Market Competition and Political Influence: An Integrated Approach[*]

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Abstract

The operation of markets and of politics are in practice deeply intertwined. Political decisions set the rules of the game for market competition and, conversely, market competitors participate in and influence political decisions. We develop an integrated model to capture the circularity between the two domains. We show that a positive feedback loop emerges such that market power begets political power in a positive feedback loop, but that this feedback loop is bounded. With too much market power, the balance between politics and markets itself becomes lopsided and this drives a wedge between the interests of a policymaker and the dominant firm. Although such a wedge would seem pro-competitive, we show how it can exacerbate the static and dynamic inefficiency of market outcomes. More generally, our model demonstrates that intuitions about market competition can be upended when competition is intermediated by a strategic policymaker.

Keywords: market and political power, political influence, market competition, Arrow effect.

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1 Introduction

The operation of markets and of politics are in practice deeply intertwined. Political decisions set the rules of the game for market competition and, conversely, market competitors participate in and influence political decisions. Since at least the time of Stigler (1971), the connection between the two domains has been formalized in economics, and the flourishing literature that emerged has deepened our understanding of how special interests can distort political outcomes and how political decisions shape market outcomes.

What has been less explored is the circularity of this connection. If political decisions affect market structure, and that market structure, in turn, determines the power of firms to participate in and influence political decisions, a circularity develops in which market and political outcomes are codetermined. The endogeneity of both market and political outcomes leads to sharp questions about the origins, persistence, and welfare effects of market power.

These questions have come to the forefront of debate in recent years in both academic writing and the public forum. Recent evidence establishes that market power has increased in the US in the past few decades (De Loecker, Eeckhout & Unger 2020). An open question is why. Was the increase due to efficiency gains that were rewarded with market leadership—what Covarrubias, Gutierrez & Philippon (2019) refer to as "good concentration"—or is it "bad concentration" derived from anti-competitive practices and, in particular, the wielding of political power to handicap market rivals?

In this paper we develop a model to explore and analyze the circularity between markets and politics. Competition in the market is Cournot between two firms repeated without end. The essential element of the model is that firms can obtain market power from two distinct sources. Market power can come from a competitive advantage that firms invest in, be it through R&D and technological superiority, from higher managerial competence, or some combination thereof. This *capability*-based market power builds a competitive advantage that makes the market as a whole more efficient.

The second source of market power is political protection. We endow a self-interested policymaker with the ability to intervene in the market to advantage one firm over its competitors. For concreteness, we model this power via a minimum standard, a regulatory tool common in practice. The policymaker can impose a standard to separate the firms, choosing a level that only the leader can meet and excluding the trailing firm from the market. The protected firm benefits from the removal of competition and passes along a share of the surplus that is gained as payment to the policymaker.[1] This *political*-based market power enables a competitive advantage by disabling competition, which, in contrast to capability-based market power, comes at the expense of efficiency.

We study this model dynamically. We show that a positive feedback loop emerges between the two sources of market power—that a capability advantage begets a political advantage and so on in a reinforcing cycle. In this way, an initial capability advantage can be parlayed over time into a larger advantage and a dominant market position.

We show, however, that this feedback loop is bounded and conditional on market power itself. We identify a threshold in capability-based advantage beyond which the feedback loop turns negative. Beyond this threshold, therefore, greater capability-based market power leads to the removal of protection and less politically-based market power. This removal restores a degree of competition and bounds the ability of firms to dominate the market through political protection.

The core insight driving this result is that the interests of the market leading firm and the policymaker are aligned but not perfectly aligned. Within each period their interests are aligned on political protection—monopoly power maximizes the surplus available for them to share. Across periods, however, the degree of market power changes, and so too does the balance of power in their relationship. If the market leader gains a large capability advantage over its competitor, the need for, and thus the value of, political protection declines, and as this declines, the ability of the policymaker to extract rents from the market leader declines. Capability-based and politically-based market power are substitutes, in effect, such that the more the market leader has of one, the less it needs of the other.

This generates dynamic incentives for the policymaker that are very different from her static incentives. Dynamically, the policymaker seeks to "manage competition." She wants to protect the leading firm so that she can extract rents, but she doesn't want the leader to get so far ahead technologically that political protection becomes obsolete. It is her desire to remain relevant that causes her to stop protecting the leader and encourage competition, hoping that this allows the trailing firm to catch up and make her protection valuable once more.

At first blush, managed competition appears promising as it bounds political intervention in the market and restores a semblance of competition. We show, however, that this is not the case. In an otherwise standard model of duopolistic competition, we show that

[1] This tool can only separate firms that have a technological difference. The tool(s) available to the policymaker are fundamental to the outcome of market and political interaction. We return to this point later in the paper.

managed competition can lead to the worst of both worlds. We characterize the unique renegotiation-proof subgame perfect equilibrium and show that play eventually stabilizes at a configuration in which technology stagnates and the policymaker protects the leading firm. The steady state is inefficient both because the leading firm is a protected monopolist and because investment stops at a low level. In fact, the capability level at the steady state is never greater than, and typically lower, than if the policymaker always protected the leading firm. Investment with political interventions is lower, therefore, than even if monopoly were guaranteed.

This result shares a deep connection with Arrow's (1962) famous "replacement effect" from markets. Arrow observed that investment in technology will be higher with competition than in monopoly. The reason is that a monopolist obtains only an efficiency gain from investment whereas a duopolist has the additional benefit of capturing greater market share.[2]

The connection of Arrow to politics is that, by intervening in the market, the policymaker affects the degree of competition and, thus, the firms' incentive to invest. Our result shows that political intervention turns Arrow's logic on its head, creating what we refer to as a *reverse Arrow effect*. Precisely because the policymaker wants to manage competition—to remove protection should the leading firm's advantage exceed a threshold—the leading firm is incentivized to stop investing early. At the threshold, investment will not decrease competition, as Arrow suggests, rather it will increase as the policymaker removes protection, allowing the follower firm to enter the market. With Arrow's logic reversed, the leading firm stops investing at the precipice of the threshold, and as the policymaker protects at this point, the market stabilizes at a steady state with no competition and low investment.

A general lesson from this analysis is that the impact of political intervention on markets is a function of the structure of market competition itself. The insight from managed competition is that a self-interested policymaker seeks market competition not for its own sake, but so that the threat of even more competition increases the value of protection to the leading firm. This implies that a standard market intuition—that competitive pressure translates into more efficient markets—need not hold when that competition is intermediated by a strategic policymaker.

To explore this idea, we consider a market in which competitive pressure is reduced. Specifically, we suppose that a firm will give up and leave the market permanently if it

[2]A duopolist "escapes competition" in the terminology of Aghion et al. (2005). We discuss this idea in more detail in Section 2.

has been excluded by political protection for some period of time. This change nominally reduces competitive pressure on the leading firm as exit by the trailing firm removes competition altogether. However, to understand the impact of this change on a market intermediated by a policymaker, we must understand how it changes the incentives of the policymaker.

We show that this reduction in competition pressure weakens the leverage of the policymaker and improves market outcomes. In fact, we show that investment is higher in the steady state than it is in monopoly and even duopoly. The reason for this reversal and efficiency gain again comes back to Arrow. The reverse Arrow effect still emerges in this setting, although now only temporarily, and the problem of underinvestment that it causes is eventually, albeit slowly, overcome. As the policymaker can only extract rents when competitive pressure is there, a weakening of that pressure reduces her influence on the market, enabling investment to reemerge.

On top of this, we show that a separate, distinct variant of the Arrow effect emerges—what we refer to as the *politically enhanced Arrow effect*—in which political protection serves as a reward to investment rather than a punishment. In this way political intervention enhances investment and is able to correct, in part, the standard market failure in which firms underinvest. Ultimately, however, political protection causes the trailing firm to exit the market and monopoly prevails.

These results illuminate a novel economic mechanism when markets and politics intersect. This mechanism goes beyond the truism that politics affect markets. Rather, it lays out a specific channel through which the structure of market competition links to the degree of political influence. We show how the power of this mechanism rests on the substitutability of the two sources of market power, that the value of political power varies inversely with the technological state of the market. Tracing through the logic of this mechanism, we then see how heightened competitive pressure can generate political inefficiency, and to such a degree that it overshadows the standard market-based efficiency of increased competition, leaving society worse off. This result poses a challenge to the standard benchmark of a competitive market. If more competition only provides fertile ground for a self-interested policymaker to extract rents, there is little reason to expect that overall efficiency will increase. As Lerner (1972, p.259) observed, "An economic transaction is a solved political problem." When politics is itself a live variable—a yet unsolved problem—the market transaction must be viewed through a broader lens.[3]

[3]Lerner (1972, p.259) goes further and argues that "Economics has gained the title of queen of the social

A more practical and immediate lesson from our model is for the current debates around market power and politics. One line of argument is that concentration must be “good” if a dominant firm has a capability advantage over competitors. What our results show is that the path to dominance matters. If the leading firm benefited from political protection along the path to dominance, the resulting market concentration need not be “good.” Indeed, in a market with softer competitive pressure, we show that the steady state of the industry involves a monopolist with a high level of capability and no political protection. Yet that position was not earned through market competition. Rather, the outcome was preordained from the moment the firm gained an initial capability advantage. In equilibrium, the firm parlays that initial advantage into lasting dominance through political protection in a reinforcing cycle. Our model informs this debate by providing a structure to understand the empirical and practical connection between market concentration and political power over time.

1.1 Connections to the Literature

Competition within the market and the dynamics of market structure have been extensively analyzed in the economics literature. While government intervention to affect market structure has been a core element of economic models, for instance in analyzing the effects of antitrust policies (e.g., Segal & Whinston 2007, Asker & Bar-Isaac 2020), most of these analyses assume a benevolent social planner or simply exogenous government interventions. Our contribution is to introduce politically motivated strategic market interventions into the standard model of firm competition.[4] Once political economy considerations are incorporated, standard intuitions about the evolution of market structures are altered. Similarly, firms and industries have been at the core of political economy models, as actors who lobby for favored policies. Yet their interests and capabilities have been generally taken as given without accounting for how they coevolve dynamically with policy (e.g., Grossman & Helpman 1994). Our paper is a small step toward bringing these literatures closer together and exploring their interdependence.[5]

Our model is closest in spirit to Coate & Morris (1999). They explicitly connect lob-

sciences by choosing *solved* political problems as its domain.”

[4]Firm exit in our model comes from political intervention, therefore, rather than due to the firm learning about its type, as in the classic model of Hopenhayn (1992).

[5]A more distant connection is to the small literature that combines industrial organization with organizational economics (Barron & Powell (2018) provide an overview). In particular, Powell (2019) focuses on commitment and how the interplay of current and future rents affects market performance.

bying and political influence to private sector investment, showing how political choices influence private sector decisions that, in turn, influence politics. In their model there is a single firm that decides which of two sectors to operate in.[6] We differ in emphasizing competition between firms and the dynamics of competition within a single market, showing the importance to a policymaker of deciding when and not just whether to extract rents.

More broadly, our model relates to the literature on the role of commitment and limited contracting tools in political economy. Our result that investment stops at a level lower than monopoly is, at heart, a commitment problem. A higher capability level for the leader would increase the surplus available, but because the firm and the policymaker cannot commit to a sharing of the surplus going forward and lack the contracting tools to front-load rents, the policymaker attempts to manage competition and the reverse Arrow effect take hold. Acemoglu (2003) views the lack of commitment in politics through the lens of the Coase theorem and argues that a political version of the theorem does not hold. Closer to our work, Acemoglu & Robinson (2000), and more thoroughly, Acemoglu (2006) and Acemoglu, Golosov & Tsyvinski (2008), build models in which economic outcomes impact politics and show how the inability to commit to who holds political power—and, therefore, the inability to commit to the sharing of future surplus—distorts economic outcomes away from efficiency.[7]

Relative to this literature, our contribution is to enrich the private sector and study competition *within* the market. This allows us to see how private sector actors compete for political influence, and how a strategic policymaker can trade their interests off against each other to maximize her own gain. By building a market structure, our model provides a microfoundation for how and why political and market outcomes are codetermined. Indeed, in contrast to the literature, formal political power in our model is not contested—it is always held by the same policymaker. What changes instead is the *value* of policy making power itself. As the conditions of market competition change, as the leading firm gains a capability-based advantage, the value of political power declines, and we show how the stakeholders wage a contest to manage that balance. A benefit to modeling the market microstructure is that we can better identify where inefficiencies come from and how they can be corrected. We pinpoint the mechanism through which higher competitive pressure in the market may augment political influence in that market. By identifying this specific

[6] Besley & Coate (1998) present a related idea in a repeated elections model in which private investment and public tax decisions are interrelated.

[7] Shleifer & Vishny (1994) develop a related model of state owned firms in which the government directly participates in the market.

mechanism of codetermination of market and political outcomes, we can consider policies to address market failures and improve overall efficiency.

The feedback loop between politics and markets has recently come into focus in the empirical literature, as most clearly and forcefully articulated in Zingales (2017) (see Philippon (2019) and Wu (2018) for related book-length treatments). Zingales provides many historical examples of inefficient outcomes caused by market participants' ability to capture political power. He coins the phrase a "Medici vicious cycle" to describe a positive feedback loop and identifies six broad factors that drive this feedback.[8] We develop a formal model of market and political competition that complements Zingales and we identify a novel channel through which the feedback loop operates. By including the policymaker as a strategic self-interested player, we show that the feedback loop is bounded, that it varies in the structure of market competition, and that it can potentially be harnessed to improve rather than harm market efficiency. At a more abstract level, the insights from our model reinforce and put structure to Zingales's (2017) argument that a 'goldilocks' balance is required between the power embedded in politics and in markets for the system to have any hope of efficient and fair progress.

2 The Model

The environment consists of two firms, indexed $j \in \{1, 2\}$, and a policymaker, P. In each period $t = 1, 2, ...$ the firms compete in the market and lobby the policymaker for protection.

The Market: Competition is Cournot in each period. Firm j in period t chooses quantity q_t^j, where $q_t^j = 0$ if the firm does not compete in the market. Total market quantity is then $Q_t = q_t^1 + q_t^2$. Market demand is constant across periods and the inverse demand function is given by:

$$p = a - b \cdot Q.$$

Each firm has a technology level given by the integer τ that allows it to produce at a constant marginal cost that decreases linearly in τ. We denote technology levels by l and f for the leader and follower firms, respectively, where $l \geq f$. The state of the market

[8] Zingales's (2017) six factors are: the main source of political power, the conditions of the media market, the independence of the prosecutorial and judiciary power, the campaign finance laws, and the dominant ideology.

in period t is then $(l_t, f_t) \in \mathbb{Z}_+ \times \mathbb{Z}_+$. Within-period Cournot profits are $\pi^L(l, f)$ and $\pi^F(l, f)$, respectively. A monopolist's profit at technology level l is denoted $\hat{\pi}^M(l)$.

The firms can improve their technology level through investment. Investment incurs a fixed cost $c(\tau) > 0$ that is increasing in τ such that $c(\tau) \to \infty$ as τ gets large. Technological advancement is deterministic and one-step per investment.[9] The step sizes in technology are small in the sense that $\hat{\pi}^M(l) > \pi^L(l+1, f)$ for all f; that is, the leader prefers to have the follower firm excluded from the market than advance a technology level and have to compete.

We generally consider the situation in which both firms begin at technology level 0, although the analysis holds should the market begin at any state of technology. Indeed, one can view a different starting state as resulting from a disruptive innovation, with the model describing incremental competition thereafter.

Political Influence: The policymaker can intervene in the market and impose a minimum technology standard. The standard can be adjusted from period to period. It is outcome relevant only if it separates the firms.[10] When a standard is imposed, the follower firm is excluded from the market, earning zero profit, and the leader obtains monopoly power.[11]

The protected firm pays rents to the policymaker, which we assume to be a fixed share of the value of protection. The value is the difference between monopoly and duopoly profits, which for the policymaker's share $\rho \in [0, 1]$ and technology levels l and f, gives rents of:

$$\pi^P(l, f) = \rho \cdot \left[\hat{\pi}^M(l) - \pi^L(l, f)\right]. \tag{1}$$

The protected firm's profit is then monopoly profit less rents:

$$\pi^M(l, f) = \hat{\pi}^M(l) - \pi^P(l, f), \tag{2}$$

which, by construction, exceeds the duopoly profit. Note that the policymaker and leading firm cannot commit to a rent-sharing agreement beyond the present period. We return to

[9] Step-by-step advancement is standard in the literature; see Aghion et al. (2005). It is straightforward to prove that our results are robust to stochastic advance in capability.

[10] It could, in principle, exclude both firms. As that delivers zero rents to the policymaker and is a dominated strategy, we set this possibility aside.

[11] Formally, investment in our model is a cost reduction and so we model the regulatory intervention as a technology or capability standard. Modeling investment as a quality improvement on the final goods would permit an analogous application to quality floor regulations. As Tirole (1997, p. 389) points out, "a product innovation can generally be regarded as a process innovation—imagine that the new product existed prior to the innovation, and that the innovation simply reduced its production cost."

this important assumption in the discussion.

Timing & Equilibrium: The timing of the play within each period is as follows. For $l_t > f_t$:

1. *Investment*. The leading firm invests ($i_t = 1$) or not ($i_t = 0$) and the state is $(l_t + i_t,\ f_t)$.
2. *Protection*. The policymaker imposes a technology standard ($a_t = 1$) or not ($a_t = 0$).
3. *Market competition*. The firms compete (if $a_t = 0$) or L is a monopolist (if $a_t = 1$).
4. *Transition*. The state in period $t + 1$ will begin at $(l_t + i_t,\ f_t + 1 - a_t)$.

The transition in stage 4 implies that the follower firm moves up one technological step whenever it competes in the market. This captures the idea that catch-up growth is easier than frontier-expanding innovation. The follower firm can more easily imitate the leading firm than the leader can come up with new ideas.[12] We assume this catch-up growth is dependent on market participation reflecting the fact that much innovation, and even imitation, comes from market interactions and experience, as documented in the literature on learning-by-doing. Our results do not require this distinction to be so sharp, nor that catch-up growth be guaranteed. We require only that catch-up growth is more likely when a firm is in the market than outside. This is important as it implies political protection impacts the market in two ways: It removes competition *and* it restrains technological catch-up by the trailing firm. Both aspects will play a role in our analysis.

When the firms are equal technologically and $l_t = f_t$, nature selects in step 1 one of the firms to invest, and play proceeds identically otherwise.[13] This is a simple tie-breaking rule that creates the opportunity for capability gaps to open up between the firms.[14]

Competition and the Incentive to Invest: The incentives of firms to invest depend on market structure and political intervention. In a purely market setting, Arrow (1962) argues that the incentive to invest is lower in monopoly than with competition. This has come to be known as the Arrow replacement effect (Tirole 1997) and led to an enormous amount of research on the impact of competition on investment and innovation. In our model, as in

[12] A version of this assumption appears in many other models of competition and innovation, such as in the influential work by Aghion et al. (2005) and Bessen & Maskin (2009).

[13] With the exception that only the selected firm can push the technological frontier; i.e., if the selected firm does not invest and protection is not offered, the follower firm does not advance technologically in step 4.

[14] Alternatively, if success is stochastic we could allow both firms the opportunity to invest as this would permit a capability gap to open up.

Arrow (1962), only a single firm has the opportunity to invest and, by so doing, it lessens the degree of competition with the follower firm, thereby "escaping competition" (Aghion et al. 2005). Empirical evidence strongly points to competition increasing the incentive to invest and innovate when the comparison is between monopoly and duopoly, as it is here (Shapiro 2012, Holmes & Schmitz 2010).[15]

Arrow's effect is intuitive although it does not follow directly from Cournot competition, and so we impose the following condition on relative profits.

Assumption 1: $\frac{\partial}{\partial l}\hat{\pi}^M(l) \leq \frac{\partial}{\partial l}\pi^L(l, f)$.

Arrow's effect is simply that, for a technology level l, the marginal gain from stepping up a level is higher for the duopolist than the monopolist. The duopolist improves efficiency *and* gains market share from its competitor. This implies that the gap between profits in monopoly and duopoly is narrowing. That as the leader's technology level grows, competition restrains its profits to a lesser degree.[16]

This property is important as it is the gap in profit between monopoly and duopoly that determines the rents paid to the policymaker. The policymaker receives a share of the value of protection, which is exactly this difference in profit. That this gap declines in the leader's technology level implies, therefore, that the policymaker's rents also decline in leading firm's capability level.

Assumption 1 is for a duopolist and a pure monopolist. The case of a protected monopolist—who shares rents with the policymaker—lies between these cases. The fixed proportion rent sharing rule we assume implies, immediately from Assumption 1, that the incentive to invest of a protected monopolist satisfies: $\frac{\partial}{\partial l}\hat{\pi}^M(l) \leq \frac{\partial}{\partial l}\pi^M(l, f) \leq \frac{\partial}{\partial l}\pi^L(l, f) \leq 0$ for each f. The incentive of the protected monopolist equals that of the pure monopolist at $\rho = 0$, that of the duopolist at $\rho = 1$, and is strictly increasing in ρ.

Final Details: The policymaker and the firms discount utility at rates, δ and β, respectively. Throughout our analysis the policymaker is far-sighted with $\delta \in (0, 1)$. For simplicity, we present the model when the firms are short-sighted ($\beta = 0$). In the appendix we

[15] Schmutzler (2009) provides a thorough theoretical treatment of the connection between competition and innovation. Shapiro (2012) synthesizes ideas from various models and identifies the critical property as market *contestability*—that the prospect of gaining or protecting profitable sales spurs innovation. This property holds for investment in our model, consistent with the Arrow effect.

[16] To the extent that competition is relaxed completely for a large enough capability gap (if, for example, the monopoly price for the leading firm is below the follower firm's cost of production), then the Arrow effect *must* hold for at least large parts of the technology range.

establish the robustness of the results for any $\beta \in (0,1)$. Note that the firms receive the benefit of investment within a period, so even when myopic, investment can have a positive return.

We identify a renegotiation proof subgame perfect equilibrium and fully characterize it. We prove in the appendix that the equilibrium is unique within this class and that it has the structure of a Markov Perfect Equilibrium. We refer to it as the equilibrium throughout the paper.

3 Market Incentives

To illuminate the market incentives in the model we begin by shutting down the policymaker as a strategic actor. We consider two benchmarks. One in which the policymaker does not exist or, equivalently, never intervenes in the market, and a second in which the policymaker always intervenes to protect the leading firm.[17]

The Policymaker Never Intervenes: Without political intervention, both firms compete in each period and the market is a duopoly. A firm invests if the improvement in capability increases profit enough to justify the cost. For firms with a single period horizon, investment is profitable if:

$$\pi^L(l+1,\ f) - c(l) \geq \pi^L(l,\ f). \tag{3}$$

The decision to invest depends on the capability level of the leader as well as the follower. This generates a threshold level of capability for the follower, denoted by $\mathrm{IC}_D(l)$, at which equality holds in (3) and the leader is indifferent between investing and not. We then have the following result.

Lemma 1 *The leader invests if and only if $f < IC_D(l)$, where $\frac{d}{dl} IC_D(l) < 0$.*

The leading firm's willingness to invest is decreasing in its own capability level and also the capability of the follower firm. The more capable is the leader itself, the cost of further advancement is higher and the increase in profit it produces is decreasing. The impact of the follower firm's capability level is solely through the market effect. As the trailing firm catches up to the leader, competition is more intense and the leader is able to capture less

[17] We consider additional benchmarks in the discussion section following the presentation of results.

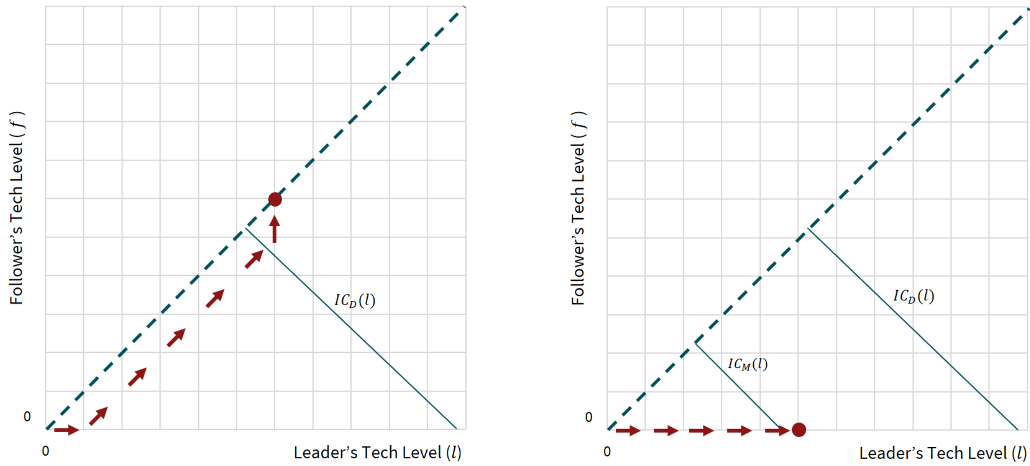

Figure 1: Market Incentives. The red arrows illustrate the equilibtrium transitional path and the red dot the equilibrium steady state. The left panel shows the case when the policymaker never intervenes. The leader invests as long as f is below the threshold IC_D. The right panel shows the case where the policymaker always intervenes. The leader invests as long as f is below the threshold IC_M.

of the value of its investment and, thus, is less willing to invest. The $\text{IC}_D(l)$ threshold is depicted in the left panel of Figure 1, where each point in the positive quadrant corresponds to a state (l, f).

The figure also depicts the dynamic path of the market when starting at the origin. One firm invests in the first period, becoming the market leader. In every subsequent period that firm invests, advancing one level, and the follower firm also advances while remaining one step behind. This continues until the state reaches the $\text{IC}_D(l)$ threshold, at which point the leader no longer finds it worthwhile to invest and stops. The follower catches up one final step and the market stabilizes at equal technology levels, as marked by the dot.

The Policymaker Always Intervenes: In this case the leading firm benefits from political protection in every period and operates as a monopolist. Investing at capability level l is profitable if:

$$\pi^M(l+1, f) - c(l) \geq \pi^M(l, f). \tag{4}$$

Although the leader is a monopolist whether it invests or not, the profitability of investment depends on the follower's capability level. This is because we are considering a protected monopolist. The leader pays rents to the policymaker proportional to the value of protection, and this depends on profitability should the leader have to compete. As in the duopoly case, this leads to a threshold in investment, denoted by $\text{IC}_M(l)$, at which equality holds in

(4) and the leader is indifferent between investing and not.

Lemma 2 *The leader invests if and only if $f < IC_M(l)$, where $\frac{d}{dl} IC_M(l) < 0$ and $IC_M(l) < IC_D(l)$.*

This is depicted in the right-side panel of Figure 1. The threshold is downward sloping as it is for duopoly. The leader is more willing to invest the further behind is the follower firm as it then pays smaller rents to the policymaker and captures more of the efficiency gains of investment. The leader's willingness to invest is lower than in duopoly, as implied by Arrow's effect, and the protected monopolist stops investing earlier than does the duopolist.

The dynamic path of the market moves only horizontally (as the follower is never in the market and never catches up). Starting at the origin, the market moves along the $l-$axis and stabilizes at $\lceil \mathrm{IC}_M^{-1}(0) \rceil$, the first capability level beyond the IC_M threshold, as marked by the dot.[18]

4 Market & Political Equilibrium

To market competition we now add the strategic policymaker. The policymaker will choose to protect only when it is in her interest. Protection delivers rents today, but it also excludes the follower firm from the market and this gives the leading firm an opportunity to advance its capability advantage, which lowers the policymaker's rents in future periods. The policymaker's optimal strategy depends, therefore, on the investment decisions of the firms which, in turn, depend on the policymaker's decision to protect or not. The market and political equilibrium is the balance between these different incentives.

Our main result is that in equilibrium this balance leads to the worst of both worlds. The policymaker's effort to extract rents from the leading firm causes that firm to stop investing when it is at a low capability level, often at a level strictly lower than in duopoly and even monopoly. Moreover, the policymaker protects the leader in every period. The equilibrium path, therefore, is inefficient both within period and across periods. When the policymaker protects and the leader does not invest, the market stabilizes and remains in a steady state thereafter.

The policymaker's incentives enter the equilibrium in the form of a simple indifference condition on the rate at which her rents increase when the follower is allowed to catch up

[18] This steady state is inefficient relative to that for duopoly as the market is uncompetitive. It also involves a lower level of investment than duopoly when ρ, the share of surplus that goes to the policymaker, is sufficiently small. We consider welfare comparisons in detail in Section 4.2.

technologically. Although she is far-sighted, this condition distills down to a one-period trade-off given by:

$$\pi^P (l, f) = \delta \pi^P (l, f + 1). \tag{5}$$

This defines, for each l, the level of f at which the policymaker is indifferent between the rents available today from protection and the higher rents available tomorrow should she not protect and the follower catches up one step. It follows from Cournot competition that $\mathrm{IC}_P(l)$ is strictly increasing in l, and that above the threshold the policymaker prefers to take rents today whereas below the threshold she is willing to be patient and prefers the higher discounted rents tomorrow. Denote the critical value by $\mathrm{IC}_P(l)$, reflecting that this is the policymaker's indifference condition.

The balance of equilibrium depends on the interaction of the policymaker's and the leading firm's incentives. In particular, equilibrium depends on where the $\mathrm{IC}_P(l)$ and $\mathrm{IC}_M(l)$ thresholds intersect and where $\mathrm{IC}_P(l)$ meets the $l-$axis. Let the intersection of the curves be at the point $\left(l^I, f^I\right)$ and the intersection of $\mathrm{IC}_P(l)$ with the $l-$axis be at $\hat{l}$. When $\hat{l}$ is positive we have the following.[19]

Proposition 1 *For $\hat{l} \geq 0$ the steady state beginning from the origin is $(l^*, 0)$, where $l^* \leq \min\left\{\lceil IC_M^{-1}(0)\rceil, \max\left\{l^I - f^I, \hat{l}\right\}\right\}$.*

The equilibrium steady state and dynamic path starting at the origin are depicted in the panels of Figure 2. The proposition establishes that the steady state is at or lower than the investment level under protected monopoly, given by the red dot. As is evident in the figure, when $\hat{l}$ and $l^I - f^I$ are lower than the monopoly level, the steady state is also strictly lower and the under investment caused by political intervention can be severe.

The left panel depicts the case when $\hat{l}$ is the binding constraint in equilibrium, whereas in the right side panel it is $l^I - f^I$ that is binding. The difference between the cases is the slope of the $\mathrm{IC}_P(l)$ threshold near the $l-$axis. If the slope is equal or greater than one then the left panel applies; the right panel applies if the slope is less than one. (Observe that $l^I - f^I$ is where the line of slope one from $\left(l^I, f^I\right)$ meets the $l-$axis). It is only if both of these values are to the right of the protected monopoly level, $\lceil \mathrm{IC}_M^{-1}(0)\rceil$, and the IC_M and IC_P curves do not intersect, that the monopoly level provides the upper bound on equilibrium investment.

[19] The alternative case of $\hat{l} < 0$ implies that the IC_P threshold intersects the 45 degree line. When this holds, there may be periods of competition as the policymaker allows the follower to catch up. Nevertheless, in the steady state, as in Proposition 1, protection is applied and investment is suppressed by the reverse Arrow effect. We provide complete details in the Appendix.

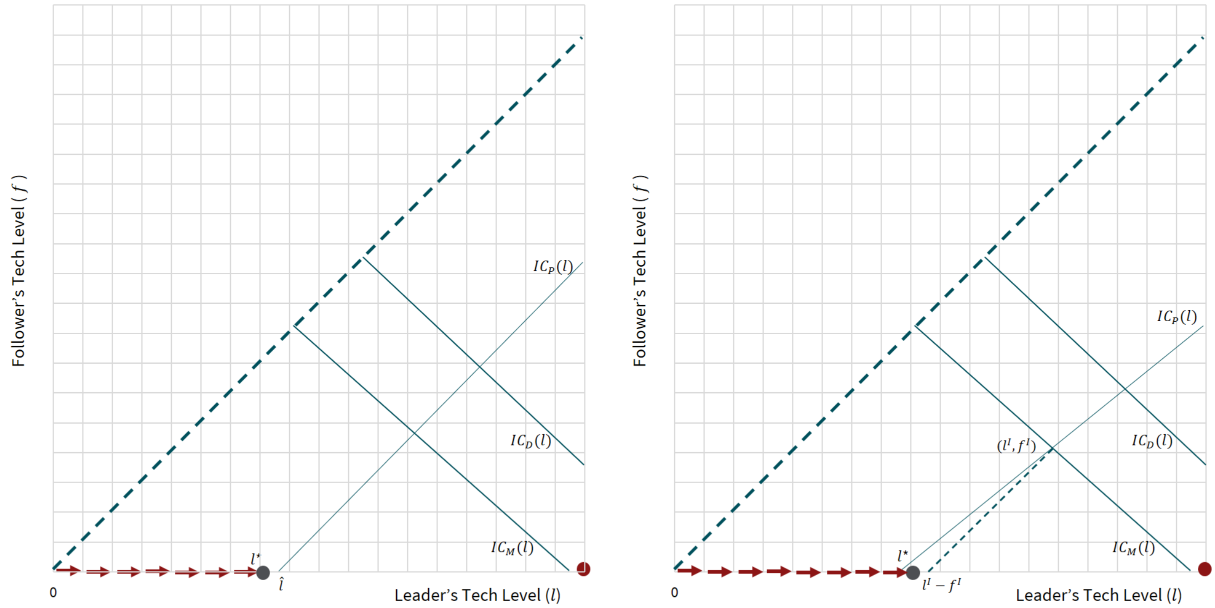

Figure 2: Market and political equilibrium. The red arrows illustrate the transitional path and the gray dot shows the equilibrium steady state. The red dot shows the steady state under the benchmark where the policymaker always intervenes. The left panel shows the case when $\hat{l} \geq l^I - f^I$ and the right panel the reverse.

The equilibrium path represents a positive feedback loop between markets and politics. One firm gains an initial capability advantage and uses that advantage to obtain political protection that it parlays into a larger capability advantage. The entire market outcome, including the steady state, is preordained once the identity of the firm with the initial advantage is realized.[20]

The positive feedback loop does raise the question of why investment stops at such a low capability level. If a monopolist invests at this capability level, why wouldn't a protected monopolist invest?

The reason why investment stops is because the feedback loop turns negative. A crucial feature of the equilibrium is that at state $(l^* + 1, 0)$ the equilibrium calls for the policymaker to not protect. Therefore, if the leader obtains one more step of capability-based market power, the feedback loop will reverse and the firm's political-based market power will be removed.

It is at this state that the policymaker tries to "manage competition." At this state she

[20]The path dependence of the equilibrium is particularly stark here and there is no overtaking in equilibrium. This strict determinancy can be relaxed with the addition of appropriate noise without upsetting the core intuition of the result.

decides that forgoing rents today is worth the benefit of allowing the trailing firm to stay in touch with the leader. By not protecting, the policymaker ensures a higher degree of potential competition tomorrow that will, therefore, allow her to extract higher rents. The policymaker manages competition not for competition's sake but to ensure her own relevance.

Managed competition undermines investment as it generates a reverse Arrow effect. Because the policymaker will remove protection at state $(l^* + 1, 0)$, the leading firm anticipates at state $(l^*, 0)$ that investment will cause it to lose protection and switch from a protected monopolist to a duopolist. This flips Arrow's logic on its head. Contra Arrow, investment does not reduce competition—it does not allow the firm to "escape competition"—as, in fact, it actually increases competition. This pro-competition effect suppresses investment and induces market stagnation at a low level of firm capability.

To this point we have explained why the steady state exists given the equilibrium behavior at higher states, but we haven't yet explained why that equilibrium behavior is what it is. If the leader instead invested at state $(l^*, 0)$, won't the policymaker be tempted to take the rents on offer? Why is her threat to not protect credible? Why does the simple condition in Proposition 1 reflect a one-period trade-off for the policymaker when she is far-sighted?

The answers to these questions depend on the full structure of the equilibrium. The policymaker is credible in her desire to manage competition because of what she anticipates in the future. To see what that is, we need to solve the model via backward induction. This produces the full characterization of equilibrium, allowing for an understanding of dynamic paths starting at any state. The details are provided in the appendix. We focus here on the intuition that generates the path in Proposition 1.

For a high enough capability level, the leading firm will not invest regardless of the degree of competition and whether it has political protection.[21] Denote this state by $l^{\max}$. It follows that state $(l^{\max}, l^{\max})$ is stable. Neither firm invests, the policymaker cannot protect, and there is no catch-up in capabilities. From here, it follows that state $(l^{\max}, l^{\max} - 1)$ is also stable. If the policymaker doesn't protect, the follower catches up, the state transitions to $(l^{\max}, l^{\max})$, and the policymaker never obtains rents again. It is, therefore, optimal to protect today to obtain rents $\pi^P (l^{\max}, l^{\max} - 1)$, and as the leader doesn't invest at capability level $l^{\max}$, state $(l^{\max}, l^{\max} - 1)$ is stable.

The one-period trade-off in the threshold $\mathrm{IC}_P (l)$ matters for how backward induction proceeds from this point. Consider state $(l^{\max}, l^{\max} - 2)$. If the policymaker does not

[21] That such a state exists follows from our assumptions on the cost of investment in capability.

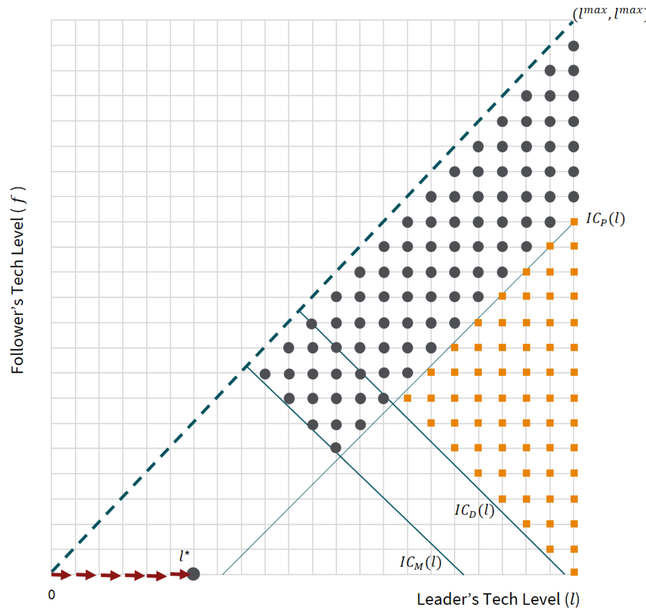

Figure 3: Equilibrium behavior for high capability states. The gray dots are steady states as above both IC_P and IC_M the policymaker protects and the leader does not invest. At the orange squares below IC_P and above IC_D the leader does not invest and policymaker does not protect, and the state progresses upwards.

protect then she receives no rents today, the state transitions to $(l^{\max}, l^{\max} - 1)$, which is stable, and she receives rents $\pi^P (l^{\max}, l^{\max} - 1)$ tomorrow and every period thereafter. If instead she protects, she receives rents $\pi^P (l^{\max}, l^{\max} - 2)$ this period, and as this state is then stable, the same rents thereafter. Her choice reduces down to the simple one-period comparison of $\pi^P (l^{\max}, l^{\max} - 2)$ and $\delta\pi^P (l^{\max}, l^{\max} - 1)$.[22]

This trade-off recurs as we backward induct for lower values of f and the leader at $l^{\max}$. This leads to a unique transition point in equilibrium behavior, as depicted in Figure 3. Above the value of $\text{IC}_P (l^{\max})$, the policymaker protects, takes rents today, and each state is stable. For lower levels of f, the policymaker is more patient, does not protect, forgoes rents, and allows the follower to catch up one step. These states are not stable and the state transitions until it reaches the first state above the $\text{IC}_P (l)$ threshold, at which it stabilizes.

Backward inducting from here is relatively straightforward for the upper regions of the state space. For states above the $\text{IC}_P (l)$ threshold and above the monopoly threshold $\text{IC}_M (l)$, the firm does not invest, the policymaker protects in equilibrium, and the state is

[22]Protection delivers the stream of rents $\sum \delta^t \pi^P (l^{\max}, l^{\max} - 2) = \frac{1}{1-\delta}\pi^P (l^{\max}, l^{\max} - 2)$, whereas not protecting delivers rents $\sum \delta^{t+1}\pi^P (l^{\max}, l^{\max} - 1) = \frac{\delta}{1-\delta}\pi^P (l^{\max}, l^{\max} - 1)$.

stable, as represented by black circles. Below the $\text{IC}_P(l)$ threshold and above the duopoly threshold $\text{IC}_D(l)$, the firm doesn't invest and the policymaker does not protect in equilibrium. These states are not stable, represented by orange squares.

Throughout this region the leader does not invest in steady states because it is protected if it does not and, as the state is above the $\text{IC}_M(l)$ threshold, investment as a monopolist is not profitable.[23] The leader likewise does not invest when it isn't protected because doing so will only transition the state to the right to another unprotected state and, when above the $\text{IC}_D(l)$ threshold, investment as a duopolist is not profitable. This behavior by the firms reinforces the equilibrium behavior of the policymaker. Throughout this region she faces the same trade-off as she faces when the leader's technology level is $l^{\max}$. Either protect and take rents today and thereafter, or forgo rents today to receive a higher stream of rents beginning tomorrow. Consequently, the single-period trade-off represented by $\text{IC}_P(l)$ demarcates behavior throughout the region.

We are now in a position to explain the genesis of the reverse Arrow effect. Consider the steady state $(l, f+1)$ immediately above the $\text{IC}_M(l)$ and $\text{IC}_P(l)$ thresholds, as marked in Figure 4. Now consider state (l, f) that is below both thresholds. The policymaker will not protect at this state because she prefers to defer rents today, allow the state to transition to $(l, f+1)$, and receive $\pi^P(l, f+1)$ in every period thereafter.

This is important not for what the leading firm will do at state (l, f), rather it is important for what the firm can expect if it invests when at state $(l-1, f)$. If the firm invests at $(l-1, f)$ it knows it will not be protected.[24] The state is below the monopoly, and thus the duopoly, threshold, and investment without protection remains profitable. This choice, however, must be compared to what happens should the leader not invest at $(l-1, f)$.

If it does not invest, the leader anticipates that the policymaker will protect. Not protecting will transition to the steady state $(l-1, f+1)$ and a higher stream of rents beginning tomorrow, whereas protecting will earn rents today and cause the present state, $(l-1, f)$, to be stable. It is optimal for the policymaker to protect as the state is above the $\text{IC}_P(l)$ threshold and her trade-off is the same as discussed above for higher states.[25]

[23] This holds even though there are reachable states that we have yet to characterize equilibrium behavior for; i.e., states just below $\text{IC}_D(l)$ and $\text{IC}_P(l)$. As the state is above $\text{IC}_M(l)$ and the policymaker protects, it follows that the leader does not invest regardless of the policymaker's behavior should the leader not invest.

[24] We indicate these states with triangles rather than the squares used in Figure 3 as, for purposes of the discussion, we specify here only that the policymaker does not protect should the market arrive at this state. The full details of equilibrium behavior are left to the appendix.

[25] This argument does not depend on the particular arrangement of states relative to the thresholds depicted in the figure. If state $(l-1, f+1)$ is stable then protecting at state state $(l-1, f)$ follows directly from the logic of IC_P. If, instead, the leader invests at state $(l-1, f+1)$ then protecting remains optimal as

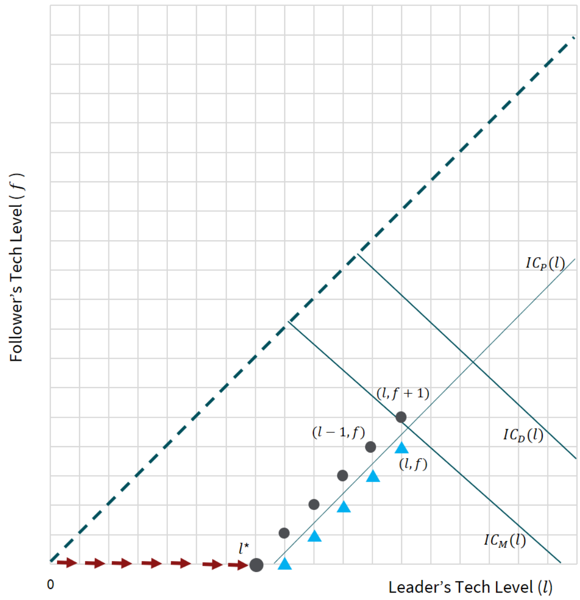

Figure 4: Backwards induction. The unravelling logic is shown given states on either side of the policymaker's incentive contraint IC_P. The gray dots are steady states where the policymaker protects and the leader does not invest. The blue triangles are states at which the policymaker does not protect and the market is a duopoly.

This simple calculus creates the reverse Arrow effect. The leading firm would invest at state $(l-1,\ f)$ if it were sure of being a monopoly or a duopoly regardless of its investment. But its investment causes the level of competition to change by changing whether the policymaker imposes protection or not. If the leader invests, protection is removed and competition is high, whereas if it doesn't invest, protection is applied and the firm has monopoly power. As the value of monopoly outweighs the benefit of a higher capability level, the firm doesn't invest. This is the reverse of Arrow's classic argument and reveals the distortion to market outcomes from political intervention.

The behavior at state $(l-1,\ f)$ is interesting but by itself of marginal impact. Its true importance is in how, by backward induction, it affects behavior at preceding states. The logic of the reverse Arrow effect recurs. Because state $(l-1,\ f)$ is stable, the policymaker will not protect at state $(l-1,\ f-1)$ and, in turn, this induces the leader to not invest and for the policymaker to protect at state $(l-2,\ f-1)$.

We establish in the proof that this unraveling continues all the way to the $l-$axis where

forgoing rents today does not cause the follower to close the capability gap with the leader and increase rents tomorrow; we make this argument precise in the appendix.

the trailing firm has a capability level of zero. If the $\mathrm{IC}_P(l)$ threshold has slope at least equal to one, unraveling follows the path of this curve, as depicted in Figure 4, or to the left of it. If the slope of $\mathrm{IC}_P(l)$ is less than one, the unraveling proceeds along a line of slope 1 or to the left of that line, and the relevant upper bound on equilibrium investment is $l^I - f^I$, as depicted in the right side panel of Figure 2. In either case, investment by the leading firm in equilibrium is strictly below that of a protected monopolist, and potentially significantly below depending on the nature of unraveling in equilibrium, and in no case is investment greater than that under monopoly.

4.1 Relaxing Competitive Pressure

The insight of 'managed competition' is that the policymaker seeks market competition purely for the threat value. She allows competition only because it enables the follower firm to catch up and increase the threat of further competition. In this sense, competitive pressure translates not into more efficient markets, but into leverage for the policymaker to extract rents. This induces the reverse Arrow effect that undermines market efficiency.

The lesson from this is that standard intuitions about market competition need not hold when markets and politics are intertwined. Counterintuitively, therefore, it may be that outcomes are improved if the degree of competition is relaxed. We explore this possibility in this section.

One dimension of competitive pressure is the willingness of the trailing firm to enter into the market and compete if political protection is removed. It is a striking feature of the equilibrium in Proposition 1 that the trailing firm never competes in the market yet nevertheless stands ever ready to do so. Although this is a reasonable description of some markets (e.g., foreign competitors and trade barriers), it is less appropriate in other markets, and one might think that the trailing firm will, at some point, give up and abandon the market altogether.

To formalize this idea, we amend the model as follows. We suppose that a firm that has been *excluded* from the market for κ consecutive periods will permanently *exit* the market. That is to say, if a firm has not been allowed to compete for κ periods it gives up and pursues opportunities elsewhere.

This assumption captures competitive pressure in a way that is simple and realistic and that resonates with the interdependence of politics and markets that is the focus of our paper. A firm exits the market if political intervention is excessive, but if it does so, the value of political protection disappears, costing the policymaker her leverage over the remaining firm. Although this is a simple variant, it complicates the analysis considerably. The state space is now the technology levels of the firms plus the number of periods of consecutive protection. As it is possible for the firms to remain at technology levels for multiple periods before advancing, we say a state is a steady state only if the capability levels have not changed for κ periods and are permanently stable. For this environment we characterize the steady states of market competition but do not provide a full description of the equilibrium path.

A market with potential exit changes the incentives of the policymaker. The policymaker must now remove protection at least once every κ periods else she loses her leverage. The wisdom of doing this is clear when the difference in capability levels of the firms is more than a single step. In this case, after protecting for $(\kappa - 1)$ periods, the policymaker faces a simple trade-off: Protect and receive rents for a final period or forgo rents today, allow competition, and renew a fresh stream of rents for κ periods. Indeed, if the leader doesn't invest while protection is off, tomorrow's rents are certain to be higher. For κ and δ not too small, the benefit of waiting is clear.[26] This implies, therefore, that the market cannot stabilize unless the firms' capability levels are close.

Lemma 3 *For κ and δ sufficiently large, the firm capability pair (l, f) is not a steady state if $f < l - 1$.*

The logic of this result does not necessarily hold within one step of the 45 degree line. The difference is that, if the leader doesn't invest when protection is removed, the state will transition to the 45 degree line. This matters because on the 45 degree line the technology standard has no bite and the policymaker cannot extract rents. The optimal behavior of the policymaker in this case depends, then, on the strategy of the firm that is given the opportunity to invest when on the 45 degree line.

The need for the policymaker to refresh competition also changes the incentives of the firms. Because in some periods the leading firm knows that protection will be removed *regardless* of whether it invests or not, the reverse Arrow effect is relaxed in those periods. In those periods, the leader invests knowing it will have to compete and, therefore, the relevant threshold is that of duopoly, $\text{IC}_D(l)$. This is not to say the reverse Arrow effect does not bind in other periods when protection is a choice for the policymaker, only that in some periods it is relaxed, and that is enough to ensure that eventually technology advances to the duopoly level.

[26]Similar forces are at work for κ small or δ small, although the analysis is more complicated and identifying steady states would require the full characterization of the equilibrium path.

Lemma 4 *For κ and δ sufficiently large, the firm capability pair $(l,\ f)$ is not a steady state if $f\ < IC_D\,(l)$.*

Combining the two lemmas provides a broader picture of equilibrium behavior. Lemma 3 shows that a steady state must be either on or adjacent to the 45 degree line where firm capabilities are equal, and Lemma 4 shows that a steady state cannot exist below the duopoly threshold.

A reasonable conjecture is that investment stops as soon as the duopoly threshold is passed. Were this true, the policymaker would, upon first reaching a state $(l, l-1)$ beyond the 45 degree line, take the κ^{th} period of rents and let the follower exit the market, all to avoid reaching the 45 degree line and stagnation.

We show that this conjecture is not true. The firms are willing to invest beyond the duopoly threshold, although only when the state is on the 45 degree line and their capability levels are equal. The reason for this willingness comes from Arrow once again. In this context, however, the logic of Arrow is enhanced rather than reversed. The firms are willing to invest on the 45 degree line because investment is the only way they can obtain protection.

To see this, observe that although the policymaker cannot protect when the firms' capabilities are equal, she can protect if one firm were to invest and obtain a technological advantage. She will protect, therefore, if and only if the leader invests. This enhances the firm's incentive to invest and the standard Arrow effect as not only is competition reduced by investment, it is entirely eliminated and the investing firm becomes a monopolist. We refer to this as the *politically enhanced Arrow effect*.

In this situation, therefore, investment is profitable for a firm on the 45 degree line if:

$$\pi^{M}\,(l+1, l) - c\,(l) \geq \pi^{L}\,(l, l)\,. \tag{6}$$

This is similar to the conditions for duopoly and monopoly in Equations (3) and (4), respectively. The difference here is that the firm receives the profit of a protected monopolist when it invests but the duopoly profit otherwise. Thus, the threshold at which (6) is satisfied by equality, which we denote by IC_{EA} for 'enhanced Arrow,' is higher than even duopoly.

The enhanced Arrow effect applies only on the 45 degree line and, thus, the IC_{EA} threshold is defined only in that case. With firms willing to invest on the 45 degree line, it implies that the logic of Lemma 3 holds one step away from the 45 degree line as long as condition (6) holds. This delivers the following result.

Proposition 2 *With relaxed competitive pressure, for κ and δ sufficiently large, every steady state is given by $(l^{**}, l^{**}-1)$ for some $l^{**} \geq IC_{EA}$, and the follower firm exits the market.*

This result can be seen in Figure 5. It depicts a potential dynamic path for the equilibrium in which investment passes the duopoly threshold with the leading firm holding a large capability advantage. Beyond the duopoly threshold the equilibrium behavior becomes clear. The leader no longer finds it profitable to invest and the state transitions vertically until reaching the 45 degree line. Progress to this point is staggered, with stretches of protection and temporary stability interspersed with periods of competition as the policymaker renews her leverage. As this path intersects the 45 degree line below the threshold IC_{EA}, the policymaker is happy to let the trailing firm fully catch up. She knows, through the enhanced Arrow effect, that the firms will invest on the 45 degree line when given the opportunity. This creates a ratchet effect as the state moves off the 45 degree line and back to it repeatedly, with investment increasing along the path. This sequence finally ends once the IC_{EA} threshold is crossed. The steady state, $(l^{**}, l^{**}-1)$, is off the diagonal and the trailing firm permanently exits the market. At this state the policymaker protects the leader for a full κ periods and accepts the exit of the following firm as she knows that, should she remove protection and let the follower catch up, neither firm will invest any more, the technology standard will not have any bite, and she wouldn't be able to extract any more rents.

The steady state is striking for what it implies about competition and protection. In contrast to Proposition 1, the leading firm is not protected in the steady state. Moreover, it faces no competition—as the trailing firm exits the market—and it has attained a high level of capability. To an observer, this outcome would suggest a firm that has earned market dominance from having a high capability. However, the full equilibrium path belies this interpretation. Political intervention is a mainstay along the equilibrium path and the final outcome is predetermined once the initial advantage is obtained even when, as in our model, it is determined by luck.[27] This is not to deny the efficiency of the final steady state, but it does imply that fairness had little to do with it.

[27] With myopic firms this conclusion depends on the tie-breaking rule when the state returns to the 45 degree line. If tie-breaking is random then the crucial stroke of luck for the firms is the final random selection of a firm to invest rather than the first. Our preferred tie-breaking rule is that the firm that was leading previously is given the opportunity to invest. This ensures the importance of the stroke of luck at the origin of the market. We prefer this rule as it reflects the market outcome when firms are forward-looking. If the tie-breaking rule at the 45 degree line were random, a leading firm that is forward-looking would invest preemptively to always maintain its lead and avoid being subject to that randomization.

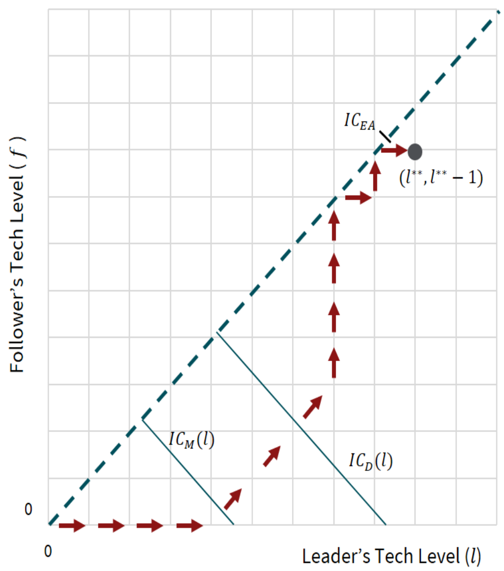

Figure 5: The steady state under relaxed competitive pressure and a potential equilibrium path. Threshold IC_{EA} is such that above it investment is no longer profitable at state (l, l) if protection will be offered at state $(l+1, l)$. The red arrows illustrate the equilibrium transitional path. The gray dot at $(l^{**}, l^{**}-1)$ represents the steady state. The policymaker protects and the follower eventually permanently exits.

4.2 Welfare

In the steady states of both Propositions 1 and 2 there is no competition and the leading firm operates as a monopolist; in Proposition 1 because the leader is protected politically, and in Proposition 2 because the trailing firm has exited the market. Therefore, a welfare comparison between these states depends only on the level of investment by the leading firm.

We establish here that the steady state in Proposition 2 strictly dominates that in Proposition 1 when ρ, the share of surplus going to the policymaker, is sufficiently small. In this case, weaker competitive pressure leads to strictly greater market efficiency.

This comparison is not immediate more generally—even though the steady state in Proposition 2 is above the duopoly threshold and the steady state in Proposition 1 is below the monopoly threshold—as the thresholds themselves are downward sloping. However, for ρ sufficiently small the welfare ranking is clear. In this case the thresholds are sufficiently separated that the right-most point of the monopoly threshold involves a smaller capability level than the left-most point of the duopoly threshold.

The logic for this result follows from the original Arrow effect. Assumption 1 requires the monopolist to have a lower incentive to invest than a duopolist, regardless of the follower firm's capability level. This assumption is for unprotected markets, however, whereas the $\text{IC}_M(l)$ constraint of Lemma 2 is for a protected monopolist who shares rents with the policymaker. As discussed in Section 2, the incentive to invest in this case is higher than for the unconstrained monopolist. The following strengthens Assumption 1 to cover this case.[28]

Assumption 1': $\frac{\partial}{\partial l}\left((1-\rho)\,\hat{\pi}^M(l)+\rho\pi^L(l,\,f)\right) \leq \frac{\partial}{\partial l}\pi^L\left(l,\,f'\right)$ for all $l,\,f,\,f'$.

This assumption converges to Assumption 1 as ρ converges to zero. This implies that the difference in incentive between a protected and an unconstrained monopolist disappears for ρ small. Conversely, therefore, if Arrow's replacement effect holds, then for values of ρ sufficiently small, the steady state in Proposition 1 is strictly dominated by that in Proposition 2. We state this formally as follows. Recall that the leader's steady state capability level is l^* in Proposition 1 and l^{**} in Proposition 2.

Proposition 3 *Suppose the premises of Propositions 1 and 2 hold. When Assumption 1' additionally holds,* $l^{**} > l^*$.

The range of ρ values for which $l^{**} > l^*$ holds is broad. Assumption 1' is a sufficient condition for this to be true but it is not necessary. Assumption 1' implies the stronger result that the leading firm's capability level in duopoly strictly dominates that in monopoly; that is, it guarantees that the leader's investment is strictly higher in Lemma 1 than in Lemma 2. This isn't necessary for $l^{**} > l^*$ as l^* may be strictly less than the monopoly threshold and l^{**} is strictly above the duopoly threshold. Indeed, as ρ decreases, the threshold IC_{EA} (that provides the lower bound for l^{**}) increases, whereas the monopoly threshold decreases, expanding the potential gap between investment levels in the steady states.

Proposition 3 implies that relaxing competitive pressure can improve market efficiency when that market is intermediated by a strategic policymaker. This ordering reflects a balance of distortions. When competitive pressure is reduced there is a direct negative effect on welfare through the standard economic forces (if the competitor disappears the market switches from competitive to monopoly). In addition, there is an indirect political effect, which is that the power of the policymaker is weakened. Both forces affect market

[28]Recall that the profit for the regulated monopolist can be rearranged as $\pi^M(l,\,f) = (1-\rho)\,\hat{\pi}^M(l) + \rho\pi^L(l,\,f)$.

efficiency, with the economic effect decreasing efficiency whereas the political effect increases it. Proposition 3 establishes that the latter effect dominates when the policymaker's ability to extract rents is not too great.

Proposition 3 compares the steady states but not the path to reach these points. A welfare analysis that includes the path only strengthens the ranking in Proposition 3. Along the path the market is competitive for many periods with the relaxed competition of Proposition 2, whereas the market is never competitive on the path of Proposition 1.[29] The same is true for the comparison of duopoly and an always-protected monopoly (Lemma 1 versus. Lemma 2). Comparison of duopoly in Lemma 1 with the relaxed competition case in Proposition 2 is less clear as duopoly delivers constant competition but a lower capability level for the leading firm.

5 Discussion

The Distortions of Politics. The policymaking side of the model combines two standard elements of political environments. First, the policymaker is self-interested. Second, she lacks commitment power. Both elements play an important role in the mechanism we uncover. To disentangle the role of each, we discuss briefly the resulting behavior when one or both of these elements does not hold. We focus on the specification of the model in which the follower never exits the market and keep the discussion informal (formal statements and proofs are contained in Appendix E).

Full commitment power. Commitment power better enables the policymaker to reward or punish investment. In so doing, and as she is far-sighted, this means a self-interested policymaker has more ability to maneuver the market to her preferred steady state. This does not negate the reverse Arrow effect, rather it magnifies it, and causes market inefficiency to increase.

To see why, observe that the policymaker's rents increase in the tightness of market competition, conditional on a difference existing and protection remaining effective, and decrease in the leading firm's technology level. Thus, the optimal state for the self-interested policymaker is $(1, 0)$ where investment is minimized and inefficiency maximized.

[29] This comparison is unambiguous under our assumption that catch-up growth is costless. A small cost would not upset this conclusion, although such a cost may alter the equilibrium itself; see the discussion in the following section.

The policymaker is able to ensure this state is reached and never left by committing to remove protection should the leader invest beyond it. Thus, by committing to the reverse Arrow effect early, the policymaker can use it to her own advantage.

Commitment power increases market inefficiency because it doesn't solve the bargaining problem between the policymaker and the leading firm. Their joint surplus is maximized by the level of investment a monopolist would undertake, yet with a sharing rule dependant on the value of protection, the monopoly outcome is not optimal for the policymaker. Thus, commitment power does not remove the wedge between the interests of the two players, rather it allows the policymaker to leverage her response to that wedge even at the expense of overall market efficiency.

Benevolent social planner. A social welfare maximizing policymaker wishes to leverage Arrow as well, although in this case it is the enhanced Arrow effect that she can use to her advantage. The enhanced Arrow effect helps the benevolent social planner as market competition, even without political intervention, suffers from the classic failure of underinvestment. Firms don't internalize the consumer benefit of investment, and therefore the duopoly investment level is below the social welfare maximizing level.

To maximize welfare the policymaker needs to push investment beyond the duopoly level. The enhanced Arrow effect points to how this can be done. By rewarding investment with protection, a patient policymaker can improve welfare by paying the price of monopoly today to attain higher market efficiency tomorrow.

This is not straightforward, however. The policymaker faces a commitment problem. Once the investment is undertaken, she would prefer to renege, to not protect and thereby avoid the cost of monopoly. Anticipating this, firms do not invest beyond the duopoly threshold and the logic of enhanced Arrow unravels. It follows that protection never occurs in equilibrium, and the equilibrium trajectory and steady state is simply that given by duopoly in Lemma 1.

Benevolent social planner with commitment. Commitment power allows the policymaker to solve her time-inconsistency problem and leverage the enhanced Arrow effect to her benefit. By committing to reward investment with protection, the policymaker can incentivize investment beyond the duopoly threshold. This is not the first best as the cost of this incentive is temporary monopoly power, yet in the steady state the policymaker restores competition and, as long as she is sufficiently patient, the net effect is positive.[30]

[30] This is logically equivalent to a patent mechanism. This shows how the logic of patents may be more widespread throughout policymaking than the narrow confines of the formal patent system.

The dual political distortions. The reverse and the enhanced Arrow effects represent two distortions in markets caused by political intervention. The reverse Arrow effect follows from the self-interest of the policymaker and plays no role without it. In contrast, the enhanced Arrow effect follows from neither self-interest nor lack of commitment alone, but rather from their interaction.[31] It is relevant to market outcomes when the policymaker is benevolent and can commit, and when she is self-interested without commitment power, but not in the absence of one or the other political features.[32]

Model Robustness. Our model provides a simple framework to illustrate a mechanism through which market and political outcomes are linked. This mechanism requires only some basic ingredients: a policymaker who extracts rents by intervening in markets and where those rents depend on the state of market competition. These ingredients are sufficient to incentivize the policymaker to want to manage competition. Our model captures this mechanism and we explored how the mechanism manifests in two different market contexts. The exact nature of the equilibrium depends on the details of the model, and changing those details will, naturally, lead to different equilibria. The underlying mechanism that we identify remains relevant more broadly and, to this end, we discuss here (and consider formally in the Appendix) several variations on our basic model. We begin with variations of the market environment, before turning to the political environment.

Far-sighted firms. The results of our model are qualitatively unchanged if we relax the assumption that firms are myopic. One impact of far-sighted firms is standard: the firms have a stronger incentive to invest as they internalize the long-run benefit, and the investment thresholds for both duopoly and monopoly increase. A second impact is more subtle. A far-sighted firm anticipates the policymaker's desire to manage competition. The firm cannot avoid the policymaker managing competition, and this continues to distort the firm's incentive to invest. Nevertheless, investment does not stop immediately when the policymaker first manages competition as the firm may invest through some of these periods. When the firm is forward looking it has preferences over which state proves stable, and it strategically invests to move the market to that point. Ultimately, the policymaker's desire to manage competition causes investment to stop prematurely, at or below the investment level of a protected monopolist.

[31] The welfare-enhancing effect of political intervention also shows that the institutional design solution is not simply to prohibit political intervention altogether.

[32] A self-interested policymaker with the power to commit would leverage the enhanced Arrow effect were the state to hit the 45 degree line above the duopoly threshold, although such a policymaker would do everything in her power to avoid reaching such a position.

Catch-up costs. Anticipating the policymaker's desire to manage competition can also be relevant to the follower firm. We have assumed that the follower catches up to the leading firm automatically when it participates in the market. This may hold in some industries, where imitation is easier and more prevalent, but is less descriptive of other industries. A cost of investment makes it more difficult for the follower firm to challenge the leader and represents a decrease in competitive pressure. We show in Appendix G that this change does not overcome the inefficiency of Proposition 1. Although the steady state investment level may improve, it remains bounded by the level of an always-protected monopolist (Lemma 2).

The logic for this result again flows through managed competition and the reverse Arrow effect. In the equilibrium of Proposition 1, the follower is poised to enter and close the technological gap with the leading firm. However, a forward-looking follower anticipates that it will never be allowed to fully catch up. That once it gets close technologically, the policymaker will again install protection and extract rents. Being reluctant to play the foil perennially, this dampens the follower's incentive to invest and compete in the market, even to an extent greater than if the market were a standard duopoly. In turn, this undermines the ability of the policymaker to manage competition. She forgoes rents should she remove protection, but with the follower not investing, she does not gain greater rents tomorrow. The leader can then see that the policymaker's threat to remove protection no longer binds in the same way.

This result does not depart from the equilibrium of Proposition 1 as radically as that in Proposition 2. This reinforces the conclusion that while political intervention into a market may upend standard intuitions, it does not reverse them. It is not that lower competitive pressure necessarily leads to more efficient market outcomes. Rather, that the effect is contingent on how it affects the incentives of the policymaker and that thinking through her incentives is a necessary step to understanding market competition.

Partially self-interested policymaker. We have assumed for simplicity that the policymaker cares only about the rents she can extract from protection (although see discussion of the opposite case earlier in this section). In precluding market competition, political protection harms consumer surplus, and a partially socially (or electorally) minded policymaker would weigh that effect against the rents she receives. Our results are robust to such motivations for the policymaker. In the appendix we show that all of our results hold if the weight the policymaker places on consumer welfare is small (that is, if the policymaker is sufficiently selfish). The conclusion for general policymaker preferences is more difficult to pin down. The mechanism that we identify in our main model carries through to this general case, although as the cost of protection for the policymaker then varies in the firms' capability levels, her willingness to protect will vary more the greater weight she places on consumer welfare.[33]

Policymaker tools. Our model is built on one particular tool that is available to the policymaker—a minimum standard. This allows us to see how the value of that tool—and the balance of power between the policymaker and firms—ebbs and flows depending on the degree of market competition. In practice, other tools are also available to policymakers, and the predictions of the model will depend on the nature of these tools, and not just the degree of market competition.

The policymaker's toolkit can be used to serve the industry's interests or those of the policymaker. Stigler (1971) provides the example of an ascendant oil industry using regulatory power to not only capture government subsidy but also protect the industry from new entrants.[34] A more common example is the creation of licensing standards that protect incumbents by making entry costly.

To serve the interests of the policymaker, the ideal tool is one that maintains a heightened threat of entry. One such regulatory structure would allow the follower firm some access but not total access to the market, thereby allowing the policymaker to extract rents from protection while keeping the follower firm incentivized to invest and not exit the market altogether. Such a market may seem competitive from the outside as the follower would appear as a competitive fringe. Yet viewed through the lens of our model, this form of limited market access can be identified properly as serving only the interests of the policymaker. We discuss below the example of the home construction industry that exhibits such a form of restrained competition.

Political bargaining. The balance of power between the firms and the policymaker depends also on the bargaining protocol. In Stigler's (1971) original view, it was the industry cartel that held all of the bargaining power, making demands of policymakers and extracting all of the benefit of political protection. McChesney (1987) shows that if instead the policymakers were proactive and could make demands of the firms, then they would extract all of the surplus. Reality lies somewhere between these extremes. We have sought to thread the gap by fixing the relative bargaining power with the parameter ρ, which we model in reduced form. The size of ρ reflects many factors, including the willingness of

[33]In this case it is possible that this change would feed through to a change in ρ. See the discussion of ρ below.

[34]Government mandated barriers to entry offer the additional advantage of avoiding antitrust scrutiny.

the policymaker to accept rents for protection or of the firm to share them, the cost to the policymaker of protection, as well as the degree of political competition. We set this to be a constant for simplicity. Fixing bargaining power also enables us to focus on how outcomes vary in the degree of market competition, independent of other institutional features. Modeling the process by which institutional design influences the bargaining power of a policymaker and, therefore, how that feeds through to market outcomes, offers many interesting possibilities.

The Model and Practice. We have developed and analyzed a stylized model that steps back from specific contexts in order to capture a mechanism that is broadly applicable. One takeaway from this exercise is that the details do matter, that while the underlying mechanism may appear in many settings, the outcome it produces depends on the details of market and political competition. The key variable is the extent of the competitive threat that the policymaker can hold over a firm or industry. When this threat is high, the power of political intervention is high, and the policymaker's impact on markets is its most extensive.

International competition and trade protection offers perhaps the cleanest example of a market in which the threat of competition is unwavering. The historical example of the East India Company illustrates this structure. Throughout the long history of the company, competitors were eager to begin trade but blocked by the grant of monopoly to the company (Erikson 2016, Zingales 2017). A striking feature of this history is that the grant of monopoly was repeatedly threatened but consistently renewed, and each renewal was made only once those with political power had extracted some surplus. This cycle repeated up until the time that the Company's interests clashed directly with those of the Crown. The history of the East India Company resonates with the equilibrium in Proposition 1 and the policymaker's need to retain relevance in the bargaining relationship with the market leader.

Other examples of markets in which competitive pressure is high are those with neighboring industries that employ similar technology. A firm with a profitable home market remains a threat to neighboring markets even when excluded for lengthy periods of time. A similar threat emerges for industries that are geographically separated. A striking example of this is the home construction industry (Schmitz 2020). In this case the distinction is between industry segments rather than individual firms, with the relevant actors the trade association for each segment. The stick-built housing sector (homes constructed on-site), through their trade association, the National Association of Home Builders (NAHB), has successfully won political protection for over a century from competition from the factory-built (modular) housing industry, with regulations imposing minimum standards on housing quality that effectively block factory-built housing other than in specially designated areas (e.g., trailer parks). Interestingly, and in accordance with our model, the ability of the factory-built industry to operate at the margins of the industry and in some isolated locations, ensured that it remained a competitive threat to the stick-built industry throughout this time.

The connection between market dominance and political protection is also evident at the aggregate level. Faccio & McConnell (2020) provide evidence from 75 countries showing that when an incumbent dominates a market for an extended period of time it is most frequently due to political protection. This resonates with our equilibrium prediction of industry lock-in, that an early advantage is a strong predictor of long-term dominance. Evidence from the trade literature shows, consistent with these findings, that the frequency and intensity of lobbying varies with the degree of product market competition (Bombardini & Trebbi 2012, Kim 2017).

Our model reinforces the idea that political intervention into markets has real impacts on firms beyond the degree of competition in markets. Evidence of real distortions in resource allocation within firms is more difficult to identify, although it has begun to accumulate. Huneeus & Kim (2020) show that politically connected firms in the U.S. attract both capital and labor, and thus grow more. From a different angle, Aghion, Bergeaud & Van Reenen (2021) begin with a specific labor regulation in France and show how it distorts the development of firms at both the intensive and extensive margin of innovation.[35]

Policy Implications. In practice, the connection between the market's competitive structure and political influence is a tricky identification problem. Is concentration good because of investment and a technological breakthrough? Or is concentration bad because of lobbying and other anti-competitive behavior? In allowing for both sources of market power, our model provides a theoretical structure to inform that decision. A core insight of our model is that standard market analyses may not hold up when political intervention is allowed for. If we place this insight in the context of antitrust policy, it leads to a different interpretation of a policymaker who points to a competitive fringe in a market as justification for approving a merger. If that fringe empowers a policymaker to more effectively impose a technology standard and extract rents, the competitive fringe can be seen as serving the interests of the policymaker rather than consumers and the logic of the merger is reversed.

[35] At the macro level, Parente & Prescott (1999) show how concentration of market power restrains economic development.

That political power and monopoly power are intertwined has been long acknowledged in practice, and particularly so in discussions around one type of policy, that of antitrust. Franklin Roosevelt's head of the Antitrust Division in the Department of Justice, Thurman Arnold, wrote in 1943 that:[36]

> "Monopolies enter into politics using money and economic coercion to maintain themselves in power, making alliances with other powerful groups against the interests of consumers and independent producers."

This connection has fallen out of focus over the years, and only recently has it that it has returned to the forefront of economists' and policymakers' minds. Wu (2018) and Philippon (2019) argue that lobbying and lax antitrust enforcement are a causal factor in the increased market concentration of recent decades (see also Gutieerrez & Philippon (2019)).

The challenge for antitrust policy is that it is not the only relevant policy intervention. In practice, other policymakers have the ability to intervene in markets, less dramatically perhaps but no less significantly. Our analysis emphasizes that when antitrust policy is set, even if it is evaluated through the lens of welfare maximization, it must account for the intervention of other, potentially less socially minded, policymakers into the market.

Another implication of our model for antitrust policy is that a static analysis is insufficient to understand the degree to which market concentration is good or bad. As noted in the previous section, the steady state in Proposition 2 describes a leading firm that has attained a high capability and faces no market competition. It does not benefit from political protection at the steady state, but only because it doesn't need it. It got to this position, however, due to protection along the equilibrium path. Therefore, even though the political patronage that a dominant company receives today may not seem to matter so much to its market performance, and its lobbying expenditures are relatively small, it does not follow that political power was not instrumental to its rise, helping critically when the technology gap was smaller and the competitive threat greater.

An intriguing corollary to this observation is that it may provide a rationale for the long-standing puzzle as to why there is so little money in politics (Ansolabehere, de Figueiredo & Snyder 2003). Although large amounts are spent, the absolute levels are small given what is at stake. Proposition 2 describes a case in which, at the steady state, there is no money spent on politics, not because the firms wouldn't spend it and political intervention couldn't be important, but because the market has reached a state of asymmetry in capability-based

[36] Arnold (1943); quoted in Schmitz (2020).

market power that political intervention has simply lost its relevance given the tools available. The equilibrium is stark, to be sure, although in aggregate it resonates with practice.[37] This suggests that insight into lobbying, as well as market competition, may come from more closely matching political spending to the state of market competition and the threats policymakers hold over firms.[38]

6 Conclusion

The focus of the political economy literature is on the choice of a policy, in which political power varies in the design of institutions and the identity of those who make the decisions. We have shown in this paper that the value of political power—the power of politics itself—varies as the market environment varies. For a fixed set of political tools, the command of policymakers over the economy and society changes as market conditions change. This, in turn, alters the impact of business on society. Eighty years ago, Thurman Arnold (1943) argued that the power of business had grown to such an extent that it subsumed part of the functions of the state:

> "In short, they will become a sort of independent state within a state, making treaties and alliances, expanding their power by waging industrial war, dealing on equal terms with the executive and legislative branches of the government and defying governmental authority if necessary with the self-righteousness of an independent sovereign."

This point has been picked up recently by Zingales (2017) from the perspective of the firm and Wu (2018) in critiquing the application of antitrust policy.[39] Our paper builds a formal structure that allows more detailed analysis of the problem and potential remedies.

The core insight of our model is that when markets and politics coevolve, the interests of firms and the policymaker are aligned but not perfectly aligned. This has ramifications

[37] We interpret here corporate political expenditures, whether direct contributions to general expenditure, reflect the value at stake from political protection. The model abstracts from these details and presumes the sharing of rents with the policymaker is direct and seamless.

[38] Suggestive evidence in this regard is that the industries with the highest political expenditures anecodtally are those with high capital expenditures, where competition is fierce and political advantage most valuable. Examples include the airline and package delivery (FedEx/UPS) industries.

[39] Zingales (2017, p.117) argues that economics has erred in focusing on the technological rather than the power dimension of firms. Wu (2018), channeling Thurman Arnold and Louis Brandeis, argues for a return to an antitrust policy in which political concerns are dominant.

for the outcomes in both domains. We build a model to capture these incentives and characterize the outcomes they produce. The policymaker cares about rents and the firms care about market power, and the market and political outcomes reflect how these forces balance out. Many practical details are left out or included in a reduced form. Adding richness to the model will affect this balance and add nuance to the predictions, to be sure, but not fundamentally change the logic for how markets and politics interact.

There are many natural ways to extend our model beyond those discussed in the previous section. On the market side, the number of firms and the structure of market competition are promising directions to explore. On the policy side, the natural extension is to multiple policymakers and instituting a degree of political competition. Political institutions are often structured hierarchically, from legislator down to regulator. Incorporating this into the model not only adds an agency problem, it opens up the question of where and not just how much firms lobby and transfer rents.

The motivations of policymakers also offers scope to broaden the applicability of the underlying insights. In addition to rents and consumer welfare, policymakers care about their careers, about policy itself, or building bureaucratic empires. These motivations can generate the same incentive to 'manage competition' as emerges for a rent-seeking policymaker. For example, a regulator will be out of a job if it solves the underlying policy issue. To avoid rendering herself obsolete, a career-minded policymaker may 'manage the policy issue' in the same way that the self-interested policymaker here manages competition.

A particularly intriguing extension is to explore a balance between political and economic goals. In our model political power is the means to the end of market power. Some important political goals stand aside from economic outcomes, and market power may be the means toward those ends. Exploring the interdependence of politics and markets more deeply is of considerable importance.

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Online Appendix

A Proofs from Section 3

A.1 Proof of Lemma 1

The duopoly profit is $\pi^{L}(l, f) = \left[(a - 2\mu(l) + \mu(f))^2\right]/9b$. Notice that

$$\text{sign}\left(\pi^{L}_{lf}(l, f)\right) = -\text{sign}\left[\mu'(l) \cdot \mu'(f)\right] \leq 0.^{40} \tag{7}$$

We define $IC_D(l)$ such that, for each l, at $f = IC_D(l)$,

$$\pi^{L}(l+1, f) - c(l) = \pi^{L}(l, f). \tag{8}$$

Then, conditions (7) and (8) imply that the leader invests if and only if $f < IC_D(l)$. In addition, $c'(l) > 0$ implies $\frac{d}{dl} IC_D(l) < 0$.

A.2 Proof of Lemma 2

The monopoly profit is $\hat{\pi}^{M}(l) = (a - \mu(l))^2/4b$. The policymaker's rent is $\pi^{P}(l, f) = \rho\left(\hat{\pi}^{M}(l) - \pi^{L}(l, f)\right)$, and the leader's payoff is

$$\pi^{M}(l, f) = \hat{\pi}^{M}(l) - \pi^{P}(l, f).$$

Notice that, by (7),

$$\text{sign}\left(\pi^{M}_{lf}(l, f)\right) = \text{sign}\left(\frac{d^2}{dldf}\left(\hat{\pi}^{M}(l) - \pi^{P}(l, f)\right)\right) = \text{sign}\left(\pi^{L}_{lf}(l, f)\right) < 0. \tag{9}$$

We define $IC_M(l)$ such that, for each l, at $f = IC_M(l)$,

$$\pi^{M}(l+1, f) - c(l) = \pi^{M}(l, f). \tag{10}$$

Then, conditions (9) and (10) imply that the leader invests if and only if $f < IC_M(l)$. Moreover, since $c'(l) > 0$, we have $\frac{d}{dl} IC_M(l) < 0$.

[40] Each subscript denotes that the derivative is taken with respect to that variable.

B Proof of Proposition 1

B.1 Preliminaries

We prove Proposition 1 in a more general setup, where the policymaker's utility is a convex combination of the rent and social welfare (the relative weight on social welfare is not zero), with weight α on social welfare. Now the policymaker's utility depends on the leader's technology level l, the follower's technology level f, and the policymaker's choice $a \in \{0, 1\}$ such that

$$\pi^P(l, f, a) = \begin{cases} \alpha \cdot SW^D(l, f) & \text{if } a = 0 \text{ (no protection)} \\ \alpha \cdot SW^M(l) + (1-\alpha) \cdot \pi^P(l, f) & \text{if } a = 1 \text{ (protection),} \end{cases}$$

where $SW^D(l, f)$ denotes social welfare under duopoly given (l, f), and $SW^M(l)$ denotes social welfare under monopoly given l. The payoffs for the firms stay the same as in the main text. The product market is Cournot, which leads to

$$\begin{aligned} SW^M(l) &= \frac{3}{8b}(a - \mu(l))^2; \\ SW^D(l, f) &= \frac{\left[a - 2\mu(l) + \mu(f)\right]^2}{9b} + \frac{\left[a - 2\mu(f) + \mu(l)\right]^2}{9b} \\ &\quad + \frac{1}{18b}(2a - \mu(l) - \mu(f))^2. \end{aligned}$$

It is straightforward to verify that $SW_l^M(l) > 0$, $SW_l^D(l, f) > 0$, and $SW_f^D(l, f) > 0$.

Let $\bar{L}$ be the smallest l such that there does not exist $l \geq f \geq 0$ for which

$$\pi^M(l+1, f) - c(l) \geq \pi^L(l, f). \tag{11}$$

The myopic leader never invests if $l \geq \bar{L}$. Hence, we will focus on $(l, f) \in \left[0, \bar{L}\right]^2$.

Note that $\pi^P(l, f, a)$ is well-defined as a function for each $a \in \{0, 1\}$ even if $l \leq f$. In the model, we label firms such that it is always the case that $l \geq f$ and setting $a = 1$ is not feasible if $l = f$. For the properties of function π^P that hold for each $(l, f) \in \left[0, \bar{L}\right]^2$ (including $l < f$), we will write "for all $(l, f) \in \mathcal{L} := \left[0, \bar{L}\right]^2$." By contrast, for the statement of the equilibrium that holds only for $l \geq f$, we will write "for all $(l, f) \in \mathcal{L}^* := \left\{(l', f') \in \left[0, \bar{L}\right]^2 : l' \geq f'\right\}$." Finally, the feasible technology profile in the model is in $\mathbb{Z}_+^2$. Thus, for the statement of the equilibrium property in state $(l, f) \in \mathcal{L}^*$ that is feasible, we will write "for all $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$."

To simplify notation, we henceforth call the technology level profile at the beginning of the period (that is, the technology level at the timing of the leader's decision) "*the ex ante state*," and we call the technology level profile after the leader's investment (that is, the technology level at the timing of the policymaker's decision) "*the interim state*."

Derivatives

We assume $\mu(t)$ is linear: $\mu(t) = \gamma_0 - \gamma_1 t$ for all $t \in [0, \bar{L}]$ and the first order condition characterizes the Cournot equilibrium. We list the derivatives of the profit functions that will be referenced in the proofs that follow: For all $(l, f) \in \mathcal{L}$,

$$\text{sign}\left(\pi^P_{lf}(l, f)\right) = \text{sign}\mu'(l)\,\mu'(f) > 0. \tag{12}$$

$$\text{sign}\left(\pi^M_{lf}(l, f)\right) = -\text{sign}\left(\pi^P_{lf}(l, f)\right) < 0. \tag{13}$$

$$\pi^P_l(l, f, 0) = -\alpha \cdot \frac{1}{9b} \cdot (4a - 4\mu(l) + 7(\mu(f) - \mu(l))) \cdot \mu'(l) > 0. \tag{14}$$

$$\pi^P_f(l, f, 0) = -\alpha \cdot \left(2\frac{a - 2\mu(l) + \mu(f)}{9b} + 4\frac{a - 2\mu(f) + \mu(l)}{9b} + \frac{2}{18b}(2a - \mu(l) - \mu(f))\right) \cdot \mu'(f) > 0. \tag{15}$$

$$\pi^P_f(l, f, 1) = -(1-\alpha) \cdot \rho \cdot 2 \cdot \frac{(a - 2\mu(l) + \mu(f))}{9b} \cdot \mu'(f) > 0. \tag{16}$$

$$\pi^P_{lf}(l, f, 0) = \alpha \cdot \frac{7}{9b} \cdot \mu'(f) \cdot \mu'(f) > 0. \tag{17}$$

$$\pi^P_{lf}(l, f, 1) = (1-\alpha) \cdot \rho \cdot \frac{4}{9b} \cdot \mu'(l) \cdot \mu'(f) > 0. \tag{18}$$

$$\pi^P_{ff}(l, f, 1) = -(1-\alpha) \cdot \rho \cdot \frac{2}{9b} \cdot \left(\mu'(f)\right)^2 < 0. \tag{19}$$

Policymaker and Investment Thresholds

For the policymaker, for each l, we define $IC_P(l)$ such that, at $f = IC_P(l)$,

$$\frac{\pi^P(l, f, 1)}{1-\delta} = \pi^P(l, f, 0) + \delta \cdot \frac{\pi^P(l, f+1, 1)}{1-\delta}. \tag{20}$$

That is, at $f = IC_P(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader in the current period (given that she will protect the leader from next period on and the leader's technology level is not changing).

In addition, for each l, we define $\overline{IC}_P(l)$ such that, at $f = \overline{IC}_P(l)$,

$$\pi^P(l, f, 1) + \frac{\delta}{1-\delta} \cdot \pi^P(l+1, f, 1) = \pi^P(l, f, 0) + \frac{\delta}{1-\delta} \cdot \pi^P(l, f+1, 1).$$

That is, at $f = \overline{IC}_P(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader currently (given that she will protect the leader from next period on), even if (i) the current protection makes the leader invest and (ii) the current non-protection makes the leader not invest.

Similarly, for each $(l, f) \in \mathcal{L}$, define $T_{l,f}$ as a smallest non-negative integer T such that

$$\pi^P(l, f, 0) + \delta \frac{\pi^P(l, f+1, 1)}{1-\delta} \geq \sum_{k=0}^{T-1} \delta^k \pi^P(l+k, f, 1) + \delta^T \frac{\pi^P(l+T, f, 1)}{1-\delta}. \quad (21)$$

That is, not-protection today and going to the steady state tomorrow is (weakly) better than protection today if protection today leads to future investment and protection until the leader's level reaches $l+T$.

Finally, given a Markov perfect equilibrium, for each $l \in [0, \bar{L}] \cap \mathbb{Z}_+$, we define $\mathcal{IC}_P^*(l) \subset [0, \bar{L}] \cap \mathbb{Z}_+$ as the set of follower's technology levels f such that

$$\pi^P(l, f, 0) + \delta \cdot V^P(l, f+1) \leq \frac{\pi^P(l, f, 1)}{1-\delta},$$

where V^P is the equilibrium value for the policymaker.

Results about the Thresholds

We derive the two results about $IC_P(l)$. First, $IC_P(l)$ is a proper threshold even if $\alpha > 0$.

Lemma B.1 *For each $(l, f) \in \mathcal{L}$, the following inequality is satisfied if and only if $f \geq IC_P(l)$:*

$$\frac{\pi^P(l, f, 1)}{1-\delta} \geq \pi^P(l, f, 0) + \delta \frac{\pi^P(l, f+1, 1)}{1-\delta}.$$

Moreover, for each $l \in [0, \bar{L}]$, $\frac{d}{dl} IC_P(l) \geq 0$.

Proof. All proofs for the auxiliary lemmas are in Section B.4. ■

We next prove that $f = IC_P(l)$ crosses the 45 degree line at most once from above, when we represent l along the horizontal axis and f along the vertical axis.

Lemma B.2 *There exists at most one $l \in [0, \bar{L}]$ such that $IC_P(l) = l$. Moreover, at such l, $\frac{d}{dl} IC_P(l) < 1$.*

Finally, we prove the following result about threshold $\overline{IC}_P(l)$.[41]

[41] Recall that, at $f = IC_P(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader currently (given that she will protect the leader from next period on), keeping the leader's technology level fixed. By contrast, at $f = \overline{IC}_P(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader currently (given that she will protect the leader from next period on), even if (i) the current protection makes the leader invest and (ii) the current non-protection makes the leader not invest. This "even if" part makes protection less attractive, and hence threshold $\overline{IC}_P(l)$ requires that the follower is stronger compared to threshold $IC_P(l)$.

Lemma B.3 *For each $(l, f) \in \mathcal{L}$, we have*

$$\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l+1, f, 1) \geq \pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)$$

if and only if $f \geq \overline{IC}_P(l)$. Moreover, for each $l \in [0, \bar{L}]$, $\overline{IC}_P(l) > IC_P(l)$.

Small Step Size

As stated in the main text, we analyze the model under the assumptions that hold if Assumption 1 (in the main text) holds and the step size is small. To formally state the result when the step size is small, we consider the following limit of the step size getting smaller. Suppose we consider the model where the step size is Δ and the discount factor is $\delta = e^{-r\Delta}$. We assume that the cost of investment is proportional to Δ: to increase l to $l + \Delta$, the cost is $c(l) \cdot \Delta$. Given the model where the step size is Δ, we can create another model with step size 1 with the same strategic incentive: simply re-define the marginal cost of production at technology level t in the new model as the marginal cost at technology level Δt in the model with step size Δ and re-define the cost of investment from technology level l to $l + 1$ in the new model as the cost of investment from the technology level Δl to $\Delta l + \Delta$ in the original model. Given this equivalence between the models, we will show that our assumptions hold in the model with a small step size Δ.

In particular, in the model with step size Δ, the threshold $IC_{P,\Delta}(l)$ is the solution for

$$\frac{\pi^P(l, x, 1)}{1 - e^{-r\Delta}} = \pi^P(l, x, 0) + e^{-r\Delta}\frac{\pi^P(l, x+\Delta, 1)}{1 - e^{-r\Delta}}.$$

As $\Delta \to 0$, we have $IC_{P,\Delta}(l) \to IC_{P,0}(l)$, where $IC_{P,0}(l)$ is the solution for

$$r\left(\pi^P(l, x, 1) - \pi^P(l, x, 0)\right) = \pi^P_f(l, x, 1). \tag{22}$$

Similarly, we have $\overline{IC}_{P,\Delta}(l) \to \overline{IC}_{P,0}(l)$, where $\overline{IC}_{P,0}$ is the solution for

$$r\left(\pi^P(l, x, 1) - \pi^P(l, x, 0)\right) = -\pi^P_l(l, x, 1) + \pi^P_f(l, x, 1). \tag{23}$$

For the leader, $\bar{L}_\Delta$ is the smallest l such that, for each f,

$$c(l)\Delta > \pi^M(l+\Delta, f) - \pi^L(l, f).$$

We have $\bar{L}_\Delta \to \bar{L}$, where $\bar{L}$ is the solution for

$$c(l) = \max_{f \leq l}\left(\pi^M(l, f) + \pi^M_l(l, f) - \pi^L(l, f)\right).$$

For a small $\varepsilon > 0$, we restrict attention to $(l, f) \in [0, \bar{L} + \varepsilon] \times [0, \bar{L} + \varepsilon]$ since, given Δ, we restrict attention to $(l, f) \in [0, \bar{L}_\Delta] \times [0, \bar{L}_\Delta]$.

In addition, $IC_{M,\Delta}(l)$ is the solution for

$$\pi^M(l+\Delta, f) - c(l)\Delta = \pi^M(l, f).$$

We have $IC_{M,\Delta}(l) \to IC_{M,0}(l)$, where $IC_{M,0}(l)$ is the solution for $\pi_l^M(l,x) = c(l)$. Similarly, $IC_{D,\Delta}(l) \to IC_{D,0}(l)$, where $IC_{D,0}(l)$ is the solution for $\pi_l^L(l,x) = c(l)$.

Assumptions

We analyze the model under assumptions A1–A8 listed below. We will show that these assumptions are implied by Assumption 1 (in the main text) if $\alpha > 0$ is small and $\Delta > 0$ is small:

Lemma B.4 *Suppose Assumption 1 (in the main text) holds. Then, for sufficiently small* $\alpha > 0$*, conditions A1–A4 stated below hold.*

Lemma B.5 *For each* $\varepsilon > 0$*, for sufficiently small* $\alpha > 0$ *and* $\Delta > 0$*, in the model with step size* Δ*, for each* $(l, f) \in \left[0, \bar{L}+\varepsilon\right] \times \left[0, \bar{L}+\varepsilon\right]$*, conditions A5–A8 stated below hold.*

Thus, for the rest of the appendix, we focus on proving Proposition 1 under the following conditions A1–A8.

A1. For each $(l, f) \in \mathcal{L}$,

$$\pi_l^P(l, f, 1) < 0. \tag{24}$$

(That is, a stronger leader reduces the policymaker's utility.)

A2. For each $(l, f) \in \mathcal{L}$, we have

$$\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1) < 0 \tag{25}$$

and

$$(1-\delta)\pi_l^P(l, f, 0) + \delta\pi_l^P(l, f+1, 1) < 0. \tag{26}$$

(The former means that a stronger leader reduces the policymaker's utility, and the latter means that the cost of a stronger leader with protection is higher than the benefit of a stronger leader without protection under proper discounting.)

A3. For each Markov perfect equilibrium and for each $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ with $f \geq IC_P(l)$,

$$\frac{\pi^P(l, f, 1)}{1-\delta} > V^P(l+1, f). \tag{27}$$

(That is, the policymaker prefers the leader not investing.)

There are two observations that will be useful later in the proof. First, condition (27) implies

$$\mathcal{IC}_P^*(l) \subseteq \{f \in \left[0, \bar{L}\right] \cap \mathbb{Z}_+ : f \geq IC_P(l)\}. \tag{28}$$

Second, we have

$$\begin{aligned}\pi^P(l, f, 1) &\geq \max\left\{\pi^P(l+1, f, 0), \delta\pi^P(l+1, \min\{f+1, l+1\}, 0)\right\} \\ &\geq \max\left\{\pi^P(l, f, 0), \delta\pi^P(l, \min\{f+1, l\}, 0)\right\}, \qquad (29)\end{aligned}$$

that is, protection is better than non-protection if the leader does not move. To see why (29) follows from (27), note that the policymaker can guarantee $V^P(l+1, f) \geq \sum_{t=1}^{\infty}\delta^{t-1}\pi^P(l_t, f_t, 0)$ where $(l_1, f_1) = (l+1, f)$ by not protecting the leader forever. Since $\pi^P(l_t, f_t, 0) = \alpha SW^D(l_t, f_t)$ is increasing in both l_t and f_t, we have

$$\sum_{t=1}^{\infty}\delta^{t-1}\pi^P(l_t, f_t, 0) \geq \max\left\{\pi^P(l+1, f, 0), \delta\pi^P(l+1, \min\{f+1, l+1\}, 0)\right\}.$$

A4. For each $l \in [0, \bar{L}]$,

$$\pi^P(l, l-1, 0) + \delta\frac{\pi^P(l, l, 0)}{1-\delta} < \sum_{\tilde{l}=l}^{\infty}\delta^{\tilde{l}-l}\pi^P\left(\min\left\{\tilde{l}, \bar{L}\right\}, l-1, 1\right). \qquad (30)$$

(That is, suppose the current state is $(l, l-1)$. If this is the last chance to protect the leader since otherwise the state will be absorbed in (l, l), then it is better to protect even though the leader will keep investing after protection.)

A5. For each $(l, f) \in \mathcal{L}^*$,

$$\pi^L(l+1, f) - c(l) - \pi^M(l, f) < 0. \qquad (31)$$

(That is, if investment leads to competition while non-investment leads to monopoly, the leader does not invest.)

In the model with step size Δ, (31) is equivalent to

$$\pi^L(l+\Delta, f) - c(l)\Delta - \pi^M(l, f) < 0, \qquad (32)$$

A6. For each f, if $\overline{IC}_P^{-1}(f) \geq f+1$, then for each (l, f) with $l \leq \overline{IC}_P^{-1}(f)$, we have

$$T_{l,f} \leq IC_P^{-1}(f) - 1 - \overline{IC}_P^{-1}(f). \qquad (33)$$

(That is, suppose (l, f) is above $\overline{IC}_P$ curve. If the policymaker has a choice between (i) non-protection today and then going to the steady state $(l, f+1)$ tomorrow and (ii) protection until (l', f) hits IC_P curve, then she prefers choice (i).)

In the model with step size Δ, (33) is equivalent to: For f satisfying $\overline{IC}_{P,\Delta}^{-1}(f) \geq$

$f + \Delta$ and $T\Delta \geq IC^{-1}_{P,\Delta}(f) - l$, we have

$$\left(1 - e^{-r\Delta}\right)\pi\,(l,\ f, 0) + e^{-r\Delta}\pi\,(l,\ f + \Delta, 1) \quad \geq \quad \left(1 - e^{-r\Delta}\right)\sum_{k=0}^{T-1} e^{-rk\Delta}\pi\,(l + k\Delta,\ f, 1) \qquad - e^{-rT\Delta}\pi\,(l + T\Delta,\ f, 1) \tag{34}$$

A7. For each $t > 0$, either (i)

$$\frac{\pi^P(l, l - t - 1, 1)}{1 - \delta} - \left(\pi^P(l, l - t - 1, 0) + \delta\frac{\pi^P(l + 1, l - t, 1)}{1 - \delta}\right) \geq 0 \tag{35}$$

for all $l \in \left[t, \bar{L}\right]$, or (ii) there exists $\tilde{l} \in [t, \bar{L}]$ such that

$$\begin{aligned}\frac{\pi^P(l, l - t - 1, 1)}{1 - \delta} - \left(\pi^P(l, l - t - 1, 0) + \delta\frac{\pi^P(l + 1, l - t, 1)}{1 - \delta}\right) &\leq 0\ \forall l \in \left[t, \tilde{l}\right], \\ \frac{\pi^P(l, l - t - 1, 1)}{1 - \delta} - \left(\pi^P(l, l - t - 1, 0) + \delta\frac{\pi^P(l + 1, l - t, 1)}{1 - \delta}\right) &\geq 0,\ \forall l \in \left[\tilde{l}, \bar{L}\right].\end{aligned} \tag{36}$$

(That is, the marginal gain of staying at $(l, l - t - 1)$ versus receiving no protection at $(l, l - t - 1)$ today and going to the steady state $(l + 1, l - t)$ tomorrow satisfies the single crossing condition.)

In the model with step size Δ, (35)–(36) are equivalent to: For each t, either (i)

$$\frac{\pi^P(l, l - t - \Delta, 1)}{1 - e^{-r\Delta}} - \left(\pi^P(l, l - t - \Delta, 0) + \frac{\pi^P(l + \Delta, l - t, 1)}{1 - e^{-r\Delta}}\right) > 0\ \forall l \in \left[t, \bar{L}_\Delta\right], \tag{37}$$

or (ii) there exists $\tilde{l}$ such that

$$\begin{aligned}\frac{\pi^P(l, l - t - \Delta, 1)}{1 - e^{-r\Delta}} - \left(\pi^P(l, l - t - \Delta, 0) + \frac{\pi^P(l + \Delta, l - t, 1)}{1 - e^{-r\Delta}}\right) &\leq 0\ \forall l \in \left[t, \tilde{l}\right], \\ \frac{\pi^P(l, l - t - \Delta, 1)}{1 - e^{-r\Delta}} - \left(\pi^P(l, l - t - \Delta, 0) + \frac{\pi^P(l + \Delta, l - t, 1)}{1 - e^{-r\Delta}}\right) &\geq 0\ \forall l \in \left[\tilde{l}, \bar{L}_\Delta\right].\end{aligned} \tag{38}$$

A8. Either $IC_P^{-1}(0) \geq 1$ or there exists f such that

$$\begin{array}{l}\text{(i) } f \text{ is the smallest } f' \text{ with } f' \geq IC_P\left(f' + 1\right) \text{ and} \\ \text{(ii) for all } f' \geq f, \text{ we have } f \geq IC_P(f + 1).\end{array} \tag{39}$$

In the model with step size Δ, (39) is equivalent to: either $IC_P^{-1}(0) \geq \Delta$ or there exists f such that

$$\begin{array}{l}\text{(i) } f \text{ is the smallest } f' \text{ with } f' \geq IC_P\left(f'+\Delta\right) \text{ and} \\ \text{(ii) for all } f' \geq f \text{, we have } f \geq IC_P\left(f+\Delta\right).\end{array} \tag{40}$$

Cutoffs

We define the cutoff $(l_1, l_1) \in \mathcal{L}^*$ as the intersection of the 45 degree line with the curve

$$\left\{(l,\, f) \in \mathcal{L}^* : f = IC_P\,(l)\right\}.$$

If the intersection does not exist, then define $l_1 = \emptyset$. We next define $(l_2, 0)$ as the intersection of the l-axis and $\{(l,\, f) \in \mathcal{L}^* : f = IC_P\,(l)\}$. If the intersection does not exist, then define $l_2 = \emptyset$.

Given Lemmas B.1 and B.2, exactly one of the following two conditions is satisfied: $l_1 = \emptyset$ or $l_2 = \emptyset$. If $l_1 \neq \emptyset$, then define f_3 as the smallest integer $f \in \mathbb{Z}_+$ such that, for each $f \geq f_3$, we have $f \geq IC_P\,(f+1)$. By (39), for each $f < f_3$,

$$f < IC_P\,(f+1)\,. \tag{41}$$

If $l_2 \neq \emptyset$, define $f_3 = 0$.

Let l_4 be the solution for $IC_P\,(x) = IC_M\,(x)$. Again, if the solution does not exist in $\left[0, \bar{L}\right]$, define $l_4 = \emptyset$. There is at most one solution since $\frac{d}{dl} IC_P\,(l) \geq 0$ and $\frac{d}{dl} IC_M\,(l) \leq 0$ by Lemmas 1 and B.1.

If $l_4 \neq \emptyset$, then there exists $(l_5,\, f_5) \in \mathcal{L}^*$ such that

$$IC_M\,(l_5) \leq f_5 \leq IC_P\,(l_5)\,. \tag{42}$$

Moreover, if $\frac{d}{dl} IC_P\,(l) \leq 1$ for all l, then we can take $(l_5,\, f_5) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ such that additionally we have

$$f_5 - 1 \leq IC_P\,(l_5 - 1)\,. \tag{43}$$

Fix such $(l_5,\, f_5)$.[42] We define a function $L^*\,(f)$ as

$$L^*\,(f) = \max\left\{IC_P^{-1}\,(f) + 1, l_5 - (f_5 - f)\right\}. \tag{44}$$

If $l_4 = \emptyset$, then we proceed as follows. Suppose further that $l_1 \neq \emptyset$. Let l_6 be the solution for $IC_P\,(x) = IC_D\,(x)$. Again, if the solution does not exist in $\left[0, \bar{L}\right]$, define $l_6 = \emptyset$. In addition, let (l_7, l_7) be the intersection of the 45 degree line with the curve

$$\left\{(l,\, f) \in \mathcal{L}^* : f = IC_D\,(l)\right\}.$$

[42] Technically speaking, due to the integer problem, such $(l_5,\, f_5) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ may not exist, although $(l_5,\, f_5) \in \mathcal{L}^*$ always does. However, for a sufficiently small step size, we can find $(l_5,\, f_5) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ that satisfy the conditions.

Next, let l_8 be the solution for

$$\pi^M (x+1, x) - c(x) = \pi^L (x, x).$$

Note that the leader invests (does not invest) at the ex ante state (l, l) if $l \leq l_8$ (or $l \geq l_8$, respectively) if it knows it will be protected at the interim state $(l+1, l)$. In addition, let l_9 be the solution for

$$\frac{\pi^P (x, x-1, 1)}{1-\delta} = \pi^P (x, x-1, 0) + \delta \frac{\pi^P (x+1, x, 1)}{1-\delta}.$$

By (35) and (36), we have

$$\frac{\pi^P (l, l-1, 1)}{1-\delta} \geq \pi^P (l, f, 0) + \delta \frac{\pi^P (l+1, l, 1)}{1-\delta} \text{ for } l \geq l_9, \tag{45}$$

$$\frac{\pi^P (l, l-1, 1)}{1-\delta} \leq \pi^P (l, f, 0) + \delta \frac{\pi^P (l+1, l, 1)}{1-\delta} \text{ for } l \leq l_9. \tag{46}$$

If $l_9 = \emptyset$, then (45) holds for all l.

Suppose instead $l_2 \neq \emptyset$. Then, let l_{10} be the smallest l such that $IC_M(l) \geq 0$. This l_{10} is less than l_2 since otherwise there would exist l with $IC_P(l) = IC_M(l)$.

Finally, we define l^{***} as follows:

1. If $l_1 \neq \emptyset$ and $l_4 \neq \emptyset$, then $l^{***} = L^*(f_3)$.
2. If $l_1 \neq \emptyset$, $l_4 = \emptyset$, and $l_6 \neq \emptyset$, then $l^{***} = f_3 + 1$.
3. If $l_1 \neq \emptyset$, $l_4 = \emptyset$, and $l_6 = \emptyset$, then $l^{***} = l_7 + 1$.
4. If $l_2 \neq \emptyset$ and $l_4 \neq \emptyset$, then $l^{***} = L^*(f_3)$.
5. If $l_2 \neq \emptyset$ and $l_4 = \emptyset$, then $l^{***} = l_{10}$.

B.2 Equilibrium Concept and Uniqueness

We consider the subgame perfect equilibrium that satisfies the following form of *renegotiation proofness*: after each history h (this can be either at the timing of the leader's investment decision or at the timing of the policymaker's protection decision), if there are two equilibria that are Pareto ranked for the policymaker and the leader, then we pick the Pareto efficient one. We show that the outcome of SPE satisfying renegotiation proofness is unique:

Lemma B.6 *The set of subgame perfect equilibrium payoffs that satisfy renegotiation proofness is unique at each ex ante state $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ and also at each interim state $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$. In this renegotiation-proof subgame perfect equilibrium, the strategy is*

Markov: the leader's investment decision depends only on the ex ante state and the policymaker's protection decision depends only on the interim state. Moreover, at each ex ante state $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$*, if (no investment, protection) is incentive compatible, then (no investment, protection) is the equilibrium outcome.*

Given this result, in what follows, we refer to "equilibrium" as the unique renegotiation proof SPE. Let $\text{eqm}(l, f) \in \{I, NI\} \times \{P, NP\}$ be the equilibrium outcome given the ex ante state $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$.

B.3 Equilibrium

We state our main result as follows:

Proposition B.1 *There exists* $l^* \leq l^{***}$ *such that the leader's technology level at the steady state is* l^* *in equilibrium. In addition, the policymaker is protecting the leader in the steady state. Moreover, when* $l_2 = \emptyset$*, the steady state satisfies* $(l^*, 0)$ *with* $l^* \leq l^{***}$*.*

Note that since the state space is effectively finite, the steady state exists. Moreover, the leader is protected at the steady state, as otherwise the follower's state would move.[43] Thus, we will focus on proving that (i) $l^* \leq l^{***}$ in the steady state and (ii) if $l_2 = \emptyset$, then the steady state satisfies $(l^*, 0)$.

In what follows, we will prove this proposition. For simplicity, we assume that there is no $(l, f) \in \mathcal{L} \cap \mathbb{Z}_+^2$ such that $f = IC_P(l)$, $f = IC_M(l)$, or $f = IC_D(l)$. Without this assumption, all the proofs go through with more tedious tie-breaking analysis based on renegotiation proofness.

Classify the ex ante state $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ into the following three regions.

1. Region 1: $f \leq IC_P(l)$ and $f \geq IC_D(l)$. In this region, the leader does not invest (NI) and the policymaker does not protect (NP), except near the 45 degree line, where the leader may invest (I) or the policymaker may protect (P), as explained below.

2. Region 2: $f \geq IC_P(l)$ and $f \geq IC_M(l)$. In this region, the leader does not invest (NI) and the policymaker protects (P) whenever $l > f$.

3. Region 3: $f \leq IC_P(l)$ and $f \leq IC_D(l)$ or $f \geq IC_P(l)$ and $f \leq IC_M(l)$. The dynamics in this region are discussed below.

Region 2. We first prove that, in Region 2, $\text{eqm}(l, f) = (NI, P)$.

Lemma B.7 *For each ex ante state* $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ *with* $f \geq IC_P(l)$ *and* $f \geq IC_M(l)$*, then* $\text{eqm}(l, f) = (NI, P)$.

[43] Or $l = f$ in the steady state. However, in that case, the policymaker cannot protect the leader at the steady state. Thus, she would rather protect the leader in the interim state $(l, f - 1)$ given (30), unless $l = f = 0$ is the steady state (in that case, $l \leq l^{***}$ is trivially satisfied).

Region 1. We next prove that, in Region 1, $\text{eqm}(l, f) = (NI, NP)$, except near the 45-degree line.

Lemma B.8 *For each ex ante state* $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ *with* $f \leq IC_P(l)$ *and* $f \geq IC_D(l)$*, then*

1. *If* $f = l$*, then* $\text{eqm}(l, f) = (I, P)$ *if* $\text{eqm}(l+1, f) = (NI, P)$ *and* $\text{eqm}(l, f) = (NI, NP)$ *otherwise.*
2. *If* $f = l - 1$*, then* $\text{eqm}(l, f) = (NI, P)$ *if* $l \geq \min\{l_7, l_8\}$ *or* $\text{eqm}(l+1, f+1) \neq (NI, P)$ *and* $\text{eqm}(l, f) = (NI, NP)$ *if* $l \leq \min\{l_7, l_8\}$ *and* $\text{eqm}(l+1, f+1) = (NI, P)$.
3. *If* $f \leq l - 2$*, then* $\text{eqm}(l, f) = (NI, NP)$.

Region 3. For Region 3, we first show that, at ex ante state (l, f), if $f \geq IC_P(l)$ and the policymaker does not protect the leader at the interim state $(l+1, f)$, then the equilibrium outcome at (l, f) is (NI, P).

Lemma B.9 *For each* $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ *satisfying* $f \geq IC_P(l)$*, if the policymaker does not protect the leader at the interim state* $(l+1, f)$*, then* $\text{eqm}(l, f) = (NI, P)$.

Next, we show that, once the leader invests at the ex ante state (l, f) and the policymaker protects the leader at the interim state $(l+1, f)$, then protection will be offered for the rest of the game.

Lemma B.10 *For each* $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$*, suppose either the policymaker protects the leader at the interim state* $(l+1, f)$ *and with* $f \leq IC_M(l)$*, or* $\text{eqm}(l, f) = (I, P)$*. Then, there exists* $l' \geq l + 1$ *such that* $\text{eqm}\left(\tilde{l}, f\right) = (I, P)$ *for all* $l \leq \tilde{l} \leq l' - 1$ *and* $\text{eqm}\left(l', f\right) = (NI, P)$.

Further, at (l, f) with $f \leq \overline{IC}_P(l)$, if the leader does not invest once f increases to $f+1$, then at state (l, f), the leader either does not invest or it invests but the policymaker does not protect at the interim state (l, f).

Lemma B.11 *For each* $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$*, if* $\text{eqm}(l, f+1) \in \{(NI, P), (NI, NP)\}$ *and* $f \leq \overline{IC}_P(l)$*, then either* $\text{eqm}(l, f) \in \{(NI, P), (NI, NP)\}$ *or the policymaker does not protect the leader at the interim state* (l, f)*. Moreover, if* $f \leq IC_P(l)$*, then it cannot be the case that* $\text{eqm}(l, f) = \text{eqm}(l, f+1) = (NI, P)$.

The next lemma shows that, at the interim state $(f+1, f)$, the policymaker protects the leader if $f \geq f_3$.

Lemma B.12 *The policymaker protects the leader at the interim state* $(f+1, f)$ *for each* $f \geq f_3$.

The results in these lemmas lead to the following unravelling results:

Lemma B.13 *Suppose $l_4 \neq \emptyset$ (and hence (l_5, f_5) is well-defined). Let l_3 be the smallest $l \geq f_3 + 1$ with* eqm $(l, f_3) = (NI, P)$. *Such l_3 exists and $l_3 \leq L^*(f_3)$.*

Given Lemma B.13, we can show the following:

Lemma B.14 *When $l_1 \neq \emptyset$ ($IC_P(l)$ intersects 45-degree line), the steady state satisfies (l^*, f^*) with $l^* \leq l^{***}$.*

Lemma B.15 *When $l_2 \neq \emptyset$ ($IC_P(l)$ does not intersect 45 degree line), the steady state satisfies $(l^*, 0)$ with $l^* \leq l^{***}$.*

This concludes the proof of Proposition B.1 (and hence Proposition 1).

B.4 Proofs of Auxiliary Lemmas

B.4.1 Proof of Lemma B.1

Note that

$$\begin{aligned} & \frac{d}{df}\left(\frac{\pi^P(l, f, 1)}{1-\delta} - \pi^P(l, f, 0) - \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}\right) \\ = \; & \frac{d}{df}\left(-\left(\pi^P(l, f, 0) - \pi^P(l, f, 1)\right) - \delta\frac{\pi^P(l, f+1, 1) - \pi^P(l, f, 1)}{1-\delta}\right). \end{aligned}$$

Since $\frac{d}{df}\left(\pi^P(l, f, 0) - \pi^P(l, f, 1)\right) \leq 0$ by (25) and $\frac{d}{df}\left(\pi^P(l, f+1, 1) - \pi^P(l, f, 1)\right) \leq 0$ by (19), we have

$$\frac{d}{df}\left(\frac{\pi^P(l, f, 1)}{1-\delta} - \pi^P(l, f, 0) - \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}\right) \geq 0.$$

Together with the definition of $IC_P(l)$, we have $\frac{\pi^P(l, f, 1)}{1-\delta} \geq \pi^P(l, f, 0) + \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}$ if and only if $f \geq IC_P(l)$. Moreover, by the implicit function theorem, the slope of $IC_P(l)$ satisfies

$$\frac{d}{dl}IC_P(l) = \frac{(1-\delta)\left(\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1)\right) + \delta\left(\pi_l^P(l, f+1, 1) - \pi_l^P(l, f, 1)\right)}{-(1-\delta)\left(\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1)\right) - \delta\left(\pi_f^P(l, f+1, 1) - \pi_f^P(l, f, 1)\right)}. \tag{47}$$

Since $\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1) \geq 0$ by (14) and (24), $\pi_{lf}^P(l, f, 1) \geq 0$ by (18),

$$\frac{d}{df}\left(\pi^P(l, f, 0) - \pi^P(l, f, 1)\right) \leq 0$$

by (25), and $\pi_{ff}^P(l, f, 1) \leq 0$ by (19), (47) is non-negative.

B.4.2 Proof of Lemma B.2

It suffices to prove that, for each l satisfying $IC_P(l) = l$, we have $\frac{d}{dl} IC_P(l) < 1$. The numerator of the fraction in (47) equals

$$(1-\delta)\cdot\left(\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1)\right) + \delta\cdot\pi_{lf}^P(l, f, 1) \geq 0$$

and the denominator equals

$$-(1-\delta)\cdot\left(\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1)\right) - \delta\cdot\pi_{ff}^P(l, f, 1) \geq 0.$$

When $f = IC_P(l) = l$, we can write $\mu(l) = \mu(f) = c$ and $\mu'(l) = \mu'(f) = \gamma$. Then, the numerator minus the denominator equals

$$\begin{aligned}
& (1-\delta)\cdot\left(\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1)\right) + \delta\cdot\pi_{lf}^P(l, f, 1) \\
& + (1-\delta)\cdot\left(\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1)\right) + \delta\cdot\pi_{ff}^P(l, f, 1) \\
= \; & (1-\delta)\cdot\left[-\frac{\alpha}{9b}(4a-4c) + \alpha\frac{3}{4b}(a-c) + (1-\alpha)\rho\frac{1}{18b}(a-c)\right]\gamma \\
& + (1-\delta)\cdot\left[-\alpha\left(2\frac{a-c}{9b} + 4\frac{a-c}{9b} + \frac{2}{18b}(2a-2c)\right)(1-\alpha)\rho 2\frac{a-c}{9b}\right]\gamma \\
& + \delta\cdot\alpha\cdot\frac{7}{9b}\cdot\gamma^2 - \delta\cdot(1-\alpha)\cdot\rho\cdot\frac{2}{9b}\gamma^2.
\end{aligned} \tag{48}$$

We would like to show that this is less than zero. Given $f = IC_P(l) = l$, we have

$$(1-\delta)\cdot\left(\pi^P(l, l, 1) - \pi^P(l, l, 0)\right) = \delta\cdot\pi_f^P(l, l, 1).$$

Solving this equality for l and substituting it to (48), we obtain that (48) equals

$$-\frac{2(1-\alpha)\rho\gamma^2\delta}{9b} < 0,$$

as desired.

B.4.3 Proof of Lemma B.3

Note that

$$\begin{aligned}
& \frac{d}{df}\left(\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l+1, f, 1) - \pi^P(l, f, 0) - \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)\right) \\
= \; & \frac{d}{df}\left(\pi^P(l, f, 1) - \pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l+1, f, 1)\right. \\
& \left. -\frac{\delta}{1-\delta}\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l, f, 1) - \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)\right).
\end{aligned} \tag{49}$$

Recall that

$$\pi_f^P(l, f, 1) - \pi_f^P(l, f, 0) \geq 0, \pi_{lf}^P(l, f, 1) \geq 0, \pi_{ff}^P(l, f, 1) \leq 0$$

by (25), (18), and (19). Hence, the sign of (49) is positive. Thus, we have

$$\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l+1, f, 1) \geq \pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)$$

if and only if $f \geq \overline{IC}_P(l)$. Moreover, at $f = IC_P(l)$, we have

$$\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l, f, 1) = \pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l, f+1, 1).$$

Since $\pi_l^P(l, f, 1) \leq 0$, we have

$$\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l+1, f, 1) < \pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l, f+1, 1),$$

which implies $\overline{IC}_P(l) > IC_P(l)$.

B.4.4 Proof of Lemma B.4

Since we focus on $(l, f) \in [0, \bar{L}]^2$ and all values are continuous in α, it suffices to show that given $\alpha = 0$, for each (l, f), the conditions (24)–(27) assumed in A1–A3 hold. First, with $\alpha = 0$, Assumption 1 (in the main text) implies

$$\begin{aligned} 0 &> \left(\pi^M(l+1, f) - \pi^M(l, f)\right) - \left(\pi^L(l+1, f) - \pi^L(l, f)\right) \\ &= \hat{\pi}^M(l+1) - \hat{\pi}^M(l) - \rho\left(\hat{\pi}^M(l+1) - \pi^L(l+1, f)\right) + \rho\left(\hat{\pi}^M(l) - \pi^L(l, f)\right) \\ &\quad - \left(\pi^L(l+1, f) - \pi^L(l, f)\right) \\ &= \frac{1-\rho}{\rho}\left(\pi^P(l+1, f, 1) - \pi^P(l, f, 1)\right). \end{aligned}$$

Hence, (24) and (26) hold.

Second, with $\alpha = 0$, we have $\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1) = -\pi_f^P(l, f, 1) < 0$ by (16). Hence, (25) holds.

Third, for (27), we will show that, for each $(l, f) \in \mathcal{L}$ satisfying $f \geq IC_P(l)$, we have $\frac{\pi^P(l, f, 1)}{1-\delta} > V^P(l+1, f)$. Fix any path of $\{l_t, f_t, i_t, a_t\}_{t=1}^{\infty}$ such that $l_1 + i_1 \geq l+1$, $f_1 = f$, $l_{t+1} = l_t + i_t$ for each t, and $f_{t+1} = \max\{f_t + (1-a_t), l_t + i_t\}$ for each t. We will show that

$$\frac{\pi^P(l, f, 1)}{1-\delta} > \sum_{t=1}^{\infty}\delta^{t-1}\pi^P(l_t + i_t, f_t, a_t). \tag{50}$$

Define t_τ as the period in which the policymaker takes $a_t = 1$ for the τ^{th} time (with the convention that $t_0 = -1$ and $f_{t_0} = f$). Then,

$$\begin{aligned} V^P(l+1, f) &= \sum_{\tau=1}^{\infty} \delta^{t_\tau - 1} \pi^P\left(l_{t_\tau} + i_{t_\tau}, f_{t_\tau}, 1\right) \text{ since } \alpha = 0 \\ &< \sum_{\tau=1}^{\infty} \delta^{t_\tau - 1} \pi^P\left(l, f_{t_\tau}, 1\right) \text{ by (24).} \end{aligned}$$

Thus, defining $\tilde{V}^P(l, f) = \sum_{\tau=1}^{\infty} \delta^{t_\tau - 1} \pi^P\left(l, f_{t_\tau}, 1\right)$, it suffices to show that $\frac{\pi^P(l, f, 1)}{1-\delta} \geq \tilde{V}^P(l, f)$.

Since (i) f_t increases if and only if $a_t = 0$ and (ii) $t_\tau - t_{\tau-1} - 1$ is the number of periods with $a_t = 0$ between $t_{\tau-1}$ and t_τ, we can write

$$\tilde{V}^P(l, f) = \pi^P(l, f, 1) + \sum_{\tau=2}^{\infty} \delta^{t_\tau - 1} \pi^P\left(l, f_{t_{\tau-1}} + t_\tau - t_{\tau-1} - 1, 1\right). \tag{51}$$

Since $f \geq IC_P(l)$, we have

$$\begin{aligned} \delta^{t_\tau - 1} \pi^P\left(l, f_{t_{\tau-1}} + t_\tau - t_{\tau-1} - 1, 1\right) &= \delta^{t_{\tau-1} - 1} \times \delta^{t_\tau - t_{\tau-1}} \pi^P\left(l, f_{t_{\tau-1}} + t_\tau - t_{\tau-1} - 1, 1\right) \\ &\leq \delta^{t_{\tau-1} - 1} \times \delta \pi^P\left(l, f_{t_{\tau-1}}, 1\right) \end{aligned}$$

and hence

$$\begin{aligned} \tilde{V}^P(l, f) &\leq \pi^P(l, f, 1) + \delta \sum_{\tau=2}^{\infty} \delta^{t_{\tau-1} - 1} \pi^P\left(l, f_{t_{\tau-1}}, 1\right) \\ &= \pi^P(l, f, 1) + \delta \sum_{\tau=1}^{\infty} \delta^{t_\tau - 1} \pi^P\left(l, f_{t_\tau}, 1\right) \\ &= \pi^P(l, f, 1) + \delta \tilde{V}^P(l, f). \end{aligned}$$

Thus,

$$\frac{\pi^P(l, f, 1)}{1-\delta} \geq \tilde{V}^P(l, f),$$

as desired.

Finally, (30) holds with $\alpha = 0$ since the left hand side is zero while the right hand side is strictly positive.

B.4.5 Proof of Lemma B.5

First, (32) is equivalent to

$$\frac{\pi^L(l+\Delta, f) - \pi^L(l, f)}{\Delta} - c(l) - \frac{\pi^M(l, f) - \pi^L(l, f)}{\Delta} < 0.$$

Note that the first term converges to $\pi_l^L(l, f)$ while the last term diverges to ∞ as $\Delta \to 0$. Since $\left[0, \bar{L} + \varepsilon\right]$ is compact, we have (32) for sufficiently small Δ.

Second, for (34), first note that (24) implies $\pi_l^P \leq 0$. Thus, (22) and (23) imply $\overline{IC}_{P,0}(l) > IC_{P,0}(l)$ for each l, or equivalently, $\overline{IC}_{P,0}^{-1}(f) > IC_{P,0}^{-1}(f)$ for each f.

Now, for each f, take $l = \overline{IC}_{P,\Delta}^{-1}(f)$ and $T\Delta \geq IC_{P,\Delta}^{-1}(f) - l$. By (24), for each $T' \leq T$, we have

$$\begin{aligned}
& \left(1 - e^{-r\Delta}\right)\pi(l, f, 0) + \delta\pi(l, f + \Delta, 1) \\
& - (1-\delta)\sum_{k=0}^{T-1}\delta^{k\Delta}\pi(l + k\Delta, f, 1) - \delta^{T\Delta}\pi(l + T\Delta, f, 1) \\
\geq \;& (1-\delta)(\pi(l, f, 0) - \pi(l, f, 1)) + (1-\delta)\sum_{k=1}^{T'-1}\pi(l, f + \Delta, 1) + \delta^{T'\Delta}\pi(l, f + \Delta, 1) \\
& - (1-\delta)\sum_{k=1}^{T'-1}\delta^{k\Delta}\pi(l + k\Delta, f, 1) - \delta^{T'\Delta}\pi\left(l + T'\Delta, f, 1\right) \\
\geq \;& (1-\delta)(\pi(l, f, 0) - \pi(l, f, 1)) + \delta^{T'\Delta}\left(\pi\left(l + T'\Delta, f, 1\right) - \pi\left(l + T'\Delta, f, 1\right)\right).
\end{aligned}$$

Fix $T'\Delta = IC_{P,\Delta}^{-1}(f) - \overline{IC}_{P,\Delta}^{-1}(f)$. For sufficiently small Δ, the above expression is strictly positive since

$$\lim_{\Delta\to 0}(1-\delta)(\pi(l, f, 0) - \pi(l, f, 1)) = \lim_{\Delta\to 0}\left(1 - e^{-r\Delta}\right)(\pi(l, f, 0) - \pi(l, f, 1)) = 0$$

and

$$\begin{aligned}
& \lim_{\Delta\to 0}\delta^{T'\Delta}\left(\pi(l, f + \Delta, 1) - \pi\left(l + T'\Delta, f, 1\right)\right) \\
= \;& \lim_{\Delta\to 0} e^{-r\left(IC_{P,0}^{-1}(f) - \overline{IC}_{P,0}^{-1}(f)\right)}\left(\pi(l, f, 1) - \pi\left(l + IC_{P,0}^{-1}(f) - \overline{IC}_{P,0}^{-1}(f), f, 1\right)\right) > 0
\end{aligned}$$

by (24). Thus, (34) holds for sufficiently small Δ.

For (37), define

$$F(l, t, \alpha, \Delta) := \frac{\pi^P(l, l - t - \Delta, 1)}{1 - e^{-r\Delta}} - \left(\pi^P(l, l - t - \Delta, 0) + e^{-r\Delta}\frac{\pi^P(l + \Delta, l - t, 1)}{1 - e^{-r\Delta}}\right).$$

This function is quadratic and convex in l for sufficiently small α and Δ since

$$\lim_{\Delta\to 0, \alpha\to 0} F_{ll}(l, t, \alpha, \Delta) = \frac{5\gamma_1^2\rho}{18b}.$$

We have

$$\lim_{\Delta\to 0,\alpha\to 0} F_l(l,t,\alpha,\Delta) = \frac{\gamma_1\left(5ar - 5r\gamma_0 - (5 - 5lr + 4rt)\gamma_1\right)\rho}{18br}.$$

Let $l^d = \frac{-5ar+5r\gamma_0+5\gamma_1+rt\gamma_1}{5r\gamma_1}$ be the solution for

$$\lim_{\Delta\to 0,\alpha\to 0} F_l(l,t,\alpha,\Delta) = 0.$$

If this $l^d \leq t$, then for all $l \geq t$, $\lim_{\Delta\to 0,\alpha\to 0} F(l,t,\alpha,\Delta)$ is increasing in l. If this $l^d \geq t$, then it means that $r \leq \frac{5\gamma_1}{5a-5\gamma_0+t\gamma_1}$. We have

$$\frac{d}{dr}F(t+\Delta,t,\alpha,\Delta) = \frac{\gamma_1\left(5a - 5\gamma_0 + t\gamma_1\right)\rho}{18br^2} > 0$$

and

$$F(t+\Delta,t,\alpha,\Delta)\big|_{r=\frac{5\gamma_1}{5a-5\gamma_0+t\gamma_1}} = -\frac{\left(25\left(a-\gamma_0\right)^2 + 10t\left(a-\gamma_0\right) + 37t^2\gamma_1^2\right)\rho}{180b} < 0.$$

Thus, for each r with $l^d \geq t$, $F(t+\Delta,t,\alpha,\Delta) < 0$. Hence, there is at most one $\tilde{l}$ such that $\lim_{\Delta\to 0,\alpha\to 0} F\left(\tilde{l},t,\alpha,\Delta\right) = 0$ and, for all $l \geq \tilde{l}$, $\lim_{\Delta\to 0,\alpha\to 0} F\left(\tilde{l},t,\alpha,\Delta\right) \geq 0$ and for all $t \leq l \leq \tilde{l}$, $\lim_{\Delta\to 0,\alpha\to 0} F\left(\tilde{l},t,\alpha,\Delta\right) \leq 0$.

In total, either (37) or (38) holds for sufficiently small α and Δ.

Finally, (40) holds for sufficiently small Δ. To see why, by the same proof as Lemma B.2, we have $\frac{d}{dl}IC_{P,0}(l) < 1$ for l such that $IC_P(l) = l$. Thus, the single crossing holds.

B.4.6 Proof of Lemma B.6

For sufficiently large l such that $\pi^M(l+1,f) - \pi^L(l,f) < c(l)$ for each f, the leader never invests. Hence, the policymaker is the single decision maker and the result holds. Fix (l,f). Suppose the result holds for each ex ante state (l',f') with $(l',f') \geq (l,f)$ and $(l',f') \neq (l,f)$ and each interim state (l'',f'') such that there is a feasible transition path from (l',f') to (l'',f''). Let $\mathcal{V}^P(l,f)$ be the set of SPE payoffs at the ex ante state (l,f) for the policymaker.

1. After the leader invests, that is, at the interim state $(l+1,f)$, $V^P(l+1,f+1)$ and $V^P(l+1,f)$ are determined by the inductive hypothesis. Hence, the continuation payoff for the policymaker is unique. Thus, the policymaker takes the strategy that gives a higher payoff. In case of a tie, since the leader prefers protection, we break the tie for protection.

2. After the leader does not invest, that is, at the interim state (l, f), the policymaker's payoff without protection is determined by $\pi^P(l, f, 0) + \delta V^P(l, f+1)$. With protection, it will be $\pi^P(l, f, 1) + \delta \upsilon$ for some $\upsilon \in \mathcal{V}^P(l, f)$. If $\min_{\upsilon \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta \upsilon \geq \pi^P(l, f, 0) + \delta V^P(l, f+1)$ or $\max_{\upsilon \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta \upsilon \leq \pi^P(l, f, 0) + \delta V^P(l, f+1)$, then by breaking the tie for protection, the optimal strategy for the policymaker is unique and her continuation payoff is determined. Hence, we assume that

$$\min_{\upsilon \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta \upsilon < \pi^P(l, f, 0) + \delta V^P(l, f+1) < \max_{\upsilon \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta \upsilon.$$

   For υ satisfying $\pi^P(l, f, 1) + \delta \upsilon > \pi^P(l, f, 0) + \delta V^P(l, f+1)$, the policymaker protects the leader. Hence, $\upsilon^* \in \arg\max_{\upsilon \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta \upsilon$ should be attained by protection. Given this property of the policymaker's value and incentives, it suffices to show that the equilibrium path that achieves $\max_{\upsilon \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta \upsilon$ also maximizes the leader's payoff. This holds since the leader prefers the protection. Thus, the renegotiation-proof strategy for the policymaker is unique and her continuation payoff is determined.

In total, at the interim state that is reachable from (l, f), the renegotiation proof SPE is unique. Thus, at the ex ante state (l, f), the leader's investment strategy is also determined. Moreover, multiplicity may only come from whether or not the policymaker protects the leader after no investment, and we select protection. Thus, by the one-shot deviation principle, if (no investment, protection) is a subgame perfect equilibrium (SPE) outcome, then we select that for the renegotiation proof SPE.

B.4.7 Proof of Lemma B.7

As seen in the proof of Lemma B.6, it suffices to show that $\text{eqm}(l, f) = (NI, P)$ is an equilibrium outcome. Given this conjecture of $\text{eqm}(l, f)$ and the specification of the continuation payoff, with protection, the policymaker obtains $\frac{1}{1-\delta} \pi^P(l, f, 1)$ while without protection, she obtains $\pi^P(l, f, 0) + \frac{\delta}{1-\delta} \pi^P(l, f+1, 1)$. By (20), it is better to protect. Therefore, the policymaker protects the leader if it does not invest. Therefore, the leader is at least guaranteed $\pi^M(l, f)$ without investment. Since $f \geq IC_M(l)$, even if investment will lead to protection, it will not invest. Hence, $\text{eqm}(l, f) = (NI, P)$ is an equilibrium outcome.

B.4.8 Proof of Lemma B.8

It will be useful to note that

$$\text{if } (l, f) \text{ is in region 1, then so is } (l+1, f) \tag{52}$$

since $f \leq IC_P(l) \Rightarrow f \leq IC_P(l+1)$ and $f \geq IC_D(l) \Rightarrow f \geq IC_D(l+1)$.

We prove the statement of the lemma by induction with respect to l. The statement clearly holds for $l = \bar{L}$. Suppose the statement holds for $l + 1$.

We first show that, at the ex ante state (l, f) with $f \leq l - 1$ in region 1, after the leader invests, that is, at interim state $(l + 1, f)$, the policymaker does not protect the leader. With protection, the policymaker obtains

$$\pi^P (l + 1, f, 1) + \delta \pi^P (l + 1, f, 0) + \delta^2 V^P (l + 1, f + 1)$$

since the leader will not invest and the policymaker will not protect it in the next period (this follows from (i) (52), (ii) $f \leq l - 1 \Rightarrow f \leq (l + 1) - 1$, and (iii) the inductive hypothesis). By contrast, without protection, she obtains the payoff at least

$$\pi^P (l + 1, f, 0) + \delta \pi^P (l + 1, f + 1, 1) + \delta^2 V^P (l + 1, f + 1)$$

(by protecting the leader in the next period) since the leader will not invest at the ex ante state $(l + 1, f + 1)$. This follows from (i) $f \geq IC_D (l) \Rightarrow f + 1 \geq IC_D (l + 1) \geq IC_M (l + 1)$ and (ii) either (a) $f+1 \geq IC_P (l + 1)$, and then Lemma B.7 implies eqm $(l + 1, f + 1) = (NI, P)$ or (b) $f + 1 < IC_P (l + 1)$, then the inductive hypothesis implies that the leader will not invest at the ex ante state $(l + 1, f + 1)$). Since $f < IC_P (l + 1)$, we have

$$(1 - \delta) \pi^P (l + 1, f, 0) + \delta \pi^P (l + 1, f + 1, 1) > \pi^P (l + 1, f, 1)$$

and hence it is better not to protect at the interim state $(l + 1, f)$.

We next show that, at the ex ante state (l, f) with $f \leq l-1$ in Region 1, the leader does not invest. This follows since, by the above argument, the policymaker does not protect the leader after investment. Thus, by definition of $IC_D (l)$, the leader does not invest at the ex ante state (l, f).

We then show that, at the ex ante state (l, f) with $f \leq l - 2$ in Region 1, we have eqm $(l, f) = (NI, NP)$. By the above argument, it suffices to show that it is optimal for the policymaker not to protect the leader after no investment. Suppose otherwise: protection is the optimal action at the interim state (l, f). Then, the policymaker's equilibrium value at the ex ante state (l, f) is $\frac{1}{1-\delta} \pi^P (l, f, 1)$ since the leader does not invest. By contrast, without protection, given the specification of the continuation play, she obtains the payoff at least $\pi^P (l, f, 0) + \delta \frac{\pi^P (l, f+1,1)}{1-\delta}$ by protecting the leader from the next period. By the definition of $IC_P (l)$, no protection is better, as desired.

It remains to consider the ex ante states (l, l) and $(l, l - 1)$ in Region 1. We first show that, for each l, if $l \geq l_8$ or eqm $(l + 1, l) = (NI, NP)$, then eqm $(l, l) = (NI, NP)$. To see why, first, note that, if $l \geq l_8$, then the firm with the investment opportunity does not invest at the ex ante state (l, l). In addition, eqm $(l + 1, l) = (NI, NP)$ also implies that the firm with the investment opportunity does not invest at the ex ante state (l, l) since eqm $(l + 1, l) = (NI, NP)$ implies that at the interim state $(l + 1, l)$, the policymaker does not protect. In total, if $l \geq l_8$ or eqm $(l + 1, l) = (NI, NP)$, then at the ex ante state (l, l), we have eqm $(l, l) = (NI, NP)$ and this state is absorbing.

We next show that, for each l such that $l \geq l_8$ or eqm $(l+1, l) = (NI, NP)$, we have eqm $(l, l-1) = (NI, P)$ if $(l, l-1)$ is in Region 1. By Lemma B.6, it suffices to show that no players have incentives to deviate. Given that we have $l - 1 \geq IC_D(l) \geq IC_M(l)$ in Region 1, the leader does not have an incentive to invest (even if it is protected after investment). Since eqm $(l, l) = (NI, NP)$ is absorbing after no protection, the policymaker protects the leader at interim state $(l, l-1)$ by (30).

We then show, for each l, if $l \leq l_8$, $l \geq l_9$, and eqm $(l+1, l) = (NI, P)$, then we have eqm $(l, l-1) = (NI, P)$ if $(l, l-1)$ is in Region 1. To this end, by Lemma B.6, it suffices to prove that no players have incentives to deviate from eqm $(l, l-1) = (NI, P)$. In turn, it suffices to show that the policymaker protects the leader at interim state $(l, l-1)$ since we have $f \geq IC_D(l) \geq IC_M(l)$ in Region 1, and hence the leader does not invest at ex ante state $(l, l-1)$ (even if it is protected after investment). Therefore, it remains to show that the policymaker protects the leader at interim state $(l, l-1)$. At the interim state $(l, l-1)$, without protection, the next ex ante state is (l, l). Since eqm $(l+1, l) = (NI, P)$ (and hence protection is offered at the interim state $(l+1, l)$) and $l \leq l_8$, the firm with an investment opportunity will invest at the ex ante state (l, l) and the next interim state is $(l+1, l)$. Then, the state stays at $(l+1, l)$. Thus, the policymaker obtains $\pi^P(l, l-1, 0) + \delta \frac{\pi^P(l+1, l, 1)}{1-\delta}$. With protection, the policymaker obtains $\frac{\pi^P(l, l-1, 1)}{1-\delta}$. By (45), it is optimal to protect the leader at the interim state $(l, l-1)$ given $l \geq l_9$.

We next show that, if $l \leq l_8$, $l \leq l_9$, and eqm $(l+1, l) = (NI, P)$, then eqm $(l, l-1) = (NI, NP)$ if $(l, l-1)$ is in Region 1. To this end, it suffices to prove that the policymaker does not protect the leader at interim state $(l, l-1)$. To see why this is sufficient, note that there is no protection at the interim state $(l+1, l-1)$ from (52), the inductive hypothesis and since we have $l - 1 \geq IC_D(l)$ given that $(l, l-1)$ is in Region 1. Thus, the leader does not invest at the ex ante state $(l, l-1)$.

By the same argument as above, no protection at the interim state $(l, l-1)$ gives payoff $\pi^P(l, f, 0) + \delta \frac{\pi^P(l+1, l, 1)}{1-\delta}$ while protection gives payoff $\frac{\pi^P(l, l-1, 1)}{1-\delta}$. By (46), it is optimal not to protect the leader at the interim state $(l, l-1)$.

B.4.9 Proof of Lemma B.9

Since the leader will lose protection after investment, the leader does not have an incentive to deviate. In addition, since $f \geq IC_P(l)$ implies $f \in \mathcal{IC}_P^*(l)$, the policymaker does not have an incentive to deviate. Thus, by Lemma B.6, the equilibrium outcome is (NI, P).

B.4.10 Proof of Lemma B.10

Suppose first that the policymaker protects the leader at the interim state $(l+1, f)$ and $f \leq IC_M(l)$. Then, we have eqm $(l, f) = (I, P)$ since the leader invests regardless of the policymaker's strategy at the interim state (l, f). Thus, it suffices to prove that, if eqm $(l, f) = (I, P)$, then, there exists $l' \geq l+1$ such that eqm $\left(\tilde{l}, f\right) = (I, P)$ for all $l \leq \tilde{l} \leq l' - 1$ and eqm $\left(l', f\right) = (NI, P)$.

At the ex ante state $(l+1,\ f)$, no investment leads to protection since the policymaker protects the leader at the interim state $(l+1,\ f)$ given eqm $(l,\ f) = (I, P)$. This means that if the leader invests at the ex ante state $(l,\ f)$, then it needs to be protected after investment. Thus, the equilibrium outcome is either eqm $(l+1,\ f) = (NI, P)$ or eqm $(l+1,\ f) = (I, P)$. Moreover, the latter implies that the policymaker protects the leader at the interim state $(l+2,\ f)$ given (31).

Recursively, we have eqm $(l,\ f) = \cdots =$ eqm $\left(l',\ f\right) = (I, P)$ and eqm $\left(l',\ f\right) = (NI, P)$ for some $l' \geq l+1$.

B.4.11 Proof of Lemma B.11

Suppose eqm $(l,\ f) = (I, P)$. Suppose the leader deviates and does not invest. Then, protection gives the policymaker a payoff of no more than $\pi^P(l,\ f, 1) + \frac{\delta}{1-\delta}\pi^P(l+1,\ f, 1)$ (by Lemma B.10, once eqm $(l,\ f) = (I, P)$, the state transits to $(l,\ f)$, $(l+1,\ f)$, ...) while non-protection gives the policymaker a payoff of no less than $\pi^P(l,\ f, 0)+\frac{\delta}{1-\delta}\pi^P(l,\ f+1, 1)$. By the definition of $\overline{IC}_P(l)$, if $f \leq \overline{IC}_P(l)$, then the latter is optimal.

Suppose eqm $(l,\ f) = (I, NP)$. Since the leader invests even though it loses protection after investment, the policymaker must not offer protection at the interim state $(l,\ f)$ given (31). Finally, if $f \leq IC_P(l)$, then $\pi^P(l,\ f, 1) < \pi^P(l,\ f, 0) + \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}$. Thus, eqm $(l,\ f+1) =$ eqm $(l,\ f) = (NI, P)$ cannot be the case.

B.4.12 Proof of Lemma B.12

Suppose there exists $\bar{f} \geq f_3$ such that the policymaker does not protect the leader at the interim state $\left(\bar{f}+1,\ \bar{f}\right)$. We will show that it implies that the policymaker does not protect the leader at the interim state $\left(\bar{f}+2,\ \bar{f}+1\right)$. This will lead to a contradiction since recursively, this implies that, for each $f \geq \bar{f}$, the policymaker does not protect the leader at the interim state $(f+1,\ f)$. However, for $f = \bar{L}-1$, since $\bar{L}-1 \geq IC_P\left(\bar{L}\right)$ and the leader does not invest, the policymaker protects the leader.

We now prove that, if the policymaker does not protect the leader in the interim state $\left(\bar{f}+1,\ \bar{f}\right)$ for $\bar{f} \geq f_3$, then she does not protect it in the interim state $\left(\bar{f}+2,\ \bar{f}+1\right)$.

Suppose otherwise: the policymaker does not protect the leader in the interim state $\left(\bar{f}+1,\ \bar{f}\right)$ for $\bar{f} \geq f_3$ but she protects it in the interim state $\left(\bar{f}+2,\ \bar{f}+1\right)$. On the one hand, at interim state $\left(\bar{f}+1,\ \bar{f}\right)$, by protecting the leader at the interim state $\left(\bar{f}+2,\ \bar{f}\right)$, protection gives the payoff no less than

$$\pi^P\left(\bar{f}+1,\ \bar{f}, 1\right) + \delta\pi^P\left(\bar{f}+2,\ \bar{f}, 1\right) + \delta^2 V^P\left(\bar{f}+2,\ \bar{f}\right).$$

Here, we assume that the leader invests at the ex ante state $\left(\bar{f}+1,\ \bar{f}\right)$, which gives us a lower bound of the policymaker's payoff given $\bar{f} \geq IC_P\left(\bar{f}+1\right)$, (24), and (27).

By contrast, no protection gives the payoff of

$$\pi^P\left(\bar{f}+1,\ \bar{f}, 0\right) + \delta\pi^P\left(\bar{f}+2,\ \bar{f}+1, 1\right) + \delta^2 V^P\left(\bar{f}+2,\ \bar{f}+1\right).$$

Here, we assume that the leader invests at the ex ante state $\left(\bar{f}+1,\ \bar{f}+1\right)$, since otherwise protection would be infeasible forever and it would be clearly suboptimal not to protect the leader at interim state $\left(\bar{f}+1,\ \bar{f}\right)$ by (30).

Since no protection is optimal at interim state $\left(\bar{f}+1,\ \bar{f}\right)$, we have

$$\begin{aligned} & \pi^{P}\left(\bar{f}+1,\ \bar{f},1\right)+\delta\pi^{P}\left(\bar{f}+2,\ \bar{f},1\right)+\delta^{2}V^{P}\left(\bar{f}+2,\ \bar{f}\right) \\ < \ & \pi^{P}\left(\bar{f}+1,\ \bar{f},0\right)+\delta\pi^{P}\left(\bar{f}+2,\ \bar{f}+1,1\right)+\delta^{2}V^{P}\left(\bar{f}+2,\ \bar{f}+1\right). \end{aligned} \tag{53}$$

On the other hand, at interim state $\left(\bar{f}+2,\ \bar{f}\right)$, protection gives a payoff of no less than

$$\pi^{P}\left(\bar{f}+2,\ \bar{f},1\right)+\delta V^{P}\left(\bar{f}+2,\ \bar{f}\right)$$

while non-protection gives a payoff of

$$\pi^{P}\left(\bar{f}+2,\ \bar{f}+1,0\right)+\delta V^{P}\left(\bar{f}+2,\ \bar{f}+1\right).$$

Since protection is optimal at interim state $\left(\bar{f}+2,\ \bar{f}\right)$ (by Lemma B.9, otherwise, $\bar{f}\geq IC_{P}\left(\bar{f}+1\right)$ implies $\text{eqm}\left(\bar{f}+1,\ \bar{f}\right)=(NI,\ P)$ and hence we would have protection at interim state $\left(\bar{f}+1,\ \bar{f}\right)$), it follows that

$$\delta\pi^{P}\left(\bar{f}+2,\ \bar{f},1\right)+\delta^{2}V^{P}\left(\bar{f}+2,\ \bar{f}\right)>\delta\pi^{P}\left(\bar{f}+2,\ \bar{f},0\right)+\delta^{2}V^{P}\left(\bar{f}+2,\ \bar{f}+1\right). \tag{54}$$

Given (53) and (54),

$$\pi^{P}\left(\bar{f}+1,\ \bar{f},1\right)+\delta\pi^{P}\left(\bar{f}+2,\ \bar{f},0\right)<\pi^{P}\left(\bar{f}+1,\ \bar{f},0\right)+\delta\pi^{P}\left(\bar{f}+2,\ \bar{f}+1,1\right).$$

Since $\bar{f}\geq IC_{P}\left(\bar{f}+1\right)$, we have

$$\pi^{P}\left(\bar{f}+1,\ \bar{f},1\right)\geq(1-\delta)\,\pi^{P}\left(\bar{f}+1,\ \bar{f},0\right)+\delta\pi^{P}\left(\bar{f}+1,\ \bar{f}+1,1\right).$$

Thus,

$$\delta\left(\pi^{P}\left(\bar{f}+2,\ \bar{f},0\right)-\pi^{P}\left(\bar{f}+1,\ \bar{f},0\right)\right)<\delta\left(\pi^{P}\left(\bar{f}+2,\ \bar{f}+1,1\right)-\pi^{P}\left(\bar{f}+1,\ \bar{f}+1,1\right)\right).$$

This is a contradiction since the left hand side is positive while the right hand side is negative.

B.4.13 Proof of Lemma B.13

We first prove the following claims:

Claim B.1 *For each* $(l,\ f)\in\mathcal{L}^{*}\cap\mathbb{Z}_{+}^{2}$ *such that* $\text{eqm}\,(l,\ f+1)=(NI,\ P)$ *and* $f\leq\overline{IC}_{P}\,(l)$*, there exists* $\min\left\{IC_{P}^{-1}\left(f\right),l-1\right\}\leq l'\leq\min\left\{\max\left\{IC_{P}^{-1}\left(f\right),l-1\right\},l\right\}$ *such that* $\text{eqm}\left(l',\ f\right)=(NI,\ P)$.

Proof. (i) Since $f \leq \overline{IC}_P(l)$, Lemma B.11 implies that, at (l, f), we have either (i) $\text{eqm}(l, f) \in \{(I, P), (I, NP)\}$ and the policymaker does not protect at the interim state (l, f), (ii) $\text{eqm}(l, f) = (NI, NP)$, or (iii) $\text{eqm}(l, f) = (NI, P)$. Moreover, (iii) implies $f \geq IC_P(l)$ since $\text{eqm}(l, f + 1) = (NI, P)$.

Suppose (i) or (ii) is the case. In particular, the policymaker does not protect the leader at the interim state (l, f). Thus, at the ex ante state $(l - 1, f)$, for the leader to invest, the policymaker has to not protect at the interim state $(l - 1, f)$ given (31). Hence, at the ex ante state $(l - 1, f)$, we have either (i) $\text{eqm}(l - 1, f) \in \{(I, P), (I, NP)\}$ and the policymaker does not protect at the interim state $(l - 1, f)$, (ii) $\text{eqm}(l - 1, f) = (NI, NP)$, or (iii) $\text{eqm}(l - 1, f) = (NI, P)$.

Recursively, if (i) or (ii) is the case for the ex ante state $(l - k, f)$, then at the ex ante state $(l - k - 1, f)$, we have either (i) $\text{eqm}(l - k - 1, f) \in \{(I, P), (I, NP)\}$ and the policymaker does not protect at the interim state $(l - k - 1, f)$, (ii) the equilibrium $\text{eqm}(l - k - 1, f) = (NI, NP)$, or (iii) $\text{eqm}(l - k - 1, f) = (NI, P)$. By Lemma B.9, if $f \geq IC_P(l - k - 1)$, then (i) or (ii) at the ex ante state $(l - k, f)$ implies $\text{eqm}(l - k - 1, f) = (NI, P)$.

In total, there exists $k \geq 0$ such that $\text{eqm}(l - k, f) = (NI, P)$ and either [$k = 0$ and $f \geq IC_P(l)$] or [$k \geq 1$ and k is no more than the smallest $k' \geq 1$ with $f \leq IC_P(l - k')$]. Thus, $l' = l \leq IC_P^{-1}(f)$ if $k = 0$ and $\min\left\{IC_P^{-1}(f), l - 1\right\} \leq l' \leq l - 1$ if $k \geq 1$. ■

Claim B.2 *Suppose* $\text{eqm}(l, f + 1) = (NI, P)$ *and* $f \geq \overline{IC}_P(l)$. *Then, there exists* $l' \leq IC_P^{-1}(f)$ *such that* $\text{eqm}(l', f) = (NI, P)$.

Proof. By Lemma B.6, if no player has incentives to deviate from $\text{eqm}(l, f) = (NI, P)$, then it is an equilibrium. If it is an equilibrium, then the statement holds with $l' = l$. Otherwise, since $f \geq \overline{IC}_P(l)$ implies $f \geq IC_P(l)$, the policymaker does not have an incentive to deviate from $\text{eqm}(l, f) = (NI, P)$ by (50). Thus, the leader must have an incentive to deviate, which requires the policymaker to protect the leader at the interim state $(l + 1, f)$ and hence $\text{eqm}(l, f) = (I, P)$. By Lemma B.10, the ex ante state will transit through $(l, f), (l + 1, f), ...,$ until it stops at some $(\hat{l}, f)$: $\text{eqm}(\hat{l}, f) = (NI, P)$. Since $\text{eqm}(l, f + 1) = (NI, P)$, the definition (21) implies $\hat{l} \leq l + T_{l,f}$. Moreover, (33) and $f \geq \overline{IC}_P(l)$—that is, $l \leq \overline{IC}_P^{-1}(f)$—implies $\hat{l} \leq IC_P^{-1}(f)$. ■

From these two claims, suppose $(l_0^*, f_0^*) = (l_5, f_5)$. Recall that $\text{eqm}(l_0^*, f_0^*) = (NI, P)$ and $f_0^* - 1 \leq \overline{IC}_P(l_0^*)$. We can recursively define $(l_n^*, f_n^*)_{n=1}^{f_3 - f_5}$ with $f_n^* = f_0^* - n$ such that l_n^* is the smallest l with $\text{eqm}(l, f) = (NI, P)$. For each n, the two claims above imply the following:

1. If $f_{n+1}^* \leq IC_P(l_n^* - 1)$, then this implies $f_{n+1}^* \leq \overline{IC}_P(l_n^* - 1)$ by Lemma B.3, Claim B.1 implies that (l_{n+1}^*, f_{n+1}^*) satisfies $l_{n+1}^* \leq \max\left\{IC_P^{-1}(f_{n+1}^*), l_n^* - 1\right\} = l_n^* - 1$ (the last equality follows from $f_{n+1}^* \leq IC_P(l_n^* - 1) \Leftrightarrow IC_P^{-1}(f_{n+1}^*) \leq l_n^* - 1$).

2. Otherwise—that is, only if $f_{n+1}^* \geq IC_P\left(l_n^* - 1\right)$—

   (a) If $f_{n+1}^* \leq \overline{IC}_P\left(l_n^* - 1\right)$, then Claim B.1 implies that $\left(l_{n+1}^*, f_{n+1}^*\right)$ satisfies $l_{n+1}^* = l_n^*$ or $l_{n+1}^* = l_n^* - 1$.

   (b) Otherwise—that is, only if $f_{n+1} \geq \overline{IC}_P\left(l_n^* - 1\right)$, Claim B.2 implies that $\left(l_{n+1}^*, f_{n+1}^*\right)$ satisfies $l_{n+1}^* \leq IC_P^{-1}\left(f_{n+1}^*\right)$.

In total, $\left(l_{n+1}^*, f_{n+1}^*\right)$ satisfies $l_{n+1}^* \leq l_n^* - 1$ and $f_{n+1}^* = f_n^* - 1$ until f_{n+1}^* becomes larger than $IC_P\left(l_n^* - 1\right)$. When f_{n+1}^* is larger than $IC_P\left(l_n^* - 1\right)$, either $\left(l_{n+1}^*, f_{n+1}^*\right) = \left(l_n^* - 1, f_n^* - 1\right)$ or $\left(l_{n+1}^*, f_{n+1}^*\right) = \left(l_n^*, f_n^* - 1\right)$ until f_{n+1}^* becomes larger than $\overline{IC}_P\left(l_n^* - 1\right)$. When f_{n+1}^* is larger than $\overline{IC}_P\left(l_n^* - 1\right)$, $\left(l_{n+1}^*, f_{n+1}^*\right)$ satisfies $l_{n+1}^* \leq IC_P^{-1}\left(f_{n+1}^*\right)$. Therefore, we have

$$l_n^* \leq \max\left\{ IC_P^{-1}\left(f_n^*\right) + 1, \quad \underbrace{l_5 - \left(f_5 - f_n^*\right)}_{\substack{\text{as } f \text{ decreases by one from } f_5, \\ l \text{ decreases at least by one from } l_5}} \right\}.$$

Given (44), we have $l_n^* \leq L^*(f)$.

B.4.14 Proof of Lemmas B.14

Consider the following three cases:

1. At $(f_3 + 1, f_3)$, we have $IC_M(f_3 + 1) \geq f_3$. In this case, by Lemma B.13, eqm $(l_3, f_3) = (NI, P)$ with $l_3 \leq L^*(f_3)$. Define $(l_{11}, f_{11}) = (l_3, f_3)$. By Lemma B.12,

$$\text{eqm}(f_3, f_3) = (I, P). \tag{55}$$

   Moreover, by definition of $L^*(f)$,

$$f_{11} \leq IC_M(l_{11}). \tag{56}$$

2. At $(f_3 + 1, f_3)$, $IC_D(f_3 + 1) \geq f_3 \geq IC_M(f_3 + 1)$. In this case, by Lemma B.7, eqm $(f_3 + 1, f_3) = (NI, P)$. This also implies

$$\text{eqm}(f_3, f_3) = (I, P). \tag{57}$$

   Define $(l_{11}, f_{11}) = (f_3 + 1, f_3)$.

3. At $(f_3 + 1, f_3)$, $f_3 \geq IC_D(f_3 + 1)$. In this case, by Lemma B.8, there exists (l_{11}, f_{11}) with $f_{11} = l_{11} - 1$ such that we have

$$f_{11} \leq IC_P(l_{11}), \tag{58}$$

$$\text{eqm}\left(f_{11},\ f_{11}\right) = (I, P)\,, \tag{59}$$

eqm $(l_{11},\ f_{11}) = (NI, P)$, and either (i) $IC_D(l) \geq f$ for each $(l,\ f) \leq (l_{11}-1,\ f_{11}-1)$ or (ii)(a) $(l_{11}-1,\ f_{11}-1)$ is in region 1, (ii)(b) eqm $(l_{11}-1,\ f_{11}-1) = (NI, NP)$, (ii)(c) we have

$$\frac{\pi^P\left(l_{11},\ f_{11}, 1\right)}{1-\delta} \leq \pi^P\left(l_{11},\ f_{11}, 0\right) + \delta\frac{\pi^P\left(l_{11}+1,\ f_{11}+l, 1\right)}{1-\delta}, \tag{60}$$

and (ii)(d) $IC_D(l) \geq f$ for each $(l,\ f) \leq (l_{11}-2,\ f_{11}-2)$.

To see why such $(l_{11},\ f_{11})$ exists, Lemma B.8 ensures that we have eqm $(l, l-1) = (NI, P)$ for every $l \geq l_8$ and also for at least every other l among $l \leq l_8$. Thus, taking $(l_{11},\ f_{11})$ close to the intersection of the 45 degree line with the IC_D-curve, that is, close to (l_7, l_7-1) (which is below the IC_P curve given the definition of f_3), we satisfy the condition.

Since $l_{11} \leq l^{***}$ for all the cases, it suffices to show that, at the steady state, $l^* \leq l_{11}$. Note that given $(l_{11},\ f_{11})$, we have eqm $(l_{11},\ f_{11}) = (NI, P)$,

$$f_{11} - 1 \leq IC_P\left(l_{11}\right), \tag{61}$$

$$\text{eqm}\left(f_{11},\ f_{11}\right) = (I, P)\,, \tag{62}$$

$$\text{either } f_{11} \leq IC_M\left(l_{11}\right) \text{ or } f_{11} = l_{11} - 1, \tag{63}$$

and either (i)

$$IC_D(l) \geq f \text{ for each } (l,\ f) \leq (l_{11}-1,\ f_{11}-1) \tag{64}$$

or (ii) (60). To see why (61) holds, note that either we have (58) or $f_{11} = f_3$—recall that f_3 is the smallest f such that there exists $l \geq f+1$ with $f \geq IC_P(l)$ given (41). Moreover, (63) holds since we have either (56) or $f_{11} = l_{11} - 1$.

We have eqm $(l_{11},\ f_{11}) = (NI, P)$ by definition. For each $l \leq l_{11}$, either eqm $(l, l) = (I, NP)$, (I, P), or (NI, NP) since (NI, P) is impossible by feasibility.

Let $\bar{l}$ be the smallest $\bar{l}$ with eqm $\left(\bar{l}, \bar{l}\right) \neq (I, NP)$. By (62), $\bar{l} \leq f_{11}$. If eqm $\left(\bar{l}, \bar{l}\right) = (NI, NP)$, then $\left(\bar{l}, \bar{l}\right)$ is the steady state. Thus, suppose eqm $\left(\bar{l}, \bar{l}\right) = (I, P)$. By Lemma B.10, the state transits from (0, 0) to $\left(\bar{l}, \bar{l}\right)$, and then $\left(\bar{l}+1, \bar{l}\right)$, ..., $\left(l', \bar{l}\right)$. It remains to show that

$$l' \leq l_{11}. \tag{65}$$

We first show the following claim. Let $\mathcal{IC}_P^{**}$ be the set of $(l,\ f) \in \mathcal{L}^*$ such that

$$\frac{\pi^P\left(l,\ f, 1\right)}{1-\delta} < \pi^P\left(l,\ f, 0\right) + \delta\frac{\pi^P\left(l+1,\ f+1, 1\right)}{1-\delta}.$$

Claim B.3 *For each $k \geq 0$, we have either* eqm $(l_{11}-k,\ f_{11}-k) = (NI, P)$ *or* $(l_{11}-k,\ f_{11}-k) \in \mathcal{IC}_P^{**}$.

Proof. If (60) holds for (l_{11}, f_{11}), then by (36), we have $(l_{11} - k, f_{11} - k) \in \mathcal{IC}_P^{**}$ for each $k \geq 0$. Thus, we focus on the case in which (60) does not hold. In this case, we have (64).

Given eqm $(l_{11}, f_{11}) = (NI, P)$, at the interim state $(l_{11}, f_{11} - 1)$, the policymaker does not protect the leader since (61) implies $f_{11} - 1 \leq IC_P(l_{11}) \leq \overline{IC}_P(l_{11})$ and hence Lemma B.11 implies that the policymaker does not protect the leader at the interim state $(l_{11}, f_{11} - 1)$.

Thus, at the ex ante state $(l_{11} - 1, f_{11} - 1)$, either eqm $(l_{11} - 1, f_{11} - 1) = (NI, P)$ or eqm $(l_{11} - 1, f_{11} - 1) = (I, NP)$ since eqm $(l_{11} - 1, f_{11} - 1) = (I, P)$ contradicts having no protection at the interim state $(l_{11}, f_{11} - 1)$ and eqm $(l_{11} - 1, f_{11} - 1) = (NI, NP)$ contradicts (64).

Moreover, if eqm $(l_{11} - 1, f_{11} - 1) = (I, NP)$, then $(l_{11} - 1, f_{11} - 1) \in \mathcal{IC}_P^{**}$. To see why, by Lemma B.6, if no players have incentives to deviate from eqm $(l_{11} - 1, f_{11} - 1) = (NI, P)$, then the equilibrium is eqm $(l_{11} - 1, f_{11} - 1) = (NI, P)$. Since there is no protection at the interim state $(l_{11}, f_{11} - 1)$, the leader does not have an incentive to deviate. Thus, the policymaker has to have a deviation gain:

$$\frac{\pi^P(l_{11} - 1, f_{11} - 1, 1)}{1 - \delta} < \pi^P(l_{11} - 1, f_{11} - 1, 0) + \delta \underbrace{\frac{\pi^P(l_{11}, f_{11}, 1)}{1 - \delta}}_{\substack{\text{at the ex ante state } (l_{11}-1, f_{11}), \\ \text{given eqm}(l_{11}, f_{11}) = (NI, P) \text{ and (63),} \\ \text{the leader invests.}}} .$$

For $f_{11} - 2$, if eqm $(l_{11} - 1, f_{11} - 1) = (NI, P)$, then by the same proof as above, we have either eqm $(l_{11} - 2, f_{11} - 2) = (NI, P)$ or $(l_{11} - 2, f_{11} - 2) \in \mathcal{IC}_P^{**}$. Otherwise, $(l_{11} - 1, f_{11} - 1) \in \mathcal{IC}_P^{**}$ and hence $(l_{11} - 2, f_{11} - 2) \in \mathcal{IC}_P^{**}$ by (36). Recursively, the statement holds for each $k \geq 0$. ■

We now finish the proof of (65). For $k = f_{11} - \bar{l}$, if eqm $(l_{11} - k, f_{11} - k) = (NI, P)$, then we have (65), as desired. Thus, suppose $(l_{11} - k, f_{11} - k) \in \mathcal{IC}_P^{**}$ for $k = f_{11} - \bar{l}$. We will show that (65) holds. In particular, we will show that $l' \leq l_{11} - k$.

Suppose otherwise: $l' > f_{11} - k$. Then, we have $V(l_{11} - k, f_{11} - k) < \frac{\pi^P(l_{11} - k, f_{11} - k, 1)}{1 - \delta}$ since, from the state $(l_{11} - k, f_{11} - k)$, l keeps increasing until l', and f does not increase. However, by not protecting the leader until the leader stops investing, the policymaker can obtain the payoff of

$$\begin{aligned}
&\pi^P(l_{11} - k, f_{11} - k, 0) + \delta \pi^P(l_{11} - k + 1, f_{11} - k + 1, 0) + \cdots \\
&+ \delta^{k'} \pi^P\left(l_{11} - k + k', f_{11} - k + k', 0\right) \\
&+ \frac{\delta^{k'+1} \pi^P\left(l_{11} - k + k' + 1, f_{11} - k + k' + 1, 1\right)}{1 - \delta},
\end{aligned}$$

Note that $\left(l_{11}-k+k',\ f_{11}-k+k'\right)\notin \mathcal{IC}_P^{**}$ implies eqm $\left(l_{11}-k+k',\ f_{11}-k+k'\right)=(NI,\ P)$ by Claim B.3. Thus, we have

$$\begin{aligned}
&\underbrace{\begin{array}{c}\pi^P\left(l_{11}-k,\ f_{11}-k,0\right)+\delta\pi^P\left(l_{11}-k+1,\ f_{11}-k+1,0\right)+\cdots\\ +\delta^{k'}\pi^P\left(l_{11}-k+k',\ f_{11}-k+k',0\right)\end{array}}_{(l,\,f)\in\mathcal{IC}_P^{**}}\\
&+\frac{\delta^{k'+1}\pi^P\left(l_{11}-k+k'+1,\ f_{11}-k+k'+1,1\right)}{1-\delta}\\
\geq\ &\pi^P\left(l_{11}-k,\ f_{11}-k,0\right)+\delta\pi^P\left(l_{11}-k+1,\ f_{11}-k+1,0\right)+\cdots\\
&+\delta^{k'-1}\pi^P\left(l_{11}-k+k'-1,\ f_{11}-k+k'-1,0\right)+\frac{\delta^{k'}\pi^P\left(l_{11}-k+k',\ f_{11}-k+k',1\right)}{1-\delta}\\
\geq\ &\cdots\\
\geq\ &\frac{\pi^P\left(l_{11}-k,\ f_{11}-k,1\right)}{1-\delta}>V\left(l_{11}-k,\ f_{11}-k\right).
\end{aligned}$$

Here, the first inequality follows from the definition of $\mathcal{IC}_P^{**}$:

$$\begin{aligned}
(l,\ f)\ \in\ &\mathcal{IC}_P^{**}\Rightarrow\pi^P\left(l_{11}-k+k',\ f_{11}-k+k',0\right)+\frac{\delta\pi^P\left(l_{11}-k+k'+1,\ f_{11}-k+k'+1,1\right)}{1-\delta}\\
\geq\ &\frac{\pi^P\left(l_{11}-k+k',\ f_{11}-k+k',1\right)}{1-\delta}.
\end{aligned}$$

This is a contradiction.

B.4.15 Proof of Lemmas B.15

By Lemma B.12, at the interim state $(1,0)$, the policymaker protects the leader and hence eqm $(0,0)=(I,\ P)$. Thus, the ex ante state transits through $(0,0)$, $(1,0)$, $(2,0)$, ..., $(l^*,0)$ where l^* is the smallest integer with eqm $(l^*,0)=(NI,\ P)$. By Lemma B.7 and B.11, we have $l^*\leq L^*(0)$ if $l_4\neq\emptyset$ and $l^*\leq l_{10}$ if $l_4=\emptyset$.

C Proofs from Section 4.1

This section presents the argument to prove Lemmas 3 and 4, and Proposition 2.

Thresholds and Assumptions

We define $IC_{EA}(l)$ as the solution for $\pi^M(l+1,x)-\pi^L(l,x)-c(l)=0$ and $x\geq 0$. By the same proof as Lemmas 1 and 2, for each $(l,\ f)\in\mathcal{L}$, we can show that

$$\pi^M(l+1,x)-\pi^L(l,x)-c(l)\geq 0$$

if and only if $f \leq IC_{EA}(l)$. Given the definition of $\bar{L}$ in (11), we have $IC_{EA}(l) \neq \emptyset$ if and only if $l \leq \bar{L}$. Relatedly, let IC^*_{EA} be the leader's technology level such that $\pi^M(l+1,l) - \pi^L(l,l) - c(l) = 0$ at $l = IC^*_{EA}$.

For a simple analysis, we assume

$$IC_P(l) \leq l - 2 \tag{66}$$

for all $0 \leq l \leq \bar{L}$. In addition, to avoid a tedious tie-breaking, we assume that there is no $(l, f) \in \mathcal{L} \cap \mathbb{Z}^2_+$ such that $f = IC_D(l)$.

We make the following assumptions, which we show below are implied by Assumption 1 (in the main text) if α is small and the step size is small.[44]

A1. For each $(l, f) \in \mathcal{L}$,

$$\pi^P_l(l, f, 1) < 0. \tag{67}$$

(That is, the stronger leader reduces the policymaker's utility given protection.)

A2. For each $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}^2_+$, we have

$$\begin{aligned} & \pi^P(l, f, 0) + \sum_{t=1}^{\kappa} \delta^t \pi^P(l+t, f+1, 1) \\ & + \min_{(l_\tau, f_\tau) \in \mathcal{L}^* \cap \mathbb{Z}^2_+ \text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f+1} \sum_{\tau=\kappa+1}^{\infty} \delta^\tau \pi^P(l_\tau, f_\tau, 0) \\ \geq \; & \pi^P(l, f, 1) + \max_{(l_t, f_t) \in \mathcal{L}^* \cap \mathbb{Z}^2_+ \text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} \delta^t \pi^P(l_t, f_t, 0). \end{aligned} \tag{68}$$

(That is, no matter what happens in the continuation play, if the leader has been protected for $\kappa - 1$ consecutive period and the follower is about to disappear, it is better not to protect the leader.)

In the model with the small step size, this condition is equivalent to

$$\begin{aligned} & \pi^P(l, f, 0) + \sum_{t=1}^{\kappa} e^{-rt\Delta} \pi^P(l+t\Delta, f+\Delta, 1) \\ & + \min_{(l_\tau, f_\tau) \in \mathcal{L}^*_\Delta \text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f+1} \sum_{\tau=\kappa+1}^{\infty} e^{-r\tau\Delta} \pi^P(l_\tau, f_\tau, 0) \\ \geq \; & \pi^P(l, f, 1) + \max_{(l_t, f_t) \in \mathcal{L}^*_\Delta \text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} e^{-r\tau\Delta} \pi^P(l_t, f_t, 0), \end{aligned} \tag{69}$$

where $\mathcal{L}^*_\Delta = \{(l, f) : \exists \left(\tilde{l}, \tilde{f}\right) \in \mathbb{Z}^2_+ \text{ such that } \tilde{l} \geq \tilde{f} \text{ and } l = \Delta\tilde{l} \text{ and } f = \Delta\tilde{f}\}$.

[44] See page B.1 for the mapping between the model with step size 1 and the one with a small step size.

A3. For each $(l, f) \in \mathcal{L}^*$,

$$\begin{cases} \pi^P(l+1, f, 1) > \pi^P(l, f, 0), \\ \pi^P(l, f-1, 1) > \pi^P(l, f, 0) \end{cases}. \tag{70}$$

(That is, protection with a slightly stronger leader brings a higher payoff than no protection with a slightly weaker leader; and protection with a slightly weaker follower brings a higher payoff than no protection with a slightly stronger follower.)

Lemma C.1 *Suppose Assumption 1 (in the main text) holds. Then, there exists $\bar{\Delta} > 0$ such that, for each $\Delta \leq \bar{\Delta}$, there exists $\bar{\alpha}$ such that for $\alpha < \bar{\alpha}$, conditions (67)–(70) hold.*

The proofs to this lemma and all auxiliary lemmas are provided in Section C.1.

Equilibrium steady state characterization

We use renegotiation proof subgame perfect equilibrium as our equilibrium concept. In equilibrium, we show that the steady state technology level is no less than IC^*_{EA}. Given the above lemma, this then implies Proposition 2 for sufficiently small step size since in the baseline model, we set $\alpha = 0$.[45] To prove this result, we first provide a counterpart of Lemma B.6.

Lemma C.2 *The unique renegotiation-proof subgame perfect equilibrium is Markov perfect. Moreover, we use renegotiation proofness only to break a tie in favor of the other player.*

Given this lemma, we write the policymaker's value function as $V^P(l, f, k)$ for each (l, f) and k. Here, $k \in \{0, ..., \kappa\}$ indicates how many consecutive periods the leader has been protected. $k = \kappa$ means that the follower has disappeared.

We next pin down the state transition for (l, f) with $f \geq IC_P(l)$ and $f > IC_D(l)$.

Lemma C.3 *For each (l, f, k) with $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}^2_+$ and $k \in \{0, ..., \kappa\}$,*

1. *If two firms have different technology levels at the ex ante state (l, f, k) (that is, $l > f$), the leader does not invest if $f \geq IC_P(l)$ and $f \geq IC_D(l)$.*
2. *If the ex ante state (l, f, k) is head-to-head competition (that is, $l = f$), then the firm with an investment opportunity invests if $l \leq IC^*_{EA}$.*
3. *If $k = \kappa - 1$ and either $l - 1 > f$ or $l \leq IC^*_{EA}$, then the policymaker does not protect the leader at the interim state (l, f, k).*
4. *If $f \geq IC_P(l)$, $f \geq IC_D(l)$, and $k < \kappa - 1$, at the interim state reached from the ex ante state (l, f, k) on the equilibrium path, the policymaker protects the leader whenever feasible.*

[45] If Δ is small, then $\delta = e^{-r\Delta}$ is large.

5. *The value* $V^P(l, f, k)$ *at the ex ante state* (l, f, k) *is decreasing in k and* $V^P(l, f, k) \leq \frac{\pi^P(l+1,l,1)}{1-\delta}$ *if* $f = l$ *and* $V^P(l, f, k) \leq \frac{\pi^P(l,f,1)}{1-\delta}$ *if* $f \leq l-1$.

Given Lemma C.3, to show that, in the long run, the leader's technology level is no less than $IC^*_{EA} - 1$, it suffices to show that the equilibrium path reaches a state (l, f, k) with $f \geq IC_P(l)$, $f \geq IC_D(l)$, and $k \leq \kappa - 1$.

We use the following auxiliary lemma, which will be useful for the final step of the proof.

Lemma C.4 *For each state* $(l, f, \kappa-1)$ *with* $(l, f) \in \mathcal{L}^* \cap \mathbb{Z}^2_+$ *and the leader's investment decision* $\iota \in \{0, 1\}$, *if the leader invests at the ex ante state* $(l+\iota, f+1, 0)$ *or* $l+\iota > f+1$, *then the policymaker does not protect the leader at the interim state* $(l+\iota, f, \kappa-1)$.

Finally, the following lemma concludes the proof that in the steady state, the leader's technology level is no less than IC^*_{EA}:

Lemma C.5 *The equilibrium path reaches a state* (l, f, k) *either with* $l \geq IC^*_{EA}$ *or with* $f \geq IC_P(l)$, $f \geq IC_D(l)$, *and* $k \leq \kappa - 1$.

C.1 Proofs of Auxiliary Lemmas

C.1.1 Proof of Lemma C.1

Since we focus on $(l, f) \in \left[0, \bar{L}\right]^2$ and all values are continuous in α (once we fix Δ), it suffices to show that, for sufficiently small Δ, given $\alpha = 0$, the assumptions (67)–(70) hold.

First, with $\alpha = 0$, Assumption 1 (in the main text) implies (67), as in the proof of Lemma B.4. Second, with $\alpha = 0$, (69) is equivalent to

$$\sum_{t=1}^{\kappa} e^{-rt\Delta} \pi^P(l + t\Delta, f + \Delta, 1) \geq \pi^P(l, f, 1).$$

Since the left hand side converges to $\kappa\pi^P(l, f, 1)$, for sufficiently small Δ, for $\alpha = 0$, (69).

Finally, (70) clearly holds with $\alpha = 0$ since $\pi^P(l, f, 0) = 0$.

C.1.2 Proof of Lemma C.2

For $l = \bar{L}$, the leader never invests. Hence, the policymaker is the only decision maker and the result holds. We now proceed inductively. Fix $l \leq \bar{L} - 1$ arbitrarily. Suppose the result holds for $l+1, ..., \bar{L}$. In particular, $V^P(l+1, f, k)$ is well-defined value function at the ex ante state $(l+1, f, k)$ for each f and k.

Fix (l, f, k) for arbitrarily f and k. We will show that at the interim state $(l+1, f, k)$ (that is, after the leader's investment), the policymaker's value is uniquely determined.

To see why, if $k = \kappa$, then the policymaker has no choice. If $k \leq \kappa - 1$, the policymaker protects the leader if and only if

$$\pi^P (l+1, f, 1) + \delta V^P (l+1, f, k+1) \geq \pi^P (l+1, f, 0) + \delta V^P (l+1, f+1, 0).$$

Note that all the values are determined. Hence, by letting the policymaker break her tie in favor for the leader, the result holds for l.

We will next show that at the interim state (l, f, k) (that is, after the leader's non-investment), the policymaker's value is uniquely determined. We proceed inductively with respect to f.

1. For each k, for $f = l$, the policymaker has no choice at the interim state (l, f, k). Moreover, from the discussion above, the policymaker's strategy at the interim state $(l+1, f, k)$ is uniquely determined. Thus, the leader's investment decision at the ex ante state (l, f, k) is uniquely pinned down. Thus, the policymaker's value $V^P (l, f, k)$ at the ex ante state (l, f, k) and also her value at the interim state (l, f, k) are uniquely determined.

2. For each $f \leq l - 1$, suppose that the policymaker's value $V^P (l, f+1, k)$ at the ex ante state $(l, f+1, k)$ is uniquely determined for each k. Then, we will show that $V^P (l, f, k)$ is uniquely determined. We proceed recursively with respect to k.

    (a) If $k = \kappa$, then the policymaker has no choice. Hence, the result holds.

    (b) For each $k \leq \kappa - 1$, suppose that the result holds for $k+1, ..., \kappa$. In particular, $V^P (l, f, k+1)$ is well-defined value function for each f.
    At the interim state (l, f, k), the policymaker protects the leader if and only if

    $$\pi^P (l, f, 1) + \delta V^P (l, f, k+1) \geq \pi^P (l, f, 0) + \delta V^P (l, f+1, 0).$$

    Note that all the values are determined. Hence, by letting the policymaker break her tie in favor for the leader, the policymaker's value $V^P (l, f, k)$ at the ex ante state (l, f, k) and also her value at the interim state (l, f, k) are uniquely determined.

Given that the policymaker's strategy is determined, the leader's optimal strategy is determined since the leader is myopic. We break this tie in favor for the policymaker.

C.1.3 Proof of Lemma C.3

As seen in Lemma C.2, the only source of multiplicity in the subgame perfect equilibrium is the tie breaking, and renegotiation proofness breaks the tie in favor for the other player. In the candidate strategy specified in the lemma, the policymaker breaks the tie in favor for the leader (namely, she protects unless $k = \kappa - 1$). Thus, it suffices to prove the optimality of each player's action and the strictness of the leader's incentive, assuming that the other player acts as prescribed in the statement of the lemma. For $l = \bar{L}$, the statement obviously holds. For each $l \leq \bar{L} - 1$, suppose all the statements 1–5 hold for $l + 1$. We now prove that statements 1–5 hold for l. This will conclude the proof by mathematical induction.

Proof of Statement 1. If $k < \kappa - 1$, then since the leader expects that the policymaker protects it if it does not invest, given $f \geq IC_D(l) \geq IC_M(l)$, the leader does not invest. If $k = \kappa - 1$, then since the leader expects that the policymaker does not protect it after its investment decision (since investment creates the interim state with $l - 1 > f$), given $f \geq IC_D(l)$, the leader does not invest.

Proof of Statement 2. Given the current state (l, l, k) with $k \leq \kappa - 1$, feasibility implies $k = 0$ since the current head-to-head competition implies that the policymaker did not protect the leader in the previous period (either f increased from the previous period or the previous state was (l, l) and protection was not feasible). Hence, without investment, the firm who receives an investment opportunity obtains the payoff of $\pi^L(l, l)$, while with investment, it obtains the payoff of $\pi^M(l + 1, l) - c(l)$ since we have assumed $l \geq IC_P(l + 1)$. Hence, it is optimal to invest if and only if $l \leq IC^*_{EA}$.

Proof of Statements 3. If $l = f$, then the policymaker has no choice. Thus, we focus on the case with $l \geq f + 1$.

With protection, the policymaker obtains a payoff of no more than

$$\pi^P(l, f, 1) + \max_{(l_t, f_t) \text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} \delta^t \pi^P(l_t, f_t, 0).$$

We next calculate the lower bound of the payoff without protection. In this case, the next ex ante state becomes $(l, f + 1)$. If $l - 1 > f$, then the next interim period state is either $(l, f + 1)$ or $(l + 1, f + 1)$. In both cases, protection is feasible. If $l - 1 \leq f$ but $l \leq IC^*_{EA}$, then the firm with an investment opportunity invests at the ex ante state $(l, f + 1)$ and the next period interim state is $(l + 1, f + 1)$. At the interim state $(l + 1, f + 1)$, protection is feasible. In total, by always protecting the leader until $k = \kappa$ from the next period, the policymaker can obtain a payoff of at least

$$\pi^P(l, f, 0) + \sum_{t=1}^{\kappa} \delta^t \pi^P(l + t, f + 1, 1) + \min_{(l_\tau, f_\tau) \text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f+1} \sum_{\tau=\kappa+1}^{\infty} \delta^\tau \pi^P(l_\tau, f_\tau, 0).$$

By (68), the policymaker does not protect the leader.

Proof of Statements 4 and 5. Suppose $l = f > IC_D(l)$. Again, feasibility implies $k = 0$ or $k = \kappa$. We first prove Statement 4 for $k = 0$. Protection is feasible only if the leader invests at the interim state $(l, l, 0)$ and in that case, the interim state will be $(l + 1, l)$. With protection at the interim state $(l + 1, l)$, the current payoff for the policymaker is $\pi^P(l + 1, l, 1)$. Then, since $\frac{d}{dl} IC_D \leq 0$ by Lemma 1, we have $l > IC_D(l) \geq IC_D(l + 1)$ and hence the leader will not invest at the ex ante state $(l + 1, l)$ and hence the next interim state will stay at $(l + 1, l)$. Thus, by protecting at the interim state $(l + 1, l)$, her total payoff is at least $\pi^P(l + 1, l, 1) + \delta \pi^P(l + 1, l, 0) + \delta^2 V^P(l + 1, l + 1, 0)$. By contrast, without protection at the interim state $(l + 1, l)$, the payoff is $\pi^P(l + 1, l, 0) +$

$\delta V^P(l+1,l+1,0)$.

The former is no less than the latter since

$$\begin{aligned} & \pi^P(l+1,l,1)+\delta\pi^P(l+1,l,0)+\delta^2 V^P(l+1,l+1,0) \\ & -\pi^P(l+1,l,0)-\delta V^P(l+1,l+1,0) \\ = \;& \pi^P(l+1,l,1)+\delta\pi^P(l+1,l,0)-\pi^P(l+1,l,0) \\ & -\delta(1-\delta)V^P(l+1,l+1,0). \end{aligned} \tag{71}$$

By the inductive hypothesis,

$$\delta(1-\delta)V^P(l+1,l+1,0)\leq\delta\pi^P(l+2,l+1,1)\leq\delta\pi^P(l+1,l+1,1).$$

Since $l+1\geq IC_P(l+1)$,

$$\delta\pi^P(l+1,l+1,1)\leq-(1-\delta)\pi^P(l+1,l,0)+\pi^P(l+1,l,1).$$

Thus, (71) is no less than 0, as desired.

We now turn into Statement 5. For $l=f$, again, only feasible k is $k=0$ or $k=\kappa$. We have $V^P(l,l,0)\geq V^P(l,l,\kappa)$. To see why, for $V^P(l,l,\kappa)$, given Assumption 1 of the main text, the remaining firm will not invest given $l\geq IC_D(l)$. Thus, the policymaker's payoff is $\frac{1}{1-\delta}SW^M(l)$. She can obtain this payoff at state $(l,l,0)$ by not protecting the leader forever.

Moreover, since the firm with an investment opportunity will invest at the ex ante state (l,l) if and only if $l\leq IC^*_{EA}$, we have

$$\begin{aligned} V^P(l,l,0) &= \begin{cases} \pi^P(l+1,l,1)+\delta V^P(l+1,l,1) & \text{if } l\leq IC^*_{EA} \\ \frac{\pi^P(l,l,0)}{1-\delta} & \text{if } l> IC^*_{EA} \end{cases} \\ &\leq \begin{cases} \pi^P(l+1,l,1)+\delta\frac{\pi^P(l+1,l,1)}{1-\delta} & \text{if } l\leq IC^*_{EA} \\ \frac{\pi^P(l,l,0)}{1-\delta} & \text{if } l> IC^*_{EA} \end{cases} \quad \text{(inductive hypothesis)} \\ &\leq \frac{\pi^P(l+1,l,1)}{1-\delta} \text{ by (70).} \end{aligned}$$

For $f=l-1\geq IC_D(l)$, suppose Statements 1-5 hold for each $f'\geq f+1$. The interim state that arises on equilibrium path is $(l,l-1)$ by Statement 1.

If the policymaker protects the leader at the interim state $(l,l-1)$, then the next ex ante state is $(l,l-1)$. Again, the leader invests at the ex ante state $(l,l-1)$ and hence the next interim state is $(l,l-1)$. Thus, by not protecting the leader in the next period, the policymaker obtains a payoff of at least

$$\begin{cases} \pi^P(l,l-1,1)+\delta\pi^P(l,l-1,0)+\delta^2 V^P(l,l,0) & \text{if } l\leq IC^*_{EA}, \\ \pi^P(l,l-1,1)+\delta\pi^P(l,l-1,0)+\delta^2\frac{\pi^P(l,l,0)}{1-\delta} & \text{otherwise.} \end{cases} \tag{72}$$

Without protection, the policymaker's payoff is

$$\begin{cases} \pi^P(l, l-1, 0) + \delta V^P(l, l, 0) & \text{if } l \le IC^*_{EA}, \\ \pi^P(l, l-1, 0) + \delta \frac{\pi^P(l,l,0)}{1-\delta} & \text{otherwise.} \end{cases} \tag{73}$$

Suppose $l \le IC^*_{EA}$. We first prove Statement 4. By the inductive hypothesis, we have

$$\begin{aligned} \delta^2 V^P(l, f+1, 0) - \delta V^P(l, f+1, 0) &\ge -\delta\pi^P(l+1, f+1, 1) \\ &\ge -\delta\pi^P(l, f+1, 1). \end{aligned}$$

Since $f \ge IC_P(l)$, we have

$$-\delta\pi^P(l, f+1, 1) \ge -\pi^P(l, f, 1) + (1-\delta)\pi^P(l, f, 0).$$

Hence, (72) minus (73) is no less than 0. Therefore, protection is optimal.

We next prove Statement 5. As before, it is clear that $V^P(l, f, k) \ge V^P(l, f, \kappa)$ for each $k \le \kappa - 1$. Moreover, by the equilibrium strategy,

$$V^P(l, f, k) = \sum_{t=0}^{\kappa-1-k} \delta^t \pi^P(l, f, 1) + \delta^{\kappa-k}\pi^P(l, f, 0) + \delta^{\kappa-k+1} V^P(l, f+1, 0).$$

Thus, for each $k \le \kappa - 2$,

$$\begin{aligned} V^P(l, f, k) - V^P(l, f, k+1) = {} & \delta^{\kappa-1-k}\pi^P(l, f, 1) + \delta^{\kappa-k}\pi^P(l, f, 0) \\ & + \delta^{\kappa-k+1} V^P(l, f+1, 0) \\ & - \delta^{\kappa-k-1}\pi^P(l, f, 0) - \delta^{\kappa-k} V^P(l, f+1, 0). \end{aligned}$$

By the inductive hypothesis,

$$\begin{aligned} \delta^{\kappa-k+1} V^P(l, f+1, 0) - \delta^{\kappa-k} V^P(l, f+1, 0) &\ge -\delta^{\kappa-k}\pi^P(l+1, f+1, 1) \quad (74) \\ &\ge -\delta^{\kappa-k}\pi^P(l, f+1, 1). \end{aligned}$$

Thus,

$$\begin{aligned} V^P(l, f, k) - V^P(l, f, k+1) \ge {} & \delta^{\kappa-1-k}\pi^P(l, f, 1) + \delta^{\kappa-k}\pi^P(l, f, 0) \\ & - \delta^{\kappa-k-1}\pi^P(l, f, 0) - \delta^{\kappa-k}\pi^P(l, f+1, 1). \end{aligned}$$

Since $f \ge IC_P(l)$, we have $\pi^P(l, f, 1) \ge (1-\delta)\pi^P(l, f, 0) + \delta\pi^P(l, f+1, 1)$. Therefore,

$$V^P(l, f, k) - V^P(l, f, k+1) \ge 0$$

and the value $V^P(l, f, k)$ is decreasing in k. Moreover, for each k,

$$\begin{aligned}
V^P(l, f, k) &\leq V^P(l, f, 0) \\
&= \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \pi^P(l, f, 0) + \delta^{\kappa+1} V^P(l, f+1, 0) \\
&\leq \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \pi^P(l, f, 0) + \delta^{\kappa+1} \frac{\pi^P(l+1, f+1, 1)}{1-\delta} \\
&\qquad \text{by the inductive hypothesis} \\
&\leq \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \pi^P(l, f, 0) + \delta^{\kappa+1} \frac{\pi^P(l, f+1, 1)}{1-\delta} \\
&\leq \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \frac{\pi^P(l, f, 1)}{1-\delta} \text{ since } f \geq IC_P(l) \\
&\leq \frac{\pi^P(l, f, 1)}{1-\delta},
\end{aligned}$$

as desired.

Suppose next $l > IC_{EA}^*$. Again, we first prove Statement 4. If $l > IC_{EA}^*$, (72) minus (73) equals

$$\begin{aligned}
& \pi^P(l, l-1, 1) + \delta \pi^P(l, l-1, 0) + \delta^2 \frac{\pi^P(l, l, 0)}{1-\delta} \\
& - \left(\pi^P(l, l-1, 0) + \delta \frac{\pi^P(l, l, 0)}{1-\delta} \right) \\
\geq\; & \pi^P(l, l-1, 1) + \delta \pi^P(l, l-1, 0) - \delta \pi^P(l, l, 0) - \pi^P(l, l-1, 0) \\
\geq\; & \pi^P(l, l-1, 1) + \delta \pi^P(l, l-1, 0) - \delta \pi^P(l, l-1, 1) - \pi^P(l, l-1, 0) \\
& \text{by (70)} \\
=\; & (1-\delta) \left(\pi^P(l, l-1, 1) - \pi^P(l, l-1, 0) \right) \\
\geq\; & 0.
\end{aligned}$$

Hence, protection is optimal.

We next prove Statement 5. Given the equilibrium strategy, for each k, we have

$$V^P(l, l-1, k) = \sum_{t=0}^{\kappa-1-k} \delta^t \pi^P(l, l-1, 1) + \delta^{\kappa-k} \frac{\pi^P(l, l, 0)}{1-\delta}.$$

This is decreasing in k and bounded by $\frac{\pi^P(l, l-1, 1)}{1-\delta}$ by (70), as desired.

The proof for f with $IC_D(l) \leq f \leq l-2$ is the same as the proof for $f = l-1$ and

$l \leq IC^*_{EA}$, except that (i) for (72), we have

$$\pi^P(l, f, 1) + \delta\pi^P(l, f, 0) + \delta^2 V^P(l, f+1, 0),$$

(ii) for (79), we have

$$\pi^P(l, f, 0) + \delta V^P(l, f+1, 0),$$

and (iii) for (74), the inductive hypothesis directly implies

$$\delta^{\kappa-k+1} V^P(l, f+1, 0) - \delta^{\kappa-k} V^P(l, f+1, 0) \geq -\delta^{\kappa-k}\pi^P(l, f+1, 1).$$

C.1.4 Proof of Lemma C.4

If $l + \imath = f$, then protection is not feasible by definition. If $l + \imath > f$, then after no protection, the leader invests if $l + \imath = f + 1$ given the premise of the lemma. Thus, the policymaker can obtain at least

$$\underbrace{\sum_{t=1}^{\kappa} \delta^t \pi^P(l+t, f, 1)}_{\substack{\text{A lower bound of the protection payoff can be attained} \\ \text{by assuming that the leader always invests given (67)}}} + \min_{(l_\tau, f_\tau) \text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f} \sum_{\tau=\kappa+1}^{\infty} \delta^\tau \pi^P(l_\tau, f_\tau, 0).$$

By contrast, with protection, she obtains at most

$$\pi^P(l, f, 1) + \max_{(l_t, f_t) \text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} \delta^t \pi^P(l_t, f_t, 0).$$

Thus, no protection is optimal by (68).

C.1.5 Proof of Lemma C.5

Consider the steady state (l, f, k) (the existence is obvious). If $l > f$ and $k < \kappa - 1$, then the state (l, f, k) is not a steady state since either protection is offered and k increases or protection is not offered and f increases. Thus, we have either $l = f$ or $k = \kappa$ in the steady state. Suppose we have $l < IC^*_{EA}$. We will prove either (i) we have $f \geq IC_P(l)$ and $f \geq IC_D(l)$, or (ii) it leads to a contradiction.

Suppose $l = f < IC^*_{EA}$ but $k \neq \kappa$. Then since protection is not feasible at state (l, f) with $l = f$, we have $k = 0$. As the leader is not investing, we have $f > IC_D(l)$. Moreover, (66) implies that, at (l, f) with $l = f$, we have $f \geq IC_P(l)$, as desired.

Suppose next that $k = \kappa$. We will prove that this would lead to a contradiction. Let $\left(\hat{l}, \hat{f}, \kappa - 1\right)$ be the last ex ante state before the equilibrium transits to $k = \kappa$. Let $\imath$ be the investment decision in that state (thus, the steady state l equals $\hat{l} + \imath$). By Lemma C.4, for the state to transit from $\kappa - 1$ to κ (that is, for the policymaker to protect the leader), we have $\hat{l}+\imath = \hat{f}+1$ and the leader does not invest in $\left(\hat{l}+\imath,\ \hat{f}+1,0\right)$. (66) and $\hat{l}+\imath = \hat{f}+1$ imply that $\hat{f} \geq IC_P\left(\hat{l}\right)$ and $\hat{f}+1 \geq IC_P\left(\hat{l}+\imath+1\right)$.

Since $\hat{l}+\imath = \hat{f}+1$, protection is not feasible in state $\left(\hat{l}+\imath,\ \hat{f}+1,0\right)$ if the leader does not invest at $\left(\hat{l}+\imath,\ \hat{f}+1,0\right)$. Thus, the fact that the leader does not invest at $\left(\hat{l}+\imath,\ \hat{f}+1,0\right)$ implies that $\hat{f}+1 > IC_D\left(\hat{l}+\imath\right)$. By contrast, recall that the steady state l equals $\hat{l}+\imath$. This means we have $\hat{l}+\imath < IC^*_{EA}$. Hence, the fact that the leader does not invest at $\left(\hat{l}+\imath,\ \hat{f}+1,0\right)$ implies that it will not be protected after investment in the interim state $\left(\hat{l}+\imath+1,\ \hat{f}+1,0\right)$. Since $\hat{f}+1 \geq IC_P\left(\hat{l}+\imath+1\right)$, Lemma C.3 implies that $\hat{f}+1 \leq IC_D\left(\hat{l}+\imath+1\right)$ (otherwise, the leader will be protected after investment), which in turn implies $\hat{f}+1 \leq IC_D\left(\hat{l}+\imath\right)$. This is a contradiction.

D Proof of Proposition 3

By Propositions 1 and 2, l^* is no higher than the solution to

$$IC_M(l) = 0 \Leftrightarrow \frac{\partial}{\partial l}\left((1-\rho)\,\hat{\pi}^M(l) + \rho\pi^L(l,0)\right) = c_l(l),$$

while l^{**} is no less than the solution to

$$IC_D(l) = l \Leftrightarrow \frac{\partial}{\partial l}\pi^L(l,l) = c_l(l).$$

Thus, Assumption 1’ implies $l^{**} > l^*$.

Supplementary Materials

E Policymaker Commitment versus Self-Interest

First, suppose the policymaker's objective is $\sum_{t=1}^{\infty} a_t \pi^P(l_t, f_t)$ and there exists $\hat{l} \geq 1$ such that $\pi^P(\hat{l}, 0) = \delta \cdot \pi^P(\hat{l}, 1)$ (that is, the IC_P curve never intersects the 45 degree line). If she can commit to a dynamic-game strategy, then the steady state is $(1, 0)$.

Proposition E.1 *Suppose there exists $\hat{l} \geq 1$ such that $\pi^P(\hat{l}, 0) = \delta \cdot \pi^P(\hat{l}, 1)$. If the policymaker can commit to a dynamic-game strategy, then the steady state is $(1, 0)$.*

Second, suppose the policymaker does not have a commitment power and her objective is to maximize the discounted sum of each period's social welfare: $\sum_{t=1}^{\infty} \delta^{t-1} SW^P(l_t, f_t)$. We will show that the protection is not offered after each (possibly off-path) history.

We need one assumption: For each (l, f) with $f \leq IC_D(l) \leq f + 1$, the following condition holds

$$\begin{aligned} & SW^M(l) + \delta SW^D(l+1, f) + \delta^2 \frac{SW^D(l+1, f+1)}{1-\delta} \\ < \; & SW^D(l, f) + \delta \frac{SW^D(l, f+1)}{1-\delta}. \end{aligned} \tag{75}$$

To see the implication of this assumption, suppose that the current interim state (after the leader's investment) is (l, f) with $f \leq IC_D(l) \leq f + 1$. If the policymaker protects the leader (as a deviation), then the follower stays at f and hence the next period the leader will invest again given $f \leq IC_D(l)$ (assuming that the policymaker will not protect the leader). Instead, if the policymaker does not protect the leader, then the follower moves up to $f + 1$ and hence next period the leader will not invest given $f + 1 \geq IC_D(l)$ (again assuming that the policymaker will not protect the leader). Condition (75) guarantees that the policymaker prefers the latter path.

Proposition E.2 *Suppose the policymaker does not have commitment power and her objective is to maximize $\sum_{t=1}^{\infty} \delta^{t-1} SW^P(l_t, f_t)$. If (75) holds, then the policymaker does not protect after any (possibly off-path) history.*

In particular, the state transition goes from $(0, 0)$ to $(1, 0)$ as a result of investment and then to $(1, 1)$ at the beginning of the next period. The next state is $(2, 1)$ as a result of investment and then $(2, 2)$ at the beginning of the next period. This chain continues to (l_{SW}, l_{SW}), where l_{SW} is the smallest l_{SW} satisfying $IC_D(l_{SW}) < l_{SW}$.

Third, suppose the policymaker has commitment power and her objective is to maximize the discounted sum of each period's social welfare $\sum_{t=1}^{\infty} \delta^{t-1} SW^P(l_t, f_t)$. Assume also that the policymaker is sufficiently long run such that, for each $l \leq \bar{L}$,

$$SW^M(l+1) + \delta SW^D(l+1, l) + \delta^2 \frac{SW^D(l+1, l+1)}{1-\delta} > \frac{SW^D(l, l)}{1-\delta}. \tag{76}$$

That is, suppose the policymaker compares the following two scenarios: the first one is to stay at (l, l) forever without protection. The other is to (i) increase the leader's technology level by one and grant monopoly for one period, (ii) increase the follower's technology level by one without protecting the leader, and (iii) stay at $(l+1, l+1)$ forever without protection. She prefers the latter (note that SW^D is increasing in both l and f). Under this assumption, the steady state technology level is no less than IC_{EA}.

Proposition E.3 *Suppose the policymaker has commitment power and her objective is to maximize* $\sum_{t=1}^{\infty} \delta^{t-1} SW^P(l_t, f_t)$. *Then, the steady state is on the* 45 *degree line. Moreover, if (76) holds, then the steady state technology level is no less than* IC_{EA}.

E.1 Proof of Proposition E.1

The policymaker can implement the steady state $(1, 0)$ by committing to the following strategy (*): Protect the leader after the leader's investment in state $(0, 0)$, protect the leader after the leader's non-investment in state $(1, 0)$, and do not protect the leader in all the other combinations of state and investment.

Moreover, since the policymaker's payoff is 0 if the leader does not invest at state $(0, 0)$, it remains to show the following: take any feasible path of $\{l_t, f_t, i_t, a_t\}_{t=1}^{\infty}$ with $l_1 = 1$ and $f_1 = 0$. The payoff $\frac{\pi^P(1,0)}{1-\delta}$ from strategy (*) is higher than the payoff from that (arbitrarily fixed) feasible path.

Let t^* be the first period in which $l_t + i_t \geq 2$. If such a period does not exist, then $l_t + i_t = 1$ forever and it is obvious that $\frac{\pi^P(1,0)}{1-\delta}$ is no worse (and strictly better unless $f_t = 0$ for all t). If such a period exists, then the payoff from the feasible path is

$$\sum_{t=1}^{t^*-1} \delta^{t-1} \pi^P(1, f_t, a_t) + \delta^{t^*-1} \sum_{t=t^*}^{\infty} \delta^{t-t^*} \pi^P(2, f_t, a_t). \tag{77}$$

In the proof of Lemma B.4, (50) shows that, for each $(l, f) \in \mathcal{L}$ satisfying $f \geq IC_P(l)$ and for each feasible path of $\{l_t, f_t, i_t, a_t\}_{t=1}^{\infty}$ with $l_1 + i_1 \geq l + 1$ and $f_1 = f$, we have

$$\frac{\pi^P(l, f)}{1-\delta} > \sum_{t=1}^{\infty} \delta^{t-1} \pi^P(l_t + i_t, f_t, a_t).$$

Thus,

$$\frac{\pi^P(1, f_{t^*})}{1-\delta} > \sum_{t=t^*}^{\infty} \delta^{t-t^*} \pi^P(2, f_t, a_t). \tag{78}$$

As mentioned on page 41, we allow $f \geq l$ when we write $(l, f) \in \mathcal{L}$.

If $f_{t^*} = 0$, then (77) and (78) imply that

$$\sum_{t=1}^{t^*-1} \delta^{t-1} \pi^P (1, f_t, a_t) + \delta^{t^*-1} \sum_{t=t^*}^{\infty} \delta^{t-t^*} \pi^P (2, f_t, a_t) < \frac{\pi^P (1, 0)}{1-\delta},$$

as desired. If $f_{t^*} = 1$, then let $t^{**} \leq t^* - 1$ be the period such that $f_{t^{**}} = 0$ and $f_{t^{**}+1} = 1$. By (77) and (78),

$$\begin{aligned}
&\sum_{t=1}^{t^*-1} \delta^{t-1} \pi^P (1, f_t, a_t) + \delta^{t^*-1} \sum_{t=t^*}^{\infty} \delta^{t-t^*} \pi^P (2, f_t, a_t) \\
< \; & \sum_{t=1}^{t^{**}-1} \delta^{t-1} \pi^P (1, 0) + \delta^{t^*-1} \frac{\pi^P (1, 1)}{1-\delta} \\
& \text{since protection is not feasible when } l = f = 1 \\
\leq \; & \sum_{t=1}^{t^{**}-1} \delta^{t-1} \pi^P (1, 0) + \delta^{t^*-2} \frac{\pi^P (1, 0)}{1-\delta} \text{ by } 0 \geq IC_P (1) \\
\leq \; & \frac{\pi^P (1, 0)}{1-\delta} \text{ since } t^{**} - 1 \leq t^* - 2.
\end{aligned}$$

E.2 Proof of Proposition E.2

We prove that, for each interim state $(l + i, f)$, where (l, f) is the state at the beginning of the period and i is the investment decision, the policymaker does not protect the leader.

Suppose $l + i = \bar{L}$. Since the leader stops investing, the policymaker, who maximizes social welfare, does not protect the leader, regardless of the history. Thus, the claim holds.

Suppose the claim holds for $l + i \geq \hat{l} + 1$, and assume that the policymaker is now at a history with $l + i = \hat{l}$. We will prove that the claim holds for $\hat{l}$ in two steps:

First, for $f = \hat{l}$, the claim holds. To see this, at the interim state $\left(\hat{l}, \hat{l}\right)$, the policymaker's unique feasible strategy is no protection. Thus, until the interim state $\left(\hat{l} + 1, \hat{l}\right)$ is reached, the policymaker has to play no protection. Moreover, the policymaker's continuation strategy from the interim state $\left(\hat{l} + 1, \hat{l}\right)$ is uniquely determined by the inductive hypothesis, and so the policymaker does not protect the leader forever.

Second, fix $\hat{f} + 1 \leq \hat{l}$ arbitrarily and suppose that the claim holds for $(l + i, f)$ with $l + i = \hat{l}$ and $f \geq \hat{f} + 1$. We will show that the claim holds for the interim state $\left(\hat{l}, \hat{f}\right)$.

Let $\mathcal{V}^{SW} \left(\hat{l}, \hat{f}\right)$ be the set of subgame perfect equilibrium payoffs for the policymaker given that the ex ante state is $\left(\hat{l}, \hat{f}\right)$.

The policymaker's payoff without protection is uniquely determined by

$$SW^D\left(\hat{l},\ \hat{f}\right)+\delta V^{SW}\left(\hat{l},\ \hat{f}+1\right), \tag{79}$$

where $V^{SW}\left(\hat{l},\ f+1\right)$ is the continuation payoff given the ex ante state $\left(\hat{l},\ f+1\right)$, which is uniquely determined by the inductive hypothesis. Thus, it remains to show that the policymaker's highest payoff at the interim state $\left(\hat{l},\ \hat{f}\right)$ is not achieved by protection. Suppose otherwise. With protection, the highest payoff that the policymaker can obtain at interim state $\left(\hat{l},\ \hat{f}\right)$ is

$$SW^M\left(\hat{l}\right)+\delta \max_{v'\in\mathcal{V}^{SW}\left(\hat{l},\hat{f}\right)} v'. \tag{80}$$

Suppose first that $\max_{v'\in\mathcal{V}^{SW}\left(\hat{l},\hat{f}\right)} v'$ is achieved when the leader does not invest in the ex ante state $\left(\hat{l},\ \hat{f}\right)$. In that case, the interim state will again be $\left(\hat{l},\ \hat{f}\right)$ and hence

$$\max_{v'\in\mathcal{V}^{SW}\left(\hat{l},\hat{f}\right)} v' = SW^M\left(\hat{l}\right)+\delta \max_{v'\in\mathcal{V}^{SW}\left(\hat{l},\hat{f}\right)} v',$$

or

$$\max_{v'\in\mathcal{V}^{SW}\left(\hat{l},\hat{f}\right)} v' \le \frac{SW^M\left(\hat{l}\right)}{1-\delta}.$$

By contrast, since social welfare is increasing in both l and f, by not protecting the leader at all, the policymaker can obtain $\frac{SW^D\left(\hat{l},\hat{f}\right)}{1-\delta}$. This is a contradiction since $SW^D\left(\hat{l},\ \hat{f}\right)\ge SW^M\left(\hat{l}\right)$.

Suppose second that $\max_{v'\in\mathcal{V}^{SW}\left(\hat{l},\hat{f}\right)} v'$ is achieved when the leader invests in the ex ante state $\left(\hat{l},\ \hat{f}\right)$. By the inductive hypothesis, the policymaker will not protect the leader once the interim state $\left(\hat{l}+1,\ \hat{f}\right)$ is reached, so (80) equals

$$SW^M\left(\hat{l}\right)+\delta SW_D\left(\hat{l}+1,\ \hat{f}\right)+\delta^2 V^{SW}\left(\hat{l}+1,\ \hat{f}+1\right). \tag{81}$$

Note that the fact that the leader investing implies $\hat{f}\le IC_D\left(\hat{l}\right)$.

If $\hat{f} \leq IC_D\left(\hat{l}\right) \leq \hat{f}+1$, then (81) equals

$$SW^M\left(\hat{l}\right)+\delta SW_D\left(\hat{l}+1,\ \hat{f}\right)+\delta^2\frac{SW^D\left(\hat{l}+1,\ \hat{f}+1\right)}{1-\delta}.$$

By contrast, (79) equals

$$SW^D\left(\hat{l},\ \hat{f}\right)+\delta V^{SW}\left(\hat{l},\ \hat{f}+1\right)=SW^D\left(\hat{l},\ \hat{f}\right)+\delta\frac{SW^D\left(\hat{l},\ \hat{f}+1\right)}{1-\delta}.$$

By Assumption 75, (79) is higher, which is a contradiction.

If $\hat{f}+1 \leq IC_D\left(\hat{l}\right)$, then (79) equals

$$\begin{aligned} & SW^D\left(\hat{l},\ \hat{f}\right)+\delta V^{SW}\left(\hat{l},\ \hat{f}+1\right) \\ = \ & SW^D\left(\hat{l},\ \hat{f}\right)+\delta SW_D\left(\hat{l}+1,\ \hat{f}+1\right)+\delta^2 V^{SW}\left(\hat{l}+1,\ \hat{f}+1\right), \end{aligned}$$

which is greater than (81), leading to a contradiction.

E.3 Proof of Proposition E.3

We first prove that the steady state is on the 45 degree line. Take a history $\hat{h}^t$ such that $l_{t'} = l_t$ for all $t' \geq t$ (the leader stops investing). Since the leader is myopic, if the policymaker changes her strategy after history $\hat{h}^t$, then it changes the path of play only after it reaches history $\hat{h}^t$. Since social welfare is increasing in both l and f, it is optimal not to protect the leader after history $\hat{h}^t$. Thus, eventually, the state reaches the 45 degree line.

We next prove that the steady state technology level is no less than IC_{EA}. Take a history $\bar{h}^t$ such that $(l_{t'},\ f_{t'}) = (l_t,\ f_t)$ for all $t' \geq t$. From the discussion above, we have $l_t = f_t$. We will prove that $l_t \geq IC_{EA}$. Suppose otherwise: $l_t < IC_{EA}$.

The policymaker's continuation payoff from history $\bar{h}^t$ is

$$\frac{SW^D\left(l_t, l_t\right)}{1-\delta}.$$

Again, since the leader is myopic, if the policymaker changes her strategy after history $\bar{h}^t$, then it changes the path of play only after it reaches history $\bar{h}^t$. If the policymaker changes the strategy after $\bar{h}^t$ such that (i) if the leader invests in period t, then the policymaker protects the leader in period t (and then no more protection from period $t+1$ on) and (ii) if the leader does not invest in period t, then no protection forever. Since $l_t < IC_{EA}$, (6) implies that the leader invests in period t. Thus, the policymaker's continuation payoff from history $\bar{h}^t$ is

$$SW^M(l_t+1)+\delta SW^D(l_t+1,l_t)+\delta^2\frac{SW^D(l_t+1,l_t+1)}{1-\delta}.$$

By (76), the policymaker is better off in the revised strategy, which is a contradiction.

F Extension: Far-Sighted Firms

In this extension, we relax the assumption that $\beta=0$. Suppose the leader's discount factor β is strictly greater than zero.[46] We now define the cutoffs IC_D^β and IC_M^β as follows: We define $IC_D^\beta(l)$ such that, for each l, at $f=IC_D^\beta(l)$,

$$\frac{1}{1-\beta}\pi^L(l+1,f)-c(l)=\frac{1}{1-\beta}\pi^L(l,f).$$

As in Lemma 1, if the policymaker never protects the leader, then, the leader is willing to invest if and only if $f\le IC_D^\beta(l)$ given its own technology level l.

Similarly, we define $IC_M^\beta(l)$ such that, for each l, at $f=IC_M^\beta(l)$,

$$\frac{1}{1-\beta}\pi^M(l+1,f)-c(l)=\frac{1}{1-\beta}\pi^M(l,f).$$

As in Lemma 2, if the policymaker protects the leader forever, then the leader is willing to invest if and only if $f\le IC_M^\beta(l)$ given its own technology level l.

Since we do not change the policymaker's preference, the definition of $IC_P(l)$ stays the same as in the main model. For a simple analysis, we assume that $IC_P(l)$ never intersects with the 45 degree line: $l-1\ge IC_P(l)$ for all l. In addition, the IC_P and IC_M curves intersect. We also focus on finding a Markov Perfect Equilibrium, where we select (NI,P) whenever it is an equilibrium outcome, as in Lemma B.6.

As in Appendix B, we classify (l,f) as follows:

1. Region 1: $f\le IC_P(l)$ and $f\ge IC_D(l)$. By the same proof as Lemma B.8, the leader does not invest (NI) and the policymaker does not protect (NP). Note that, since we assume $l-1\ge IC_P(l)$ for all l, Region 1 does not include any part of the 45 degree line (Precisely, for each l, neither (l,l) nor $(l,l-1)$ are in Region 1).

2. Region 2: $f\ge IC_P(l)$ and $f\ge IC_M(l)$. In this region, as in Lemma B.7, the leader does not invest (NI) and the policymaker protects (P) whenever $l>f$.

[46] As in the main model, the follower automatically catches up and always exists in the market. That is, the follower is not a strategic player and we do not need to model its discount factor.

3. Region 3: $f \le IC_P(l)$ and $f \le IC_D(l)$ or $f \ge IC_P(l)$ and $f \le IC_M(l)$. This is the region where we have the backward induction argument in Appendix B. We now focus on how this backward induction argument is robust to far-sighted firms.

Recall that the logic of backward induction relies on the assumption that the leader does not invest if it loses protection for one period by doing so: condition (31). With far-sighted firms, this is not necessarily true, since the firm may invest even if it loses protection for one period if there is enough long-run gain. However, near the indifference curve $IC_M^{\beta}(l)$, since the leader is almost indifferent between no investment and investment if it is protected forever, it will not invest if (i) not investing leads to permanent protection and (ii) investing leads to no protection for at least one period.

More formally, define $IC_L(l)$ such that, for each l, at $f = IC_L(l)$,

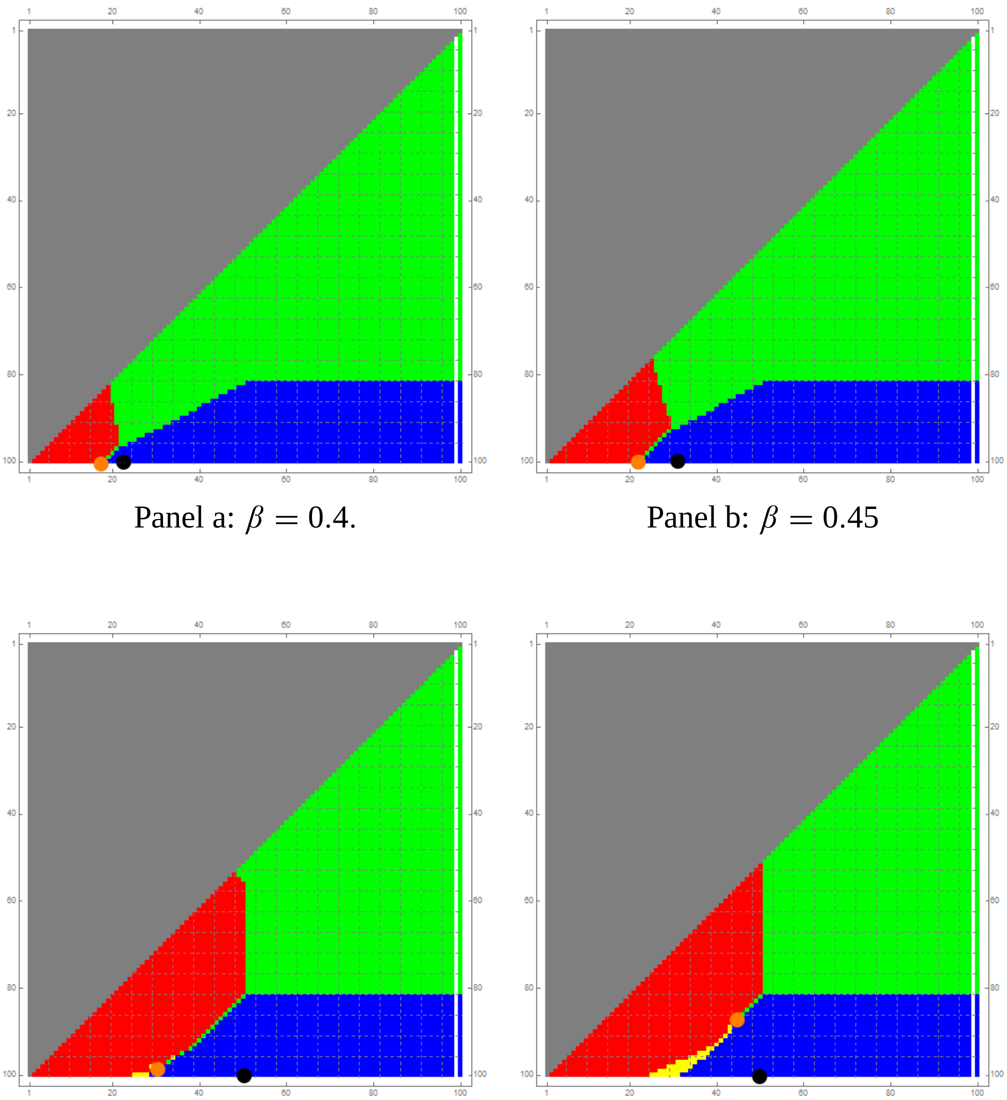
$$\pi^L(l+1, f) + \frac{\delta_L}{1-\delta_L}\pi^M(l+1, f+1) - \frac{1}{1-\delta_L}\pi^M(l, f) = c(l).$$

That is, suppose that if the leader invests, it loses protection for one period. All the statements in Lemmas B.9–B.13 hold if we restrict attention to $\{(l, f) : f \ge IC_L(l)\}$. Thus, near the intersection of the IC_P and IC_M^{β} curves, we can find (l, f) with $\text{eqm}(l, f) = (NI, P)$.

For example, in the following numerical simulation, the state transits from $(0, 0)$ to $(1, 0)$, $(2, 0)$. Then, it either hits the green region and the steady state is (NI, P) on the l-axis, or it hits the yellow region and the state moves along the left boundary of the yellow region until it hits to the first green dot. This green dot is the steady state: $\text{eqm}(l, f) = (NI, P)$. Thus, it is still true that the long-run investment level with a strategic policymaker is lower than the case where the policymaker protects the leader (in that case, as in 2, the steady state will be the intersection between the l-axis and IC_M^{β}-curve, denoted by the black dot in the figure below).

G Extension: Adding Costly Catch-Up

In this extension, we incorporate a notion of costly investment for the follower. Specifically, we assume that a follower who is allowed to compete in the market increases its technology level conditional on also being allowed to compete in the following period. This implicitly assumes that the expected payoff from moving up one technology step must be strictly positive (which is equivalent to requiring that the follower pays a small cost of moving up technologically, which makes moving up preferable only if some profit is expected in the future). Formally, we assume that, in each period t, the follower's technology level increases from f_t to $f_t + 1$ at the beginning of period $t+1$ if and only if protection is not offered in period $t+1$ (with a positive probability). The rest of the model is the same as the baseline model. In particular, we focus on renegotiation proof subgame perfect



Panel a: $\beta = 0.4$.

Panel b: $\beta = 0.45$

Panel c: $\beta = 0.55$.

Panel d: $\beta = \delta = 0.66$

Figure 6: Numerical illustration of the model with far-sighted firms in the (l,f) space. The model is simulated with parameter values a=10, b=1/10, $\rho = 0.5$, $\gamma_0 = 0$, $\gamma_1 = 12$, and $\delta = 0.66$. Each panel displays the equilibrium regions when the discount factor for the leader is as mentioned in the panel caption. The equilibrium regions are (NI,P) in green, (NI,NP) in blue, (I,P) in red, and (I,NP) in yellow. The steady state is depicted by the orange dot. The steady state under the protected monopoly benchmark is depicted by the black dot.

equilibrium, and assume that, for each $(l,\ f)$ with $f \geq IC_P(l)$, for any MPE, we have

$$\frac{\pi^P(l,\ f,1)}{1-\delta} > V^P(l+1,\ f). \tag{82}$$

We can show that the renegotiation proof subgame perfect equilibrium is unique.

Lemma G.1 *The set of subgame perfect equilibrium payoffs that satisfy renegotiation proofness is unique after each ex ante state $(l,\ f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ and also after each interim state $(l,\ f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$. In this renegotiation-proof subgame perfect equilibrium, the strategy is Markov: the leader's investment decision depends only on the ex ante state and the policymaker's protection decision depends only on the interim state.*

Note that (82) implies $\frac{\pi^P(l,f,1)}{1-\delta} > \frac{\pi^P(l+1,f,0)}{1-\delta}$ since the policymaker can obtain at least $\frac{\pi^P(l+1,f,0)}{1-\delta}$ by not protecting the leader forever (recall that $\pi^P(l,\ f,0)$ is increasing in both l and f). Thus,

$$\pi^P(l,\ f,1) > \pi^P(l+1,\ f,0) > \pi^P(l,\ f,0). \tag{83}$$

For the simple proof, we assume that, for each $l > IC_{EA}$, we have $l-1 > IC_P(l)$ (that is, the policymaker prefers to protect the leader if the follower is sufficiently strong and the leader's technology level stays at $l > IC_{EA}$). This holds, for example, when IC_P curve never intersects with the 45 degree line, since it guarantees $l-1 > IC_P(l)$ for all l.

Recursively, we will show that the follower's technology level never increases and protection is always offered.

Lemma G.2 *In the model of costly catch-up, the policymaker protects for all $(l,\ f) \in \mathcal{L}^* \cap \mathbb{Z}_+^2$ with $l > f$, and hence f never increases.*

G.1 Proofs of Auxiliary Lemmas

G.1.1 Proof of Lemma G.1

For sufficiently large l such that $\pi^M(l+1,\ f) - \pi^L(l,\ f) < c(l)$ for each f, the leader never invests. Hence, the policymaker is the single decision maker and the result holds. Fix $(l,\ f)$. Suppose the result holds for each ex ante state $(l',\ f')$ with $(l',\ f') \geq (l,\ f)$ and $(l',\ f') \neq (l,\ f)$ and interim state $(l'',\ f'')$ such that there is a feasible transition path from $(l',\ f')$ to $(l'',\ f'')$. Let $\mathcal{V}^P(l,\ f)$ be the set of SPE payoffs at the ex ante state $(l,\ f)$ for the policymaker.

1. After the leader invests, that is, at the interim state $(l+1,\ f)$, the follower's catchup transition is determined since the equilibrium path from the ex ante state $(l+1,\ f+1)$ is determined by the inductive hypothesis. Moreover, the values $V^P(l+1,\ f+1)$ and $V^P(l+1,\ f)$ are determined by the inductive hypothesis. Hence, the continuation payoff for the policymaker is unique. Since the leader prefers protection, in the renegotiation proof SPE, we break the tie for protection.

2. After the leader does not invest, that is, at the interim state (l, f), whether the follower catches up if the policymaker does not protect the leader is determined since the equilibrium path from the ex ante state $(l, f+1)$ is determined by the inductive hypothesis (or $l = f$ and no automatic catch-up happens).

   (a) Suppose the follower catches up if the policymaker does not protect the leader. Then, the policymaker's payoff without protection is determined by $\delta V^P(l, f+1)$. With protection, it will be $\pi^P(l, f, 1) + \delta v$ for some $v \in \mathcal{V}^P(l, f)$. If

   $$\min_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta v \geq \pi^P(l, f, 0) + \delta V^P(l, f+1)$$

   or

   $$\max_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta v \leq \pi^P(l, f, 0) + \delta V^P(l, f+1),$$

   then the equilibrium payoff for the policymaker is unique (again, if the policymaker is indifferent, we break the tie for protection). Hence, we assume that

   $$\min_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1)+\delta v < \pi^P(l, f, 0)+\delta V^P(l, f+1) < \max_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1)+\delta v.$$

   For v satisfying $\pi^P(l, f, 1) + \delta v > \pi^P(l, f, 0) + \delta V^P(l, f+1)$, the policymaker protects the leader. Hence, $v^* \in \arg\max_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta v$ should be attained by protection. Given this property of the policymaker's value and incentives, it suffices to show that the equilibrium path that achieves payoff $\max_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta v$ also maximizes the leader's payoff. This holds since the leader prefers protection. Therefore, at the interim state (l, f), the renegotiation proof SPE is unique.

   (b) Suppose the follower does not catch up if the policymaker does not protect the leader. After the leader does not invest, the policymaker's payoff without protection is $\pi^P(l, f, 0) + \delta v$ for some $v \in \mathcal{V}^P(l, f)$. With protection, it will be $\pi^P(l, f, 1) + \delta w$ for some $w \in \mathcal{V}^P(l, f)$. Since $\pi^P(l, f, 1) \geq \pi^P(l, f, 0)$ by (83), there are the following two possibilities. First, if $\pi^P(l, f, 1)+\delta \min_{w \in \mathcal{V}^P(l,f)} w \geq \pi^P(l, f, 0)+\delta \max_{w \in \mathcal{V}^P(l,f)} v$, then the equilibrium payoff for the policymaker is unique (again, if the policymaker is indifferent, we break the tie for protection). Second, if

   $$\pi^P(l, f, 1) + \delta \min_{w \in \mathcal{V}^P(l,f)} w \leq \pi^P(l, f, 0) + \delta \max_{v \in \mathcal{V}^P(l,f)} v,$$

   then since

   $$\pi^P(l, f, 0) + \delta \max_{v \in \mathcal{V}^P(l,f)} v \leq \pi^P(l, f, 1) + \delta \max_{w \in \mathcal{V}^P(l,f)} w,$$

   the optimal value $v^* \in \arg\max_{w \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1)+\delta w$ should be attained by protection. Given this property of the policymaker's value and incentives, it suffices to show that the equilibrium path that achieves $\max_{w \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta w$ also maximizes the leader's payoff. This holds since the leader prefers protection.

In total, at the interim state that is reachable from (l, f), the renegotiation proof SPE is unique. Thus, at the ex ante state (l, f), the leader's investment strategy is also determined.

G.1.2 Proof of Lemma G.2

We will show the following claim by backward induction: For each interim state (l, f), the policymaker protects the leader whenever feasible (that is, unless $l = f$).

For $l \geq \bar{L}$, since the leader will never invests, even if the follower catches up, the policymaker protects the leader at the interim state (l, f) if $f \geq IC_P(l)$. Given this continuation play, at the interim state (l, f) with $f \geq IC_P(l) - 1$, the follower does not catch up (if it does, then the next ex ante state will be $(l, f+1)$; since the leader does not invest, the next interim state is $(l, f+1)$ and the policymaker protects the leader). Hence, it is optimal for the policymaker to protect the leader given discounting at the interim state (l, f) with $f \geq IC_P(l) - 1$. Recursively, protection is always offered and the follower does not catch up.

Suppose the statement holds for each $(l', f') \geq (l, f)$ with $(l', f') \in \mathcal{L}^*$ and $(l', f') \neq (l, f)$. We will prove the statement for (l, f).

We first prove that unless $l = f + 1$ and the leader does not invest at the ex ante state $(l, f+1)$, the follower does not catch up at the interim state (l, f) and the next state stays at (l, f). To see why, once the ex ante state becomes $(l, f+1)$, the interim state satisfies $(l', f') \geq (l, f+1)$. Thus, the policymaker protects the leader unless $l = f + 1$ and the leader does not invest at the ex ante state $(l, f+1)$.

Thus, the protection leads to the payoff of $\pi^P(l, f, 1) + \delta V^P(l, f)$ and the non-protection leads to the payoff of $\pi^P(l, f, 0) + \delta V^P(l, f)$. Thus, protection is optimal.

Next, suppose that $l = f+1$ and the leader does not invest at the ex ante state $(l, f+1)$. Since the leader does not invest even though the policymaker protects the leader at interim state $(l+1, l) = (l+1, f+1) > (l, f)$, we have $l > IC_{EA}$. As $l > IC_{EA}$ implies $l - 1 > IC_P(l)$, the policymaker protects the leader given (50).