Delegated Bargaining*

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Abstract

I consider a model of delegated bargaining where an uninformed principal bargains with an opponent and can delegate negotiation to a biased agent who is privately informed about the cost of agreement. Applications of the model include diplomacy and bargaining on behalf of a firm by division managers. I characterize the equilibria that result, showing that there are equilibria with inefficient delay, as well as with immediate agreement. Any equilibrium with delay has an atom of acceptances followed by smooth screening. The probability of delegation decreases over time. As long as there are circumstances in which the principal would rather disagree, the Coase conjecture fails in every equilibrium. Using an agent can benefit the principal, but only in the initial stages of negotiation. If the conflict of interest between the principal and the agent increases, the payoff of the opponent increases.

1 Introduction

Reliance on experts to conduct bargaining is commonplace. Diplomacy is a high-profile example of this practice. The tendency of Trump to rely on direct talks with foreign leaders, bypassing the usual diplomatic channels, sparked concern in the administration and beyond.[1] Yet the consequences of delegating bargaining to experts are poorly understood. How much can a leader gain from using delegates, as compared to negotiating herself? When does negotiating through delegates yield no benefits? How does reliance on delegates affect the speed of reaching agreement? The model I develop answers these questions.

The negotiations over the Iranian nuclear program illustrate the dynamics that can

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[1] See Salama and Nicholas (2019).

arise. The negotiations that occurred in the 2000s and 2010s had two phases. In the first one, the negotiators' preferences were closely aligned with the preferences of the Supreme Leader, and little progress was made. In the second phase, the less extreme Hassan Rouhani became the new president in 2013 and, appointing Javad Zarif as a foreign minister, took charge of the negotiations (Davenport 2013: 33). This led to a breakthrough in the negotiations and a historic deal signed in 2015. This pattern is consistent with the predictions of my model: first the delegate was closely aligned with the principal and an equilibrium with delay was played, and then a new delegate who was more eager to agree took over and the equilibrium with delay disappeared.

The main elements of the model are as follows. An uninformed principal is engaged in bargaining with an opponent, and, in each period, can delegate the negotiation to an informed agent. The opponent then makes an offer to the entity conducting the negotiation in this period. If agreement is made, the principal gets the share that is agreed upon and pays the cost of implementing the agreement that is privately known by the agent. This cost may be the expense of installing environmentally-friendly production technologies to comply with an environmental treaty or the fallout from domestic jobs lost due to a new trade treaty. The agent gets the benefit from the agreement and bears the cost only if the agent is the one who made the agreement.[2] The costs of implementing the agreement borne by the principal and the agent are interdependent but not the same. Applications of the model include diplomacy and bargaining on behalf of a firm by division managers, as well as real estate development negotiations and labor negotiations.

I characterize the (weak-Markov and Pareto undominated)[3] equilibria in this setting. I show that there exist equilibria with immediate agreement, as well as equilibria with delay. I characterize the offers made to the agent in immediate agreement (IA) equilibria, showing that they are equal to the costs for which the principal obtains a higher payoff from agreement than the agent. As long as there are circumstances in which the principal would rather not make an agreement, the Coase conjecture fails in every sequential (and not just weak-Markov Pareto undominated) equilibrium: the share that the principal accepts is strictly less than one.

Why doesn't the Coase conjecture hold here? Naively, one might expect it to hold because if the principal just delegated in every period, the proposer would face Coasean incentives and offer 1 to the agent immediately. Then, since the agent receives and accepts

[2] I discuss an extension where the agent also gets a payoff from agreements made by the principal in Section 7.

[3] See Section 3 for more details on the class of equilibria that I restrict my attention to.

the offer of 1 immediately, the principal would indeed want to delegate. It turns out, however, that the principal will not delegate in every period: in particular, she will not delegate if enough agent types rejected the offers and the remaining agents' costs are too large. Anticipating this, the proposer will not offer 1, which, in turn, makes the principal even less willing to delegate close to the end of the game. This leads to unraveling where the highest offer the proposer ever makes is much lower than 1.

I uncover the structure of equilibria with delay. In any such equilibrium, first the principal delegates with probability one, and the proposer makes an offer that is accepted by all agents with sufficiently small costs. After this, slow and smooth screening starts, with the principal not delegating negotiation at some rate and, conditional on delegation, the agent accepting the offer at some rate. During the smooth screening phase, the offers the proposer makes decrease continuously over time.

I show that, in any equilibrium with delay, the initial offer of the proposer to the agent is equal to some cost for which the principal is less eager to agree than the agent. I will refer to the minimum of the costs of the agents who haven't agreed yet as the state. If the initial offer is rejected, smooth screening obtains in an interval of states where the principal is less eager to agree. The state in which screening ends is the lowest state above the initial offer for which there is no conflict of interest. Delay and delegation end by a certain time with probability one, with the principal taking charge.

The model provides insight into bargaining dynamics in equilibria with delay. I show that the principal delegates with a higher probability in lower states. This means that over time, as the state goes up, the probability of delegation decreases. Thus, as time passes without successful agreement, the principal "loses patience" and starts negotiating the agreement herself. I further show that, whenever the principal negotiates herself, she makes the agreement immediately, whereas delegation may be associated with delay.

When deciding whether to use a delegate, the principal faces a tradeoff between utilizing the superior information that the delegates has and avoiding the cost associated with the misalignment of her and the delegate's interests. This results in delegates securing agreement at worse terms than the principal: in the same state, the offers that delegates accept are lower than the offer the principal accepts. This cost of using delegates is made up for by the fact that, because delegates are informed, they make agreements only when the cost is low enough, whereas the uninformed principal makes agreements even when the cost is high.

I show that, even though delegates accept lower offers than the principal would, they still obtain a strictly positive payoff from agreement. The fact that the proposer is not able to fully expropriate the delegates in an equilibrium with screening is due to the need to provide incentives for the principal to delegate.

What is the cause of delay in agreement? I identify the threat of the principal bypassing the delegate, as well as complex conflict of interest between the principal and the delegate, as causes of delay. In an equilibrium with delay, the proposer is indifferent between making an acceptable offer to the agent and waiting for a chance to make an deal with the principal. Thus the reason for delay is the principal mixing between making a deal with the proposer and delegating. The conflict of interest is complex if there are both circumstances in which the principal is more eager to agree than the agent, and circumstances in which the principal is less eager to agree. The complexity of the conflict of interest contributes to delay: under such conflict, there is a cost for which the interests of the principal and the agent are perfectly aligned. It turns out that the smooth screening phase of an equilibrium with delay can only end at such a cost.

The results I obtain shed light on the usefulness of delegates in negotiations. I show that using a delegate may help the principal secure a better agreement than she would had a delegate not been available. Yet the benefits from using delegates are accrued in the beginning of negotiations. That is, advantageous agreements are made immediately, and if we observe long negotiations with parties unable to reach an agreement, we can infer that the principal obtains no benefit from using a delegate during these negotiations.

The results also highlight the cost to the principal from being unable to commit to perpetual delegation: the highest payoff the principal can get from using a delegate when deciding to delegate or not in every period is strictly lower than the payoff she would get had she been able to commit to delegate once and for all.

I establish comparative statics that hold in any equilibrium with delay. I show that the payoff of the proposer increases as the conflict of interest between the principal and the agent increases. The proposer also benefits if higher costs become more likely.[4] Greater patience results in more delegation and slower screening. If the conflict of interest in states where the principal is less eager to agree than the agent increases, then the probability of delegation decreases, so the principal tends to rely less on agents whose interests are more misaligned. Finally, if the principal becomes less eager to agree for high enough costs, then in an equilibrium with smooth screening in low states there is less delegation and screening is faster.

I show that changing the support of the distribution of agent's costs by adding lower costs may hurt the principal and the agent (and benefit the proposer). This is because

[4]See section 6 for more details.

adding lower costs may cause an equilibrium with lower offers and delay to exist.[5] This is true regardless of how likely these lower costs are: even a small probability of lower costs can decrease the payoff of the principal by a substantial amount. In terms of applications, the result suggests that increasing military strength may, surprisingly, worsen the terms of a military agreement a country is able to negotiate and improving economic conditions may hamper negotiating trade deals.

Finally, I consider extensions. I first consider common values: whereas in the main model the agent gets the payoff from the deal only if she makes the deal, in the extension I allow the agent to also get a fraction of the payoff when the principal makes the deal. The structure of equilibria is preserved under common values, the set of offers in IA equilibria expands and the length of intervals on which smooth screening happens in equilibria with delay shrinks. I also look at a setting where the principal can communicate with the agent before making the delegation decision. In general, communication expands the set of equilibrium outcomes. I show, however, that equilibria with informative communication lack robustness: if informative communication is costly, then, no matter how small the cost is, no equilibria with informative communication exist.

To the best of the author's knowledge, the model in the present paper is the first model of delegation in incomplete information bargaining. The model sheds light on the dynamics of delegated bargaining, providing testable implications and comparative statics.

The rest of the paper proceeds as follows. Section 2 discusses the applications. I introduce the model in section 3. Section 4 considers equilibria with delay. Section 5 characterizes immediate agreement equilibria. Results about welfare and comparative statics are presented in section 6. Section 7 discusses extensions of the model. Section 8 reviews the related literature.

2 Applications

2.1 Diplomacy

The first application of the model I discuss is diplomacy. Here the principal is the head of state (for simplicity, let us call her the president), the agent is a diplomat, and the proposer is a representative of a foreign country.

There is an agreement to be negotiated. Because the diplomat possesses special

[5] Depending on the initial cost functions, adding lower costs may decrease the principal's payoff in the principal-optimal equilibrium.

expertise, she knows more than the president about the costs and benefits of the agreement. For example, if a military treaty is to be negotiated, then the military personnel has more information about the details of the treaty, while bureaucrats from an environmental agency know more about the consequences of an environmental agreement. Career diplomats are better at evaluating proposals in the area that they specialize in.

The president can either negotiate the agreement herself or delegate the negotiation to a diplomat. Knowing who she will negotiate with, the representative of the foreign country makes a proposal. Should the parties fail to agree at the meeting, the president will again either delegate or negotiate herself. Importantly, the president cannot commit to delegate to the diplomat permanently.

The diplomat cares about getting credit for having brokered the agreement, and so only gets the payoff from the agreement if she was the one who negotiated it. If instead the president sidelined the diplomat and made the agreement herself, then the diplomat does not get any payoff.[6]

During the negotiations, the representative of the foreign country makes all the offers. This is a stylized reflection of the observation that in international negotiations it is often the most powerful countries that put forward the proposals. During the Paris Peace Conference after World War I, for example, the "Big Four" powers – France, Great Britain, United States and Italy – came up with all the major points of the Treaty of Versailles signed with Germany (René 1958: 363). Thus the theory of delegated bargaining in the present paper applies to bargaining between countries whose power differs substantially.

The interests interests of the president and the diplomat are partially aligned: deals that are better for the president also yield greater benefit to the diplomat. The interests are misaligned because the nature of benefits differs. While a diplomat mainly cares about getting credit for negotiating a good agreement, the president may be concerned about the impact of a trade deal on campaign donations by the affected companies or the impact of a military agreement on the likelihood of war.

This portrayal of diplomacy is in line with the way in which political science literature understands some aspects of diplomacy and offers a novel perspective on other aspects. Most of this literature views diplomacy as means to communicate private information, especially information about the resolve to fight (Fearon 1997, Sartori 2002). This literature does not view diplomats as distinct actors nor explains why delegation does or does not occur. An

[6]The model can be extended to a common values setting where the agent gets a payoff from the agreement regardless of who makes the deal but the payoff is larger if the deal was made by the agent. Section 7 discusses this extension.

exception is Lindsey (2017) who argues that committing to delegate to a biased diplomat can improve information transmission. Jost and Strange (2018) emphasize that the expertise that certain agents possess induces leaders to delegate to these agents diplomacy on the issues they are experts on.[7]

2.2 Negotiations by Firms

The second application of the model I discuss is negotiations by firms. One example is as follows: the government would like to have a public transit project implemented and negotiates with a consulting firm, such as McKinsey, about this project. Here the principal is the director of the firm, the agent is a firm division manager, and the proposer is the government.

Because the division manager knows more about the division specialty than the firm director, she knows more about just how costly implementing the project is going to be for the firm and the benefits the project might yield. For example, if the project involves building new roads, the division manager will have a better idea of how the new roads are going to impact property taxes, opportunities for local suppliers and economic activity.

The director can either negotiate himself or delegate the negotiation to the manager. Because the manager cares about getting credit for securing the project, being the one to make the deal brings her payoff rewards. The interests of the director and the manager are only imperfectly aligned: while both of them would like the company to do well, the manager cares about bringing in profitable business for her division, whereas the director wants the company as a whole to thrive. The public transit project, for example, might be unprofitable for the division in charge of the project but more appealing to the firm director if it ensures the goodwill of the government for the firm.

2.3 Other Applications

Another application is real estate development negotiations where, for instance, a large company negotiating with a city the terms on which it can build its headquarters there. One example is Amazon's failed negotiations about a second headquarters in the New York City in 2019. Jeff Bezos could conduct these negotiations himself or delegate them to the company executives who would know more about the benefits and pitfalls involved, such as potential tax benefits and opposition from New York residents.

[7] See section 8 for a review of other related literature.

The model also applies to buyers hiring agents to acquire objects such as houses for them. Here the principal is the buyer and the proposer is the seller. The agent knows more about the value of the object than the potential buyer. The agent only gets a payoff from agreement if the agent negotiates the agreement. The structure of the agent's compensation is fixed and the interests of the buyer and the agent are misaligned: the agent only cares about getting the maximal compensation and possibly avoiding litigation, while the buyer cares about acquiring a high-quality object at a low price.

Finally, the model applies to labor relations experts representing trade unions in labor negotiations. Here the principal is the trade union and the proposer is the management of a company. The labor relations expert knows more than the union about how costly implementing the proposed deal is. The expert gets a benefit only if she brokers the deal successfully. The interests of the union and the expert are misaligned: the expert cares about her compensation and reputation and possibly puts some weight on the interests of the management, while the union only cares about getting the best possible deal.

3 Model

3.1 Strategies, Timing and Payoffs

There is a proposer S, an agent A, and a principal P. They can agree to implement a project producing a benefit of size 1. The agent and the principal have costs c and $k(c)$ of implementing the project respectively, while the cost of the proposer is normalized to 0. c follows a distribution F on $[0, 1]$ that admits a continuous and strictly positive density f. The agent is privately informed about the realization of c.[8]

k is strictly increasing and differentiable, so the preferences of the principal and the agent are aligned but only imperfectly. For much of the paper, I also assume that the conflict of interest between the principal and the agent is complex: k crosses the 45-degree line at least once. That is, there are circumstances in which the principal is more eager to agree than the agent, and circumstances in which she is less eager to agree.

The timing within a period is as follows. First the principal decides whether to delegate or not. Importantly, the principal cannot commit to delegation policies beyond the current period. After observing the principal's delegation decision, the proposer makes an offer x

[8] This model is equivalent to a model where the agent is a buyer who is privately informed about her value for a good and the proposer is the seller of the good. I describe the model in terms of costs of implementing a project because this description fits the applications better.

that gives the share x of the benefit to the responder side and leaves the share $1-x$ to herself. If the principal did not delegate, the principal makes the decision to accept or reject himself. If the principal delegated, then the agent makes the decision to accept or reject. If the offer is accepted, the game ends and the players collect their payoffs. If the offer is rejected, the game continues and we go to the next period, where the above timing is repeated. The proposer thus makes all the offers. The game lasts forever, and if no agreement is reached, all agents receive the payoff of 0.

All players discount the future at rate r. The length of the time period is Δ. For tractability reasons, as is standard in the recent literature on dynamic bargaining with private information, I focus on the limits of sequences of discrete-time equilibria as the length of the time period Δ goes to 0.

A strategy of the principal is a mapping from the history of rejected offers and delegation decisions to the probability of delegation in the current period and to an acceptance strategy specifying the set of offers that the principal accepts in the current period. A strategy of the proposer is a mapping from the history of rejected offers and delegation decisions to the distribution over offers to the agent in the current period and to the offer to the principal in the current period. A strategy of an agent with cost c is a mapping from the history of rejected offers and delegation decisions to an acceptance strategy specifying the set of offers that the agent accepts in the current period.

The agent gets the share offered by the proposer and pays the cost only if the agent makes the deal. The principal gets the share offered by the proposer and pays the cost whenever there is an agreement. The interpretation is that the proposer offers to the principal and the agent something that they both value, so an offer of x accepted by the agent results in both the principal and the agent getting the payoff of x minus the cost. Thus if the cost of the agent is c and there is agreement at time t with the proposer offering share x, then (evaluating at time 0) the payoff of the principal is $e^{-rt}(x-k(c))$, the payoff of the proposer is $e^{-rt}(1-x)$ and the payoff of the agent is $e^{-rt}(x-c)$ if the agent made the deal and is 0 if the principal made the deal.[9]

3.2 Equilibrium Definition

We can show using standard arguments that in any (sequential) equilibrium after any history for any offer x there exists a threshold cost θ (depending on the history, the offer x

[9] Section 7 discusses the extension of the model to the case where the agent also gets a payoff if the principal made the deal.

and the length of the period Δ) such that all agents with costs less than θ accept the offer x and all agents with costs greater than θ reject it. This result is known as the "skimming property". The result implies that after any history the proposer and the principal believe that the set of the possible agent's costs is of the form $[\theta, 1]$ for some $\theta \in [0, 1]$. I will henceforth refer to θ, the lower bound on the agent's costs, as the state.

Following Fudenberg et al. (1985), we say that an equilibrium is weak-Markov if in this equilibrium the principal conditions her delegation decision only on the state θ and on the offer made by the proposer to the agent in the previous period and conditions her acceptance decisions only on the state θ and the current offer made by the proposer, the proposer conditions her offers to the agent only on the state θ and on the offer made by the proposer to the agent in the previous period and conditions her offers to the principal only on the state θ, and the acceptance strategy of the agent depends only on the current offer made by the proposer. An equilibrium is strong-Markov if it is weak-Markov and, in addition, the principal conditions her delegation decisions and the proposer conditions her offers to the agent only on the state θ.

As Fudenberg et al. (1985) and others have shown, strong-Markov equilibria may not exist. I restrict our attention to weak-Markov equilibria. Ausubel and Deneckere (1989), among others, prove the existence of weak-Markov equilibria. Ausubel and Deneckere (1989) show that, in a classical model of bargaining with private information, in a weak-Markov equilibrium the proposer's offer strategy on the equilibrium path is pure and depends only on the state.[10] In my setting, the proof in Ausubel and Deneckere (1989) can be adapted to show that for all $\Delta > 0$ there exists a weak-Markov equilibrium in which on the equilibrium path (i) the proposer's offer strategy is pure and depends only on the state and (ii) the principal conditions her delegation decision only on the state θ. Moreover, as $\Delta \to 0$, any limit of weak-Markov equilibria must have these properties. I will thus write the strategy of the proposer (on the equilibrium path) as a mapping from the state to the offer and the strategy of the principal (on the equilibrium path) as a mapping from the state to the probability of delegation.

Formally, a weak-Markov equilibrium is a set of functions (ν, χ, y, π, a)[11] where $\nu_\Delta(x)$ is the agent's acceptance rule that specifies the highest type that accepts the offer x, $\chi_\Delta(\theta)$ is the rule that specifies the offers the proposer makes to the agent given that the state is θ,

[10] Except possibly for the first price. See Proposition 4.3 in Ausubel and Deneckere (1989) for the proof.

[11] For convenience, I am going to use different notation for the strategies in equilibria with immediate agreement (in the limit as $\Delta \to 0$). In particular, I will use $X(\theta)$ to denote the take-it-or-leave-it (TILI) offer of the proposer to the agent in state θ and we will use J to denote the feasible TILI offers. See the Appendix for more details.

$y_\Delta(\theta)$ is the rule that specifies the offers the proposer makes to the principal given that the state is θ, $\pi_\Delta(\theta)$ is the probability that the principal does not delegate given that the state is θ, and $a_\Delta(x;\theta)$ is the probability that the principal accepts the offer x given that the state is θ such that (i) the proposer chooses $\chi_\Delta(\theta)$ and $y_\Delta(\theta)$ to maximize her expected discounted payoff, (ii) the acceptance rule ν maximizes the expected discounted payoff of the agent, (iii) the principal chooses the non-delegation probability $\pi_\Delta(\theta)$ and the acceptance probability $a_\Delta(x;\theta)$ to maximize her expected discounted payoff.[12]

Finally, I focus on equilibria in which every continuation is Pareto undominated (an equilibrium is Pareto undominated if there does not exist another equilibrium in which each player gets a weakly higher payoff and at least one player gets a strictly higher payoff). This is going to rule out equilibria in which, after negotiation is delegated to the agent, the proposer does not make an offer that is acceptable to some agent types and instead waits until there is an opportunity to strike a deal with the principal.[13] Lemma 4 in the Appendix provides more details.

I assume that $1 - E[k(c)] > 0$. This means that if the principal gets no additional information about the cost, there is an offer she would be willing to accept.[14] I let $\overline{c} = \sup\{c > 0 : k(c) \leq c\}$ denote the highest cost at which the principal is more eager to agree than the agent. Letting c_0 denote the state in which the principal's expected cost is 1 and letting $Y(\theta)$ denote the proposer-optimal TILI offers in state θ, I assume that if $\overline{c} < 1$, then the state c_1 such that $c_0 \in conv(Y(c_1))$ satisfies $c_1 > \overline{c}$.[15] This amounts to assuming that the set of states for which the principal does not want to agree even if the proposer offers the whole surplus is small enough. Clearly, if this set is so large as to include all states, no agreement is possible. In general, as this set grows larger, some equilibria with delegation cease to exist. I thus focus on parameters that support the largest set of equilibria.

I let $W(\theta, x) = \frac{F(x)-F(\theta)}{1-F(\theta)}(1-x) + \frac{1-F(x)}{1-F(\theta)}(1-y(x))$ for $x \in (\theta, c_0)$ denote the proposer's payoff from making a TILI offer to the agent and, if the offer is rejected, making a deal with the principal. I assume that second order condition holds, so that $\frac{\partial^2}{\partial x^2}W(\theta, x) < 0$.[16]

[12] I may need to also allow the proposer to randomize among prices off the equilibrium path to make the principal indifferent between delegating and not. See the proof of Lemma 9 in the Appendix for more details.

[13] Unlike in other papers on bargaining with incomplete information (see, for example, Fuchs and Skrzypacz (2010)), in my setting requiring stationarity is not enough to rule out equilibria in which there is zero probability of agreement with the agent in some periods. This is because my model admits multiple stationary equilibria and the threat of reverting to stationary equilibria that are bad for the proposer can sustain equilibria in which the proposer makes no acceptable offers to the agent.

[14] If this fails, then there is a unique equilibrium in which there is no agreement.

[15] Here *conv* denotes the convex hull.

[16] We can show that the second order condition is $f(x)(k'(x) - 2) + f'(x)(k(x) - x) < 0$. Thus it requires that k does not increase too steeply, f does not increase too steeply when the agent is more eager to agree,

It can be shown that if the second order condition fails, some equilibria may disappear but no equilibria other than the ones I characterize can exist. Thus requiring the second order condition amounts to focusing on parameters that support the largest set of equilibria.

4 Equilibria with Delay

4.1 Benchmarks

I first discuss two simpler benchmark models: the one without a principal, and the one without an agent. If there is no principal, then the game is standard incomplete information bargaining where the uninformed proposer makes all the offers. In this case, the Coase conjecture obtains: agreement is immediate and the informed party, the agent, gets all the surplus. If there is no agent, then bargaining proceeds under complete information. In this case, agreement is also immediate and, since the proposer makes all the offers, the proposer gets all the surplus. Thus a natural question is, what happens when there is both the principal and the agent?

4.2 Delay

I start describing equilibria with delay by considering a simple specification of conflict of interest in section 4.3. There, for high enough costs, the interests of the principal and the agent are aligned, while for lower costs the agent is more eager to agree. I use this case to explain the intuition behind an equilibrium with delay. I then characterize the class of equilibria with delay under a general conflict of interest in section 4.4.

The functions (ν, χ, y, π, a) determine the sequence of offers on the equilibrium path starting in any state θ. In particular, the offer to the agent in state θ is $x = \chi_\Delta(\theta)$, and the offer to the agent in the next period is $\chi_\Delta(\nu_\Delta(x))$. We let $\theta_+ = \nu_\Delta(\chi_\Delta(\theta))$ denote the threshold cost in the next period given that the threshold cost today is θ.[17]

We let $\dot{\theta} = \lim_{\Delta\to 0} \frac{\theta_+ - \theta}{\Delta}$ denote the rate at which the state changes on the equilibrium path. We say that there is *smooth screening* in state θ if $\dot{\theta} \in (0, \infty)$. Note that, because $\dot{\theta} < \infty$, our definition of (smooth) screening entails *no atoms* in agreement: as the length of a time interval goes to 0, the probability of agreement in this interval also goes to 0. At the

and f increases steeply enough when the agent is less eager to agree. For example, if f is uniform, it requires that $k'(x) < 2$.

[17] We will suppress the dependence of θ_+ on Δ.

same time, there is no waiting to agree: the probability of agreement in any fixed interval of time is strictly positive. We are in a *smooth screening phase* of an equilibrium in an interval of states I if there is smooth screening in all states $\theta \in I$.

$\dot{\theta}$ indicates how fast the state moves up, with higher values of $\dot{\theta}$ corresponding to faster screening. We say that there there is *delay* if the expected time to agreement is strictly greater than 0 (in the limit as Δ goes to 0). Note that smooth screening and delay are limit notions that apply only to the frequent-offer limit of discrete-time games and not to discrete-time games directly.

For simplicity, we assume that the set $\{c : k(c) > c\}$ is a finite union of intervals. We say that the principal is *less (more) eager to agree* than the agent in state c if $k(c) < c$ ($k(c) \geq c$).

4.3 Delay when Agent is More Eager to Agree

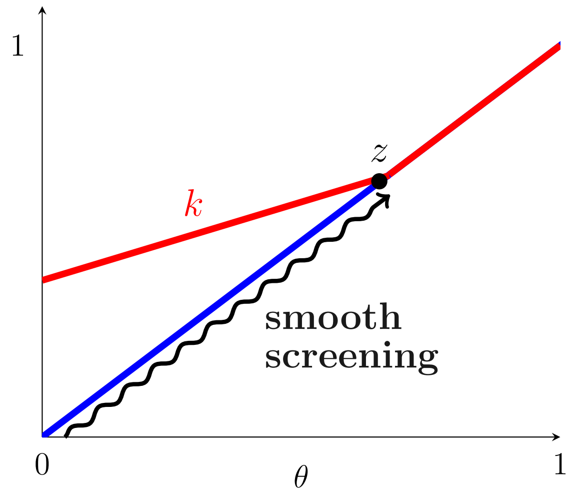
I start by considering a simple case where the agent is always more eager to agree than the principal. In particular, when the cost is low enough, the agent is strictly more eager to agree. For higher costs, the payoff of the agent is equal to the payoff of the principal, so there is no conflict of interest. Proposition 1 provides a result on an equilibrium with delay in this case.

Proposition 1. *If $k(c) > c$ for $c < z$ and $k(c) = c$ for $c > z$ for some $z \in (0, 1)$, there exists an equilibrium with smooth screening starting in state 0. There is no other equilibrium with smooth screening starting in state 0. In this equilibrium, screening ends in state z.*

I now describe the dynamics that arise in the equilibrium with smooth screening starting in state 0 and provide intuition for them. In this equilibrium, the principal randomizes between delegating and not in every period, taking the decision herself at some rate. The state moves slowly from 0 to z. When the state reaches z, there is no more delegation. In state z, the principal accepts with probability one the offer $y(z) = E[k(c)|c \geq z]$ made by the proposer. Figure 1 illustrates the equilibrium dynamics. In the figure, the state θ is on the x-axis, the agent's cost is the 45-degree line in blue and the principal's cost k is in red.

Several observations will help us shed light on equilibrium dynamics. First, if the proposer makes an offer to the principal, then the principal must be indifferent between accepting and rejecting the offer. This is because if the principal strictly preferred to accept, then the proposer can decrease the offer slightly and still get the principal to accept.

Figure 1: Delay when Agent is More Eager to Agree



Moreover, the principal must accept the offer of the proposer with probability one: if the principal was rejecting the offer with a positive probability, the proposer could increase the offer slightly and get the principal to accept with probability one. Thus, whenever the principal does not delegate in state θ, the proposer makes the offer $y(\theta)$ and the principal accepts.

Second, randomization by the principal between delegating and not is key to sustaining delay. If the principal always delegated, then the problem facing the proposer would be the same as when there is no principal. Then the proposer would want to screen the agents too fast, in line with the Coase conjecture. If the principal never delegated, there would be immediate agreement between the proposer and the principal. Thus, as long as there is delay, the principal must be indifferent between delegating and not.

Because the principal must be indifferent before delegation and after delegating and receiving an offer from the proposer, the payoff of the principal is 0 in every period. I show that this implies that the offer that the agent with cost c accepts is $\chi(c) = k(c)$. Thus, as long as smooth screening continues, the agent accepts the offer equal to the cost of the principal, which is strictly higher than the cost of the agent on the states where the screening happens. Observe that the type of the agent is identifiable during screening: the principal knows that, should she delegate in state θ, the agents accepting the offer will be the agents with costs very close to θ. Yet, in spite of the agent's cost being identifiable, neither the principal nor the proposer are able to extract all surplus from the agent in equilibrium.

4.3.1 Proposer's Problem

We now explain the problem the proposer faces after the principal delegates. To do this, we first introduce some helpful notation. Recall that, given that the length of the time period is Δ, $\pi_\Delta(\theta)$ is the probability that the principal takes the decision herself in state θ.

We let $V_\Delta(\theta)$ denote the ex-post value of the proposer in state θ (after it is known that the principal delegated in this period). Given an equilibrium, we let Θ_Δ denote the thresholds on the equilibrium path.

After the principal delegates in state θ, the proposer offers $\nu_\Delta^{-1}(\theta_+) = \chi_\Delta(\theta)$ to the agent. Because agents with costs in $[\theta, \theta_+]$ accept this offer, the probability that the offer is accepted is $\frac{F(\theta_+)-F(\theta)}{1-F(\theta)}$. With a complementary probability, the offer is rejected. Then, in the next period, with probability $\pi_\Delta(\theta_+)$, the principal does not delegate. In this case, the proposer offers $y(\theta_+)$ to the principal, and this offer is accepted with probability one. With probability $1 - \pi_\Delta(\theta_+)$, the principal delegates, in which case the proposer gets the payoff $V_\Delta(\theta_+)$.

Note that the proposer choosing which offer to make is equivalent to her choosing the agent's acceptance threshold. Then a necessary condition for an equilibrium is that

$$\begin{aligned} V_\Delta(\theta) = \; & \max_{\theta_+ \in \Theta_\Delta} \frac{F(\theta_+) - F(\theta)}{1 - F(\theta)} \left(1 - \nu_\Delta^{-1}(\theta_+)\right) + \\ & \frac{1 - F(\theta_+)}{1 - F(\theta)} e^{-r\Delta} (\pi_\Delta(\theta_+)(1 - y(\theta_+)) + (1 - \pi_\Delta(\theta_+)) V_\Delta(\theta_+)) \end{aligned} \tag{1}$$

We let $V(\theta) = \lim_{\Delta \to 0} V_\Delta(\theta)$ denote the limit of the proposer's value function and let $\pi(\theta) = \lim_{\Delta \to 0} \frac{\pi_\Delta(\theta)}{\Delta}$ denote the rate of non-delegation. Note that choosing the threshold θ_+ is the same as choosing $\frac{\theta_+ - \theta}{\Delta}$, which converges to $\dot{\theta}$ as Δ goes to 0. Subtracting $e^{-r\Delta} V_\Delta(\theta)$ from both sides of (1), dividing by Δ and taking the limit as $\Delta \to 0$, we find that the proposer's HJB equation is

$$rV(\theta) = \max_{\dot{\theta} \in [0,\infty]} \pi(\theta)(1 - y(\theta) - V(\theta)) + \dot{\theta} \left(\frac{f(\theta)}{1 - F(\theta)} (1 - \chi(\theta)) - \frac{f(\theta)}{1 - F(\theta)} V(\theta) - V'(\theta) \right)$$

The proposer chooses $\dot{\theta}$ to maximize her payoff. By the standard arguments,[18] the proposer must be indifferent between speeding the screening up and slowing it down, which implies that the coefficient on $\dot{\theta}$ must be 0. This observation yields the following two equations: $(1 - F(\theta))V'(\theta) + f(\theta)V(\theta) - f(\theta)(1 - k(\theta)) = 0$ and $\pi(\theta) = \frac{rV(\theta)}{1 - E[k(c)|c \geq \theta] - V(\theta)}$.

[18] See Fuchs and Skrzypacz (2010).

Because the proposer is indifferent between speeding the screening up and slowing it down, the payoff of the proposer is equal to the payoff she would get from never making an acceptable offer to the agent and instead waiting for the principal to not delegate so that the proposer can make the deal with the principal directly. An important reason why delay is possible here is that if the proposer could choose whether the principal delegates in this period, she would prefer that, with probability one, the principal does not delegate. Thus, because the principal randomizes, non-delegation by the principal becomes an event that the proposer is willing to *wait for*.

Because the principal takes the decision herself at some positive rate, which means that $\pi(\theta)$ is strictly positive, the proposer gets a strictly positive payoff in equilibrium. This implies that the part of the Coase conjecture that requires the proposer to get a payoff of 0 fails. Moreover, because there is delay in agreement, the part of the Coase conjecture that requires immediate agreement fails.

4.3.2 Agent's Incentives

If an agent with cost θ_+ accepts the offer $\chi_\Delta(\theta)$, she gets the payoff $\chi_\Delta(\theta) - \theta_+$. If she rejects the offer, then in the next period, with probability $\pi_\Delta(\theta_+)$, the principal takes the decision herself, which gives the payoff of 0 to the agent. With a complementary probability, the principal delegates, in which case the proposer makes the offer $\chi_\Delta(\theta_+)$, which the agent with cost θ_+ accepts with probability one. Because the agent with cost θ_+ has to be indifferent about accepting $\chi_\Delta(\theta)$, we have[19]

$$\chi_\Delta(\theta) - \theta_+ = e^{-r\Delta}(1 - \pi_\Delta(\theta_+))(\chi_\Delta(\theta_+) - \theta_+)$$

Dividing by Δ, taking the limit as $\Delta \to 0$ and recalling that $\chi(\theta) = k(\theta)$, we obtain

$$\dot{\theta} = \frac{(r + \pi(\theta))(k(\theta) - \theta)}{k'(\theta)}$$

If we hold the rate of non-delegation $\pi(\theta)$ fixed, then the speed of screening increases as k, the cost of the principal, increases. This is because the lower the offer k the proposer makes to the agent is, the fewer agents are willing to accept this offer and the lower the state θ_+ after a rejection is. Moreover, the speed of screening increases as $k'(\theta)$, the derivative

[19] By a standard argument, since the agent's objective function is supermodular in the agent's cost c and the cost c' of the type the agent imitates, the skimming property implies that this condition is not only necessary but also sufficient for the optimality of the agent's strategy.

of the cost of the proposer in state θ, decreases. This is because if k' is smaller, then the next offer upon rejection is closer to the current offer. Thus the benefit to rejecting the current offer and pretending to have a higher cost is smaller. This means that more agents are willing to accept the current offer, which raises the state θ_+ after a rejection.

4.4 Delay under Complex Conflict of Interests

Oftentimes the issue at stake in bargaining is complex, which engenders complex conflict of interests. We next characterize the class of equilibria with delay under such conflict of interests. That it, in this section we assume that there is preference reversal: k crosses the 45-degree line at least once.

It is important to consider complex conflict of interests because under such conflict, for a range of parameters, there does not exist a Coasean equilibrium: the payoff of the proposer is bounded away from zero in all sequential equilibria. This is in contrast to much of the literature on bargaining with private information where the Coase conjecture obtains.

The discussion in sections 4.3.1 and 4.3.2 yields the following result about the smooth screening phase of an equilibrium with delay.

Proposition 2 (Smooth screening phase). *Consider a sequence of games indexed by the period length and a corresponding sequence of equilibria with a frequent-offer limit that has delay. Then $V_\Delta(\theta)$, $\chi_\Delta(\theta)$ and $\pi_\Delta(\theta)$ for this sequence of equilibria converge to $V(\theta)$, $\chi(\theta)$ and $\pi(\theta)$ satisfying $\chi(\theta) = k(\theta)$, $(r + \pi(\theta))(k(\theta) - \theta) = \dot{\theta}k'(\theta)$, $(1 - F(\theta))V'(\theta) + f(\theta)V(\theta) - f(\theta)(1 - k(\theta)) = 0$ and $\pi(\theta) = \frac{rV(\theta)}{1 - E[k(c)|c \geq \theta] - V(\theta)}$ for all states θ in which there is smooth screening in the limit.*

Proposition 2 pins down the proposer's value function, the offers to the agent, the speed at which the state moves up and the rate of non-delegation in the smooth screening phase of any equilibrium with delay. We now let

$$\mathcal{I} = \{c : k(c) \geq c \text{ and there exists } c' > c \text{ such that } k(c') = c'\}$$

denote the set of all states where the principal is less eager to agree than the agent except for the highest such interval.

An equilibrium with delay in which smooth screening starts in state $s > 0$ has the following form: first the principal delegates (with probability one), then the proposer makes the offer $k(s)$, and all agents with costs below s accept. If the offer is rejected, then the equilibrium with smooth screening starting in state s and ending in the lowest state s' above s in which there is no conflict of interest ($k(s') = s'$) is played. This equilibrium has the form described in Proposition 2. In the equilibrium with delay with smooth screening starting in state 0, screening starts immediately, and there is no atom of acceptances in the beginning. Theorem 1 characterizes equilibria with delay.

Theorem 1. *Every equilibrium with delay has the following form:*

1. *breakthrough phase: principal delegates with probability one and proposer offers $k(s) \in \mathcal{I}$ that is accepted by all agents with cost $c \leq s$;*
2. *smooth screening phase.*

There exists an equilibrium with smooth screening phase starting in state s if and only if $s \in \mathcal{I}$. For each $s \in \mathcal{I}$, the equilibrium with smooth screening starting in state s is unique.

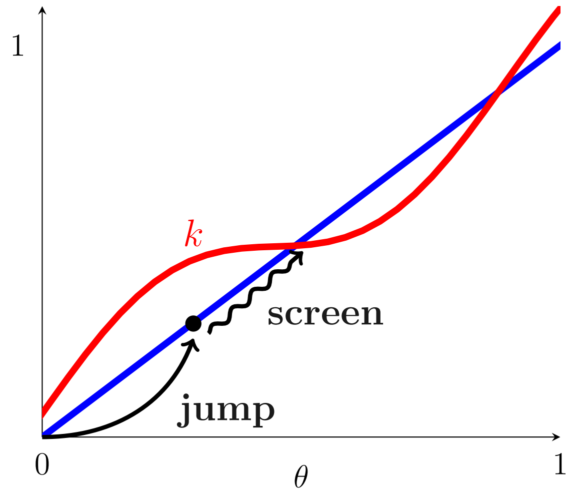
Theorem 1 provides a tight characterization of all equilibria with delay, showing that *every* equilibrium with delay must start with an atom of acceptances and then continue with smooth screening. That is, there cannot be atoms in agreement after the first instance, nor can there be stretches of time without a positive probability of agreement with the agent.

Theorem 1 implies that, given any interval where the principal is less eager to agree and there is no conflict of interest in the highest state, there exists an equilibrium in which smooth screening happens on exactly this interval. Conversely, if we take an equilibrium with delay, then the set of states on which smooth screening happens must be an interval of this form.

Figure 2 illustrates the dynamics in an equilibrium with delay. In the figure, the cost of the agent is in blue, and the cost of the principal is in red. In the equilibrium depicted in the figure, first there is delegation with probability one and the proposer makes an offer accepted by a positive mass of agents with costs in the lowest interval where the agent more eager to agree. If the offer is rejected, then smooth screening starts and the state moves slowly to the first point in which there is no conflict of interest.

I now provide some intuition for why any equilibrium with delay must have the form described above. Given a state s such that $k(s) > s$, I let $B(s) = \inf\{c : c \geq s, k(c) \leq c\}$ denote the lowest state above s where the principal is more eager to agree than the agent. I will explain why screening must end in state $B(s)$. Observe that in the state s' where the screening ends, if there is delegation, the proposer makes a take-it-or-leave-it (TILI) offer $X(s')$ to the agent. In this case, the agent with cost s' has to be indifferent between accepting the last screening offer $\chi(s') = k(s')$ and waiting for the TILI offer, so we must

Figure 2: Equilibrium with Delay



have $X(s') = k(s')$. This means that the offer $X(s')$ cannot be higher than $B(s)$ because $B(s) = k(B(s))$ and k is increasing.

Moreover, it can be shown that, whenever the principal is less eager to agree than the agent in state s, the most preferred TILI offer of the proposer in s is between $B(s)$ and s. In general, the proposer may not be able to make her most preferred TILI offer: if in state s the principal strictly prefers to delegate, a TILI offer s is not consistent with equilibrium. However, if a state s in which the principal is less eager to agree is a feasible TILI offer, then all offers $s'' \leq s$ in the interval where the principal is less eager to agree containing s are feasible TILI offers. Because an offer $X(s')$ lower than $B(s)$ is feasible by the argument outlined above, this implies that, in particular, the offer s' is a feasible TILI offer in state s'.

It can also be shown that, whenever the proposer can make a TILI offer equal to the current state, the principal does not want to delegate. Therefore, the principal does not want to delegate in state s'. However, if the principal does not delegate in the terminal state s', then the agent with cost s' has to be indifferent between accepting the last screening offer $\chi(s') = k(s')$ and getting a payoff of 0 from no delegation in the next period. This implies that we must have $k(s') - s' = 0$, which means that screening ends in state $B(s)$.

5 Immediate Agreement Equilibria

5.1 Immediate Agreement under Complex Conflict of Interests

I now turn to characterizing equilibria with immediate agreement under complex conflict of interest. Here I maintain the assumption that k crosses the 45-degree line at least once. I show that any IA equilibrium has the following structure. If the principal delegates, she delegates with probability one. If the offer the proposer makes to the agent is rejected, then there is no further delegation and the proposer makes a deal with the principal.

I let $D = \{c : k(c) \leq c\}$ denote the set of costs for which the principal is more eager to agree than the agent. Theorem 2 characterizes the IA equilibria under complex conflict of interest.

Theorem 2. *There exists an IA equilibrium in which offer x is made after delegation if and only if $x \in D$. There exists an IA equilibrium in which the principal does not delegate.*

Theorem 2 says that the proposer makes an offer x to the agent in some IA equilibrium if and only if the principal is more eager to agree than the agent when the cost is x. Theorem 2 also says that there is an IA equilibrium without delegation. In this equilibrium, the principal agrees with the proposer with the offer $y(0)$.

I now provide the intuition for the structure of the IA equilibria. It can be shown that, in any state θ where the principal is more eager to agree that the agent, the TILI offer to the agent that the proposer would most like to make is θ. This result is driven by the fact that, after the agent rejects the TILI offer, the proposer gets to make a deal with the principal. Thus the proposer faces a tradeoff between having some agents accept the offer and then making a deal with the principal in a higher state and making the deal with the principal right away in the current state. It turns out that in states where the principal is more eager to agree, this tradeoff is resolved in favor of making the deal with the principal immediately.

We can also show that, anticipating a TILI offer,[20] the principal is willing to delegate in state θ if and only if the payoff that the TILI offer gives to the proposer is smaller than the payoff $1-y(\theta)$ the proposer would obtain from making a deal with the principal immediately. Moreover, the payoff that the proposer gets in state θ from offers close enough to θ is greater than $1 - y(\theta)$, while the payoff from higher offers is smaller than $1 - y(\theta)$. This implies that the principal is willing to delegate only if she anticipates that the proposer will make a

[20] This reasoning applies only to TILI offers that result in the proposer making a deal with the principal if the offer is rejected by the agent.

sufficiently high TILI offer.

We can further show that in states where the principal is more eager to agree, the principal is willing to delegate in state θ *no matter* which offer $x > \theta$ the proposer makes. Only if the principal anticipates that the proposer will make the offer $x = \theta$ is the principal indifferent between delegating and not. Then whether the principal delegates in θ depends on what the principal expects the proposer to do. If the principal expects the proposer to make the offer $x = \theta$, then it is consistent with equilibrium for the principal not to delegate, whereas if the principal expects the proposer to make the offer $x > \theta$, then the principal must delegate. Thus, by conjecturing that the proposer makes the offer $x = \theta$ in state θ, we can render any state $\theta \in D$ a feasible TILI offer.

As mentioned in the previous section, if some offer in an interval where the principal is less eager to agree is a feasible TILI offer, then all offers below it in this interval are also feasible TILI offers. If instead some offer x in this interval was feasible while an offer x' slightly below it was not, then in state x', the principal would delegate. However, we can show that if x' is close enough to x, then the proposer would rather make the offer x or lower than some offer above x. Moreover, if the principal anticipates that the proposer will make the offer x or lower, then the principal does not want to delegate. Thus all offers in the interval that are below x must be feasible.

If the principal is less eager to agree than the agent when the cost is 0, then no offers in the lowest interval where the principal is less eager to agree can be made in an IA equilibrium with delegation. This is because if some offer in such an interval was feasible, so would all offers above it. Then the proposer will make one of these offers, which, in turn, would make the principal unwilling to delegate – because these offers are not high enough.

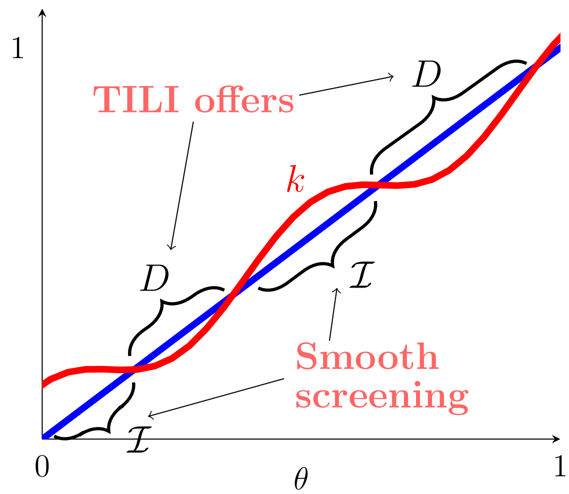
It can be shown that, whenever the principal delegates, (if the state is sufficiently low) the offer that the proposer makes is the lowest feasible offer. Moreover, any subset of states where the principal is more eager to agree can be rendered feasible. Finally, the feasible part of any interval where the principal is less eager to agree (except for the lowest such interval) must be a lower part of this interval by the argument above. These observations imply that the offer made must be in some interval where the principal is more eager to agree. This is exactly what Theorem 2 says.

We conclude this section by observing that the equilibria we have described so far are the only equilibria that exist (in the class of equilibria that we have restricted our attention to).

Corollary 2.1. *Every equilibrium is either an equilibrium with delay with smooth screening* *starting in $s \in \mathcal{I}$, or an IA equilibrium with no delegation, or an IA equilibrium with offer $s \in D$.*

Figure 3 illustrates the equilibria that exist for a given configuration of costs.

Figure 3: Equilibria



5.2 Immediate Agreement under One-Directional Conflict of Interest

Proposition 3 describes the IA equilibria when the agent is strictly more eager to agree than the principal, and when the agent is strictly less eager to agree.

Proposition 3. *If $k(c) < c$ for all c, then for all $x \in (0, 1]$ there exists an IA equilibrium with this offer. If $k(c) > c$ for all c, then if there is delegation in an IA equilibrium, the offer to the agent is strictly less than* 1*.*

Proposition 3 says that if the conflict of interest is one-directional and strict, and the principal is more eager to agree, then there exists a Coasean equilibrium: there agreement happens immediately with offer 1. Other IA equilibria exist too: in fact, for any offer $x \in (0, 1]$ we can find an IA equilibrium where this is the offer the proposer makes to the agent. If the agent is more eager to agree, then in any equilibrium there is immediate agreement and no equilibrium is Coasean: even though agreement happens immediately, the offer made after delegation is strictly less than 1. In this case, for some parameters the equilibrium does not involve delegation.

Moreover, we can show that if $k(1) > 1$, then there is no Coasean equilibrium in the class of all sequential equilibria, and not just in the class of all weak-Markov Pareto undominated equilibria. The reason is as follows. Recall that c_0 is the state in which the expected cost of the principal is 1. If the state is above c_0, then the principal is not willing to agree even if offered the whole surplus, and so is not willing to delegate no matter which offer she expects the proposer to make. This means that in any state above c_0 there is no further delegation, so any such state is a feasible TILI offer. This implies that the proposer making the offer 1 cannot be part of any equilibrium: the proposer strictly prefers to make a TILI offer between c_0 and 1 instead, and such offers are feasible.

I next explain why the highest offer made in an IA equilibrium can be much lower than 1. Recall that in state c_1, c_0 is a preferred offer for the proposer. Because it is feasible, the proposer is willing to make this offer in state c_1 if given a chance. By the reasoning explained earlier, whenever the principal expects the proposer to make the proposer's preferred offer (provided that this offer is at most c_0), the principal does not want to delegate. Then the principal does not want to delegate in c_1, so c_1 is a feasible TILI offer. We can proceed to the states below c_1 in a similar manner and, in doing so, may reach a state much lower than 1 that is a feasible TILI offer.

The results imply that there is a discontinuity in the attainable equilibrium payoffs as we change the conflict of interest, in the following sense. If we take k that crosses the 45-degree line at least once and satisfies $k(1) > 1$ and $c_1 > \bar{c}$, we know that the highest offer that can be made in an IA equilibrium is $\bar{c}$. If we now keep k below $\bar{c}$ the same but decrease k above $\bar{c}$ such that $k(1) \leq 1$, then the Coasean equilibrium appears, so the highest offer made in an IA equilibrium is 1. This exposes a lack of robustness of the Coasean outcome in settings where bargaining can be delegated: if there is even a slight possibility that the principal may not be willing to agree when offered the whole surplus (which happens when $k(1) > 1$), the Coasean outcome disappears.

A consequence of our results is that the principal and the agent may be hurt if the support of the distribution of costs is expanded by adding lower costs. For instance, if the distribution of costs is only supported on $c \in [c', 1]$ satisfying $k(c) > c$, then in equilibrium there is immediate agreement at an offer x greater than c'. If, however, we expand the support to $[0, 1]$ and assume a complex conflict of interest, then equilibria with delay appear, and in all equilibria the offers are strictly lower than x. This result holds regardless of the probability assigned to the lower costs that we add. Substantively, the result suggests that improving military or economic strength of one side to the bargaining may worsen the outcomes for that side.

6 Comparative Statics and Welfare

6.1 Welfare

How do the players rank payoffs from different equilibria? In this section I summarize the results on the welfare ranking of the equilibria from the point of view of different players.[21] The proposer likes the IA equilibrium with no delegation best. The proposer prefers the IA equilibrium with offer θ to the equilibrium with delay with screening starting in state θ. Among the equilibria with screening, the proposer prefers equilibria with screening starting in lower states. Among the IA equilibria with delegation, the proposer likes the ones with lower offers better. The payoff of the proposer from any equilibrium with delay with screening ending in state θ is greater than her payoff from any IA equilibrium with offer above θ.

The IA equilibrium with no delegation is the least preferred one for the agent because the agent gets a payoff of 0 in this equilibrium. Among the IA equilibria with delegation, the agent prefers the equilibria with higher offers. The agent prefers the equilibrium with delay with screening starting in state θ to the IA equilibrium with offer θ. This is because more types of agents strike a strictly profitable agreement in the equilibrium with delay. Finally, among the equilibria with screening, the agent prefers equilibria with screening starting in higher states.

In general, equilibria with higher initial offers to the agent may be worse for the principal. However, we can establish that these equilibria are better for the principal if we make an assumption on the parameters. In particular, if $f(x)k(x) < 1$ for all $x \in [0, 1]$, then among the IA equilibria, the principal prefers the equilibrium with the highest offer and among the equilibria with delay, the principal prefers the equilibrium with screening starting in the higher state. This is because, as the initial offer increases, the share that the principal gets increases but, at the same time, agents with higher costs start accepting the offer. If the mass of the agents accepting this higher offer is too high (which happens if f is too high) or the cost of the principal increases too steeply (which happens if k is too high), then the increase in cost overwhelms the benefit of increasing the offer.

6.2 Comparative Statics

In this section I discuss comparative statics in equilibria with delay. Given the principal's cost functions k and $\tilde{k}$, we say that the *conflict of interest is greater* under $\tilde{k}$

[21]Some of the results are proven in Proposition 7 in the Appendix.

than under k on a set Ω if, for $c \in \Omega$, $\tilde{k}(c) > k(c)$ if $k(c) > c$, $\tilde{k}(c) < k(c)$ if $k(c) < c$, $\tilde{k}(c) = k(c)$ if $k(c) = c$, and, for $c \notin \Omega$, $\tilde{k}(c) = k(c)$.[22] Intuitively, the conflict of interest is greater if the costs for which the principal is more or less eager to agree than the agent are the same but the cost difference is greater.

Recall that, given an equilibrium with delay, V is the value function of the proposer, π is the rate at which the principal takes the decision herself and $\dot{\theta}$ is the rate at which the state moves up in the smooth screening phase.[23] To state the comparative statics, we fix an equilibrium with delay with screening starting in some state θ. Proposition 4 presents the first set of comparative statics.

Proposition 4.

1. *If the conflict of interest is greater under $\tilde{k}$ than under k, then V is larger under $\tilde{k}$.*
2. *If the conflict of interest is greater on $\{c : k(c) > c\}$ under $\tilde{k}$ than under k, then π is larger under $\tilde{k}$.*
3. *π is increasing in r and in θ. $\dot{\theta}$ is increasing in r.*

Proposition 4 shows that delegation is affected by the extent of the conflict of interest: when there is more conflict, delegation is used less, and if it is used, it benefits the opponent more. In particular, if we keep the states in which the principal and the agent disagree the same but increase the intensity of conflict, the payoff of the proposer increases. The reason for this is as follows. Because in an equilibrium with delay the proposer has to be indifferent about speeding up screening or slowing it down, the derivative of the proposer's value function in a given state has to be proportional to the difference between the state and the offer made to the agent in this state. As the conflict of interest increases, the offer k made to the agent increases. This is because, since the payoff from agreement is now lower for the principal, a higher offer keeps her indifferent between delegating and not. This means that the value function of the proposer now increases more steeply, which leads to the higher payoff of the proposer.

The second part of the Proposition says that if we increase the conflict of interest only on the states where the principal is less eager to agree (so that the principal becomes even less willing to agree on these states), then the probability of delegation decreases in any equilibrium with delay. This is because in an equilibrium with delay the payoff of the proposer is the same as in the circumstance where she never strikes a deal with the agent and

[22] If the conflict of interest is greater for all costs, we just say that conflict of interest is greater.

[23] Here we suppress the dependence of the parameters on the state in which the screening starts in the equilibrium with delay.

instead waits for the principal to make an agreement with her. This means that, keeping the value function of the proposer fixed, the probability of delegation is decreasing in the offer y that the principal receives. As the principal becomes less eager to agree, the offer $y(\theta)$ equal to the expected cost of the principal from agreement in state θ increases. Because increasing the conflict of interest also increases the value function of the proposer, the probability of delegation decreases.

The lower the state is, the more the principal delegates. Thus the probability of delegation decreases over time as the state goes up. There are two ways in which an increase in the state affects the probability of delegation: first, the offer y the principal accepts increases, and, second, the value function V of the proposer decreases. Because the probability of delegation is decreasing in y and in V, these effects go in the opposite directions. Proposition 4 shows that the first effect dominates.

Finally, the more patient the agents are, the more delegation there is and the slower the state moves up. The intuition for this result is that delegation causes delay. This is because, if there is no delegation, the principal would agree immediately, while the agent's cost may be too high to agree today. Patient agents are able to bear the delay more easily, so delegation becomes more likely as agents become more patient.

We now present the second Proposition on comparative statics. Here we examine how the probability of delegation and the rate at which the state moves up change as we make the principal less willing to agree and as we change the distribution of costs.

Proposition 5.

1. *For all $s \in \mathcal{I}$, if $\hat{k}(\theta) \geq k(\theta)$ for $\theta > B(s)$ and $\hat{k}(\theta) = k(\theta)$ for $\theta \leq B(s)$, then π and $\dot{\theta}$ are larger on $[s, B(s)]$ under $\hat{k}$ than under k.*
2. *If $g(\theta) > f(\theta)$ for $\theta < B(s)$ and F FOSD dominates G, then the payoff of the proposer on $[s, B(s)]$ is lower under G.*

The first part of Proposition 5 says that if the principal becomes less eager to agree on all high enough states, then in any equilibrium with delay with screening happening on lower states, where the principal's willingness to agree is unchanged, the probability of delegation is smaller and the state moves up faster. The intuition for the impact on the probability of delegation is the same as in the previous Proposition. The intuition for the impact of the speed of screening is that, as above, delegation causes delay, and, because the principal's willingness to agree is unchanged on low enough states, the change in the conflict of interest affects the speed of screening only through the probability of delegation.

Informally, the second part of Proposition 5 says that the payoff of the proposer is higher if higher cost realizations have larger probability. This is because higher cost realizations having larger probability increases the probability that high offers made to the agent by the proposer are accepted.

We conclude the discussion of comparative statics by considering how changes in parameters affect the IA equilibria. If the principal is less eager to agree at the lowest cost (so that $k(0) > 0$) and becomes less eager to agree everywhere, then the lowest offer made in an IA equilibrium increases. This is because, in this case, the smallest element of the set D of the feasible offers in IA equilibria is farther away from the lowest state 0. If we change the magnitude of the conflict of interest, then the set of IA equilibria does not change. This is because this change does not affect the set D of the feasible offers in the IA equilibria.

7 Discussion

7.1 Dynamic Commitment

Suppose that the principal has dynamic commitment. Then, if the principal commits to delegating forever, we are in the classical Coase conjecture setting, so the agreement happens immediately with the proposer offering the whole surplus. This means that, because the expected value of agreement is positive for the principal (that is, $1 - E[k(c)] > 0$), the principal would find it strictly profitable to indeed commit to delegating forever. Moreover, as long as there are some circumstances in which the principal would not want to agree even if offered the whole surplus (that is, $k(1) > 1$), the payoff the principal gets with dynamic commitment is strictly higher than her highest payoff without dynamic commitment. This is because, whenever $k(1) > 1$, the offers made in any equilibrium without dynamic commitment are bounded away from 1.

Substantively, this highlights the cost of the lack of long-term commitment to delegation in diplomacy and suggests that changes to the political system that improve ability to commit, such as less turnover of power, should improve bargaining outcomes. Similarly, changes to corporate governance that enable firm directors to retain their positions for extended periods of time may be beneficial.

7.2 Common Values

We might consider a different specification of payoffs where the agent gets the payoff from agreement regardless of whether this agreement was negotiated by the agent or the principal. In particular, suppose that when the principal accepts the offer x, instead of getting a payoff of 0, the agent with cost c gets $\alpha(x - c)$ for some $\alpha \in (0, 1)$.

The model is robust to this extension, in the sense that the structure of equilibria remains the same as in the case of private values. In particular, there is a set D' of offers that are TILI offers in IA equilibria. D' is obtained by taking each interval that D, the set of feasible TILI offers under private values, consists of and increasing the upper bound of this interval. Thus the set of feasible TILI offers D' is a strict superset of D. We can show that if α is small enough, so that the extent of common values is small, then D' is a union of the same number of intervals as D. If α is larger, then some intervals may merge and D' may be a union of a smaller number of (larger) intervals than D.

Moreover, each equilibrium with delay has a smooth screening phase that is some interval of states not in D ending at a point not in D. In particular, any screening interval under common values can be obtained by taking a screening interval under private values and decreasing the upper bound. For a small enough extent of common values α equilibria with delay are preserved, while if the common values component is substantial the upper bounds of screening intervals may have to decrease enough that equilibria with delay disappear.

We can summarize the above discussion as saying that under common values more offers are feasible in IA equilibria and the intervals of states on which smooth screening happens are smaller.[24] In terms of the application to diplomacy, if diplomats care about the agreement independently of how it advances their career, then delay in reaching agreement is smaller and better deals are possible. In this manner, public-mindedness of diplomats improves bargaining outcomes. Similarly, if employees that bargaining is delegated to care about the company's performance beyond their own division (for example, if they are also shareholders), the outcome of bargaining may be better.

7.3 Communication

Here we consider an extension in which, before deciding whether to delegate, the principal can communicate with the agent. In particular, in the beginning of each period, before making the delegation decision, the principal asks the agent to make a report. The

[24] Proposition 8 in the Appendix proves some facts about equilibria under common values. The observations we list in this section can be proven using the facts in Proposition 8.

principal commits either to the probability of delegation or to a binary decision to delegate or not in this period as a function of the agent's report. We continue to maintain the assumption that the principal does not have dynamic commitment and thus cannot commit to delegation decisions beyond the current period.

We first observe that there exists an equilibrium in which in the first period the principal commits not to delegate no matter which report the agent makes; the agent reveals the cost to the principal, and the principal then plays the Coasean equilibrium in the continuation, securing immediate agreement and getting the whole surplus. Because the agent is indifferent about the reports she makes since no delegation ensures that she gets a payoff of 0, this equilibrium lacks robustness: it breaks down if the agent has a vanishingly small cost of engaging in informative communication or if there is a small common values component to the agent's payoff, as in section 7.2.

There also exists an equilibrium in which no informative communication occurs. In particular, if the principal commits to not delegate no matter what, then it is an equilibrium for all agents to report the same message. In general, because agents that never accept offers on the equilibrium path are indifferent among all messages, each communication mechanism the principal might choose can be played by the agent in multiple ways. As a consequence, a range of equilibria might be sustained by a threat of playing the mechanism in a particular way. These issues persist if we consider mechanisms where communication happens only once in the beginning of the game or cheap talk where the principal cannot commit to the delegation decisions conditional on reports. If we assume that any communication mechanism the principal commits to induces a principal-optimal equilibrium, then the principal would choose the no-delegation mechanism inducing the Coasean equilibrium as described above since this yields the highest possible payoff to the principal.

The principal's ability to obtain the highest payoff using communication hinging on the indifference of some types of the agent is undesirable. Motivated by this, I introduce communication costs. In particular, I suppose that a communication mechanism has the option of refusing to answer, which is costless, as well as some costly messages. Proposition 6 shows that in this setting there does not exist an equilibrium with informative communication. This result holds no matter how small the cost of communication is, as long as it is strictly positive.

Proposition 6. *If communication is costly, then there does not exist an equilibrium with informative communication.*

The reason we cannot have informative communication with communication costs is unraveling: the agents choosing the costless message include all agents with 0 payoff, as well as some agents with a very small positive payoff. Because some agents with a very small positive payoff select out, the proposer has to make a smaller offer, which leads even more agents to select out and results in unraveling.

The results suggest that we should not expect to see much communication between the president and the diplomats and then when this communication happens, it should not make a difference to the outcomes. Analogously, little communication of consequence should happen between the firm director and division managers engaged in bargaining.

7.4 Delegation Set

I have assumed that the principal can only choose whether to delegate or not. In principle, a richer delegation choice is possible: the principal might be able to commit to a set of offers that she allows the agent to accept in this period, possibly conditional on the offer of the proposer. In this case, it is best for the principal to commit to only accept the offer of the whole surplus. Allowing the principal to have such commitment power is functionally equivalent to allowing the principal to be the proposer. The principal then would use this proposal power to extract all surplus from the opponent.

8 Related Literature

The paper is related to two strands of bargaining literature: the literature on bargaining with incomplete information and the Coase conjecture, and the literature on delegation in bargaining with complete information.

The first strand of literature the paper is related to is the literature on bargaining with incomplete information. The most prominent result in this literature is the Coase conjecture: in a setting where the buyer is privately informed and the uninformed seller makes all the offers, agreement happens immediately at price zero (in the limit as the offers become frequent). It has been shown that the Coase conjecture holds in this setting (Stokey 1981, Bulow 1982, Fudenberg, Levine, and Tirole 1985, Gul, Sonnenschein, and Wilson 1986).

The Coase conjecture has also been shown to fail in various settings, which include the presence of outside options (Board and Pycia 2014), capacity constraints (McAfee and Wiseman 2008), higher-order uncertainty (Feinberg and Skrzypacz 2005), two-sided private information (Cramton 1984, Cho 1990), interdependent values (Deneckere and Liang 2006), interdependent values and arriving news (Daley and Green 2020) and arriving buyers (Fuchs and Skrzypacz 2010), including when the entry of new buyers is endogenous (Chaves 2020). In some cases, even though the classical Coase conjecture fails, the generalized version of the Coase conjecture requiring that the payoff of the uninformed party be the same as if this party was unable to make offers holds (Fuchs and Skrzypacz 2010, Daley and Green 2020, Chaves 2020). In the present paper, formulating the generalized Coase conjecture is not straightforward because the payoff the proposer gets if she is not able to make offers to the agent depends on what the principal does, and multiple strategies of the principal are consistent with equilibrium in this case. Moreover, regardless of how the generalized Coase conjecture is formulated, it fails in the IA equilibria.

The literature on delegation in bargaining centers around the observation that it would be beneficial for a principal to commit to use an agent tougher than herself to conduct negotiations. The idea that commitment to tougher stances in bargaining, not necessarily through the use of delegates, is valuable, can be traced back to Schelling (1956) and has been more recently explored by Crawford (1982) and others. In the literature that considers the value of commitment through using delegates, the source of commitment is the ability to design contracts promising monetary compensation to the delegates conditional on the outcomes (Fershtman, Judd, and Kalai 1991, Katz 1991, Cai and Cont 2004), or the ability to choose supermajority agreement ratification requirements (Haller and Holden 1997), or the choice of different bargaining protocols (Perry and Samuelson 1994), or the ability to vote out a representative with a high cost of bargaining (Cai 2000).

In contrast to this literature, the present paper focuses on a novel tradeoff between using the information the agent has and mitigating the effect of the agent's bias on the outcomes. That is, the reason a delegate is valuable in the present model is the delegate's specialized knowledge which she can use to make better agreements, and not that only the delegate is able to reject offers that are too low.

Much of the literature on delegation in bargaining features complete information (Fershtman, Judd, and Kalai 1991, Katz 1991). When there is incomplete information, either the delegate learns the opponent's value before the bargaining commences (Cai and Cont 2004) or the delegate possesses private information about her cost of effort (Cai 2000, Cai and Cont 2004), or the principal's disagreement payoffs are stochastic and realized after the agreement is presented for ratification (Haller and Holden 1997, Perry and Samuelson 1994). This is in contrast to the present paper where the private information the delegate has is about the principal's value, is available to the delegate from the beginning and persists throughout negotiations. Finally, unlike the present paper, none of the papers in the literature on delegation in bargaining obtain real delay: as the offers become arbitrarily frequent, agreement is obtained immediately.

Appendix

A.1 Payoffs from TILI offers

I will analyze IA equilibria directly in continuous time instead of taking a limit of discrete-time games as $\Delta \to 0$. For this reason, no discounting appears in the formulas that follow (since in IA equilibria the game lasts for two periods, each of length Δ). Note that $y(\theta) = E[k(c)|c \geq \theta]$. Let c_0 be defined by $1 - y(c_0) = 0$. Let $V_x(\theta) = \frac{F(x)-F(\theta)}{1-F(\theta)}(1-x) + \frac{1-F(x)}{1-F(\theta)}(1-y(x))$ if $x \leq c_0$ and $V_x(\theta) = \frac{F(x)-F(\theta)}{1-F(\theta)}(1-x)$ if $x > c_0$. $V_x(\theta)$ is the payoff of the proposer from making a TILI offer x in state θ. Let $Y(\theta) = \text{argmax}_{x\in[\theta,1]} V_x(\theta)$.

Lemma 1 shows that the proposer's most preferred offer is increasing in the state.

Lemma 1. *Y is increasing.*

Proof of lemma 1. We will show that if x, $x' \in Y(\theta)$ for $x > x'$, then $x' \notin Y(\theta')$ for any $\theta' > \theta$. Suppose for the sake of contradiction that $x' \in Y(\theta')$ for some $\theta' > \theta$.

For $x > x'$, we have $V_x(\theta) - V_{x'}(\theta) \propto \mathcal{C}_{x,x'}(\theta) := F(\theta)(x-x') + F(x)(1-x) - F(x')(1-x') + (1-F(x))\max\{0, 1-y(x)\} - (1-F(x'))\max\{0, 1-y(x')\}$ and $\mathcal{C}'_{x,x'}(\theta) = f(\theta)(x-x') > 0$.

Then $\mathcal{C}_{x,x'}(\theta) \geq 0$ and $\mathcal{C}_{x,x'}(\theta') \leq 0$. However, because $\mathcal{C}'_{x,x'}(\theta') < 0$ and $\theta' > \theta$, this is a contradiction. ■

Let $T(\theta, x) = x - E[k(c)|c \in [\theta, x]]$. The principal's payoff from a TILI offer x to the agent in state θ is positive if and only if $T(\theta, x)$ is positive. For $\theta \notin D$ such that $\theta < \bar{c}$, let $\kappa(\theta) \in (\theta, \bar{c})$ be defined by $T(\theta, \kappa(\theta)) = 0$. Lemma 2 shows that $\kappa(\theta)$ is well-defined.

Lemma 2 records some facts about proposer-optimal TILI offers and the principal's payoff from TILI offers. In particular, lemma 2 shows that the principal's payoff from a TILI offer $x \leq c_0$ in state $\theta < \bar{c}$ is positive if and only if the offer is not in $(0, \kappa(\theta))$. The lemma also shows that in states $\theta < \bar{c}$ where the principal is less eager to agree the proposer-optimal TILI offer is in $(0, \kappa(\theta))$.

Lemma 2. *For all $\theta \in [0, \bar{c}) \setminus D$ there exists $\kappa(\theta) \in (\theta, B(\theta))$ such that $T(\theta, x) < 0$ for $x \in [\theta, \kappa(\theta))$, $T(\theta, \kappa(\theta)) = 0$ and $T(\theta, x) > 0$ for $x \in (\kappa(\theta), c_0)$. If $Y(\theta) \leq c_0$ and $\theta \in [0, \bar{c}) \setminus D$, then $Y(\theta) \in (\theta, \kappa(\theta))$. $x > \theta$ for all $x \in Y(\theta)$ and $\theta \notin D$. $Y(\theta) \leq c_0$ for all $\theta \leq \bar{c}$.*

Proof of lemma 2. We first show that, for $x \in (\theta, c_0)$, we have $V_x(\theta) > 1 - y(\theta)$ if and only if $T(\theta, x) < 0$. We have $V_x(\theta) = \frac{F(x)-F(\theta)}{1-F(\theta)}(1-x) + \frac{1-F(x)}{1-F(\theta)} \max\{0, 1-y(x)\}$. If $1 - y(x) \geq 0$, then $V_x(\theta) > 1 - y(\theta)$ is equivalent to $(F(x) - F(\theta))x < \int_\theta^x k(c)dF(c)$. Note that $T(\theta, x) < 0$ is equivalent to $(F(x) - F(\theta))x < \int_\theta^x k(c)dF(c)$, as required.

Claim 2.1. If $T(\theta, x) \geq 0$ for some $x \leq c_0$ and $\theta < x$, then $T(\theta, x') > 0$ for all $x' \in (x, c_0)$.

Proof of claim 2.1. $T(\theta, x) \geq 0$ implies that the proposer weakly prefers to make the offer θ rather than x. The second order condition on $[0, c_0)$ implies that the payoff of the proposer is single-peaked in x, so the proposer strictly prefers to make the offer θ rather than any offer $x' \in (x, c_0)$. Because $T(\theta, x') > 0$ is equivalent to $V_{x'}(\theta) < y(\theta)$, this implies that $T(\theta, x') > 0$. ■

Next observe that, for $\theta \notin D$, because $T(\theta, \theta) = \theta - k(\theta) < 0$, $T(\theta, B(s)) > 0$ since $T(\theta, B(s)) = B(s) - E[k(c)|c \in [\theta, B(s)]] > 0$ and k is increasing, and $T(\theta, x)$ is continuous in x, there exists $\kappa(\theta) \in (\theta, B(s))$ such that $T(\theta, x) < 0$ for all $x \in [\theta, \kappa(\theta))$ and $T(\theta, \kappa(\theta)) = 0$. Moreover, because $T(\theta, x) \geq 0$ for some $x \leq c_0$ and $\theta < x$ implies that $T(\theta, x') > 0$ for all $x' \in (x, c_0)$ by claim 2.1, we have $T(\theta, x) > 0$ for all $x \in (\kappa(\theta), c_0)$.

The previous two paragraphs imply that if $Y(\theta) \leq c_0$ and $\theta \in [0, \overline{c}) \setminus D$, then $Y(\theta) \in (\theta, \kappa(\theta))$. For $\theta \notin D$, if $Y(\theta) \leq c_0$, $Y(\theta) > \theta$ follows from the argument above, and if $x > c_0$ for $x \in Y(\theta)$, then $x \in \operatorname{argmax}_{x \in [\theta, 1]} \frac{F(x) - F(\theta)}{1 - F(\theta)}(1 - x)$, so that $x > \theta$. Because Y is increasing by lemma 1 and c_1 such that $c_0 \in conv(Y(c_1))$ satisfies $\overline{c} < c_1$, we have $Y(\theta) \leq c_0$ for all $\theta \leq \overline{c}$. ■

Lemma 3 records some facts about proposer-optimal TILI offers and the principal's payoff from TILI offers when the principal is more eager to agree. In particular, the lemma says that the proposer prefers to not make an acceptable offer to the agent if and only if the principal is more eager to agree, and in states where the principal is more eager to agree, the principal's payoff from a TILI offer below c_0 is positive no matter what the offer is.

Lemma 3. *$Y(\theta) = \theta$ if and only if $\theta \in D$. For all $\theta \in D$, $T(\theta, x) > 0$ for all $x \in (\theta, c_0)$.*

Proof of lemma 3.

Claim 3.1. $Y(\theta) = \theta$ if and only if $\theta \in D$.

Proof of claim 3.1. Because for $\theta \notin D$, $x > \theta$ for all $x \in Y(\theta)$ by lemma 2, $Y(\theta) = \theta$ implies that $\theta \in D$. It remains to show that $\theta \in D$ implies that $Y(\theta) = \theta$. Fix $\theta \in D$ and note that $T(\theta, \theta) \geq 0$. Then claim 2.1 in the proof of lemma 2 implies that $T(\theta, x') > 0$ for all $x' \in (\theta, c_0)$. By the proof of lemma 2, for $x \leq c_0$, we have $V_x(\theta) < 1 - y(\theta)$ if and only if $T(\theta, x) > 0$. Thus $T(\theta, x) > 0$ for all $x \in (\theta, c_0)$ implies that $V_x(\theta) < 1 - y(\theta)$ for such x. Since $1 - y(\theta) = V_\theta(\theta)$, we have $V_x(\theta) < V_\theta(\theta)$ for all $x \in (\theta, c_0]$. Because $Y(\theta) \leq c_0$ for all $\theta \leq \overline{c}$ by lemma 2, this implies that $Y(\theta) = \theta$. ■

Claim 3.2. For all $\theta \in D$, $T(\theta, x) > 0$ for all $x \in (\theta, c_0)$.

Proof of claim 3.2. Because $Y(\theta) = \theta$ by claim 3.1, we have $V_x(\theta) < 1 - y(\theta)$ for all

$x > \theta$. We can use arguments similar to the ones in the proof of lemma 2 to show that this implies that $T(\theta, x) > 0$ for all $x \in (\theta, c_0)$. ■ ■

A.2 Main Proofs: Existence and Structure of Equilibria

Proposition 2 establishes the necessary conditions that must hold in a smooth screening phase in any equilibrium with delay.

Proof of Proposition 2. In the proof that follows, we suppress the dependence of θ_+ on Δ.

We first show that, after the first period, the payoff of P is 0 in every period. Observe that, upon not delegating and receiving an offer from S, P must be indifferent about accepting the offer. Suppose that in state θ after the previous offer x' P does not delegate at time t and then rejects the offer of S. Because the equilibrium is weak-Markov, since the state does not change with P's rejection, if in the next period P again does not delegate, then S must make the same offer x' to P. If the payoff of P in state θ after the offer x' is strictly positive, this is a contradiction because (since the equilibrium is weak-Markov) P could obtain a higher payoff from playing at t the continuation starting at $t + \Delta$. Then the payoff of P is 0.

Note that, because we consider a sequence of equilibria with a frequent-offer limit that has smooth screening, we may assume without loss of generality that in every state in which there is smooth screening in the limit, there is a strictly positive probability of agreement with the agent for all $\Delta > 0$.

Claim 1. The offer S makes to the agent in state θ is $\chi_\Delta(\theta) = E[k(c)|c \in [\theta, \theta_+]]$. Moreover, as $\Delta \to 0$, $\chi_\Delta(\theta)$ converges to $\chi(\theta)$ such that $\chi(\theta) = k(\theta)$.

Proof of claim 1. Recall that, by the argument above, after the first period, the payoff of P is 0 in every period. Because P has to delegate with a positive probability, this implies that, in each period, the payoff of P from delegating is 0. The probability that the agent accepts the offer is $\frac{F(\theta_+)-F(\theta)}{1-F(\theta)}$. Conditional on acceptance, the expected cost of P is $E[k(c)|c \in [\theta, \theta_+]]$. Then, because the continuation payoff of P is 0, the payoff from delegating is $\frac{F(\theta_+)-F(\theta)}{1-F(\theta)}(\chi_\Delta(\theta) - E[k(c)|c \in [\theta, \theta_+]])$. Because the payoff of P from delegating in the current period is also 0, we have $\frac{F(\theta_+)-F(\theta)}{1-F(\theta)}(\chi_\Delta(\theta) - E[k(c)|c \in [\theta, \theta_+]]) = 0$. Because there is a strictly positive probability of agreement with the agent (which implies that $\theta_+ > \theta$), this implies that in any θ in which there is smooth screening in the limit we have $\chi_\Delta(\theta) = E[k(c)|c \in [\theta, \theta_+]]$. Therefore, $\chi(\theta) = \lim_{\Delta \to 0} E[k(c)|c \in [\theta, \theta_+]] = k(\theta)$. ■

We claim that in a smooth screening phase of any equilibrium with delay P must take the decision herself at a strictly positive rate. This is because, following the discussion in the text in section 4.3.1, S must be indifferent between making an acceptable offer to the agent and waiting to make the deal with P. If P did not take the decision herself at a strictly positive rate, then the payoff of S from waiting would be 0. This is a contradiction because S can obtain a strictly positive payoff from making an acceptable offer to the agent.

Claim 2 (Agent's indifference). $\pi_\Delta(\theta)$ converges to $\pi(\theta)$ such that $(r+\pi(\theta))(k(\theta)-\theta) = \dot{\theta}k'(\theta)$.

Proof of claim 2. Some of the proof follows from the discussion in text and the fact that $\chi(\theta) = k(\theta)$ by claim 1. The rest of the proof is analogous to the proof of lemma 7 in Fuchs and Skrzypacz (2010) and is thus omitted. ■

Claim 3 (Proposer). $V_\Delta(\theta)$ and $\pi_\Delta(\theta)$ converge to $V(\theta)$ and $\pi(\theta)$ such that $(1-F(\theta))V'(\theta) + f(\theta)V(\theta) - f(\theta)(1-k(\theta)) = 0$ and $\pi(\theta) = \frac{rV(\theta)}{1-E[k(c)|c\geq\theta]-V(\theta)}$.

Proof of claim 3. Some of the proof follows from the discussion in text and the fact that $\chi(\theta) = k(\theta)$ and $y(\theta) = E[k(c)|c \leq \theta]$. The rest of the proof is analogous to the proof of lemmas 4 and 5 in Fuchs and Skrzypacz (2010) and is thus omitted. ■ ■

Lemma 4. *For all $\Delta > 0$, in any equilibrium, if P delegates, then there is agreement with a strictly positive probability in this period. Moreover, in every equilibrium with delay, after smooth screening starts, every state θ on the equilibrium path satisfies* $\lim_{\Delta\to 0}(\theta_+ - \theta) = 0$.

Proof of lemma 4.

Claim 4.1. For all $\Delta > 0$, in any equilibrium, if P delegates, then there is agreement with a strictly positive probability in this period.

Proof of claim 4.1. We will show that, in any weak-Markov equilibrium, the continuation starting at time t in state θ in which agreement happens with probability 0 at time t in state θ is Pareto dominated.

Suppose first that A never accepts the offers of S on the equilibrium path. This continuation equilibrium is Pareto dominated by the IA equilibrium with no delegation (in which S and P agree immediately at the offer $y(\theta)$). In both equilibria, the payoff of P is 0 and the payoff of A is 0. Moreover, the payoff of S is strictly higher in the equilibrium with immediate agreement.

Suppose next that A accepts some offers of S on the equilibrium path. Let t' denote the first time after t at which there is strictly positive probability of agreement with the agent. Note that the payoff of S in a weak-Markov equilibrium (after a history where P delegated) depends only on the state. This implies that the payoff of S after delegation is the same at t and t'. This continuation equilibrium σ is Pareto dominated by the equilibrium σ' in which S plays at t the continuation that she played starting at t' in the equilibrium σ. This is because in the equilibrium σ' the payoff of S is the same, the payoff of A is strictly higher and the payoff of P is weakly higher. ■

Claim 4.2 (No atoms after starting screening). In every equilibrium with delay, after smooth screening starts, every state θ on the equilibrium path satisfies $\lim_{\Delta\to 0}(\theta_+ - \theta) = 0$.

Proof of claim 4.2. Suppose for the sake of contradiction that $\lim_{\Delta\to 0}(\theta_+ - \theta) > 0$ for some state θ on the equilibrium path of an equilibrium with delay such that there is smooth screening on all states $[\theta, \theta - \epsilon]$ for some $\epsilon > 0$. Let $\tilde{\chi}_\Delta(\theta)$ denote the offer made by S to the agent in state θ on the equilibrium path. Then, because P has to get the payoff 0 from delegation, $\lim_{\Delta\to 0}(\theta_+ - \theta) > 0$ implies that we must have $\lim_{\Delta\to 0} \tilde{\chi}_\Delta(\theta_+) > \lim_{\Delta\to 0} \tilde{\chi}_\Delta(\theta)$.

Note that, by claim 4.1, there cannot be a positive length of time that passes without agreement with some agent types. This and the fact that $\lim_{\Delta\to 0} \tilde{\chi}_\Delta(\theta_+) > \lim_{\Delta\to 0} \tilde{\chi}_\Delta(\theta)$ imply that agent with cost θ strictly prefers to wait for $\tilde{\chi}_\Delta(\theta_+)$ rather than accept $\tilde{\chi}_\Delta(\theta)$, which is a contradiction. ■ ■

Immediate Agreement Equilibrium as a Feasible Pair

Given an equilibrium, we let C_0 denote the set of states in which there is smooth screening and let $\tilde{C}$ denote the set of offers leading to states in which there is smooth screening. Let $\tilde{V}_x(\theta)$ denote the payoff of S in state θ given that she makes an offer x leading to states in which there is smooth screening. Let $W_x(\theta) = V_x(\theta)$ if $x \notin \tilde{C}$ and $W_x(\theta) = \tilde{V}_x(\theta)$ if $x \in \tilde{C}$. Given an equilibrium, a feasible pair (J, X) is $J \subseteq [0,1]$ and $X : [0,1] \to [0,1]$ such that $X(\theta) \in \operatorname{argmax}_{x\in[\theta,1]\cap\left(J\cup\tilde{C}\right)} W_x(\theta)$ and, for $\theta \notin C_0$, we have $\theta \in J$ if and only if $X(\theta) - E[k'|k' \in [k(\theta), X(\theta)]] \leq 0$ if $X(\theta) \in \tilde{C}$ and $T(\theta, X(\theta)) \leq 0$ if $X(\theta) \notin \tilde{C}$. We will refer to J as a feasible set.

Given an IA equilibrium in which there is immediate agreement in all states (on and off the equilibrium path), a feasible pair (J, X) is $J \subseteq [0,1]$ and $X : [0,1] \to [0,1]$ such that $X(\theta) \in \operatorname{argmax}_{x\in[\theta,1]\cap J} V_x(\theta)$ and $T(\theta, X(\theta)) \leq 0$ if and only if $\theta \in J$.

In any IA equilibrium, (in the limit as $\Delta \to 0$) there is exactly one offer made after delegation. We will find it convenient to use a feasible pair (J, X) to describe (most of) the play in an IA equilibrium.[25]

[25] (J, X) does not describe the play in an IA equilibrium fully because we also need to describe what happens if S makes an offer that is not in J and not in $\tilde{C}$. Lemma 9 provides this description.

We let $z = \inf\{\theta \in [0,1] : \theta \in D\}$ denote the lowest state in which the principal is more eager to agree than the agent. We say an equilibrium is *smooth screening* if $\lim_{\Delta\to 0} \frac{\theta_+ - \theta}{\Delta} \in (0, \infty)$ for all states θ on the equilibrium path.

Lemma 5. *Suppose that $k(0) > 0$. If in all states $\theta \geq z$ there exists an IA equilibrium with $z \in J$, then there exists a smooth screening equilibrium with screening starting in state* 0.

Proof of lemma 5. In the smooth screening equilibrium with screening starting in state 0, the following happens. If S makes an offer $x \in [\chi(\theta), \chi(z)]$ in state θ, then the state upon rejection of the offer is θ' such that $x = \chi(\theta')$ and the smooth screening equilibrium continuation is played from state θ'. If S makes an offer $x > \chi(z)$, then an IA equilibrium is played. As long as smooth screening continues, after delegation S offers $\chi(\theta)$ in state θ. If P does not delegate in θ, then S offers $y(\theta)$, which is accepted by P with probability one. Moreover, in state $\theta \in [0, z]$ the principal takes the decision herself at rate $\pi(\theta)$.

Because $\chi(\theta) = k(\theta)$, by the proof of claim 1 in Proposition 2, in every period P is indifferent about delegating and is thus willing to randomize. Because P is indifferent, P is willing to take the decision herself with probability one in z. Because $z < c_0$, we have $1 - y(\theta) \geq 0$ for all $\theta \leq z$. Because $\chi(\theta) = k(\theta)$, we have $\chi(\theta) - \theta \geq 0$ for all $\theta \leq z$. This implies that on the screening path, P accepts the offers of S with probability one when P takes the decision herself, and the offers made by S during screening give a positive payoff to the agents who accept them. Because $\chi(z) = z$, the agent with cost z is indifferent about accepting the offer $\chi(z)$. Since the proposer's unconstrained optimal offer in state z is $Y(z) = z$ by claim 3.1 in lemma 3, because $z \in J$, the proposer does not want to make a different offer in z. The condition in claim 2 in the proof of Proposition 2 ensures that the agent with cost θ is indifferent about accepting the offer $\chi(\theta)$.

Recall that $V(\theta)$ is the value of the proposer in state θ in a smooth screening equilibrium. Because in state z, P takes the decision herself with probability one, the terminal condition is $V(z) = 1 - y(z)$. Then the proposer's HJB equation in claim 3 in the proof of Proposition 2 ensures that in $\theta < z$ the proposer does not want to deviate to offering any $\chi(\theta')$ for $\theta' \in [0, z]$. Moreover, the proof of lemma 1 implies that for all $\theta < z$ we have $V_{X(z)}(\theta) > V_s(\theta)$ for all $s > X(z)$. Then in $\theta < z$ the proposer prefers $X(z)$ when choosing among TILI offers $x \in [X(z), 1]$. Because $V_{X(z)}(z) = 1 - y(z) = V(z)$ and S does not want to deviate to any $\chi(\theta')$ for $\theta' \in [0, z]$, this implies that in any state $\theta < z$, S does not want to deviate to offering $X(z)$. ■

We let $b(s) = \sup\{\theta : \theta \leq s, \theta \in D\}$ if such state exists and $b(s) = 0$ otherwise.

Lemma 6 shows that if the principal prefers not to delegate in some state, then there cannot be screening in any states above. Lemma 6 also shows that if some cost c at which the principal is less eager to agree than the agent is a feasible TILI offer and there is immediate agreement in an interval of states right below c (where the principal is less eager to agree), then all costs in this interval are also feasible TILI offers.

Lemma 6. *If $\theta \in J$, then $(\theta, 1] \cap C_0 = \varnothing$. Moreover, if $s \in J \setminus D$, $X(s) \leq c_0$ and there is immediate agreement in all states $[s_1, s]$ for $s_1 \geq b(s)$, then $\theta \in J$ for all $\theta \in [s_1, s]$. If $s \in [0, \overline{c}] \setminus D$ and $x \geq c_0$, $V_x(s) < 1 - y(s)$.*

Proof of lemma 6.

Claim 6.1. If $\theta \in J$, then $(\theta, 1] \cap C_0 = \varnothing$.

Proof of claim 6.1. If an offer $x \in \tilde{C}$ is made in state θ, the agents accepting x have costs in $[\theta, k^{-1}(x)]$, which implies that $x - E[k' | k' \in [k(\theta), x]] > 0$. ■

Claim 6.2. If $s \in J \setminus D$, $X(s) \leq c_0$ and there is immediate agreement in all states $[s_1, s]$ for $s_1 \geq b(s)$, then $\theta \in J$ for all $\theta \in [s_1, s]$.

Proof of claim 6.2. Claim 6.1 and $s \in J$ imply that $C_0 \cap (s, 1] = \varnothing$. Because there is immediate agreement in $[s_1, s]$, this implies that $C_0 \cap [s_1, 1] = \varnothing$.

We claim that all states $\theta \in [s_1, s]$ such that $s \in (\theta, \kappa(\theta))$ satisfy $\theta \in J$. Suppose for the sake of contradiction that for some $\theta \in [s_1, s]$ such that $s \in (\theta, \kappa(\theta))$ we had $\theta \notin J$. We will show that in θ, after delegation S makes the offer $X(\theta)$ such that $X(\theta) \in (\theta, \kappa(\theta))$ and $X(\theta) \leq c_0$.

$X(s) \leq c_0$ and $\theta < s$ imply that $X(\theta) \leq X(s) \leq c_0$. By lemma 2, in state θ any TILI offer in $(\theta, \kappa(\theta))$ is preferred by S to any TILI offer outside of $(\theta, \kappa(\theta))$. Because $s \in (\theta, \kappa(\theta))$ and $s \in J$, in state θ there is a feasible offer in $(\theta, \kappa(\theta))$. Then $X(\theta) \in (\theta, \kappa(\theta))$.

By lemma 2, $T(\theta, s') < 0$ for all $s' \in (\theta, \kappa(\theta))$ such that $s' \leq c_0$ and $\theta \notin D$. Then $T(\theta, X(\theta)) < 0$, which implies that $\theta \in J$, a contradiction. Next, observe that for all $\theta \notin D$ we have $\kappa(\theta) > \theta$. Because there is immediate agreement in $[s_1, s]$, we can use a proof by induction.

In particular, there exists $\theta_1 < s$ such that for all $\theta \in [\theta_1, s)$, we have $s \in (\theta, \kappa(\theta))$, so the argument above implies that $[\theta_1, s) \subseteq J$. Then there exists $\theta_2 < \theta_1$ such that for all $\theta \in [\theta_2, \theta_1)$, we have $\theta_1 \in (\theta, \kappa(\theta))$. Observe that, because $s_1 \geq b(s)$, the induction cannot terminate above s_1. Thus, proceeding by induction, we obtain the result. ■

Claim 6.3. If $s \in [0, \overline{c}] \setminus D$ and $x \geq c_0$, $V_x(s) < 1 - y(s)$.

Proof of claim 6.3. $V_x(s) < 1 - y(s)$ is equivalent to $M(s, x) := F(x) - x(F(x) -$

$F(s)) - 1 + \int_s^1 k(c)dF(c) < 0$. We have $\frac{\partial}{\partial s}M(s,x) = f(s)(x - k(s)) > 0$ because $x \geq c_0 > \bar{c} > k(s)$. Because $Y(\theta) = \theta$ for $\theta \in D$ by lemma 3, we have $V_x(\theta) < 1 - y(\theta)$ for $\theta \in D$, so $M(\theta, x) < 0$ for $\theta \in D$. Then for all $s < \theta$, $M(s,x) < 0$, so $V_x(s) < 1 - y(s)$, as required. ∎ ∎

Lemma 7. *Suppose that $k(0) > 0$. There is no smooth screening equilibrium (with smooth screening starting in state 0) other than the one described in the proof of lemma 5. In this equilibrium, smooth screening ends in state z. In z, there is no delegation and S makes a deal with P.*

Proof of lemma 7.

Claim 7.1. The state s in which smooth screening ends satisfies $V(s) \leq 1 - y(s)$.

Proof of claim 7.1. By claim 3 in Proposition 2, $\pi(\theta) = \frac{rV(\theta)}{1-y(\theta)-V(\theta)}$. Because $\pi(\theta) > 0$, this implies that $V(\theta) < 1 - y(\theta)$ for all $\theta < s$. It follows that $V(s) \leq 1 - y(s)$. ∎

Claim 7.2. In any equilibrium with smooth screening starting in $\theta' \in (0, z)$, the terminal state is z.

Proof of claim 7.2. Suppose that the terminal state was $s > z$. Note that we cannot have $k(s) - s < 0$ because in this case offers made by S during screening would give a strictly negative payoff to the agents who are supposed to accept them. Thus if $s > z$, then $\theta = k(\theta)$ for all $\theta \in [z, s]$. Because $s > z$, we have $\pi(z) < \infty$, which implies that $\dot{\theta} = 0$ for $\theta \in (z, s)$ by Proposition 2. Then state s is never reached, so s cannot be the terminal state. Thus suppose that $s < z$.

Suppose first that in s P delegates with probability 0. Then the payoff of an agent with cost s from accepting the offer $\chi(s)$ is $\chi(s) - s$, while the payoff to rejecting is 0. Because the agent with this cost must be indifferent, we must have $s = k(s)$. Because $s \leq z$, this can only hold at $s = z$.

Thus suppose that in s P delegates with a strictly positive probability. Suppose that in the IA equilibrium that is played after delegation $[s, z) \cap J = \varnothing$. Then either $X(s) \in \tilde{C}$ or $X(s) \geq z$. Note first that we cannot have $X(s) \in \tilde{C}$: if we did, then there would be an atom in screening in state s, which contradicts claim 4.2 in lemma 4. Then $X(s) \geq z$ and, by lemma 2, the principal strictly prefers to delegate in state s. This implies that in s P delegates with probability one. Then the payoff of an agent with cost s from rejecting the offer $\chi(s)$ is $X(s) - s$. Because the agent with this cost must be indifferent, we must have $k(s) = X(s)$. However, because $X(s) \geq z$ and $k(s) < z$ (because $k(z) = z$, $z > s$ and k is increasing), this cannot hold.

Note that claim 6.3 in the proof of lemma 6 implies that $V_x(s) < 1-y(s)$ for $s \in [0, \bar{c}]\backslash D$ and $x \geq c_0$. Then $X(s') \leq c_0$ for $s' = \sup[s, z) \cap J$. The argument above implies that $[s, z) \cap J \neq \varnothing$. Since claim 6.1 in lemma 6 implies that $s = \sup C_0$, lemma 6 implies that $\theta \in J$ for all $\theta \in [s, s']$ because $X(s') \leq c_0$. Therefore, $s \in J$.

Because $s \in J$, by making the TILI offer s in state s, S can ensure the payoff $1 - y(s)$. Thus we must have $V(s) \geq 1 - y(s)$. Because $V(s) \leq 1 - y(s)$ by claim 7.1, this implies that $V(s) = 1 - y(s)$.

If $V(s) = 1 - y(s)$ and P delegates with a strictly positive probability in s, then $V_{X(s)}(s) = 1 - y(s)$. Because $X(s) < c_0$ by lemma6, this can only hold at $X(s) \in \{s, \kappa(s)\}$. We cannot have $X(s) \neq s$ because then $(s, \kappa(s)) \cap J \neq \varnothing$, which implies that $X(s) \in (s, \kappa(s))$, a contradiction. Thus we must have $X(s) = s$ and $J \cap (s, z] = \varnothing$. Then, because the agent with cost s is indifferent, we must have $s = k(s)$, so $s = z$. ■ ■

Lemma 8. *Every equilibrium with delay has the following form:*

1. *breakthrough phase: the principal delegates with probability one and the proposer offers $k(s) \in \mathcal{I}$ that is accepted by all agents with cost $c \leq s$;*
2. *smooth screening phase.*

Proof of lemma 8. In the proof, we continue suppressing the dependence of offers and acceptance cutoffs on Δ.

To see that that every equilibrium with delay has the form in the lemma, note that, by claim 4.1 in lemma 4, there cannot be a positive length of time that passes without agreement with some agent types. Then in any equilibrium with delay in each state either there is smooth screening or there is an atom in agreement. Claim 4.2 in lemma 4 shows that atoms cannot occur after smooth screening starts. This implies that there can only be an atom in agreement in the beginning, before smooth screening starts.

Note that in any equilibrium with smooth screening starting in state s the agent with cost s has to be indifferent between accepting the offer x and waiting for the offer $k(s)$. This implies that $x = k(s)$. Because there can be smooth screening in state θ only if $k(\theta) - \theta \geq 0$, this means that if smooth screening starts in s, we must have $s \in \mathcal{I} \cup [\bar{c}, 1]$. Because screening must end in some state θ such that $\theta = k(\theta)$ by lemma 7, smooth screening can only start in $s \in \mathcal{I}$. ■

Lemma 9 shows that if the principal is less eager to agree than the agent in all states, then there is a feasible pair (J, X). Lemma 9 also shows that, for any feasible pair (J, X) such that there is immediate agreement in every state, an IA equilibrium exists.

Lemma 9. *If the state is $\bar{c}$ and $k(\theta) > \theta$ for all $\theta > \bar{c}$, there exists a feasible pair (J, X). If (J, X) is such that there is immediate agreement in every state, there exists an IA equilibrium with (J, X) as a feasible pair.*

Proof of lemma 9.

Claim 9.1. If the state is $\bar{c}$ and $k(\theta) > \theta$ for all $\theta > \bar{c}$, there exists a feasible pair (J, X).

Proof of claim 9.1. Note that $T(c_0, 1) = 0$ and $T(c_0, x) < 0$ for all $x \in [c_0, 1)$. Thus for $\theta \in [c_0, 1]$, $T(\theta, x) \leq 0$ for all $x \in [\theta, 1]$. This implies that for all $\theta \in [c_0, 1]$, $T(\theta, x) \leq 0$ for all $x \in Y(\theta)$. Thus we must have $[c_0, 1] \subseteq J$ and $X(\theta) \in Y(\theta)$ for all $\theta \in (c_1, 1]$. We let $X(c_1) = \inf Y(c_1)$, $J_1 = \{\theta \in [c_1, c_0] : T(\theta, X(\theta)) \leq 0\}$ and $n_1 = \{\theta : Y(\theta) = c_1\}$.

For $\theta \in (n_1, c_1)$, let $X(\theta) = \inf \operatorname{argmax}_{x \in J_1} V_x(\theta)$. By lemma 2, for all θ such that $\theta \notin D$ and $x \leq c_0$, we have $T(\theta, x) \leq 0$. Thus, because $X(c_1) = \inf Y(c_1) \leq c_0$, we have $c_1 \in J$. Then $X(n_1) = Y(n_1) = c_1$.

Claim 7.2 in lemma 7 implies that, because $k(\theta) > \theta$ for all $\theta > \bar{c}$, for all $s \geq \bar{c}$ there is immediate agreement in states $[b(s), s]$ (since any screening must end at a point such that $k(\theta) = \theta$). By lemma 6, if $s \in J \setminus D$ and $X(s) \leq c_0$, then, because there is immediate agreement in $[b(s), s]$, $\theta \in J$ for all $\theta \in [b(s), s]$. Because $c_1 \in J$, $X(c_1) = \inf Y(c_1) \leq c_0$ and $b(c_1) = \bar{c}$, this implies that $(\bar{c}, c_1] \subseteq J$. Thus $J = [\bar{c}, c_1] \cup J_1 \cup [c_0, 1]$. For $\theta \in [\bar{c}, n_1]$, let $X(\theta) = \operatorname{argmax}_{x \in J \cap [\theta, 1]} V_x(\theta)$ and observe that X is a function. ■

Claim 9.2. Suppose that (J, X) is such that there is immediate agreement in every state. Then there exists an IA equilibrium with (J, X) as a feasible pair.

Proof of claim 9.2. By construction, when S is restricted to making offers in J, making the offer $X(\theta)$ in state θ is optimal for S. To show that an equilibrium exists, it is sufficient to specify the off-equilibrium-path play and show that S prefers making the offer $X(\theta) \in J$ to any $x \notin J$.

Thus suppose that in state θ there is delegation and S offers $x \notin J$. Suppose first that $[\theta, x) \cap J = \varnothing$, which implies that $X(\theta) > x$. Then all agents reject x, in the next period the state is θ, P delegates and S makes the offer $X(\theta)$. Because $x < X(\theta)$, all agents indeed prefer to reject x.

Suppose next that $[\theta, x] \cap J \neq \varnothing$. If P delegates in θ, then S makes the offer $X(\theta)$. Now suppose that S deviates and makes an offer x. Let $s = \sup\{\theta' \in J : \theta' < x\}$. Because $[\theta, x] \cap J \neq \varnothing$, $s \in [\theta, x)$. Suppose first that $T(s, X(s)) = 0$.

The following happens off the equilibrium path after S offers x. All agents with $c < s$ accept, all agents with $c > s$ reject, and the state after a rejection is s. In state s, P randomizes, not delegating with probability π' and delegating with probability $1 - \pi'$. If P

does not delegate, the agent gets 0. If P delegates, the agent with cost c gets $\max\{0, X(s) - c\}$. Then the payoff of an agent with cost $c \leq X(s)$ to accepting x is $x - c$, while the payoff to rejecting is $(1-\pi')(X(s)-c)$. Because the agent with cost s is indifferent, $x - s = (1-\pi')(X(s)-s)$, which implies that $1-\pi' = \frac{x-s}{X(s)-s}$. Note that, because $X(s) \in J$ and $s = \sup\{\theta' \in J : \theta' < x\}$, we must have $X(s) > x$. The fact that $x < X(s)$ implies that $1-\pi' < 1$. $s < x$ implies that $\pi' > 0$. Therefore, $\pi' \in (0,1)$.

Because $s < c_0$, the payoff of S after offering x in state θ is $v = \rho(1-x)+(1-\rho)(\pi'(1-y(s)) + (1-\pi')V_{X(s)}(s))$ where $\rho = P[c \in [\theta, s] | c \geq \theta]$. Because $x = \pi's + (1-\pi')X(s)$,

$$v = \pi'(\rho(1-s) + (1-\rho)(1-y(s))) + (1-\pi')\left(\rho(1-X(s)) + (1-\rho)V_{X(s)}(s)\right)$$

We have $V_{X(s)}(s) = \rho'(1-X(s)) + (1-\rho')\max\{0, 1-y(X(s))\}$ where $\rho' = P[c \in [s, X(s)] | c \geq s]$. This implies that $\rho(1-X(s))+(1-\rho)V_{X(s)}(s) = (1-X(s))(\rho+(1-\rho)\rho') + (1-\rho)(1-\rho')\max\{0, 1-y(X(s))\} = (1-X(s))P[c \in [\theta, X(s)] | c \geq \theta] + P[c \geq X(s) | c \geq \theta]\max\{0, 1-y(X(s))\} = V_{X(s)}(\theta)$. This and the fact that $\rho(1-s)+(1-\rho)(1-y(s)) = V_s(\theta)$ (because $s < c_0$) imply that $v = \pi'V_s(\theta) + (1-\pi')V_{X(s)}(\theta)$.

Because $V_s(\theta) < V_{X(\theta)}(\theta)$ and $V_{X(s)}(\theta) \leq V_{X(\theta)}(\theta)$ since s, $X(s) \in J$, we have $v < V_{X(\theta)}(\theta)$. Therefore, offering x is not optimal for S.

Suppose next that $T(s, X(s)) \neq 0$. Then at $\theta = s$, $T(\theta, X(\theta))$ is discontinuous in θ and S is indifferent between TILI offers $x_1 > x_2$, with x_1, $x_2 \in J$ such that $T(s, x_1) > 0$ and $T(s, x_2) < 0$. After delegation in state s, S offers x_1 with probability π'' and offers x_2 with probability $1-\pi''$ such that $\pi''P[c \in [s, x_1] | c \geq s](x_1 - E[k(c) | c \in [s, x_1]]) + (1-\pi'')P[c \in [s, x_2] | c \geq s](x_2 - E[k(c) | c \in [s, x_2]]) = 0$. Because $T(s, x_1) > 0$ and $T(s, x_2) < 0$, $\pi'' \in (0,1)$. Then in state s, P is indifferent between delegating and not and so finds it incentive compatible to randomize. Note that, because $x_1 > x_2 \geq s$ and $\pi'' \in (0,1)$, we have $\pi''x_1 + (1-\pi'')x_2 > s$. The rest of the proof is similar to the proof above for the case where $T(s, X(s)) = 0$. ■ ■

A.3 Proofs of Theorems and Propositions

Proof of Theorem 2.

Claim 2.1. If $\theta \in J$ for some $\theta \in [0, z)$, then in the IA equilibrium there is no delegation.

Proof of claim 2.1. Note that smooth screening cannot start (off the equilibrium path) in any state $s' \in [0, \theta)$. This is because, by claim 4.2 in lemma 4, screening has no atoms and, by claim 7.2 in lemma 7, any smooth screening starting in $s' \in [0, \theta)$ must end in state z (which would contradict $\theta \in J$). Therefore, in $s' \in [0, \theta)$ there is immediate agreement.

Lemma 6 implies that if $\theta \in J$ for some $\theta \in [0, z)$ and there is immediate agreement in all states in $[0, \theta)$, then, because $X(\theta) \leq c_0$ by claim 6.3 in the proof of lemma 6, we have $[0, \theta] \subseteq J$. $0 \in J$ implies that $T(0, X(0)) \leq 0$. Moreover, by lemma 2, $T(0, X(0)) < 0$, which implies that in the IA equilibrium there is no delegation. ■

Claim 2.2. If there is delegation in state 0, S makes the offer $x = \inf J$.

Proof of claim 2.2. If there is delegation in state 0, then, by claim 2.1, for all $\theta \in [0, z]$, $\theta \notin J$. Lemma 2 implies that if $k(0) > 0$, then $Y(0) < z$, and lemma 3 implies that if $k(0) \leq 0$, then $Y(0) = 0$. Then $Y(0) \leq \inf J$ and the fact that $V_x(\theta)$ is single-peaked in x on $[0, c_0]$ due to the second order condition imply that S makes the feasible offer x closest to $Y(0)$, which is $x = \inf J$. ■

Claim 2.3. For all $D_0 \subseteq D$, there exists a feasible pair (J, X) satisfying $J \cap D \setminus \bar{c} = cl(D_0) \setminus \bar{c}$ such that there is immediate agreement in every state.

Proof of claim 2.3. Fix $D_0 \subseteq D$. Conjecture that $J \cap D \setminus \bar{c} = cl(D_0) \setminus \bar{c}$ and $(J \setminus D) \cap [0, \bar{c}) = \varnothing$.[26] Fix $\theta \in cl(D_0) \setminus \bar{c}$. By lemma 3, in state θ, the offer θ is the proposer's preferred offer. Because $\theta \in J$, S offers θ if P delegates. This implies that delegating in state θ yields a payoff of 0 to P. Thus P prefers not to delegate in θ, so that $\theta \in J$, as required. Fix $\theta \in D \setminus (cl(D_0) \cup \bar{c})$. Because $\theta \notin J$, S offers $x > \theta$ if P delegates. By the proof of claim 9.1 in lemma 9, there exists $J \cap [\bar{c}, 1]$ such that $\bar{c} \in J$. This implies that $X(\theta) < c_0$. Then lemma 3 implies that $T(\theta, X(\theta)) > 0$ for $X(\theta) < c_0$, so delegating in state θ yields a strictly positive payoff to P. Thus P prefers to delegate in θ, so that $\theta \notin J$, as required. Finally, fix $\theta \in [0, \bar{c}) \setminus D$. Because $(J \setminus D) \cap [0, \bar{c}) = \varnothing$, P prefers to delegate in θ, so that $\theta \notin J$, as required. ■

Claim 2.4. Any feasible set J in an IA equilibrium satisfies either $\inf J \in D$ or $\inf J = 0$. For all $D_0 \subseteq D$, there exists an IA equilibrium with a feasible set J satisfying $J \cap D \setminus \bar{c} = cl(D_0) \setminus \bar{c}$.

Proof of claim 2.4. We let $\bar{c}_0 = \sup\{\theta \in [0, 1] : \theta \notin D\}$. We choose no more than one point from each interval comprising the set $[0, \bar{c}) \setminus int(D)$[27] if $k(1) > 1$ and from each interval comprising the set $[0, \bar{c}_0) \setminus int(D)$ if $k(1) \leq 1$. Let R denote the resulting set of points. Let $I_s = (b(s), s]$ if $s < B(s)$ and let $I_s = (b(s), B(s))$ if $s = B(s)$.

We first show that any IA equilibrium has a feasible set J satisfying $(J \setminus D) \cap [0, \bar{c}) =$

[26]Lemma 9 implies that it is enough to specify $J \cap [0, \bar{c})$.
[27]Here int denotes the interior.

$(\cup_{s\in R} I_s)$ (it is enough to specify $J \cap [0, \bar{c})$ because claim 9.1 in lemma 9 shows that there exists $J \cap [\bar{c}, 1]$). Fix $s \in J \cap [0, \bar{c})$ and note that $X(s) \leq c_0$ by lemma 6. Note that smooth screening cannot start (off the equilibrium path) in any state $s' \in [b(s), s]$. This is because, by claim 4.2 in lemma 4, screening has no atoms and, by claim 7.2 in lemma 7, any smooth screening starting in $s' \in [b(s), s]$ must end in $B(s)$. Therefore, in states $s' \in [b(s), s]$, there must be immediate agreement. The fact that any IA equilibrium has a feasible set J satisfying the above condition follows from the fact that, by lemma 6, if $s \in J \setminus D$, $X(s) \leq c_0$ and there is immediate agreement in all states $[b(s), s]$, then $\theta \in J$ for all $\theta \in [b(s), s]$. This implies that any feasible set J in an IA equilibrium satisfies either $\inf J \in D$ or $\inf J = 0$.

It remains to show that for all $D_0 \subseteq D$, there exists an IA equilibrium with a feasible set J satisfying $J \cap D \setminus \bar{c} = cl(D_0) \setminus \bar{c}$. By claim 2.3, for all $D_0 \subseteq D$ there exists a feasible pair (J, X) satisfying $J \cap D \setminus \bar{c} = cl(D_0) \setminus \bar{c}$ such that there is immediate agreement in every state. By claim 9.2 in lemma 9, an IA equilibrium with (J, X) as a feasible pair exists for all feasible pairs (J, X) such that there is immediate agreement in every state. ■ ■

Proof of Theorem 1 and Corollary 2.1. The first part of Theorem 1 follows from lemma 8.

Fix $s \in \mathcal{I}$. We will show that an equilibrium with smooth screening starting in s exists. Theorem 2 implies that the hypothesis of lemma 5 (that in all states $\theta \geq z$ there exists an IA equilibrium with $z \in J$) is satisfied. Then the fact that, in state s, there exists an equilibrium with smooth screening starting in s follows from lemma 5.

The following happens off the equilibrium path in an equilibrium with delay with smooth screening starting in s. If S offers $x < k(s)$ in state 0, then all agents reject, and in the next period P delegates with probability one and S offers $k(s)$. Because $x < k(s)$, it is indeed incentive compatible for all agents to reject the offer. If S offers $x \in (k(s), B(s))$ in state 0, then all agents with costs $c \leq k^{-1}(x)$ accept the offer, and a rejection is followed by a smooth screening equilibrium starting in state $k^{-1}(x)$. Finally, if S makes an offer $x \geq B(s)$, then an IA equilibrium with $J = D \cap [B(s), 1]$ is played.

We now show that P delegates with probability one in state 0. For $s \in \mathcal{I} \setminus [0, z]$, the payoff of P in the equilibrium with smooth screening starting in s is weakly larger than her payoff in the IA equilibrium with offer s. This implies that, for $s \in \mathcal{I} \setminus [0, z]$, the payoff of P in the equilibrium with smooth screening starting in s is strictly positive, so P is willing to delegate. For $s \in [0, z]$, the initial offer S makes is $x = k(s)$, so that $T(0, x) = x - E[k'|k' \in [k(0), x]] > 0$. This implies that the payoff of P in the equilibrium with smooth screening starting in s is also strictly positive, so P is willing to delegate.

Claim 2.5. S prefers to offer $k(s)$ rather than $x > k(s)$ in state 0.

Proof of claim 2.5. We first show that S prefers to offer $k(s)$ rather than $x \in (k(s), B(s))$ in state 0. Let $\rho = P[c \in [0,s]]$. The proposer's payoff in the equilibrium with smooth screening starting in s (evaluated in state 0) is $v = \rho(1-k(s)) + (1-\rho)V(s)$. Suppose that S offered $x \in (k(s), B(s))$ in state 0. This leads to a smooth screening equilibrium starting in state $s' = k^{-1}(x)$. Let $\rho' = P[c \in [0,s']]$ and $\tilde{\rho} = P[c \in [s,s']|c \geq s]$. Then the payoff of S is $v' = \rho'(1-k(s')) + (1-\rho')V(s')$. By the proof of lemma 5, $V(s) > \tilde{\rho}(1-k(s')) + (1-\tilde{\rho})V(s')$. Then $v = \rho(1-k(s)) + (1-\rho)V(s) > \rho(1-k(s')) + (1-\rho)\tilde{\rho}(1-k(s')) + (1-\rho)(1-\tilde{\rho})V(s') = v'$, as required.

We next show that S prefers to offer $k(s)$ rather than $x > B(s)$ in state 0. By the proof of lemma 5, $V(s) > V_{B(s)}(s)$. Then $v = \rho(1-k(s)) + (1-\rho)V(s) > \rho(1-k(s)) + (1-\rho)V_{B(s)}(s) > \rho(1-B(s)) + (1-\rho)V_{B(s)}(s) = V_{B(s)}(0)$ because $k(s) < B(s)$. Therefore, the proposer's payoff from offering $k(s)$ is greater than the payoff from a TILI offer $B(s)$, which implies that it is greater that any feasible TILI offer in J. ■

The fact that the smooth screening phase starting in each $s \in \mathcal{I}$ is unique follows from Proposition 2. Observe that any equilibrium with smooth screening starting in s must end in state $B(s)$ with S making a deal with P with probability one. Then the corollary follows from Theorem 2 and the fact that, as shown above, smooth screening can only start in $s \in \mathcal{I}$. ■

Proof of Propositions 1 and 3. Proposition 1 follows from Theorem 1. The part of Proposition 3 dealing with the case in which $k(c) < c$ for all c follows from Theorem 2. The part of Proposition 3 dealing with the case in which $k(c) > c$ for all c follows from the proof of lemma 9. ■

A.4 Welfare, Comparative Statics and Extensions

Proof of Propositions 4 and 5.

Claim 1. If conflict of interest is greater under $\tilde{k}$ than under k, then $V(\theta)$ is larger under $\tilde{k}$ for all $\theta \in [0, \bar{c}] \setminus D$.

Proof of claim 1. Recall that $V'(\theta) = \frac{f(\theta)}{1-F(\theta)}(1 - k(\theta) - V(\theta))$ for all $\theta \in [0, \bar{c}] \setminus D$ by Proposition 2. Because $\tilde{k}(\theta) > k(\theta)$ for all $\theta \in [0, \bar{c}] \setminus D$ and the ODE satisfies a terminal condition, the statement in the claim follows from the comparison theorem for ODEs. ■

Claim 2. If conflict of interest is greater on $\{\theta : k(\theta) > \theta\}$ under $\tilde{k}$ than under k, π is larger under $\tilde{k}$. Moreover, π is increasing in r and increasing in θ. For all $s \in \mathcal{I}$, if $\hat{k}(\theta) \geq k(\theta)$ for $\theta > B(s)$ and $\hat{k}(\theta) = k(\theta)$ for $\theta \leq B(s)$, then π is larger on $[s, B(s)]$ under $\hat{k}$ than under k.

Proof of claim 2. Because $\pi(\theta) = \frac{rV(\theta)}{1-E[k(c)|c\geq\theta]-V(\theta)}$ by Proposition 2, π is increasing in V for $\theta < c_0$. By claim 1, V is larger under $\tilde{k}$ than under k. Because $\tilde{k}(\theta) \geq k(\theta)$ for all θ (since conflict of interest is greater on $\{\theta : k(\theta) > \theta\}$ under $\tilde{k}$ than under k), we have $E\left[\tilde{k}(c)|c \geq \theta\right] \geq E[k(c)|c \geq \theta]$ for all θ. Then, because π is increasing in $E[k(c)|c \geq \theta]$, π is larger under $\tilde{k}$ than under k.

Because $\hat{k}(\theta) = k(\theta)$ for all $\theta \in [0, B(s)]$, V on $[s, B(s)]$ is the same under k and $\hat{k}$. Because $\hat{k}(\theta) \geq k(\theta)$ for all θ, π is larger on $[s, B(s)]$ under $\hat{k}$ than under k by the same argument as above.

The fact that $(1 - F(\theta))V'(\theta) + f(\theta)V(\theta) - f(\theta)(1 - k(\theta)) = 0$ and $V(z) = 1 - y(z)$ (in the equilibrium with delay with smooth screening starting in state 0) by Proposition 2 implies that V is independent of r. Then $\pi(\theta) = \frac{rV(\theta)}{1-E[k(c)|c\geq\theta]-V(\theta)}$ implies that π is increasing in r.

We have $\pi'(\theta) \propto (1-y(\theta))V'(\theta)+y'(\theta)V(\theta)$. Because $V'(\theta) = \frac{f(\theta)}{1-F(\theta)}(1-k(\theta)-V(\theta))$, $\pi'(\theta) > 0$ is equivalent to $(1 - y(\theta))(1 - k(\theta) - V(\theta)) + V(\theta)(y(\theta) - k(\theta)) > 0$. Because $V(\theta) < 1 - y(\theta)$, it is enough to show that $k(\theta) < y(\theta)$, which is satisfied. ■

Claim 3. $\dot{\theta}$ is increasing in r. For all $s \in \mathcal{I}$, if $\hat{k}(\theta) \geq k(\theta)$ for $\theta > B(s)$ and $\hat{k}(\theta) = k(\theta)$ for $\theta \leq B(s)$, then π and $\dot{\theta}$ are larger on $[s, B(s)]$ under $\hat{k}$ than under k.

Proof of claim 3. We have $\dot{\theta} = \frac{(r+\pi(\theta))(k(\theta)-\theta)}{k'(\theta)}$ by Proposition 2. The fact that $\dot{\theta}$ is increasing in r follows from the fact that $\theta - k(\theta) < 0$ for all $\theta \notin D$, $k'(\theta) > 0$ and the fact that π is increasing in r by claim 2.

π is larger on $[s, B(s)]$ under $\hat{k}$ than under k by claim 2. Note that $k(\theta) = \hat{k}(\theta)$ for all $\theta \in [0, B(s)]$. Then $\dot{\theta} = \frac{(r+\pi(\theta))(k(\theta)-\theta)}{k'(\theta)}$ and $\theta - k(\theta) < 0$ for all $\theta \notin D$ imply that $\dot{\theta}$ is larger on $[s, B(s)]$ under $\hat{k}$ than under k. ■

Claim 4. If $g(\theta) > f(\theta)$ for $\theta < B(s)$ and F FOSD dominates G, then the payoff of the proposer on $[s, B(s)]$ is lower under G than it is under F.

Proof of claim 4. Because F FOSD dominates G, $1 - G(\theta) \leq 1 - F(\theta)$ for all θ. For $\theta \in [0, B(s)]$, we have $\frac{f(\theta)}{1-F(\theta)}(1 - k(\theta) - V(\theta)) < \frac{g(\theta)}{1-G(\theta)}(1 - k(\theta) - V(\theta))$. Because $V'(\theta) = \frac{f(\theta)}{1-F(\theta)}(1-k(\theta)-V(\theta))$ and the ODE satisfies a terminal condition, the comparison theorem for ODEs yields the result in the claim. ■ ■

Let σ^s denote the equilibrium with delay with smooth screening starting in state s, and let σ_x denote the IA equilibrium with delegation in which offer x is made to the agent. We will use σ^0 to denote the IA equilibrium without delegation. Let $v(\sigma)$ denote the payoff of the proposer in equilibrium σ and let $u(x)$ denote the payoff of the principal in an equilibrium in which the initial offer to the agent is x.

Proposition 7. *$v\left(\sigma_{b(s)}\right) > v(\sigma^s) > v(\sigma_x)$ for all $s \notin D$ and $x \in D \cap [B(s), 1]$. If $k(x)f(x) \leq 1$ for all $x \in [0,1]$, then $u'(x) \geq 0$ for all $x \in [0,1]$. The equilibria described in Theorems 1 and 2 are Pareto undominated.*

Proof of Proposition 7. Proposition 2 implies that $v\left(\sigma_{b(s)}\right) > v\left(\sigma^{b(s)}\right)$. Note that $v\left(\sigma^{b(s)}\right) \geq v(\sigma^s)$. It follows that $v\left(\sigma_{b(s)}\right) > v(\sigma^s)$. Observe that in the equilibrium with delay with smooth screening starting in s the proposer can always make a TILI offer $x = B(s)$. Because the proposer chooses not to, $v(\sigma^s) > v(\sigma_x)$ for all $x \in D \cap [B(s), 1]$.

We now show that none of the equilibria we describe are Pareto dominated. In the proof, we use the discussion in Section 6 explaining how players rank equilibrium payoffs. No IA equilibrium with delegation is Pareto dominated because P obtains a strictly lower payoff in the IA equilibrium without delegation. The IA equilibrium without delegation is not Pareto dominated because S obtains a strictly lower payoff in the IA equilibria with delegation. No equilibrium with delay with smooth screening starting in state θ is Pareto dominated because A obtains a strictly lower payoff in the IA equilibrium without delegation.

Because the principal gets the payoff 0 in the smooth screening phase of an equilibrium, we have $u(x) = F(x)\left(x - \int_0^x k(c)dF(c)\right)$ if $x \in D$ and $u(x) = F(k^{-1}(x))\left(x - \int_0^{k^{-1}(x)} k(c)dF(c)\right)$ if $x \notin D$. Suppose first that $x \in D$. Then $u'(x) = f(x)\left(x - \int_0^x k(c)dF(c)\right) + F(x)(1 - k(x)f(x))$. Because $x - \int_0^x k(c)dF(c) \geq 0$ whenever P delegates, to have $u'(x) \geq 0$ it is sufficient to have $k(x) \leq \frac{1}{f(x)}$ for all $x \in D$. Suppose next that $x \notin D$. Then $u'(x) = \frac{1}{k'(x)}\left(f(k^{-1}(x))\left(x - \int_0^{k^{-1}(x)} k(c)dF(c)\right)\right) + F(k^{-1}(x))\left(1 - xf(k^{-1}(x))\right)$. Because $x - \int_0^{k^{-1}(x)} k(c)dF(c) \geq 0$, to have $u'(x) \geq 0$ it is sufficient to have $x \leq \frac{1}{f(k^{-1}(x))}$ for all $x \notin D$. This is the same as $k(x) \leq \frac{1}{f(x)}$ for all $x \notin D$. Thus to have $u'(x) \geq 0$ for all x it is sufficient to have $k(x)f(x) \leq 1$ for all $x \in [0,1]$. ■

Proposition 8 (Common Values). *Under common values, the following is true. A TILI offer x is accepted by all agents with cost less than $\omega(x) = \frac{1}{1-\alpha}x - \frac{\alpha}{1-\alpha}y(\omega(x))$. For $x \in (\theta, c_0)$, $V_x(\theta) > 1 - y(\theta)$ if and only if $\omega(x) \in (\theta, \kappa(\theta))$. In a smooth screening equilibrium the terminal state s satisfies $s = \frac{1}{1-\alpha}k(s) - \frac{\alpha}{1-\alpha}y(s)$.*

Proof of Proposition 8. Suppose that an agent with cost c is indifferent between accepting and rejecting a TILI offer x. The payoff from accepting the offer is $x - c$, while the payoff to rejecting is $\alpha(y(c)-c)$, so all agents with $c \leq \frac{1}{1-\alpha}x - \frac{\alpha}{1-\alpha}y(c)$ accept. Let $\omega(x) = \frac{1}{1-\alpha}x - \frac{\alpha}{1-\alpha}y(\omega(x))$ denote the cost threshold for an agent to accept the TILI offer x. Observe that $\omega(x) < x$. Then the payoff of S from making a TILI offer $x > \theta$ in state θ is $V_x(\theta) = \frac{F(\omega(x))-F(\theta)}{1-F(\theta)}(1-x) + \frac{1-F(\omega(x))}{1-F(\theta)} \max\{0, 1-y(\omega(x))\}$. We have $T(\theta, x) = x - E[k(c)|c \in [\theta, \omega(x)]]$. Using an argument similar to the one in lemma 2, we can show that for $x \in (\theta, c_0)$, $T(\theta, x) < 0$ if and only if $V_x(\theta) > 1 - y(\theta)$ (which happens if and only if $\omega(x) \in (\theta, \kappa(\theta))$). Observe that in state θ, all TILI offers $x \in [\theta, \omega^{-1}(\theta))$ are rejected by all agents and lead to state θ upon rejection. Using a proof similar to the proof of lemma 7, we can show that in a smooth screening equilibrium the terminal state s satisfies $\chi(s) - s = \alpha(y(s) - s)$, so that $s = \frac{1}{1-\alpha}k(s) - \frac{\alpha}{1-\alpha}y(s)$. ■

Proof of Proposition 6. Fix an equilibrium and let c' denote the highest type that accepts an offer on the equilibrium path (note that the agents' acceptance strategies must still be threshold). Let σ_0 denote the costless message and let $\epsilon > 0$ denote the cost of sending the a costly message. Suppose that a costly message is sent by some types of agents with a strictly positive probability on the equilibrium path. Note that the set of types sending σ_0 must include $[1, c' - \epsilon]$. Then the higher offer S can make upon delegation is $c' - \epsilon$. This offer will not be accepted by types $(c', c' + \epsilon)$, contradicting our assumption that c' is the highest type that accepts an offer on the equilibrium path. Thus there does not exist an equilibrium with informative communication. ■

References

Ausubel, Lawrence M., and Raymond J. Deneckere. 1989. "Reputation in Bargaining and Durable Goods Monopoly." *Econometrica* 57(3): 511–531.

Board, Simon, and Marek Pycia. 2014. "Outside Options and the Failure of the Coase Conjecture." *American Economic Review* 104(2): 656–671.

Bulow, Jeremy I. 1982. "Durable-Goods Monopolists." *Journal of Political Economy* 90(2): 314–332.

Cai, Hongbin. 2000. "Bargaining on Behalf of a Constituency." *Journal of Economic Theory* 92(2): 234–273.

Cai, Hongbin, and Walter Cont. 2004. "Agency Problems and Commitment in Delegated Bargaining." *Journal of Economics and Management Strategy* 13(4): 703–729.

Chaves, Isaias N. 2020. "Privacy in Bargaining: The Case of Endogenous Entry." Working Paper.

Cho, In-Koo. 1990. "Uncertainty and Delay in Bargaining." *Review of Economic Studies* 57(4): 575–595.

Cramton, Peter C. 1984. "Bargaining with Incomplete Information: An Infinite-Horizon Model with Two-Sided Uncertainty." *Review of Economic Studies* 51(4): 579–593.

Crawford, Vincent P. 1982. "A Theory of Disagreement in Bargaining." *Econometrica* 50(3): 607–637.

Daley, Brendan, and Brett Green. 2020. "Bargaining and News." *American Economic Review* 110(2): 428–474.

Davenport, Kelsey. 2013. "Rouhani wants 'serious' nuclear talks." *Arms Control Today* 43(7): 32–33.

Deneckere, Raymond, and Meng-Yu Liang. 2006. "Bargaining with Interdependent Values." *Econometrica* 74(5): 1309–1364.

Fearon, James D. 1997. "Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs." *Journal of Conflict Resolution* 41(1): 68–90.

Feinberg, Yossi, and Andrzej Skrzypacz. 2005. "Uncertainty about Uncertainty and Delay in Bargaining." *Econometrica* 73(1): 69–91.

Fershtman, Chaim, Kenneth L. Judd, and Ehud Kalai. 1991. "Observable Contracts: Strategic Delegation and Cooperation." *International Economic Review* 32(3): 551–559.

Fuchs, William, and Andrzej Skrzypacz. 2010. "Bargaining with Arrival of New Traders." *American Economic Review* 100(3): 802–836.

Fudenberg, Drew, David K. Levine, and Jean Tirole. 1985. "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information." In *Game-Theoretic Models of Bargaining*, ed. Alvin E. Roth. Cambridge, UK: Cambridge University Press pp. 73–98.

Gul, Faruk, Hugo Sonnenschein, and Robert Wilson. 1986. "Foundations of Dynamic Monopoly and the Coase Conjecture." *Journal of Economic Theory* 39(1): 155–190.

Haller, Hans, and Steinar Holden. 1997. "Ratification Requirement and Bargaining Power." *International Economic Review* 38(4): 825–851.

Jost, Tyler, and Austin Strange. 2018. "Delegated Diplomacy: Why China Uses the Military for Face-to-Face Exchanges." Working Paper.

Katz, Michael L. 1991. "Game-Playing Agents: Unobservable Contracts as Precommitments." *The RAND Journal of Economics* 22(3): 307–328.

Lindsey, David. 2017. "Diplomacy Through Agents." *International Studies Quarterly* 61(3): 544–556.

McAfee, R. Preston, and Thomas Wiseman. 2008. "Capacity Choice Counters the Coase Conjecture." *Review of Economic Studies* 75(1): 317–332.

Perry, Motty, and Larry Samuelson. 1994. "Open- versus Closed-Door Negotiations." *The RAND Journal of Economics* 25(2): 348–359.

René, Albrecht-Carrié. 1958. *A Diplomatic History of Europe Since the Congress of Vienna.* London: Methuen.

Salama, Vivian, and Peter Nicholas. 2019. "Talk to Trump, skip the diplomats: World leaders' new U.S. tactic." In *Wall Street Journal (Online).* 14 March.

Sartori, Anne E. 2002. "The Might of the Pen: A Reputational Theory of Communication in International Disputes." *International Organization* 56(1): 121–149.

Schelling, Thomas C. 1956. "An Essay on Bargaining." *American Economic Review* 46(3): 281–306.

Stokey, Nancy L. 1981. "Rational Expectations and Durable Goods Pricing." *The Bell Journal of Economics* 12(1): 112–128.