receiving children decrease approximately by 0.10 of a standard deviation. The results suggest that children who remained in the sending environment benefit from the outflow of the most disadvantaged companions.

Panel B in Table 10 shows the effects of the policy change on parental investment decisions. I find that parents of receiving children reduce their engagement with children due to a lower expected level of peers' skills. On the contrary, parents of moved and remaining children increase their investments due to the policy (positive) change in the expected future peers. Indeed, as suggested by the instrumental variable results, a positive (negative) *permanent* change in peer composition induces a positive (negative) change in investments due to the dynamic complementarity. Figure 7 illustrates how the policy change has affected the equilibrium endogenous distribution of peers' skills and the relative parents' expectations, causing parents with better (worse) expectations to increase (decrease) their investments.

Finally, Table 9 and Table 10 underline the importance of accounting for equilibrium effects in this type of policy analysis. I find that equilibrium feedback effects tend to amplify the policy effects, and ignoring those would lead to biased policy predictions for children's final skills of approximately seven times smaller. Part of this gap is due to the erroneous predictions for investment decisions: in the absence of dynamic-equilibrium feedback effects, the *static* complementarity between parents and peers dominates the dynamic effects of the policy. In this case, a positive (negative) change in peer composition induces a negative (positive) change in investments.

[40]The national population's standard deviation of children's log-skills at age 16 is 1.37

9.1.1 Heterogenous Effects

In this section, I analyze whether the larger counterfactual policy (when the moved children group is 30% of the receiving cohort) generates heterogeneous effects between children due to the new counterfactual social network. I focus my attention to the two sources of potential homophily bias: skills and race. Figure 8 shows the return of policy (treatment effect) in % of skills at age 16 for different children in terms of their initial endowment of skills at age 13. The x-axis displays the percentiles of initial skill endowment for each subgroup. I find that children with lower skills at age 13 benefit the most when moved to a better environment. The treatment effect for children in the first decile of skills within the group of moved children is between 32%-35% in skills at age 16 (approximately 0.25 of a standard deviation for log-skills at age 16). This result is clear evidence of the role of segregated social interactions in child development: in the absence of context-specific network formation, the CES technology in (4) would predict that return of policy would increase in a child's endowment.[41] However, a discontinuous policy-induced change in peers creates higher benefits for children who are moved away from adverse social interactions. The policy induces a lower return for children with better initial skills, with the bottom of the effects of approximately 24% for children above the median of the moved group.

The heterogeneous effects in Figure 8 for receiving children reveal the importance of the counterfactual endogenous network formation. I find that children in the lower part of the skills distribution have the most sizable adverse effects within the group of receiving children. The treatment effect in this case is -0.14 of a standard deviation of log-skills at age 16. In fact, this group of children is the most exposed to social interactions with the new potential peers. For the same reason as the counterfactual change in social interactions, Figure 8 suggests that the policy return for children in the sending (low-income) environment are higher for low-skilled children, with a policy effect of approximately 0.20 of a standard deviation of log-skills at age 16 for children in the first skill decile, in contrast with 0.13 of a standard deviation for children in the highest decile.

Evidence from previous empirical studies indicates a potential racial difference in peer effects in children, pointing out that peer effects seem to be stronger *intra*-race and for minorities (see for example Hoxby, 2000; Angrist and Lang, 2004; Imberman et al., 2012).[42] My quantitat-

[41]This is because of the assumed complementarity between a child's endowment and other inputs: $\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial \overline{H}_{i,t}} \geq 0$.

[42]Imberman et al. (2012) exploit the Katrina natural experiment to evaluate peer effects in receiving schools in Louisiana and Houston. The authors find negative peer effects on school attendance, disciplinary infractions and math scores for black children, although the last effect is statistically imprecise

ive exercise confirms the previous literature's results. Table 11 shows the decomposed results for receiving children by race. Children from minority groups are most adversely affected by the policy, with a reduction of approximately 42% and 35% (-0.30 and -0.25 of a standard deviation) of log-skills at age 16 for black and Hispanic children, respectively. On the other hand, I find that the effects on white children are smaller, with a reduction of log-skills at age 16, on average, of approximately 11% (0.08 of a standard deviation).[43] Panel B in Table 11 shows the counterfactual parental investment behavior. Again, groups that are more exposed to the *permanent* change in cohort composition (through homophily bias in social interactions) sizably reduce their parental involvement due to the dynamic complementarity between their current choice and the expected future peers.

9.1.2 The Scale Effects of Policy

During the previous policy analysis, I analyzed the effects of moving two different fractions of disadvantaged children into a high-income environment: 5% and 30% of the receiving population. Equilibrium effects on both receiving and sending environments were quantitatively different. In this section, I analyze the different implications for the same counterfactual policy as a function of different fractions of moved children. Specifically defining the original moved group as the *eligible* children, I now compute the equilibrium effects of the policy relative to the fraction of *eligible* children who are actually moved.

Figure 9 reports the average effects on children's log-skills at age 16 in the counterfactual economy as a function of the fraction of *eligible* moved children (x-axis) for the three subgroups of interest (moved, receiving and remaining children). The first result is that both the moved and receiving children are better off if the policy provides a relatively small group of moved children. For moved children, the return of the policy rapidly drops as the fraction of eligible children rises in the new environment. Moving 3% or 30% of children creates a difference of approximately 20% in the final skills for moved children. This suggests that the probability of children from the same original environment continuing to interact with each other in the new social context is still high. For the receiving children, the decline is more gradual. The total change between moving nobody versus moving 30% of eligible children is approximately 15%, and it monotonically declines as more eligible children are moved into the environment. The second result is that the remaining children gain increasingly more out of the policy if the fraction of disadvantaged children who are moved out from that environment increases. A gain of

[43]The heterogeneous treatment effects by race for the smaller policy are qualitatively similar, see Table B-4

40% is guaranteed for remaining children if 30% of children are moved out from that environment. An increased outflow of the most disadvantaged children from the sending environment benefits children who remain, which is a result of the positive effects of the new peers' skills.

9.2 The Persistent Effects of Social Environments on Skills Dynamics

The last quantitative analysis focuses on underlining the persistent effects on skill dynamics of growing up in disadvantaged environments. To answer this question, I perform a simulated counterfactual policy which targets the same disadvantaged group of children as before, but now I boost their initial endowment at age 13 while keeping them living in the low-income environment. I compare their dynamics of skills throughout childhood with children with similar initial skills but living in better environments.

Table 12 reports the counterfactual results. I find that living in different social contexts permanently shapes the developmental trajectories of children. In particular, after starting from the same initial endowment, the dynamics of skills in the disadvantaged environment fail to keep up with the skills dynamics of children from the high-income environment. The growth rate of skills is approximately 60% higher for children in the higher-income environment during the age of 13–14 years, while at the end, the two groups' growth rates in skill converge. This leads to a total difference, at age 16, of approximately 57% in final skills.

Social interactions and children's skills composition play an important role in explaining this result. First, children living in the low-income environment have, on average, lower-skilled peers than children living in the high-income environment. Secondly, the two different social contexts determine parents' different expectations about their child's social interactions with peers, thus affecting their investment behavior.

Panel B in Table 12 shows that parents increase engagements with their offspring as the result of a higher initial skill endowment of their child. However, they are far from the parental investment levels of parents in high-income environments. The difference is approximately between 4 and 6 percentage points in terms of time allocated to child development. Figure 10 shows that part of this investment gap is due to differences in the expected peer effects throughout childhood. Hence, social influences determine patterns of skill inequality and, again, one key mechanism is the dynamic complementarity between parents and expected peers.

10 Conclusion

This paper studies the role of children's social interactions in the dynamics of children's skills. I estimate a tractable dynamic equilibrium model of parental investment and endogenous formation of peer groups. The model is estimated using information about friendships, children's test scores and parental investments in the National Longitudinal Study of Adolescent Health (Add Health). I exploit within school / across cohort variations in peers' composition to identify the degree of complementarity between parents and peers in producing a child's skills. I find that parents and peers are *static* substitutes and *dynamic* complementary inputs in child development. After validating my estimated model using findings in Chetty and Hendren (2016a) on environment exposure effects in children, I assess the importance of social interactions in skills dynamics with various policies.

This article underlines three main points: (i) social interactions and social context permanently shape the developmental trajectories of children; (ii) changing cohort composition and the relative social interactions generates *winners* and *losers* and the heterogeneous effects are due to the endogenous formation of new peers; (iii) neglecting the dynamic equilibrium effects of skill formation and social interactions would lead to biased predicted effects of policies.

I want to conclude this paper by considering an extension of this work. Specifically, one potential type of parental investment can be the choice of neighborhood where the family lives. In this case, parents have alternative margins in response to changes in peer composition, and to a certain extent, they can also decide to change where they live as a response to the previously considered policy. Modeling this second channel is challenging, because now the environment composition is also endogenous, and it becomes part of the equilibrium solution of the model. However, understanding the extent to which neighborhood decisions are influenced by children's social interactions is an important question in considering the effects of socioeconomic segregation on intergenerational mobility. Therefore, future work is needed.

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Table 1: Sample Statistics

	Mean (1)	Standard Deviation (2)
Child's Age	15.65	1.74
Fraction black	0.16	0.37
Fraction hispanic	0.17	0.38
Fraction white	0.67	0.47
N of reported friends (In-School)	4.48	3.58
Measures for skills:		
PPVT	64.26	11.14
English	2.83	0.98
Math	2.72	1.03
History	2.86	1.01
Science	2.82	1.01
Family's characteristics:		
Income ($ 1994)	42,844	27,724
Mother's education	13.13	2.35
Measures for parental investments : (activities in the last 4 weeks with mother)		
Gone shopping	0.72	0.44
Played a sport	0.08	0.28
Gone to a religious service	0.38	0.49
Talked about someone you are dating (or a party you went to)	0.47	0.50
Gone to a movie, play, museum, concert, or sports event	0.25	0.44
Had a talk about a personal problem you were having	0.39	0.49
Had a serious argument about your behavior	0.33	0.47
Talked about your school work or grades	0.63	0.48
Worked on a project for school	0.13	0.34
Talked about other things you are doing in school	0.54	0.50

Notes: This table reports the descriptive statistics for the sample I use in the estimation of the model. The number of reported friends is the number of nominated friends during the survey. The measures for parental investments are binary variables, which take value one if the activity was done, zero otherwise.
Data source: National Longitudinal Survey of Adolescent Health (Add Health).

Table 2: Parental Investments, Child's and Peers' Skills

	Dependent Variable Fraction (%) of Invested Parental Time	
	(1) Measurement Error Adjusted	(2) Measurement Error Adjusted and Instrumental Variables (IV)
Child Skills (Log)	2.660	2.120
	(0.316)	(0.668)
Peers' Skills (Log)	-1.441	0.720
	(0.650)	(0.354)
N of Children	14,267	14,267
Age Fixed Effects	✓	✓
School's Fixed Effects	✓	✓
Panel B: First Stage		
$Z_{1,i,t}$ (Minorities Children)		-0.104
		(0.052)
$Z_{2,i,t}$ (White Children)		0.082
		(0.037)
F-Stat Excl. Instruments		11.78
P-value		0.000

Notes: This table shows estimates for both model 2 (column 1) and 2 (column 2). The dependent variable is the fraction of invested parental time at age t and the covariates (log skills and log peers' skills) are at also at time t. All models also include controls for children's race, mother's skills and lagged family income. Standard errors in parenthesis are computing using a cluster bootstrap. The first stage statistics in column 2 shows the coefficients (and standard errors in parenthesis) of both excluded instruments for the first stage as well as the F-statistic of the joint null hypothesis that both coefficients are zero. Stock and Yogo provide critical values to test weak IV condition based on the F-stat of excluded instruments. Those critical values can be interpreted as a test with a 5 % significance level, of the hypothesis that the maximum relative bias (with respect to the OLS estimates) is 10% or at least 15%. In this case, the Stock and Yogo critical values for the F-stat of the excluded instruments are 19.93 (10%) and 11.59 (15%).
Data source: National Longitudinal Survey of Adolescent Health (Add Health).

Table 3: Estimates for the Network Formation Model

Parameter	Estimate	S.E.
Constant (δ_1)	-0.246	0.0172
Child's Log-Skills (δ_2)	0.088	0.0048
Black ($\delta_{3,1}$)	0.075	0.0023
Hispanic ($\delta_{3,2}$)	-0.005	0.0001
Both Black ($\delta_{4,1}$)	0.763	0.0317
Both Hispanic ($\delta_{4,2}$)	0.701	0.0298
Both White ($\delta_{4,3}$)	0.559	0.0475
Distance in Children's Skills (δ_5)	-0.038	0.0014
N of Children (Hundreds, $\delta_{6,1}$)	-0.890	0.0003
N of Children Squared (Hundreds, $\delta_{6,2}$)	0.001	0.0000
Distance in Children's Skills · %White ($\delta_{6,3}$)	- 0.063	0.0032
Distance in Children's Skills · %Black ($\delta_{6,4}$)	0.042	0.0025
Age (δ_7)	-0.050	0.0010
Additional Unobserved Heterogeneity ($\zeta_{i,j,t}$)		
Correlation with Skill Shocks	-0.404	0.0212
Standard Deviation	0.110	0.0095

Notes: This table shows the structural estimates for the child's utility for friendships in Equation (5). The child's utility shock is defined as $\nu_{i,j,t} = \widetilde{\nu}_{i,j,t} + \zeta_{i,j,t}$, where $\zeta_{i,j,t}$ and is correlated with the production function shock ($\xi_{i,t}$). The last part of the table shows the estimated correlation and standard deviation σ_ζ. The standard errors are computed using a cluster bootstrap.

Table 4: Estimates for the Technology of Skill Formation

Parameter	Estimate	S.E.
Child's Skills (α_1)	0.744	0.0682
Investments (Yearly Hours, α_2)	0.009	0.0014
Elasticity Investment vs Peers (α_3)	0.944	0.0270
Return to Scale (α_4)	0.767	0.0283
Std of Shocks (σ_ξ)	0.700	0.0461
Panel B: TFP		
Constant ($\gamma_{0,tfp}$)	-1.329	0.1256
Neighborhood Quality ($\gamma_{1,tfp}$)	0.008	0.0003
Age Trend ($\gamma_{2,tfp}$)	0.030	0.0008
Panel C: School-Quality Effects		
Low Income Neighborhood		
Mean ($\eta_{s,1}$)	-0.033	0.0350
Standard Deviation ($\sigma_{s,1}$)	0.262	0.0264
Medium Income Neighborhood		
Mean ($\eta_{s,2}$)	0.006	0.0277
Standard Deviation ($\sigma_{s,2}$)	0.244	0.0278
High Income Neighborhood		
Mean ($\eta_{s,3}$)	0.041	0.0318
Standard Deviation ($\sigma_{s,3}$)	0.188	0.0249

Notes: This table shows the estimates for the technology of children skill formation in Equation (17). Panel B reports the parameter estimates for the neighborhood-specific TFP defined in Section 5.2. Panel C reports the mean and standard deviation of school-quality for each neighborhood type. The standard errors are computed using a cluster bootstrap.

Table 5: Estimates for the Preferences and Income Variables

Parameter	Estimate	S.E.
Panel A: Auxiliary Coefficients for Investments		
Curvature on consumption (γ_1)	0.786	0.0046
Weight on Child's Skills (γ_2)	0.901	0.0030
Weight on Final Child's Skills (γ_4)	2.475	0.2455
Curvature on Child's Skills (γ_3)	0.562	0.0256
Curvature on Final Child's Skills (γ_5)	0.465	0.0011
Panel B: Parameters of Labor and Non-Labor Income		
Constant (Wage, $\kappa_{1,0}$)	2.750	0.0067
Mother's Skills (Wage, $\kappa_{1,1}$)	0.438	0.0048
Constant (Non-Labor Income, $\kappa_{2,0}$)	9.992	0.0174
Mother's Skills (Non-Labor Income, $\kappa_{2,1}$)	1.033	0.0113

Notes: Panel A shows the estimates for the utility parameters in Equation (9). Panel B reports the estimates for the wage and income process described in Section 5.2. The standard errors are computed using a cluster bootstrap.

Table 6: Estimates for Initial Conditions

Panel A: Mean Initial Child‘s and Mother‘s Skills

	Neighborhood 1		Neighborhood 2		Neighborhood 3	
	Child	Mother	Child	Mother	Child	Mother
Black	-0.47	-0.07	-0.40	0.36	-0.30	0.44
	(0.08)	(0.15)	(0.27)	(0.25)	(0.29)	(0.20)
Hispanic	-0.49	-0.93	-0.48	-0.77	-0.34	-0.36
	(0.11)	(0.19)	(0.26)	(0.19)	(0.25)	(0.19)
White	0.00	0.00	0.02	0.26	0.22	0.58
	(-)	(-)	(0.24)	(0.18)	(0.24)	(0.19)

Panel B: Variance-Covariance Initial Child‘s and Mother‘s Skills

	Neighborhood 1		Neighborhood 2		Neighborhood 3	
	Child	Mother	Child	Mother	Child	Mother
	0.65		0.87		0.89	
	(0.05)		(0.08)		(0.15)	
Black	0.20	0.61	0.31	0.67	0.30	0.64
	(0.08)	(0.14)	(0.09)	(0.17)	(0.16)	(0.14)
	0.84		1.10		0.78	
	(0.09)		(0.10)		(0.12)	
Hispanic	0.22	1.59	0.26	1.58	0.28	1.33
	(0.08)	(0.32)	(0.08)	(0.35)	(0.10)	(0.34)
	1.00		1.09		0.99	
	(-)		(0.09)		(0.13)	
White	0.48	1.00	0.37	0.74	0.36	0.78
	(0.07)	(-)	(0.04)	(0.19)	(0.06)	(0.17)

Notes: This table shows the estimates of initial conditions parameters by neighborhood-quality type (low-medium-high income) and race as described in Equation (16). The standard errors are computed using a cluster bootstrap.

Table 7: Model Validation from Exposure Effects in Children from Chetty and Hendren (2016a)

Panel A: Exposure Effects

	Baseline	
	Chetty-Hendren	Model
Neighborhood Exposure Effect	0.044	0.048
	(0.008)	

Panel B: Exposure Effects by Parental Income

	Below Median Income		Above Median Income	
	Chetty-Hendren	Model	Chetty-Hendren	Model
Neighborhood Exposure Effect	0.031	0.023	0.047	0.047
	(0.003)		(0.003)	

Notes: This table shows the comparison between model's predictions and findings in Chetty and Hendren (2016a) about the childhood exposure effects (baseline and heterogeneous effects by family income).

Table 8: Decomposition of Exposure Effects: Peers, School Quality and Neighborhood Quality

Panel A: Baseline Decomposition

	Baseline	Decomposition		
	Overall	Peers	Peers + School Quality	Peers + School Quality + Neighborhood Quality
Exposure Effect	0.048	0.027	0.038	0.048
(Percent)		(+55.58%)	(+23.81%)	(+20.60%)

Panel B: Decomposition for Disadvantaged Children

	Baseline	Decomposition		
	Overall	Peers	Peers + School Quality	Peers + School Quality + Neighborhood Quality
Low Family Income	0.056	0.034	0.046	0.056
(Percent)		(+61.49%)	(+20.13%)	(+18.38%)
Low Skills	0.054	0.030	0.042	0.054
(Percent)		(+56.40%)	(+22.54%)	(+21.05%)

Notes: This table shows the decomposition of childhood exposure effects in Chetty and Hendren (2016a) by: peers, school and neighborhood quality. Panel B shows the decomposition for disadvantaged children (both in terms of family income and child's skills).

Table 9: Counterfactual Effects on Skills and Investments (moved children are 5% of the receiving cohort)

Panel A: Effects on Children's Log-Skills (Mean)

	Moved Children				Receiving Children		
	Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)		Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)
Age 13	-0.94	-0.94	-0.94	Age 13	0.12	0.12	0.12
Age 14	-0.43	-0.23	-0.38	Age 14	0.87	0.85	0.87
Age 15	0.12	0.48	0.23	Age 15	1.55	1.52	1.55
Age 16	0.45	1.00	0.76	Age 16	2.12	2.09	2.11

Panel B: Effects on Parent's Investment Decision (Mean)

	Moved Children				Receiving Children		
	Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)		Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)
Age 13	17.73	22.16	17.22	Age 13	26.75	26.34	26.86
Age 14	17.50	21.32	16.33	Age 14	26.63	26.37	26.97
Age 15	10.83	10.48	9.18	Age 15	22.99	23.07	23.26

Notes: This table shows the counterfactual policy effects for moved and receiving when a fraction of moved children (5% of the receiving population) are moved into a high-income environment. For each subgroup, I compare the baseline results with the policy predictions (equilibrium effects). I also compute the predicted policy effects without equilibrium effects. The latter one is the predicted policy effects if I ignore equilibrium adjustments after the policy implementations. Panel B focus on the investments decisions for moved and receiving children. For each subgroup, I compare the mean predicted parental investments in the baseline case with the equilibrium and no-equilibrium effects.

Table 10: Counterfactual Effects on Skills and Investments (moved children are 30% of the receiving cohort)

Panel A: Effects on Children‘s Log-Skills (Mean)

	Moved Children				Receiving Children				Remained Children		
	Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)		Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)		Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)
Age 13	-1.00	-1.00	-1.00	Age 13	0.12	0.12	0.12	Age 13	-0.05	-0.05	-0.05
Age 14	-0.37	-0.28	-0.33	Age 14	0.87	0.80	0.87	Age 14	0.42	0.52	0.43
Age 15	0.24	0.40	0.34	Age 15	1.55	1.43	1.55	Age 15	0.87	1.05	0.88
Age 16	0.53	0.84	0.80	Age 16	2.12	1.97	2.11	Age 16	1.26	1.49	1.29

Panel B: Effects on Parent‘s Investment Decision (Mean)

	Moved Children				Receiving Children				Remained Children		
	Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)		Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)		Baseline	Counterfactual (Equilibrium)	Counterfactual (No Equilibrium)
Age 13	17.89	19.52	17.89	Age 13	26.75	24.74	27.22	Age 13	18.57	21.00	18.04
Age 14	18.00	18.62	17.15	Age 14	26.63	24.84	27.72	Age 14	18.84	21.16	18.16
Age 15	11.77	11.35	10.98	Age 15	22.99	22.82	23.92	Age 15	18.11	18.69	17.53

Notes: This table shows the counterfactual policy effects for moved and receiving when a fraction of moved children (30% of the receiving population) are moved into a high-income environment. For each subgroup, I compare the baseline results with the policy predictions (equilibrium effects). I also compute the predicted policy effects without equilibrium effects. The latter one is the predicted policy effects if I ignore equilibrium adjustments after the policy implementations. Panel B focus on the investments decisions for moved, receiving and remained children. For each subgroup, I compare the mean predicted parental investments in the baseline case with the equilibrium and no-equilibrium effects.

Table 11: Counterfactual Effects on Receiving Children by Race (moved children are 30% of the receiving cohort)

Panel A: Effects on Children‘s Log-Skills (Mean)

	Black			Hispanic			White	
	Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)
Age 13	-0.30	-0.30	Age 13	-0.34	-0.34	Age 13	0.22	0.22
Age 14	0.28	0.08	Age 14	0.45	0.30	Age 14	0.97	0.93
Age 15	0.96	0.61	Age 15	1.08	0.81	Age 15	1.66	1.58
Age 16	1.49	1.07	Age 16	1.59	1.24	Age 16	2.24	2.13

Panel B: Effects on Parent‘s Investment Decision (Mean)

	Black			Hispanic			White	
	Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)
Age 13	20.09	15.30	Age 13	21.97	17.72	Age 13	27.92	26.41
Age 14	18.85	14.46	Age 14	20.67	16.62	Age 14	28.02	26.72
Age 15	16.32	14.97	Age 15	16.69	14.99	Age 15	24.29	24.40

Notes: This table shows the counterfactual policy effects for receiving children (by race) when a fraction of moved children (30% of the receiving population) are moved into a high-income environment. For each subgroup, I compare the baseline results in skills and parental investments (Panel B) with the equilibrium counterfactual predictions.

Table 12: Counterfactual Effects on Skills and Investments of Earlier Interventions

Panel A: Effects on Children‘s Log-Skills (Mean)

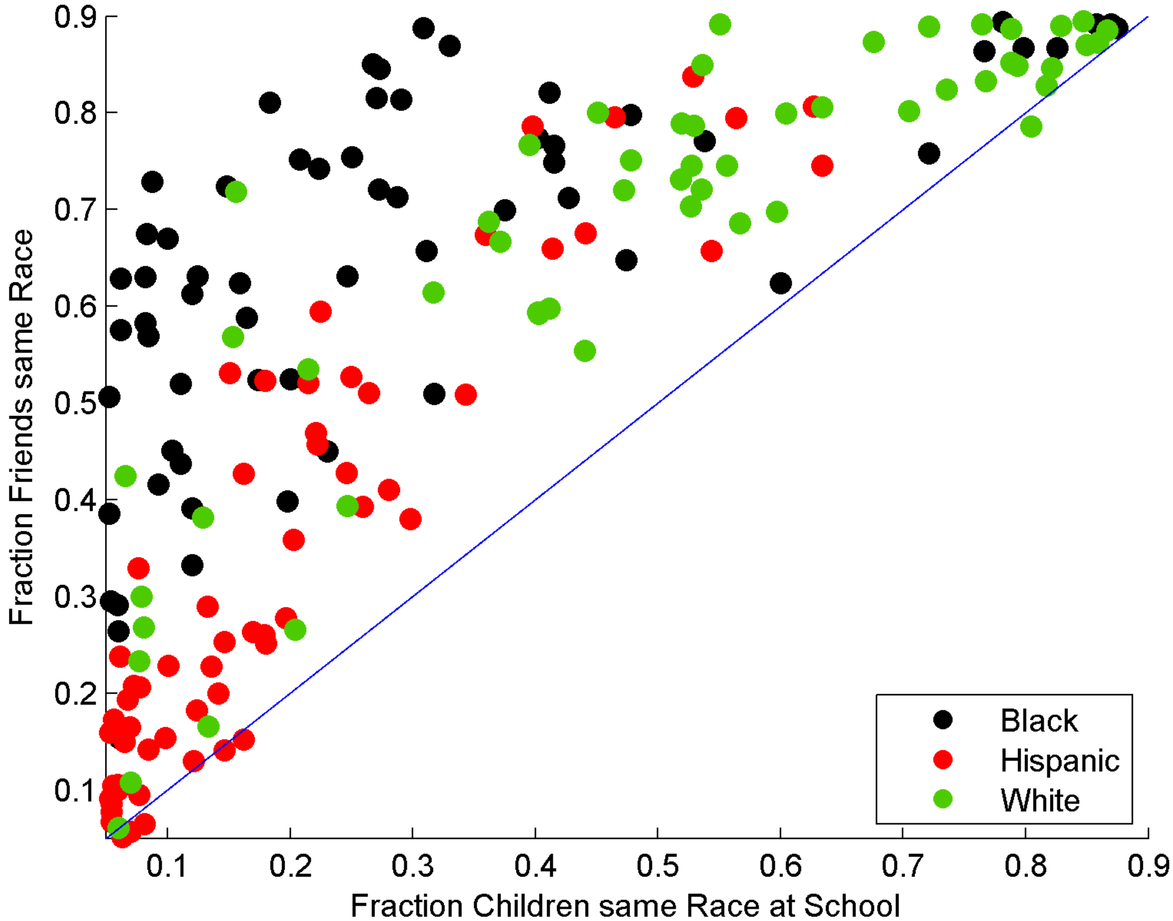
	Baseline	Counterfactual	Similar Children in Richer Environment
Age 13	-1.00	0.46	0.46
Age 14	-0.37	0.85	1.09
Age 15	0.24	1.31	1.74
Age 16	0.53	1.73	2.31

Panel B: Effects on Parent‘s Investment Decision (Mean)

	Baseline	Counterfactual	Similar Children in Richer Environment
Age 13	17.89	20.82	26.47
Age 14	18.01	20.76	26.20
Age 15	11.77	20.24	24.85

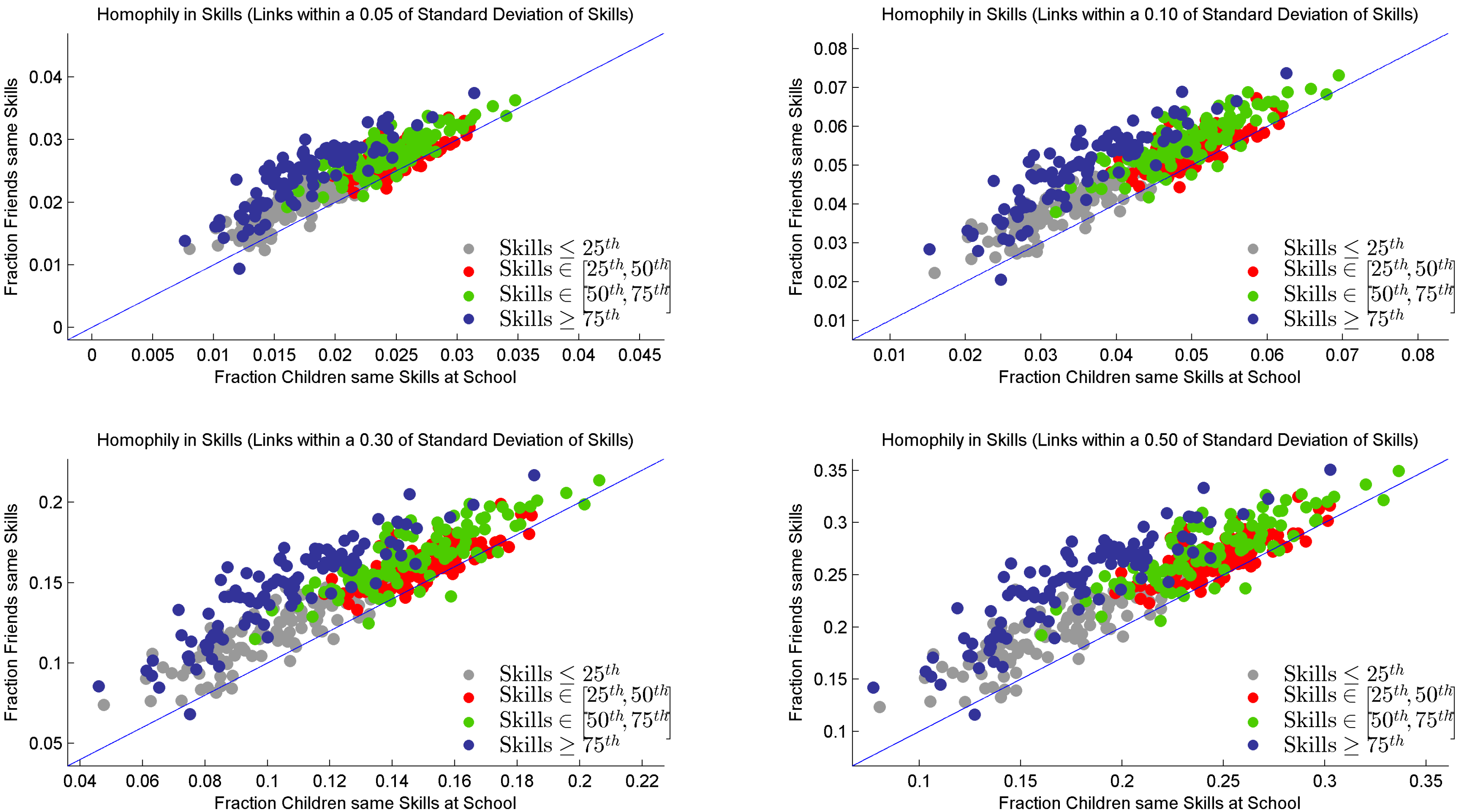
Notes: This table shows the counterfactual policy effects of fostering initial skills (age 13) in skill formation and parental investments (Panel B) for disadvantaged children. Column 1 and 2 report the baseline and counterfactual predictions. Column 3 reports the dynamics of skills and parental investments for children with same mean initial skills at age 13 but who are living in the high-income environment.

Figure 1: Homophily in Network Formation by Race



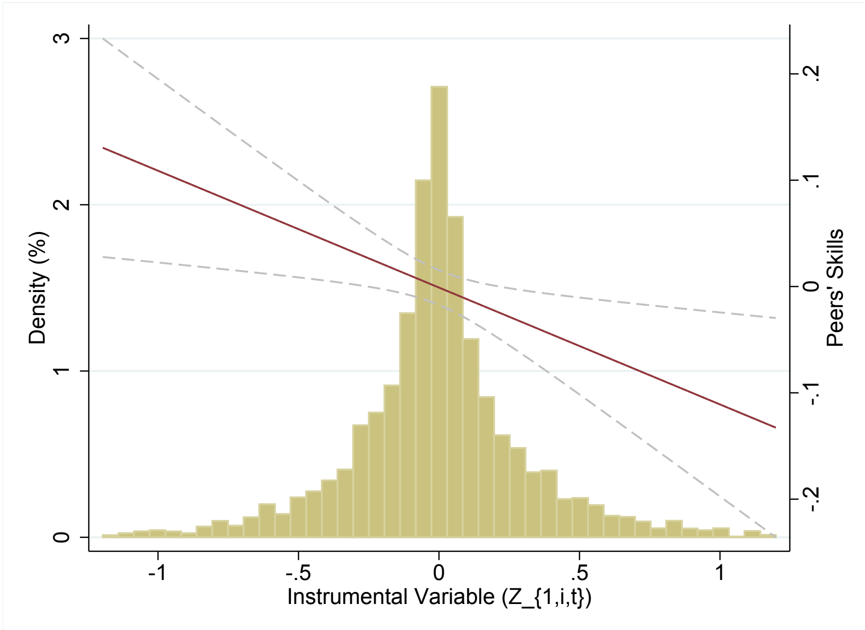
Notes: This figure displays analysis on network formation using the homophily index (see Coleman (1958)). In detail, letting $f_{x,s}$ be the average fraction of friends who are of the same race x at school s and $q_{x,s}$ to be the total fraction of children of race x in a given school s, the homophily-bias index looks as $HBI_{x,s} = \frac{f_{x,s}}{q_{x,s}}$.

Figure 2: Homophily in Network Formation by Skills

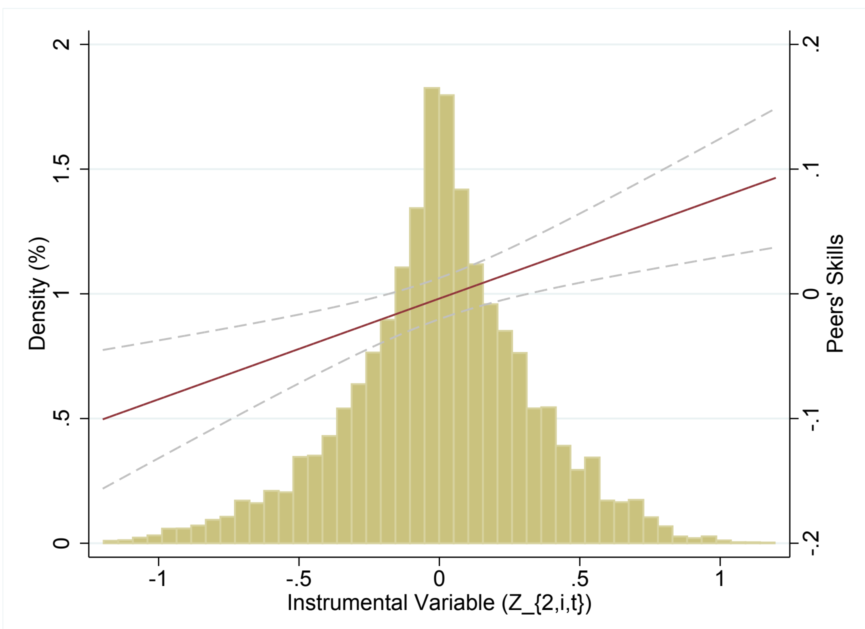


Notes: This figure displays analysis on network formation using the homophily index (see Coleman (1958)). In detail, letting $f_{x,s}$ be the average fraction of friends who are of similar skills level x at school s and $q_{x,s}$ to be the total fraction of similar children with skills level x in a given school s, the homophily-bias index looks as $HBI_{x,s} = \frac{f_{x,s}}{q_{x,s}}$.

Figure 3: First Stage Graphs



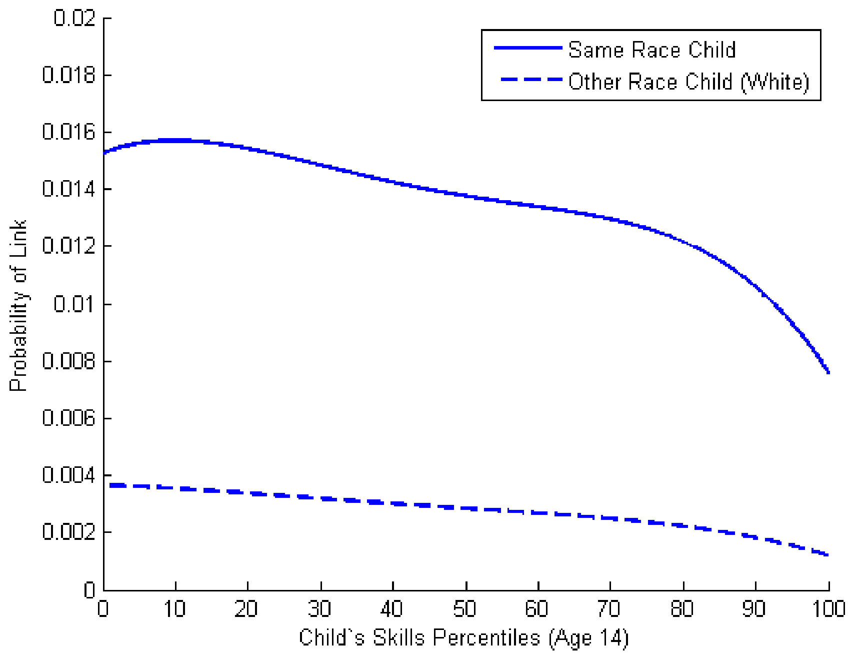
(a) First instrument: Fraction of Minority Children



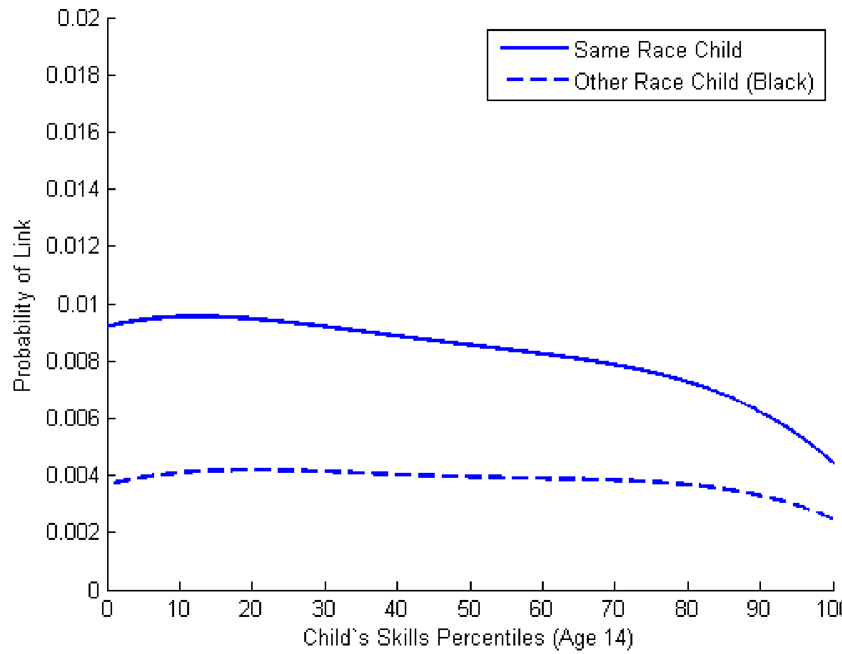
(b) Second instrument: Fraction of White Children

Notes: This figure shows graphically the first stage effects of the instrumental variables. Both graphs show the within-school variation of IV (x-axis) and the peers' skills (y-axes). Solid lines represents a linear regression of peer's skills on each of the two instrumental variables, after controlling for school fixed effects and all the covariates in Table 2. Dashed lines show 90% confidence intervals. The plotted values on the background show the overall variation of both instrumental variables (top and bottom 1% excluded).

Figure 4: The Probability Two Children Become Friends by Skills and Race



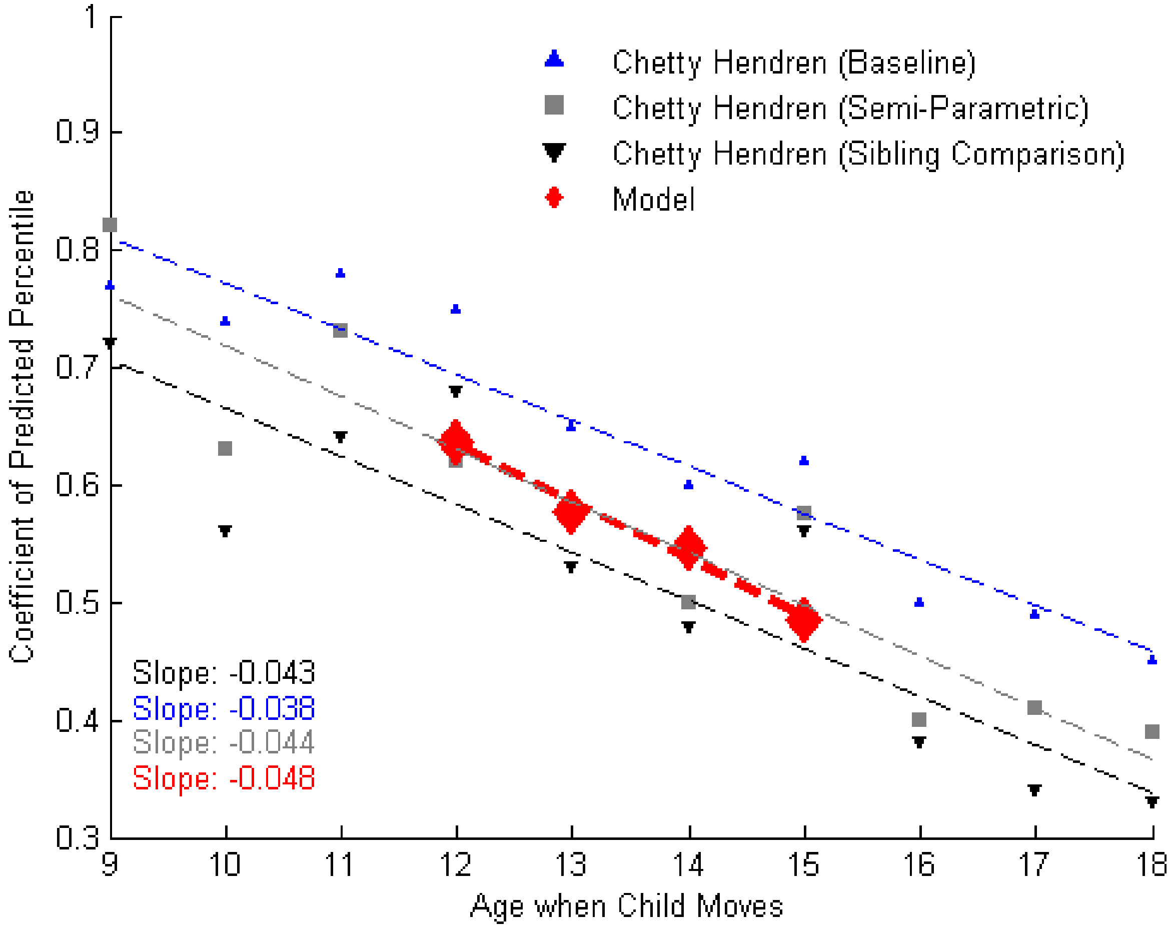
(a) Low Skill (First Quartile) Black Children



(b) Low Skill (First Quartile) White Children

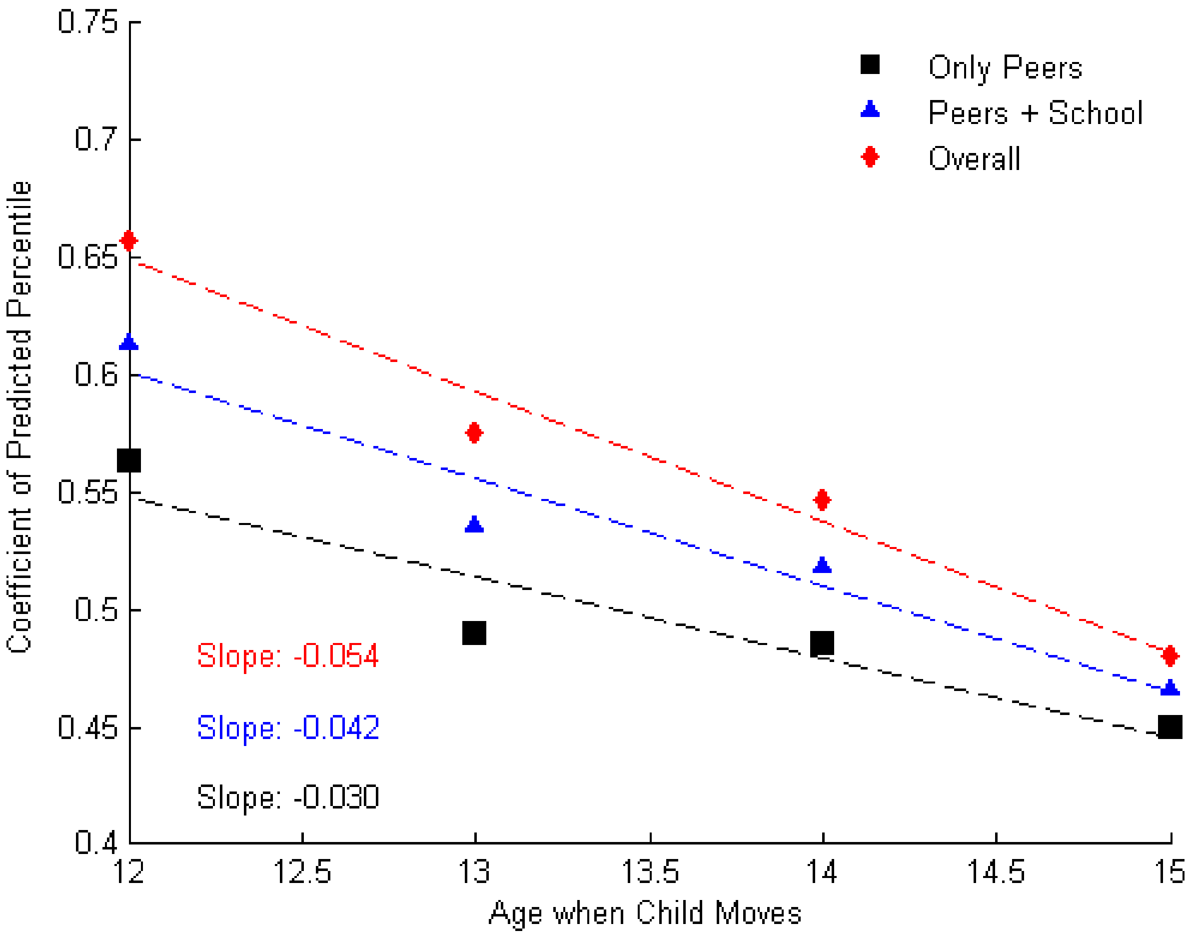
This figure displays the probability that two children become friends by children skills and race. Figure (a) shows the marginal probability for a black child with low skills (in the first quartile of skill distribution at age 14) to become friends with different children over the spectrum of skills and for different races. Figure (b) shows the same graph but for a white child with low skills (in the first quartile of skill distribution at age 14).

Figure 5: Model Validation from Exposure Effects in Children from Chetty and Hendren (2016a)



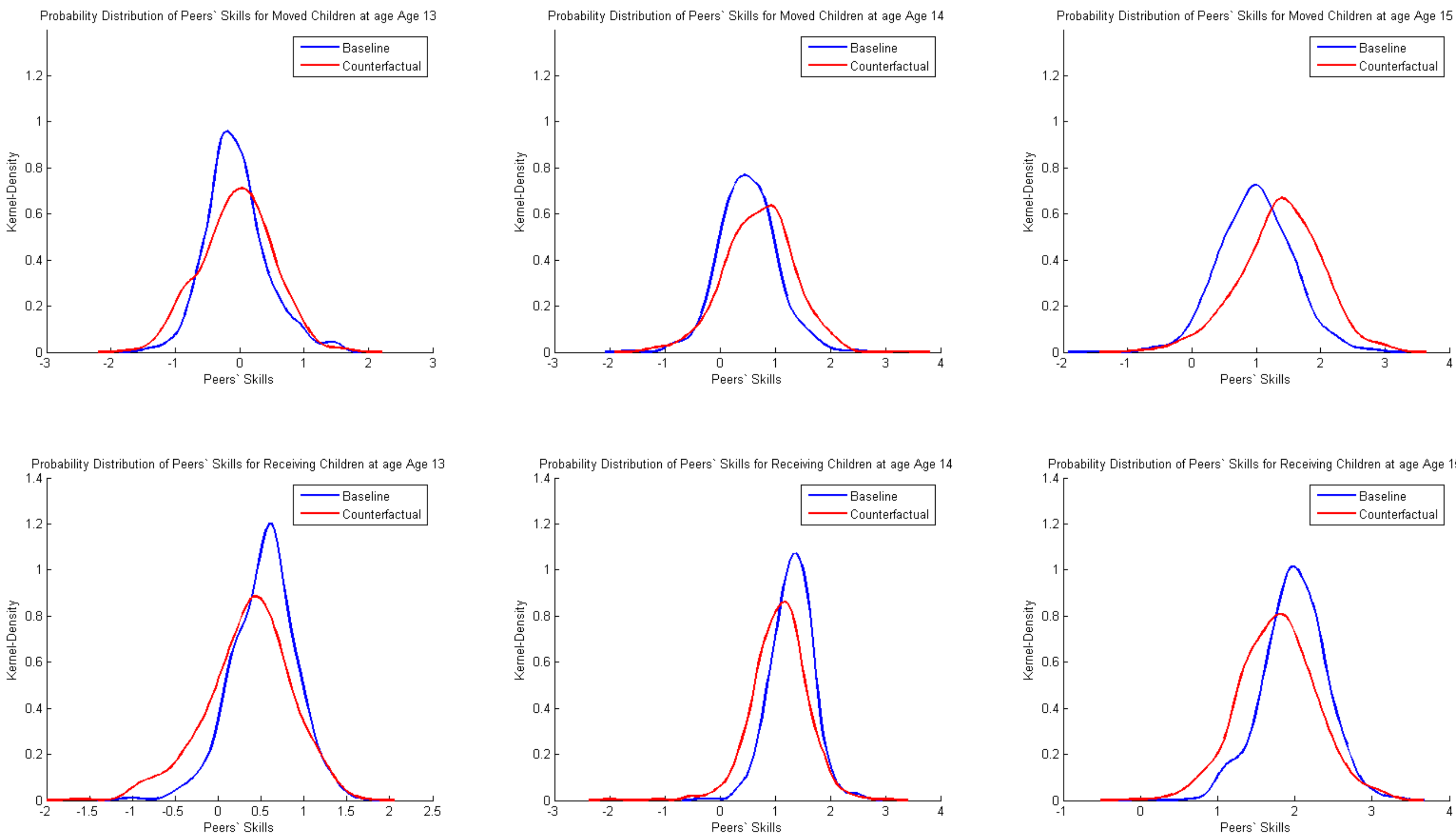
Notes: This figure displays the comparison between model's predictions and findings in Chetty and Hendren (2016a) about childhood exposure effects.

Figure 6: Decomposition Exposure Effects for Disadvantaged Children



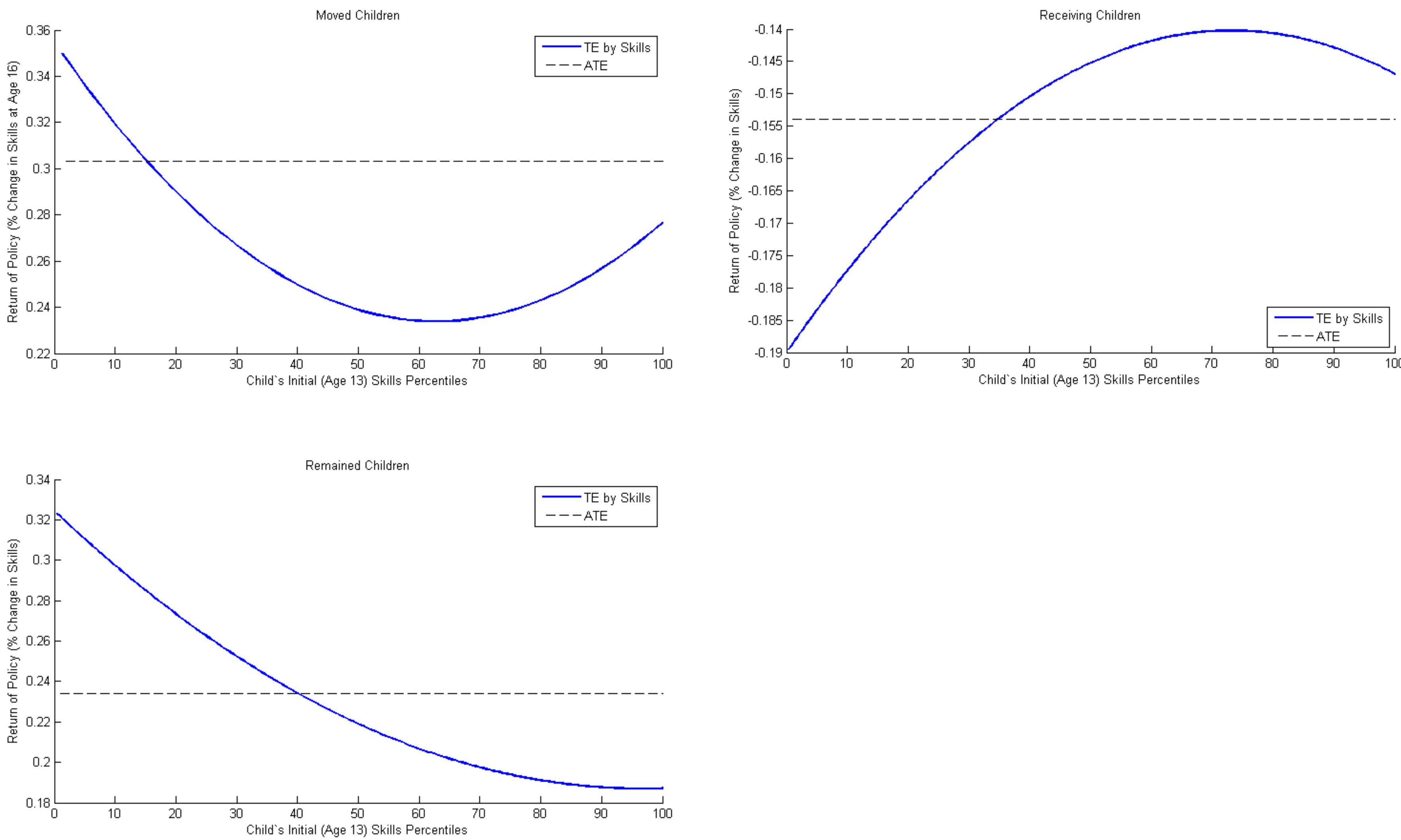
Notes: This figure displays the graphical decomposition for disadvantaged children of childhood exposure effects by: peers, school and neighborhood quality.

Figure 7: Peers' Skills Distribution (Baseline vs Counterfactual Economy)



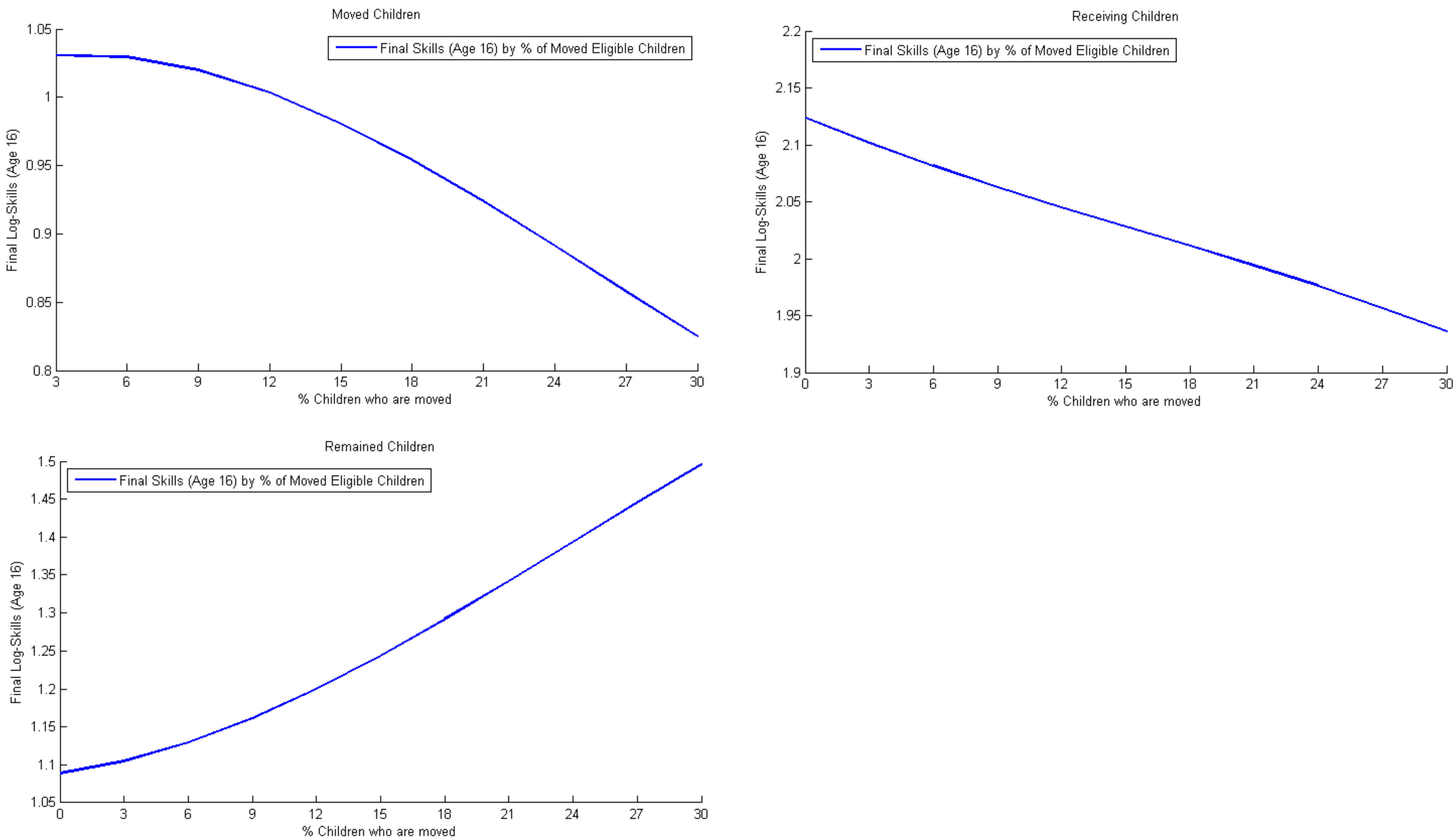
Notes: This figure shows the equilibrium distribution of peers' skills for moved and receiving children by age, before and after the policy is implemented. Parent's expectations about future peers' skills are based on the above distributions. The policy moved a fraction of disadvantaged children into a high-income environment (approximately 30% of the population of the receiving cohort).

Figure 8: Treatment Effect by Skills (Moved-Receiving-Remained Children)



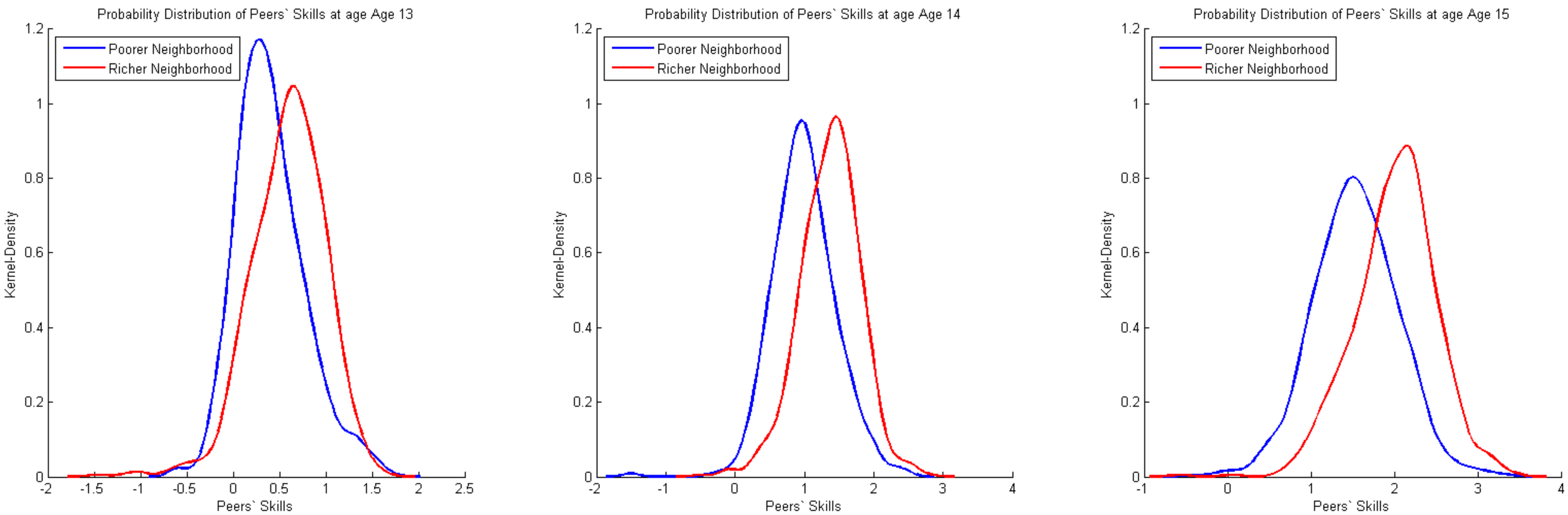
Notes: This figure shows the heterogeneous treatment effects on child's age 16 skills by initial (age 13) child's skills percentile for moved, receiving and remained children. The policy moved a fraction of disadvantaged children into a high-income environment (approximately 30% of the population of the receiving cohort).

Figure 9: Treatment Effect by Fraction of Moved Eligible Children (Moved-Receiving-Remained Children)



Notes: This figure shows the effect of the size of moved children for the policy returns for moved, receiving and remained children. Eligible children live in low-income environments and are below first quartile of age 13 skill distribution.

Figure 10: Peers' Skills Distribution (Low-Income vs High-Income)



Notes: This figure shows the equilibrium distribution of peers' skills for the children who received the initial boost of skills but still live in the low-income environment and their similar counterpart living in high-income environment. Parent's expectations about future peers' skills are based on the above distributions.

A Data Appendix

The Add Health database was designed to study the impact of the social environment (i.e. friends, family, neighborhood and school) on adolescents' behavior in the United States of America. Add Health's original sample comprises students of 132 representative schools in the United States. There are 90,118 students, ranging between grades 7 and 12 in the 1994–1995 school year (Wave I). A subsample of students (20,745) was selected for an additional home interview (in-home). The home interview includes new questions for the children and a questionnaire for one of their parents. Information related to school quality and area of residence makes the data set attractive. Those surveyed were interviewed again in 1995–96 (Wave II), 2001–2002 (Wave III), and 2007–2008 (Wave IV). The data set includes specific information on family background, students' school grades and their scores in the Add Health Picture Vocabulary Test (AHPVT – a revised version of the Peabody Picture Vocabulary Test [PPVT]), as well as information about children's peers.

A main source of information which makes the Add Health data set particularly attractive for achieving the objective of this project is the friendship nomination. During the first two waves, children were asked, both during the in-home and in-school interviews, to nominate their best five male and best five female friends. This detailed information helped me to reconstruct the structure of friendship for every child in the sample by simply matching their identifier.

A.1 Measures for Children's Skills and Parental Investments

The Peabody Picture Vocabulary Test (PPVT) was developed in 1959 as a test of receptive vocabulary and is oriented to give an estimation of verbal ability and school aptitude. More generally, it intends to provide a measure of intelligence. The test is age standardized, and performance does not depend on the reading ability of the test-taker. The test-giver reads a word, and the test-taker selects the image she thinks best fits the meaning of the word from among four simple illustrations.

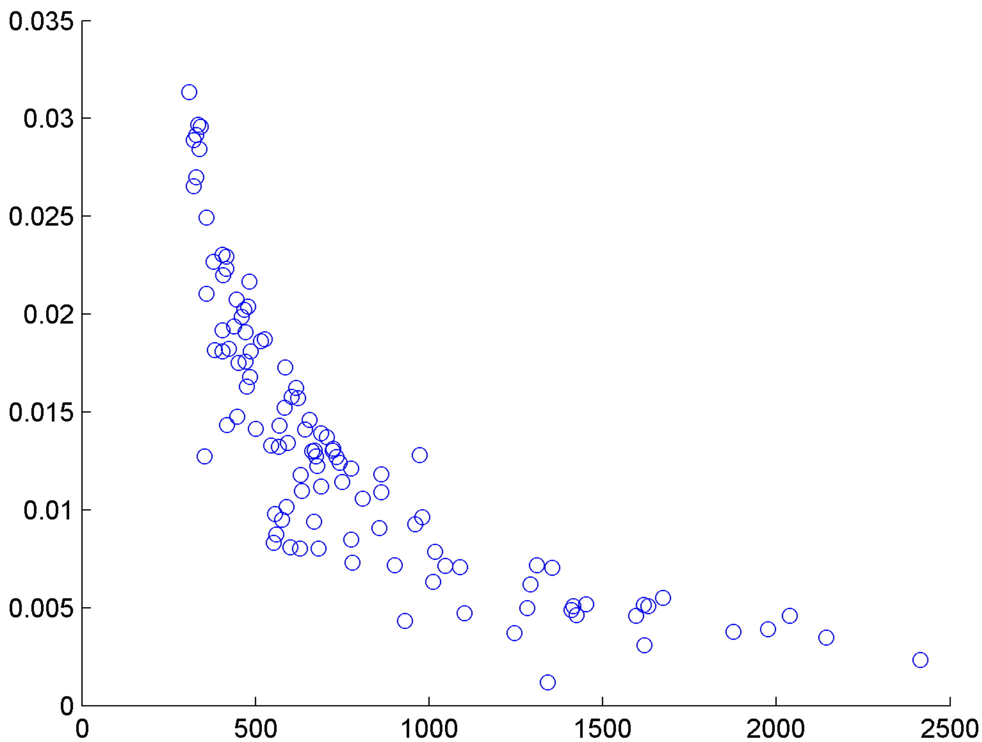
In the Add Health dataset, a computerized and shorter version of the Peabody Picture Vocabulary Test, the AHPVT, has been implemented. The AHPVT includes half of the questions of the original PPVT (every other item in the original sequence was selected for use). Add Health provides two versions of the AHPVT: raw and standardized test scores. The standardized version has a mean of 100 and a standard deviation of 15 and is standardized by age (for further technical details about the AHPVT, see Halpern, 2000).

Add Health provides information about the AHPVT test scores for Waves I, III and IV. During Wave I, respondents are between 11 and 21 years old. I consider this piece of information as one of the measures of children's latent skills. Additionally, I consider children's grades at school for both Wave I and Wave II. Analyzing all these multiple measurements, I am able to combine both the cross-sectional as well as the longitudinal information about child development.

During the in-home interview in Wave I and Wave II, children provided information about activities they had engaged in with their parents during the previous four weeks. These activities include: going shopping, sport activities, going to a movie/museum/concert or sport event, talking about personal problems or school, or working on a project for school. There are a total of nine activities each child can do with their parents. These types of activities provide information about the level of parental engagement with their child. During Wave I, a parent, preferably the resident mother, of each adolescent respondent was interviewed (in-home interview). The parent questionnaire included a question about the achieved level of education for the respondent, which is considered as one proxy for the mother's skills. During the same survey, the respondent provided information about total family income during the previous calendar year.

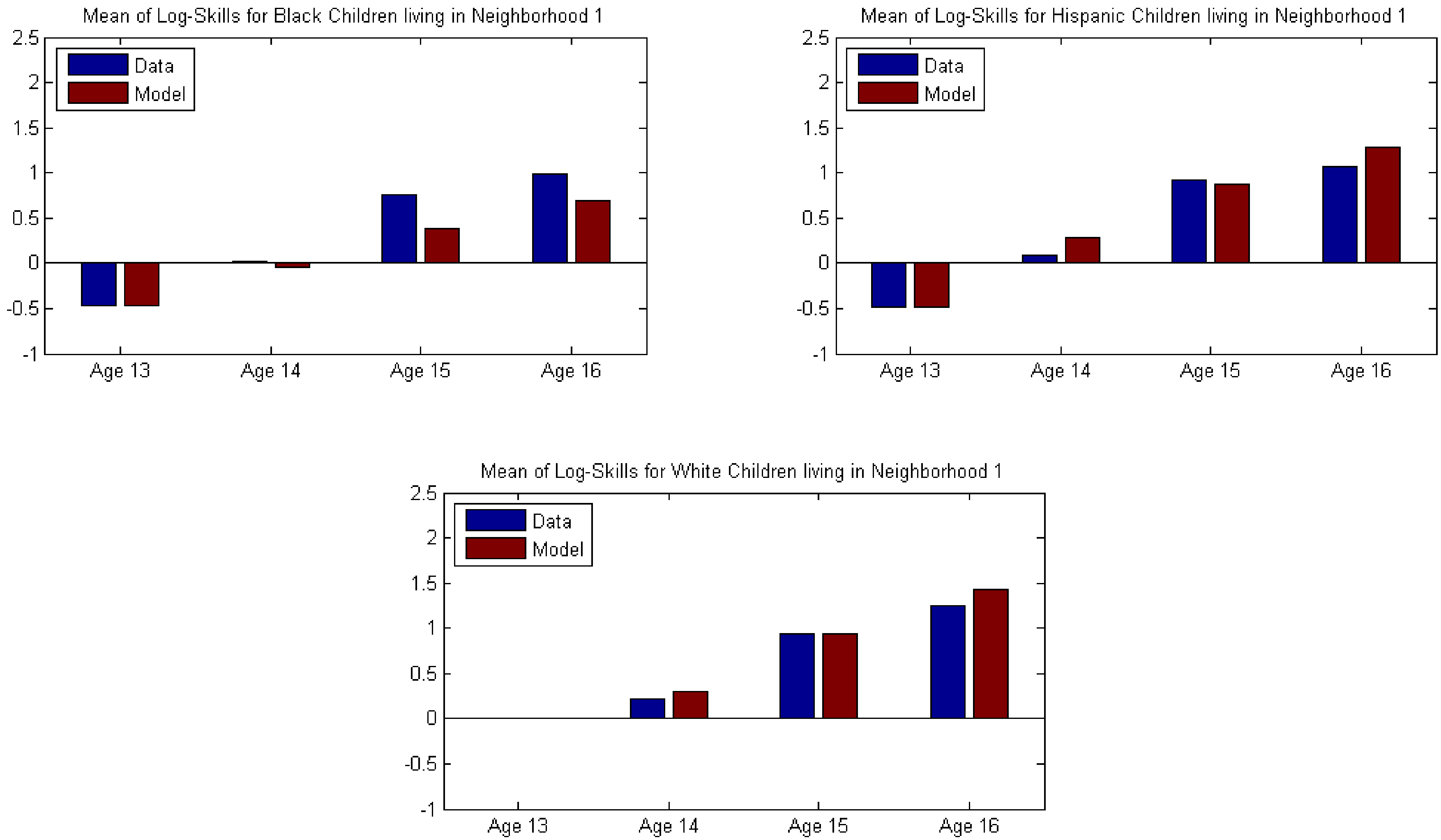
B Additional Figures and Tables

Figure B-1: Probability of the Friendships and School's Size



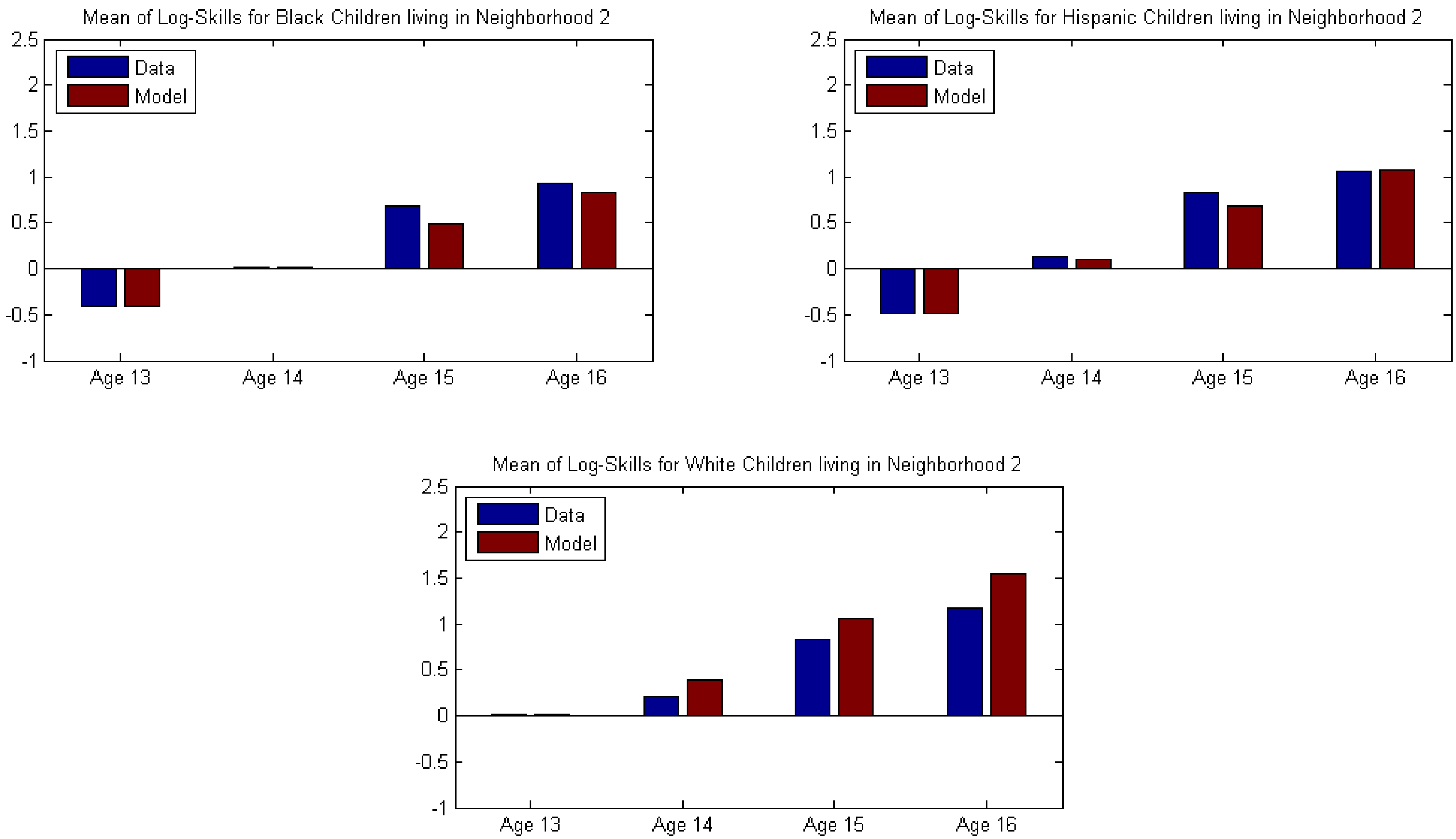
Notes: This figure shows the probability, for each school, that a child becomes friend with another child as function of school's size.

Figure B-2: Mean of Children Skills by Race (Low-Income Neighborhood)



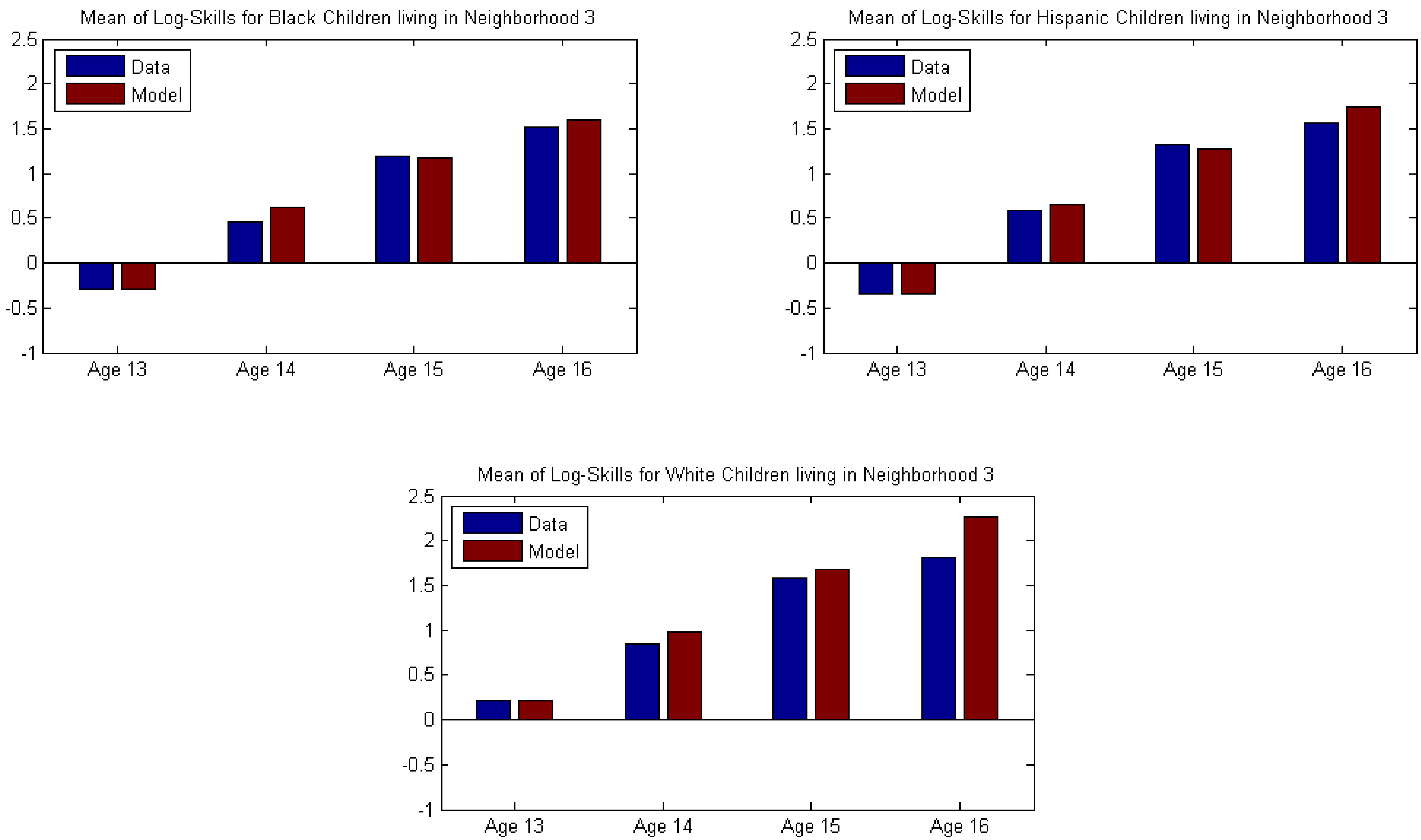
Notes: This figure shows the sample fit for the mean of children's skills for low-quality (low-income) neighborhood by race.

Figure B-3: Mean of Children Skills by Race (Medium-Income Neighborhood)



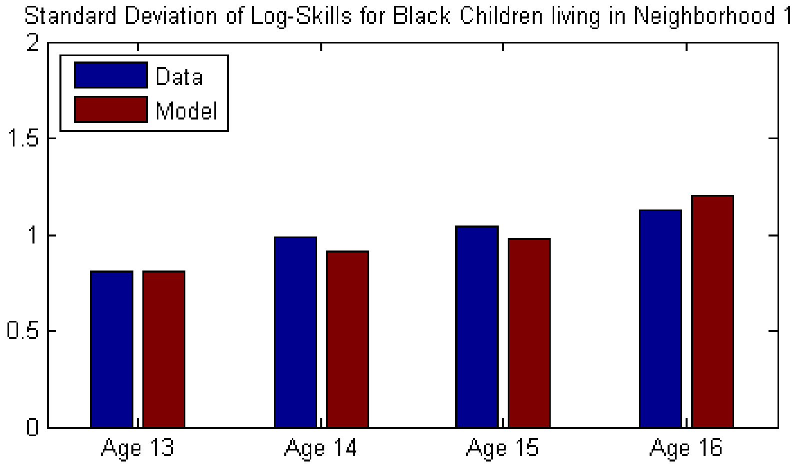
Notes: This figure shows the sample fit for the mean of children's skills for medium-quality (medium-income) neighborhood by race.

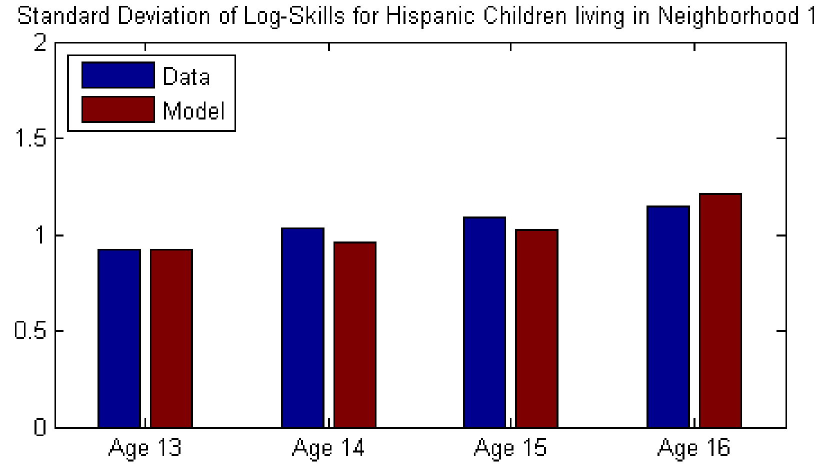
Figure B-4: Mean of Children Skills by Race (High-Income Neighborhood)

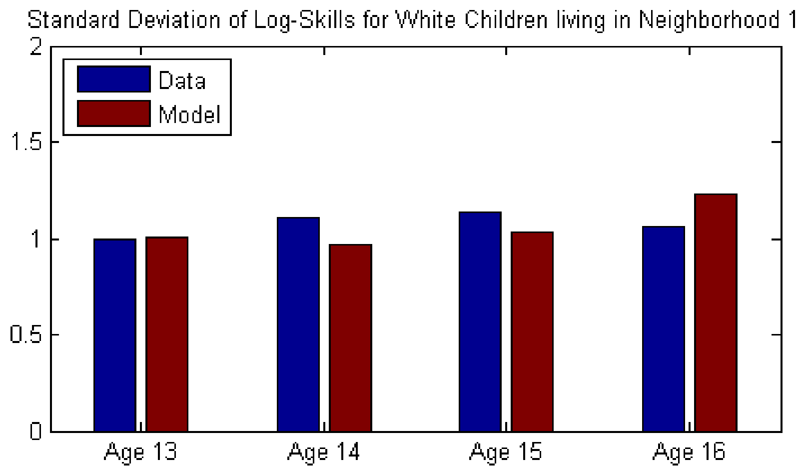


Notes: This figure shows the sample fit for the mean of children's skills for high-quality (high-income) neighborhood by race.

Figure B-5: Standard Deviation of Children Skills by Race (Low-Income Neighborhood)

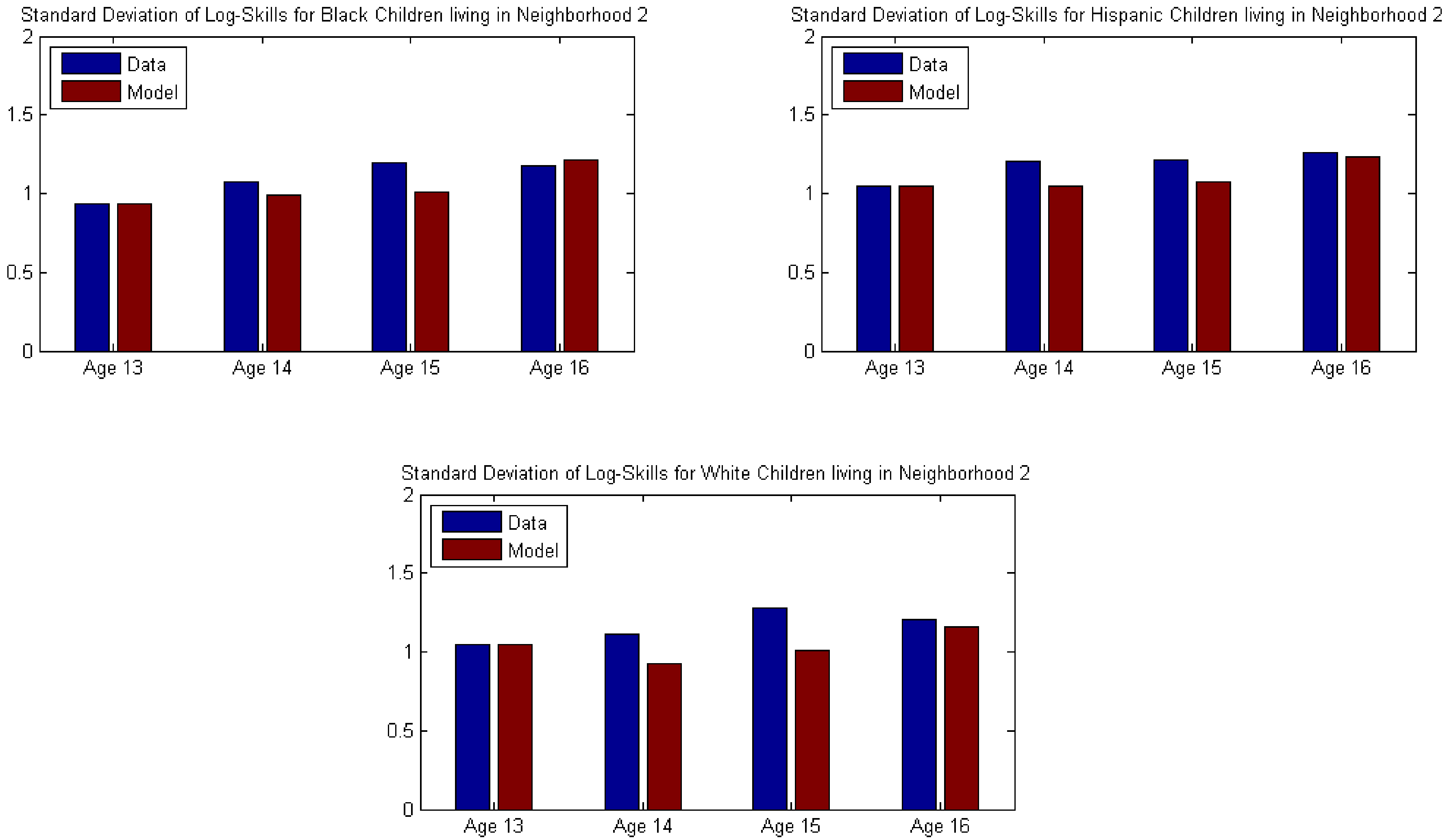






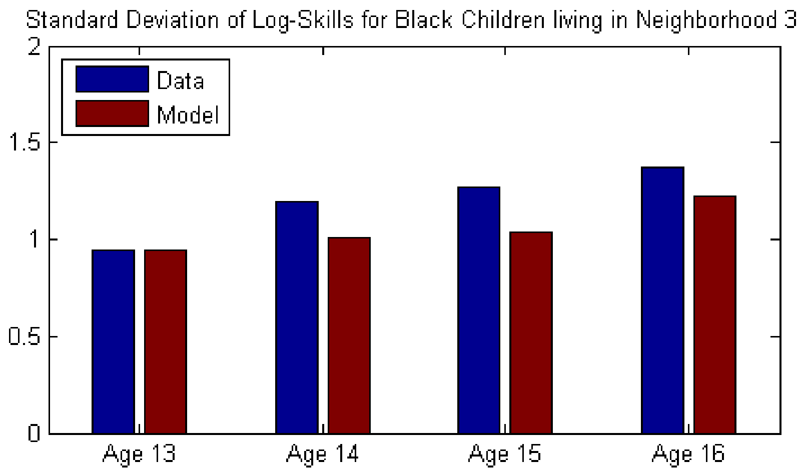
Notes: This figure shows the sample fit for the standard deviation of children's skills for low-quality (low-income) neighborhood by race.

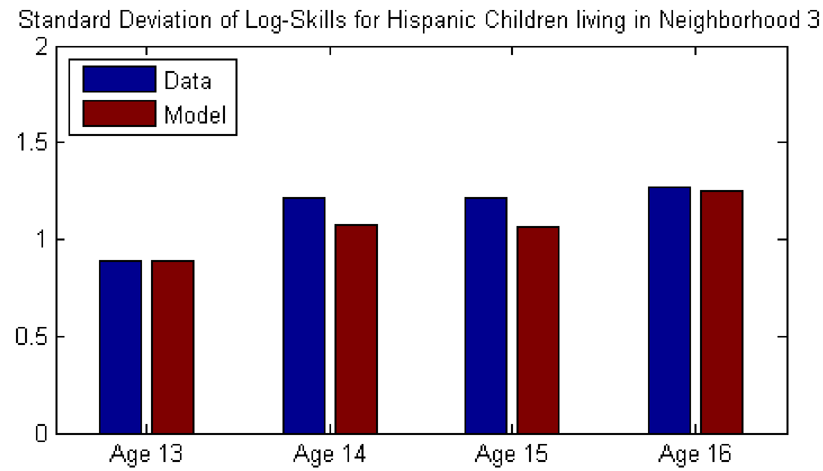
Figure B-6: Standard Deviation of Children Skills by Race (Medium-Income Neighborhood)

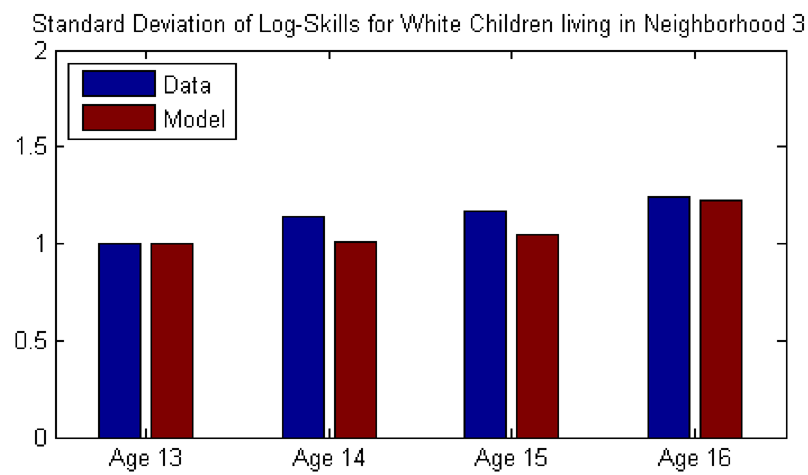


Notes: This figure shows the sample fit for the standard deviation of children's skills for medium-quality (medium-income) neighborhood by race.

Figure B-7: Standard Deviation of Children Skills by Race (High-Income Neighborhood)

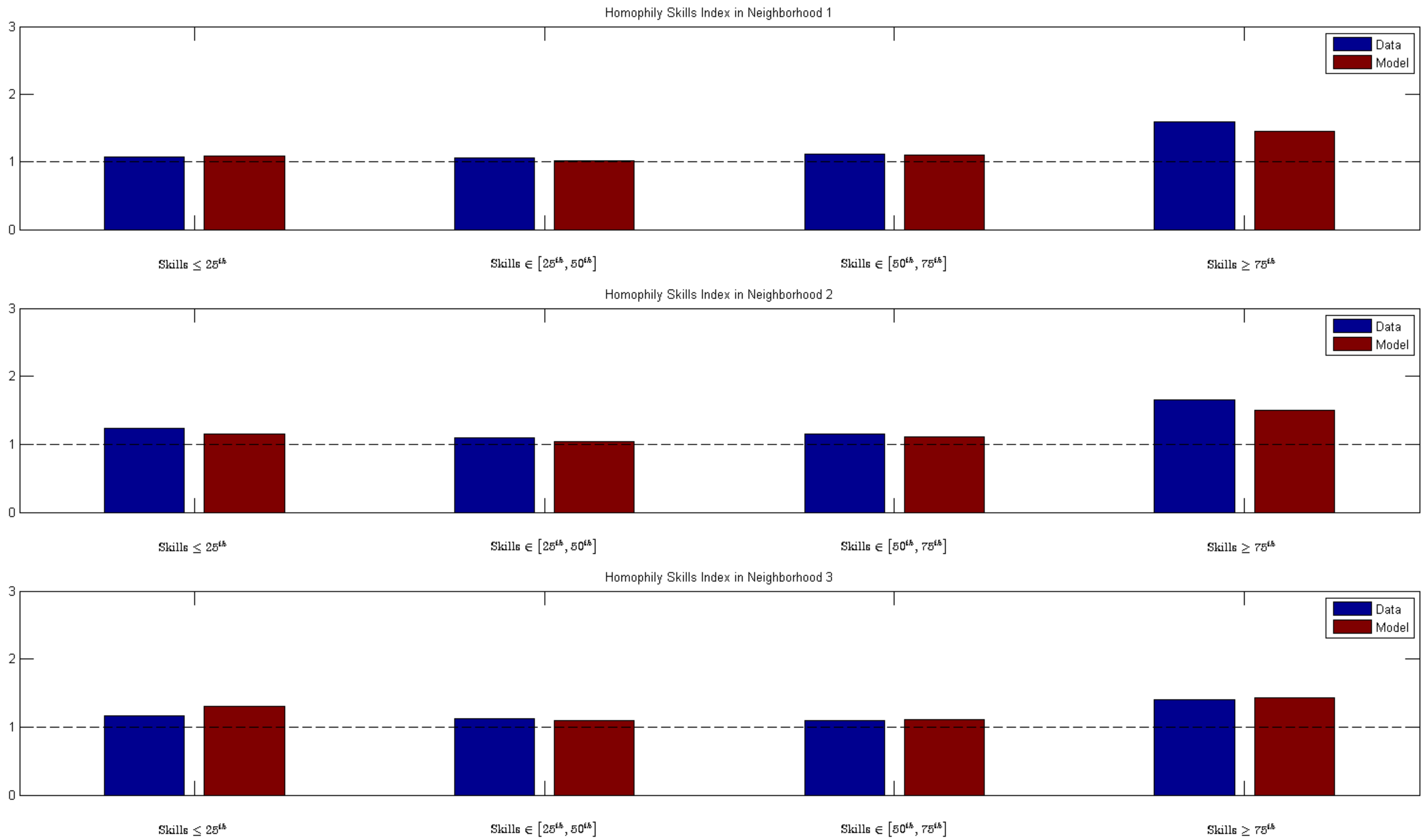






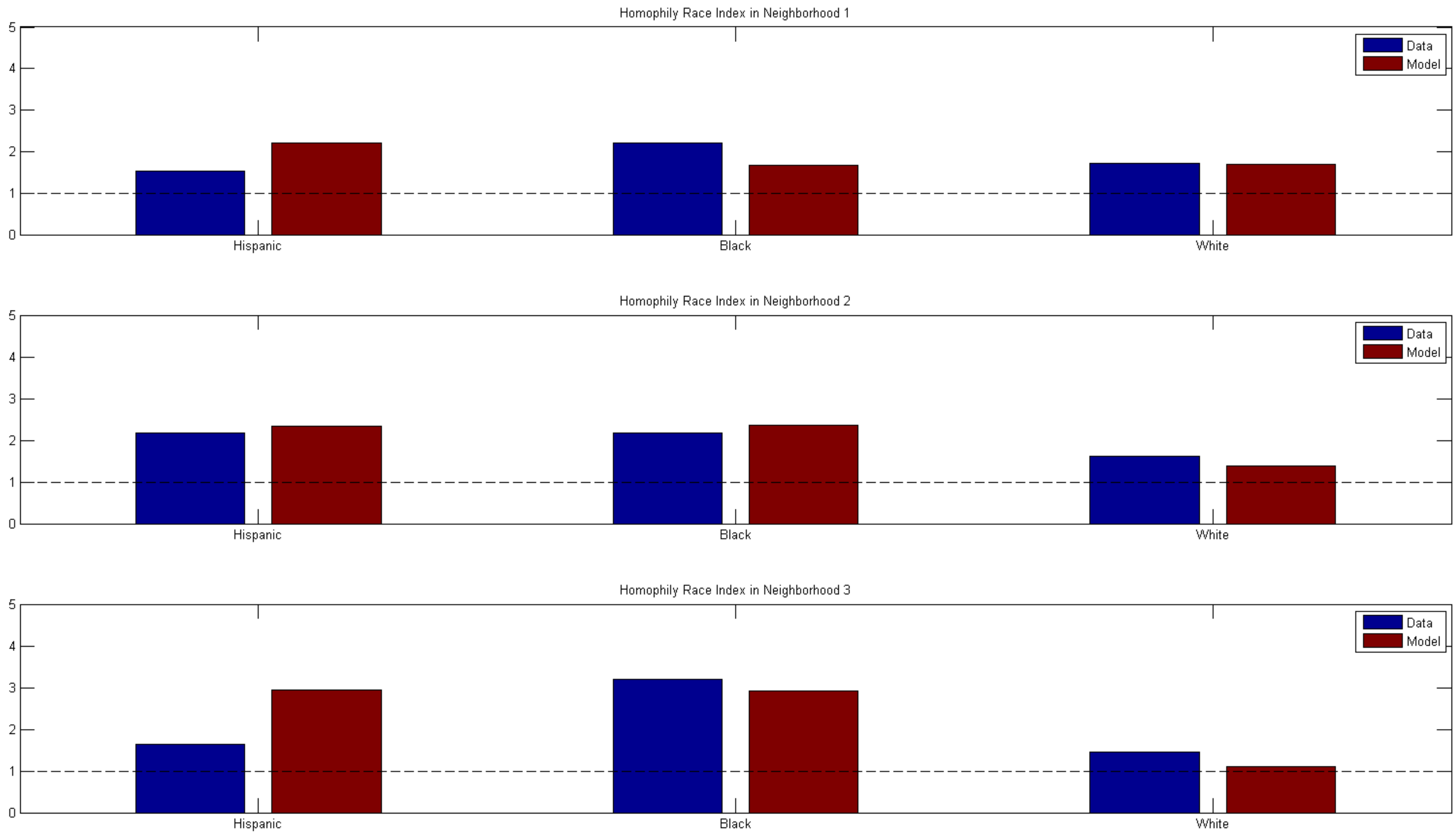
Notes: This figure shows the sample fit for the standard deviation of children's skills for high-quality (high-income) neighborhood by race.

Figure B-8: Index for Skills Homophily by Neighborhoods



Notes: This figure shows the sample fit for the homophily index for skills by by neighborhood-quality type (low-median-high income).

Figure B-9: Index for Race Homophily by Neighborhoods



Notes: This figure shows the sample fit for the homophily index for race by by neighborhood-quality type (low-median-high income).

Table B-1: Sample Statistics by Neighborhood Types

Neighborhood's Type	(1)	(2)	(3)
Panel A: Fraction of Families by Race			
Fraction Black (%)	31.74	14.00	5.36
Fraction Hispanic (%)	26.77	14.50	10.86
Fraction White (%)	41.50	71.50	83.78
Panel B: Family Income (in 1994 $)			
	Mean Family Income		
Family Income	29,637	43,458	57,512
	Mean Family Income by Race		
Black Family	26,917	41,778	46,592
Hispanic Family	26,167	33,749	49,940
White Family	34,046	45,632	59,224
Panel C: Children PPVT Achievements			
	Mean PPVT by Race		
Black Children	57.02	62.37	64.39
Hispanic Children	58.06	60.33	64.51
White Children	63.87	66.46	69.98
	PPVT Gaps between Neighborhoods		
Black Children	66 (% SD)	18 (% SD)	-
Hispanic Children	58 (% SD)	37 (% SD)	-
White Children	55 (% SD)	32 (% SD)	-
	PPVT Gaps within Neighborhoods		
Black Children	61 (% SD)	37 (% SD)	50(% SD)
Hispanic Children	52 (% SD)	55 (% SD)	49(% SD)
White Children	-	-	-

Notes: This table reports descriptive statistics by by neighborhood-quality type (low-medium-high income).

Table B-2: Sample Fit for Auxiliary Coefficients (Investments)

	Dependent Variable Fraction (%) of Invested Parental Time			
	Measurement Error Adjusted	Measurement Error Adjusted and Instrumental Variables (IV)	Measurement Error Adjusted	Measurement Error Adjusted and Instrumental Variables (IV)
	Data		Model	
Child Skills (Log)	2.660 (0.316) [2.041,3.280]	2.120 (0.668) [0.810,3.430]	2.152	3.124
Peers' Skills (Log)	-1.441 (0.650) [-2.715,-0.167]	0.720 (0.354) [0.026,1.414]	-1.272	0.895
		First Stage		First Stage
$Z_{1,i,t}$ (Minorities Children)		-0.104 (0.052) [-0.206,-0.002]		-0.127
$Z_{2,i,t}$ (White Children)		0.082 (0.037) [0.009,0.155]		0.105

Notes: This table shows the sample fit for auxiliary regression coefficients for models in (2). Columns 1 and 2 shows the same estimated coefficients as in Table 2. Both standard errors in parenthesis and the 95% confidence interval in square brackets are computed using a cluster bootstrap.
Data source: National Longitudinal Survey of Adolescent Health (Add Health).

Table B-3: Sample Fit for Auxiliary Coefficients (Dynamics of Skills)

	Data	Model
Investments (Log)	0.408	0.343
	(0.197)	
	[0.021,0.794]	
Child‘s Skills (Log)	0.750	0.760
	(0.238)	
	[0.283,1.216]	
Peer‘s Skills (Log)	0.366	0.167
	(0.167)	
	[0.038,0.693]	

Notes: This table shows the sample fit for auxiliary regression coefficients of the auxiliary model for the dynamics of children's skills. The dependent variable is the next period (t+1) child's log-skills. The covariates in the table are: log-investments, current child's log-skills, current peers' log-skills. The regression also include as controls: age fixed effects, race, last period family's income, mother's skills, school's fixed effects. Both standard errors in parenthesis and the 95% confidence interval in square brackets are computed using a cluster bootstrap.
Data source: National Longitudinal Survey of Adolescent Health (Add Health).

Table B-4: Counterfactual Effects on Receiving Children by Race (moved children are 5% of the receiving cohort)

Panel A: Effects on Children‘s Log-Skills (Mean)

	Black			Hispanic			White	
	Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)
Age 13	-0.30	-0.30	Age 13	-0.34	-0.34	Age 13	0.22	0.22
Age 14	0.28	0.23	Age 14	0.45	0.41	Age 14	0.97	0.96
Age 15	0.96	0.87	Age 15	1.08	1.03	Age 15	1.66	1.64
Age 16	1.49	1.39	Age 16	1.59	1.53	Age 16	2.24	2.21

Panel B: Effects on Parent‘s Investment Decision (Mean)

	Black			Hispanic			White	
	Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)		Baseline	Counterfactual (Equilibrium)
Age 13	20.09	19.01	Age 13	21.97	20.71	Age 13	27.92	27.67
Age 14	18.85	17.94	Age 14	20.67	20.25	Age 14	28.02	27.85
Age 15	16.32	16.36	Age 15	16.69	16.63	Age 15	24.29	24.37

Notes: This table shows the counterfactual policy effects for receiving children (by race) when a fraction of moved children (5% of the receiving population) are moved into a high-income environment. For each subgroup, I compare the baseline results in skills and parental investments (Panel B) with the equilibrium counterfactual predictions.

C The Latent Factor Models

I consider the following latent factor model for either the child's skills ($h_{i,t}$) or the mother's skills (m_i):

$$
\begin{aligned}
Z^h_{i,t,k} &= \mu^h_{t,k} + \lambda^h_{t,k} \cdot \ln h_{i,t} + \epsilon^h_{i,t,k} \\
Z^m_{i,k} &= \mu^m_{t,k} + \lambda^m_k \cdot \ln m_i + \epsilon^m_{i,k}
\end{aligned}
\tag{C-1}
$$

where μs and λs are, respectively, the location and scale (or loading factor) parameters for each considered measure k at any age t for each child i. The distribution of the latent factors is identified by exploiting multiple measures in the data. One commonly used condition for identification is the independence of the joint distribution of latent variables and measurement errors.

Assumption 1. *Measurement model assumptions:*

(i) $\epsilon^h_{i,t,k} \perp \epsilon^h_{i,t,k'}$ *and* $\epsilon^m_{i,k} \perp \epsilon^m_{i,k'}$ *for all* t *and* $k \neq k'$

(ii) $\epsilon^h_{i,t,k} \perp \epsilon^h_{i,t',k'}$ *and* $\epsilon^m_{i,k} \perp \epsilon^m_{i,k'}$ *for all* $t \neq t'$ *and all* k *and* k'

(iii) $\epsilon^h_{i,t,k} \perp h_{i,t'}$ *and* $\epsilon^m_{i,k} \perp m_i$ *for all* t *and* t' *and all* k

(iv) $\epsilon^h_{i,t,k} \perp \epsilon^h_{j,t,k}$ *and* $\epsilon^m_{i,k} \perp \epsilon^m_{j,k}$ *for all* k *and for any family* j *different from family* i

(v) $\epsilon^h_{i,t,k}, \epsilon^m_{i,k} \perp X_i$*, for all* t *and all* k

Assumption 1 (i) is that measurement errors are independent contemporaneously across measures. Assumption 1 (ii) is that measurement errors are independent over time. Assumption 1 (iii) is that measurement errors in any period are independent of the latent variables in any period. Assumption 1 (iv) that measurement errors are independent between observations. Finally, assumption 1 (v) assures that errors in measuring skills are independent of the observable characteristics of children and mothers. While these assumptions are strong, they are common in the current literature (see Cunha et al., 2010; Agostinelli and Wiswall, 2016). Assumptions (i)–(v), together with the specification in (C-1), guarantee the non-parametric identification of the latent distribution. However, in this work, I consider a parametric model of skill formation, hence each of the conditions in (i)–(v) can be relaxed to statements about the zero correlation between the considered variables, instead of the full independence conditions.

D Proof of Theorem 1

In this proof, I consider the two-period case. The four-period case follows by the induction hypothesis. The proof is based on backward induction. The goal is to show that for each period t, I can compute the policy functions for both parents and children that solve the fixed point associated to the equilibrium conditions in (4.5). For the purpose of exposition, I employ only the endogenous state variables of the problem $(h, \overline{H})$ and define the technology of skill formation in (4) as $f(\cdot)$.[44] Finally, the proof is executed for the case of log utility ($\gamma_3 = \gamma_5 = 1$) and perfect substitution between parents and peers ($\alpha_4 = 1$). This parametric case is the harder case to prove an existence of a fixed point through Tarski's fixed point theorem, and this is because of the non-trivial preservation of monotonicity and supermodularity in the value function. Any other case where either utility function is more convex or the technology provides higher complementarity between endogenous inputs follows by construction. Hence, by proving that this case admits a fixed point, I prove that a fixed point exists for any other admissible parameterization of the model .

- Last period case ($t = T$):

  During the last period, children decide their own peer-solving problem (6) based on their current level of skills. Parents observe the realization of their child's network formation and then make their last investment decision. No equilibrium conditions here are necessary, since during the next period, no children's network is formed. In other words, during the last period, the equilibrium policy functions to solve the two (parent's and child's) decision problems. The associated last period value function for parent i is:

$$
\begin{aligned}
V_T^P(h_{i,T}, \overline{H}_{i,T}) = \Big(\big(\tau - I_T^*(h_{i,T}, \overline{H}_{i,T})\big) \cdot w_{i,T} + y_{i,T}\Big)^{\gamma_1} + (\gamma_2 + \alpha_1)\, log(h_{i,T}) \\
+ \beta\gamma_4 log\Big(\alpha_3 I_T^*(h_{i,T}, \overline{H}_{i,T}) + (1 - \alpha_3)\overline{H}_{i,T}\Big) \qquad \text{(D-1)}
\end{aligned}
$$

  Lemma 1. *The value function $V_T^P(h_{i,T}, \overline{H}_{i,T})$ is monotone, increasing in both arguments and supermodular.*
  Proof: It is easy to show that the policy function is monotone in both dimensions (monotone increasing in the first argument and decreasing in the second argument). Hence, the value

[44] For clarity and without loss of generality, I ignore all the exogenous variables which are irrelevant for the equilibrium analysis, such as the mother's skills and the family's characteristics.

function clearly has monotone increasing in the first argument. Additionally, because of the homothetic preferences, the overall peer effects H on children's skills is positive, e.g., the change (decrease) of the policy function due to higher H does not dominate the initial change in H.[45] *Hence, the value function is also increasing with the peer effects. To show that the value is supermodular, consider the derivative of V_T^P with respect to $h_{i,T}$:*

$$\frac{\partial V_T^P(h_{i,T},\overline{H}_{i,T})}{\partial h_{i,T}} \equiv \frac{\gamma_2+\alpha_1}{h_{i,T}} \tag{D-2}$$

where equation (D-2) is derived applying the principle of optimality. Given equation (D-2) is independent of $H_{i,T}$, it follows trivially that V_T^P is supermodular in $(h_{i,T},\overline{H}_{i,T})$.

- First period case ($t = T-1$):

  In order to solve the parent's problem and child's problem differently (remember that parents take a child's decision regarding the next period's network formation as a given), let us define $\widetilde{h}_{i,t+1}$ as the skills that children care about once they decide upon their friends in the next period. Parents take $\widetilde{h}_{i,t+1}$ as a given – that is, they think the process of the formation of peer groups is independent of their investment decisions. In this case, the parent's problem is:

$$V_{T-1}^P(h_{i,T-1},\overline{H}_{i,T-1};\widetilde{h}_{i,T}) =$$

$$\max_{I_{i,T-1}\in[0,\tau]} \left(\left(\tau-I_{i,T-1}\right)\cdot w_{i,T-1}+y_{i,T-1}\right)^{\gamma_1}+\gamma_2\, h_{i,T-1}+\beta E\left[V_T^P\left(h_{i,T}\left(I_{i,T-1}\right),\overline{H}_{i,T}(\widetilde{h}_{i,T})\right)\middle|\widetilde{h}_{i,T}\right] \tag{D-3}$$

  where $\overline{H}_{i,T}(\widetilde{h}_{i,T})$ is the stochastic mapping about the next period's peer effects $\overline{H}_{i,T}$ given $\widetilde{h}_{i,T}$. Given the empirical evidence on children's social interactions, I consider the case when this mapping is stochastically ordered in $\widetilde{h}_{i,T}$,

$$E\left[\overline{H}_{i,T}(\widetilde{h}_{i,T})\middle|\widetilde{h}_{i,T}\right] \le E\left[\overline{H}_{i,T}(\widetilde{h}_{i,T}')\middle|\widetilde{h}_{i,T}'\right] \text{ if } \widetilde{h}_{i,T} < \widetilde{h}_{i,T}',$$

  which means that the higher the child's skills, the higher the probability of becoming friends with higher-skilled children. The fix-point problem here comes from the equi-

[45] This can be proved through the comparative statics of the problem, using the concavity of utility over consumption and children's skills.

librium consistency conditions, which require that the endogenous network formation is consistent with the parental decisions about the child's development. In other words, in the equilibrium path, I am imposing that the optimal level of children's skills decided by the parents are the same level of skills governing the network-formation decision

$$\widetilde{h}_{i,T} = h^*_{i,T}.$$

To show that this fix point has a solution, I am applying standard results in the dynamic lattice programming literature.

Lemma 2. *Under Lemma 1, there exists a policy function $I_{T-1}(\cdot)$ which solves the equilibrium fix-point problem in (D-3).*
Proof: This result follows directly from Tarski's fixed point theorem. The continuation value is both increasing and supermodular in the two endogenous variables (individual endogenous children's skills and peers' skills). The stochastic process governing the network formation stochastically ascends with respect to the children's skills, and the choice set is a complete lattice.

The Markovian equilibrium of the model is defined as the sequence of the policy functions solves each of the two periods' equilibrium problems.

E Indirect Inference

Part of the selected moments in the estimation procedure includes coefficients of auxiliary regressions. In particular, I consider separate auxiliary models to analyze the *parental investments* and the *dynamics of skills*. The first set of coefficients consider parental investments as the dependent variable in the following linear regression

$$I_{i,s,t} = \beta_0 + \beta_1 \ln h_{i,s,t} + \beta_2 \ln \overline{H}_{i,s,t} + X_i' \beta_3 + \beta_s + u_{i,s,t}\,, \tag{E-1}$$

where $I_{i,s,t}$ is the parental investment (as fraction of time) for parent of child i, in school s when she is t years old, which is recovered through a latent factor model (see Section 5.1) using data on parental engagement described in previous section. The child's skills are defined as $h_{i,s,t}$, while $\overline{H}_{i,s,t}$ is the mean of her peers' skills. X_i is a vector of child and parents' exogenous characteristics, which includes race, age fixed-effects, lagged family income and mother's skills. Finally, β_s is the school fixed effects. The coefficients of interests are related to how parental investments respond to changes in child skills (β_1) and peers' skills (β_2). In the estimation procedure, I include as targeted moments the parameters in (E-1), for both the school fixed-effects estimator, as well as for the IV estimator explained in Section 3.4.

Additionally, I consider an auxiliary model of skill dynamics, where the next period skills are the dependent variable of the following regression

$$\ln h_{i,s,t+1} = \beta_0^h + \beta_1^h \ln h_{i,s,t} + \beta_2^h \ln \overline{H}_{i,s,t} + \beta_3^h \ln I_{i,s,t} + X_i' \beta_4^h + \beta_s^h + u_{i,s,t}^h\,, \tag{E-2}$$

where $(\beta_0^h, \beta_1^h, \beta_2^h, \beta_3^h)$ are the specific auxiliary parameters I selected as targeting moments for estimation. They represent respectively the elasticity of the next period skills with respect to the stock of current skills, the peers' skills and the parental investments. X_i is a vector of child and parents' exogenous characteristics, which includes race, age fixed-effects, lagged family income and mother's skills. Finally, β_s is the school fixed effects. The estimated distribution of school fixed-effects are used in the estimation as the school quality distributions in the model.