Rational Inattention and the Business Cycle Effects of Productivity and News Shocks*

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Abstract

In the standard neoclassical model, anticipated fluctuations in productivity fail to cause business cycle comovement. In response to news about higher future productivity, consumption rises but employment and investment fall. Suppose that firms are subject to rational inattention. They choose an optimal signal about the state of the economy. The optimal signal turns out to confound current with expected future productivity. Labor and investment demand rise after a news shock, causing an output expansion. Rational inattention also improves the propagation of a standard productivity shock, by inducing persistence.

Keywords: information choice, rational inattention, business cycles, news shocks, productivity shocks (*JEL*: D83, E32, E71).

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1 Introduction

The basic challenge for any business cycle model is to specify an impulse and a propagation mechanism that produce business cycle comovement. This challenge is difficult, as Barro and King (1984) first explained.[1] A key insight from the Real Business Cycle model is that fluctuations in productivity generate comovement in the standard neoclassical economy. Employment, investment, output, and consumption move together after a productivity shock, as they do in the data in a business cycle expansion or contraction.[2]

This insight is fragile, however. In the model it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. *Anticipated* fluctuations in productivity do *not* cause comovement. Suppose productivity will rise in the future (while current productivity is unchanged). The news causes a wealth effect. Firms have no incentive to increase labor demand before productivity improves, while households *reduce* labor supply due to the wealth effect. As a result, hours worked *fall*. With capital predetermined and current productivity unchanged, output contracts. Households, wanting to smooth consumption, choose to reduce the capital stock before productivity improves. Consumption *rises* while investment *declines*. The model fails to produce comovement in response to news about future productivity. It predicts an output *contraction* after news that productivity will improve.[3]

It is convenient to model anticipated fluctuations in productivity as "news shocks about productivity" ("news shocks" for short). A shock drawn by nature in period t affects productivity in period $t + h$, where h is a strictly positive integer. The question is how the economy responds to a news shock before period $t + h$. In the standard neoclassical model, labor input, investment, and output fall while consumption rises. Labor input, investment, and output increase only once productivity improves. In New Keynesian models each firm commits to supply output at a fixed

[1] Much more recently, Jaimovich and Rebelo (2009), p.1097, write that "the ability to generate comovement is a natural litmus test for macroeconomic models. It is a test that most models fail."

[2] Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and King, Plosser, and Rebelo (1988) are classic references on the RBC model.

[3] With a high elasticity of intertemporal substitution, the model predicts a rise in employment and investment and a *fall* in consumption. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up.

price, and therefore a rise in consumption exerts upward pressure on the demand for labor and investment. The response of the economy to a news shock depends on monetary policy. With optimal monetary policy the response is identical to the flexible-price neoclassical benchmark.[4]

This paper asks how a single friction, rational inattention, changes the propagation of a news shock in the neoclassical setting. Rational inattention is the idea that people cannot process all available information and they allocate attention optimally (Sims, 2003). In a rational inattention model, an agent chooses an optimal signal about the state of the economy, recognizing that a more informative signal requires more attention, which is costly. The agent takes actions based on the optimal signal, rather than based on perfect information or some exogenous incomplete information set. How does a news shock propagate when people have a limited ability to process information and can *choose* what information to attend to?

We consider a baseline RBC model. Neoclassical firms produce homogeneous output with capital and labor. There are no adjustment costs. Households have standard preferences for consumption and leisure. The perfect information equilibrium is familiar. We focus on the equilibrium when firms are subject to rational inattention and households have perfect information.[5]

We find that rational inattention induces an increase in the firms' demand for labor and investment in response to a news shock. This rational inattention effect helps the model produce comovement. The finding of *front-loaded* demand may be surprising, if one expects actions based on a history of noisy signals to be *delayed* relative to actions under perfect information. However, the optimal signal under rational inattention causes *a combination of delay in actions and forward-looking actions.*

Let us explain. The decision-maker in a firm takes repeated actions (chooses capital and labor *in every period*) and has memory (remembers past signals). Since the agent has limited attention, the optimal signal contains noise, which induces delay in actions. With repeated actions and memory, however, the optimal signal is not only about the current optimal action (or current productivity).

[4] With suboptimal monetary policy a standard New Keynesian model (Smets and Wouters, 2007) produces comovement after news about future productivity, but the impulse response of employment turns negative once productivity improves. The same is true in a heterogeneous agent version of the model (we thank Christian Wolf for this observation). For a review of the literature on news-driven business cycles, see Lorenzoni (2011), Beaudry and Portier (2014), and Jaimovich (2017).

[5] We add rational inattention on the side of households in a later section of the paper.

The signal is about the current *and future* optimal actions (or current *and future* productivity). Forward-looking information choice is beneficial because it lowers the prior uncertainty about the optimal action. It is desirable to enter well-informed into future periods. In an economy with news shocks, the optimal signal *confounds current productivity with expected future productivity.* Labor and investment demand react immediately to a news shock, *as if productivity has already changed with some probability.*

Consider an example. Suppose a firm uses one input, labor. A shock drawn by nature in period t affects productivity in period $t+1$. Productivity follows an ARMA(1,1) process $a_{t+1} = \rho a_t + \varepsilon_t$ (ρ lies between 0 and 1, and ε_t follows a Gaussian white noise process). Assume that the profit-maximizing labor input of the firm is proportional to productivity. The immediate response of labor input to a news shock is *zero.* There is no incentive to hire labor until productivity improves. If the firm is subject to rational inattention, its actual labor input deviates from the profit-maximizing labor input. The immediate response of the actual labor input to a news shock is *different from zero.* The reason is that the optimal signal under rational inattention is about current *and future* productivity. The innovation ε_t helps predict future productivity. Therefore, the optimal signal is on *a linear combination of* a_t *and* ε_t. A positive realization of the signal raises labor demand immediately. The firm *has chosen* to be imperfectly aware of the timing of productivity changes, even if in principle information about the timing of such changes is available. This intuition carries over to the case with two inputs, capital and labor.

Think of Lucas (1972). In that classic business cycle model with imperfect information, firms are assumed to observe a one-dimensional signal about nominal aggregate and relative demand. In the rational inattention RBC model with news shocks, firms choose to observe a one-dimensional signal about current and future productivity. Focusing on a single linear combination of the elements of the state vector, $(a_t, \varepsilon_t)'$ in the ARMA(1,1) example, saves on attention.[6]

With standard preferences (we focus on log utility from consumption and linear disutility from work), a news shock causes the usual wealth effect. Households want to consume more (save less) and work less after a positive news shock. The wealth effect exerts downward pressure on hours

[6] It seems plausible that in a dynamic world, in which in principle many variables affect optimal actions, people find attractive a low-dimensional summary statistic that combines information about the current and expected future state of the economy.

worked and investment. In general equilibrium, is the rational inattention effect on labor and investment demand weaker or stronger than the wealth effect on labor and saving supply? Does output contract or expand?

We find that the rational inattention effect on labor demand *more than offsets* the wealth effect on labor supply. Hours worked *rise* in response to a news shock. The rational inattention effect on investment demand *approximately offsets* the wealth effect on saving supply. Investment is close to zero in response to a news shock. The effect of rational inattention on investment is weakened in general equilibrium because the cost of capital increases as firms invest more. The impulse response of output before productivity improves is pinned down by the impulse responses of labor and capital.[7] The rational inattention effect on input demand is strong enough to turn an output contraction from the baseline model into an output *expansion*: Output rises in response to a news shock.

Hence, the single assumption of rational inattention by firms makes the model predict an output expansion after news that productivity will improve. By maintaining that households have perfect information, we stack the deck against us because in this case the wealth effect that reduces labor supply and saving supply is fully operating. We also solve a version of the model with rational inattention by firms *and* rational inattention by households. We find that comovement strengthens.

In addition, we ask if rational inattention improves the propagation of a standard productivity shock (a shock that affects productivity in the same period in which the shock is drawn). It has been a challenge for the RBC model to reproduce the persistence in the data. The first-order autocorrelation of employment, investment, and output growth is positive in the data but negative in the baseline model.[8] In the perfect information equilibrium, the impulse responses of employment, investment, and output to a productivity shock peak on impact and then decline monotonically. They inherit the shape of the impulse response of exogenous productivity. We find that when firms are subject to rational inattention, these impulse responses become hump-shaped. Since the optimal signal contains noise, the firms' beliefs are anchored on the steady state and evolve slowly. As a result, employment, investment, and output respond with delay to a productivity shock. The first-order autocorrelation of employment, investment, and output growth

[7] Output also depends on productivity but the short-run impulse response of productivity to a news shock is zero.

[8] This shortcoming of the RBC model was first noted by Cogley and Nason (1995) and Rotemberg and Woodford (1996).

in the model become positive and are approximately in line with the data. This finding holds true even though rational inattention is the only source of inertia and the marginal cost of attention is small.

The literature has explored a number of ways to obtain a model that predicts comovement in response to news about future productivity. Jaimovich and Rebelo (2009) modify the baseline RBC model by adding investment adjustment costs, variable capital utilization, and a new class of preferences. Investment adjustment costs and variable capital utilization produce an increase in input demand in response to a news shock, whereas the new preferences control the wealth effect on input supply. Beaudry and Portier (2004, 2007) move to a multi-sector neoclassical setting. They introduce a cost complementarity so that higher output in one sector makes production more efficient in other sectors, leading to a rise in input demand. Another approach has been to combine nominal stickiness with suboptimal monetary policy. Lorenzoni (2009) analyzes a New Keynesian economy with a Taylor rule where noise in a public signal about productivity causes comovement of employment and output.[9] By contrast, we explore how a *single* new assumption, rational inattention, changes the propagation of a news shock in the baseline RBC model. The assumption of rational inattention seems well suited to apply to the question if people have an incentive to be perfectly aware of the timing of productivity changes.[10]

Turning to standard productivity shocks, the literature has pursued the idea that moving away from perfect information rational expectations can improve the propagation mechanism relative to the baseline RBC model. Eusepi and Preston (2011) abandon rational expectations altogether, replacing it by adaptive learning. They find that the first-order autocorrelation of employment, investment, and output growth in the model become positive. We add rational inattention, a form of imperfect information rational expectations, to the baseline RBC model. Surprisingly, the single assumption of rational inattention turns out to be sufficient to bring the first-order autocorrelation of employment, investment, and output growth in the model approximately into line with the data. Business cycle models face the challenge of matching the persistence in the macro data more

[9] Angeletos and La'O (2010) study a neoclassical model with strategic complementarity and dispersed information in which a similar noise shock makes employment and output comove.

[10] There is a large empirical literature on news and noise shocks: Beaudry and Portier (2006), Barsky and Sims, (2012), Schmitt-Grohé and Uribe (2012), Blanchard, L'Huillier, and Lorenzoni (2013), Görtz and Tsoukalas (2017), Chahrour and Jurado (2018), and others.

generally, not only conditional on a productivity shock.[11] Our finding may therefore be helpful also for model builders who allow for sources of fluctuations other than productivity.

Solving a dynamic stochastic general equilibrium model with rational inattention is challenging. One needs to solve attention problems (signal choice problems) of individual agents *in a dynamic model*. Furthermore, one needs to find a fixed point of an economy in which the optimal signal of an agent depends on the signals chosen by other agents. Sims (2003), Maćkowiak and Wiederholt (2009), Sims (2010), Maćkowiak, Matějka, and Wiederholt (2018), Miao, Wu, and Young (2020), and Afrouzi and Yang (2020) make progress solving attention problems of individual agents in a dynamic environment with Gaussian shocks. Maćkowiak and Wiederholt (2015) solve a DSGE model with rational inattention where the physical environment is similar to a simple New Keynesian model (for example, there is no capital). By contrast, here the physical environment is a neoclassical business cycle model. We use a guess and verify method to find the fixed point, at each iteration employing the results of Maćkowiak, Matějka, and Wiederholt (2018) to solve agents' attention problems. One issue in the literature on rational inattention is how to define equilibrium. We assume that prices, which all agents take as given, adjust to guarantee market clearing.[12]

The next section defines the physical environment. Section 3 introduces rational inattention. Section 4 develops intuition for the effects of rational inattention, by considering special cases of the model. Section 5 shows the effects of productivity shocks and news about future productivity in the complete model. Section 6 studies a version of the model in which all agents, firms and households, are subject to rational inattention. Section 7 concludes and outlines further research.

2 Model – physical environment

We consider a baseline RBC model that allows for an additional factor of production ("an entrepreneurial input") in fixed supply. The production function is Cobb-Douglas and exhibits decreasing returns to scale in the variable factors, capital and labor. We introduce a third factor in fixed

[11] See Sims (1998) for a general discussion, Christiano, Eichenbaum, and Evans (2005), and Smets and Wouters (2007) in the context of New Keynesian models, and Auclert, Rognlie, and Straub (2020) in the context of a Heterogeneous Agent New Keynesian model.

[12] In Maćkowiak and Wiederholt (2015), in each market one side of the market sets the price and the other side of the market chooses the quantity.

supply because to formulate the attention problem of a firm we need the firm's choice of capital and labor under perfect information, not only the capital-labor ratio, to be determinate.

Time is discrete. There is a continuum of firms indexed by $i \in [0,1]$. All firms produce the same good using an identical technology represented by the production function

$$Y_{it} = e^{a_t} K_{it-1}^{\alpha} L_{it}^{\phi} N_i^{1-\alpha-\phi}$$

where Y_{it} is output of firm i in period t, K_{it-1} is capital input, L_{it} is labor input, and e^{a_t} is total factor productivity, common to all firms. N_i is an entrepreneurial input, specific to firm i, in fixed supply. The parameters α and ϕ satisfy $\alpha \geq 0$, $\phi \geq 0$, and $\alpha + \phi < 1$.

The capital stock of firm i evolves according to the law of motion

$$K_{it} - K_{it-1} = I_{it} - \delta K_{it-1}$$

where $\delta \in (0,1]$ is the depreciation rate. The firm maximizes the expected discounted sum of profits or dividends. The profit of firm i in period t, D_{it}, is given by

$$D_{it} = Y_{it} - W_t L_{it} - I_{it}$$

where W_t is the wage rate. The profits of all firms flow to a mutual fund. Households own and trade shares in the mutual fund.[13]

Total factor productivity is determined according to the law of motion

$$a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h}$$

where ε_t follows a Gaussian white noise process with unit variance, $\rho \in (0,1)$, $\sigma > 0$, and $h \geq 0$. A shock drawn by nature in period t affects productivity in period $t+h$. We solve the model either with $h = 0$ (a standard productivity shock) or with $h \geq 1$ (a news shock). For ease of exposition we abstract from long-run growth.

There is a continuum of households indexed by $j \in [0,1]$. Each household j maximizes the expected discounted sum of utility. The discount factor is $\beta \in (0,1)$. The utility function is

$$U(C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{jt}^{1+\eta}}{1+\eta}$$

[13] When firm i was sold to the mutual fund, the entrepreneurial input was paid the present value of its future marginal products and in return committed to supply its service without additional payments.

where C_{jt} is consumption by household j in period t, L_{jt} is hours worked, $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution, and $\eta \geq 0$ is the inverse of the Frisch elasticity of labor supply. Typically, we will set $\gamma = 1$ and $\eta = 0$. The budget constraint in period t is

$$V_t Q_{jt} - V_t Q_{jt-1} = W_t L_{jt} + D_t Q_{jt-1} - C_{jt}$$

where V_t is the price of a share in the mutual fund in period t, Q_{jt} is household j's share in the mutual fund, and $D_t \equiv \int_0^1 D_{it} di$ is the dividend from the mutual fund.

Aggregate output is $Y_t \equiv \int_0^1 Y_{it} di$. Aggregate capital and investment are defined analogously. Aggregate consumption is $C_t \equiv \int_0^1 C_{jt} dj$.

In equilibrium in every period the wage adjusts so that labor demand equals labor supply, $\int_0^1 L_{it} di = \int_0^1 L_{jt} dj \equiv L_t$, and the price of a share in the mutual fund adjusts so that asset demand equals asset supply normalized to one, $\int_0^1 Q_{jt} dj = Q \equiv 1$.

The non-stochastic steady state of this economy is described in Appendix A. To solve the model when firms and households have perfect information, we log-linearize the first-order conditions of firms and households and the other equilibrium conditions at the non-stochastic steady state. This yields the completely standard log-linear equilibrium conditions stated in Appendix B. We refer to the solution as the perfect information equilibrium.

3 Model – rational inattention by firms

Under rational inattention the decision-maker in firm i chooses a signal about the state of the economy.[14] The decision-maker maximizes the expected discounted sum of profits, recognizing that a more informative signal requires more attention, which is costly. This section begins by deriving the decision-maker's objective. We then state the decision-maker's attention problem. Finally, we define the equilibrium in the economy in which firms are subject to rational inattention and households have perfect information.

3.1 Loss in profit from suboptimal actions

We derive an expression for the expected discounted sum of losses in profit when actions of firm i deviate from the profit-maximizing actions – the actions the firm would take if it had perfect

[14] The optimal signal may follow a multivariate stochastic process.

information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of profits at the non-stochastic steady state.

Recall that the profit of firm i in period t is given by $Y_{it} - W_t L_{it} + (1-\delta) K_{it-1} - K_{it}$. We assume that the mutual fund instructs each firm to value profits according to the marginal utility of consumption.[15] The profit function can be written in terms of log-deviations from the non-stochastic steady state:

$$C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta} \right) \left[(1-\delta) e^{k_{it-1}} - e^{k_{it}} \right] \right\}$$

where an upper-case letter without a time subscript denotes the value of a variable in the non-stochastic steady state, and a lower-case letter denotes the log-deviation of a variable from its value in the non-stochastic steady state. The term $C^{-\gamma} e^{-\gamma c_t}$ is the marginal utility of consumption.

Taking the quadratic approximation to the expected discounted sum of profits, we obtain the following expression for the expected discounted sum of losses in profit from suboptimal actions:

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta_0 (x_t - x_t^*) + (x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right) \right] \tag{1}$$

where $x_t \equiv (k_{it}, l_{it})'$, $x_t^* \equiv (k_{it}^*, l_{it}^*)'$, the matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha (1-\alpha) & 0 \\ 0 & \phi (1-\phi) \end{bmatrix}$$

$$\Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}$$

and the stochastic process x_t^* satisfies the equations

$$E_t a_{t+1} - (1-\alpha) k_{it}^* + \phi E_t l_{it+1}^* = \frac{\gamma E_t (c_{t+1} - c_t)}{1 - \beta (1-\delta)} \tag{2}$$

$$a_t + \alpha k_{it-1}^* - (1-\phi) l_{it}^* = w_t \tag{3}$$

and the initial condition $k_{i,-1}^* = k_{i,-1}$. See Appendix C.

The vector x_t^* is *the profit-maximizing* input choice when the decision-maker in the firm has perfect information in every period. Equations (2)-(3) are the usual optimality conditions for capital and labor, where E_t denotes the expectation conditional on the entire history up to and including

[15] All households have the same consumption level so long as households have perfect information.

period t. Equation (2) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital. Equation (3) states that the profit-maximizing labor input equates the marginal product of labor to the wage. The vector x_t is *an alternative* input choice. Expression (1) gives the expected discounted sum of losses in profit when the stochastic process for the firm's actions, x_t, differs – for whatever reason – from the stochastic process for the profit-maximizing actions, x_t^*. After the quadratic approximation this loss is quadratic in $x_t - x_t^*$. The interaction term $(x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^*\right)$ appears because bringing too much capital into a period raises the optimal labor input in that period.[16]

Maćkowiak, Matějka, and Wiederholt (2018) derive analytical results for attention problems in a dynamic environment with Gaussian shocks. We now rewrite objective (1) so that it matches the objective in that paper. This requires only that we redefine x_t and x_t^*.

We show in Appendix C that expression (1) is equivalent to

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta (x_t - x_t^*) \right] \tag{4}$$

where $x_t \equiv (k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})'$, $x_t^* \equiv (k_{it}^*, l_{it}^* - \frac{\alpha}{1-\phi} k_{it-1}^*)'$, the matrix Θ is given by

$$\Theta = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha \left(1 - \alpha - \frac{\alpha\phi}{1-\phi}\right) & 0 \\ 0 & \phi(1-\phi) \end{bmatrix}$$

and the stochastic process x_t^* satisfies

$$x_t^* = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t a_{t+1} - \phi E_t w_{t+1} - (1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \right] \\ \frac{1}{1-\phi} (a_t - w_t) \end{pmatrix}. \tag{5}$$

The first entry of the new vector x_t^* is the profit-maximizing capital stock to be carried into period $t+1$, k_{it}^*. The profit-maximizing capital stock, k_{it}^*, is proportional to the difference between expected productivity and a weighted average of expected factor prices.[17] The second entry of the new vector x_t^* is the profit-maximizing labor input for a given capital stock, $l_{it}^* - [\alpha/(1-\phi)] k_{it-1}^*$. The profit-maximizing labor input for a given capital stock, $l_{it}^* - [\alpha/(1-\phi)] k_{it-1}^*$, is proportional to the difference between productivity and the wage. The advantage of rewriting equations (2)-(3)

[16] Objective (1) is written so that the firm wants to *maximize* it.

[17] By expected factor prices we mean the expected wage and the expected consumption growth rate. The latter pins down the cost of capital.

as equation (5) is that the right-hand side of equation (5) depends only on variables exogenous to the firm. Moreover, expression (1) collapses to expression (4) once it is written in terms of the new vectors x_t and x_t^*.

It follows from expression (4) that the best response of firm i in period t given any information set $\mathcal{I}_{it}$ is the conditional expectation of x_t^*, $x_t = E\left(x_t^* | \mathcal{I}_{it}\right)$.

3.2 The attention problem of a firm

In period $t = -1$, the decision-maker in firm i chooses the stochastic process for the signal to maximize the expected discounted sum of profits, (4), net of the cost of attention. We assume that the marginal cost of attention per period is constant, $\lambda > 0$. In every period $t = 0, 1, 2, ...$, the decision-maker observes a realization of the optimal signal and takes actions – chooses capital and labor.

The statement of the attention problem can be simplified, without loss of generality, based on Maćkowiak, Matějka, and Wiederholt (2018). Let x_{1t}^* denote the first element and x_{2t}^* the second element of x_t^*, $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - \left[\alpha / \left(1 - \phi\right)\right] k_{it-1}^*$. Suppose that x_{1t}^* and x_{2t}^* each follows a finite-order ARMA process. The vector x_t^* has a first-order VAR representation

$$\xi_{t+1} = F \xi_t + v_{t+1}$$

where v_t is a Gaussian vector white noise process, F is a square matrix, and ξ_t is a vector containing x_{1t}^* and x_{2t}^* and, if appropriate, lags of x_{1t}^* and x_{2t}^* and current and lagged ε_t. The state vector ξ_t contains all information available in period t about the current and future profit-maximizing actions. The analytical results of Maćkowiak, Matějka, and Wiederholt (2018) imply that *the optimal signal is a signal about the state vector* ξ_t. Furthermore, the optimal signal is at most two-dimensional.

The decision-maker in firm i solves:

$$\max_{G, \Sigma_\psi, \kappa} \left\{ \sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^*\right)' \Theta \left(x_t - x_t^*\right) \right] - \left(\frac{\beta}{1 - \beta} \right) \lambda \kappa \right\} \tag{6}$$

subject to

$$\xi_{t+1} = F \xi_t + v_{t+1} \tag{7}$$

$$x_t = E\left(x_t^* | \mathcal{I}_{it}\right) \tag{8}$$

$$\mathcal{I}_{it} = \mathcal{I}_{i,-1} \cup \{S_{i0}, \ldots, S_{it}\} \tag{9}$$

$$S_{it} = G'\xi_t + \psi_{it} \tag{10}$$

and

$$H\left(\xi_t | \mathcal{I}_{it-1}\right) - H\left(\xi_t | \mathcal{I}_{it}\right) \leq \kappa. \tag{11}$$

Expression (6) states that the decision-maker maximizes the expected discounted sum of profits net of the cost of attention. He or she takes as given the law of motion for the profit-maximizing actions (equation (7)). The agent's actual actions are equal to the conditional expectation of the profit-maximizing actions given the period t information set (equation (8)). The period t information set $\mathcal{I}_{it}$ consists of the sequence of signal realizations $S_{i0}, \ldots, S_{it}$ and initial information $\mathcal{I}_{i,-1}$ (equation (9)). The optimal signal is a signal about the state vector ξ_t (equation (10)). The noise in the signal ψ_{it} follows a Gaussian vector white noise process with variance-covariance matrix Σ_ψ. The decision-maker chooses the signal weights G and the variance-covariance matrix of the noise Σ_ψ. The noise ψ_{it} is assumed to be independently distributed across firms.[18] The agent faces the information flow constraint (11). The left-hand side is the difference between prior uncertainty and posterior uncertainty about the state vector ξ_t in any period t. $H\left(\xi_t | \mathcal{I}_{i\tau}\right)$ denotes the entropy of ξ_t conditional on $\mathcal{I}_{i\tau}$, $\tau = t-1, t$. $H\left(\xi_t | \mathcal{I}_{it-1}\right)$ is the prior uncertainty, before receiving the period t signal, and $H\left(\xi_t | \mathcal{I}_{it}\right)$ is the posterior uncertainty. The information flow constraint is binding at an optimum. A choice of G and Σ_ψ implies a choice of the amount of attention κ.

Both the expected discounted sum of profits and the cost of attention in expression (6) depend on conditional second moments. The conditional second moments can in principle vary over time because the decision-maker conditions on more signal realizations as time passes. To abstract from transitional dynamics in the conditional second moments, we assume that after the agent has chosen the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. The conditional second moments can then be computed using the steady-state Kalman filter, with state equation (7) and observation equation (10), and problem (6)-(11) can be solved numerically in a straightforward way.[19]

[18] Woodford (2003) and Maćkowiak and Wiederholt (2015) make the same assumption. This assumption implies that information is dispersed: In every period, each firm i has a different conditional expectation $E\left(x_t^* | \mathcal{I}_{it}\right)$.

[19] Maćkowiak, Matějka, and Wiederholt (2018) make the same assumption. Woodford (2003) also uses the steady-state Kalman filter to compute conditional second moments in a model in which agents observe exogenously given

3.3 Equilibrium

We focus on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. For simplicity, until Section 6 we refer to this equilibrium as the rational inattention equilibrium.[20]

The rational inattention equilibrium can be defined as follows. In every period $t = 0, 1, 2, ...,$ the wage w_t adjusts so that labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj = l_t$, and the price of a mutual fund share v_t adjusts so that asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$.

Each firm solves problem (6)-(11). The choice of G and Σ_ψ by firm i together with equations (8)-(10) yield the input choices of the firm, k_{it} and l_{it}. Firm-level output, investment, and profit satisfy $y_{it} = a_t + \alpha k_{it-1} + \phi l_{it}$, $\delta i_{it} = k_{it} - (1 - \delta) k_{it-1}$, and $(D/Y) d_{it} = y_{it} - (WL/Y)(w_t + l_{it}) - (I/Y) i_{it}$.[21] Aggregate variables satisfy $y_t = \int_0^1 y_{it} di$, $k_t = \int_0^1 k_{it} di$, $i_t = \int_0^1 i_{it} di$, and $d_t = \int_0^1 d_{it} di$.[22]

Households who have perfect information satisfy the usual first-order conditions

$$\beta E_t v_{t+1} - v_t + (1 - \beta) E_t d_{t+1} = \gamma E_t (c_{t+1} - c_t) \tag{12}$$

and

$$w_t - \gamma c_t = \eta l_t. \tag{13}$$

Since households are identical, $c_{jt} = c_t$ and $l_{jt} = l_t$ for each j.

Finally, the resource constraint

$$y_t = (C/Y) c_t + (I/Y) i_t \tag{14}$$

holds. The relevant steady-state ratios appear in Appendix A. To obtain the resource constraint, we log-linearize the period budget constraint of household j and we aggregate, imposing $\int_0^1 q_{jt} dj = 0$, to obtain

$$c_t = (WL/C)(w_t + l_t) + \left(\beta^{-1} - 1\right)(V/C) d_t.$$

We combine this equation with the equation for d_t from the firms' side of the model.

signals.

[20] In Section 6 we add rational inattention on the side of households.

[21] These equations follow from log-linearization of the production function, the law of motion of capital, and the definition of profit. See Section 2. All relevant steady-state ratios appear in Appendix A.

[22] These equations follow from log-linearization of the definitions of the aggregate variables. See Section 2.

4 Developing intuition

How does rational inattention affect the propagation of productivity shocks and news about future productivity? To develop intuition, this section studies special cases of the model. In the first special case, labor is the only variable input. In the second special case, capital is the only variable input. Section 5 analyzes the rational inattention equilibrium of the complete model.

4.1 The case with labor only

Suppose that labor is the only variable input, $\alpha = 0$. The attention problem of a firm simplifies. The firm's action (labor input choice) is one-dimensional with $x_t = l_{it}$, $x_t^* = l_{it}^*$, and $\Theta = -C^{-\gamma} Y \phi (1 - \phi)$. Households live hand-to-mouth because there is no capital and all households are identical. Consumption is equal to output, $c_t = y_t$. Labor supply is governed by equation (13).

The perfect information equilibrium can be solved for analytically. Labor input and the wage are proportional to productivity:

$$l_t = \left(\frac{1 - \gamma}{1 - \phi + \gamma \phi + \eta} \right) a_t \tag{15}$$

and

$$w_t = \left(\frac{\gamma + \eta}{1 - \phi + \gamma \phi + \eta} \right) a_t.$$

Moreover, $y_t = a_t + \phi l_t$ and $c_t = y_t$. The endogenous variables inherit the stochastic properties of exogenous productivity. In particular, the growth rate of employment is *negatively* autocorrelated. Furthermore, the impulse responses of all variables to a news shock are *zero* until productivity changes. Firms have no incentive to change labor demand until productivity changes. Similarly, households have no incentive to change labor supply in this special case of the model. The wealth effect on labor supply vanishes because hand-to-mouth households cannot vary saving and consumption in response to a news shock.

Consider the rational inattention equilibrium. Finding a fixed point of an economy in which agents are subject to rational inattention can be challenging. In a rational expectations equilibrium, actions of an agent generally depend on the actions of other agents. In a rational expectations equilibrium with rational inattention, in addition, the optimal signal of an agent depends on the signals chosen by other agents. To find the fixed point, we guess that in equilibrium the profit-maximizing labor input l_{it}^* follows a finite-order ARMA process. We solve the attention problem of firm i, and we compute the firm's labor input from the equation $l_{it} = E\left(l_{it}^* | \mathcal{I}_{it}\right)$ and the solution of the attention problem. We verify the guess from the optimality condition $l_{it}^* = \left[1/\left(1-\phi\right)\right]\left(a_t - w_t\right)$, where the wage depends on the signal and labor demand choices of all firms as well as the labor supply decisions of households. We can compute the market-clearing wage from the equilibrium condition $w_t = \gamma c_t + \eta l_t = \gamma(a_t + \phi l_t) + \eta l_t$, where $l_t = \int_0^1 l_{jt} dj = \int_0^1 l_{it} di$.[23]

One period in the model equals one quarter. As an example, we assume $\gamma = 0.5$, $\eta = 0$, $\phi = 0.6$, $\beta = 0.99$, $\rho = 0.9$, and $\sigma = 0.01$. The amount of inattention in the model depends on the marginal cost of attention, λ. Here we set $\lambda = (4/100,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is to $4/100,000$ of steady-state output.[24]

The upper-left panel in Figure 1 shows the impulse response of aggregate labor input l_t to a productivity shock ($h = 0$).[25] In the perfect information equilibrium, the impulse response peaks on impact and then declines monotonically (line with points). The impulse response is hump-shaped in the rational inattention equilibrium (line with circles).

To develop intuition, assume that a measure zero of firms are subject to rational inattention. Other firms have perfect information, implying that the profit-maximizing labor input l_{it}^* is equal to labor input in the perfect information equilibrium and thus proportional to productivity (equation (15)). Productivity follows an AR(1) process, $a_t = \rho a_{t-1} + \sigma \varepsilon_t$. Hence, the optimal action l_{it}^* follows an AR(1) process. Since the optimal action follows an AR(1) process, the optimal signal is $S_{it} = l_{it}^* + \psi_{it}$ where ψ_{it} follows a Gaussian white noise process with variance σ_ψ^2.[26] The action in any period is a weighted sum of the prior and the signal in that period, $l_{it} = E\left(l_{it}^* | \mathcal{I}_{it}\right) = (1-\omega)\, E\left(l_{it}^* | \mathcal{I}_{it-1}\right) + \omega S_{it}$. Rational inattention ($\lambda > 0$) implies that the signal is noisy ($\sigma_\psi^2 > 0$). Therefore, the firm puts weight on the prior ($0 < \omega < 1$), implying that the action l_{it} is delayed

[23] We verify that l_{it}^* follows the finite-order ARMA process that we guessed, up to a very small numerical error. We proceed analogously in every rational inattention fixed point solution below.

[24] $C^{-\gamma}Y$ is the marginal utility of consumption multiplied by output, in the steady state. When we divide the firm's marginal cost of attention by $C^{-\gamma}Y$, we express it as a share of steady-state output. In Section 5 we report what different values of λ imply for the model's fit to survey data on expectations.

[25] Throughout the paper an impulse response of 1 is a 1 percent deviation from the steady state.

[26] See Maćkowiak, Matějka, and Wiederholt (2018), Proposition 3. We normalize to one the coefficient on l_{it}^* in the optimal signal.

relative to – peaks later than – the optimal action l_{it}^*. It turns out that this mechanism can explain the first-order autocorrelation of employment growth in the data, even though rational inattention is the only source of inertia and the marginal cost of attention is small. See Section 5.1.

In general equilibrium where all firms are subject to rational inattention, the argument must be modified slightly. In the rational inattention equilibrium, the profit-maximizing labor input l_{it}^* no longer follows an AR(1) process (l_{it}^* depends on the wage which changes when all firms are rationally inattentive) and hence the optimal signal changes. Firms still put weight on their priors, and their actions l_{it} are delayed relative to their optimal actions l_{it}^*. Aggregate labor input $l_t = \int_0^1 l_{it} di$ peaks later than in the perfect information equilibrium. See the upper-left panel in Figure 1.

The upper-right panel in Figure 1 shows the impulse response of l_t to a news shock ($h = 6$, as an example). The shock is drawn in period 0 while productivity changes in period $h = 6$. In the perfect information equilibrium, the impulse response of employment is zero until productivity changes in period 6 (line with points). In the rational inattention equilibrium, hours worked *rise in the period in which the news arrives*, period 0, and keep rising thereafter (line with circles). *Rational inattention induces an increase in labor demand in response to a news shock.* As a result, labor input rises in equilibrium.

To gain intuition, consider the same partial equilibrium argument. Assume that a measure zero of firms are subject to rational inattention. Other firms have perfect information, implying that the profit-maximizing labor input l_{it}^* is equal to labor input in the perfect information equilibrium and thus proportional to productivity (equation (15)). For simplicity set $h = 1$ so that productivity follows an ARMA(1,1) process, $a_{t+1} = \rho a_t + \sigma \varepsilon_t$. Hence, the optimal action l_{it}^* follows an ARMA(1,1) process. Since the optimal action follows an ARMA(1,1) process, the state vector is $\xi_t = (l_{it}^*, \varepsilon_t)'$ and the optimal signal is $S_{it} = l_{it}^* + g\varepsilon_t + \psi_{it}$ *with* $g \neq 0$.[27] The signal is not only about the current optimal action (or current productivity). The signal is about the current *and future* optimal actions (or current *and future* productivity). The innovation ε_t enters the signal because it helps predict future optimal actions. A one-dimensional signal on the state vector $\xi_t = (l_{it}^*, \varepsilon_t)'$ confounds current productivity with expected future productivity. Therefore, the action $l_{it} = E(l_{it}^* | \mathcal{I}_{it})$ is *front-loaded* relative to the optimal action l_{it}^*. The response of l_{it}^* on

[27] See Maćkowiak, Matějka, and Wiederholt (2018), Proposition 5. With the parameter values assumed here $g = 0.0055$ and $\sigma_\psi = 0.0195$.

impact of a news shock is *zero.* The response of l_{it} is *different from zero.* Labor demand reacts immediately *as if productivity has already changed with some probability.* Firms *have chosen* to be imperfectly aware of the timing of productivity changes, even if in principle information about the timing of such changes is available.

With $h = 6$ rather than $h = 1$ and in general equilibrium, the argument must be modified slightly. With $h = 6$ productivity follows an ARMA(1,6) process rather than an ARMA(1,1). In addition, when all firms are subject to rational inattention, the wage is no longer proportional to productivity. Hence, the profit-maximizing labor input l_{it}^{*} follows a more complicated process than an ARMA(1,6) and the optimal signal changes. The optimal signal still confounds current productivity with expected future productivity, implying that aggregate labor input l_t changes on impact of a news shock. See the upper-right panel in Figure 1.

Why is the optimal signal a one-dimensional signal about the state vector? The agent takes repeated actions (chooses labor *in every period*) and has memory (remembers past signals). All relevant information is available, but the agent with limited attention seeks to simplify and summarize the available information. Forward-looking information choice is beneficial because it lowers the prior uncertainty about the optimal action. It is desirable to enter well-informed into future periods. When the optimal action follows an AR(1) process, learning about the present and learning about the future are the same thing. Outside of this special case, learning about the future calls for including in the signal all elements of the state vector beyond the current optimal action. These elements of the state vector help predict future optimal actions. In addition, focusing on a single linear combination of the elements of the state vector saves on attention. Observing a one-dimensional summary statistic is more efficient than observing separate signals about all elements of the state vector. When productivity is driven by news shocks, choosing a one-dimensional signal about the state vector implies being imperfectly aware of the timing of productivity changes.[28]

4.2 The case with capital only

Suppose that capital is the only variable input, $\phi = 0$. The attention problem of a firm is analogous to Section 4.1. The firm's action (capital input choice) is one-dimensional with $x_t = k_{it}$, $x_t^{*} = k_{it}^{*} = \frac{1}{1-\alpha}\left[E_t a_{t+1} - \frac{\gamma E_t(c_{t+1}-c_t)}{1-\beta(1-\delta)}\right]$, and $\Theta = -C^{-\gamma} Y \beta \alpha (1-\alpha)$. In the absence of labor income

[28] See also the proof and discussion of Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018).

consumption is equal to the dividend process, $c_t = d_t$. Saving and consumption behavior is governed by equation (12).

Assume log utility from consumption, $\gamma = 1$, and full capital depreciation, $\delta = 1$. The perfect information equilibrium can be solved for analytically: $k_t = \alpha k_{t-1} + a_t$, $k_t = i_t = y_t = c_t = d_t = v_t$. Thus, in this special case the model can produce some positive autocorrelation in the growth rates of investment and output. However, the impulse responses of all variables to a news shock are *zero* until productivity changes. With $\gamma = 1$ and $\delta = 1$, there is no incentive to change consumption or investment until productivity changes.

Consider the rational inattention equilibrium. To find the fixed point, we guess that in equilibrium the profit-maximizing capital input k_{it}^* follows a finite-order ARMA process. We solve the attention problem of firm i, and we compute the firm's capital input from the equation $k_{it} = E\left(k_{it}^* | \mathcal{I}_{it}\right)$ and the solution of the attention problem. Aggregating across firms yields k_t, i_t, y_t, and d_t, while the budget constraint implies that $c_t = d_t$. We verify the guess from the optimality condition $k_{it}^* = \frac{1}{1-\alpha}\left[E_t a_{t+1} - E_t\left(c_{t+1} - c_t\right)\right]$, where the expected consumption growth rate depends on the signal and investment demand choices of all firms as well as on the saving decisions of households. The market-clearing mutual fund share price v_t can be calculated from equation (12) and the solution for c_t. As an example, we assume $\gamma = 1$, $\alpha = 0.33$, $\beta = 0.99$, $\delta = 1$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (8/100,000)C^{-\gamma}Y$.

The lower-left panel in Figure 1 displays the impulse response of aggregate investment i_t to a productivity shock ($h = 0$). The impulse response is hump-shaped (line with circles). The model yields more first-order autocorrelation in the growth rate of investment compared with the perfect information equilibrium (line with points).

The lower-right panel in Figure 1 shows the impulse response of i_t to a news shock ($h = 6$). In the perfect information equilibrium, the impulse response of investment is zero until productivity changes in period 6 (line with points). In the rational inattention equilibrium, investment *rises in the period in which the news arrives*, period 0, and keeps rising thereafter (line with circles). *Rational inattention induces an increase in investment demand in response to a news shock.* As a result, investment rises in equilibrium.

Since the attention problem of a firm is analogous to Section 4.1, the intuition for what happens to investment demand is the same as the intuition given there. In particular, forward-looking information choice leads investment demand to react immediately to a news shock, as if productivity has already changed with some probability.

In the case of news shocks ($h \geq 1$), rational inattention makes very different predictions than the alternative model in which firms receives a signal of the form "the optimal action plus i.i.d. noise."[29] For simplicity, suppose that a measure zero of firms are subject to rational inattention (other firms have perfect information). Since the optimal signal confounds current productivity with expected future productivity, the rationally inattentive firms change their investment on impact of a news shock. In the alternative model, a measure zero of firms choose capital input based on a history of signals of the form "the optimal action plus i.i.d. noise." The firms solve the same attention problem *subject to the restriction* that the signal is of the form "the optimal action plus i.i.d. noise." While a signal of this form provides only a noisy estimate of current productivity, it does *not* confound current productivity with expected future productivity. Therefore, the firms change their investment only when productivity changes. They record a larger profit loss from suboptimal actions, even though they pay more attention to the state of the economy compared with the rationally inattentive firms. The rationally inattentive firms *benefit from being imperfectly aware of the timing of productivity changes*.[30]

With $h = 0$ actions based on the optimal signal are also different from actions based on the restricted signal, except when the optimal action follows an AR(1) process. How much difference there is depends on the details of the model. In this model the difference turns out to be modest.[31]

Let us summarize Section 4. In a dynamic environment rational inattention induces a combination of delay in actions and forward-looking actions. As a result, the impulse responses to productivity shocks and news about future productivity change significantly. Employment and investment react with delay to a productivity shock. They *rise* in response to news that productivity will improve.

[29] Or "current productivity plus i.i.d. noise," or "the news shock plus i.i.d. noise."

[30] For the parameter values used here with $h = 6$, the expected profit loss from suboptimal capital input in the alternative model is 27 percent greater and κ is 39 percent higher compared with the rationally inattentive firms.

[31] Consider the partial equilibrium analysis with $h = 0$ and the same parameter values. The profit-maximizing capital input follows an AR(2) process. The growth rate of investment of rationally inattentive firms has a serial correlation of 0.67, whereas with the restricted signal the serial correlation rises to 0.72.

5 Predictions of the model

What does rational inattention imply about the business cycle effects of productivity shocks and news about future productivity? We return to the complete model with variable capital and labor, $\alpha > 0$ and $\phi > 0$.

Throughout this section we set $\gamma = 1$, $\eta = 0$, $\alpha = 0.33$, $\phi = 0.65$, $\beta = 0.99$, $\delta = 0.025$, $\rho = 0.9$, and $\sigma = 0.01$. Thus, we assume log utility from consumption and linear disutility from work, $\alpha + \phi$ close to 1, a depreciation rate of 2.5 percent per quarter, and a persistent productivity process with an innovation of 1 percent.[32]

5.1 The effects of productivity shocks

Let $h = 0$. Consider the perfect information equilibrium. Figure 2 shows the impulse responses to a productivity shock (lines with points). Aggregate labor input, investment, output, and consumption move in the same direction, consistent with a business cycle. The impulse responses of labor input, investment, and output peak on impact and then decline monotonically. They inherit the shape of the impulse response of exogenous productivity.[33]

Following common practice, we compare unconditional second moments in the model and in the data. Table 1 reports selected unconditional moments for the model (column "Perfect information") and for the quarterly post-war data from the United States.[34] The comparison is familiar. Let us focus on the persistence of growth rates. The first-order autocorrelation of employment, investment, and output growth are positive in the data but negative in the model. In the model these variables inherit the autocorrelation of exogenous productivity growth.[35]

[32] Below we state the value of the marginal cost of attention λ. It is interesting to note that only the ratio σ^2/λ matters for the equilibrium impulse responses because the first term in objective (6) is linear in σ^2 and the second term in objective (6) is linear in λ.

[33] Figure 2 contains a panel with the impulse response of the conditional expectation of productivity by firms. In the perfect information equilibrium, this impulse response equals the impulse response of productivity.

[34] The unconditional moments of the data come from Eusepi and Preston (2011), Table 2. The sample period is 1955Q1-2007Q4. The unconditional moments from the model are computed from the equilibrium MA representation of each variable. Eusepi and Preston (2011) use the measure of hours worked by Francis and Ramey (2009). Productivity is measured as real GDP divided by hours worked. See the Data Appendix in Eusepi and Preston (2011).

[35] The model matches well the standard deviation of consumption, investment, and productivity relative to output,

Consider the rational inattention equilibrium. Finding the fixed point is more difficult than in Section 4 because we must consider two inputs, capital and labor, and two factor prices, the cost of capital and the wage. In equilibrium the factor prices depend on the signal and input demand choices of all firms as well as on the labor supply and saving decisions of households. To find the fixed point, we guess that in equilibrium consumption c_t follows a finite-order ARMA process. With $\gamma = 1$ and $\eta = 0$, the optimality condition (13) simply states that the wage equals consumption. Therefore, a guess about consumption implies a guess about both factor prices, the cost of capital (the expected consumption growth rate) and the wage. We calculate the implied ARMA representations of the optimal inputs $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - [\alpha/(1-\phi)]\, k_{it-1}^*$ from equation (5). We solve the attention problem of firm i, and we compute the firm's capital and labor inputs from the equations $x_t = E\left(x_t^*|\mathcal{I}_{it}\right)$, $k_{it} = x_{1t}$, $l_{it} = x_{2t} + [\alpha/(1-\phi]k_{it-1}$, and the solution of the attention problem. Aggregating across firms yields k_t, i_t, l_t, y_t, and d_t. We verify the guess by solving for c_t from the resource constraint (14). The market-clearing mutual fund share price v_t can be calculated from equation (12) and the solution for d_t and c_t.

What are the effects of rational inattention on the propagation of a productivity shock? We set $\lambda = (1/10,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $1/10,000$ of steady-state output.[36] In the rational inattention equilibrium, the impulse responses of employment, investment, and output become hump-shaped (Figure 2, lines with circles). These impulse responses are hump-shaped even though there are no adjustment costs. The first-order autocorrelations of employment, investment, and output growth become positive (Table 1, column "Rational inattention"). The model matches well the first-order autocorrelation of employment growth in the data, even though rational inattention is the only source of inertia and the marginal cost of attention is small. The model underpredicts somewhat the serial correlation of output and investment growth.

In Figure 2 note also that consumption declines somewhat when firms become subject to ra-

while underpredicting the volatility of hours. The model matches well the correlation of consumption, hours, and investment with output, while overstating the correlation of productivity with output. Finally, the model matches well the first-order autocorrelation of consumption growth. It turns out that rational inattention has little effect on these predictions of the model. See Table 1.

[36] In the rational inattention equilibrium, we can compute the expected profit loss of firm i from suboptimal actions. This is equal per period to $4/100,000$ of steady-state output, even less than the marginal cost of attention λ.

tional inattention. Households consume less because rationally inattentive firms underestimate productivity and produce less than in the perfect information equilibrium.

Section 4 explained the effects of rational inattention one input at a time. In this section the new feature is that rational inattention induces delay in the demand for *both inputs*, capital and labor, *at the same time*. Figure 2 shows the impulse response of the conditional expectation of productivity by firms to a productivity shock. The impulse response is hump-shaped, indicating that the firms' beliefs are anchored on the steady state and evolve slowly. The rational inattention effect turns out to be sufficient to bring the first-order autocorrelation of employment, investment, and output growth in the model approximately into line with the data.

The amount of inattention in the model, governed by the parameter λ, can be compared to survey data on expectations. Coibion and Gorodnichenko (2015) show that models with an informational friction predict a regression relationship between the average forecast error and forecast revision in a cross-section of agents. Suppose that firms in this model report their forecasts of output. Let $\hat{y}_{t+\tau|t}$ denote the period t average forecast of output in period $t+\tau$, where τ is a positive integer. The average forecast error, $y_{t+\tau} - \hat{y}_{t+\tau|t}$, is positively related to the average forecast revision, $\hat{y}_{t+\tau|t} - \hat{y}_{t+\tau|t-1}$. The regression coefficient increases in the size of the informational friction, pinned down by the value of λ. Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2019) estimate this regression relationship using survey data on forecasts of a number of variables. Typically, these authors report coefficients in the range of 0.3-1.4.[37] We repeat their estimation using quarterly data on forecasts of output (real GDP) from the U.S. Survey of Professional Forecasters for the period 1968Q4-2019Q4 obtained from the Federal Reserve Bank of Philadelphia. Focusing on forecasts three periods ahead, $\tau = 3$, we estimate a regression coefficient of 0.91 with a t-statistic of 2.57.[38] Next, we simulate data from our model with the parameter values used in this section, including the value of λ. When we run the same regression on the simulated data, on average across the simulations we obtain a coefficient of 1.07. We conclude that the amount of inattention in this version of the model is consistent with the survey data on expectations.

[37] See in particular Coibion and Gorodnichenko (2015), Table 1 and Figures 1-2, and Bordalo, Gennaioli, Ma, and Shleifer (2019), Table 3.

[38] Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2019) also focus on $\tau = 3$. Both papers report results for forecasts of output *growth* but not output *level*.

5.2 The effects of news about future productivity

Let $h \geq 1$. We focus here on $h = 2$ (the same case that Jaimovich and Rebelo, 2009, focus on) and $h = 4$ (one of two cases in Schmitt-Grohé and Uribe, 2012).

Consider the perfect information equilibrium. Figures 3 and 4 show the impulse responses with $h = 2$ and $h = 4$, respectively (lines with points). The shock is drawn in period 0 while productivity changes in period h. A news shock causes a wealth effect. Consumption and leisure are normal goods, and therefore households want to consume more (save less) and work less after a positive news shock. Firms have no incentive to increase labor demand before productivity improves, while households *reduce* labor supply due to the wealth effect. As a result, hours worked *fall*. With capital predetermined and current productivity unchanged, output contracts. Households, wanting to smooth consumption, choose to reduce the capital stock before productivity improves. Consumption *rises* while investment *declines*. The model fails to produce comovement in response to news about future productivity. It predicts an output *contraction* after news that productivity will improve. Note also that, after falling on impact, employment, investment, and output *keep falling* between when the news arrives (period 0) and when productivity changes (period h). This is particularly clear in Figure 4 ($h = 4$). Employment, investment, and output increase only once productivity improves.

With a high elasticity of intertemporal substitution, the model predicts a fall in consumption and a rise in labor input and investment. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up. "However, no combination of parameters can generate a joint increase in consumption, investment, and employment." (Lorenzoni, 2011, p.539.)

Consider the rational inattention equilibrium (Figures 3-4, lines with circles).[39] In both figures employment *rises in the period in which the news arrives*, period 0, and keeps rising thereafter. The conditional expectation of productivity by firms increases on impact, which pushes up labor demand. In general equilibrium, the desire of households to reduce labor supply is pulling employ-

[39] To find the fixed point we proceed as in Section 5.1. In the economy with $h = 2$, we assume the same value of λ as in Section 5.1. This yields a per period expected profit loss equal to $6/100,000$ of steady-state output. With $h = 4$ we set $\lambda = (3.5/10,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $3.5/10,000$ of steady-state output. The per period expected profit loss turns out to equal $2/10,000$ of steady-state output.

ment *down*. It turns out that the rational inattention effect on labor demand is strong enough to *more than offset* the wealth effect on labor supply. Employment rises in equilibrium.

Figures 3-4 show the impulse response of investment in general equilibrium ("RI general equilibrium") and the impulse response of investment by rationally inattentive firms of measure zero when other firms have perfect information ("RI partial equilibrium," line with asterisks). In partial equilibrium investment *rises in the period in which the news arrives*, and keeps rising thereafter. The conditional expectation of productivity by rationally inattentive firms increases on impact, which pushes up investment demand. In general equilibrium, the desire of households to decrease saving for a given level of output is pulling investment *down*. It turns out that the rational inattention effect on investment demand *approximately offsets* the wealth effect on saving. Investment is close to zero in equilibrium. Furthermore, investment *rises* between period 0 and period h. This is particularly clear in Figure 4 ($h = 4$). In the economy with $h = 4$, investment becomes positive in period $h - 1$ (the period before productivity improves).

With capital predetermined and a rise in labor input in period 0, the period 0 impulse response of output to a news shock is positive (Figures 3-4). Between period 0 and period h, with investment close to zero and labor input on the rise, output continues to expand. *The rational inattention effect on input demand induces an output expansion in response to a news shock.* Note also that in the economy with $h = 4$, employment, investment, and output are all positive in period $h - 1$ (the period before productivity improves).

Consider in more detail what affects investment in general equilibrium. Investment rises in response to a news shock relative to the perfect information equilibrium. The cost of capital increases (the expected consumption growth rate rises). The profit-maximizing capital input of an individual firm *falls*. See the first line in equation (5). Capital is a strategic substitute. An individual firm demands less capital when other firms invest more. This general equilibrium feedback effect turns out to be very strong. The coefficient on the expected consumption growth rate in the first line of equation (5) equals -504.[40] The coefficient on the expected consumption growth rate increases in the depreciation rate, δ, and decreases in the elasticity of output with respect to labor, ϕ. In Section 4.2, with full capital depreciation and without labor input ($\delta = 1$, $\phi = 0$), this coefficient

[40] Labor is also a strategic substitute. However, the general equilibrium dampening of labor demand due to a higher wage is weak. The coefficient on the wage in the second line of equation (5) equals -2.9.

rises by more than two orders of magnitude, to -1.5, implying that the strategic substitutability is much weaker. The impulse response of investment to a news shock is positive in this case (Section 4.2).[41]

To summarize, rational inattention induces an increase in the demand for labor and investment in response to news that productivity will improve. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. The rational inattention effect on investment demand approximately offsets the wealth effect on saving. As a result, output rises.

In Figures 3-4 note also that consumption increases somewhat when firms become subject to rational inattention. Households consume more because rationally inattentive firms overestimate productivity and produce more than in the perfect information equilibrium.

What is the optimal signal? In problem (6)-(11) the firm can in principle choose a high-dimensional signal process, consisting of signals on elements of the state vector ξ_t and/or signals on linear combinations of the elements of ξ_t. We find that a univariate signal that confounds current productivity with expected future productivity is optimal.[42] The upper-left panel in Figure 5 shows the impulse response of the optimal signal to a news shock in the economy with $h = 4$. The signal rises after a news shock. To simplify, the message to firms from a positive signal realization is: "Hire and invest, productivity is either already up or about to rise (and it is not that important precisely when productivity rises)." Furthermore, the impulse response of the signal has the same non-monotonic shape as the impulse responses of employment and investment.

The optimal signal is analogous to an asset price. An asset price is also a univariate statistic that combines information about the current and expected future state of the economy. The upper-right panel in Figure 5 shows the impulse response of the price of a mutual fund share v_t to a news shock in the same economy with $h = 4$. In the rational inattention equilibrium (line with circles), the stock price rises in response to a news shock, like the optimal signal, even though productivity

[41] The model with labor input ($\phi > 0$) requires a sufficiently small depreciation rate δ to produce positive impulse responses of both labor input and investment in partial equilibrium (otherwise, only the impulse response of investment is positive). In general equilibrium the strategic substitutability in capital is then very strong, which keeps the impulse response of investment to a news shock close to zero.

[42] That is, we find that a univariate signal on all elements on the state vector is optimal. Whenever an element of the state vector can be written as a linear combination of the other elements, this element can be dropped from the signal without affecting the solution. Both statements apply when $h = 0$ and when $h \geq 1$.

has not yet improved. The signal and the stock price are not identical, however. The signal reduces efficiently the firms' uncertainty about their optimal inputs. The stock price reflects the current and expected future cash flows and discount rates, via equation (12). In particular, the increase in the expected consumption growth rate in period $h-1$ (the period before productivity improves) exerts downward pressure on v_t after a news shock. As a result, v_t falls in response to a news shock in the perfect information equilibrium (line with points).[43]

The solution of this model reflects some general features of rational inattention. Agents focus on important variables – here the state vector ξ_t. Agents recognize that information acquired at present can also reduce their uncertainty about the future. Agents engage in dimensionality reduction – here the optimal signal is one-dimensional even though the state vector, in general, is multi-dimensional.[44]

Again we can compare the amount of inattention in the model to the SPF data. In the economy with $h = 2$, we assume the same value of λ as in Section 5.1 ($\lambda = (1/10,000)C^{-\gamma}Y$). When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 2$ (with $\tau = 3$), on average we obtain a coefficient of 1.23. The amount of inattention in this version of the model is consistent with the survey data on expectations.[45] As h rises, the model requires more inattention to produce an increase in employment after a news shock. In the economy with $h = 4$, we raise the marginal cost of attention to $\lambda = (3.5/10,000)C^{-\gamma}Y$. When we run the Coibion-Gorodnichenko regression on data simulated from this economy (with $\tau = 3$), on average we obtain a coefficient of 2.87. Thus, the amount of inattention in this version of the model is somewhat greater than found in the SPF data.[46]

The effects of rational inattention are different from the effects of investment adjustment costs. It is straightforward to add convex investment adjustment costs to the perfect information version of the model. Now in the perfect information equilibrium investment and hours rise in response to a news shock but consumption *falls*. Jaimovich and Rebelo (2009) add *three* assumptions to the baseline RBC model to obtain comovement: investment adjustment costs, variable capital

[43]Beaudry and Portier (2006) use stock prices to identify news shocks in the data. In the rational inattention equilibrium, the stock price rises in response to a news shock as in Beaudry and Portier (2006).

[44]See Maćkowiak, Matějka, and Wiederholt (2020) for general features of rational inattention.

[45]Recall that in the SPF data the analogous regression coefficient is 0.91 with a t-statistic of 2.57.

[46]It seems plausible that in the real world decision-makers in small and medium firms pay less attention to the aggregate economy than professional forecasters.

utilization, and a new class of preferences.

6 Rational inattention by firms and households

We focused on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. To obtain comovement in response to a news shock, it seems critical to find a mechanism leading to a shift in labor demand and investment demand for a given level of productivity. Rational inattention *on the side of firms* is such a mechanism. To illustrate in the most transparent way the effects of rational inattention by firms, we assumed that households have perfect information.

Finding a fixed point of an economy in which firms *and* households are subject to rational inattention is even more difficult than what we have considered so far. Now equilibrium depends on the signals chosen by firms *and* on the signals chosen by households. In a special case of the model, however, we can solve for equilibrium in which *all* agents, firms and households, are subject to rational inattention. We assume that labor is the only variable input ($\alpha = 0$, as in Section 4.1) and households do not trade shares in the mutual fund.[47] Each household j chooses a signal about the state of the economy to maximize the expected discounted sum of period utility. The household recognizes that a more informative signal requires more attention. The attention problem of each firm i is unchanged.[48]

We substitute the period budget constraint into the utility function of household j and rewrite the utility function in terms of log-deviations from the non-stochastic steady state. Taking the quadratic-approximation to the expected discounted sum of utility, following the same steps as in Appendix C and assuming the same regularity conditions, we obtain an expression for the expected discounted sum of losses in utility from suboptimal actions. Suppose that the utility-maximizing labor supply l_{jt}^* follows a finite-order ARMA process. This process has a first-order VAR representation in terms of the state vector $\tilde{\xi}_t$. The optimal signal is a one-dimensional signal

[47] The latter assumption ensures that households live hand-to-mouth while holding different beliefs about the state of the economy.

[48] Households no longer have the same consumption level in this version of the model. We assume that firm i values profits according to the marginal utility of consumption of the representative (average) household.

about the state vector $\tilde{\xi}_t$ (Maćkowiak, Matějka, and Wiederholt, 2018). Household j solves:

$$\max_{\tilde{g},\tilde{\sigma}_{\psi}^{2},\tilde{\kappa}}\left\{\sum_{t=0}^{\infty}\beta^{t}E_{j,-1}\left[\frac{-C^{1-\gamma}\left(\gamma+\eta\right)}{2}\left(l_{jt}-l_{jt}^{*}\right)^{2}\right]-\left(\frac{\beta}{1-\beta}\right)\mu\tilde{\kappa}\right\}$$

subject to the law of motion for the state vector $\tilde{\xi}_t$, $l_{jt}=E(l_{jt}^{*}|\mathcal{I}_{jt})$, $\mathcal{I}_{jt}=\mathcal{I}_{j,-1}\cup\{S_{j0},\ldots,S_{jt}\}$, $S_{jt}=\tilde{g}'\tilde{\xi}_t+\psi_{jt}$, and $H(\tilde{\xi}_t|\mathcal{I}_{jt-1})-H(\tilde{\xi}_t|\mathcal{I}_{jt})\leq\tilde{\kappa}$, where $\mu>0$ is the marginal cost of attention. The household's problem is analogous to problem (6)-(11) except that the household takes a single action (decides how much to work), and thus the signal weights $\tilde{g}$ form a vector rather than a matrix and the noise ψ_{jt} simply follows a univariate Gaussian white noise process with variance $\tilde{\sigma}_{\psi}^{2}$. The noise ψ_{jt} is assumed to be independently distributed across households. We compute the conditional second moments using the steady-state Kalman filter.

To find the fixed point, we guess that in equilibrium the wage w_t follows a finite-order ARMA process. We calculate the implied ARMA representations of l_{it}^{*} and l_{jt}^{*} from the optimality conditions $l_{it}^{*}=\left[1/\left(1-\phi\right)\right]\left(a_t-w_t\right)$ and $\eta l_{jt}^{*}=w_t-\gamma c_{jt}$, where $c_{jt}=w_t+l_{jt}^{*}$ from the budget constraint. We solve the attention problem of firm i, and we compute labor demand from the equation $l_{it}=E\left(l_{it}^{*}|\mathcal{I}_{it}\right)$ and the solution of the attention problem. We solve the attention problem of household j, and we compute labor supply from the equation $l_{jt}=E(l_{jt}^{*}|\mathcal{I}_{jt})$ and the solution of the attention problem. We adjust the guess for the wage so that $\int_0^1 l_{it}di=\int_0^1 l_{jt}dj$ in every period.

This appears to be the first time in the literature when a general equilibrium model is solved in which all agents are subject to rational inattention and prices, which the agents take as given, adjust so that markets clear (here, the wage adjusts to equate labor demand and supply in every period).[49]

How does rational inattention by households affect the dynamics of employment? Figure 5 shows the new equilibrium with firms and households subject to rational inattention (lower row, lines with asterisks).[50] The lower row of Figure 5 also reproduces the old equilibrium from Section 4.1 with firms subject to rational inattention and perfectly informed households. Consider a productivity shock, $h=0$ (Figure 5, lower-left panel). With inattentive households the wage response must be stronger for labor supply to change by a given amount. The stronger responsiveness of the wage

[49] In Maćkowiak and Wiederholt (2015), all firms and households are also subject to rational inattention. In each market one side of the market sets the price and the other side of the market chooses the quantity.

[50] We assume the same parameter values as in Section 4.1. In addition we set $\mu=(2/100,000)C^{1-\gamma}$, which means that the per period marginal cost of attention to a household is equal to $2/100,000$ of steady-state consumption.

reduces the profit-maximizing and the actual labor demand. Labor input in the new equilibrium is lower than in the old equilibrium (while the wage is higher). The serial correlation of the growth rate of employment rises.

Consider a news shock, $h = 6$ (Figure 5, lower-right panel). With inattentive households the labor supply decision becomes forward-looking, which makes households more willing to supply labor in response to a news shock. The payoffs from future work rise, and the optimal signal of households confounds the payoff from current work with the payoffs from future work. This effect reduces the responsiveness of the current wage rate, raising the profit-maximizing and the actual labor demand. Labor input in the new equilibrium is higher than in the old equilibrium immediately after a news shock (while the wage is lower).

The model with all agents subject to rational inattention produces a rise in labor input after a news shock, just like in the main part of the paper. Furthermore, rational inattention by households *strengthens* this result (labor input is *even higher* immediately after a news shock).

7 Conclusions and outlook

In the neoclassical business cycle model, it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. Under rational inattention firms choose to be imperfectly aware of the timing of productivity changes. This effect helps the model produce business cycle comovement, causing an output expansion after news that productivity will improve. Rational inattention also improves the propagation of standard productivity shocks, by inducing persistence.

It would be interesting to add rational inattention by households to the complete model with capital and labor. Rational inattention by households could generate a slower response of consumption growth and the cost of capital, making the model predict an increase in investment immediately after a news shock. Alternatively, one could add to the model one of the several features considered in the literature on news shocks. It would also be interesting to compare the model in greater detail to the data, in particular to the evidence in the empirical literature on news and noise shocks. Hopefully, advances in solution methods and computational speed will make this further research feasible soon.

A Non-stochastic steady state

The non-stochastic steady state is the solution of the model when total factor productivity e^{a_t} is equal to 1 in every period and this is common knowledge.

Let an upper-case letter without a time subscript denote the value of a variable in the non-stochastic steady state. Profit maximization implies that $\alpha K_i^{\alpha-1} L_i^{\phi} N_i^{1-\alpha-\phi} = \beta^{-1} - 1 + \delta$ and $\phi K_i^{\alpha} L_i^{\phi-1} N_i^{1-\alpha-\phi} = W$ for each firm i, which determines K_i and L_i as functions of W and parameter values (including N_i):

$$K_i = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1-\phi}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{\phi}{1-\alpha-\phi}} N_i$$

$$L_i = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{1-\alpha}{1-\alpha-\phi}} N_i.$$

Suppose that N_i is constant across i, $N_i = N$. It follows that K_i and L_i are constant across i, $K_i = K$, $L_i = L$. Moreover, Y_i, I_i and D_i are also constant across i, $Y_i = Y = K^{\alpha} L^{\phi} N^{1-\alpha-\phi}$, $I_i = I = \delta K$, $D_i = D = Y - WL - I$.

Utility maximization implies that $V = [\beta/(1-\beta)]\, D$ and $W C_j^{-\gamma} = L_j^{\eta}$ for each household j. Suppose that in the non-stochastic steady state each household holds an equal share of the mutual fund, $Q_j = 1$ for each j. C_j and L_j are then constant across j, $C_j = C$, $L_j = L$, and the budget constraint implies that $C = WL + D$. Combining this equation with $D = Y - WL - I$ yields the resource constraint $Y = C + I$.

One can solve the system of equations:

$$K = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{1-\phi}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{\phi}{1-\alpha-\phi}} N$$

$$L = \left(\frac{\alpha}{\beta^{-1} - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{\phi}{W}\right)^{\frac{1-\alpha}{1-\alpha-\phi}} N$$

$$W = L^{\eta} \left(K^{\alpha} L^{\phi} N^{1-\alpha-\phi} - \delta K\right)^{\gamma}$$

for K, L and W for given parameter values (including N). The last equation comes from combining the equilibrium condition $WC^{-\gamma} = L^{\eta}$ with the resource constraint. One can then compute the other endogenous variables (Y, I, C, D, and V) from the equations $Y = K^{\alpha} L^{\phi} N^{1-\alpha-\phi}$, $I = \delta K$, $C = Y - I$, $D = Y - WL - I$, $V = [\beta/(1-\beta)]\, D$.

The following steady-state ratios are useful: $WL/Y = \phi$, $I/Y = \alpha\beta\delta/[1-\beta(1-\delta)]$, $C/Y = 1 - I/Y$, $D/Y = 1 - WL/Y - I/Y$, $WL/C = (WL/Y)(Y/C)$, $V/C = [\beta/(1-\beta)]\,(D/Y)(Y/C)$.

B Perfect information benchmark

Suppose that all agents have perfect information. Let a lower-case letter denote the log-deviation of a variable from its value in the non-stochastic steady state. The firms' first-order conditions imply that

$$a_t + \alpha k_{t-1} - (1-\phi)\, l_t = w_t$$

and

$$E_t a_{t+1} - (1-\alpha)\, k_t + \phi E_t l_{t+1} = \frac{\gamma\, (E_t c_{t+1} - c_t)}{1-\beta\,(1-\delta)}.$$

From the production function, the law of motion of capital and the profit function, we have

$$y_t = a_t + \alpha k_{t-1} + \phi l_t$$

$$\delta i_t = k_t - (1-\delta)\, k_{t-1}$$

and

$$(D/Y)\, d_t = y_t - (WL/Y)\,(w_t + l_t) - (I/Y)\, i_t.$$

The households' first-order conditions imply that

$$\beta E_t v_{t+1} - v_t + (1-\beta)\, E_t d_{t+1} = \gamma E_t\, (c_{t+1} - c_t)$$

and

$$w_t - \gamma c_t = \eta l_t.$$

Finally, the resource constraint reads

$$y_t = (C/Y)\, c_t + (I/Y)\, i_t.$$

C Expected loss in profit from suboptimal actions

Proposition 1 *Let $E_{i,-1}$ denote the expectation operator conditioned on information of the decision-maker of firm i in period -1. Let g denote the functional that is obtained by multiplying the profit function by β^t and summing over all t from zero to infinity. Let $\tilde{g}$ denote the second-order Taylor approximation of g at the non-stochastic steady state. Let x_t, z_t and v_t denote the following vectors*

$$x_t = \begin{pmatrix} k_{it} \\ l_{it} \end{pmatrix} \quad z_t = \begin{pmatrix} a_t \\ w_t \\ c_t \end{pmatrix} \quad v_t = \begin{pmatrix} x_t \\ z_t \\ 1 \end{pmatrix}.$$

Suppose that the decision-maker of firm i knows in period -1 the firm's initial capital stock, $k_{i,-1}$. Suppose also that there exist two constants $\delta < (1/\beta)$ and $A \in \mathbb{R}$ such that, for each period $t \geq 0$, for $\tau = 0, 1$, and for all $m, n \in \{1, 2, 3, 4, 5, 6\}$,

$$E_{i,-1} \left| v_{m,t} v_{n,t+\tau} \right| < \delta^t A. \tag{16}$$

Here $v_{m,t}$ and $v_{n,t}$ denote the mth and nth element of the vector v_t. Then, the expected discounted sum of losses in profit when the law of motion for the actions differs from the law of motion for the optimal actions under perfect information equals

$$\begin{aligned} & E_{i,-1} \left[\tilde{g} \left(k_{i,-1}, x_0, z_0, x_1, z_1, \ldots \right) \right] - E_{i,-1} \left[\tilde{g} \left(k_{i,-1}, x_0^*, z_0, x_1^*, z_1, \ldots \right) \right] \\ = & \sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^* \right)' \Theta_0 \left(x_t - x_t^* \right) + \left(x_t - x_t^* \right)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right) \right]. \end{aligned} \tag{17}$$

The matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha (1-\alpha) & 0 \\ 0 & \phi (1-\phi) \end{bmatrix} \quad \Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}. \tag{18}$$

The optimal actions under perfect information are given by

$$x_t^* = \begin{pmatrix} k_{it}^* \\ l_{it}^* \end{pmatrix} = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t \left[a_{t+1} \right] - \phi E_t \left[w_{t+1} \right] - (1-\phi) \frac{\gamma E_t \left[c_{t+1} - c_t \right]}{1-\beta(1-\delta)} \right] \\ \frac{\alpha}{1-\phi} k_{it-1}^* + \frac{1}{1-\phi} \left(a_t - w_t \right) \end{pmatrix}, \tag{19}$$

where E_t denotes the expectation operator conditioned on the entire history of the economy up to and including period t, and the initial capital stock is given by the initial condition $k_{i,-1}^* = k_{i,-1}$.

Proof. First, we introduce notation. The profit of firm i in period t depends on three sets of variables: (i) variables that the decision-maker of firm i chooses in period t (here k_{it} and l_{it}), (ii) variables that the decision-maker chose in the past (here k_{it-1}), and (iii) variables that the decision-maker takes as given (here a_t, w_t and c_t). The first set of variables is collected in the vector x_t, the second set of variables is an element of x_{t-1} for all $t \geq 0$ once we define the vector $x_{-1} = \begin{pmatrix} k_{i,-1} & 0 \end{pmatrix}'$, and the third set of variables is collected in the vector z_t.

The next steps are word for word identical to steps "Second" to "Seventh" in proof of Proposition 2 in online Appendix D of Maćkowiak and Wiederholt (2015). The reason is that these steps only require that the payoff in period t depends only on own current actions (collected in x_t), own previous-period actions (collected in x_{t-1}), and variables that the decision-maker takes as given (collected in z_t) and that the initial condition $k_{i,-1}$ and the vector v_t satisfy conditions (40)-(42) in online Appendix D of Maćkowiak and Wiederholt (2015). The payoff in period t in Proposition 1 is profit, whereas the payoff in period t in online Appendix D of Maćkowiak and Wiederholt (2015) is period utility, but in both cases this payoff depends only on own current actions (x_t), own previous-period actions (x_{t-1}), and variables that the decision-maker takes as given (z_t). Conditions (40)-(41) in online Appendix D of Maćkowiak and Wiederholt (2015) are satisfied because of the assumption in Proposition 1 that the decision-maker knows the initial condition x_{-1}. Condition (42) in online Appendix D of Maćkowiak and Wiederholt (2015) is equal to condition (16) in Proposition 1. These steps "Second" to "Seventh" yield equation (17), where Θ_0 is defined as the Hessian matrix of second derivatives of g with respect to x_t evaluated at the non-stochastic steady state and divided by β^t, Θ_1 is defined as the Hessian matrix of second derivatives of g with respect to x_t and x_{t+1} evaluated at the non-stochastic steady state and divided by β^t, and x_t^* is defined as the actions that the decision-maker would take if he or she had perfect information in every period $t \geq 0$.

Eighth, the functional g in Proposition 1 is the discounted sum of profit

$$g\left(x_{-1}, x_0, z_0, x_1, z_1, \ldots\right) = \sum_{t=0}^{\infty} \beta^t f\left(x_t, x_{t-1}, z_t\right),$$

where the function f is the profit function

$$f\left(x_t, x_{t-1}, z_t\right) = C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta} \right) \left[\left(1 - \delta\right) e^{k_{it-1}} - e^{k_{it}} \right] \right\}.$$

Computing the matrices Θ_0 and Θ_1 for this functional g yields equation (18).

Ninth, we characterize the optimal actions under perfect information. Formally, the process $\{x_t^*\}_{t=0}^{\infty}$ is defined by the initial condition $x_{-1}^* = \begin{pmatrix} k_{i,-1} & 0 \end{pmatrix}'$ and the optimality condition

$$\forall t \geq 0 : E_t \left[\theta_0 + \Theta_{-1} x_{t-1}^* + \Theta_0 x_t^* + \Theta_1 x_{t+1}^* + \Phi_0 z_t + \Phi_1 z_{t+1} \right] = 0. \tag{20}$$

Here θ_0 is defined as the vector of first derivatives of g with respect to x_t evaluated at the non-stochastic steady state and divided by β^t, Θ_{-1} is defined as the matrix of second derivatives of g with respect to x_t and x_{t-1} evaluated at the non-stochastic steady state and divided by β^t, Φ_0 is defined as the matrix of second derivatives of g with respect to x_t and z_t evaluated at the non-stochastic steady state and divided by β^t, Φ_1 is defined as the matrix of second derivatives of g with respect to x_t and z_{t+1} evaluated at the non-stochastic steady state and divided by β^t, and E_t denotes the expectation operator conditioned on the entire history of the economy up to and including period t. See the step "Fourth" in proof of Proposition 2 in online Appendix D of Maćkowiak and Wiederholt (2015). Computing the vector θ_0 and the matrices Θ_{-1}, Φ_0, and Φ_1 for the functional g defined in the previous step yields

$$\begin{array}{cc} \theta_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \Theta_{-1} = C^{-\gamma} Y \begin{bmatrix} 0 & 0 \\ \alpha\phi & 0 \end{bmatrix} \\ \Phi_0 = C^{-\gamma} Y \begin{bmatrix} 0 & 0 & \gamma \frac{\alpha}{\beta^{-1} - 1 + \delta} \\ \phi & -\phi & 0 \end{bmatrix} & \Phi_1 = C^{-\gamma} Y \begin{bmatrix} \alpha\beta & 0 & -\gamma \frac{\alpha}{\beta^{-1} - 1 + \delta} \\ 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Substituting the equations for θ_0, Θ_{-1}, Φ_0, Φ_1, Θ_0, Θ_1, x_t and z_t into equation (20) yields

$$E_t \left[a_{t+1} \right] - (1 - \alpha) k_{it}^* + \phi E_t \left[l_{it+1}^* \right] = \frac{\gamma E_t \left[c_{t+1} - c_t \right]}{1 - \beta (1 - \delta)} \tag{21}$$

$$a_t + \alpha k_{it-1}^* - (1 - \phi) l_{it}^* = w_t. \tag{22}$$

Equations (21)-(22) are the usual optimality conditions for capital and labor. Equation (21) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital. Equation (22) states that the profit-maximizing labor input equates the marginal product of labor to the wage. Rearranging equations (21)-(22) yields the closed-form solution (19) for the actions that the firm would take in period t if the firm had perfect information in every period $t \geq 0$. ■

Proposition 2 *Under condition (16), we have*

$$\sum_{t=0}^{\infty}\beta^t E_{i,-1}\left[\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)\right]$$
$$= -C^{-\gamma}Y\sum_{t=0}^{\infty}\beta^t E_{i,-1}\left[\frac{\beta\alpha\left(1-\alpha-\frac{\alpha\phi}{1-\phi}\right)}{2}\left(k_{it}-k_{it}^*\right)^2+\frac{\phi\left(1-\phi\right)}{2}\left(\zeta_{it}-\zeta_{it}^*\right)^2\right], \qquad (23)$$

where $\zeta_{it}\equiv l_{it}-\frac{\alpha}{1-\phi}k_{it-1}$ *and* $\zeta_{it}^*\equiv l_{it}^*-\frac{\alpha}{1-\phi}k_{it-1}^*$.

Proof. First, we have

$$\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)$$
$$= C^{-\gamma}Y\left[-\frac{\beta\alpha\left(1-\alpha\right)}{2}\left(k_{it}-k_{it}^*\right)^2-\frac{\phi\left(1-\phi\right)}{2}\left(l_{it}-l_{it}^*\right)^2+\beta\alpha\phi\left(k_{it}-k_{it}^*\right)\left(l_{it+1}-l_{it+1}^*\right)\right] \qquad (24)$$

because $x_t=\begin{pmatrix} k_{it} & l_{it}\end{pmatrix}'$, $x_t^*=\begin{pmatrix} k_{it}^* & l_{it}^*\end{pmatrix}'$ and the matrices Θ_0 and Θ_1 are given by equation (18). Second, define

$$\zeta_{it}=l_{it}-\frac{\alpha}{1-\phi}k_{it-1},$$

and

$$\zeta_{it}^*=l_{it}^*-\frac{\alpha}{1-\phi}k_{it-1}^*.$$

This definition implies

$$l_{it}-l_{it}^*=\zeta_{it}-\zeta_{it}^*+\frac{\alpha}{1-\phi}\left(k_{it-1}-k_{it-1}^*\right).$$

Substituting the last equation into equation (24) yields

$$\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)$$
$$= C^{-\gamma}Y\left[\begin{array}{c} -\frac{\beta\alpha(1-\alpha)}{2}\left(k_{it}-k_{it}^*\right)^2-\frac{\phi(1-\phi)}{2}\left(\zeta_{it}-\zeta_{it}^*\right)^2 \\ -\alpha\phi\left(\zeta_{it}-\zeta_{it}^*\right)\left(k_{it-1}-k_{it-1}^*\right)-\frac{\alpha^2\phi}{2(1-\phi)}\left(k_{it-1}-k_{it-1}^*\right)^2 \\ +\beta\alpha\phi\left(k_{it}-k_{it}^*\right)\left(\zeta_{it+1}-\zeta_{it+1}^*\right)+\frac{\beta\alpha^2\phi}{1-\phi}\left(k_{it}-k_{it}^*\right)^2 \end{array}\right].$$

Multiplying by β^t and summing over all t from zero to T yields

$$\sum_{t=0}^{T}\beta^t\left[\frac{1}{2}\left(x_t-x_t^*\right)'\Theta_0\left(x_t-x_t^*\right)+\left(x_t-x_t^*\right)'\Theta_1\left(x_{t+1}-x_{t+1}^*\right)\right]$$
$$= C^{-\gamma}Y\left[\begin{array}{c} -\frac{\beta\alpha}{2}\left(1-\alpha-\frac{\alpha\phi}{1-\phi}\right)\sum\limits_{t=0}^{T}\beta^t\left(k_{it}-k_{it}^*\right)^2-\frac{\phi(1-\phi)}{2}\sum\limits_{t=0}^{T}\beta^t\left(\zeta_{it}-\zeta_{it}^*\right)^2 \\ +\beta^T\frac{\beta\alpha^2\phi}{2(1-\phi)}\left(k_{iT}-k_{iT}^*\right)^2+\beta\alpha\phi\beta^T\left(k_{iT}-k_{iT}^*\right)\left(\zeta_{iT+1}-\zeta_{iT+1}^*\right) \end{array}\right],$$

where we have used $k_{i,-1}^{*} = k_{i,-1}$ and the fact that several terms on the right-hand side cancel. Taking the expectation $E_{i,-1}$ and the limit as $t \to \infty$ yields

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^*\right)' \Theta_0 \left(x_t - x_t^*\right) + \left(x_t - x_t^*\right)' \Theta_1 \left(x_{t+1} - x_{t+1}^*\right) \right] \\
= \;& C^{-\gamma} Y \left[\begin{array}{c} -\frac{\beta\alpha}{2} \left(1 - \alpha - \frac{\alpha\phi}{1-\phi}\right) \sum_{t=0}^{\infty} \beta^t E_{i,-1} \left(k_{it} - k_{it}^*\right)^2 - \frac{\phi(1-\phi)}{2} \sum_{t=0}^{\infty} \beta^t E_{i,-1} \left(\zeta_{it} - \zeta_{it}^*\right)^2 \\ + \frac{\beta\alpha^2\phi}{2(1-\phi)} \lim_{T\to\infty} \beta^T E_{i,-1} \left(k_{iT} - k_{iT}^*\right)^2 + \beta\alpha\phi \lim_{T\to\infty} \beta^T E_{i,-1} \left(k_{iT} - k_{iT}^*\right) \left(\zeta_{iT+1} - \zeta_{iT+1}^*\right) \end{array} \right].
\end{aligned}$$

Under condition (16) the two infinite sums on the right-hand side of the last equation converge to an element in $\mathbb{R}$ and the third and fourth term on the right-hand side of the last equation equal zero. ■

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Figure 1: Impulse responses with a single variable factor

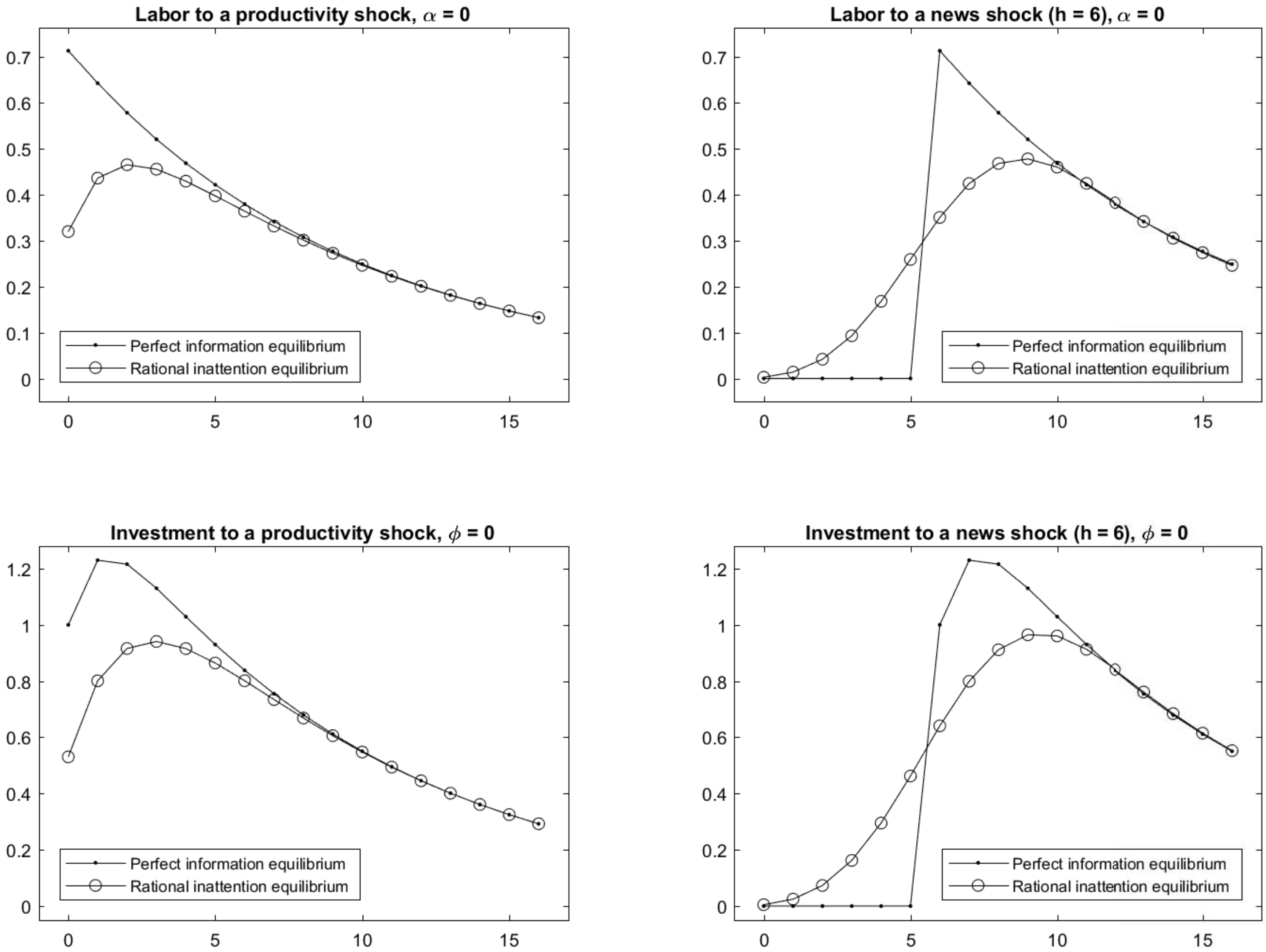

Figure 2: Impulse responses to a productivity shock

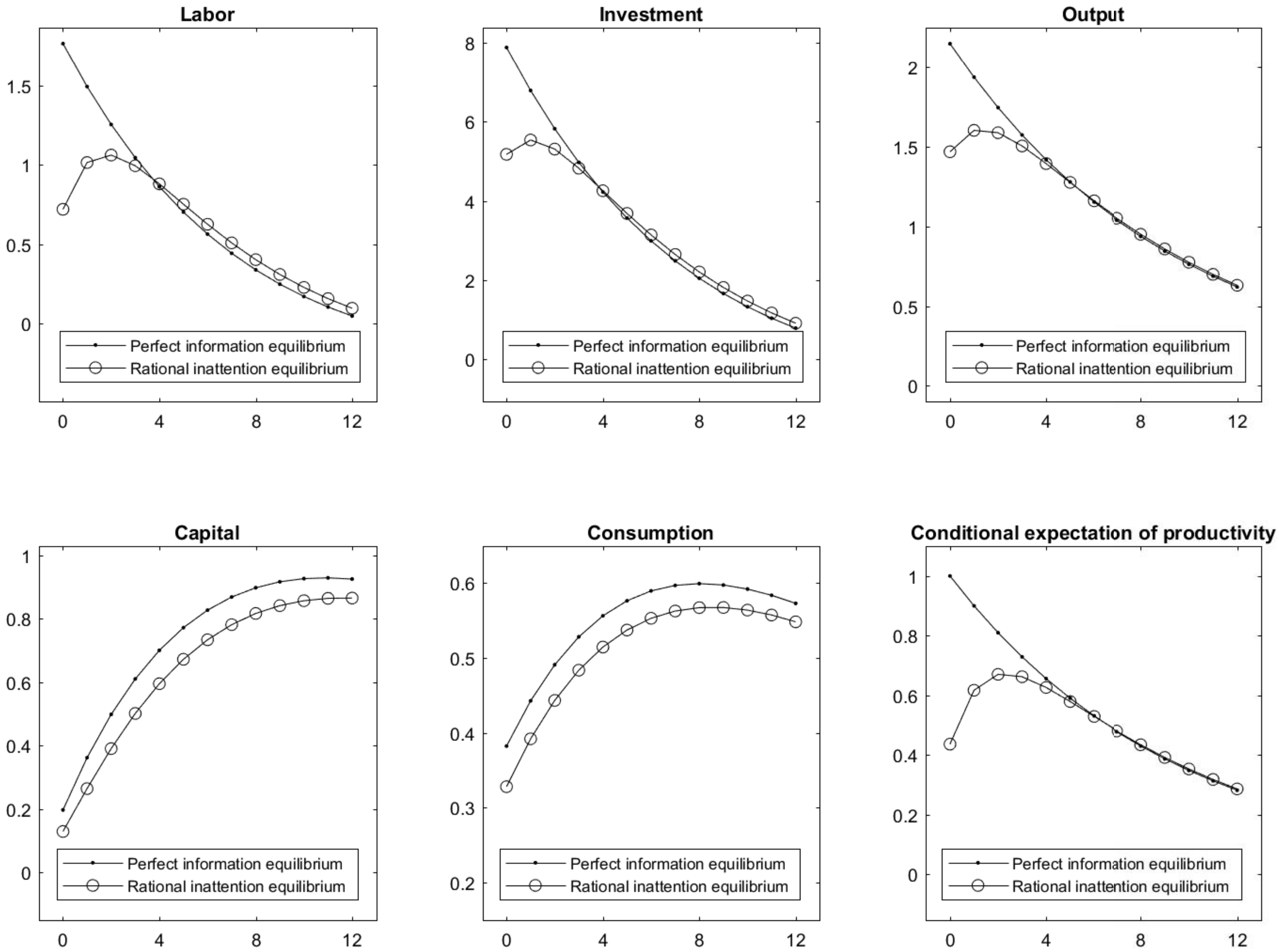

Figure 3: Impulse responses to a news shock (h = 2)

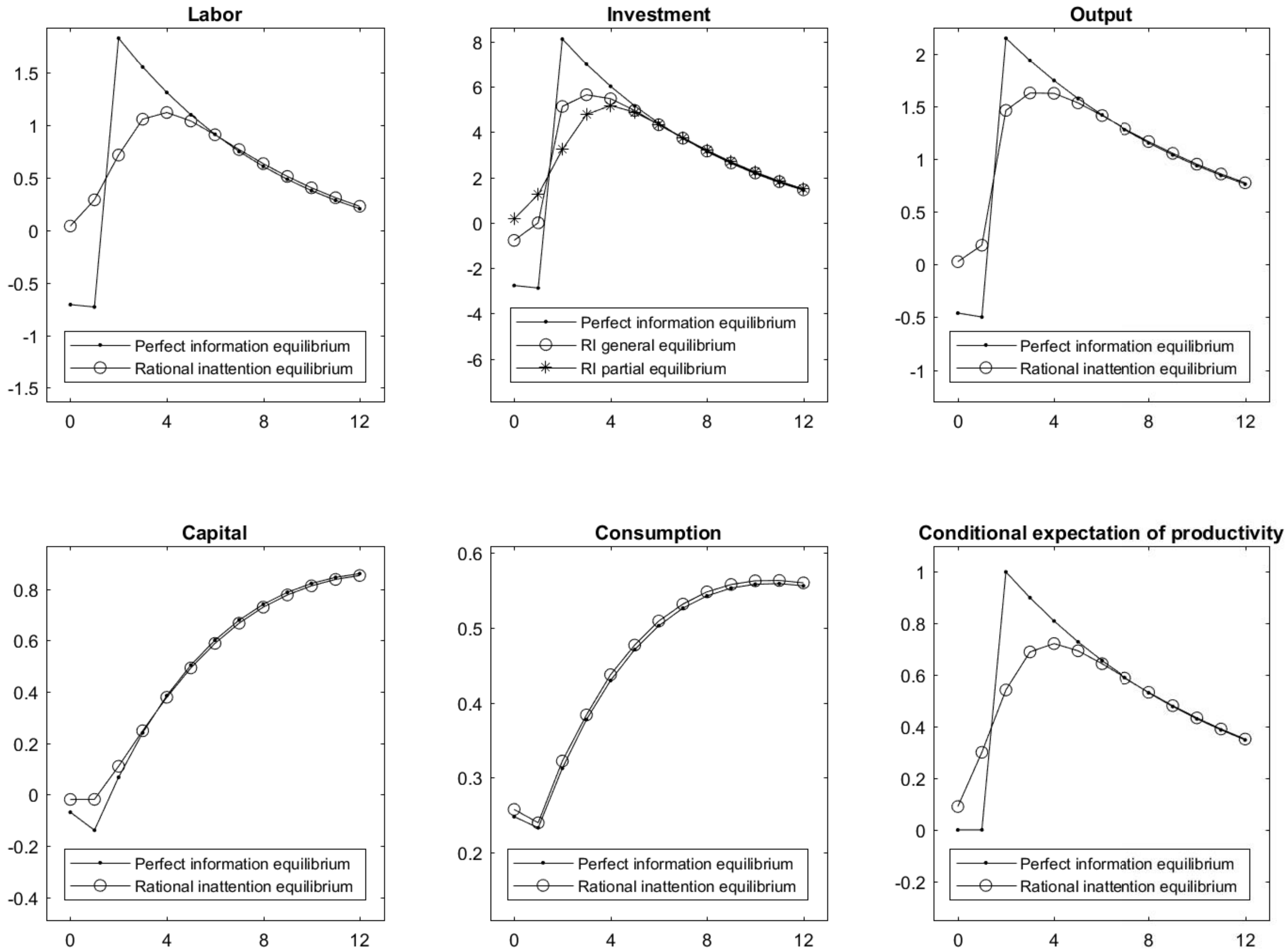

Figure 4: Impulse responses to a news shock (h = 4)

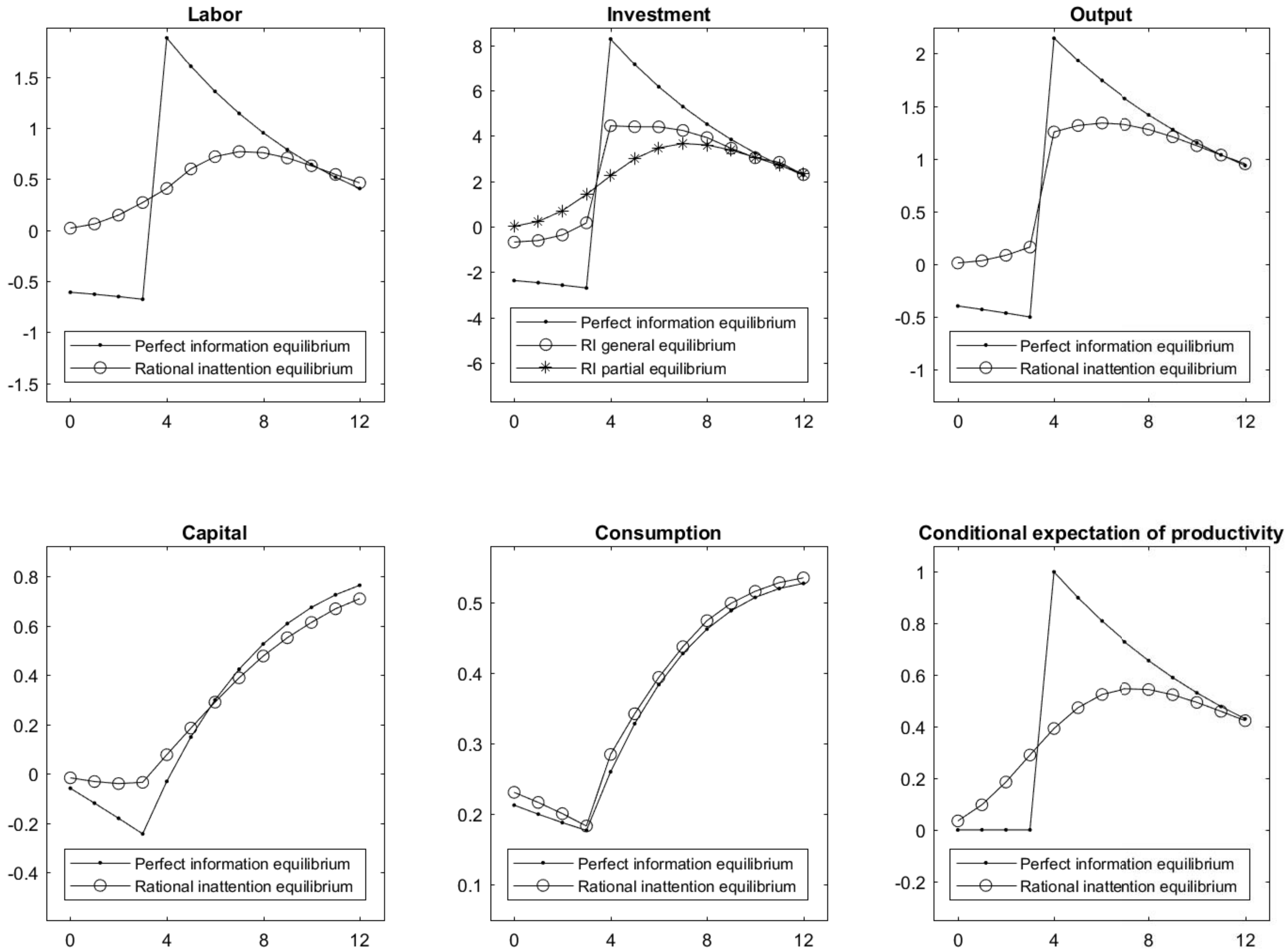

Figure 5: Additional impulse responses

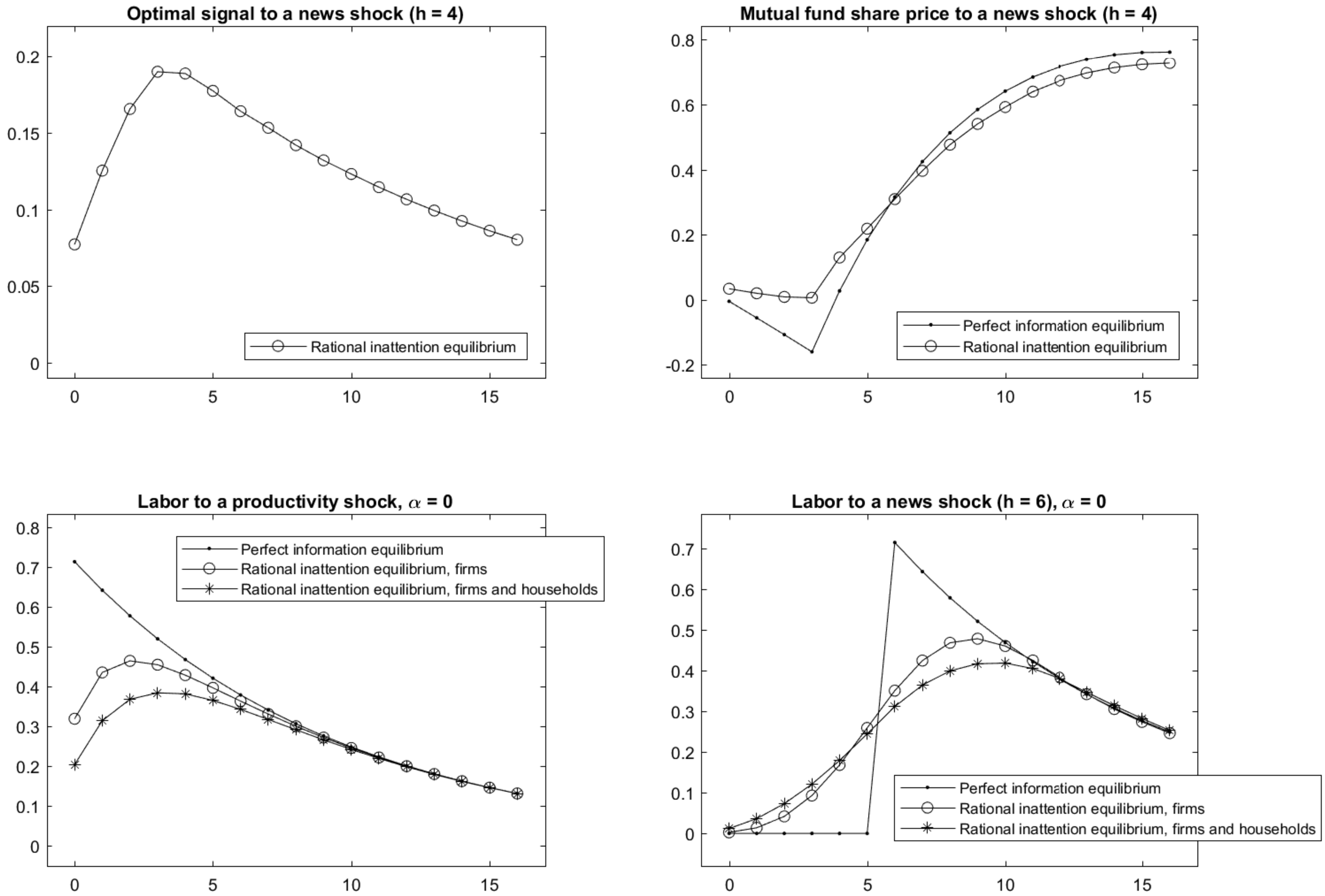

Table 1: Business cycle statistics

	Data	Model, *h = 0*	
		Perfect information	Rational inattention
Relative standard deviation			
σ_c/σ_y	0.55	0.56	0.59
σ_l/σ_y	0.92	0.66	0.57
σ_i/σ_y	2.88	3.05	2.93
σ_a/σ_y	0.52	0.46	0.51
Correlation			
$\rho_{c,y}$	0.78	0.78	0.81
$\rho_{l,y}$	0.85	0.85	0.83
$\rho_{i,y}$	0.90	0.93	0.92
$\rho_{a,y}$	0.40	1.00	0.99
First-order serial correlation			
Δc	0.27	0.23	0.28
Δl	0.41	-0.06	0.46
Δi	0.35	-0.06	0.14
Δy	0.30	-0.05	0.13
Δa	-0.06	-0.05	-0.05

Data: United States, 1955Q1-2007Q4, from Eusepi and Preston (2011).
Model: Unconditional moments computed from equilibrium MA representation of each variable.