Rents and Intangible Capital: A Q+ Framework*

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Abstract

In recent years, US investment has been lackluster, despite rising valuations. Key explanations include growing rents and growing intangibles. We propose and estimate a framework to quantify their roles. The gap between valuations — reflected in average Q — and investment — reflected in marginal q — can be decomposed into three terms: the value of installed intangibles; rents generated by physical capital; and an interaction term, measuring rents generated by intangibles. The intangible-related terms contribute significantly to the gap, particularly in fast-growing sectors. Our findings suggest care in a pure-rents interpretation, given the rising role of intangibles.

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1 Introduction

Recent research highlights two apparently contradictory, medium-run facts about the US economy: returns to business capital, and corporate profits more generally, have been either stable or growing (Gomme et al., 2011); yet investment has been lackluster, in particular relative to corporate valuations (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018). *Ceteris paribus*, investment theory would predict a rise in investment in response to higher returns to capital and corporate valuations.

In neoclassical models, the divergence between returns and investment can be cast as a rising gap between the average value of business capital, or Tobin's *average* Q, and its marginal value, or Tobin's *marginal* q. We directly observe rising average Q in the data, via market values, while marginal q is a shadow value measured implicitly by lackluster investment. A gap between the average value of capital and its marginal value can arise and grow for a number of reasons. Two leading explanations have recently emerged: intangible capital and rents.

Over the last several decades, intangible capital has grown as a share of investment and as a share of assets (Corrado et al., 2005, 2009). A shift toward intangibles in production could cause physical investment to appear low relative to valuations. Typical measures, such as Tobin's average Q, increasingly underestimate the true stock of assets, and thus increasingly overstate the incentive to invest in physical capital (Gutiérrez and Philippon, 2017; Alexander and Eberly, 2018; Crouzet and Eberly, 2019).

Alternatively, the gap between average Q and marginal q may be explained by market power. Rising market power and its corresponding rents can account for a stable or rising rate of return on assets despite a falling user cost of capital. Rising rents also reduce the marginal return to additional capital, consistent with a weaker incentive to invest. Several recent papers indeed document a rise in the measured capital share over the last three decades, which, along with declining required returns to capital, is consistent with higher rents (Barkai, 2020; Gutiérrez and Philippon, 2018a).

From a positive perspective, both intangibles and rents have the potential to explain the divergence between returns and investment. However, the normative implications of the two mechanisms may differ. Rising intangibles reflect supply-side changes in the organization of production (Haskel and Westlake, 2018), with no clear implications for welfare. By contrast, rising rents could be associated with deadweight losses, for instance if they are due to price markups (De Loecker et al., 2020) or wage markdowns (Benmelech et al., 2018).[1]

[1] The normative implications of rising rents and reduced competition can however depend on the economic environment. Among many others, Aghion et al. (2005), for instance, provide an example of a model in which reduced competition may be associated with increased innovation.

Any normative or policy conclusion drawn from the divergence between investment and returns thus requires a careful assessment of which of the two mechanisms is most relevant in practice. However, most of the literature has considered each of these mechanisms in isolation, which tends to overstate their respective explanatory power. The goal of this paper is to assess them jointly, and in doing so, to provide a quantitative estimate of the role of each in the divergence between returns and investment. To do this, we extend the Q-theory model (Hayashi, 1982; Abel and Eberly, 1994) to simultaneously allow for the presence of economic rents and the accumulation of a stock of intangible assets. We call this model the "Q+" framework.

Using this framework, we make two main contributions. First, from a theoretical perspective, we show that the gap between average Q and marginal q for physical capital, which we call the "investment gap", can be decomposed into three distinct terms: a term capturing rents to physical capital, a term capturing the value of installed intangible capital, and a term capturing rents to intangible capital. The last element of this decomposition, an interaction term that is new to our analysis, is particularly important: it clarifies the fact that rising rents and rising intangibles cannot be meaningfully analyzed in isolation, as their interaction contributes to the gap between investment and returns. Moreover, this decomposition is very general, as our framework nests a number of existing investment models.

Second, we show that this interaction term is empirically important to the recent rise in the investment gap. Importantly, we show how each term in our decomposition can be quantified using data on profits, investment, valuations, and estimates of the intangible capital stock within the structure of the model. In aggregate data, the interaction term accounts for between one-quarter and one-half of the investment gap, depending on how broad the definition of intangibles is. In addition, our approach leads to lower estimates of the *increase* in total rents than existing work. As we show, this is equivalent to a smaller estimate of the *decrease* in total user costs of capital. This occurs because while including intangibles raises valuations, it also boosts the user cost of capital due to higher depreciation rates (hence reducing rents).

Finally, we move beyond the aggregate data, recognizing that economy-wide increases in rents and intangibles may be driven by composition effects across sectors. In fact, we find that the aggregate investment gap is driven by fast-growing industries, such as Healthcare and Tech. Moreover, these industries' investment gaps are mostly explained by intangibles, even when intangibles are narrowly measured. We also show that among the subsectors of Healthcare, Tech, and Manufacturing, only a subset experienced rising rents, and those that did generally also experienced a rise in intangible intensity. Taken together, these empirical results suggest that the investment gap in these industries reflects a change in the factors of production, rather than unequivocal and broad evidence of rising market power.

In Section 2, we develop and analyze the "Q+" framework. The gap between average Q and marginal q, which we call the "investment gap", is our main focus. We show how this gap can be decomposed into three distinct terms: a term capturing rents to physical capital, a term capturing the value of installed intangibles, and a term capturing rents to intangible capital. The first two terms would obtain, respectively, in a model without rents (but with intangibles), and in a model without intangibles (but with rents). When both are present in the model, a third term appears, which captures the economic rents earned by intangible capital. The model demonstrates how these can be identified separately from rents earned by physical capital. The result is independent of the specifics of exogenous processes and of capital adjustment cost and revenue functions, so long as they satisfy simple homogeneity assumptions. We also provide versions of the framework in which each of these terms can be solved in closed form. These analytical expressions clarify the key forces driving the effects of rents, intangibles, and their interaction. In particular, rents on intangible capital are the present value of markups multiplied by an appropriately defined user cost, which takes into account adjustment costs. This user cost is large for intangible capital because intangibles depreciate quickly, foreshadowing our findings on the quantitative importance of rents generated by intangibles.

In Section 3, we apply this decomposition to aggregate data, after showing how to estimate the components of the investment gap using moments of corporate profits, investment, valuations, and estimates of the intangible stock. We begin with data from US national accounts, which are broader in coverage, but provide a narrower definition of intangibles, as they focus on R&D capital. Two periods stand out with large investment gaps: the 1965-1975 decade, and the post-1990 period. Most interestingly, the composition of the gap is different between these two periods: whereas the 1965-1975 gap is mostly driven by rents generated by physical capital, approximately 40% of the post-1990's gap is due to the intangibles-related terms. The term capturing rents to intangibles is sizable, accounting for 25% of the gap, with the direct intangibles effect making up the other 15%. The post-1990's change is driven by three underlying trends. First, the share of intangibles approximately doubles. Second, the user costs of intangibles are not only much higher, but also more stable than those of physical capital. We infer this from the fact that *gross* intangible investment rates are stable and elevated in the data, which is consistent with high and stable depreciation rates for intangibles (a finding which is borne out independently by BEA data on intangible depreciation rates). Third, overall rents increase, though they do so more moderately than suggested by other recent work. This is driven by differences in our estimates of the decline in the user cost of capital, which we explore in detail in Section 3.

Section 4 examines the investment gap using data on publicly traded firms. While narrower in scope, these data have two advantages: we can use a broader definition of intangible capital, and we can dis-aggregate results by sector. When we expand intangibles to include the organization capital stock of firms (rather than just R&D) following Eisfeldt and Papanikolaou (2013), we find that by 2015, the two intangibles-related terms account for two thirds of the total investment gap. Including organization capital has relatively little impact on estimated user costs of intangibles — they remain elevated —, but it substantially increases the stock of intangibles, boosting both their direct effect on the investment gap, and the interaction term. Our estimates of rents as a share of value added are also roughly cut in half. Thus, empirically plausible amounts of intangible capital can explain the investment gap without requiring high rents.

Finally, in Section 4, we also estimate our decomposition at the sectoral level, in order to assess the extent to which the aggregate investment gap reflects composition effects. We divide our sample into five broad sectors: Consumer, Services, Tech, Healthcare, and Manufacturing. In the Manufacturing sector, the investment gap is small, and both rents and intangibles are declining. By contrast, in the Tech and Healthcare sectors, the investment gap has been growing rapidly since the 2000's. In both sectors, the primary driver is rents to intangible capital. In the Consumer sector, results depend on the measurement of the intangible capital stock. Reported R&D is small, so there is little role for intangibles when they are measured with this proxy. However, innovation in the consumer sector is not well-measured by R&D (see Foster et al. 2006 and Crouzet and Eberly 2018). When including organization capital, most of the gap is estimated to reflect the direct effect of large investment in intangibles in that sector — rents on either physical or intangible capital appear to have only modestly increased. The Service sector is similar in some respects to Consumer, in that R&D is small, so rents explain most of the gap. However, adding organization capital does not change this view, as there has been little growth in organization capital in the Service sector; hence, rents explain most of the gap throughout. Finally, we also study the relationship between rising rents and rising intangibles across the constituent subsectors of our five broad sectors. We find that the rise in rents was heterogeneous across subsectors, and within Manufacturing, Tech and Healthcare, subsectors that experienced a rise in rents also experienced an increase in intangible intensity. These findings suggest that the rise in rents may be both narrower than aggregate estimates and also related to changes in the underlying structure of production.

Our results caution against interpreting the gap as a broad rise in market power. Our evidence shows that intangibles play a key role, and no single mechanism provides a unified account of the gap, even across broadly defined sectors. Normative implications should hence be drawn with care.

Related research and contribution Our work first relates to the literature on the implications of rising intangible capital for macroeconomics and finance, which itself builds on work measuring intangibles and documenting their rise (Corrado et al., 2005, 2009; Eisfeldt and Papanikolaou, 2013). Closest to our approach are Hall (2001), who links the rise in intangibles to stock market valuations, and McGrattan and Prescott (2010), who examine the potential role of intangibles for macro trends in a business cycle model.[2] Relative to these papers, we study medium-run trends, emphasize sectoral heterogeneity, and, most importantly, allow for market power within our model.

Second, our work is related to a recent literature on the size and implications of rising rents. A number of researchers have interpreted the findings of Autor et al. (2020), who show that industry concentration rose in U.S. industries after 2000, as potential evidence of market power, and examined profitability and markup data for further evidence.[3] Most closely related to our work are Gutiérrez and Philippon (2018a) and Barkai (2020), who document a significant increase in pure profit shares and markups, especially after 2000. Barkai (2020), in particular, does not directly examine investment, but shows that the decline in the labor share is not offset by a rising capital share; he attributes the resulting gap to pure profits. Our approach, based on valuations, uncovers a more modest increase in rents than these papers, a point we expand on in Section 3. Similarly, Basu (2019) reviews the evidence from the rents literature, and argues that macro trends related to profitability are largely consistent with historical variation. He points instead to weak investment as the outlier and asks how to reconcile it with the apparently modest changes in rents. Our paper explains this apparent divergence as the combined effect of moderate rents with rising intangibles.

In recent and related research, Karabarbounis and Neiman (2019) find that the gap between measured capital income and estimates of the required compensation of capital is most likely explained by mismeasurement in the cost of capital. Our approach provides an alternative measure of the user cost of capital which further supports this view. Most closely related to our work is Farhi and Gourio (2018), who estimate the contribution of market power, risk premia, and intangibles to recent macro trends, as well as Corhay et al. (2020), who highlight the role of declining entry as a source of increasing market power. Relative to their work, our analysis focuses more specifically on investment and on the role that intangible capital plays in explaining weak investment relative to valuations.

A rich literature in corporate finance has discussed potential sources of wedges between average Q and marginal q, and the performance of investment-Q regressions. Most recently,

[2] See also Hansen et al. (2005) and Ai et al. (2013).

[3] De Loecker et al. (2020) and Hall (2019) use firm-level accounting data and industry data, respectively, and find both high and rising markups.

Peters and Taylor (2017) revisit the relationship between investment and Q when intangibles are present.[4] Belo et al. (2019) also provide decompositions of firm value across types of capital, including intangibles. We leverage the empirical results of both papers in our analysis, but also provide a more general framework than either, by allowing for rents, a key element in the relationship between investment and Q. Section 2 provides further comparisons of our framework with existing models.

Our results are also connected to recent findings documenting a decline in investment/cash-flow sensitivities and questions whether it reflects financing constraints (Chen and Chen, 2012). From the standpoint of our model, a potential interpretation of the results of Chen and Chen (2012) is that Tobin's Q increasingly captures cash flow effects, through the growing importance of rents, particularly those associated with intangibles. Relatedly, recent research by Falato et al. (2020) studies how the growth in corporate cash holdings relates to rising intangible intensity. They argue that the reduced reliance on physical capital has shrunk corporate debt capacity, which firms offset by increasing precautionary cash holdings. We document an additional aggregate and sectoral trend, the increase in rents. This trend, by increasing the curvature of firms' profits with respect to capital, may have exacerbated the precautionary motive, further contributing to the growth in cash holdings.

Finally, this paper is related to our own prior research, and in particular to Crouzet and Eberly (2019). Relative to that paper, the current paper differs in two important ways. First, we derive a decomposition of the investment gap that allows for *both* intangibles and rents. By contrast, the framework Crouzet and Eberly (2019) does not allow for rents. The addition of rents delivers one of the key insights of this paper: rents can *amplify* the effect of intangibles on the investment gap; or, put differently, in the more general framework studied in this paper, the slope coefficient on intangible capital is higher when rents are high. Second, on the empirical side, this paper uses the structure of the model to quantify the respective contributions of intangibles, rents, and their interaction, to the growth of the investment gap over the past three decades. We find that the contribution of the interaction term is substantial, leading us to estimate that up to 60% of the aggregate investment gap is due to the rise in intangibles. By contrast, Crouzet and Eberly (2019) provides reduced-form evidence that the investment gap is higher in industries with higher intangible intensity and higher market power, but does not allow for an interaction between the two mechanisms. As a result, Crouzet and Eberly (2019) attributes only about 30% of the gap to intangibles.[5]

[4] Related, Andrei et al. (2019) show that the correlation between Q and investment at high frequencies has recently increased. We focus on the divergence between valuations and investment at longer horizons.

[5] It is also worth noting that the approach followed in Crouzet and Eberly (2019) is not structural. The statement, in that paper, that 30% of the gap is attributable to intangibles refers to the incremental explanatory power of intangibles in reduced-form regressions.

2 Rents, intangibles, and the investment gap: theory

In this section, we derive a general decomposition of the gap between average Q and marginal q. We call this the "investment gap". For each type of capital employed by the firm, the investment gap depends on economic rents, the other forms of capital employed by the firm, and the rents they generate. We provide analytical characterizations of the gap in certain special cases, relate our results to existing work, and study extensions of our basic framework. Proofs for the results of this section are in Appendix 1.

2.1 Model

Time t is discrete. A firm uses N different capital inputs, collected in a vector $\mathbf{K}_t = \{K_{n,t}\}_{n=1}^{N}$ in production.[6] The firm's operating profits as a function of capital are $\Pi_t(K_t)$, where K_t is an aggregate of the different types of capital, given by $K_t = F_t(\mathbf{K}_t)$. Total investment costs, including adjustment costs on capital, are given by $\tilde{\Phi}_t(\mathbf{K}_t, \mathbf{K}_{t+1})$. We index the functions F_t, Π_t and $\tilde{\Phi}_t$ to indicate that they can depend arbitrarily on other unspecified exogenous variables. The discount factor of the firm is $M_{t,t+1}$. Firm value satisfies:

$$\begin{aligned} V_t^c(\mathbf{K}_t) \quad &= \quad \max_{\mathbf{K}_{t+1}} \quad \Pi_t(K_t) - \tilde{\Phi}_t(\mathbf{K}_t, \mathbf{K}_{t+1}) + \mathbb{E}_t\Big[M_{t,t+1} V_{t+1}^c(\mathbf{K}_{t+1})\Big] \\ \text{s.t.} \qquad & \qquad K_t = F_t(\mathbf{K}_t), \end{aligned} \tag{1}$$

where $V_t^c(.)$ is the value of the firm including distributions. We make the following assumptions about the primitives of the problem.

Assumption 1. *$F_t(\mathbf{K}_t)$ is homogeneous of degree 1.*

Assumption 2. *$\Pi_t(K_t)$ is increasing, concave, and homogeneous of degree $\dfrac{1}{\mu} \leq 1$.*

Assumption 3. *Investment costs satisfy $\tilde{\Phi}_t(\mathbf{K}_t, \mathbf{K}_{t+1}) = \sum_{n=1}^{N} \Phi_{n,t}\left(\frac{K_{n,t+1}}{K_{n,t}}\right) K_{n,t}$, where each function $\Phi_{n,t}$ is strictly increasing and convex.*

The parameter μ plays a central role in our analysis: it indexes the economic rents accruing to the firm, with $\mu = 1$ corresponding to no rents. We discuss the link between μ and economic rents in more detail in Section 2.2.

In Section 2.4, we provide examples of models in the literature which are particular cases of the general model just described. We also clarify which frictions this model abstracts from, some of which we tackle in the extensions described in Section 2.5.

[6] These capital inputs can be broadly understood as any quasi-fixed factor which are costly to adjust and contribute to the output of the firm over more than one period; for instance, any stock of skilled labor that is both costly to adjust and does not fully depreciate within the period.

2.2 A decomposition of the investment gap

Our main result on the investment gap uses the following lemma.

Lemma 1. *Let:*

$$V_t^e = \mathbb{E}_t\left[M_{t,t+1}V_{t+1}^c\right], \quad q_{n,t} \equiv \frac{\partial V_t^e}{\partial K_{n,t+1}}, \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t}\frac{\partial K_t}{\partial K_{n,t}}. \tag{2}$$

Firm value can be written as:

$$V_t^e = \sum_{n=1}^{N} q_{n,t}K_{n,t+1} + (\mu - 1)\sum_{n=1}^{N}\sum_{k\geq 1}\mathbb{E}_t\left[M_{t,t+k}\Pi_{n,t+k}K_{n,t+k}\right]. \tag{3}$$

This lemma decomposes firm value into two parts.

The first part is the sum of the value of the installed stocks of each capital type n. This value is equal to the replacement cost, $K_{n,t+1}$, multiplied by marginal q, $q_{n,t}$; the latter will be different from 1 so long as the corresponding capital adjustment costs $\Phi_{n,t}$ are strictly convex. This generalizes the Hayashi (1982, Proposition 1) result to multiple capital inputs; this generalization was first noted by Hayashi and Inoue (1991), in a model where $\mu = 1$.

For the second part of equation (3), note that when there is only one type of capital ($N = 1$), it boils down to a discounted sum of the terms $(\mu - 1)\Pi_{K,t+k}K_{n,t+k}$. Moreover,

$$(\mu - 1)\Pi_{K,t+k} = \frac{\Pi_{t+k}}{K_{t+k}} - \Pi_{K,t+k}, \tag{4}$$

so that these terms are difference between the average and the marginal (revenue) product of capital. We interpret this difference as the flow value of rents. When $N = 1$ the second term in equation (3) is then simply the present value of future rents. This term is non-zero only when $\mu > 1$, as first noted by Lindenberg and Ross (1981) and Hayashi (1982, Proposition 2) in models where $N = 1$. The magnitude of μ controls the overall size of rents.

When $N > 1$, the second term in Equation (3) is the sum of terms of the form:

$$(\mu - 1)\Pi_{n,t+k} = \left(\frac{\Pi_{t+k}}{K_{n,t+k}} - \Pi_{K,t+k}\right)\frac{\partial F_{t+k}}{\partial K_{n,t+k}}. \tag{5}$$

These terms capture the marginal contribution of capital of type n to overall rents earned by the firm. The flow value of rents is the gap between the average and the marginal (revenue) product of capital of type n. The intuition from the $N = 1$ case thus carries through, with the added insight that total rents are additively separable across capital types, which will be useful in quantifying the contribution of each type of capital to overall rents.

Result 1. *Define average Q for capital of type n, $Q_{n,j,t}$, as:*

$$Q_{n,t} = \frac{V_t^e}{K_{n,t+1}}.$$

Then, the investment gap for capital of type n can be written as:

$$\begin{aligned} Q_{n,t} - q_{n,t} &= (\mu - 1) \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \Pi_{n,t+k} (1 + g_{n,t+1,t+k}) \right] &(6) \\ &+ \sum_{\substack{m=1 \\ m \neq n}}^{N} S_{m,n,t+1} q_{m,t} &(7) \\ &+ (\mu - 1) \sum_{\substack{m=1 \\ m \neq n}}^{N} S_{m,n,t+1} \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \Pi_{m,t+k} (1 + g_{m,t+1,t+k}) \right], &(8) \end{aligned}$$

where $1 + g_{n,t+1,t+k} \equiv \dfrac{K_{n,t+k}}{K_{n,t+1}}$, *and* $S_{m,n,t+1} \equiv \dfrac{K_{m,t+1}}{K_{n,t+1}}$.

The investment gap is the sum of three terms, (6), (7) and (8).

When there are no rents and a single type of capital ($\mu = 1$ and $N = 1$), these three terms are zero. Average Q and marginal q are equal, as in Hayashi (1982, Proposition 1), and there is no investment gap.

If there are rents but only one type of capital ($\mu > 1$ and $N = 1$), only the term (6) is nonzero. Average Q will overstate marginal q, and the gap is equal to the present value of flow rents, that is, the term (6). This case includes the Lindenberg and Ross (1981) effect.

If there are no rents but several types of capital ($\mu = 1$ and $N > 1$), then for each type of capital, average Q will *still* overstate marginal q. Average Q for a specific type of capital reflects, in part, the value of other types of capital used by the firm, because these other types of capital contribute to firm value overall. It therefore overstates the true incentive to invest — the marginal q — of that type of capital.[7] This omitted capital effect is captured by the term (7) in the expression of the investment gap.

If there are both economic rents and several types of capital ($\mu > 1$ and $N > 1$), the rents term (6) and the omitted capital term (7) are still non-zero. But additionally, the term (8) is non-zero. It represents the interaction between the rents and the omitted capital effect. It captures how rents accruing to *other* types of capital affect total firm value and, through the omitted capital effect described above, add to the gap between average Q and marginal q. This interaction term is larger, the higher the relative importance of other types of capital, and the higher the rents generated by other types of capital.

[7] Crouzet and Eberly (2019) also make this point in a model with two types of capital and no rents.

2.3 Balanced growth

We now provide analytical expressions for the investment gap decomposition in a model with balanced growth. These expressions help build intuition for each of the components of the gap, and also anticipate our empirical applications.

Without loss of generality, we focus on the $N = 2$ case; $K_{1,t}$ is "physical capital," and $K_{2,t}$ is "intangible capital". We assume the profit function is $\Pi_t = A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}}$, where $\mu \geq 1$. A_t is an exogenous process capturing firm fundamentals and growth such that $A_{t+1}/A_t = 1+g$. We also assume $M_{t,t+1} = (1+r)^{-1}$, with $g < r$. Finally, we assume that the capital aggregator and the capital adjustment costs are time-invariant, and that investment costs satisfy the standard conditions:

$$\Phi_n(1) = \delta_n, \quad \Phi_n'(1) = 1, \quad \Phi_n''(1) = \gamma_n \geq 0.$$

Result 2. *In balanced growth, the investment gap for physical capital is given by:*

$$\begin{aligned} Q_1 - q_1 &= \frac{\mu - 1}{r - g} R_1 \quad + \quad S q_2 \quad + \quad \frac{\mu - 1}{r - g} R_2 S, \\ R_n &\equiv (r - g)\Phi_n'(1+g) + \Phi_n(1+g), \quad n = 1, 2, \end{aligned} \tag{9}$$

and where marginal q, average Q, and intangible to physical capital ratio, S, are constant.

In order to build intuition for the elements of Equation (9), consider first the special case of linear investment costs: $\gamma_n = 0$ and $\Phi_n(x) = x - 1 + \delta_n$. In that case,

$$R_n = r + \delta_n.$$

Intuitively, without convex adjustment costs, the firm behaves as though it were renting capital in perfectly competitive markets, equating the marginal revenue product of each type of capital to its Jorgensonian user cost, R_n: $\Pi_{n,t} = \Pi_n = R_n = r + \delta_n$. The two rents terms in decomposition (9) then represent the net markup over the marginal (user) cost of each type of capital, discounted by the Gordon growth term $r - g$.

When investment costs are convex ($\gamma_1 > 0, \gamma_2 > 0$), the $\{R_n\}$ can be interpreted as "internal" user costs. They satisfy:

$$R_n = r + \delta_n + \gamma_n r g + o(g),$$

where $o(g)$ is the little-o Landau notation. The additional term $\gamma_n r g + o(g)$ reflect the cost of continuously adjusting capital along the firm's growth path.

2.4 Discussion

Why is $Q_{n,t} - q_{n,t}$ an "investment gap"? We extend the terminology "investment gap" used in Gutiérrez and Philippon (2017) and Alexander and Eberly (2018). The first-order condition for investment is: $g_{n,t} = \Psi_{n,t}(q_{n,t} - 1)$, where $g_{n,t}$ is the net investment rate, and $\Psi_{n,t}(y) \equiv \left(\Phi'_{n,t}\right)^{-1}(1+y) - 1$. When the investment gap is positive ($Q_{n,t} > q_{n,t}$), we have $\Psi_{n,t}(q_{n,t} - 1) = g_{n,t} \leq \Psi_{n,t}(Q_{n,t} - 1)$. Investment predicted using average Q will exceed actual investment; that is, there will appear to be a "gap" the two.

Why not use "Total Q"? Total Q is the ratio of the value of the firm to its total (physical plus intangible) capital stock (Peters and Taylor, 2017). In our model, it is given by:

$$Q_{tot,t} \equiv V_t^e / \sum_{n=1}^{N} K_{n,t+1} = \underbrace{\sum_{n=1}^{N} s_{n,t+1} q_{n,t}}_{\equiv q_{tot,t}} + (\mu - 1) \sum_{n=1}^{N} s_{n,t+1} \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} \Pi_{n,t+k} (1 + g_{n,t+1,t+k}) \right]$$

where $s_{n,t+1} = K_{n,t+1} / \sum_{n=1}^{N} K_{n,t+1}$. Define the "total Q investment gap" as $Q_{tot,t} - q_{tot,t}$. This gap will be positive when the firm earns rents; moreover, rents can be decomposed across types of capital. However, we do not focus on $Q_{tot,t} - q_{tot,t}$ for one main reason: $q_{tot,t}$ is not a sufficient statistic for total investment, except in specific cases.[8] As a result, there is no mapping from $Q_{tot,t} - q_{tot,t}$ to the empirical shortfall in total investment. By contrast, $q_{n,t}$ is a sufficient statistic for investment in capital n, and so the capital-specific investment gap $Q_{n,t} - q_{n,t}$ entirely accounts for the relationship between $Q_{n,t}$ and investment.

How general is the model? Our model puts no restriction on functional forms other than Assumptions 1-3. It therefore nests a number of existing models; for instance Lindenberg and Ross (1981), Hayashi (1982), Abel (1983), Abel and Blanchard (1986), Hayashi and Inoue (1991), case I of Abel and Eberly (1994), and Abel and Eberly (2011), as well as the investment block of the macroeconomic models in McGrattan and Prescott (2010), Karabarbounis and Neiman (2019) or Barkai (2020).

However, it has three limitations. First, it does not allow for non-convex adjustment costs. Second, it abstracts from financial constraints. The next subsection discusses extensions in this direction. Third, it assumes that rents, μ, are exogenous. In particular, they do not depend on past investment, in contrast, for instance, with models of customer capital.[9] In this sense, our results are restricted to "neoclassical" models of the firm, and provide a benchmark against which the effects of other frictions on the investment gap can be compared.

[8] Appendix 1.3 shows that perfect substitutability and identical investment costs is one such case.
[9] See, for instance, Gourio and Rudanko (2014) and Belo et al. (2014).

2.5 Extensions

Uncertainty Closed-form solutions for the investment gap exist when fundamentals are uncertain; we discuss this in greater detail in Appendix 2.2.

Result 3. *Assume that* $N = 2$, $\Pi_t = A_t^{1-\frac{1}{\mu}} K_t^{\frac{1}{\mu}}$, *and:*

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 1 + g_{t-1} & w.p.\ 1-\lambda \\ 1 + \tilde{g} & w.p.\ \lambda \end{cases}, \qquad \tilde{g} \sim G(.)\ i.i.d.$$

Moreover, assume that $\Phi_n(x) = x - 1 + \delta_n$, $n = 1, 2$. *Then:*

$$Q_{1,t} - q_{1,t} = \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_1) + S + \frac{\mu - 1}{r - \nu(g_t)}(r + \delta_2)S, \tag{10}$$

where the expression for the function $\nu(.)$ *is reported in Appendix 2.2.*

The resulting decomposition is similar to Result 2, except that the Gordon growth term $\frac{1}{r-\nu(g_t)}$ adjusts for the possibility of regime changes in growth rates.[10] Key intuitions are similar to those discussed in Section 2.3: the two rents terms are equal to the present value of flow rents, with flow rents equal to the net markup over user costs. In Section 5.3, we use Result 3 in order to estimate a version of the model with uncertainty in our empirical applications.

Market power, decreasing returns, and rents Two natural sources of rents are market power on the goods market ("pure" rents), and decreasing returns to scale ("quasi" or "Ricardian" rents). The mapping between μ and these two sources of rents is the following.

Result 4. *Suppose that the firm uses flexible inputs that are Cobb-Douglas substitutes with capital, where* α *is the capital share. Let* ζ *index returns to scale, and let* μ_S *be the firm's markup over the cost of sales. Then:*

$$\mu \equiv 1 + \frac{\mu_S/\zeta - 1}{\alpha}.$$

Moreover, total (pure and quasi-) rents over operating surplus are given by $\frac{\mu-1}{\mu}$.

Appendix 2.3 establishes this result. Importantly, the magnitude of μ does not depend separately on μ_S or ζ, but only on their *ratio*. Our results can therefore be thought of through the lens of either type of rent. In Appendix 2.3, we also argue μ_S and ζ cannot be separately identified using only *nominal* ratios (such as cost shares, surplus ratios, user

[10] The case $\lambda = 0$ corresponds to constant growth, as in the balanced growth model of Result 2; in that case, $\nu(g_t) = g_t$. The case $\lambda = 1$ corresponds to $i.i.d.$ growth rates, with $\nu(g_t) = \mathbb{E}[\tilde{g}]$.

costs, or average returns to capital), as these ratios are all functions of μ_S/ζ instead of either parameter independently. This point is also highlighted by Basu (2019). Therefore, in Section 5.5, we will discuss what our estimates of μ imply for the value of "pure" rents under different assumptions about returns to scale.

Heterogeneous rents parameters Our results extend to the case where the rents parameters μ is allowed to be different across types of capital, as follows.

Result 5. *Assume that operating profits are given by a mapping $\Pi_t(\mathbf{K}_t)$ satisfying:*

$$\tilde{\Pi}_t(\mathbf{K}_t) = \sum_{n=1}^{N} \mu_n \tilde{\Pi}_{n,t} K_{n,t}, \quad \mu_n \geq 1 \quad \forall n. \tag{11}$$

In balanced growth with $N = 2$, we have:

$$Q_1 - q_1 \quad = \quad \frac{\mu_1 - 1}{r - g} R_1 \quad + \quad S q_2 \quad + \quad \frac{\mu_2 - 1}{r - g} R_2 S. \tag{12}$$

Similar generalizations of Lemma 1 and Result 1 are reported in Appendix 2.4.[11] Appendix 2.4 also characterizes a class of operating profit functions satisfying condition (11); it could capture, for instance, a firm with different revenue streams generated by independent divisions, each using a different type of capital. We focus on the version with $\mu_1 = \mu_2 = \mu$ because separate identification of each the rents parameters in the case $\mu_1 \neq \mu_2$ is more challenging: it requires data on the marginal revenue product of each type of capital separately. We return to this issue in Section 5.6.

Link to the production-based asset pricing literature In Appendix 2.5, we study the difference between stock returns and returns to investment in each type of capital, following Cochrane (1991, 1996).[12] When $N = 1$ and $\mu = 1$, the two are equalized, as in Cochrane (1991). But when $N > 1$ or $\mu > 1$, they need not be. Moreover, their difference is driven by the same three forces as the investment gap: omitted capital; rents; and their interaction.

An important difference is that returns depend on *changes* in firm value, whereas average Q and marginal q depend on the *level* of firm value. As a result, the returns gap is more likely to be informative about high-frequency movements in intangible intensity and rents, while the investment gap is more likely to be informative about long-run trends.[13] For instance, in

[11] The baseline model explicitly separates capital aggregation from the operating surplus function; it is a special case of this more general model, with $\tilde{\Pi}_t(\mathbf{K}_t) = \Pi_t(F_t(\mathbf{K}_t))$ and $\mu_n = \mu \ \forall n$.

[12] We thank an anonymous referee for this suggestion.

[13] This echoes Cochrane (1991, p.218): "Returns emphasize high frequency aspects of the data that the models may be better able to capture in the presence of slow moving and unobserved changes in technology."

balanced growth, the difference between stock returns and returns to investment is zero, as the two returns are equalized to the discount rate even if $N > 1$ and $\mu > 1$. By contrast, in balanced growth the investment gap remains positive, as highlighted by Result 2. Given that trends are the focus of our paper, we choose to work with the investment gap.

Financing frictions A large literature has shown financial constraints can drive a wedge between average Q and marginal q (Whited, 1992; Gomes, 2001; Hennessy et al., 2007; Bolton et al., 2011; DeMarzo et al., 2012). However, the sign and size of this wedge is a matter of debate, particularly if the firm has market power (Cooper and Ejarque, 2003). In Appendix 2.6, we study the investment gap in versions of the model with two simple financial frictions.

Result 6. *Assume that shareholders can raise debt B_{t+1}, subject to a collateral constraint of the form $B_{t+1} \leq \theta K_{1,t+1}$. Define marginal $q_{n,t}$ as $q_{1,t} \equiv q^{(E)}_{1,t} + \lambda_t \theta$ for $n = 1$ and $q_{n,t} \equiv q^{(E)}_{n,t}$ for $n = 2, ..., N$, where $q^{(E)}_{n,t}$ is the marginal value of an unit of capital to shareholders, and λ_t is the Lagrange multiplier on the leverage constraint. Then, the decomposition of the enterprise investment gap for capital n, $Q_{n,t} - q_{n,t}$, is the same as in Result 1.*

This result states that a collateral constraint with respect to physical capital does not change the expression for the investment gap, so long as one focuses on the *enterprise* investment gap, defined as the gap $Q_{n,t} - q_{n,t}$.[14] The intuition is that because debt is risk-free, there is no conflict between creditors and shareholders, and the investment policy chosen by shareholders also maximizes total *enterprise* value.

Result 7. *Assume that the flow value of dividends to shareholders is given by $K_t f(d_t)$, where $d_t = D_t/K_t$, D_t is revenue net of investment costs, and f satisfies $f(0) = 0$, $f' > 0$, $f'(0) = 1$, and $f'' \leq 0$. Then, the investment gap has the same expression as in Result 1, replacing the discount factor $M_{t,t+k}$ with $f'(d_{t+k})M_{t,t+k}$.*

The function $f(.)$ describes equity financing frictions in a reduced-form way: the fact that $f'(d_t) < 1$ when $d_t > 0$ could capture agency costs of free cash flows (Jensen and Meckling, 1976), while the fact that $f'(d_t) > 1$ when $d_t < 0$ could capture costs of seasoned equity offerings (Altınkılıç and Hansen, 2000).[15] These frictions change the way in which shareholders value future rents, but do not affect the three main elements of the decomposition.[16]

[14] Appendix 2.6 shows that, consistent with the prior literature, the investment gap for *shareholders*, that is, the difference between the ratio of equity value to the stock of physical capital, and $q^{(E)}_{n,t}$, has a similar expression as Result 1, with an additional, *negative* wedge, reflecting the fact that part of the marginal return to investment, for shareholders, comes from the fact that it relaxes the borrowing constraint.

[15] We follow Hennessy et al. (2007), except that we allow for $f'(d_t) < 1$ when $d_t > 0$. This makes equity financing costs matter on the balanced growth path, where $d_t = d > 0$.

[16] As discussed in Appendix 2.6, two definitions of marginal q are possible, depending on whether one normalizes marginal q by $f'(d_t)$ or not. Result 7 refers to an unadjusted marginal q; with the latter definition, the investment gap has an additional wedge, which we characterize in Appendix 2.6.

Thus, simple frictions to either equity or debt financing do not change the basic insights of Result 1 regarding the components of the investment gap. However, they can change the *magnitude* of these components, as well as the size of the investment gap overall, relative to the frictionless model. We discuss this point in more detail in Section 5.7.

3 The investment gap in aggregate data

We now show that the investment gap for non-financial corporate businesses has tripled since 1985, driven by the combined effects of rising rents and rising intangibles. This section uses national accounts data, which has the most coverage, but the narrowest measure of intangible capital. We broaden the intangibles measure in the next section, drawing on firm-level data.

3.1 Methodology

We use the balanced growth model to construct the investment gap and its components in the data. We have:

$$Q_1 - q_1 \quad = \quad \frac{\mu - 1}{r - g} R_1 \quad + \quad q_2 S \quad + \quad \frac{\mu - 1}{r - g} R_2 S, \tag{13}$$

where recall that, neglecting terms of order $o(g)$, $R_n = r + \delta_n + \gamma_n r g$, and $q_n = 1 + \gamma_n g$, $n = 1, 2$. We measure Q_1 and S directly from data, as described below. However, we infer values for $\{\mu, r - g, R_1, R_2, q_1, q_2\}$ from the following additional observable moments: $\{ROA_1, i_1, i_2, g\}$, where $ROA_1 = \Pi_t / K_{1,t}$ is average returns to physical capital, i_1 and i_2 are gross investment rates, and g is the net growth rate of total capital $K_{1,t} + K_{2,t}$.

We proceed as follows. First, we use the fact that:[17]

$$\mu = \frac{ROA_1}{R_1 + S R_2}. \tag{14}$$

Intuitively, rents create a wedge between average returns to physical capital and the weighted average user cost of capital. Second, we have that:

$$R_n = r - g + i_n + \gamma_n r g, \quad n = 1, 2, \tag{15}$$

where we used the fact that $i_n = g + \delta_n$ along the balanced growth path. Finally, substituting

[17] See Appendix 1 for a formal derivation of this relationship.

Equations (14) and (15) in the investment gap decomposition (13), we obtain:

$$r - g = \frac{ROA_1 - (i_1 + Si_2)}{Q_1} - \frac{\gamma_1 + S\gamma_2}{Q_1} g^2. \tag{16}$$

This expression for the Gordon growth term $r - g$ only requires estimates of the adjustment cost parameters. Given the value for $r-g$ and other data moments, values of R_1 and R_2 follow from Equation (15); and the value of μ follows from Equation (14). Finally, q_1 and q_2 are obtained from the values of g and from calibrated values for the adjustment cost parameters γ_1 and γ_2, which we discuss below.

The most important feature of this identification approach is that it matches, by construction, the empirical value of Q_1. It infers the Gordon growth term $r - g$ which, given other moments, ensures that the model produces a value of Q_1 consistent with the data. Our use of valuations, via Q_1, is a natural implication of the model, but also an important point of departure from the recent literature. We discuss this point in more detail Section 3.3.

Additionally, we note that our methodology does not make direct use of data estimates of economic rates of depreciation, δ_n. Instead, we substitute depreciations for gross and net investment rates, using the relationship $\iota_n = g + \delta_n$, $n = 1, 2$. We choose to use investment rates in our empirical approach because our main goal is to account for their behavior relative to valuations. Below, we discuss in more detail the implications of our estimated model for depreciation rates.

3.2 Aggregate Data

Our sample period is 1947-2017, and we focus our analysis on the non-financial corporate business (NFCB) sector.[18] Appendix 3 reports details on data sources and data construction. We construct time series for five of the moments used in the decomposition, $\{i_{1,t}, i_{2,t}, S_t, ROA_{1,t}, Q_{1,t}\}$, using six times series in levels, $\{K_{1,t}, I_{1,t}, K_{2,t}, I_{2,t}, \Pi_t, V_t\}$. These are the operating surplus of the NFCB sector, the stock of physical capital at replacement cost, investment in physical capital, the stock of intangibles at replacement cost, investment in intangibles, and the market value of claims on the NFCB sector.[19]

We obtain measures of $K_{1,t}, I_{1,t}, K_{2,t}$ and $I_{2,t}$ from BEA Fixed Assets tables 4.1 and 4.7. The BEA Fixed Assets tables use perpetual inventory methods to construct the stock of three specific forms of intangible capital: R&D; own-account software; and artistic originals.[20] To

[18] Appendix 3.7 shows that trends in economy-wide and NFCB average returns to capital are similar.

[19] We use current-dollar values for all time series in levels, with the exception of our proxy for g_t, the computation of which is described below.

[20] Related to Footnote 6, we note that investment in R&D capital, in the BEA, is partly estimated using compensation to R&D workers (Corrado et al., 2005). Thus, one potential re-interpretation of the BEA's

the extent that firms invest in other types of intangibles, results in this section should thought of as a lower bound on the overall role of intangibles. Section 4 expands the analysis to organization capital for the subset of publicly traded firms in the NFCB sector.

Operating surplus Π_t is obtained from NIPA Table 1.14. Consistent with the model, this series represents the difference between value added and payments to labor; expenditures categorized as intangible investment are not treated as intermediates in value added.

We construct a measure of V_t using Flow of Funds tables L.103 and F.103. In the model, V_t represents the market value of all net claims on the NFCB sector, both debt and equity. The Flow of Funds data provide an estimate for the market value of equity of the NFCB sector, but not for debt. Our approach to estimate the latter is described in detail in Appendix 3.2. It is similar to the approach of Hall (2001), except that we do not subtract *all* financial assets owned by the sector from the gross market value of claims, but *only* financial assets identified as liquid in the Flow of Funds.[21] Section 5.1 shows that this choice affects the level of the investment gap, but not its composition.

We then construct $ROA_{1,t} = \Pi_t/K_{1,t}$, $i_{1,t} = I_{1,t}/K_{1,t}$, $i_{2,t} = I_{2,t}/K_{2,t}$, $S_t = K_{2,t}/K_{1,t}$, and $Q_{1,t} = V_t/K_{1,t}$. Additionally, g_t is the annual growth rate of the quantity index for private non-residential fixed assets of the NFCB sector, provided in BEA Fixed Assets table 4.2.[22]

We use calibrated values for adjustment costs. We consider three cases: zero adjustment costs, $\gamma_1 = \gamma_2 = 0$; positive adjustment costs; and high adjustment costs. For the positive adjustment cost case, we choose values of $\gamma_1 = 3$ and $\gamma_2 = 12$, following the estimates of Belo et al. (2019). For the high adjustment cost case, we choose values $\gamma_1 = 8$ and $\gamma_2 = 18$, at the high end of existing estimates. In Section 5.2, we discuss the effect of adjustment costs in more detail. We show that our results regarding the composition of the investment gap are robust to the choice of adjustment costs, and attribute greater importance to intangibles when adjustment costs are higher.

The time series for the resulting six moments, $\{i_{1,t}, i_{2,t}, S_t, ROA_{1,t}, Q_{1,t}, g_t\}$ are reported in Appendix Figure 1. The key trends discussed in the introduction are visible in that figure.

stock of R&D intangibles is as a stock of skilled R&D labor. Separating strictly the two would require data on the composition of R&D costs between labor and other expenses.

[21] Financial assets are generally subtracted from the gross market value of claims in order to include net debt, instead of gross debt, in firm value calculations. On the other hand, financial assets can only meaningfully be counted as negative debt to the extent that they are liquid. Additionally, a large part of non-liquid financial assets in table L.103 are obtained as a residual, further complicating their interpretation.

[22] Although the balanced growth model imposes identical growth rates across capital types, the growth rate of intangibles has generally been higher than that of physical capital, and therefore higher than g_t, as reported in Appendix Figure 3. However, as that figure also shows, they were close the 1970s to the mid-1980s, and have been close since the early 2000s. Appendix 3.3 describes these time series in more detail, and Appendix 4.1 shows that the results from main decomposition are robust to allowing for heterogeneous growth rates across capital stocks.

The average return to physical capital increases after 1985, while the physical investment rate declines. The ratio of intangible to physical capital increases, particularly after 1985. Q_1 rises sharply after 1985, and after a peak in 2000, remains approximately double its value in the pre-1985 period.

Finally, we compute the decomposition using moving averages of moments over 7-year centered rolling windows. This treats each successive window as if it were generated by a different quantitative implementation of the model, allowing us to capture gradual changes in the investment gap.[23] In Section 5.3, we report results obtained by estimating a version of the model with shocks (instead of the balanced growth model) on split samples using GMM.

3.3 Baseline results

The investment gap and underlying structural changes Figure 1 reports the investment gap decomposition, Equation (13), for the NFCB sector and R&D intangible capital. The decomposition emphasizes three main findings.

First, the investment gap is large during two distinct periods: 1960-1970, and after 1985. The wedge between average Q and marginal q is therefore not strictly a hallmark of the post-1980s period. Second, rents attributable to physical capital — the first term in Equation (13) — play a sizable (though somewhat declining) role in explaining the investment gap: they account 61% of it in 2015, compared to 67% in 1965.[24] Third, rents attributable to intangibles — the third term in Equation (13) — have become markedly more important in recent years. In 2015, 25% of the investment gap reflects the combined effects of high rents and a large stock of intangibles, compared to 10% in 1965, using the BEA measure of R&D capital only, the narrow measure of intangibles available in these data.

From the standpoint of the model, these changes are driven by three underlying forces, reported in Figure 2 : a greater importance of intangibles in the production function; higher rents; and a decline in user costs, more pronounced for physical than for intangible capital.

The top left panel of Figure 2 shows that even using the relatively narrow definition of intangibles in the NFCB data, the share of intangible capital in production, η, increased substantially after 1985, from 0.17 to 0.29 in 2015.[25] The behavior of the intangible share approximately mimics the behavior of the measured ratio of intangible to physical capital at replacement cost, which increases rapidly after 1985.

[23] Using alternative window sizes from 3 to 9 years gives quantitatively similar results.

[24] These numbers, and those that follow in this discussion, refer to the model with intermediate adjustment costs, $\gamma_1 = 3$ and $\gamma_2 = 12$.

[25] This is derived assuming a Cobb-Douglas aggregator $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. The level of intangible share is sensitive to the Cobb-Douglas assumption, but not the magnitude of the change after 1985.

The effects of the intangible share on the overall investment gap are magnified by the rise in rents after 1985. The top right panel of Figure 2 reports estimates of the rents implicit in Equation (13). In order to facilitate comparison with existing estimates, we express them as the flow value of rents relative to value added, which is related to the parameter controlling rents in the model, μ, through $s = (1 - s_L)(1 - 1/\mu)$, where s_L is the labor share of value added.[26] Rents, as a fraction of value added, increase from 1.5% in 1985, to 7.7% in 2015 — a cumulative 6.2 percentage point (p.p.) change over three decades. Expressed as markups over value added, this is an increase from 1.015 in 1985, to 1.083 in 2015.

Finally, we note two other features of our time-series for the investment gap. First, the gap is elevated during the 1960s; the decomposition attributes this to a combination of low user costs (driven by the low interest rates of the period), and high rents.[27] Second, the gap is particularly small during the 1975-1985 period. The model primarily attributes this reversal to the large increase in discount rate and the decrease in growth rates around the early 1980s, which, by reducing the present value of future rents, pushes the average value of installed capital closer to its marginal value.

Comparison to existing literature These findings are *qualitatively* consistent with the recent literature arguing that pure profits as fraction of value added have been growing over the last three decades (Gutiérrez and Philippon, 2017; Barkai, 2020; Karabarbounis and Neiman, 2019). However, they differ *quantitatively*. For instance, Barkai (2020) finds that the pure profit share rose from -5.6% in 1984 to 7.9% in 2014, an increase of 13.5 p.p. over the period. Karabarbounis and Neiman (2019), in their "case Π," find that the pure profit share must have risen by about 13 p.p. over the same period. We find an increase in rents of half that magnitude.[28]

User costs are at the heart of this difference. Specifically, our approach leads to user costs that are initially lower, but that decline more slowly. Figure 2 reports these implied user costs. User costs for physical capital decline from 15.4% to 12.6% between 1985 and 2015, while user costs for intangibles decline from 36.8% to 30.4%; their weighted average only declines from

[26] We measure the labor share for the NFCB sector using NIPA data on labor payments for that sector, as described in Appendix 3.1. As discussed in Appendix 4.2, given our estimate of μ, matching the labor share in the data implies that the Cobb-Douglas elasticity of value added with respect to labor must vary over time. Alternatively, we consider fixing the Cobb-Douglas elasticity of value added with respect to labor; the results are almost identical.

[27] On the latter point, we note that, related to the recent work of Gutiérrez and Philippon (2018b), the legal literature on antitrust policy has highlighted the 1960s as a period of weak enforcement (Hovenkamp, 2018).

[28] Related to this are the markup estimates of De Loecker et al. (2020) and Hall (2019). These markups, when expressed in value added terms, are much higher than ours — approximately 1.9 and 4 in 2015, respectively —, and also far outside the range typically considered reasonable in the macroeconomics literature, as discussed in detail by Basu (2019).

17.1% to 15.2%. (By contrast, Barkai (2020), for instance, finds a required rate of return on capital that falls from approximately 20% in 1985 to approximately 14% in 2014.) The smaller decline in user costs translates to higher payments to capital (particularly to intangibles), and therefore a smaller increase in rents.

The way we infer the discount factor perceived by firms from the data is key to this result. As discussed before, we rely on valuations; by contrast, the papers mentioned above generally combine risk-free rates with imputed estimates of risk premia to obtain discount rates. Appendix Figure 2 reports the discount rate r implied by our approach. It declines from 7.9% to 5.6% between 1985 and 2015. This is a smaller decline than the risk-free rate over the same period of time, and is therefore consistent with a mild rise in risk premia over this period of time, as argued by Caballero et al. (2017), Farhi and Gourio (2018), and Karabarbounis and Neiman (2019) in their case R. In Section 5.4, we discuss the results we would obtain using a cost-of-capital approach instead of a Q approach.

User costs and rates of depreciation Our analysis implies that user costs for intangible capital have fallen by *less* than those of physical capital. This change in relative user costs explains why rents attributable to intangibles, which are the present value of net markups over their user costs, as indicated by Equation (13), account for an increasing fraction of the investment gap after 1985.

Our approach infers this from the higher gross investment rates in intangibles, which, through the relationship $\delta_n = \iota_n - g$, $n = 1, 2$, also imply high rates of depreciation. Appendix Figure 4 reports the model-implied depreciation rates, along with empirical counterparts, obtained from the BEA's Fixed Assets tables.[29] The model-implied and data series behave similarly; both show a marked increase in depreciation rates for intangibles.[30] The main difference is that our implied rate of depreciation for intangible is higher, on average, than its empirical counterpart, owing to the higher net growth rate of the intangible capital stock mentioned above.[31]

We note that depreciation estimates indirectly enter our measurement, through their impact on estimates of capital stocks.[32] However, investment rates in BEA data are not primarily

[29] Appendix 3.4 describes the computation of the empirical counterparts to the model depreciation rates.

[30] Given that the BEA's estimates of depreciation rates are based on constant depreciation rates at the asset level, the upward trend in depreciation rates reflect a shift in the composition of the capital stock toward assets with shorter service lives.

[31] As mentioned in footnote 22, these small differences in growth rates across capital stocks do not materially affect the quantitative results obtained in our decomposition.

[32] In the BEA data, estimates of economic depreciation are the residuals that reconcile measured gross investment, and estimates the net stocks of capital based on perpetual inventory methods. The net stock estimates themselves rely on assuming constant rates of economic depreciation at the asset-level, the values of which are based on microeconomic studies. Appendix 3.4 describes the methodology in detail.

driven by the value of economic depreciation rates assumed by the BEA. Appendix Figure 4 also reports gross investment rates and gross depreciation rates for both physical and intangible investment. While gross physical investment rates have trended downward, depreciation rates of physical capital, for instance, have trended upward.

3.4 Counterfactuals

In order to further illustrate the respective roles played by intangibles and rents in our estimation, Figure 3 reports results from two counterfactual exercises.

The top panel constructs the change in the share of intangibles in production, η, that would be necessary in order to fully account for the increase in the investment gap, assuming that rents remain fixed at their 1985 level. This change is 34 p.p., compared to 12 p.p. in our baseline results. This, in turn, implies that the ratio of intangible to total capital, at replacement cost, would need to be 30 % in 2015, or approximately twice its observed value of 14 % in the NFCB sector.[33] In Section 4, we show that this magnitude is comparable to the ratio of intangible to total capital *including* organization capital among publicly traded firms.[34]

The bottom panel of Figure 3 shows the increase in rents, as a fraction of value added, which would be required in order to match the observed investment gap, assuming that both the share of intangible capital, R_2, and the intangible investment rate ι_2, had remained fixed at their 1985 values. Instead of the 6.2 p.p. increase in rents as a fraction of value added which we estimate as our baseline, rents would have needed to increase by 8.4 p.p., reaching 10.0% of value added by 2015. The total contribution of intangibles to the investment gap would nevertheless remain elevated (approximately 31%, instead of 39% in our baseline), due to the rising rents generated by the (more moderate) fixed stock of intangibles. This is really an intermediate case, since it allows growth in intangibles, but not the acceleration seen in the data.

To show more extreme cases of these two counterfactuals, Appendix 4.3 reports results in

[33] Simple algebra, using the results of Section 3.1, shows that the counterfactual ratio of intangible to physical capital $\hat{S}$ under fixed rents is the smallest positive root of $Ax^2 + Bx + C = 0$, where $A = \iota_2 + \gamma_2 g(\iota_2 + g + \gamma_2 g)$, $B = \iota_1 + \iota_2 - ROA_1 - Q_1\iota_2 + \gamma_1 g(\iota_2 + g) + \gamma_2 g(\iota_2 + g - Q_1 g - ROA_1) + 2\gamma_1\gamma_2 g^3$, $C = ROA_1 Q_1/\mu^{(1985)} + \iota_1 - ROA_1 - Q_1\iota_1 + \gamma_1 g(\iota_1 + g + \gamma_1 g^2 - ROA_1 - Q_1 g)$, and $\mu^{(1985)}$ is the estimated value of the rents parameter μ in 1985 using our baseline approach. The ratio of intangible to total capital is then given by $\hat{S}/(1+\hat{S})$.

[34] This magnitude is also comparable to Karabarbounis and Neiman (2019), "case K". These authors show that, if the profit share is assumed to be zero, then unmeasured capital would need to account for approximately 40% of all business capital after 1970 in order to explain the measured capital share. Expressed in terms of value added, our estimates imply that intangibles would need to be approximately 63% of value added in the NFCB sector; this in line with similar estimates obtained by McGrattan and Prescott (2005) under perfect competition.

versions of the model with either no intangibles ($N = 1$) or no rents ($\mu = 1$). In the case of no intangibles, the model requires a 12 p.p. increase rents (as a fraction of value added) from 1980 to 2015, reaching 14% in 2015; this is almost double the magnitude obtained in our baseline approach. In the case of no rents, the implied ratio S of intangible to physical capital required to explain the level of Q_1 in 2015 is approximately 1, compared to 0.3 in the data. Additionally, the implied times series for S exhibits periods of substantial decline, particularly in the late 1970s and in the wake of the dot-com bubble. This intangible capital "destruction" is at odds with empirical measures of S, which grow consistently in the data, as shown in Appendix Figure 1.[35]

Aside from the additional results already mentioned, Section 5 provides further robustness checks and extensions to our baseline results, including: results from an approach that infers intangibles from Q_1, and from an approach that infers rents from Q_1 (Section 5.4); a discussion of the magnitude of pure rents under different assumptions about returns to scale (Section 5.5); a discussion of the implications of the model with heterogeneous rents parameters $\{\mu_n\}$ (Section 5.6); and a discussion of how the financing frictions discussed in Section 2.5 may bias our results (Section 5.7).

Summarizing, we documented a large investment gap in the NFCB sector after 1985. This gap reflects a combination of rising rents and a growing importance intangibles in production, with the latter accounting for about one-third of the gap. Additionally, though our valuation-based approach finds rising rents, the magnitude of the increase is approximately half that of existing estimates.

4 The investment gap in firm-level data

In this section, we construct investment gaps at the sectoral level, and highlight how they change when measures of intangibles are expanded beyond R&D capital. We find substantial differences across sectors in both the level of the gap and the relative contributions of rents and intangibles. Expanding measures of intangibles beyond R&D reduces the quantitative estimates of rents, and suggests that intangibles are the dominant force behind the growth in the investment gap.

[35] The no rents approach corresponds to the method used by Hall (2001) to estimate the stock of intangible capital of non-financial businesses. He also finds a decline in the stock of intangibles in the late 1970s.

4.1 Data

We use the non-financial segment of Compustat, instead of data drawn from the National Accounts. This restricts the scope of our analysis to publicly traded firms. We choose Compustat both because, to our knowledge, there is no comprehensive sectoral data on operating surplus Π_t and enterprise value V_t spanning a sufficiently long time period, and because it allows for measures of intangible capital that can be expanded beyond R&D.

Sector definitions Compustat is a dataset of publicly traded US firms, so that the scope of the analysis is similar to Section 3, but now excludes private corporations.[36] We split the sample into five broad sectors: the Consumer sector (primarily retail and wholesale trade); the High-tech sector (primarily software and IT); the Healthcare sector (producers of medical devices, drug companies, and health care service companies); the Manufacturing sector; and the Service sector (professional and business services, entertainment, and hospitality services). These groups are similar to the Fama-French 5 classification, with the main difference being that we exclude financial companies from our analysis.[37]

Data moments In order to construct the key moments needed for our analysis, we proceed similarly to Section 3; Appendix 3.5 reports the details. The two main differences are as follows. First, we consider two types of intangibles: R&D, similar to the analysis of Section 3; and organization capital, which we did not observe in the aggregate data in Section 3. R&D investment is measured using reported R&D expenditures. For investment in organization capital, we follow Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2017) and impute investment as 30% of SG&A expenditures net of R&D investment.[38] Second, for operating surplus, Π_t, we use operating income before depreciation, but we adjust for expensing of intangible investment in accounting data, consistent with our model.

[36] Details on data construction are reported in Appendix 3.5. Appendix 3.7 contains a discussion of the differences between Compustat and the National Accounts data.

[37] Appendix Tables 1 and 2 report the NAICS sectors that make up our classification. Using KLEMS data from the BLS, described in Appendix 3.6, we estimate that the sectors we study accounted for 86.0% of total value added by private, non-financial businesses in 2001. The remaining 14.0% are accounted for by Transportation, Warehousing, and Construction, which we also exclude from our analysis because they are not well represented in Compustat; there are fewer than 10 firm observations per year for a majority of their constituent NAICS subsectors.

[38] The primary source for the 30% imputation rate is the work of Hulten and Hao (2008). Appendix 3.5 discusses other existing estimates in the literature, which are generally close to this value.

4.2 The aggregate investment gap in Compustat

We start by applying our baseline analysis to pooled data from all Compustat sectors, as an initial comparison to the aggregate results. The results are summarized in Table 1.[39] Here, we highlight the two main findings of this exercise.

First, when using only R&D capital, the same trends highlighted in the introduction are apparent in both the Compustat and the NFCB data: rising returns to physical capital, rising Q_1, and declining physical investment rates. Compustat moments are very close to those of the NFCB, consistent with the fact that the fixed asset tables primarily measure intangibles as capitalized R&D. The exception are returns to physical capital, which are higher among publicly traded firms. As a result, total rents as a fraction of value added are higher among Compustat firms than in the NFCB sector as a whole. The rent share of value added is about 2 percentage points higher in the post-2001 period in the Compustat sample, as indicated in Table 1. Other than this difference, when using only R&D, the implications of our analysis look similar for Compustat and the NFCB as a whole.

Second, once organization capital is included, intangibles are the dominant force behind the investment gap. With organization capital, the ratio of intangible to physical capital more than doubles. Returns to physical capital also further increase, since operating surplus rises after adjusting for the expensing of intangible investment in organization capital. However, the effect of the higher stock of intangibles dominates. After 2001, for instance, the two intangible-related terms account for 69% of the total investment gap, on average, as opposed to 39% when only including R&D. The intangible share in production approximately doubles compared to when only R&D capital is included, reaching $\eta = 0.48$ on average after 2001. Additionally, the importance of rents overall declines. The share of rents in value added falls to 4.9% of value added after 2001, compared to 8.7% when only R&D capital is included. Thus, intangible capital of an empirically plausible magnitude can account for the majority of the investment gap and reduce the role of rents substantially.

Appendix Figure 6 also reports the time series for the components of the gap obtained when explicitly separating R&D from SG&A. This decomposition is quantitatively similar to the one obtained by taking the sum of the two measures of intangibles. As discussed in Appendix 4.4, this alternative decomposition approach indicates that rents generated by R&D capital are rising somewhat faster than those generated by SG&A capital.

[39] Additionally, Appendix Figure 5 reports the raw time series for the moments used in our baseline analysis, Appendix Figure 6 reports the time series for the investment gap and its decomposition, and Appendix Figure 7 reports the time series for the share of intangibles in production, the share of rents in value added, and the user costs of the two types of capital, all based on the aggregated data from the Compustat sample.

4.3 The investment gap at the sectoral level

Trends across sectors Table 2 reports averages of the six data moments used in the construction of the investment gap over two periods, 1985-2000 and 2001-2017.[40] There are notable differences across sectors, even with this relatively coarse sectoral classification. High-tech and Healthcare are characterized by a combination of high asset returns and high valuations, declining physical investment, and a high (and rising) share of intangibles, consistent with the aggregate data for the NFCB sector as a whole. The Consumer and Services sectors also features high returns and low physical investment. In these sectors, when measured as R&D, intangibles appear to be a negligible fraction of total capital. (As we discuss below, they are between one quarter and one half of total capital when organization capital is included.) Finally, Manufacturing is characterized by declining returns, declining valuations, declining physical investment, and a declining intangible share, in contrast to the other sectors.

Results using only R&D capital Figure 4 reports investment gaps and their decomposition for the five sectors of our analysis, when intangibles are measured only with R&D capital. The model used to construct this decomposition has positive adjustment costs of $\gamma_1 = 3$ and $\gamma_2 = 12$, as in the previous section. This figure shows that the level and the composition of the investment gap differs substantially across sectors.

One extreme is the Manufacturing sector. In that sector, the investment gap is small. Moreover, little of it is explained by intangibles. This is consistent with the fact that the stock of R&D capital (relative to the stock of physical capital) has been declining in manufacturing since the early 2000's. Accordingly, the bottom panel of Table 2 indicates that intangibles' share in the production function has decreased. Though rents have been rising in that sector — they increased by 3.8 percentage points of value added from before to after 2000, as indicated by Table 2 —, they remain small.

The other extreme is the Consumer and the Services sectors. There, the investment gap is large, in particular after 1990. However, it is almost entirely explained by rents to physical capital when using R&D capital alone — our measure of intangibles for this exercise — since measured R&D is very small.[41] The combination of high returns, high valuations, and low intangibles lead to a high (and rising) share of rents in value added, reaching 12.4% in the Consumer sector and 13.0% in the Services sector after 2000, as reported in Table 2.

The Healthcare and High-tech sectors are intermediate cases. Both experienced a large increase in the physical investment gap starting in the mid-1980's. In both cases, rents at-

[40] Appendix Figures 25 to 29 report the full time series for these moments for each sector.

[41] In the Consumer sector, intangibles rise slightly after the mid-2000's, driven primarily by Amazon's reported R&D expenditures, but remains too low to account for the physical investment gap.

tributable to physical capital have also increased. However, they only account for about one-half — in the High-tech sector — and one-third — in the Healthcare sector — of the investment gap overall. In both sectors, the key change in the composition of the investment gap after 2000 is a substantial increase in the level and rents to intangible capital. For the Healthcare sector, for instance, they account, alone, for 41% half of the total investment gap. Table 2 indicates that this is the effect of two changes: a rising intangible share; and a rise in overall rents. Rents as a fraction of value added rise by 6.6 percentage points in the High-tech sector, and 4.3 percentage points in the Healthcare sector, between the pre- and post-2000 periods. The intangible share in production also increased, particularly in the Healthcare sector, where it roughly doubles.

Results including organization capital The previous sectoral results were constructed using only R&D as a measure of the intangible capital stock. Expanding the definition of intangibles to include organization capital has two main effects, both of which are most clearly apparent in the Consumer sector. (Summary results are in Appendix Table 3.)

First, unsurprisingly, the implied share of intangibles in the production function increases substantially. The increase is particularly striking in the Consumer sector, where the stock of organization capital becomes comparable in magnitude to the stock of physical capital. (The increase in intangible intensity η is smaller, though still visible, in the Services sector.) Second, the *level* of implied rents declines substantially. In the Consumer sector, rents fall from 12.4% to 2.7% of value added after 2001. (In the Services sector, they fall from 13.0% to 8.2% of value added.) The combined effect of these two changes is to magnify the direct contribution of intangibles to the investment gap. The Consumer and Healthcare sectors are both particularly impacted; in both, intangibles measured in this way account for more than half of the investment gap.

It is worth noting, though, that while including organization capital leads to a substantial decrease in the *level* of rents, it has a more moderate impact on their *trend*. Figure 5 reports the cumulative change in the estimated share of rents in total value added from 1985 onward for each of the four sectors, measuring intangibles using either R&D (blue circled line) or the sum of R&D and organization capital (green crossed line). The Consumer sector is where including organization capital makes the sharpest difference: the cumulative change in rents falls by approximately one-third.[42] In the Services sector, including organization capital also reduces the trend increase in rents, by about a fifth. In other sectors, there is little trend

[42] Prior work (Foster et al., 2006; Crouzet and Eberly, 2018) has indeed argued that the Consumer sector relies extensively on intangible capital, particularly brand capital and, in more recent years, innovations to supply chain and logistics. Investment in these intangibles are not recorded as R&D expenditures, but instead expensed as SG&A, and so they are picked up by our measure of organization capital.

increase in organization capital relative to R&D capital after 1985, and so cumulative *changes* in rents are similar under the two measures.

Counterfactuals Figure 5 also reports a counterfactual that highlights the differential effects of the rise in intangibles across sectors. Similarly to Section 3, we compute the cumulative change in the share of rents that would have had to occur in order to explain the investment gap, had the ratio of intangible to physical capital stayed constant over the sample. In the Manufacturing sector, where intangible intensity is *declining*, the cumulative increase in rents would have been smaller. A similar finding holds for the Services sector, where intangible intensity is also slightly declining when using R&D capital only, as reported in Table 2. In the other sectors, it would have been larger, and in some substantially so. The Healthcare sector is the most striking example; there, the increase in rents needed to account for the investment gap without a rise in intangibles would have been about 50% (or 5 p.p.) larger. In the Consumer sector, the difference is approximately 30%, relative to the case where intangibles are measured including organization capital. Thus, in both of these sectors, a substantial part of the investment gap is due not purely to rising rents, but to the interaction of rising rents with high and growing intangibles.

4.4 The relationship between rents and intangibles

The previous analysis shows that sectors that experienced the sharpest increase in rents over the last three decades (Healthcare, High-Tech) were also those where intangible capital grew most rapidly. In this section, we ask whether the relationship between trends in rents and in intangible intensity is systematic, by exploring these trends at a more disaggregated level.

Results using only R&D capital Figure 6 summarizes the contrasting evolution of the five broad sectors of our analysis more succinctly. The top left panels of the figure reports the distribution of the rents parameter μ and the Cobb-Douglas share η of intangibles in production as of 1980, with μ on the vertical axis and η on the horizontal axis. The top right panel of the figure reports this distribution as of 2015.

As of 1985, rents and intangible intensity were low in all five sectors, and there was little heterogeneity across sectors — the five sectors cluster in the southwest portion of the graph. Thereafter, the five sectors diverge. In the Consumer and Services sectors, rents increased, but intangible intensity remained roughly the same — the sectors move vertically toward the northwest part of the graph. Rents and intangible intensity did not change substantially in the Manufacturing sector, which remains in the southwest corner of the graph. Finally, rents and intangible intensity increased simultaneously in the Healthcare and High-tech sectors, which move out from the origin toward the northeast part of the graph.

Figure 6 also reports the distribution of rents μ and intangible intensity η for the subsectors that make up each of the five sectors in our analysis. The subsectors correspond to the NAICS 2D/3D level and are those described in Appendix Tables 1 and 2. Each subsector is represented by a transparent dot (the shape of the dots match those of their parent sector).[43] Additionally, in order to keep the graph area compact, we have not plotted six subsectors where μ exceeds 2 in 2015.[44]

Figure 6 suggests that the evolution of the five broad sectors generally captures the more granular evolution of their subsectors. With few exceptions, subsectors are initially clustered around the southwest part of the graph, indicating limited rents and intangibles in 1985. The Consumer and Services subsectors then experienced no increase in intangible intensity but a sharp increase in rents, moving up toward the northwest. The Healthcare and High-tech subsectors also generally experienced a simultaneous increase in both rents and intangibles, moving out northeastward between the 1985 and 2015 plot.

However, the evolution of subsectors within Manufacturing seems to have been substantially more heterogeneous than the aggregate sector's evolution would suggest. Certain subsectors experienced a large increase in both intangibles and rents, while other remained physical-capital intensive and rent-free. For instance, subsector 333 (Machinery, in which the two largest companies by book assets in 2015 were John Deere and Caterpillar) experienced both a large increase in intangibles, and a large increase in rents. On the other hand, subsector 212 (Mining excluding Oil and Gas, in which the two largest companies by book assets in 2015 were Newmont Mining and Freepont McMoRan) had stable intangible intensity and no notable increase in rents over the period. The same pattern holds in the Oil and Gas subsector (324), which also had stable intangible intensity and stable rents over the period.

As a result, within Manufacturing (as also within Healthcare and High-tech), sectors which experienced a large increase in intangible intensity also experienced a high increase in rents — as in the broad Healthcare and High-tech sectors. Aggregation however obscures this coherence between the three sectors, as Manufacturing is dominated by subsectors where

[43] In this analysis, we have dropped all the subsectors that did not have at least ten firms in each year from 1985 to 2015 in Compustat; the list of the subsectors dropped for this reason is reported in Appendix Tables 1 and 2.

[44] These sectors are the following (with their NAICS code, the share of operating profits in their sector, and the implied value for μ in 2015): in High-tech, Computer Systems Designs and Related Services (5415; 2.4%; $\mu = 2.04$) in Manufacturing, Food and Beverage and Tobacco Products (311; 20%; $\mu = 2.71$), Apparel and Leather Product (315; 3.3%; $\mu = 2.95$), and Oil and Gas extraction (211; -20.2%; $\mu = 2.30$); in Services, Administrative and Support Services (561; 18.3%; $\mu = 3.25$), and Miscellaneous Professional, Scientific and Technical Services (5412; 16.0%; $\mu = 3.08$).

rents and intensity did not substantially change since 1980, while in Healthcare and High-tech, most subsectors experienced an increase in intangible intensity and rents. This pattern stands in contrast with the Consumer and Services subsectors, where rents rose in spite of little or no change in intangible intensity, at least as measured by R&D, which we generalize below.

Figure 7 expands on the differences between the Manufacturing, Healthcare, and High-tech sectors, on the one hand, and the Consumer and Services sectors, on the other. The top two panels of the figure report a scatterplot of time trends of the rents parameters $\mu_{s,t}$ and the Cobb-Douglas intangible share $\eta_{s,t}$, estimated *within* each of the 55 subsectors separately.[45] These scatterplots help evaluate whether subsectors where the trend increase in intangibles was high, also experienced a high trend increase in rents, and vice-versa.

Consistent with the previous results, the scatterplots indicate that this is the case for the Manufacturing, Healthcare, and High-tech subsectors — where the correlation between the time trends in rents and intangibles is positive —, but not for the Consumer and Services subsectors — where the correlation is negative.[46] Table 3 provides additional evidence consistent with this interpretation of Figure 7, using the simple regression framework:

$$\mu_{s,t} = \alpha_s + \beta\eta_{s,t} + \epsilon_{s,t}. \tag{17}$$

The point estimates of β — which capture the within-subsector covariance between rents and intangibles — is positive in the sample of Manufacturing, Health and High-tech subsectors, but negative in the sample of Consumer and Services subsectors.

This supports the notion that in the Manufacturing, Health and High-tech subsectors, when rents rose, they rose in tandem with intangible (R&D) capital. By contrast, the two trends were not coincident in the Consumer and Services subsectors.

Results including organization capital Figures 6 and 7, and Table 3, also report results for the case where organization capital is also used in addition to R&D capital to measure intangible intensity. The results are qualitatively similar, and quantitatively stronger, in the sample of Manufacturing, Healthcare, and High-Tech subsectors.

For Consumer and Services, the results generally still support the view that the rise in rents

[45] More precisely, Figure 7 reports the coefficients $\{\gamma_\mu, \gamma_\eta\}$ in:

$$\begin{aligned}\mu_{s,t} &= \delta_{\mu,s} + \gamma_{\mu,s}t + \epsilon_{\mu,s,t},\\ \eta_{s,t} &= \delta_{\eta,s} + \gamma_{\eta,s}t + \epsilon_{\eta,s,t}.\end{aligned}$$

[46] The slope of the simple OLS line in the top left panel of Figure 7 is 0.51, with a heteroskedasticity-robust t-statistic of 1.34; on the top right panel, the slope is -5.70, with a robust t-statistic of 2.84.

was not accompanied by a rise in intangible intensity in those sectors. The time trends in the Cobb-Douglas share $\eta_{s,t}$ and the rents parameters $\mu_{s,t}$ appear to be weakly negatively related. The point estimate of the coefficient β in Equation 17 is positive, though it is smaller than for the Manufacturing, Healthcare and High-tech subsectors, and only marginally statistically significant. It should be noted that there are relatively few subsectors belonging to the Services and Consumer sectors in the Compustat Non-Financial sample (only seven in total), making it more difficult to ascertain whether rents and intangibles rose in tandem in those subsectors. Nevertheless, even including organization capital as a measure of intangibles, the evidence of positive correlation between rising rents and rising intangibles appears to be substantially less clear-cut in the Consumer and Services subsectors than in the Manufacturing, Healthcare and High-tech subsectors.

Relation to prior work and interpretation In prior work (Crouzet and Eberly, 2019), we highlighted the fact that measures of intangible intensity and measures of markups appeared to be correlated, both in aggregate or sectoral time series, and within sector. The current analysis differs in two main ways. First, the methodology we use is different: our prior work used reduced-form proxies for intangibles and market power, while this analysis uses estimates of the rents parameter η and the intangible intensity μ derived from our structural model. Second, the results we arrive differ in some important ways from our prior analysis. In particular, the joint *increase* in intangible intensity and in rents only appears to be significant in the High-Tech, Healthcare, and Manufacturing sample. By contrast, in the Consumer and Services sectors, the relationship is either significant and negative (with R&D only), or has weak statistical significance (with R&D and organization capital).

One potential interpretation of the contrasting results between High-tech, Healthcare, and Manufacturing, on the one hand, and Consumer and services, on the other, is that intangible investment has a different economic function in each of these groups of sectors. In the first group (which contains subsectors such as machinery or medical devices), intangible investment may be associated with product differentiation, which in turn might allow firms to charge higher prices and earn higher rents. On the other hand, in Consumer and Services (which contains subsectors such as retail chains), product differentiation may be weaker. There, intangible investment might instead be associated with efficiency gains and reductions in costs (for instance, through process innovation), which could in turn lead to price competition and lower rents.

Summarizing, the three main findings of this section are the following. First, a broader empirical definition of intangibles — one that includes organization capital — reduces the contribution of rents to the investment gap, and substantially so after 2000. Second, even across broadly defined sectors, there are large differences in the composition of the investment gap. As a whole, the Manufacturing sector has a small investment gap, declining intangibles, and moderate rents, at odds with aggregate trends. By contrast, the Healthcare and High-tech sector are characterized by a larger investment gap than in aggregate, and one where intangibles play a bigger role, particularly in the Healthcare sector. Third, the rise in rents and the rise in intangibles are systematically correlated within the subsectors of Healthcare, High-Tech, and Manufacturing, but not within Consumer and Services subsectors. The latter two findings are particularly interesting: they suggest that any aggregate statement about the investment gap may be misguided, as there is substantial heterogeneity in both the aggregate investment gap itself and the underlying forces that explain it.

5 Robustness and additional results

In this section, we discuss robustness checks on our baseline results, as well as results related to the extensions to our main model that were discussed in Section 2.5.

5.1 Enterprise value

We consider an alternative measure of the enterprise value of the NFCB sector, that of Hall (2001). As mentioned above, this measure subtracts all financial assets of the NFCB sector from gross claims, instead of subtracting only liquid financial assets, as we do in our baseline. The top panel of Appendix Figure 8 reports the time series for Q_1 obtained this way (details on data construction are reported in Appendix 3.2). It is lower than in our baseline, though it displays approximately the same medium and long-run trends. The bottom panel of Appendix Figure 8 then reports the investment gap obtained using this measure of Q_1.

The main difference with our baseline is in the overall level of the gap; it is about half as large. As a result, implied rents are lower than in our baseline. For instance, without adjustment costs, rents are 4.2% of value added when using this measure of Q_1, as opposed to 7.7% in our baseline measurement exercise, and their cumulative increase from 1985 to 2015 is 5 p.p., as opposed to 6.2 p.p. in our baseline measurement exercise.[47] Moreover, the direct effect of intangibles becomes larger; and overall, intangibles account for *more* of the gap with this measure of Q_1 than in our baseline. Overall, results using this alternative measure of the enterprise value of the NFCB sector suggest that intangibles play a larger role in the investment gap.

[47] With adjustment costs, the share of rents is 3.4% in 2015, and the 1985-2015 increase is 5.1 p.p.

5.2 Adjustment costs

The value of $\gamma_1 = 3$, which we draw from Belo et al. (2019), is in the range of typical estimates of values of the convexity parameter in quadratic adjustment investment cost functions. Given the annual calibration, the value $\gamma_1 = 3$ corresponds to a doubling time of three years, between the cases of fast adjustment (2 years) and slow adjustment (8 years) considered in Hall (2001). The value $\gamma_2 = 12$ is close to the baseline estimate for knowledge capital adjustment costs in Belo et al. (2019).

Appendix Figure 9 reports four implied moments under alternative combinations of adjustment costs for physical and intangible assets. The values considered are $\gamma_2 \in [0, 20]$ and $\gamma_1 \in [0, 10]$. The four moments are the change in the overall investment gap, $Q_1 - q_1$; the contribution of intangibles to the investment gap in 2015; the implied intangible share in 2015; and the implied share of rents in total value added in 2015. Of these moments, none display significant sensitivity to changes in user costs except the share of rents in value added. That share is *highest* when adjustment costs are *lowest*. This is because taking into account adjustment costs tends to raise user costs of capital and lower implied rents.

5.3 GMM estimation on split samples

In our baseline approach, we apply moments conditions implied by the model to seven-year averages of the underlying data in order to construct our decomposition of the investment gap. Appendix 4.5 reports results from a different methodology, which consists of estimating a version of the model with *i.i.d.* shocks to fundamentals and no adjustment costs using GMM on split samples (the 1985-2000 and 2001-2017 samples, respectively).[48]

The moment conditions used in this estimation are similar to those described in Section 3.1, so the results of the estimation, which are reported in Appendix Table 4 are qualitatively in line with our baseline analysis, confirming that, in the NFCB sector, both rents and the Cobb-Douglas intangible share increased.[49]

The value added by this estimation approach over our baseline approach is that it allows to test formally whether point estimates of the structural parameters of interest (intangible intensity, the size of rents relative to value added, and user costs) changed significantly across subsamples. Appendix Table 4 shows that for the NFCB data, across the two subsamples, the increase in rents and in intangible intensity, and the decline in user cots, are all statistically significant. However, this is not the case in the Compustat sample, where changes in rents,

[48] We focus on the post-1985 period because over this period, results from aggregate data and results from the Compustat sample can be compared.

[49] Quantitatively, the results are somewhat different because the underlying data used to estimate the moments conditions is not averaged over seven-year windows.

in particular, are not significant in the specifications where intangibles are measured using organization capital in addition to $R\&D$ capital. These results thus further support the notion that including organization capital in our analysis substantially reduces the estimated increase in rents.

5.4 Alternative identification strategies

We next discuss alternative identification strategies for constructing the various elements of the investment gap decomposition (13). These alternative approaches are described in greater detail in Appendix 4.6. They use measures of the average cost of capital in order to construct r and the Gordon growth term $r - g$, and but do not necessarily match observed values of Q_1. By contrast, our approach measures infer the Gordon Growth term $r - g$ from Q_1, so that it matches Q_1 by construction.

Average cost of capital approach Appendix 4.6.1 reports the investment gap decomposition obtained using a first alternative identification strategy, which we call the "average cost of capital approach". This approach is closer to that of Barkai (2020) and Karabarbounis and Neiman (2019) (case II). We measure the average cost of capital as the leverage-weighted average of the cost of debt (obtained from average interest rates on the market value of debt of the NFCB sector), and the cost of equity (obtained from the PD ratio of public firms). We then construct the different terms on the right-hand side of Equation (13) using the same moments as in our baseline, except that we do not match the observed value of Q_1.

In this approach, the (implied) value of the investment gap (that is, the left-hand side of Equation 13) is growing faster after 1985 than the investment gap we measured in our baseline approach (that is, the left-hand side of Equation 13). By 2015, the implied investment gap is about twice as large as the measured one. This is because the discount rate r obtained using an average cost of capital approach is lower, and declining faster, than the discount rate implicit in our baseline decomposition. Consistent with Barkai (2020) and Karabarbounis and Neiman (2019) (case II), lower discount rates also lead to a higher, and more rapidly increasing profit share (approximately 9.0 p.p. over the 1985-2015 period, instead of 6.2 p.p. in our baseline approach).

However, the composition of the implied investment gap remains similar to our baseline findings. Appendix 4.6.1 reports more detailed results, compares the discount rates implied by both approaches, and expands on the interpretation of the results in terms of implicit equity risk premia, following the discussion of Section 3.3.

Inferring intangibles from the investment gap Appendix 4.6.2 describes a second alternative approach, which builds on the average cost of capital approach. In this approach, we use the values of Q_1 in order to infer the size of the intangible capital stock, instead of matching the times series for S from the BEA data. This approach is similar to the "no rents" case discussed above in that movements in the implied intangible capital stock mirror movements in Q_1. It can be thought of as an extension of the analysis of Hall (2001) that allows for rents.

This approach suggests that about two-thirds of the investment gap is due to intangibles, as opposed to one-third in our baseline analysis. However, similar to the "no rents" case, this approach leads to inferring an intangible capital stock that is sharply declining in the late 1970s and early 1980s, by contrast with most empirical measures of intangibles, including the one which our baseline approach uses.

Inferring rents from the investment gap Finally, Appendix 4.6.2 describes a third alternative approach, which consists of inferring the rents parameter μ from the value of Q_1, while again using the average cost of capital approach to measure $r - g$. This approach matches all the same moments as our baseline analysis, except average returns to capital ROA_1. Overall, this approach leads to an investment gap decomposition that is quantitatively and qualitatively similar to our baseline analysis. The main difference is that the increase in rents is somewhat smaller than in our baseline approach. Relatedly, the implied returns to capital in this approach, are on average approximately 4 p.p. lower than in the data, and increases somewhat less after 1980.

Summary While these alternative approaches lead to somewhat different values of rents and of the investment gap than in our baseline analysis, the relative contribution of intangibles to the investment gap is similar (or larger) in these alternative approaches compared to our baseline approach.

We view our Q approach as having two main advantages over these alternatives. First, it allows us to match simultaneously the two most natural metrics of the returns to investment, the average return to capital, ROA_1, and Tobin's Q, Q_1, whereas the alternative approaches generally do not match these moments or, when they do, require large and sometimes negative changes in intangibles. Second, our baseline approach does not require information on the capital structure of the firm, other than that contained in the measurement of enterprise value. This allows us to sidestep issues related to direct measurement of the cost of equity and debt capital.

5.5 Markups and returns to scale

Appendix 4.7 discusses what our estimates of total rents imply for "pure rents" (rents attributable to pricing power or markups) and for "quasi-rents" (rents attributable to decreasing returns). Specifically, we estimate the markup μ_S over the marginal cost of sales implied by different degrees of returns to scale ζ.[50]

The broad conclusion from the empirical analysis is that, at the aggregate level, a relatively modest degree of decreasing returns to scale ($\zeta = 0.95$) is sufficient to account for most of revenue in excess of capital costs and variable input costs without having to resort to markups.[51] Additionally, even under increasing returns to scale ($\zeta = 1.05$), the implied sales markup remains substantially below existing estimates. In particular, in our baseline specification, the markup over sales is 1.099 on average after 2000, increasing from 1.072 on average in the pre-2000 period. By comparison, De Loecker et al. (2020) report a revenue-weighted average markup of price over the cost of sales of approximately 1.5 after 2010.[52]

Related work has taken different approaches to estimating returns to scale, depending on data availability. Where detailed cost data are available, for example from the Census of Manufacturing, returns to scale can be estimated using data on cost shares and output. Syverson (2004) develops this methodology and estimates that a benchmark of constant returns to scale is justified in his detailed industry analysis. More recently De Loecker et al. (2020) use two approaches. First, using Compustat and hence lacking detailed cost shares, they use a demand approach and estimate slightly increasing returns to scale in their specifications. In a standard specification, similar to ours, they estimate nearly constant returns of 1.02 in 1980, rising to 1.08 by 2016. When they specify overhead in the production function, which in Compustat includes some intangibles, they have higher returns to scale of 1.07 initially rising to 1.13 at the end of the sample. When instead they approximate the Syverson (2004) cost share methodology, they obtain lower estimates of nearly constant returns, of 0.98 pre-1980 and 1.03 by 2010, using industry averages. In firm-level data, which have more heterogeneity, they find initially slightly more decreasing returns and a larger increase.

[50] In order to estimate μ_S, we additionally require data on total revenue, as opposed to operating surplus. As discussed in Appendix 4.7, this data does not appear to be available for the NFCB sector in either NIPA tables or in Flow of Funds data, so we use the Compustat sample of Section 4 instead.

[51] Recent research has argued that returns to $R\&D$ investment may have declined in recent years (Bloom et al., 2020), which would strengthen the idea that "quasi"-rents, not pure rents, may explain the growth in total rents in our estimates, particularly in R&D-intensive sectors.

[52] See their Figure III.

5.6 Heterogeneous rents parameters

In our baseline approach, the operating profit function is homogeneous of degree μ with respect to all capital types. As discussed in Section 2, it is straightforward to relax this assumption and instead construct the investment gap with heterogeneous rents parameters $\{\mu_n\}_{n=1}^N$. However, estimating these rents parameters in the data is more challenging; separate identification of each rents parameter μ_n would essentially require measuring separately revenue generated from each type of capital, which we do not observe in the data.

In Appendix 4.8, we nevertheless make two empirical points about the model with heterogeneous rents parameters $\{\mu_n\}_{n=1}^N$. First, it can be shown that if the true model featured heterogeneous rents, then the μ measured in our baseline methodology would be the user-cost weighted average rents parameter across capital types. Second, two limit cases (all rents generated by intangibles, $\mu_1 = 1$; and all rents generated by physical capital $\mu_2 = 1$) can be identified in the data using the same moments as our baseline approach. The former case requires an intangible intensity η in the order of $\eta = 0.5$, and a rent parameters in the order of $\mu_2 = 2$, to rationalize the data, both of which are substantially above our baseline estimates. The case $\mu_2 = 1$, by contrast, delivers predictions that are closer to our baseline, primarily because of the relative size of physical capital, the average rents parameter μ is close to the rents parameter for physical capital, μ_1.

5.7 Financing frictions

As mentioned in the extensions to the model described in Section 2.5, while introducing simple financing frictions does not change the insight of Result 1, frictions can affect the overall magnitude of the gap, or its composition.

In Appendix 4.9.1, we discuss the quantitative impact of equity financing frictions on the size of the investment gap. On the balanced growth path, these frictions generally imply that the investment gap is *larger* than in our baseline model. With equity financing frictions, the first-order condition for firm investment is $q_{n,t} = \Phi'_{n,t} f'(d_t)$, where $f'(d_t)$ captures the wedge between the marginal value of internally generated cash and external distributions to and from shareholders. Along the balanced growth path, it must be that $d_t < 0$, so $f'(d_t) < 1$, and so $q_{n,t}$ is lower than when there are no equity financing frictions $f'(d_t)$. Intuitively, the marginal returns to increasing capital are lower because of the wedge created the friction, $f'(d_t) < 1$.

Appendix Figure 10 reports the implied size of the investment gap, for different values of $f'(d_t)$, along the balanced growth path, along with the composition of the investment gap between total rents (attributable to intangibles and physical capital) and the omitted intangibles effect.[53] The main message of these figures is that introducing equity financing frictions will in general magnify the total contribution of rents to the gap. The intuition is that *total* rents (those due to either physical capital or intangibles) are the residual after taking into account the value of the intangible capital stock. This latter value is adjusted downward with equity financing frictions, because of the wedge $f'(d_t)$ between inside and outside finance. Thus for a given (empirical) value of Q_1, rents are magnified. Appendix Figure 11 repeats this exercise across the five sectors of the analysis of Section 4; the effects of introducing equity financing frictions are most visible for the most intangible-intensive sectors, where the omitted capital effect is initially largest.[54] However, in general, even introducing relatively large frictions ($f'(d) = 0.80$, implying that one dollar of free cash flow after investment only raises flow payoffs to shareholders by 80 cents) does not alter our qualitative conclusions, and quantitatively, only has modest effects on the overall direct contribution of intangibles to the gap.

Finally, Appendix 4.9.2 studies how frictions to debt issuance affect our estimates. The main result is that, when the collateral constraint applies to the stock of physical capital (only), total rents remain correctly estimated in our approach, but the contribution of physical rents is underestimated, while the contribution of intangibles is overestimated. The intuition for this result is that the collateral constraint reduces the "internal" user cost of capital, because part of the return to holding physical capital is that it relaxes the borrowing constraint, and helps shareholders lever up and take advantage of the wedge between their discount factor and that of debtholders. Additionally, we show that with a collateral constraint, our approach leads to total user costs that are always larger than their true value (driven by the overestimate of the user cost of physical assets). In turn this implies that our baseline approach underestimates the rents parameter μ, relative to its true value. Thus, when borrowing creates excess returns to shareholder, but is subject to a collateral constraint that applies to physical capital only, our baseline results will generally overstate the role of rents to physical capital, and understate the value of μ.

However, Appendix 4.9.2 also shows that these effects are likely to be quantitatively small. For instance, even for a wedge between shareholders' discount rate, and debtholders' discount rate, of 5%, our estimate of total rents as a share of value added is only 2 p.p. higher than its

[53] In Appendix 4.9.1, we show that the complete decomposition cannot be obtained in the model with equity issuance frictions without additional parametric assumptions about $f(.)$. However, we provide conditions on the Cobb-Douglas share of intangibles in production such that our baseline estimate of the relative contribution of intangibles to total rents (which assumes no equity issuance frictions) is an *under*estimate of their true contribution, in the presence of equity issuance frictions.

[54] This result is reminiscent of Bianchi et al. (2019), who find, using a structural approach, that equity financing shocks (as opposed to debt financing shocks) are more likely to affect R&D investment.

true value.[55] Thus, overall, the omission of this form of financing friction from our baseline model is unlikely to substantially alter our quantitative results.

5.8 Rents and productivity

Finally, we briefly discuss the relationship between our measure of rents, and measures of total factor productivity. Our baseline analysis estimates the rents parameter μ using the ratio of aggregate or sectoral returns to physical capital, to estimated user costs. One concern is that, if firms have heterogeneous marginal returns to capital, highly productive firms will produce more, have higher revenue, and push up average returns to physical capital. This could occur even if rents are relatively small.[56] To alleviate this concern, in Appendix 4.10, we use the disaggregated data from Section 4, along with estimates of total factor productivity at the subsector level obtained from the BLS KLEMS tables, to study the correlation between our estimates of rents, and measures of total factor productivity. We show that growth in the two measures are uncorrelated, both within the Healthcare/High-Tech/Manufacturing sectors and within the Consumer/Services sectors. This suggests that our rents measure is not primarily driven by heterogeneity in marginal returns to capital across firms.

6 Conclusion

This research provides a general decomposition of the gap between average Q — which is observable — and marginal q — the shadow value that drives investment. This decomposition captures the effects of unmeasured capital, such as intangibles, and also the effect of rents.

We use measurement of the gap to shed light on the growing divergence between physical investment and valuations, which our approach interprets as being driven by the combined effects of growing rents and growing intangible capital. With a relatively narrow measure of intangibles (R&D capital), one-third of the investment gap reflects a combination of growth in the intangible capital stock and rents generated by intangible capital. Expanding the definition of intangibles beyond R&D increases this contribution to about two thirds. In addition to these aggregate effects, sectoral results show that rents on intangibles are largest in some of the fastest growing sectors in the economy, such as Tech and Health, and that within these sectors, rents are highest in subsectors with rapid growth in intangibles, as well.

[55] We estimate the investment gap and its elements by matching θ to observed values of the ratio of book debt to the replacement cost of physical assets, and by calibrating the wedge in discount rates $r - r_b$.

[56] In the limit where rents are zero, for instance because products within an industry are perfect substitutes, the most productive firms would be the only ones to produce; this is highlighted, for instance, in Autor et al. (2020).

Our analysis opens several questions for future research.

First, though our general decomposition allows for risk premia, we remained deliberately agnostic about their source in our empirical applications. A more thorough treatment of their interaction with the investment gap would be a useful next step. A particularly interesting direction to explore are priced capital quality shocks specific to intangible capital, as the rise in intangibles might then contribute to the growing wedge between the risk-free rate and the implicit firm discount rates discussed in Section 3.

Second, our decomposition holds at the firm level. Exploring the distribution of the investment gap across firms of particular sectors would both help validate our findings on the sources of the investment gap, and shed further light on the reasons for its growth over the last two decades.

Third, our decomposition suggests ways in which standard investment-Q regressions might need to be adjusted in order to take into account the possibility that firms have intangible capital and earn rents. Specifically, building on Peters and Taylor (2017), the decomposition suggests that controlling for intangible intensity may not be sufficient; an additional interaction term with empirical proxies for rents may further help improve the empirical performance of the regressions, particularly in the cross-section, a dimension we have not explored in this paper.[57]

Fourth, we noted in Section 2.5 that there is a close link between the investment gap and the gap between stock returns and returns to investment. The returns gap, in our model, is driven by the same three fundamental forces that explain the investment gap. As we argued, because it captures changes in firm value, the returns gap approach is better suited to studying short-run variation in rents and omitted factors such as intangibles. Using our framework to decompose higher-frequency data on the returns gap would help connect our Q approach to the production-based asset pricing literature, and also hopefully shed light on the importance of rents and omitted factors for that literature.

Finally, and in a different vein, we have maintained a neoclassical approach to the interaction between intangibles and rents. A broader approach, however, could allow for an economic interaction; for example, investment in intangibles such as product innovation or a software platform may generate rents to the firm. These interactions would augment the neoclassical approach we take here, and could generate additional links between intangible capital and the decisions and valuation of the firm. We pursue this in future work.

[57] Relatedly, in previous work (Crouzet and Eberly, 2019), we showed that estimated time effects in a standard investment-Q panel regressions display a smaller downward trend when controlling for intangibles; the current analysis suggests expanding this with an interaction term capturing rents.

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		Non-Financial Corporate Businesses				Compustat non-financials (R&D)		Compustat non-financials (R&D+ org. cap.)	
	Targeted moments	**1947 - 1965**	**1966 - 1984**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
i_1	Physical investment rate	0.089	0.108	0.099	0.087	0.097	0.090	0.097	0.090
i_2	Intangible investment rate	0.252	0.276	0.281	0.261	0.260	0.248	0.251	0.245
S	Intangible/physical capital	0.053	0.078	0.124	0.164	0.102	0.136	0.308	0.377
ROA_1	Return on physical capital	0.208	0.211	0.211	0.221	0.225	0.256	0.276	0.314
Q_1	Av. Q for physical capital	1.184	1.413	2.032	2.479	1.764	2.177	1.764	2.177
g	Growth rate of total capital stock	0.034	0.038	0.029	0.019	0.026	0.024	0.026	0.024
	Implied moments	**1947 - 1965**	**1966 - 1984**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
$Q_1 - q_1$	Investment gap	0.072	0.308	0.908	1.439	0.620	1.173	0.859	1.446
	% rents from physical capital	*69*	*41*	*61*	*71*	*71*	*66*	*36*	*33*
	% intangibles	*25*	*52*	*21*	*14*	*15*	*15*	*43*	*41*
	% rents from intangibles	*7*	*7*	*18*	*24*	*15*	*19*	*21*	*26*
η	Intangible share in production	0.099	0.145	0.227	0.286	0.179	0.222	0.392	0.440
s	Rents as a fraction of value added	-0.008	0.014	0.035	0.067	0.029	0.076	0.005	0.043
R_1	User cost of physical capital	0.193	0.171	0.143	0.128	0.164	0.155	0.163	0.155
R_2	User cost of intangible capital	0.392	0.369	0.341	0.312	0.350	0.326	0.339	0.322
μ	Curvature of operating profit function	0.984	1.051	1.136	1.244	1.117	1.290	1.023	1.145
$\tilde{\mu}$	Markup over value added	0.993	1.014	1.037	1.072	1.030	1.083	1.005	1.045

Table 1: Summary of targeted and implied moments, for the non-financial corporate business sector (columns 3 to 5) and for the Compustat Non-Financial sample. For Compustat non-financials, columns 6 and 7 use R&D as the measure of intangibles, and columns 8 and 9 use the sum of R&D and SG&A as the measure of intangibles. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Rents as a fraction of value added are computed as $s = (1 - s_L)(1 - 1/\mu)$, where s_L is the labor share of value added for the NFCB sector. Markups over value added are computed as $\tilde{\mu} = 1/(1 - s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources are described in Sections 3 and 4.

		Compustat non-financials (Intangibles = R&D)									
		Consumer		Services		High-tech		Healthcare		Manufacturing	
	Targeted moments	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
i_1	Physical investment rate	0.128	0.098	0.142	0.084	0.139	0.101	0.105	0.082	0.094	0.093
i_2	Intangible investment rate	0.245	0.317	0.241	0.224	0.346	0.331	0.225	0.190	0.226	0.226
S	Intangible/physical capital	0.008	0.023	0.028	0.010	0.227	0.238	0.346	0.722	0.113	0.087
ROA_1	Return on physical capital	0.269	0.281	0.261	0.245	0.359	0.397	0.355	0.495	0.226	0.222
Q_1	Av. Q for physical capital	2.672	2.651	2.517	2.587	2.937	3.261	3.064	4.306	1.467	1.743
g	Growth rate of total capital stock	0.054	0.037	0.082	0.016	0.065	0.014	0.046	0.028	0.016	0.028
	Implied moments	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**	**1985 - 2000**	**2001 - 2017**
$Q_1 - q_1$	Investment gap	1.523	1.645	1.380	1.574	1.634	2.424	1.908	3.329	0.367	0.687
	% rents from physical capital	*98*	*93*	*93*	*97*	*46*	*55*	*42*	*32*	*43*	*72*
	% intangibles	*1*	*2*	*3*	*1*	*31*	*13*	*30*	*27*	*47*	*16*
	% rents from intangibles	*1*	*6*	*4*	*2*	*23*	*32*	*28*	*41*	*9*	*12*
η	Intangible share in production	0.013	0.058	0.038	0.020	0.324	0.367	0.387	0.562	0.176	0.143
s	Rents as a fraction of value added	0.087	0.124	0.070	0.130	0.044	0.110	0.060	0.103	0.016	0.052
R_1	User cost of physical capital	0.188	0.169	0.191	0.150	0.198	0.174	0.173	0.151	0.175	0.159
R_2	User cost of intangible capital	0.319	0.418	0.310	0.306	0.429	0.420	0.316	0.271	0.332	0.305
μ	Curvature of operating profit function	1.412	1.575	1.308	1.621	1.184	1.474	1.260	1.433	1.064	1.186
$\tilde{\mu}$	Markup over value added	1.095	1.142	1.075	1.150	1.047	1.124	1.064	1.115	1.017	1.055

Table 2: Summary of targeted and implied moments for the different sectors of the Compustat non-financial sample. All columns measure intangibles as the R&D capital stock. The moments are averages over the sub-period indicated in each column. The intangible share in production is estimated under the assumption that physical and intangible capital are Cobb-Douglas substitutes: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Rents as a fraction of value added are computed as $s = (1 - s_L)(1 - 1/\mu)$, where s_L is the labor share of value added for the NFCB sector. Markups over value added are computed as $\tilde{\mu} = 1/(1 - s)$. The implied moments reported are for the model with adjustment costs; the adjustment cost values are $\gamma_1 = 3$ and $\gamma_2 = 12$. In the decomposition of the investment gap, percentages may not add up due to rounding. Data sources are described in Section 4.

(a) Manufacturing, High-tech, and Healthcare sectors

	Rents ($\mu_{s,t}$)			
	Intangibles=R&D		Intangibles=R&D + org. cap.	
	(1)	(2)	(1)	(2)
Cobb-Douglas intangible share ($\eta_{s,t}$)	1.39***	1.39***	1.13***	1.13***
	(0.10)	(0.44)	(0.09)	(0.39)
Number of observations	1040	1040	1040	1040
Adjusted R-sq.	0.71	0.71	0.53	0.53
Clustering of s.e.	none	subsector	none	subsector

(b) Consumer and services sectors

	Rents ($\mu_{s,t}$)			
	Intangibles=R&D		Intangibles=R&D + org. cap.	
	(1)	(2)	(1)	(2)
Cobb-Douglas intangible share ($\eta_{s,t}$)	−6.04***	−6.04***	0.74***	0.74*
	(0.48)	(1.37)	(0.11)	(0.32)
Number of observations	294	294	294	294
Adjusted R-sq.	0.69	0.69	0.42	0.42
Clustering of s.e.	none	subsector	none	subsector

Table 3: The relationship between intangibles and rents at the sub-sector level. In both panels, the model estimated is $\mu_{s,t} = \alpha_s + \beta\eta_{s,t} + \epsilon_{s,t}$, where s is a sector and t is a year. Specifications marked (1) report heteroskedasticity-consistent standard errors, while the specifications marked (2) report standard errors clustered at the sub sector level. The data is the Compustat non-financial sample, aggregated to the level of the subsectors described in Tables 1 and 2. Both $\mu_{s,t}$ and $\eta_{s,t}$ are winsorized at the bottom and top 1%. The top panel reports results for the subsectors belonging to the Manufacturing, High-tech and Healthcare sectors (pooled together), while the bottom panel reports the results for the subsectors belonging to the Consumer and Services subsectors.

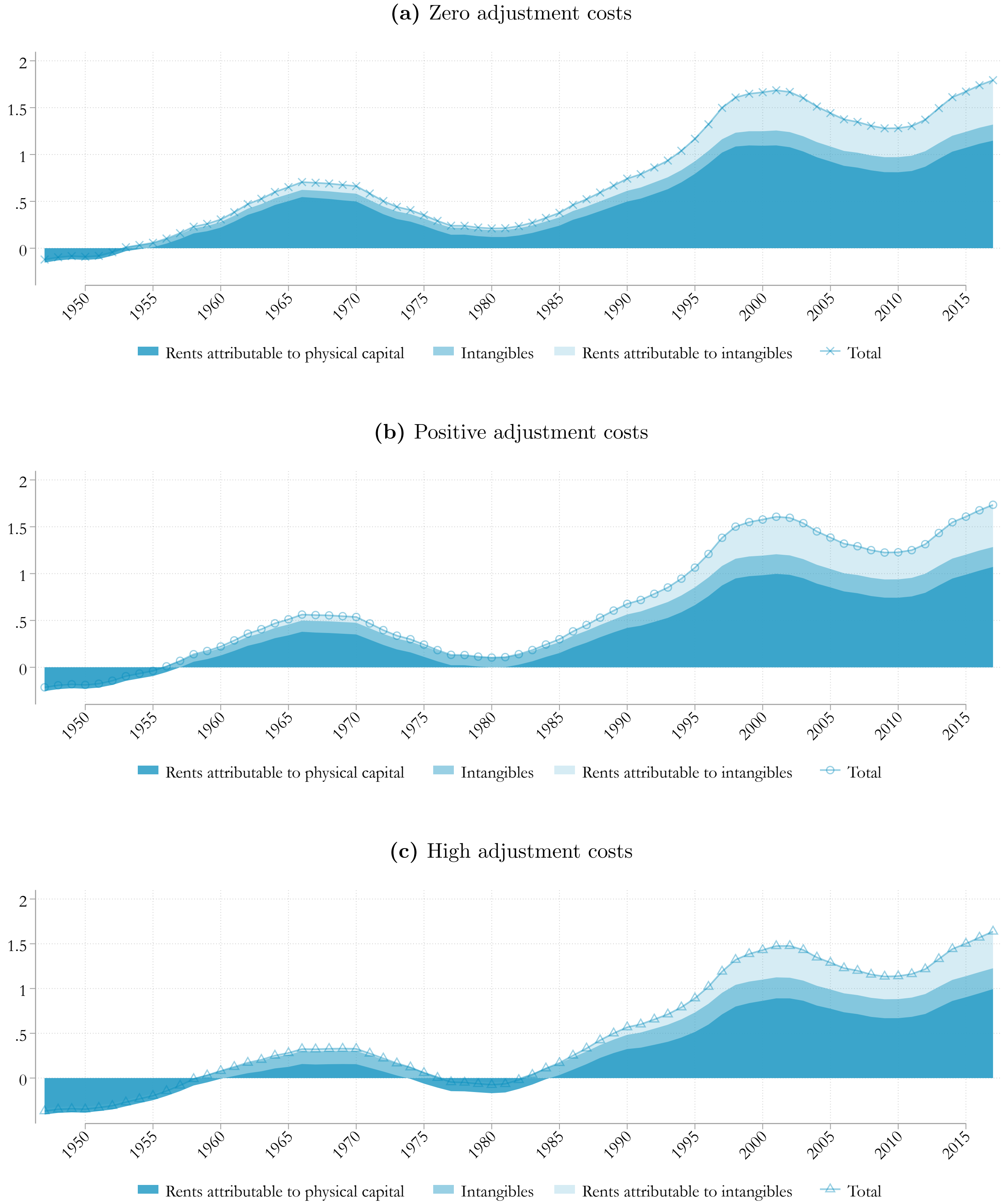


Figure 1: The investment gap $Q_1 - q_1$ for physical capital in the non-financial corporate (NFCB) sector for different values of adjustment costs. In each panel, the crossed blue line is an estimate of $Q_1 - q_1$ constructed using data from the Flow of Funds and from the BEA fixed asset tables. The shaded areas present the decomposition of the physical investment gap into three terms, corresponding to the effects of rents generated by physical capital, the omitted capital effect due to intangibles, and rents generated by intangibles. The decomposition is described in Equation (13). The top panel reports results with zero adjustment costs ($\gamma_1 = \gamma_2 = 0$); the middle panel reports results with positive adjustment costs ($\gamma_1 = 3$, $\gamma_2 = 12$); and the bottom panel reports results with high adjustment costs ($\gamma_1 = 8$, $\gamma_2 = 18$). Methodology and data sources are described in Section 3.

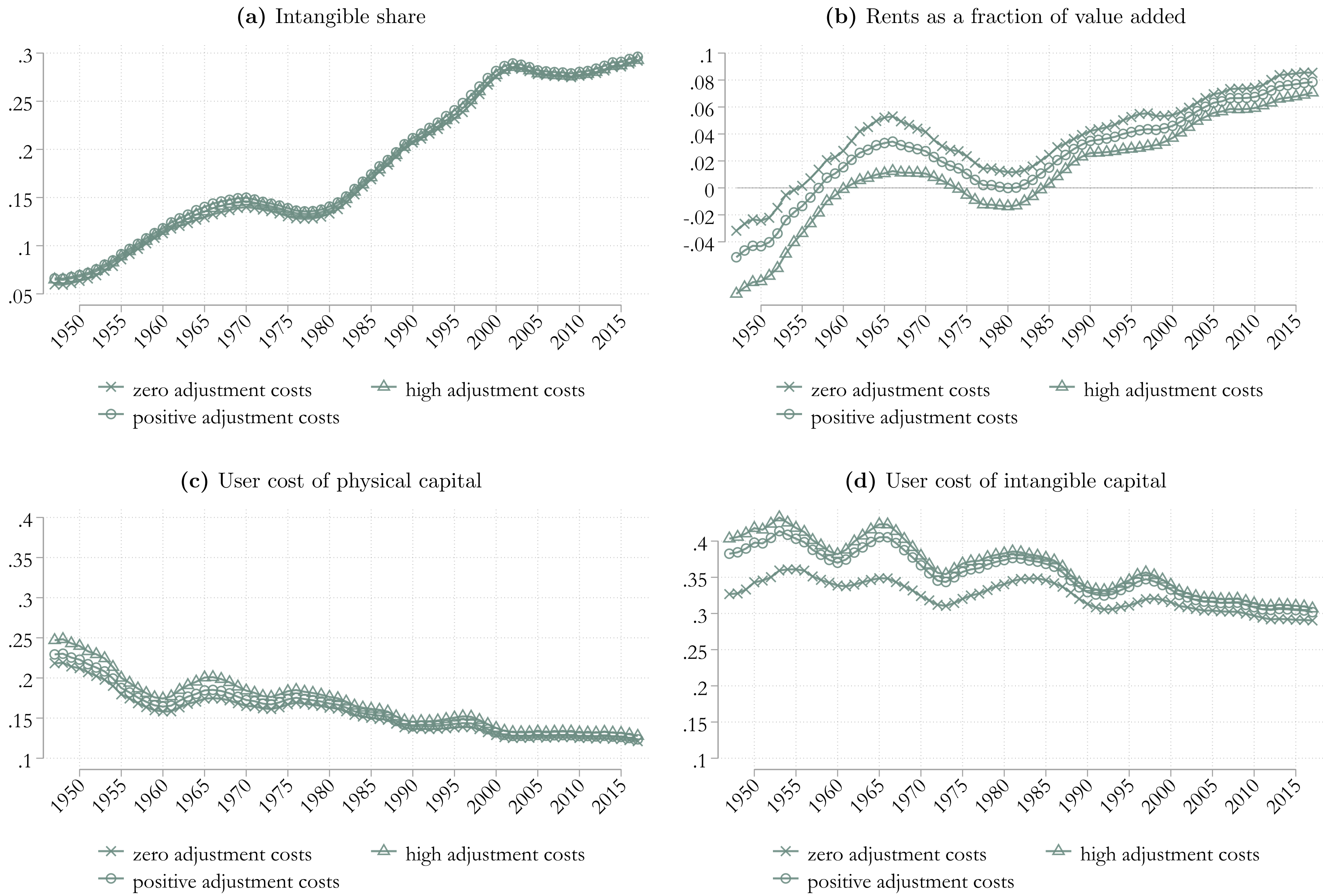
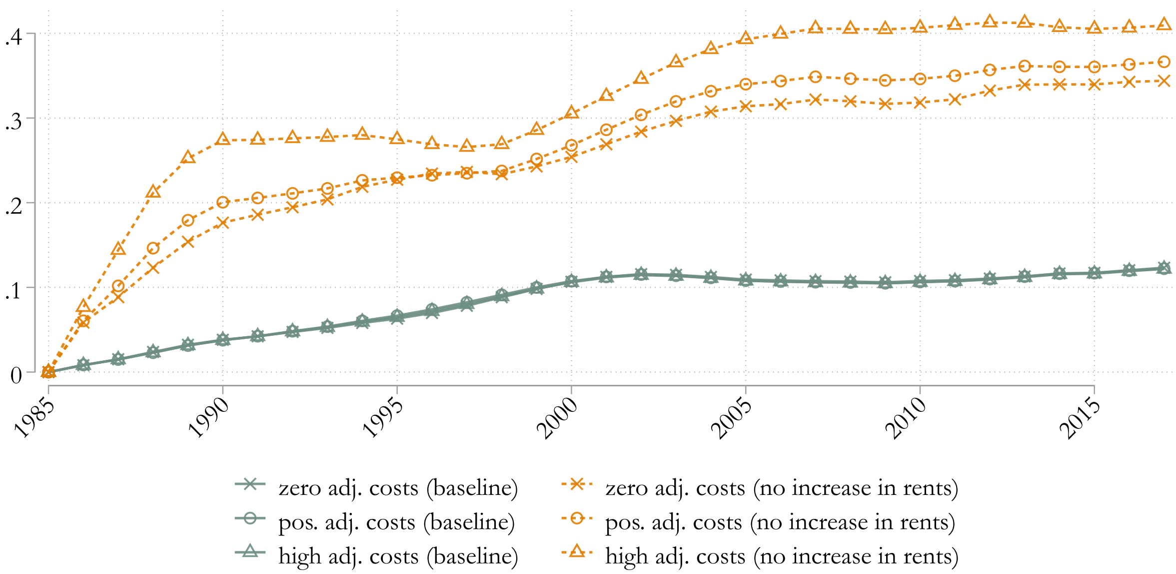


Figure 2: Other model moments for the NFCB sector. Panel (a) reports the share of intangibles in production, η, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. Panel (b) reports rents as a fraction of value added, s_{VA}, which is given by $s_{VA} = (1 - s_L)(1 - 1/\mu)$, where μ is the model parameter governing the size of rents, and s_L is labor's share of value added. Panels (c) and (d) report user costs for each type of capital, R_1 and R_2. The "zero adjustment costs" case corresponds to $\gamma_1 = \gamma_2 = 0$; the "positive adjustment costs case" corresponds to $\gamma_1 = 3$ and $\gamma_2 = 12$; the "high adjustment costs" case corresponds to $\gamma_1 = 8$ and $\gamma_2 = 18$. Methodology and data sources are described in Section 3.

(a) Change in intangible share



(b) Change in rents as a fraction of value added

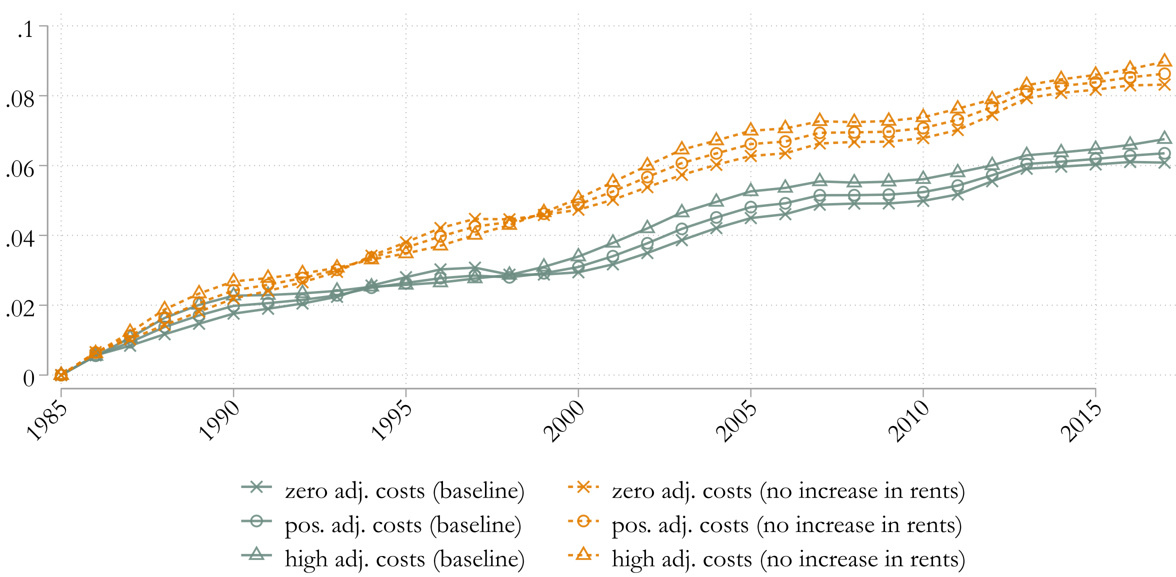


Figure 3: Counterfactual exercises for the NFCB sector. The top panel reports the change in the change in the intangible share in production, η, from 1985 to 2017, when the capital aggregator is assumed to be Cobb-Douglas: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. The blue lines report the change in the baseline decomposition; see panel (a) of Figure 2. The orange lines report the change when the parameter controlling rents, μ, is set to its estimated value in 1985. The bottom panel reports the change in rents as a fraction of value added from 1985 to 2017; rents as a fraction of value added are given by $s = (1 - s_L)(1 - 1/\mu)$, where μ is the model parameter governing the size of rents, and s_L is labor's share of value added. The blue lines report the change in the baseline decomposition; see panel (b) of Figure 2. The orange lines report the change when the ratio of intangible to physical capital, S, and the intangible investment rate, ι_2, are fixed to their 1985 values. Methodology and data sources are described in Section 3.

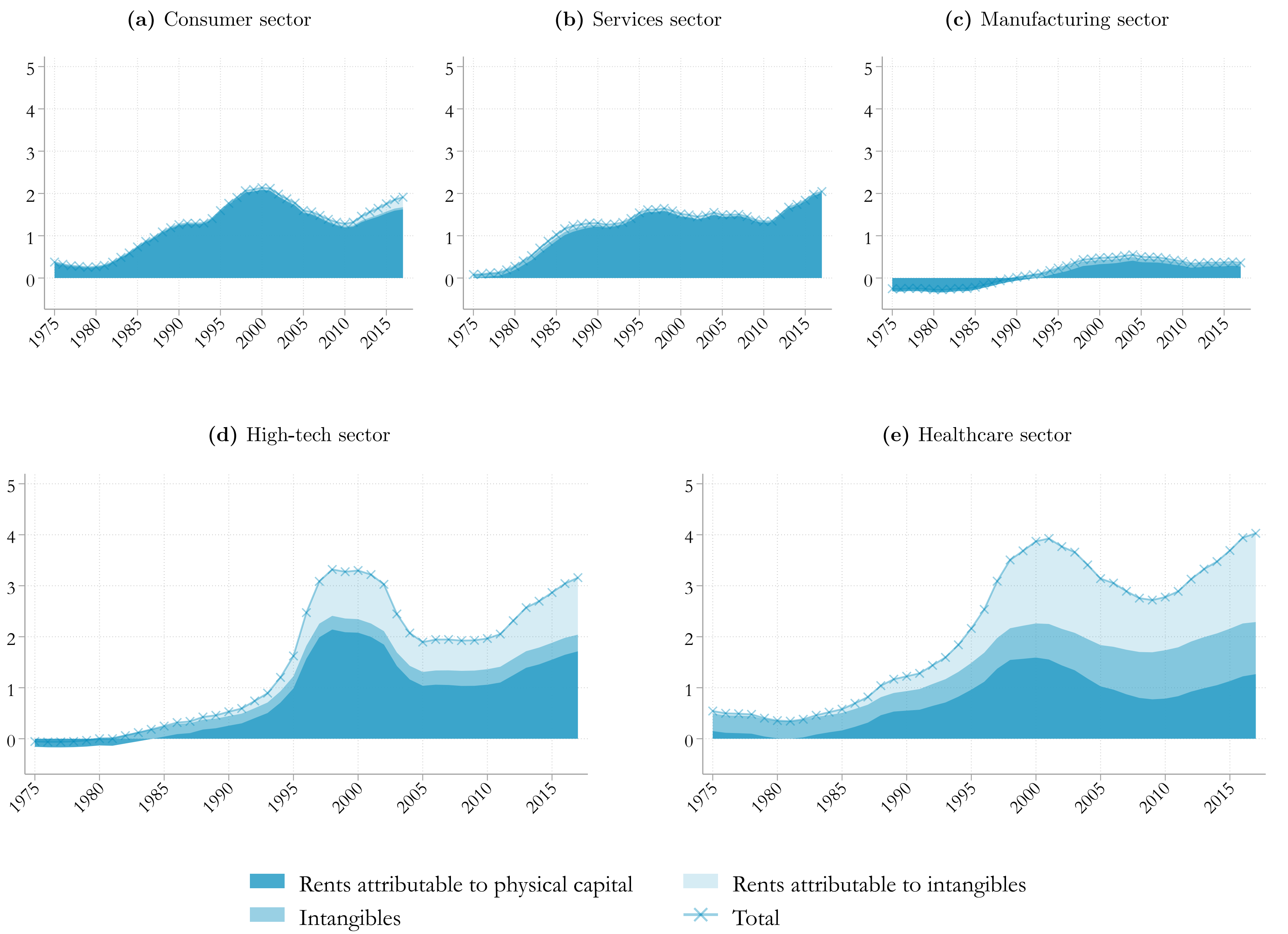


Figure 4: The investment gap $Q_1 - q_1$ for physical capital across sectors, using R&D as a measure of intangibles. Data is from the Compustat Non-Financial (NF) sample. We use the version of model with adjustment costs $\gamma_1 = 3$ and $\gamma_2 = 12$, in order to construct the components of the investment gap. Methodology and data sources are described in Section 4.

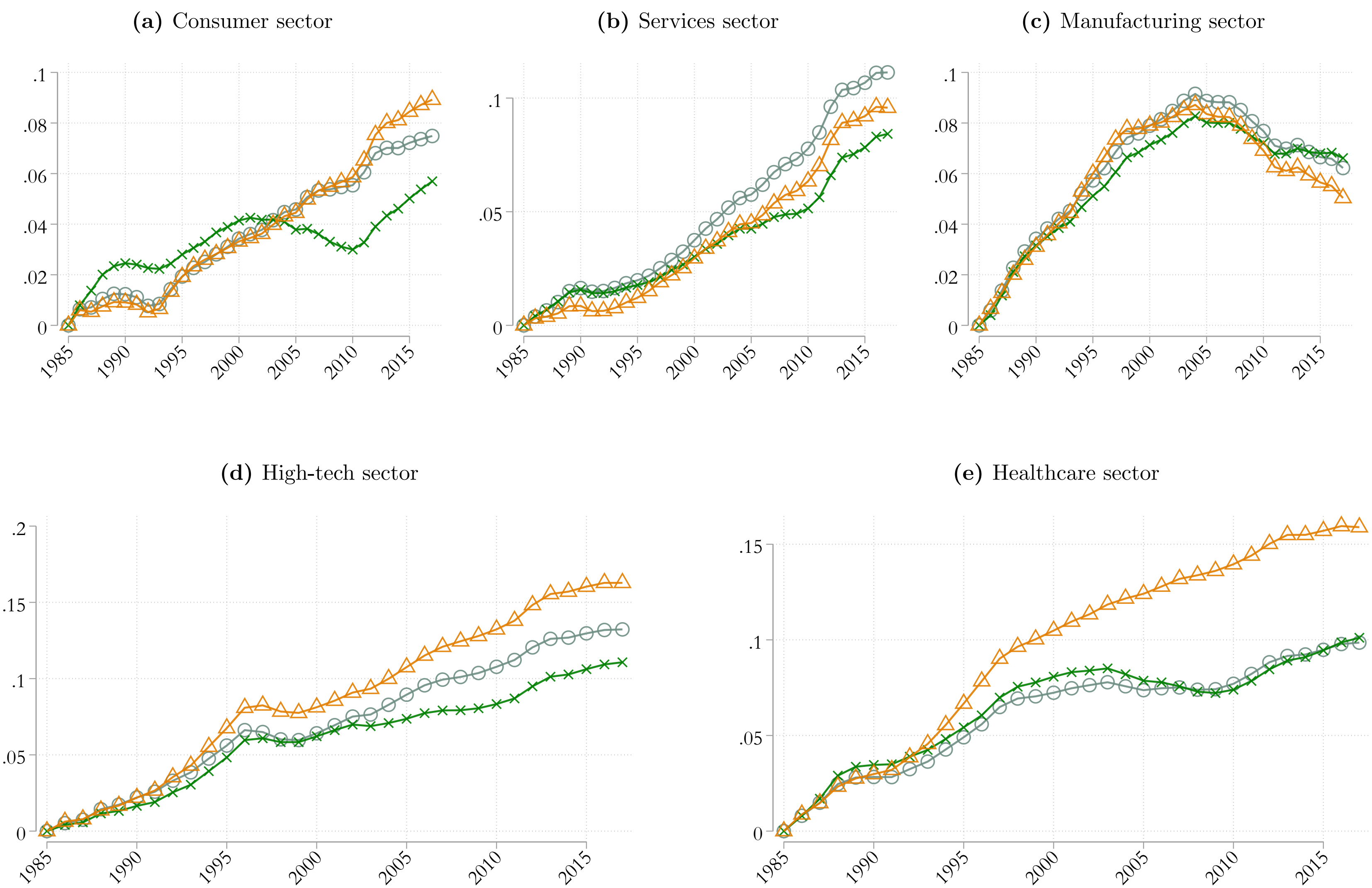


Figure 5: Rents as a fraction value added in the Compustat non-financial sample. Each panel reports the change in the change in the intangible share in production, η, from 1985 to 2017, if the capital aggregator is Cobb-Douglas: $K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta}$. The blue circled lines report the change obtained in the baseline exercise, using R&D as the measure of intangible capital. The green crossed line reports the change obtained when also including organization capital. Finally, the orange line with triangles reports the counterfactual change necessary to match the investment gap when the parameter controlling rents, μ, is kept equal to its estimated value in 1985, in the case where R&D only is used to measure intangible capital. Methodology and data sources are described in Section 4.

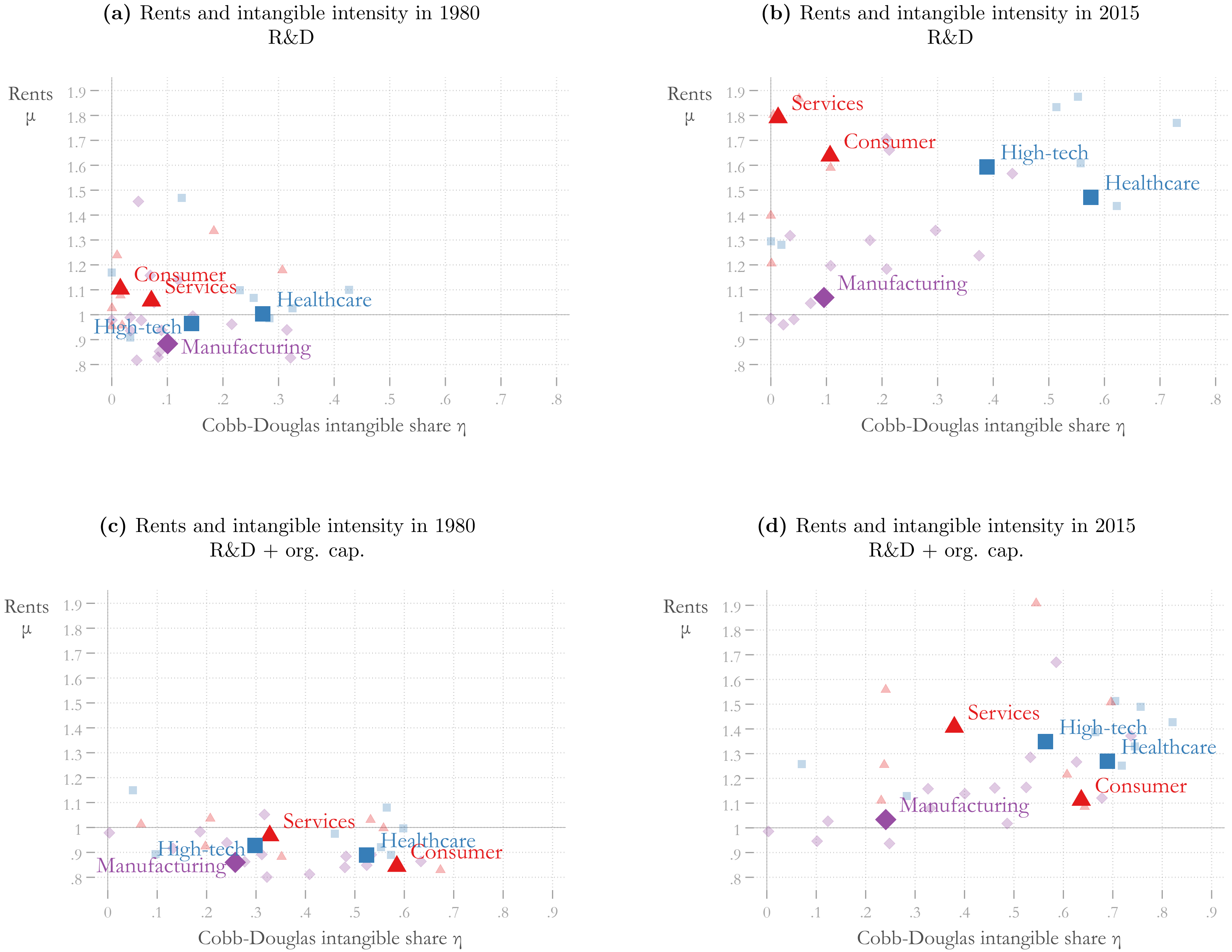


Figure 6: The evolution of rents and intangibles across sectors and underlying subsectors. The top panel reports the values of the rents parameter μ and the Cobb-Douglas intangible share η across the main sectors in our analysis in the year 1980. The bottom panel reports the same structural parameters in the year 2015. Data are from the Compustat Non-Financial (NF) sample, aggregated up to the BEA sectors (circles with size proportional to operating revenue). The graph also reports the value of rents and intangibles for the five broad sectors as in Figure 4. Appendix Tables 1 and 2 report the sectoral classification used to construct the figures. Data sources are described in Section 4.

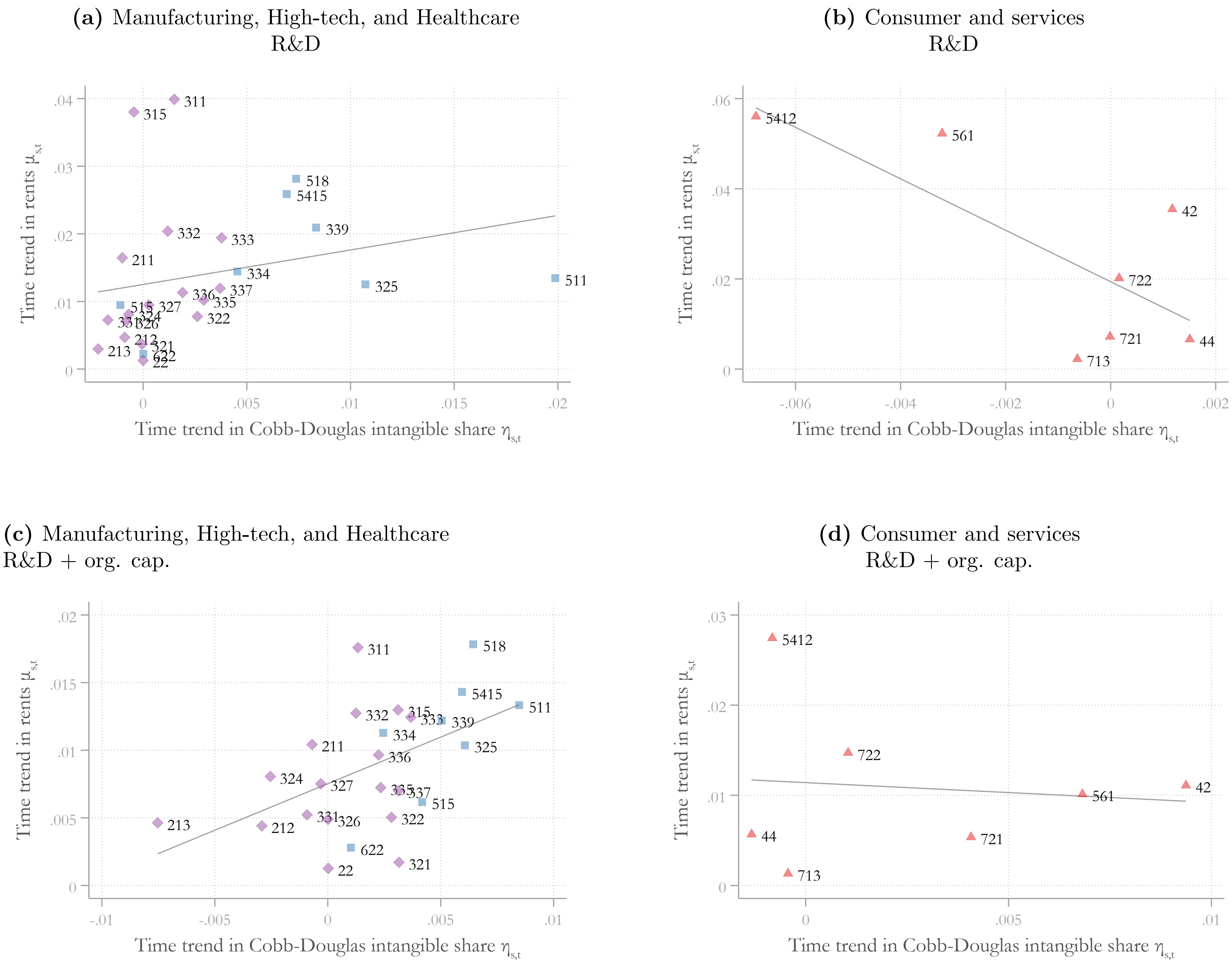


Figure 7: The relationship between rising rents and rising intangible intensity across subsectors. Each panel reports a scatter plot of the coefficients $(\gamma_{\mu,s}, \gamma_{\eta,s})$, where s is a sector, and the coefficients are the estimated time trends of the rents parameters $\mu_{s,t}$ and the Cobb-Douglas intangible intensity $\eta_{s,t}$, i.e. $\mu_{s,t} = \alpha_{\mu,s} + \gamma_{\mu,s}t + \epsilon_{\mu,s,t}$ and $\eta_{s,t} = \alpha_{\eta,s} + \gamma_{\eta,s}t + \epsilon_{\eta,s,t}$. The top left panel reports these coefficients for the Manufacturing, Healthcare, and High-tech sectors when intangibles are measured using R&D capital (the slope of the simple OLS line is 0.51, with a robust t-statistic of 1.34); the bottom left panel reports these when intangibles are measured using R&D capital plus organization capital (the slope of the simple OLS line is 0.68, with a robust t-statistic of 4.11). The top and bottom right panels are similarly constructed, but subsectors belonging to the Consumer and Services subsectors; in the top panel, the slope of the OLS line is -5.70, with a robust t-statistic of 2.84; in the bottom panel, the slope is -0.22, with a robust t-statistic of -0.28.