Leisure-Enhancing Technological Change*

Łukasz Rachel†

March 11, 2022

Abstract

Modern economies are awash with leisure-enhancing technologies: products supplied in exchange for time and attention, rather than money. This paper studies how such technologies interact with the broader macroeconomy. The theory provides a technology-based account for the decades-long downward trend in hours worked and lackluster measured productivity growth observed across developed economies. In particular, since leisure technologies crowd out "traditional" innovation, the theory sheds new light on the modern manifestation of the Solow Paradox. I show that the adverse traditional productivity effect dominates the utility gain from the free products, leaving societies persistently worse-off. The market equilibrium is inefficient: the ad-based business model of leisure innovators means that the wrong price values leisure technologies in equilibrium; moreover, the adverse impact of leisure-enhancing innovations on future traditional productivity is underpriced.

*I am grateful to and thank the Editor and three anonymous referees for their excellent comments and suggestions. I am deeply indebted to Ricardo Reis, Ben Moll and Francesco Caselli for their continuous guidance and support. I am grateful to Daron Acemoglu, Philippe Aghion, Ufuk Akcigit, Robert Barro, Charlie Bean, Olivier Blanchard, Erik Brynjolffson, Tiago Cavalcanti, Svetlana Chekmasova, Maarten De Ridder, Nicola Fontana, Emmanuel Farhi, Matthew Gentzkow, Austan Goolsbee, Shane Greenstein, Gene Grossman, Wouter den Haan, Elhanan Helpman, Per Krusell, Robin Lee, Josh Lerner, Rachel Ngai, Thomas Sampson, Larry Summers and John Van Reenen and for helpful conversations and comments. I would like to thank participants at seminars at Bocconi, UCL, Princeton, Chicago, Chicago Booth, Toronto, CREi, Oxford, Brown, FRBNY, Harvard Business School, and conferences including NBER Summer Institute, VMACS Junior Macro Conference and the RES Annual Meeting for the many comments that significantly improved the paper. I am grateful for the hospitality of the Department of Economics at Harvard University where I worked on this project as a Fulbright Visiting Fellow. Financial support of US-UK Fulbright Commission and the Bank of England are gratefully acknowledged. The views in this paper are solely mine and do not reflect the views of the Bank of England or its policy committees.

†Princeton, LSE and the Bank of England. *Email:* lukasz.rachel@gmail.com. *Web:* https://sites.google.com/site/lukaszrachel/

1 Introduction

In models of economic growth, technological change is a catch-all generalization of a large and diverse set of innovations undertaken in the real world. In this paper I distinguish between "traditional" product- or process-innovations and inventions that are *leisure-enhancing*.

The defining difference between the traditional and leisure-enhancing technologies is the way they are monetized. Improving a production process or introducing a new product tends to raise the profits of the innovator directly. Instead, leisure-enhancing products are often available for free, and are instead monetized indirectly through harnessing consumers' time and attention. Because of this, leisure-enhancing innovations are profitable to the extent they capture consumers' time.[1] The main insight of this paper is that the traditional and the leisure-enhancing technologies interact in ways that shed new light on important macroeconomic phenomena, such as dynamics of hours worked and a modern incarnation of the Solow (1987) Paradox,[2] with associated implications for welfare and efficiency.

Consider social media as a telling example. Survey estimates suggest that in 2020 an estimated 4 billion active users have spent on average over 2 hours a day using social media.[3] This great success in terms of capturing consumers' time appears to have been achieved, in part, by innovation activity in the social media sector (Figure 1 presents a stylized timeline of such innovations). Consumers can tap into social media services without reaching for their wallets: it is their time, attention and data that buys them access.[4]

These salient features carry beyond the social media platforms operating in recent years. The 'leisure sector' as a whole is an important cluster of innovation and discovery, and has likely become more so over the recent past. For example, a proxy for its share in overall R&D spending across the industrialized world has more than doubled between 2005 and 2014, according to the data produced by the OECD.[5] Furthermore, monetizing time and attention is hardly a new phenomenon. Using data for the United States, the left panel of Figure 2 shows that indirectly financed zero-price products go back decades. The share of advertising revenues in GDP follows a similar pattern. And historically leisure technologies have been instrumental in

[1]Other innovations do not exhibit such systematic bias in general. It is true that consumption of many goods and services takes time; but while some innovations are time intensive, others are time-saving.

[2]In 1987 Bob Solow famously quipped that "computer age is visible everywhere except for the productivity statistics". Computer age eventually made an appearance in the mid-90s, driving much of the pick-up in growth in capital intensity and total factor productivity in the United States (see Jorgenson (2005) for a summary). This revival was ultimately short-lived, and TFP growth since the early 2000s has again been puzzlingly sluggish. The perception of rapid technological change appears to be, once again, at odds with the official statistics.

[3]The figures are from Globalwebindex, a consultancy which runs a large survey of online behaviors.

[4]Industry estimates suggest that over 90% of social media firms' revenues comes from advertising (OfCOM, 2019). In this paper attention- and data gathering are assumed to be perfectly correlated with capturing consumers' time. Nonetheless, the distinction may play an important role in the context of the modern technologies. Complementary work of Farboodi and Veldkamp (2019) considers the long-run consequences of data gathering.

[5]See Figure A.2 in Appendix A.

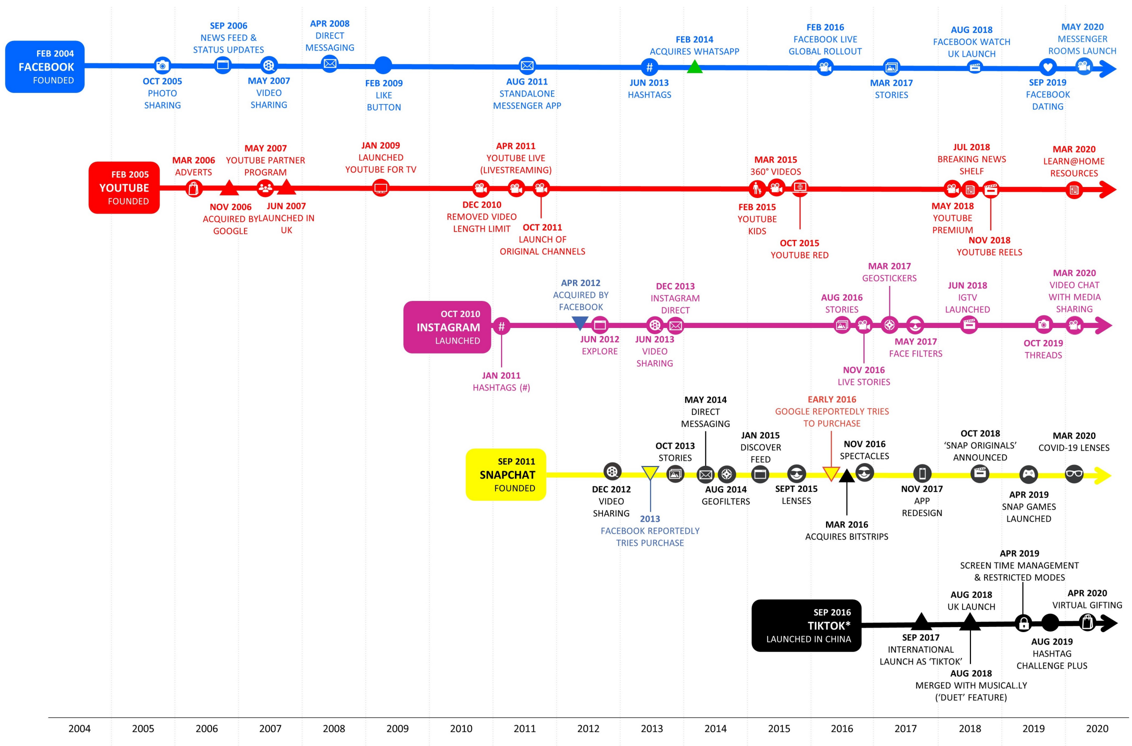

Figure 1

Timeline of Selected Innovations in the Social Media Sector

Source: Ofcom.

shifting time allocation patterns: for example, Aguiar and Hurst (2007a) and Gentzkow (2006) find evidence that the introduction of the television in the 1950s and 1960s had a large impact on time allocation patterns in the United States, and Falck *et al.* (2014) document the significant impact on leisure time of the roll-out of the internet in Germany in the 2000s. Both episodes constituted an expansion of free-of-charge, ad-financed services available to consumers.

The technological developments in leisure have occurred against the backdrop of a trend decline in hours worked (Figure 2, middle panel) and slowing growth of labor- and total factor productivity (the right panel). How, if at all, are these trends linked?

To begin thinking about this question, I use an illustrative setup with exogenous growth in leisure technology. Households derive leisure utility from various activities such as watching TV or browsing the web, and the range of activities expands exogenously over time. I show that with such an exogenous increase, hours worked decline at a constant rate and yet this decline is consistent with balanced growth and thus with the Kaldor Facts. More importantly, if the development of traditional technology is endogenous and relies on human input, the decline in hours worked has a negative effect on productivity growth in that sector. These insights suggest that, to the extent that leisure technologies can be thought of as shifting the relative weight on the utility of leisure relative to consumption, they provide a candidate explanation for the joint

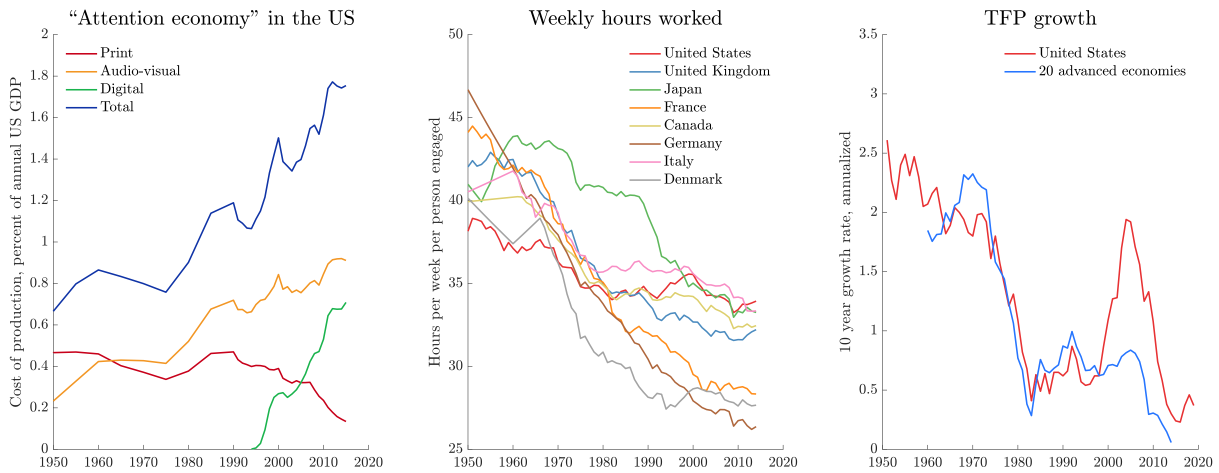

Figure 2

Motivating Trends: Free Products in the United States, and Cross-Country Trends in Hours Worked and Total Factor Productivity

Notes: Estimates of the cost of production of free consumer services are from the Bureau of Economic Analysis (Nakamura *et al.* (2017)). The figure shows the ratio of free consumer content, measured by the costs of production, to GDP. Thus, for example, it does not attempt to capture utility benefit of Facebook, but only the cost of providing it. Hours worked are from Penn World Tables 9.0 (Feenstra *et al.* (2015)). The US TFP growth rate is the utilization-adjusted series following Basu *et al.* (2006). The TFP growth rate for advanced economies is constructed by the IMF and is PPP-weighted (Adler *et al.* (2017)). Both series show 10-year growth rates.

dynamics of hours and productivity elsewhere in the economy. But why is it sensible to think of leisure technologies in this way? Where do these technologies come from? And are there any other ways through which they interact with the macroeconomy?

To study these issues I develop a tractable general equilibrium theory of an *attention economy* – the economic ecosystem that supports the existence of leisure-enhancing innovations. The essence of the attention economy is that brand equity – a form of intangible capital acquired by firms through advertising – requires consumers' time and attention. The paper thus provides a novel focus on the macroeconomic implications of how some of the intangible assets are produced.

On the consumer side, the model builds on Becker (1965), with leisure utility generated from combining users' time with market goods and services. The novel aspect is that I focus on services that are *free* (available at zero prices) and *strongly non-rival* (the marginal cost of supplying an extra user is zero)[6] – such as TV channels, web content or social media. This focus is justified given the proliferation of such services; it also plugs the gap in the existing literature, which has focused on the role of durable goods (such as TV sets, computers, smartphones) and fixed-cost expenditure (e.g. broadband subscription) in household production of leisure. I show that within

[6]I use the term "strongly non-rival" in order to highlight the parallels of "leisure-enhancing ideas" with the general notion of non-rivalry of ideas as highlighted by Romer (1990), and further underscore the fact that the marginal cost of supplying an extra user with a product based on leisure technologies is zero (which is not the case in general if non-rival ideas are embodied in rival goods such as materials).

such a framework the index of leisure technology naturally shows up as a time-varying shifter in the household utility function.

On the firm side, I derive a tractable extension to the canonical monopolistic competition setup in which firms demand brand equity in equilibrium.

Between the consumers seeking free entertainment and firms demanding brand equity are the platforms that innovate in order to capture 'eyeballs' and supply businesses with ads. I derive the equilibrium supply of leisure technologies and show that, because of the indirect monetization, it is tightly linked to the profitability of selling ads (and hence to the market size of the traditional economy).

Embedding these features in a setting with endogenous traditional innovation brings out the following insights.

Leisure-enhancing technologies emerge endogenously on the growth path, once the economy is sufficiently developed. This is driven by the interaction between a feature of household preferences (leisure technologies must be sufficiently developed for households to use them) and an aforementioned market size effect (the economy must be sufficiently large to support platforms' business model). The steady-state equilibrium thus takes a form of a *segmented balanced growth path* (sBGP). The remaining results of the paper concern the changing nature of economic growth between the two segments of the sBGP, elucidating this new kind of structural change.

One feature of the equilibrium is that hours worked decline in the presence of leisure-enhancing innovations. Ever-improving leisure options tilt the balance towards more free leisure and less work and traditional consumption. This prediction matches the trend in time use observed across countries over long periods (Aguiar and Hurst, 2007b) and provides a new way to interpret the recent dramatic shifts in time allocation (Appendix A presents more evidence on these shifts).

The growth rate of productivity in the traditional sectors of the economy declines following the entry of the platforms. There are three channels through which this effect operates. The first channel underlines the heightened competition for time and attention that is characteristic of the attention economy: better leisure leaves less time for productive activities and diminishes the market size for traditional innovation.[7] Second, the leisure R&D sector competes with the traditional R&D sector in factor markets (e.g. for talent). Third, brand equity competition results in profit shifting, away from competing firms and towards the platform sector. I pin down these effects analytically and show, using a calibrated model, that the emergence of the attention economy can account for between a third and a half of the slowdown in TFP growth

[7] The framework in this paper builds on the semi-endogenous growth paradigm (Jones, 1995) in which the long-run growth rate of total factor productivity is tied to the growth rate of the pool of (human) resources used to generate ideas. But as I explain below, the broad insights carry over into a broad range of models in which innovation and adoption of ideas require human input.

observed in the data.

The theory helps better understand the measurement challenge associated with the attention economy. Two questions arise in this context: first, is GDP significantly mismeasured? And second, is GDP becoming a less reliable guide to welfare? I answer the first question in the negative: the components that are missing from GDP are too small to make a difference. But, to the extent that increases in usage go hand-in-hand with increases in utility,[8] GDP does miss a potentially sizeable welfare effect of leisure technologies. Leisure-enhancing technologies introduce a systematically *growing* wedge between GDP and welfare.[9]

Do the leisure technologies make up for the loss of traditional productivity in terms of welfare? I show that immediately after leisure technologies emerge, the welfare response is dominated by the adverse traditional productivity effect. Intuitively, the lower productivity in the traditional sector affects a lion share of representative household's utility that is derived from consumption of traditional goods and services.

The final set of results goes deeper into the efficiency properties of the decentralized equilibrium. Because leisure technologies are monetized indirectly and do not carry a price, the equilibrium supply is suboptimal (the size of the effect is in general ambiguous; in the calibrated economy there is significant undersupply). Secondly, in a growing economy leisure technologies interact with the usual distortions present in an endogenous growth setting, exacerbating the inefficiently low labor supply.

Related literature. In proposing a directed-technology explanation for the trend in hours worked, this paper brings together the literatures on endogenous innovation[10] with that on the long-run shifts in time allocation.[11] Since the seminal paper of King *et al.* (1988) which derive the 'balanced growth' preference class, most growth models have featured constant hours worked along the balanced growth path. Yet the historical data which show a steady long-run decline of around -0.4% per annum (Jones, 2015).[12] In contributions closely related to this paper, Ngai and

[8]This qualification is an important one. For example, there is growing evidence of an association between greater use of some of the leisure technologies and higher depressive and anxiety scores, poor sleep, low self-esteem and body image concerns (Kelly *et al.* (2018); Royal Society for Public Health (2017)). The present paper assumes a simple revealed-preference perspective, but even with this assumption it finds negative welfare effects. It stands to reason that introducing habit formation and addiction into the analysis would only strengthen these results.

[9]These findings suggest that leisure time (enhanced by leisure technology) ought to be included in measures of economic wellbeing, in the spirit of Nordhaus and Tobin (1972) and Stiglitz *et al.* (2009).

[10]It is impossible to cite all, or even most, of the contributions in this vein. Some of the prominent examples include Romer (1990), Aghion and Howitt (1992), Jones (1995), Kortum (1997), Segerstrom (1998), Eicher and Turnovsky (1999), Eicher and Turnovsky (1999), Acemoglu (2002), Steger (2005), Acemoglu and Guerrieri (2008), and Aghion *et al.* (2014).

[11]Prominent contributions include Aguiar and Hurst (2007a), Ramey and Francis (2009), Aguiar *et al.* (2017), Vandenbroucke (2009), Aguiar *et al.* (2012) and Scanlon (2018).

[12]Leisure inequality has increased as poorer households increased their leisure time by more than the rich (Aguiar and Hurst (2008), Boppart and Ngai (2017a)). The free leisure technologies could be important in helping to explain this divergence. Investigating this hypothesis is left for future work.

Pissarides (2008) and Boppart and Krusell (2020) provide two alternative accounts for this trend: the former paper highlights the role of differential sectoral growth rates and non-separability of preferences while the latter characterizes the preference class that delivers an income effect larger than the substitution effect along the BGP. Both of these papers and other related contributions assume growth is exogenous. Instead, this paper assumes separable balanced growth preferences and instead focuses on the endogenous rise of the attention economy.

The present paper extends the line of research recently summarized in Aguiar and Hurst (2016) which develops a unified theory of consumption and time allocation. The contribution is to develop a tractable model for analysis of zero price services. The focus on leisure technologies brings the paper close to Aguiar *et al.* (2017) who investigate how video games have altered the labor supply of young men in the United States. Relative to that paper I cast the net more broadly.[13]

The paper also contributes to the literature on the productivity slowdown and the mismeasurement hypothesis.[14] It shows that while mismeasurement of GDP (a production-based metric) is second order, a growing disconnect between GDP and measures of economic wellbeing is likely.

Finally, this paper builds on the literature on two-sided markets, intangible capital and advertising in industrial organization and in macroeconomics.[15] Relative to these literatures its contribution is to study the consequences of how intangible assets are produced.

Roadmap. Section 2 sets the scene by illustrating the growth effects of exogenous leisure technologies. Section 3 outlines the model of the attention economy and defines the equilibrium. The main results characterizing the balanced growth equilibrium are presented in Section 4. Section 5 illustrates the magnitudes. Section 6 discusses the measurement challenges. Section 7 studies the efficiency properties of the market equilibrium. Section 8 concludes with a historical

[13]The present paper speaks to historical events such as the roll-out of the TV in the 1950s as well as the more recent digital trends and considers the whole swathe of free technologies which are used by a vast majority of the population, whereas Aguiar *et al.* (2017) focus on computer games which are used primarily by young men. This paper also goes beyond the labor supply effects and explores the implications for total factor productivity, measurement and welfare.

[14]Useful references include Brynjolfsson and Oh (2012), Byrne *et al.* (2016a), Bean (2016), Bridgman (2018), Syverson (2017), Coyle (2017), Aghion *et al.* (2017), Nakamura *et al.* (2017), Hulten and Nakamura (2017), Brynjolfsson *et al.* (2018) and Jorgenson (2018).

[15]Classic references on the economics of platforms are Rochet and Tirole (2003) and Anderson and Renault (2006) who study the equilibrium pricing decisions in two-sided markets. Relative to that literature I explore the implications of the two-sided market structure in a macro setting, drawing on the lessons that this literature has offered on optimal pricing. There is an extensive literature on the economics of advertising summarized in the IO Handbook Chapter by Bagwell (2007). Several papers analyzed theoretically the way in which ads enter the consumer problem, and what the positive and normative implications are (Dorfman and Steiner (1954), Dixit and Norman (1978), Becker and Murphy (1993), Benhabib and Bisin (2002)) as well as the businesses decisions to invest in and accumulate intangible capital (Hall (2008), Corrado and Hulten (2010), Corrado *et al.* (2012), Gourio and Rudanko (2014),Cavenaile and Roldan-Blanco (2020)).

narrative through the lens of the model and a discussion of areas for future work.

2 Exogenous leisure-enhancing technological change

To illustrate the long-run growth effects of leisure technologies and to set the stage for the analysis that follows I begin with a simple setup with *exogenous* growth in leisure-enhancing technologies (denoted M) and *endogenous* growth of traditional technology (denoted A). In this section I assume that M expands exogenously at rate γ_M (throughout the paper notation γ denotes net growth rates). The economy is populated by $N = N_0 e^{nt}$ individuals with preferences over consumption and leisure that belong to the balanced growth class (King *et al.*, 1988):

$$\int_0^\infty e^{-\rho t} \left(\log c + l\right) dt \tag{1}$$

where c is per-capita consumption and l is leisure utility (I discuss the choice of the specific functional form below).[16] To generate leisure utility l, consumers engage in a range of *leisure activities*, each of which combines consumers' time with leisure services, available at zero prices (examples of activities include watching television or spending time on a social media platform). Specifically I assume that:

$$l := \left(\int_0^M \underbrace{\left[\min\{\ell_\iota, m_\iota\}\right]}_{\text{activity } \iota}^{\frac{\nu-1}{\nu}} d\iota \right)^{\frac{\nu}{\nu-1}}, \tag{2}$$

where ℓ_ι is the time spent on activity ι and m_ι denotes the (zero-price) services required for that activity (a TV channel, a social media platform, etc.).[17] There is a continuum of M activities so that total leisure time is $\ell := 1 - h = \int_0^M \ell_\iota d\iota$.[18] Parameter $\nu > 1$ is the elasticity of substitution across activities; as long as ν is finite, there is love of variety in leisure options. Within each activity, there is no substitutability between time and free services: this is a natural assumption given that the services are available free of charge: a positive elasticity would lead to a complete substitution away from time and towards free services.[19]

[16] I assume that parameters of the model satisfy $\rho > \frac{1}{\nu-1}\gamma_M$ so that household utility is finite.

[17] The units of the input m_ι are the same as the units of time spent on an activity ℓ_ι.

[18] Time endowment is normalized to 1 and h denotes hours worked in the market. Activities that do not involve free leisure technologies, such as walking in a park or hiking, are outside of the benchmark model, but are straightforward to incorporate (see Appendix G). The baseline model abstracts from home production for simplicity, and focuses on the leisure vs. market hours margin. Incorporating home production in a fuller model would be important for the quantitative implications of the model and is left for future research.

[19] In practice, besides time and leisure services, paid-for consumption goods – broadband charges, TV sets, phones or computers, for example – are inputs in leisure production. Appendix C proposes a more general leisure production function in which there are complementarities between leisure and consumption goods, and shows that the insights continue to hold in that more general formulation.

Consumers choose how much time to spend on each activity and how much of the zero price services to consume; that is, they choose the pairs $\{\ell_\iota, m_\iota\}_{\iota\in[0,M]}$. Since the services m_ι are available at zero prices, optimality implies that $\ell_\iota \leq m_\iota \forall\iota$: consumers choose at least as much leisure services as is required in a chosen time. Consequently, the problem boils down to optimally allocating time across activities. In this simple symmetric setup, the optimal allocation calls for an equal share of leisure time to be spent on each activity, $\ell_\iota = \ell/M$, implying that leisure utility is

$$l = (1-h)M^{\frac{1}{\nu-1}}. \tag{3}$$

Relative to the standard formulation where $l = \ell = 1-h$, the framework highlights the importance of technology for generating leisure utility.

The supply side of the economy is standard: it features a constant-returns final good production function, monopolistic competition in the intermediate sector, and profit-driven horizontal innovation, as in Romer (1990) and Jones (1995). I lay out all the specific assumptions in the following section, and focus here only on the process by which traditional ideas get invented. The index of technology is given by the range of intermediate inputs denoted with A, which expands as a result of R&D activity. New ideas are developed by researchers, whose success rate depends on the existing stock of knowledge:

$$\underbrace{\dot{A}}_{\text{new ideas}} = \underbrace{L^A}_{\text{researcher-hours}} \cdot \underbrace{A^\phi}_{\text{success rate}} \tag{4}$$

where $L^A := N \cdot h \cdot s_A$ is the pool of human resources employed in generating ideas and s_A is the share of labor input employed in R&D. I assume that $0 < \phi < 1$.[20]

This coarse description of the economy omits many relevant details but is sufficient to gain insights into the interactions between leisure- and traditional- technologies. Solving problem (1) we obtain that households choice of hours satisfies

$$h = \min\left\{1, \frac{\Phi}{M^{\frac{1}{\nu-1}}}\right\}, \tag{5}$$

where $\Phi := \frac{1-\alpha}{1-s_A}\frac{Y}{C}$, which is constant on the balanced growth path – an equilibrium where all variables grow at constant rates. Note that a steady growth in wages does not alter labor supply: with balanced growth preferences assumed here, income and substitution effects cancel out, isolating the effect of leisure technology on labor supply decisions. Indeed, for M sufficiently large, (5) implies

$$\gamma_h = -\frac{1}{\nu-1}\gamma_M, \tag{6}$$

[20]This places my benchmark framework within the semi-endogenous class of growth models (Jones, 1995). The evidence does indeed suggest that ideas "are getting harder to find", supporting the assumption of $\phi < 1$ (Bloom *et al.*, 2020). But the lessons here are more general and extend beyond this particular underlying growth paradigm.

that is, hours worked decline at a rate proportional to the growth rate of M.

Differentiating (4) with respect to time yields the expression for the growth rate of A on the BGP:

$$\gamma_A = \frac{n + \gamma_h}{1 - \phi}. \tag{7}$$

Combining (6) and (7) gives the following result:

Proposition 1. ***Growth effects.*** *Suppose M_0 is large and $n > \frac{1}{\nu-1}\gamma_M$. Then growth is balanced, with hours declining at a constant rate given by (6) and A increasing at a constant rate given by*

$$\gamma_A = \frac{n - \frac{1}{\nu-1}\gamma_M}{1 - \phi}. \tag{8}$$

The growth rate of A is decreasing in γ_M.

The result is simple yet striking: leisure technology growth weighs down on the growth rate of the traditional economy not just directly through its impact on the labor input, but also indirectly through TFP growth. The mechanism is straightforward: the long-run growth rate of A is pinned down by the growth rate of the pool of resources devoted to generating ideas. Leisure technologies effectively reduce the growth of that pool.

The formula in (8) is specific to the semi-endogenous growth framework underlying the analysis, but the mechanism is present in a broader class of models with expanding varieties where innovation requires real resources such as those building on Romer (1990); Grossman and Helpman (1991). A complementary interpretation is highlighted by the Schumpeterian models of growth where the diminished market size for traditional innovations that results from consumers working and thus consume less lowers the incentives to innovate in that sector (see Appendix E).

2.1 Balanced growth preferences with growing M

Utility function in (1) is linear in leisure utility – it imposes $\eta = 0$ on separable "balanced growth preferences":[21]

$$u = \log c + \frac{l^{1-\eta}}{1-\eta}, \quad 0 \leq \eta < 1. \tag{9}$$

This restriction turns out to be important for generating balanced growth when leisure technologies improve at a steady clip. To see why, consider the intratemporal optimality condition under (3) and (9):

$$\frac{w}{c} = M^{\frac{1-\eta}{\nu-1}} (1-h)^{-\eta}. \tag{10}$$

[21]The restriction that the curvature of leisure utility is less than the curvature of consumption utility ($\eta < 1$) ensures that improved leisure technology leads to an increase in time spent on leisure, in line with the empirical evidence discussed in the introduction.

Suppose there exists a balanced growth path. Condition (10) implies that in such equilibrium the following must hold: $\gamma_w - \gamma_c = \frac{1-\eta}{\nu-1}\gamma_M - \eta\gamma_{1-h}$. The budget constraint implies that consumption and labor income must grow at the same rate: $\gamma_c = \gamma_w + \gamma_h$. Together these imply:

$$-\gamma_h = \frac{1-\eta}{\nu-1}\gamma_M - \eta\gamma_{1-h}. \tag{11}$$

Since it is impossible for h and $1-h$ to simultaneously grow at constant non-zero rates and since $\eta < 1$, it must be that either $\gamma_M = \gamma_h = \gamma_{1-h} = 0$ (the standard case without leisure technologies) or that $\eta = 0$. In other words, with growth in leisure technologies, $\eta = 0$ is the restriction that is necessary for balanced growth.[22]

While leisure technology growth imposes strong parametric restrictions on utility in (9), a utility function which takes the disutility of work as an argument can be parametrized more flexibly and still be consistent with balanced growth. For example,

$$u = \log c - \frac{\omega^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \quad \omega := M^{\frac{1}{\nu-1}}h,\ \theta > 0, \tag{12}$$

where ω is the disutility of labor yields the equivalent to equation (11):

$$-\gamma_h = \left(1+\frac{1}{\theta}\right)\frac{1}{\nu-1}\gamma_M + \frac{1}{\theta}\gamma_h. \tag{13}$$

This condition reduces to (6) – and thus is consistent with balanced growth – for *any* value of the Frisch elasticity θ. The two formulations – the one with linear utility of leisure as in (1) and the one with convex disutility of work in (12) – yield identical conclusions in terms of the proportionality between the growth of hours worked and growth in leisure technology. I work with (9) throughout, chiefly because a positively valued leisure allows for meaningful discussions of measurement and of welfare. But future applied work could consider using (12) if the flexibility in setting the Frisch elasticity is important.[23]

3 Endogenous M: the attention economy

The previous Section provided a preview of the interactions between leisure- and traditional technologies but it assumed that the leisure technologies are exogenous. I now turn to the important question of what M is and how it is determined in equilibrium.

[22] In the case with $\gamma_M, \eta > 0$ growth can still be balanced asymptotically, since in the long-run $1-h$ converges to a constant ($=1$) and so γ_{1-h} converges to zero. However, hours worked approach zero at that point limiting applicability in practice.

[23] In this paper, the long-run results are exactly identical, and the transitional dynamics are almost the same if utility (12) is used.

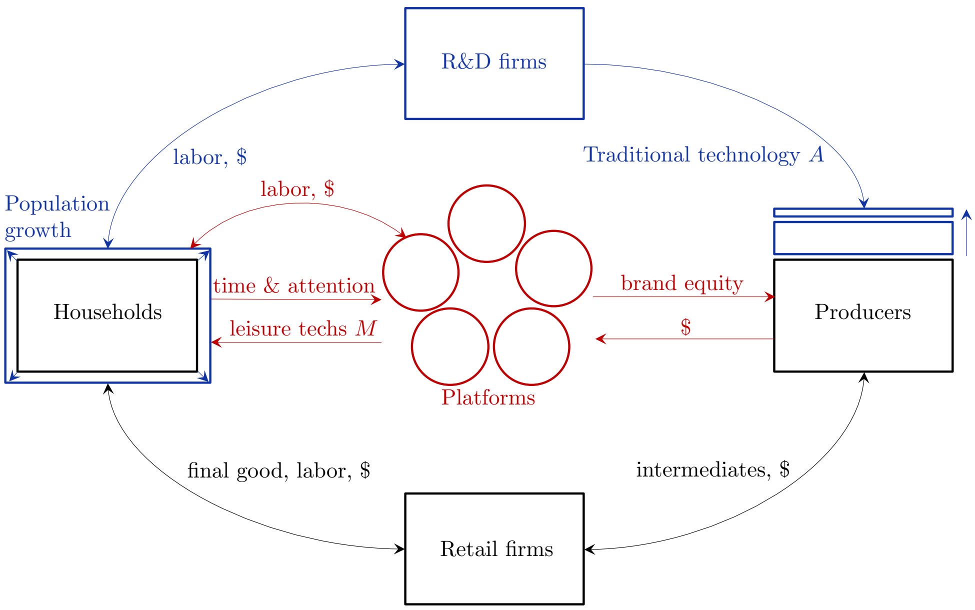

Figure 3
The Model Structure

The framework builds on the classic monopolistic competition setting (Dixit and Stiglitz, 1977) with endogenous horizontal innovation as in Romer (1990) and Jones (1995). Figure 3 illustrates the structure of the economy. I now lay out the assumptions on the behavior of the remaining agents in the economy (the behavior of households and R&D producers was outlined in the previous section).

3.1 Traditional production and brand equity competition

3.1.1 Final good

Competitive final good producers combine labor with differentiated intermediate goods $x_i,\ i \in [0, A]$. The sole departure from the benchmark expanding variety framework is that the desirability of product i is determined by the brand equity capital of its producer:[24]

$$Y = \int_0^A \left(\left(\frac{b_i}{\bar{b}} \right)^{\chi \cdot \Omega} x_i \right)^{\alpha} L_Y^{1-\alpha} di, \tag{14}$$

where L_Y is labor employed in the production of the final good, $b_i \geq 0$ is the brand equity associated with product i and $\bar{b}$ is the average brand equity across all firms: $\bar{b} := \frac{1}{A} \int_0^A b_i di$.

[24]In this simple setting each producer operates a single production line and sells only one product. With multiple production lines, large firms may find advertising more profitable than smaller firms since advertising one product has spillover effects to other products under the same brand (a phenomenon known as *umbrella advertising*). See Cavenaile and Roldan-Blanco (2020) who study this aspect of brand equity competition using a variant of Akcigit and Kerr (2018) model with advertising.

Fraction $\frac{b_i}{\bar{b}}$ measures the relative advantage of firm i due to its holdings of brand equity, as compared to its competitors.[25] Parameter $0 \leq \chi < \frac{1-\alpha}{\alpha}$ measures the perceived effectiveness of ads.[26] Indicator variable Ω equals to 1 when $\bar{b} > 0$ and 0 otherwise, making (14) well-defined when no firm invests in brand equity.

The implication of (14) is that only by investing in brand equity by more than its competitors can a firm boost demand for its product: brand equity is all about *relative* advantage. This assumption is supported by empirical evidence on advertising, starting from the early studies such as Borden (1942) and Lambin (1976), and through more recent work summarized in Bagwell (2007). This literature suggests that marketing may have some positive short-lived impact on individual firm's sales, but that the effect tends to disappear once the unit of observation is expanded to a broader sector (or to the macroeconomy).

Beyond its simplicity and empirical relevance, an advantage of this formulation is its neutrality: in a symmetric equilibrium, brand equity investments have no *direct* impact on aggregate productivity or consumer welfare (since in such equilibrium $b_i = \bar{b}\ \forall i$ and the $\frac{b_i}{\bar{b}}$ term vanishes). This is a neutral stance since there are many possible channels outside of the model but analyzed in the literature, both positive and negative, through which brand equity might affect aggregate output and welfare.[27] To give just a few examples: on the "positive" side, brand equity investments can provide consumers with useful information about available products, which might lead to fiercer competition, lowering the distortion that arises from monopoly power (Nelson, 1974; Butters, 1977; Grossman and Shapiro, 1984; Milgrom and Roberts, 1986; Stahl, 1989; Rauch, 2013); they can complement consumption goods (Becker and Murphy, 1993); or, when interpreted as accumulation of information and data, they can help firms better target consumer needs (Jones and Tonetti, 2019; Farboodi and Veldkamp, 2019). Examples of negative effects include the possibilities that the brand equity competition may lead to greater product differentiation, raising markups and exacerbating the monopoly distortion (Molinari and Turino, 2009); that aggressive advertising might become a nuisance to consumers (Johnson, 2013); that collection of mass datasets might raise privacy concerns (Tucker, 2012); and that advertising leads to envy and supports 'conspicuous consumption', ultimately diminishing consumers' utility (Veblen, 1899; Benhabib and Bisin, 2002; Michel *et al.*, 2019). Incorporating some of these channels into the theory would necessarily be somewhat ad-hoc and would detract from the focus

[25] The final-good firms anticipate any shifts in relative demand due to firms' intangible capital investments and demands more of the varieties with higher brand equity. This setup is isomorphic to the model where consumers were choosing the products directly and their relative taste for specific varieties was driven by brand equity.

[26] The framework collapses to a textbook model with $\chi = 0$. The upper limit on χ is dictated by the requirement that intermediate producers make non-negative profits in equilibrium.

[27] The literature has distinguished three broad views of advertising: the *persuasive view* which sees advertising as primarily shifting demand curves outwards or lowering the elasticities of substitution across goods; the *informative view* according to which ads help consumers make better choices; and the *complement view* which sees ads as complements to the advertised consumption goods. The formulation in this paper is most closely aligned with the persuasive view with an additional assumption that direct effects wash out in equilibrium.

of the paper. Consequently, the formulation in (14) puts these considerations aside and allows the paper to focus on the *indirect* macro effects of brand equity competition and the attention economy.[28]

3.1.2 Intermediate goods

The differentiated goods are produced by a continuum of monopolistically competitive firms. Every firm has to invest in a blueprint as the prerequisite of production. The owner of a blueprint is the only producer of the respective good. Technology is such that each unit of capital, which can be rented at net rate r and depreciates at rate δ, can be used to produce a unit of the intermediate good. Furthermore, each producer can invest in intangible capital in the form of brand equity, which can be purchased at price p_B. For simplicity, I assume that brand equity depreciates fully after use, so that producer i's problem remains static. Producer i maximizes profits:

$$\max_{x_i, p_i, k_i, b_i} p_i x_i - (r + \delta) k_i - p_B b_i \tag{15}$$

subject to the linear technology $x_i = k_i$, the demand curve for its product and taking r, p_B and $\bar{b}$ as given. The value of the maximized profit is Π.

3.1.3 Traditional R&D sector

New designs of differentiated goods are invented by the R&D sector employing researchers (equation (4)). The value of a blueprint at time t is:

$$V(t) = \int_t^\infty \Pi(\tau) e^{-\int_t^\tau r(u) du} d\tau. \tag{16}$$

There is free entry to R&D so that

$$V \cdot A^\phi = w. \tag{17}$$

3.2 Platforms

3.2.1 Market structure

I assume that there are J platforms that engage in Cournot competition in the brand equity market and that these firms take aggregate variables as given. Parameter J determines the degree of competition and mark-ups in the platform sector (this is without loss of generality as the structure of the equilibrium would be the same with infinitesimal platforms or under free entry, or if the platforms internalized the impact on their choices on the average level of brand equity). I also make the following assumption:

[28] Appendix H considers two non-neutral ways of modeling brand equity competition.

Assumption 1: Platforms do not charge for leisure services: their price is zero.

The introduction and Appendix A present empirical basis for this assumption. From a theoretical standpoint zero prices can arise a result of optimal pricing behavior in two-sided markets characterized by asymmetric externalities and differing elasticities of demand (and likely some transaction costs which prevent prices for going negative). To explore this possibility in more detail, Appendix D derives the optimal pricing strategy of a monopoly platform and shows that the optimal price charged on the consumer side might be zero or negative when consumer demand is highly elastic and when the interaction externalities are asymmetric. These are exactly the conditions that are likely to be satisfied in the context of the attention economy: consumers exert positive externalities on the advertisers and on each other (ad watching and network effects, respectively), while advertisers do the opposite (if ads are a nuisance to consumers, and if congestion limits their effectiveness).[29] Incorporating these features explicitly into the dynamic model is beyond the scope of this article, which instead studies the consequences of this business model on the macroeconomy. Future work might consider different conditions under which platforms charge a positive price, a zero price, or indeed pay their consumers for use of their services.

3.2.2 Technologies

Platforms are endowed with two technologies. First, to produce brand equity, they must capture consumers' time:

$$B_j = \ell_j \qquad \text{where} \qquad \ell_j = \ell \cdot \frac{M_j}{M}. \tag{18}$$

The amount of brand equity produced is linear in consumers' time captured by platform j,[30] and j's share of consumers' time is determined by the share of leisure technologies that platform j supplies. Second, platforms operate a technology for generating varieties of leisure activities M: *the leisure ideas production function*. I consider two formulations:

$$\text{Dynamic: } \dot{M}_j = L_j^M \cdot A^\phi \tag{19}$$

$$\text{Static: } M_j = L_j^M \cdot A^\phi. \tag{20}$$

[29] A complementary explanation relies on competition and strong non-rivalry. Since the marginal cost of providing an extra user with a leisure technology that already exists is zero, a high degree of competition between platforms could depress prices towards the marginal cost and possibly beyond (again, transaction costs might account for exactly zero prices in equilibrium). This could be an equilibrium since firms can cover for their costs on the brand equity side of the business. Another explanation could be that, in a model with firms life-cycle, entry and exit, firms may find it optimal to charge zero prices to build customer base.

[30] The particular form of (18) is chosen for parsimony. The production function of brand equity could also include other inputs, such as labor or capital, without altering the conclusions of the analysis. Clearly, the important point is that consumers' leisure time is an input in production of brand equity.

where $-1 \leq \phi < 1$ and A is the stock of existing knowledge in the economy.[31] The dynamic formulation follows the tradition in growth theory literature and assumes that new leisure technologies are added to the existing stock, mirroring the ideas production function used in the traditional R&D sector and hence putting leisure technologies on an equal modeling footing with the traditional technologies. The static alternative assumes that leisure technologies depreciate every period and so may be interpreted as content. The long-run results of the paper hold for either of these two formulations (see Proposition 4 below). The advantage of the static formulation is that it possible to derive the equilibrium supply of leisure technologies analytically, which is useful to gain the intuition. For that reason I use the static formulation (20) in the main text, and I delegate the analysis of the dynamic formulation to Appendix F.[32]

3.2.3 Aggregation

Brand equity output is homogenous across platforms, so that the aggregate supply is simply $B = \sum_J B_j$. Similarly, we have: $M = \sum_J M_j = L^M A^\phi$.

3.3 Equilibrium definition

Definition 1. The *almost-perfect foresight equilibrium* is a set of paths of aggregate quantities $\{Y, C, K, A, M, B, h, \ell, s_A, s_M\}_{t=0}^{\infty}$; micro-level quantities $\{x_i, b_i, h_\iota, B_j\}_{t=0}^{\infty} \forall i, \iota, j$; prices $\{p_i, p^B, w, r\}_{t=0}^{\infty} \forall i$ and platform activity indicator $\{\Omega\}_{t=0}^{\infty}$ such that: households choose consumption and time across leisure activities and work to maximize utility in (1) taking all aggregate variables as given; final-output producers choose $\{x_i\}$ and L_Y to maximize profits taking all aggregate variables, $\{b_i\} \forall i$ and $\bar{b}$ as given; intermediate producers choose p_i and b_i to maximize profits, taking $\bar{b}$ and other aggregate variables as given; platform j chooses B_j to maximize profits taking actions of all other platforms $B_k \forall k \neq j$, the average level of ads $\bar{b}$ and all aggregate variables as given; there is free entry to the traditional R&D sector; wages are equal across sectors; labour, goods and brand equity markets clear so that $L_Y = (1 - s_A - s_M) Nh$, $Y = \dot{K} + C + \delta K$, $A\bar{b} = B$. If $B_j(t) = 0$ then the indicator function Ω_t takes value zero and is equal to 1 otherwise. Finally, if for all $t' \leq t$, $B_j(t') = 0 \ \forall j$ then $\mathbb{E}_t B_j(t'') = 0 \ \forall j$ for at any $t'' > t$. Otherwise agents have perfect foresight.

---

[31] For the sake of transparency I assume that parameter ϕ which governs the magnitude of increasing returns to R&D is the same in the traditional- and the leisure-enhancing sector.

[32] The long-run growth results are also robust to alternative formulations of the leisure production function; for example, M could be produced using final output or there could be two-way knowledge spillovers and the long-run results would continue to hold. In the latter case we would have $M_j = L_j^M \cdot (A + M)^\phi$ and perhaps $\dot{A} = L^A \cdot (A + M)^\phi$ if the leisure technologies can affect innovation in the traditional sector. It is straightforward to show that A and M grow at the same rate in equilibrium: if $X := A + M$ then $\frac{\dot{A}}{A} = L^A \cdot X^\phi / A$, and so $0 = n + \gamma_h + \phi\gamma_X - \gamma_A$. We also have $\gamma_M = n + \gamma_h + \phi\gamma_X$. These two equations imply that $\gamma_M = \gamma_A = \gamma_X$ and $\gamma_A = \frac{n+\gamma_h}{1-\phi}$. But while the long-run results are unchanged, the formulation in (20) is more convenient as the equilibrium supply of leisure technologies can be expressed in closed form.

The final element of this definition is that agents do not anticipate leisure technologies if no leisure technologies had ever existed. It is unlikely that firms and consumers can anticipate the emergence of new kinds of technologies that have never existed. This formulation is also convenient since it ensures growth is exactly balanced when $\bar{b} = 0$ and it allows for a tractable analysis of endogenous entry of the platforms along the growth path. I study the perfect foresight equilibrium numerically in Appendix J and show that this makes little difference to the results.

4 The segmented balanced-growth path

Goal and strategy. In most models of economic growth the balanced growth path can be characterized by computing the constant growth rates of model variables. The balanced growth path in this paper instead consists of *two segments* along which growth is balanced, with a transition in between. When the economy is smaller than a certain threshold, platforms are inactive and there is no leisure-enhancing technological change (segment 1); as the economy grows, at some point leisure innovations appear, the economy adjusts, and asymptotically growth is again balanced (segment 2). The goal of this Section is to prove that the growth path indeed takes this segmented form, and to characterize segments 1 and 2 analytically (the following Section then quantifies the effects described here and numerically computes the transition path).

The strategy for characterizing the equilibrium is as follows. I first guess that some platforms are active. Under this guess I compute the equilibrium as an intersection of (1) the household optimal choice of hours for a given level of leisure technologies, with (2) the platforms' optimal supply of leisure technologies for a given level of hours. This approach lends itself to a graphical analysis which gives the intuition on the equilibrium dynamics. I then find the conditions under which it is indeed optimal for the platforms to operate.

4.1 Equilibrium time allocation and the leisure technologies

Appendix B contains the solution to the representative household's problem (1); the main result is summarized in the following lemma:

Lemma 1. Hours worked and leisure technology. *Optimal hours worked satisfy:*

$$h = 1 - \ell = \min\{1, \Phi M^{\frac{1}{1-\nu}}\} \tag{21}$$

where $\Phi := \left(\frac{Y}{C}\frac{1-\alpha}{1-s_A-s_M}\right)^{\frac{\theta}{1+\theta}}$ *is a variable that is constant when growth is balanced.*

Proof. Appendix B. □

When leisure technologies are not well developed, households optimally choose the corner solution $h = 1$, with no time spent on marketable leisure. For M sufficiently large, hours worked vary inversely with the measure of available leisure options (recall that $\nu > 1$).[33]

4.2 Equilibrium supply of leisure technologies

4.2.1 Demand for brand equity

Equilibrium supply of M is ultimately determined by the equilibrium supply of brand equity B: for the platforms, leisure technologies are strictly a means to an end. This and the next subsection compute equilibrium B.

Starting on the demand side, solving (15) gives the following results:

Lemma 2. Demand curve and intermediate profits. *Firm i's (inverse) demand for brand equity b_i satisfies:*

$$p_B = \alpha^2 \chi \frac{Y}{A} \frac{1}{\bar{b}} \left(\frac{b_i}{\bar{b}} \right)^{\frac{1}{\varepsilon}} \tag{22}$$

where $\varepsilon = -\frac{1}{1-\frac{\alpha}{1-\alpha}\chi}$.

In a symmetric equilibrium all firms choose identical brand equity investments: $b_i = \bar{b}$ and so

$$p_B = \alpha^2 \chi \frac{Y}{B}. \tag{23}$$

Equilibrium prices, quantities and revenues in the intermediate sector are "as if" there was no brand equity competition. Equilibrium profits of an intermediate firm are

$$\Pi = \alpha \frac{Y}{A} \left(1 - \alpha - \alpha\chi\right), \tag{24}$$

which is lower than $\alpha \frac{Y}{A}(1-\alpha)$, the value of profits with no brand equity competition.

Proof. Appendix B. □

Brand equity competition lowers firm profits: each firm spends money on ads, but in equilibrium this spending fails to deliver.[34] This will have important implications for the innovation

[33] The implication of the theory that there is a causal link between leisure technology and total leisure time receives strong empirical support. For example, Gentzkow and Shapiro (2008) and Aguiar and Hurst (2007b) show how the introduction of television substantially raised leisure consumption in the United States. Falck *et al.* (2014) identify exogenous geographical variation in the speed of the roll-out of broadband internet in Germany and document the significant boost to leisure consumption as high-speed internet became available. Using proprietary data on television and internet subscriptions, Reis (2015) documents that television shows and internet content are imperfect substitutes, supporting the prediction that more plentiful leisure varieties increase overall leisure time.

[34] Note also that firms do not pass the costs associated with brand equity onto consumers. The reason is that the cost of brand equity does not affect their marginal cost.

incentives, the issue I return to below.[35]

4.2.2 Platforms' cost structure

Equations (18) and (20) imply the following derived production function for brand equity:

$$B_j = L_j^M \cdot \frac{\ell}{M} A^{\phi} \tag{25}$$

Using this together with equation (21), platform $j's$ cost function is:

$$\mathbb{C}(w, B_j; M, A, \ell) = B_j \cdot w \cdot \frac{M}{\ell A^{\phi}}. \tag{26}$$

That is, at any t platform j faces a marginal cost $\mathcal{M}^B := w\frac{M}{\ell A^{\phi}}$ that it takes as given. Note that the marginal cost will be changing *over time* as the aggregate variables M, A and ℓ change, but it is independent of the quantity produced at any instance.

4.2.3 Platform's problem

Platform j solves

$$\max_{B_j \geq 0} p_B \left(B_j + \sum_{k \neq j} B_k \right) \cdot B_j - B_j \cdot \mathcal{M}_B \tag{27}$$

where $p_B \left(B_j + \sum_{k \neq j} B_k \right)$ is the inverse demand for brand equity and B_k is the output level of platform k, $k \neq j$. This is a textbook Cournot competition problem: each platform acts as a monopolist facing the demand curve $p_B \left(B_j + \sum_{k \neq j} B_k \right)$, taking the actions of its competitors as given. Since in equilibrium $b_i = \frac{\sum B_j}{A}$ by the symmetry of the choices of the intermediate firms, equation (22) implies that the demand curve can be written as follows:[36]

$$p_B \left(B_j + \sum_{k \neq j} B_k \right) = \alpha^2 \chi \frac{Y}{B_j + \sum_{k \neq j} B_k} \left(\frac{B_j + \sum_{k \neq j} B_k}{A\bar{b}} \right)^{\frac{\alpha}{1-\alpha}\chi}. \tag{28}$$

Solving the Cournot game in (27) given (28) yields the next lemma:

[35] Lemma 2 shows that competition through brand equity can be easily incorporated in the monopolistically competitive setting with tractable closed-form results such as the constant elasticity demand function in (22). Since the monopolistically competitive setting is present in a vast number of application in economics, it would be straightforward to consider brand equity competition in those models as well, demonstrating a potentially wider applicability of the formulations developed here.

[36] Note that, in line with Definition 1 of the equilibrium, each platform takes the average brand equity investments in the economy $\bar{b}$ as given. This assumption is more realistic for large J, and in particular when $J \to \infty$ and platforms are infinitesimal. This assumption is innocuous however, because if the platforms do internalize the impact of their choices on $\bar{b} := \frac{B_j + \sum_{k \neq j} B_k}{A}$, the equilibrium mark-up is simply $(1 - 1/J)^{-1}$ and all the results in the paper continue to hold.

Lemma 3. Supply of brand equity. *The price of brand equity in the Nash equilibrium of the Cournot competition game is equal to markup over the marginal cost, where the markup is:*

$$\Psi := \frac{p_B}{\mathcal{M}_B} = \frac{1}{1 - \left(1 - \frac{\alpha}{1-\alpha}\chi\right)\frac{1}{J}}. \tag{29}$$

4.2.4 Equilibrium supply of leisure technologies

Combining equations (18), (23) and (29) yields the following lemma:

Lemma 4. Equilibrium supply of leisure technologies. *When $h = 1$, platforms are inactive: $B_j = M_j = 0 \forall j$ and $\Omega = 0$.*

Whenever $h < 1$ platform j's profits are non-negative:

$$\Pi_j^B = B_j \mathcal{M}_B (\Psi - 1) \geq 0,$$

and the equilibrium supply of leisure technologies M satisfies:

$$M = \kappa L_Y A^{\phi}. \tag{30}$$

where $\kappa = \frac{\alpha^2 \chi}{\Psi(J)(1-\alpha)}$ is a constant.

Lemma 4 states that whenever households choose to spend positive amount of time on leisure, platforms can make positive profit. In that case the equilibrium supply of leisure technologies depends positively on the size of the economy (hours worked, population and technical advancement), because a larger economy generates more demand for brand equity and because it makes the leisure technologies cheaper to produce. If households spend no time on leisure, platforms have no way of making a positive return and they remain inactive.

4.2.5 Existence and uniqueness

Equations (21) and (30) readily give the following result:

Proposition 2. Existence and uniqueness. *The equilibrium exists and is unique.*

4.3 Graphical representation

Figure 4 illustrates the equilibrium graphically as the intersection of two curves: the "Time allocation" curve (equation (21)) and the "Leisure technology supply curve" (equation (30)). Since the slope of the latter depends on the levels of N and A which are grow over time, this curve continuously rotates clockwise.[37]

[37] The intuition for why this curve is upward sloping is simply that higher level of hours worked translates into higher output and thus to greater demand for brand equity, thus supporting a higher supply of leisure technologies

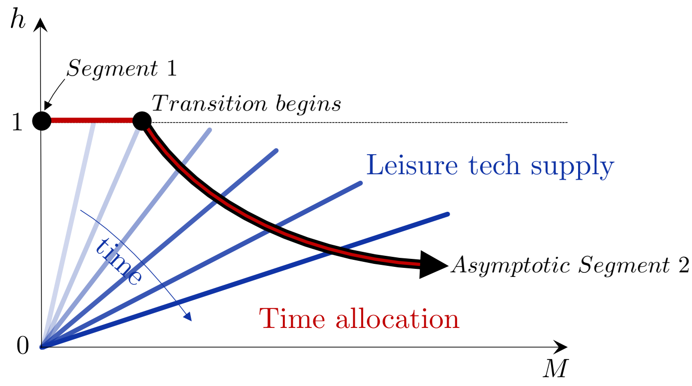

Figure 4
The segmented balanced growth path

Note: The Leisure Technology Supply curve rotates clockwise over time as the economy grows and demand for brand equity rises. When this curve crosses the Time Allocation curve on the flat segment with $h = 1$, platforms are inactive and the equilibrium is at point $[0, 1]$. The sBGP thus consists of Segment 1 with zero leisure technologies (the $[0, 1]$ point in the diagram), the transition following the entry of the platforms (the thick downward sloping curve), and segment 2 where growth is asymptotically balanced (the thick arrow). This illustrative representation is only approximate in that it ignores the shift in the Time Allocation curve during the transition (as a result of changes in $\Phi(t)$ which is time varying along the transition). I compute the full transition path numerically below.

As long as the economy is small and the two curves cross on the flat section of the "Time allocation" curve, there is no leisure-enhancing technological change and $M = 0$ in equilibrium. Once the economy is sufficiently large and the "Leisure tech supply" is sufficiently flat, the two lines cross at $h < 1$, and the equilibrium coincides with the crossing point of the two curves.

4.4 Origins of the attention economy

When do the leisure technologies first emerge?

Proposition 3. The condition for leisure-enhancing technological change. *Platforms are active and there is leisure-enhancing technological change if*

$$N(t) \geq \Gamma, \tag{31}$$

where Γ *is a variable that is constant.*

Proof. See Appendix B. □

Proposition 3 describes a watershed moment for an economy, which occurs when the "Leisure tech supply" curve first crosses the "Time allocation" curve at the interior value of h (i.e. $h < 1$). Since N grows exponentially and Γ is constant, the proposition shows that it is only a matter of time when the leisure technologies emerge in equilibrium.

Given this result, the balanced growth equilibrium can be formally defined as follows:

in equilibrium. The curve rotates for similar reasons.

Definition 2. A *segmented balanced growth path* (sBGP) is an equilibrium path along which: (i) when (31) is not satisfied, per capita consumption, output and the measure of varieties A all grow at a constant rate; (ii) as $t \to \infty$, per capita consumption, output, A and M grow at possibly distinct but constant rates, and h decreases at a constant rate.

To facilitate a tractable characterization of the sBGP I make the following assumption about the initial levels of the state variables in this economy:

Assumption 2: Initial levels of capital and technology K_0 and A_0 are such that growth is balanced and $h = 1$ for all $t < \hat{t}$, where $\hat{t}$ satisfies (31) with equality.[38]

4.5 Long-run growth effects of leisure technologies

How does the nature of growth change as a result of leisure-enhancing technologies? The following proposition shows what happens to the growth rates in segments 1 and 2 of the sBGP.

Proposition 4. Growth along the sBGP. *Suppose Assumptions 1 and 2 hold and let $\hat{t}$ be such that $N(\hat{t}) = \Gamma$.*

For $t \leq \hat{t}$, platforms are inactive, there is no leisure enhancing technological change, hours worked are constant and equal to 1, and per capita consumption, per capita output, wages and TFP all grow at the same constant rate given by:

$$g := \frac{n}{1-\phi}. \tag{32}$$

For $t \geq \hat{t}$, platforms are active and the economy transitions to segment 2 of the sBGP. Asymptotically, hours worked decline at a constant rate

$$\gamma_h = -\frac{n}{(\nu-1)(1-\phi)+1} \tag{33}$$

and the growth rates of traditional- and leisure technologies are equal and given by:

$$\gamma_A = \gamma_M = \frac{n}{1-\phi+\frac{1}{\nu-1}} < g. \tag{34}$$

Per-capita output and consumption grow at:

$$\gamma_{\bar{y}} = \gamma_A \left(\frac{\nu-2}{\nu-1}\right) < g \tag{35}$$

which is positive if $\nu > 2$.

These long-run results hold irrespectively of whether the leisure ideas production function assumes a dynamic (19) or a static (20) formulation.

[38] In the formulation with a dynamic leisure ideas production function in (19), I further assume that M_0 is zero.

Proof. See Appendix B. □

The first part of Proposition 4 derives the growth rate of the economy on segment 1 of the sBGP. The expression in (32) is familiar from the canonical semi-endogenous growth model of Jones (1995).[39] Along segment 1 platforms are inactive, M is zero, labor supply is constant, and firms and households do not anticipate the future entry of platforms (in line with the equilibrium definition above). Segment 1 thus serves as a convenient benchmark against which to compare the economy once the leisure sector emerges.

The second part of the Proposition mirrors the results in Proposition 1. In the asymptotic segment 2, hours worked are no longer constant but are instead falling at a constant rate. The speed of the decline is governed by the elasticity of substitution across leisure varieties ν. The effect vanishes in the limit as $\nu \to \infty$ and leisure varieties become perfect substitutes.

Along the sBGP leisure technologies grow at the same rate as traditional technologies. This implication is a straightforward corollary of the fact that the leisure ideas production function (equations (19) or (20)) takes the same form as the ideas production function in the traditional sector (equation (4)).

The emergence of leisure-enhancing technologies is associated with a decline in the long-run growth rate of traditional technology. The mechanics of this effect are the same as those that underlie Proposition 1, and the economic intuition is that the heightened competition for time and attention that results from leisure technologies leaves less resources available for productive activities.

Proposition 4 also says that the declining hours worked and slower growth in hourly productivity combine to deliver a potentially sizeable slowdown in the growth rate of per-capita output and consumption. Indeed, for low values of ν the effect can be so powerful as to reduce the growth rate to zero or below.

4.6 Allocative effects of leisure technologies

Following the entry of the platforms, workers can be employed in the traditional R&D sector (share s_A), the leisure R&D (share s_M) and in the production sector (residual share $1 - s_A - s_M$). The allocation of labor in the long-run is as follows:

Proposition 5. Allocation of labor on the sBGP. *For $t < \hat{t}$ (in segment 1 of the sBGP) the share of labor employed in the platform sector s_M is zero. The share of labor in the A sector is:*

$$s_A = \frac{1}{1 + \frac{1-\alpha}{\Delta_1}} \quad \textit{where} \quad \Delta_1 = \alpha\,(1-\alpha)\,\frac{g}{\rho + g} \tag{36}$$

This share is increasing in Δ_1 and thus increasing in g.

[39] The only difference is that I have implicitly assumed no R&D duplication externalities.

In segment 2 of the sBGP the share of labor employed in the R&D sector converges to

$$s_A = \frac{1}{1+\frac{1-\alpha}{\Delta_2}} \quad \textit{where} \quad \Delta_2 = \alpha\left(1-\alpha-\alpha\chi\right)\frac{\gamma_A}{\rho+\gamma_A}\left(1+\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}. \tag{37}$$

Since $\chi > 0$ and $\gamma_A < g$, the share of labor in traditional R&D is lower once the attention economy emerges. The share of labor employed by platforms in leisure-enhancing research is:

$$s_M = \frac{1-s_A}{1+\left(\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}}. \tag{38}$$

Proof. Appendix B. ☐

The leisure-driven structural change shifts the allocation of labor away from traditional R&D and towards leisure-enhancing R&D via three channels, which can be seen directly in the closed-form expression for Δ_2 in Proposition 5:

$$\Delta_2 = \alpha\left(1-\alpha-\underbrace{\alpha\chi}_{\text{lower profits}}\right)\cdot\underbrace{\frac{\gamma_A}{\rho+\gamma_A}}_{\text{fewer inventions}}\cdot\underbrace{\left(1+\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}}_{\text{competition for researchers}}.$$

The first channel reflects the hit to intermediate producers' profitability. A share of firm revenues finds its way to the platform sector. As a result, each newly invented blueprint – whose value is a discounted sum of future profits – is worth less, lowering incentives to innovate. The 'fewer inventions' channel operates via lowering the pace at which new ideas are being invented and hence diminishing the productivity of researchers.[40] Finally, there is the 'competition for researchers' channel as traditional R&D firms must compete with platforms in the labor market. A share of workers who in the absence of leisure technologies would work in the traditional R&D sector find employment in the leisure sector instead.[41]

Each of these three channels lowers the long-run value of s_A. Within the semi-endogenous framework this has no impact on the long-run growth rate of technology – the causation runs from γ_A to s_A and not vice-versa, and the effect on growth is temporary, generating persistent *level* effects.[42] In models with scale effects in which the level of profitability or the size of the pool of potential researchers has an impact on growth rates, these effects would affect the long-run growth rate.

[40]This is only partly offset by a less crowded market that results from the slowdown in growth. The first of these two effects shows up in the nominator and the second in the denominator of $\frac{\gamma_A}{\rho+\gamma_A}$.

[41]These predictions are broadly consistent with the rise of importance of R&D in software and entertainment reported in Figure 9 in Jones (2015). Further corroborating evidence on the share of R&D spending in sectors most closely related to leisure are presented in Appendix A.

[42]Since the transition takes a long time, the effects on the growth rates can be persistent.

5 Quantification

The analytical results allows for a sharp characterization of the growth process in segment 1 and in the asymptotic segment 2. This Section provides an illustrative quantification of the long-run effects and solves for the transitional dynamics between the two segments, contrasting the predictions with the data.

5.1 sBGP as a dynamic system

If an economy admits a balanced growth path, the equilibrium can be written as a system of differential-algebraic equations in normalized variables that are constant on such a path. This is also the case here.[43]

Proposition 6. Equilibrium as a dynamic system. *Let* $\gamma_A := \frac{n}{1-\phi+\Omega\frac{1}{\nu-1}}$, $\gamma_Y := n + \left(\frac{\nu-2}{\nu-1}\right)^{\Omega}\gamma_A$, $\beta_A := \gamma_A/n$ *and* $\beta_Y := \gamma_Y/n$ *where* $\Omega = 0$ *if* $t < \hat{t}$ *and* $\Omega = 1$ *otherwise. Let the lower case letters denote the variables that are constant along the two segments of the sBGP:* $a := \frac{A}{N^{\beta_A}}$, $k := \frac{K}{N^{\beta_Y}}$, $c := \frac{C}{N^{\beta_Y}}$, $v := \frac{V}{N^{\beta_Y-\beta_A}}$, $\pi := \frac{\Pi}{N^{\beta_Y-\beta_A}}$, $y := \frac{Y}{N^{\beta_Y}}$, $\tilde{h} := \frac{h}{N^{\frac{1}{1-\nu}\beta_A}}$. *For given levels of* K_0 *and* A_0, *the dynamic equilibrium is the solution to the system:*

$$\dot{k} = y - c - \delta k - \gamma_Y k \tag{39}$$

$$\dot{a} = a^{\phi} s_A \tilde{h} - \gamma_A a \tag{40}$$

$$\dot{c} = c\,(r - \rho - \gamma_Y) \tag{41}$$

$$\dot{v} = v\,(r - (\gamma_Y - \gamma_A)) - \pi \tag{42}$$

$$(1-\alpha)\frac{y}{1 - s_A - s_M} = v a^{\phi}\tilde{h} \tag{43}$$

$$y = k^{\alpha}\left((1 - s_A - s_M)\tilde{h}a\right)^{1-\alpha} \tag{44}$$

$$\tilde{h} = \left(h^{\hat{t}}\right)^{\frac{\Omega-1}{\nu}}\left(\Phi^{1-\nu}\frac{\alpha^2}{\Psi(1-\alpha)}\chi(1 - s_A - s_M)a^{\phi}\right)^{-\frac{\Omega}{\nu}} \tag{45}$$

$$r = \alpha^2\frac{y}{k} - \delta \tag{46}$$

$$\pi = \alpha\frac{y}{a}\left(1 - \alpha - \alpha\chi\Omega\right) \tag{47}$$

$$s_M = \Omega\frac{1 - s_A}{1 + \left(\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}} \tag{48}$$

and the transversality condition (see equation (75) in Appendix (B)), where $h^{\hat{t}} := \Phi(\hat{t})^{1-\nu}\frac{\alpha^2}{\Psi(1-\alpha)}\chi(1 -$

[43] In Appendix F I derive the stationary form under the assumption that the leisure ideas production is given by equation (19).

$s_A(\hat{t}) - s_M(\hat{t}))a(\hat{t})^{\phi}$ *follows from the fact that hours worked do not jump at* $\hat{t}$ *and* $\Phi = \frac{1-\alpha}{1-s-s_M}\frac{y}{c}$.[44]

Proof. Appendix B. □

The system can be used to compute the transitional dynamics following $\hat{t}$.[45]

5.2 Calibration

Parametrization corresponds to annual frequency, with the discount rate of 1% and population growth of 1% per annum. Several parameters are calibrated to standard values: the capital share α equals 0.35, the depreciation rate δ is 5% per year.

Parameter ϕ guides the degree of increasing returns to innovation and determines the steady state growth rate of the economy. Recent work by Bloom *et al.* (2017) has found that the ϕ parameter varies widely across sectors in the US economy, but is likely to be well below 1. I set it to 0.5, targeting the growth rate of the economy in segment 1 of $g = 2\%$.

The elasticity across leisure varieties ν pins down the strength of the link between leisure technologies and time allocation choices (a higher ν makes this link weaker). To get a sense of the plausible magnitudes it is useful to consider estimates of other elasticities from the existing literature. For example, Goolsbee and Klenow (2006) estimate the elasticity between internet vs. everything else of about 1.5. The calibrated value of ν needs to be significantly higher than this. In trade literature, Broda and Weinstein (2006) study the welfare gains from increased variety as a result of the rising trade penetration in the US economy. In the process, the authors estimate thousands of elasticities of substitution between similar products imported from different countries. For example, they establish that the elasticity of substitution across cars (apparel and textiles) imported from different countries is around 3 (6). Within products classified as differentiated, the median elasticity is around 2 and the mean is about 5. Given these estimates I set $\nu = 4$. This is a cautious calibration, in part on account of the fact that leisure technologies substitute for non-marketable leisure activities and not solely for work hours (see Appendix G). I explore the robustness of the numerical results to different values of ν in Appendix I.

[44] Equation (39) describes the evolution of the capital stock in the economy; (40) is a stationary ideas production function; (41) is the Euler equation; (42) is the Bellman equation for the value of the blueprint; (43) denotes equality of wages across sectors; (44) is the production function; (45) is an equation that pins down equilibrium hours worked; (46) is the capital demand equation; (47) are the profits of intermediate firms; and (48) is the share of labor in the platform sector. Note that when $\Omega = 0$, the model collapses to a stationary representation of the Jones (1995) economy. Note also that the equation for normalized hours worked follows from the fact that there can be no jump in hours worked at $\hat{t}$: if there was a jump, each platform could increase its profits by entering at $t < \hat{t}$.

[45] I compute the transition of the model using the relaxation algorithm developed by Trimborn *et al.* (2008). Once the model is parametrized, the transition path can be computed as a response of the system to a change in Ω from 0 to 1 and in the pair ν, χ from $[+\infty, 0]$ to the calibrated parameter values. This gives the values of the normalized variables over the transition. The final step is to convert the normalized variables back into original units. For this we need to compute $N(\hat{t})$, the population size at which leisure enhancing technological change first emerges. At $\hat{t}$ the optimal "shadow" choice of hours worked crosses unity from above – in other words, there is no jump in h at $\hat{t}$. Therefore, by the definition of h, $N(\hat{t})$ solves $N(\hat{t}) = \left(h^{\hat{t}}\right)^{(1-\nu)(1-\phi)-1}$.

Parameter	Description	Value	Target / source
ρ	Household discount rate	0.01	$r \approx 4\%$
n	Population growth	0.01	AEs data
α	Capital share	0.35	standard calibration
δ	Capital depreciation	0.05	standard calibration
ϕ	Returns to ideas in R&D	0.5	Bloom *et al.* (2020)
J	Number of platforms	5	high degree of concentration
χ	Perceived effectiveness of brand equity	0.08	empirical elasticities
ν	Elasticity of substitution between leisure activities	4	see text

Table 1
Model calibration

Parameter χ corresponds to the perceived effectiveness of brand equity. I set this parameter to 0.08, which means that for each producer a unilateral doubling of brand equity is expected to increase quantity sold by 5%.[46] This is motivated by the consensus in the empirical literature that estimates the effectiveness of advertising (Bagwell (2007), DellaVigna and Gentzkow (2010), Lewis and Reiley (2014), Lewis and Rao (2015)). Finally, I set J equal to 5, to capture the high degree of concentration in the market. The results are insensitive to this choice.

5.3 The magnitude of long-run growth effects

Plugging in the parameter values into the formulas in Propositions 4 and 5 revels that leisure technologies can have substantial macroeconomic effects. The model predicts hours worked declining by around -0.4% per annum. The growth of traditional technology falls from $g = 2\%$ to $\gamma_A = 1.2\%$ along the sBGP. The share of workers in traditional R&D sector is 4% initially and falls to 2.5%, with the platforms employing 1.5% of the workforce. I now turn to how these magnitudes compare to the trends observed in the data.

5.4 Confronting the model with the data

In order to compare the simulated transition path with the observed trends one must first decide on the empirical counterpart to $\hat{t}$: the point at which platforms first become active. This inevitably requires some judgement. One plausible candidate is the mass roll-out of television, which in the case of the United States started in the late 1940s in the midst of the post-war boom. Television is widely recognized to have revolutionized the world of mass-available leisure. In the United States, adoption along both the extensive and the intensive margins was rapid, with average time spent watching TV of 1.5 hours per day by mid-1960s (Figure 5).[47] The remainder

[46] To see this, note that equilibrium quantity sold is $x(i) = [\alpha^2 \left(\frac{b(i)}{b}\right)^{\alpha\chi} L_Y^{1-\alpha}]^{\frac{1}{1-\alpha}}$, thus the elasticity to intangible capital is $\frac{\alpha\chi}{1-\alpha}$, which is roughly 0.05 for $\alpha = 0.35$ and $\chi = 0.08$.

[47] Adoption of the radio in the mid-1920s would be another candidate. While adoption of radio receivers occurred before World War 2, the top right panel in Figure 6 shows that the number of radio stations grew rapidly after the war, around the same time as television was being rolled out, lending some support to the choice of 1950 as

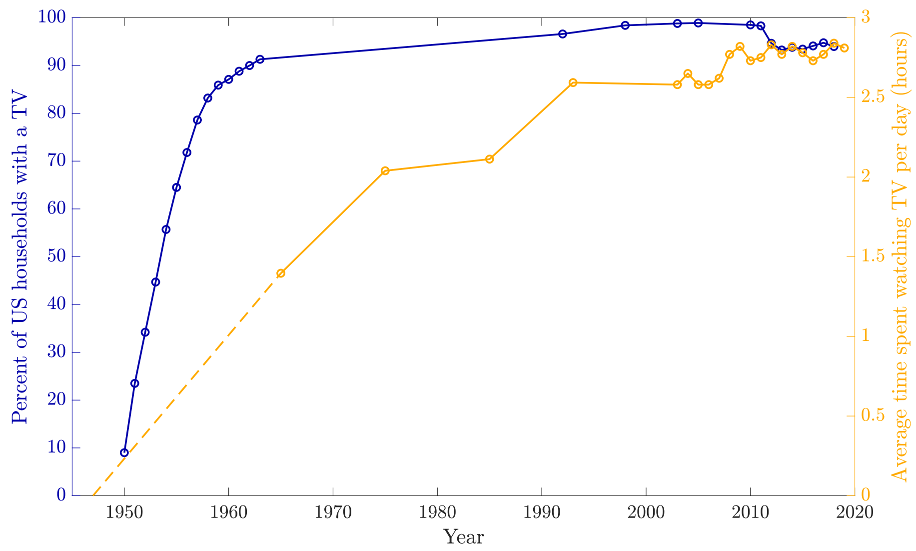

Figure 5

Adoption of TV in the United States

Sources: American Time Use Survey, Aguiar and Hurst (2007b) and Comin and Hobijn (2009). Notes: the dashed line joins the first point available in the data on time use (1965) together with 1947, when fewer than 0.5% of households had a TV set installed at home – a proportion clearly too limited to show up in average time use across the population (source: Televisor Monthly, 1948, accessed via http://earlytelevision.org/us_tv_sets.html).

of this section assumes the perspective that leisure economy has emerged in the post-war years and traces out its effects.

Figure 6 plots the transition path generated by the model (black circled lines) against the trends observed in the data starting in 1950. The model matches the modest size of the leisure sector measured by Nakamura *et al.* (2017) as the cost of production of zero-price goods and services (panel A). Despite its modest size, the leisure sector has large macroeconomic effects: the emergent leisure technologies (panel B) lead to a substantial decline in hours worked (panel C) and a slowdown in the growth rate of total factor productivity (panel D). In this illustrative calibration, leisure technologies can account for all of the decline in average hours worked and for around a half – or 1 percentage point – of the slowdown in traditional TFP growth.[48]

Simulating platform entry predicts a constant size of the leisure sector over the transition, while in the data we observe an increase that starts in the late 1970s (panel A). To provide an illustration of what might drive this increase and what the macroeconomic effects may be, the lines with filled circles in Figure 6 show the transitional dynamics if, in addition to the entry of the platforms in 1950, there are exogenous shifts within the leisure sector itself: an increase in the perceived effectiveness of brand equity χ and a rise in the degree of competition among

the point of departure. Corroborating this judgement, Vandenbroucke (2009) finds that over the period 1900-1950 only about 7% of the shift in time allocation was due to leisure technology.

[48]Section 6 shows that leisure technologies are not captured in the GDP statistics. Therefore the measured TFP growth shown in panel D corresponds to the growth rate of A in the model.

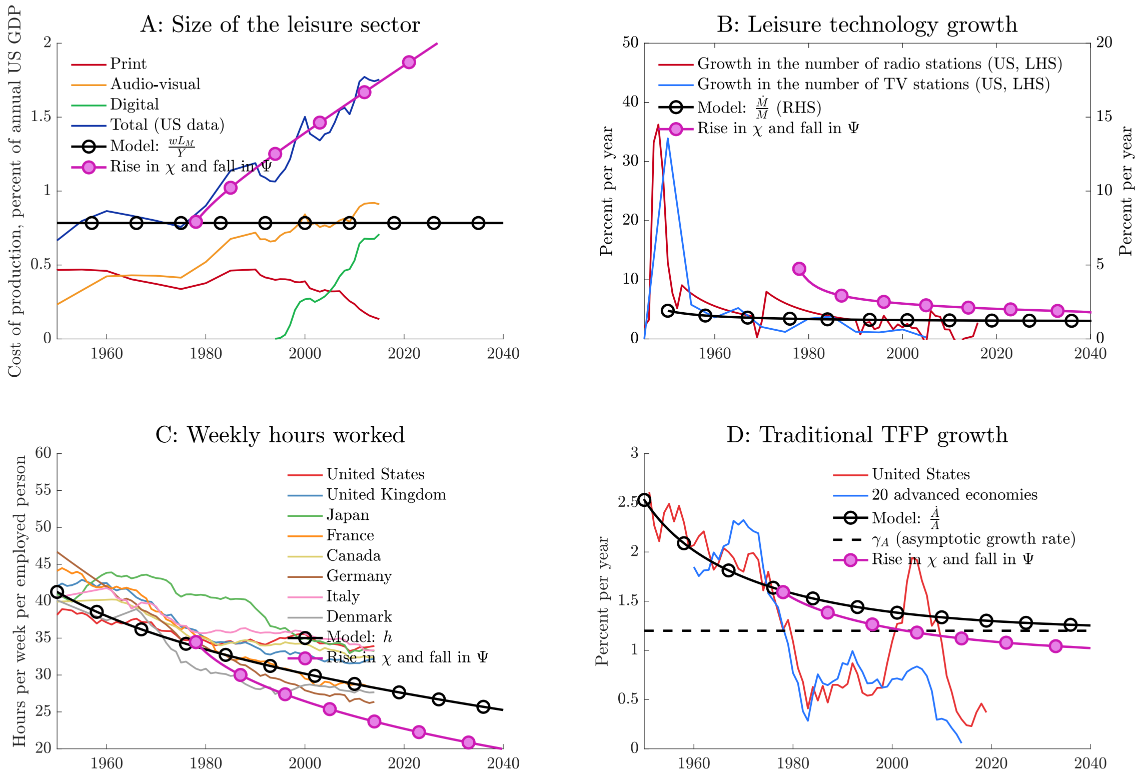

Figure 6

The Model's Growth Path versus the Trends Observed in the Data

Data sources as in Figure 2, except for the top-right panel in which the data are from the Federal Communications Commission and Statista. These data are interpolated over the missing values. The black lines with empty circles show the model's transition following the entry of platforms that is assumed to have taken place in 1950. The pink line with filled circles shows the transition with additional shocks to parameters χ (increase) and Ψ (decrease) calibrated in such a way that the size of the leisure sector matches the increase since 1978 shown in the top-left panel.

platforms (a fall in Ψ).[49] These shifts are calibrated to match by construction the increase in the relative size of the platform sector since the late 1970s in panel A.[50] The expanded production of brand equity can in equilibrium support faster growth of leisure technologies (panel B), which leads to a sharper decline in hours worked in the market (panel C). This more rapid decline in hours leads to an additional downward drag on traditional productivity (panel D).

The model is stylized and the parametrization is exploratory, so the quantitative predictions must be viewed accordingly. Nonetheless, two broad lessons emerge.

First, the effects can be quantitatively substantial: one should not discount the attention economy as an important explanation for macro trends merely because it is small as a share of GDP.

Second, it is likely that the attention economy itself has undergone technological shifts over time. Put differently, while the leisure technologies we see today represent, in part, a natural

[49] Which is isomorphic to an increase in platforms' productivity in turning consumers' time into brand equity.

[50] For simplicity, the share of labor employed in the traditional R&D sector is held constant in this simulation.

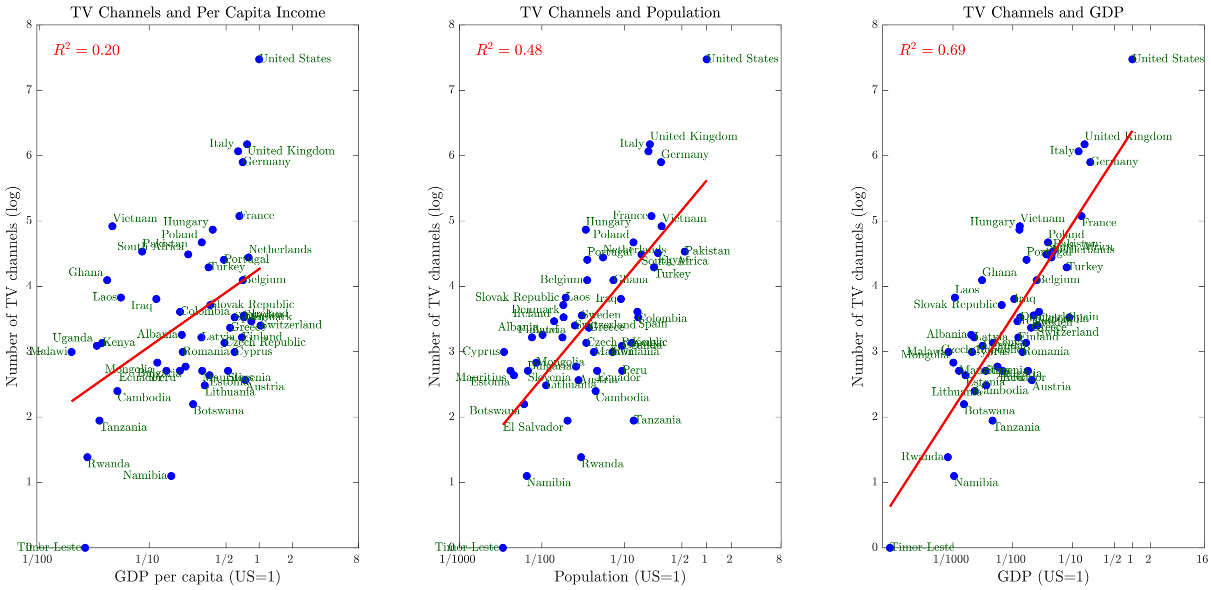

Figure 7

The Number of TV Channels and Market Size

Sources: Data on GDP per capita are from Penn World Tables 9.0 (Feenstra *et al.* (2015)). Data on population are from the World Bank. Data on the number of TV channels in each country has been hand collected from online sources. As such these data are subject to some measurement error. TV channels include state-run channels.

progression from those that we saw in the 1950s, there are also structural differences such as the emergence of data gathering, user-generated content, or portability of devices through which leisure technologies can be accessed. Future work could usefully study these and other shifts both empirically and theoretically.

5.4.1 Cross-country evidence on the market-size effect

In light of the theory, the equilibrium level of leisure technologies depends on market size. Figure 7 provides a simple test of this prediction, by plotting the number of TV channels across countries against GDP per capita, population and the level of aggregate GDP separately in the three consecutive panels. The number of TV channels is potentially a useful metric of M in the context of cross-country analysis, because of the language- and culture- barriers tend to limit the market to national borders.[51] The rising R^2 from the left to the right panel suggests that market size, both in terms of level of development and population – does indeed play an important role.

6 Measuring the leisure economy

Leisure technologies and the services that they provide are not captured in headline GDP statistics. The 2008 UN System of National Accounts views platforms as advertising agencies: their

[51] For some other leisure technologies such as mobile phone apps the market is global and cross-country exercise may be less useful. This concern could also apply to the English-speaking countries in the case of TV.

output is ads, which serve as intermediate inputs of the ad-buyers (Byrne *et al.* (2016b), Bean (2016)).[52] Two questions arise in this context. First, does this mean that GDP is significantly mismeasured? And second, do leisure technologies make GDP a less reliable guide to welfare over time? In this Section I explain why the answers to these questions are 'no' and 'yes', respectively.

6.1 Production cost-based value of leisure technologies

GDP has been designed with the intention to measure market-based production.[53] Assuming this perspective, Nakamura *et al.* (2017) propose valuing production of these services at cost, consistent with the usual treatment in the National Accounts. In the context of the present model this value is:

$$V_1 := w \cdot L_M = \frac{\alpha^2 \chi}{\Psi} Y \tag{49}$$

where the second equality follows from substituting the equilibrium value of wages.[54] One implication of (49) is that V_1 grows at the same rate as output, so that adding this production-based value to GDP will not change the measured growth rate.[55] Moreover, given the relatively small size of this sector, the level effect will not be very large. Together, these observations imply that GDP is not significantly mismeasured, at least when there are no shifts in the structural

[52] To illustrate this, it is useful to demonstrate measuring GDP using the output-, expenditure- and income approaches in the model economy. The output measure is the sum of value added in the final, intermediate and platform sectors:

$$GDP(O) = (Y - Ax) + (Ax - B) + B = Y.$$

This equation shows explicitly that brand equity output of the platforms is netted out as an intermediate in the production of differentiated goods. The expenditure measure is the sum of consumption, investment and capital consumption:

$$GDP(E) = C + \dot{K} + \delta K = Y$$

where the final equality follows from the resource constraint. Finally, the income measure is the sum of wage payments, profits, rent payments and households' outlays on patents:

$$\begin{aligned} GDP(I) &= ((1 - s_A - s_M)w_Y + s_A w_A + s_M w_M)\, hN + J\Pi_B + A\Pi + (r+\delta)K - V\dot{A} = \\ &= (1-\alpha)Y + V \cdot A^{\phi} L_A + \frac{\alpha^2\chi}{\Psi}Y + \left(\alpha^2 \chi Y - \frac{\alpha^2\chi}{\Psi}Y\right) + \alpha Y\left(1 - \alpha - \alpha\chi\right) + \alpha^2 Y - V \cdot A^{\phi} L_A = \\ &= Y. \end{aligned}$$

GDP in this economy is simply equal to final output Y. Clearly, GDP does not include leisure technologies.

[53] Nonetheless, in practice GDP does include elements that are outside of the production boundary, such as home production of goods or owner-occupied housing. Moreover, given the lack of an agreed comprehensive measure of economic wellbeing, it is often mis-used as a measure of welfare. See Jorgenson (2018) for an overview of the debate and Coyle (2017) for an extensive discussion of the production boundary in the context of digital goods.

[54] See the proof of Lemma 4 in Appendix B for a derivation.

[55] In practice, some in the literature have used the aggregate advertising spending / revenues, which is $p_b B = \Psi V_1$. Thus all the results continue to hold.

parameters of the model.

The production-based measure discussed above does not attempt to capture the utility consumers obtain from the zero price leisure technologies. For that, one must turn to measures that focus on the use of these technologies.

6.2 Use-based measures of value

One such metric is the time spent with these services valued at an ongoing wage (Goolsbee and Klenow (2006), Brynjolfsson and Oh (2012)). In addition, the model can be deployed to directly compute a compensating variation measure (a change in consumption required to compensate consumers for no access to leisure technologies):

$$V_{2a} := N \cdot w \cdot \int_0^M \ell(\iota) d\iota = \Phi \frac{1-h}{h} C \tag{50}$$

$$V_{2b} := N(\bar{c} - c) = \left(\exp\left(\Phi \frac{1-h}{h} \right) - 1 \right) C. \tag{51}$$

where $\{\bar{c}\}_{t=0}^{\infty}$ in (51) solves $u(\bar{c}, 0, 0) = u(c, \ell, M)$ and $\{c, \ell, M\}_{t=0}^{\infty}$ are equilibrium paths of these variables. The main difference between the two measures is that the compensating variation measure V_{2b} corrects for the diminishing marginal utility of consumption. This means that an increasingly large amount of traditional consumption needs to be transferred to agents to compensate them for a hypothetical loss of better leisure technologies to keep their utility unchanged.

6.3 Analysis

Proposition 7. ***Measurement of the growth rates.*** *In equilibrium, the production cost-based measure V_1 is proportional to output. The value of leisure technologies derived from the use side V_{2a} and V_{2b} grow faster than output even in the long-run:*

$$\gamma_{V_1} = \gamma_Y < \gamma_{V_2}$$

Proof. Appendix B. □

The underlying reason for the difference between the production and use-based approaches is the strong non-rivalry: in the attention economy, the use of these technologies is detached from production.

To illustrate, Figure 8 plots the level and the growth rate of GDP per capita as currently measured together with the 'enhanced' activity metrics that include the value of leisure services computed using the three different approaches.[56] In the medium-term all activity metrics fall short of the counterfactual pre-$\hat{t}$ trend, though to a different degree: adding the cost-based measure to GDP does not make much difference (the lines with diamonds lie on top of the thick solid lines), while the use-based measures translate into growth rates that are around 0.4 percentage points above the growth rate of measured GDP – a magnitude that is similar to some of the estimates reported in the literature.[57] Overall, the welfare gain from the use of free technologies makes up around half of the slowdown in the growth rate of GDP (over the horizon illustrated in the Figure).

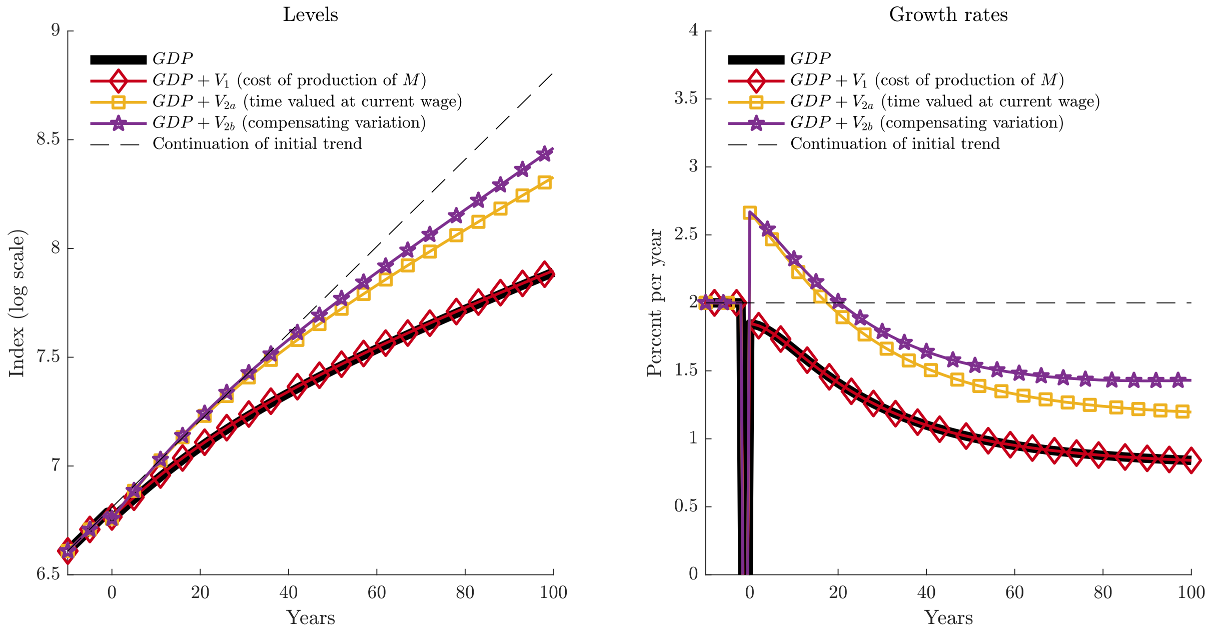

Figure 8

The Level and the Growth Rate of GDP and the Enhanced Measures of Activity

What with the other half? That is, why does the level of utility fall below the no-leisure-technologies counterfactual for some time?[58] This may appear surprising at first: the wage-based measure effectively relabels leisure time as productive time, so we might expect that adding it back to GDP recovers all the lost ground. The reason why this is not the case is that there is feedback from time allocation to productivity. Even as consumers value an extra hour spent on leisure at the current marginal product of labor, this does not recover the counterfactual trend because the marginal product of labor itself grows less rapidly. Normalizing $\hat{t} = 0$, for $t \geq \hat{t} = 0$,

[56]The near-term dynamics are driven by a jump in the share of workers in traditional R&D, which lowers the level of GDP at $\hat{t}$ and raises the growth rate over the near term.

[57]E.g. Brynjolfsson and Oh (2012) find the effect of around 0.3 percentage points per annum. Formulas in (50) and (51) The boost to growth rates is larger with the compensating variation measure, reflecting the curvature of the utility function as discussed above.

[58]In the long-run the positive effects dominates, since the consumption utility increases linearly and leisure utility increases exponentially.

instantaneous utility along the equilibrium path is equal to:[59]

$$u(t) = \log\left(\frac{C(t)}{N(t)}\right) + \Phi(t)\left(\frac{1}{h(t)} - 1\right). \tag{52}$$

Meanwhile, the counterfactual no-leisure path of utility is $\tilde{u} = \log\left(\frac{C(\hat{t})}{N(\hat{t})}e^{gt}\right)$ as the economy continues along segment 1 of the sBGP with per capita consumption growing at a constant rate g and $\ell = 0$. Focusing on the medium-term forces and so ignoring the transitional dynamics, it is straightforward to compute the loss due to lower consumption growth brought about by leisure technologies: $\log\left(\frac{C}{N}\right) - \tilde{u} = \frac{\gamma_M}{\nu-1}\frac{2-\phi}{1-\phi}t$ and the gain coming from leisure utility: $\Phi\left(e^{\frac{1}{\nu-1}\gamma_M t} - 1\right)$. Both of these are zero at $t = \hat{t}$ while for $t > \hat{t}$ the growth rate of the gain is greater than the growth rate of the loss as long as

$$t > \frac{(1-\phi)(\nu-1)+1}{n}\log\left(\frac{2-\phi}{1-\phi}\right) > 0. \tag{53}$$

In other words, there is a period of time after $\hat{t}$ where the loss (from lower C) is larger than the gain (from higher l), and the net effect of leisure technologies is to detract from welfare.[60] Conversely, it is straightforward to verify that if the growth rate of A is exogenous, the utility gain from better technologies is always larger than the loss due to lower consumption: it is the negative productivity spillover that explains why utility goes down.

7 Efficiency

The decline in welfare highlighted in the previous Section underscores the fact that the equilibrium allocation is suboptimal. This section explores the efficiency properties of the equilibrium, starting with the rat-race assumption embodied in the brand equity competition (which turns out to be inconsequential), and then discussing the inefficiencies due to (i) indirect monetization of leisure services and (ii) the interaction of leisure technologies, endogenous labor supply and the usual inefficiencies present when innovation is endogenous.

7.1 Brand equity in socially-optimal allocation

Given the combative nature of brand equity competition in the benchmark model, we have the following result:

Lemma 5. ***Optimal brand equity.*** *For a given (optimal) choice of leisure hours ℓ^*, the planner is*

[59] To see this, use equation (21) in $u = \log c + M^{\frac{1}{\nu-1}}(1-h)$.

[60] This period of time can be very long – for the calibration presented in Section (5), the value of the threshold in (53) is 275 years. It would of course take even longer for the welfare effect to turn positive.

indifferent between producing any amount of brand equity between 0 and $B(\ell^*)$. If the real resource cost of production of brand equity was positive then the planner would choose to produce none.

Proof. Follows immediately from (18) and the fact that for a given level of ℓ^* the choice of B^* in $[0, B(\ell^*)]$ has no impact on the resource constraint or utility. $\square$

In what follows I assume, without loss of generality, that $B^* = 0$.

7.2 Inefficiency due to the indirect monetization of leisure services

The indirect monetization business model of the platforms leads to an inefficient supply of leisure technologies in equilibrium.[61] I now study this inefficiency in a static economy populated by N individuals and endowed with exogenously given levels of capital stock K and knowledge A. The production side is identical to before, except there is no traditional R&D sector (since A is fixed): in particular, the final good is produced competitively with technology given by (14), intermediate firms produce output with capital and they advertise their products, maximizing profits as in (15), and platforms maximize profits by supplying brand equity and leisure technologies (equations (18), (20) and (27)). Households maximize a separable utility function:

$$U = u(c) + v(h, M), \tag{54}$$

where, with slight abuse of notation, c denotes per capita consumption.[62]

In the decentralized equilibrium, since there is no saving, market clearing requires $C := cN = Y$. Furthermore, wages are equalized across sectors and households choose hours worked optimally:

$$w_Y = w_M \tag{55}$$

$$(1 - s_M)w_Y + s_M w_M \geq -\frac{\partial v/\partial h}{\partial u/\partial c} \tag{56}$$

where s_M is the share of labor devoted to the production of leisure technologies, and $1 - s_M$ is the share of labor employed in the production of the final good. Since labor is paid its marginal revenue product, equilibrium wages satisfy:

$$w_Y = \frac{\partial Y}{\partial L_Y} = -\frac{\partial Y}{\partial s_M}\frac{1}{hN} = \frac{\partial Y}{\partial h}\frac{1}{1 - s_M}\frac{1}{N} \tag{57}$$

$$w_M = \frac{\partial}{\partial L_M}(p_B \cdot B_j) = \frac{p_B}{\Psi}\frac{1-h}{M}\frac{\partial M}{\partial L_M} = \frac{p_B}{\Psi}\frac{1-h}{M}\frac{\partial M}{\partial s_M}\frac{1}{hN} = \frac{p_B}{\Psi}\frac{1-h}{M}\frac{\partial M}{\partial h}\frac{1}{s_M}\frac{1}{N}. \tag{58}$$

[61] See section 3.2.1 for the discussion of what mechanisms likely result in this business model.

[62] Clearly, the balanced growth preferences formulated in (1) are a special case of (54).

Combining these results with equations (55) and (56) yields the following condition that must hold in a decentralized equilibrium:

$$-\left(\frac{\partial Y}{\partial h}\frac{dh}{ds_M}-\frac{\partial Y/\partial s_M}{\partial M/\partial s_M}\frac{\partial M}{\partial h}\frac{dh}{ds_M}\right)\geq N\frac{\partial v/\partial h}{\partial u/\partial c}\frac{dh}{ds_M}. \tag{59}$$

This condition holds with equality if the equilibrium choice of labor hours is interior.

Consider now the constrained planning problem: the planner chooses how much leisure technologies to produce (or equivalently how much labor resources to devote to production of M), respecting the households' time allocation rule h^*:

$$\max_{s_M} u\left(\frac{C}{N}\right)+v(h,M) \text{ s.t. } Y=F_Y(h,s_M),\ M=F_M(h,s_M),\ Y=C \text{ and } h=h^*$$

The solution satisfies:

$$-\left(\frac{\partial Y}{\partial s_M}+\frac{\partial Y}{\partial h}\frac{dh}{ds_M}-N\frac{\partial v/\partial M}{\partial u/\partial c}\frac{\partial M}{\partial h}\frac{dh}{ds_M}\right)\geq N\left(\frac{\partial v/\partial h}{\partial u/\partial c}\frac{dh}{ds_M}+\frac{\partial v/\partial M}{\partial u/\partial c}\frac{\partial M}{\partial s_M}\right) \tag{60}$$

The left-hand side of this inequality is the marginal social cost of increasing the production of leisure technologies. It consists of a direct effect in terms of the foregone production of the consumption good and an indirect cost that comes via lower hours worked. The right-hand side is the marginal social benefit which captures higher leisure utility as hours decline and the leisure technology improves.

Expressions (59) and (60) differ, illustrating the underlying inefficiency. This inefficiency is linked to the restriction that the price of leisure services is zero and these technologies are instead monetized indirectly (the market power of the platforms plays a role too). In a hypothetical equilibrium with direct monetization of leisure technologies, perfect competition among the platforms and no cross-subsidization through brand equity, the positive price of leisure services would equalize the sum of the marginal rates of substitution across consumers with the marginal rate of transformation:[63]

$$p_M=\frac{\partial v/\partial M}{\partial u/\partial c}=-\frac{1}{N}\frac{\partial Y/\partial s_M}{\partial M/\partial s_M}. \tag{61}$$

Under (61), conditions (59) and (60) become identical. Conversely, in the decentralized equilibrium where the provision of leisure technologies relies on indirect monetization (61) does not hold, since the allocation of labor across sectors is driven by the profitability of brand equity

[63] Equality of wages across sectors would imply that $p_M=-\frac{1}{N}\frac{\partial Y/\partial s_M}{\partial M/\partial s_M}$, and consumer optimality condition would imply $p_M=\frac{\partial v/\partial M}{\partial u/\partial c}$.

business and not by the utility gains from leisure services:

$$-\frac{\partial Y/\partial s_M}{\partial M/\partial s_M} = \frac{p_B}{\Psi}\frac{1-h}{M} \neq N\frac{\partial v/\partial M}{\partial u/\partial c}. \tag{62}$$

The direction of this inefficiency will depend on parameters that govern the profitability of and degree of competition in the brand equity business. To illustrate what this inefficiency means for the calibration presented in Section 5, Figure 9 compares the optimal and equilibrium allocations for economies of different sizes (varying A along the x-axis). The optimal allocation features a much higher share of labor devoted to leisure innovation, compared to the equilibrium. This under-provision in the decentralized equilibrium comes about both through the extensive margin (leisure technologies become available only at suboptimally high levels of A) and the intensive margin (even when produced, the level of leisure technologies is too low).

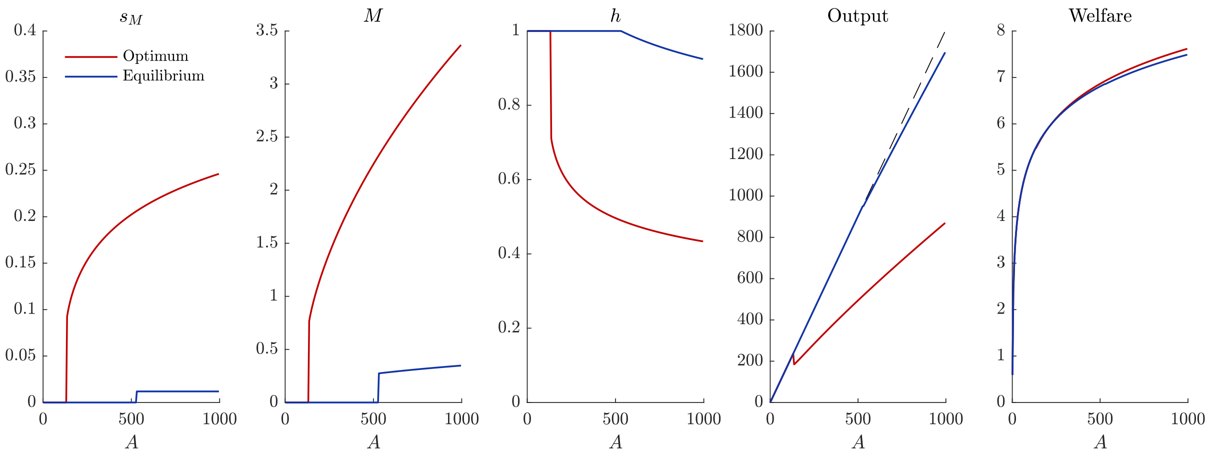

Figure 9

Socially optimal allocation and equilibrium in the static model

Note: The Figure plots the socially optimal allocation and equilibrium allocation for different values of A and implicitly K, holding the capital-to-output ratio constant at 3.

7.3 Leisure technologies and endogenous growth

In this final section I briefly discuss the way in which leisure technologies interact with the usual endogenous growth externalities. The planning problem in the dynamic economy of Section 3 is not tractable as the optimal path does not feature balanced growth. To make progress I consider a setup with exogenous and constant level of leisure technology M. In the planning problem, the representative consumer solves:

$$\max_{c,h,s_A} \int e^{-\rho t} \log c + M(1-h)dt$$

subject to

$$Y = K^{\alpha} \left(AL_A\right)^{1-\alpha} \tag{63}$$

$$\dot{K} = Y - C \tag{64}$$

$$\dot{A} = L_A A^{\phi} \tag{65}$$

$$L_A = shN,\ L_Y = (1-s)hN \tag{66}$$

$$\frac{\dot{N}}{N} = n. \tag{67}$$

Appendix B solves this program and also characterizes the decentralized equilibrium of this economy. It shows that hours worked on the socially optimal growth path and in the decentralized equilibrium are constant and can be written as:

$$h = \min\left\{1, \frac{(1-\alpha)\frac{Y}{C}\left(1+\frac{s}{1-s}\right)}{M}\right\}. \tag{68}$$

That is, the steady state labor supply is a non-decreasing function of the inverse of the consumption-to-output ratio $\frac{Y}{C}$ and the share of labor employed in R&D s, and a non-increasing function of M.

Equilibrium and socially optimal h differ due to the differences in $\frac{Y}{C}$ and s across the two allocations. The appendix derives closed-form expressions for both variables and proves that:

$$\left(\frac{Y}{C}\right)^{DC} < \left(\frac{Y}{C}\right)^{SP} \tag{69}$$

$$s^{DC} < s^{SP}. \tag{70}$$

Intuitively, in the decentralized equilibrium (DC) the market power of the intermediate producers means that capital is underpaid relative to labor, which depresses the capital-to-output ratio and raises the consumption-to-output ratio relative to the optimum (SP). Furthermore, both the monopoly mark-up in the intermediate sector and the knowledge spillovers from innovation mean that the decentralized economy underinvests in R&D.

Results in (68), (69) and (70) imply that for a given value of M, the equilibrium labor supply is weakly below the optimal level:

$$h^{DC} \leq h^{SP},$$

which holds with strict inequality if M is sufficiently high and $h^{DC} < 1$. The monopoly mark-ups and knowledge spillovers interact with endogenous labor/leisure choice.[64] In particular, the

[64]This result is similar to the findings in Eriksson (1996) who studies labor supply in an endogenous growth model of Grossman and Helpman (1991).

size of the inefficiency is (weakly) increasing in how valued leisure is: there is no inefficiency when M is low as $h^{SP} = h^{DC} = 1$ and the economy behaves as if labor supply was inelastic; when leisure is highly valued, workers provide too little labor in equilibrium as market wages do not reflect the social value of their efforts. The broad lesson is that the improved leisure technology – higher M – exacerbates the effects of the usual inefficiencies present in economies in which there is too little innovation.[65] Absent appropriate subsidies to research to counteract such inefficiencies, the planner might want to constrain the supply of leisure technology to offset these effects.

8 Conclusion

In this paper I formalized the idea of leisure-enhancing technologies: products that are available for free and are thus specifically designed to capture our time and attention. Using the theory I studied how these technologies shape the growth patterns and what the welfare consequences are. The main takeaway is that these technologies can simultaneously explain shifts in hours worked and account for the low growth observed in the data. The effect on GDP growth reflects both the measurement difficulties and the 'real' slowdown, since better leisure technologies do not fully offset the crowding out of traditional productivity.

The analysis can be extended in many interesting directions. First, the theory delivered the conclusion that the size of the economy determines whether leisure technologies are viable or not. But other factors play a role too.

One such factor is the share of time that households can feasibly allocate to marketable leisure. The first half of the 20th century saw a substantial increase in this share. Two historical events were key: the introduction of the two-day weekend in the 1930s[66] and the gradual adoption of household appliances – the washing machine, the flush toilet, the vacuum and others – from 1920s through 1940s and beyond. Both of these freed up time for other activities.[67] It is plausible that these developments have acted to direct resources towards inventions and activities that complement leisure and leisure time, in the spirit of the directed technical change

[65] The economies considered in this paper exhibit too little investment in R&D in equilibrium, as the only two inefficiencies present in the current setting both depress the private return to innovation relative to the social return: the first is the monopoly power of producers and the associated surplus appropriability problem and the second is the "standing on the shoulders of giants" externality. Other externalities – notably the duplication externality and the business stealing effect – which are absent from the model would act to push the other way, raising the private return relative to social return. In an economy where the latter two effects dominate leisure technologies would alleviate, rather than exacerbate, the inefficiencies. Most of the literature suggests that there is underinvestment in R&D, however, making what is considered here the relevant case (see e.g. Jones and Williams (2000) and Jones and Summers (2020)).

[66] In the US, Henry Ford made Saturday and Sunday days off for his staff as early as 1926, and the US as a whole adopted the five-day system in 1932 (in part to counter the unemployment caused by the Great Depression).

[67] This is especially interesting since the model presented here abstracted from home production – a margin that is clearly important in practice. See e.g. Greenwood *et al.* (2005).

literature (Acemoglu, 2002). Incorporating these mechanisms in the model and assessing their empirical validity is an interesting avenue for future work.

Second, the audio-visual entertainment revolution that started in the 1920s and rapidly accelerated in the 1950s has been propelled by the introduction of general purpose technologies that allowed for the signal to be transmitted to households: the radio receiver and the television set. More recently, invention of PCs, smartphones and tablets made it possible for the free leisure technologies to spread far and wide (Figure 10). To study the implications one could bring in the insights from Fernald and Jones (2014) on modeling general purpose technologies into the framework developed here.

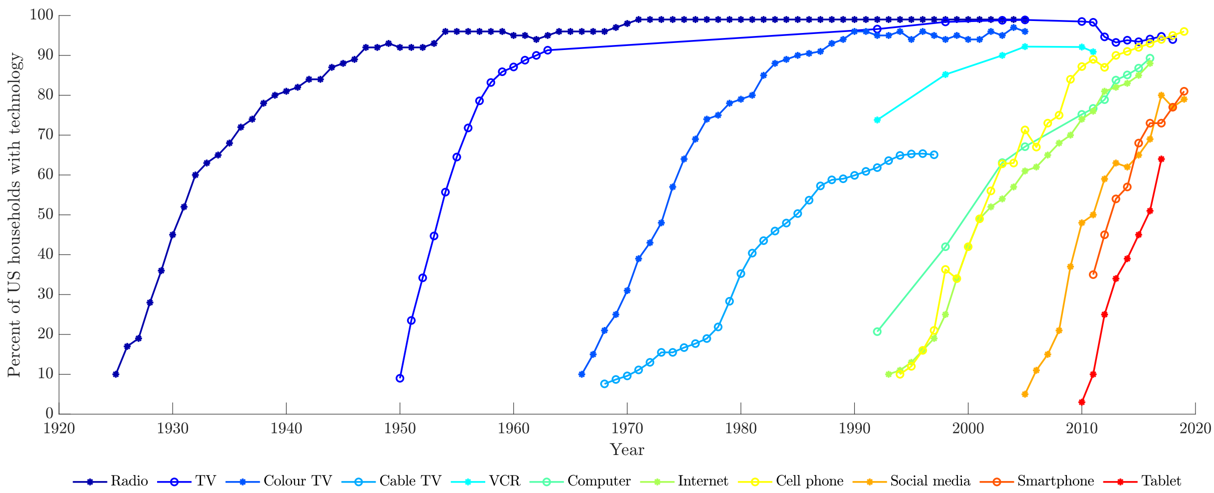

Figure 10

Adoption of leisure-linked general purpose technologies

Sources: Comin and Hobijn (2009).

An open question is how the rise of the leisure sector interacts with heterogeneity, both at the household and at the firm level. On the household side, it is interesting to study how time allocation decisions interact with income and wealth inequality. For example, disaggregated evidence on time allocation across the income distribution shows that poor individuals increased their leisure more than the rich (Boppart and Ngai, 2017b). Allowing for household and income heterogeneity in the presence of leisure-enhancing technologies could bring out new insights and aid the debate on leisure-inequality and the welfare implications. Considering firm heterogeneity may be important, too: the current setting is well suited to analyze equilibrium outcomes when heterogenous firms compete not only in prices but also in intangible assets. More productive firms may devote more resources to brand building, cementing their market share, with interesting implications for market power (De Loecker and Eeckhout, 2017). To tackle these issues, the framework developed here could be usefully incorporated into a model of firm dynamics and growth in the tradition of Klette and Kortum (2004).

The recent leisure technologies – social media, smartphone apps, etc. – tend to diffuse rapidly across the world. This raises at least two issues that future work should address. First, this phenomenon suggests that the market that guides the supply of leisure technologies becomes increasingly global. Second, it means that adoption of leisure technologies in emerging economies can be rapid even at low levels of output per capita. Such "premature adoption" would have interesting implications for growth and development prospects in these countries.

As leisure economy becomes ever more important going forward, the framework built here can be used as a base for explorations of some of the pressing policy questions, such as optimal taxation of platforms or competition- and anti-trust policy in presence of zero-price services. These ideas formulate an exciting research agenda for economics in general, and macroeconomics specifically, in the years to come.

References

Acemoglu, D. (2002). Directed Technical Change. *Review of Economic Studies*, **69** (4), 781–809.

— and Guerrieri, V. (2008). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy*, **116** (3), 467–498.

Adler, G., Duval, R., Furceri, D., Kiliç Çelik, S., Koloskova, K. and Poplawski-Ribeiro, M. (2017). Gone with the Headwinds: Global Productivity. *IMF Staff Discussion Notes*, **17** (04), 1.

Aghion, P., Akcigit, U. and Howitt, P. (2014). What Do We Learn From Schumpeterian Growth Theory? *Handbook of Economic Growth*, **2**, 515–563.

— and Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, **60** (2), 323–351.

—, Jones, B. and Jones, C. (2017). Artificial Intelligence and Economic Growth.

Aguiar, M., Bils, M., Charles, K. K. and Hurst, E. (2017). *Leisure Luxuries and the Labor Supply of Young Men*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

— and Hurst, E. (2007a). Measuring Trends in Leisure: The Allocation of Time Over Five Decades. *The Quarterly Journal of Economics*, **122** (3), 969–1006.

— and — (2007b). Measuring trends in leisure: the allocation of time over five decades. *The Quarterly Journal of Economics*, **122** (3), 969–1006.

— and — (2008). *The Increase in Leisure Inequality*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

— and — (2016). The Macroeconomics of Time Allocation. In *Handbook of Macroeconomics*, vol. 2, pp. 1–44.

—, — and Karabarbounis, L. (2012). Recent developments in the economics of time use. *Annual Review of Economics*, **4**, 373–397.

Akcigit, U. and Kerr, W. R. (2018). Growth through heterogeneous innovations. *Journal of Political Economy*, **126** (4).

Anderson, S. P. and Renault, R. (2006). Advertising Content. *American Economic Review*, **96** (1), 93–113.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics*, **00** (3), 668–691.

Bagwell, K. (2007). Chapter 28 The Economic Analysis of Advertising. In *Handbook of Industrial Organisation*, pp. 1701–1844.

Basu, S., Fernald, J. G. and Kimball, M. S. (2006). Are technology improvements contractionary? *American Economic Review*, **96** (5), 1418–1448.

Bean, C. (2016). *Independent Review of UK Economic Statistics*. Tech. rep.

Becker, G. S. (1965). A Theory of the Allocation of Time. *The Economic Journal*, **75** (299), 493–517.

— and Murphy, K. M. (1993). A Simple Theory of Advertising as a Good or Bad. *The Quarterly Journal of Economics*, **108** (4), 941–964.

Benhabib, J. and Bisin, A. (2002). Advertising, Mass Consumption and Capitalism. *Working Paper*, pp. 1–38.

Bloom, N., Jones, C., Van Reenen, J. and Webb, M. (2017). *Are Ideas Getting Harder to Find?* Tech. rep., National Bureau of Economic Research, Cambridge, MA.

—, Jones, C. I., van Reenen, J. and Webb, M. (2020). Are ideas getting harder to find? *American Economic Review*, **110** (4), 1104–1144.

Boppart, T. and Krusell, P. (2020). Labor Supply in the Past, Present, and Future: A Balanced-Growth Perspective. *Journal of Political Economy*, **128** (1), 118–157.

— and Ngai, L. (2017a). Rising inequality and trends in leisure. (DP12325).

— and Ngai, L. R. (2017b). Rising inequality and trends in leisure.

Borden, N. H. (1942). Findings of the Harvard Study on the Economic Effects of Advertising. *Journal of Marketing*, **6** (4), 89–99.

Bridgman, B. (2018). Is Productivity on Vacation? The Impact of the Digital Economy on the Value of Leisure.

Broda, C. and Weinstein, D. E. (2006). Globalization and the Gains From Variety. *The Quarterly Journal of Economics*, **121** (2), 541–585.

Brynjolfsson, E., Eggers, F. and Gannamaneni, A. (2018). *Using Massive Online Choice Experiments to Measure Changes in Well-being*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

— and Oh, J. H. (2012). The attention economy: Measuring the value of free digital services on the internet. *International Conference on Information Systems, ICIS 2012*, **4**, 3243–3261.

Butters, G. R. (1977). Equilibrium Distributions of Sales and Advertising Prices. *The Review of Economic Studies*, **44** (3), 465.

Byrne, D. M., Fernald, J. G. and Reinsdorf, M. B. (2016a). Does the United States have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*.

—, — and — (2016b). Does the united states have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*, **2016** (SPRING), 109–182.

Cavenaile, L. and Roldan-Blanco, P. (2020). Advertising, Innovation and Economic Growth.

Christensen, M. A., Bettencourt, L., Kaye, L., Moturu, S. T., Nguyen, K. T., Olgin, J. E., Pletcher, M. J. and Marcus, G. M. (2016). Direct Measurements of Smartphone Screen-Time: Relationships with Demographics and Sleep. *PLOS ONE*, **11** (11).

Comin, D. A. and Hobijn, B. (2009). *The CHAT Dataset*. Tech. rep.

Corrado, C., Haskel, J., Jona-Lasinio, C. and Iommi, M. (2012). Intangible Capital and Growth in Advanced Economies: Measurement Methods and Comparative Results.

Corrado, C. A. and Hulten, C. R. (2010). How Do You Measure a Technological Revolution? *American Economic Review*, **100** (2), 99–104.

Coyle, D. (2017). Do-it-yourself digital: the production boundary, the productivity puzzle and economic welfare Diane Coyle Professor of Economics, University of Manchester, and ONS Fellow 1. pp. 1–38.

De Loecker, J. and Eeckhout, J. (2017). *The Rise of Market Power and the Macroeconomic Implications*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

DellaVigna, S. and Gentzkow, M. (2010). Persuasion: Empirical Evidence. *Annual Review of Economics*, **2** (1), 643–669.

Dixit, A. and Norman, V. (1978). Advertising and Welfare. *The Bell Journal of Economics*, **9** (1), 1.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, **67** (3), 297–308.

Dorfman, R. and Steiner, P. O. (1954). Optimal Advertising and Optimal Quality. *The American Economic Review*, **44** (5), 826–836.

Eicher, T. S. and Turnovsky, S. J. (1999). *Non-Scale Models of Economic Growth*. Tech. Rep. 457.

Eriksson, C. (1996). Market failures in the R&D growth model with endogenous labor supply. *Journal of Public Economics*, **61** (3), 445–454.

Falck, O., Gold, R. and Heblich, S. (2014). E-lections: Voting behavior and the internet. *American Economic Review*, **104** (7), 2238–2265.

Farboodi, M. and Veldkamp, L. (2019). A Growth Model of the Data Economy.

Feenstra, R. C., Inklaar, R., Timmer, M. P., Aten, B., Atkeson, A., Basu, S., Deaton, A., Diewert, E., Heston, A., Jones, C., Rao, P., Rodríguez-Clare, A. and Végh, C. (2015). The Next Generation of the Penn World Table. *American Economic Review*, **105** (10), 3150–3182.

Fernald, J. G. and Jones, C. I. (2014). The future of US economic growth. *American Economic Review*, **104** (5), 44–49.

Gentzkow, M. (2006). Television and Voter Turnout*. *Quarterly Journal of Economics*, **121** (3), 931–972.

— and Shapiro, J. M. (2008). Preschool Television Viewing and Adolescent Test Scores: Historical Evidence from the Coleman Study. *Quarterly Journal of Economics*, **123** (1), 279–323.

Goolsbee, A. and Klenow, P. J. (2006). Valuing consumer products by the time spent using them: An application to the Internet. In *American Economic Review*, vol. 96, pp. 108–113.

Gourio, F. and Rudanko, L. (2014). Customer capital. *Review of Economic Studies*, **81** (3), 1102–1136.

Greenwood, J., Seshadri, A. and Yorukoglu, M. (2005). Engines of liberation. *Review of Economic Studies*, **72** (1), 109–133.

Grossman, G. and Helpman, E. (1991). *Innovation and Growth in the Global Economy*. Cambridge, MA: The MIT Press.

Grossman, G. M. and Shapiro, C. (1984). Informative Advertising with Differentiated Products. *The Review of Economic Studies*, **51** (1), 63.

Hall, R. E. (2008). General equilibrium with customer relationships: a dynamic analysis of rent-seeking. *Working Paper*, pp. 1–35.

Hulten, C. and Nakamura, L. (2017). Accounting for Growth in the Age of the Internet: The Importance of Output-Saving Technical Change. *Working Paper*, pp. 1–36.

Johnson, J. P. (2013). Targeted advertising and advertising avoidance. *RAND Journal of Economics*, **44** (1), 128–144.

Jones, B. and Summers, L. (2020). A Calculation of the Social Returns to Innovation.

Jones, C. I. (1995). R&D-Based Models of Economic Growth. *Journal of Political Economy*, **103** (4), 759–784.

— (2015). The Facts of Economic Growth. *The Handbook of Macroeconomics Volume 2*, pp. 0–81.

— and Tonetti, C. (2019). Nonrivalry and the economics of data.

— and Williams, J. C. (2000). *Too Much of a Good Thing? The Economics of Investment in R&D*. Tech. Rep. 1.

Jorgenson, D. W. (2005). Chapter 10 Accounting for Growth in the Information Age. *Handbook of Economic Growth*, **1** (SUPPL. PART A), 743–815.

— (2018). Production and welfare: Progress in economic measurement. *Journal of Economic Literature*, **56** (3), 867–919.

Kelly, Y., Zilanawala, A., Booker, C. and Sacker, A. (2018). Social Media Use and Adolescent Mental Health: Findings From the UK Millennium Cohort Study. *EClinicalMedicine*, **6**, 59–68.

King, R. G., Plosser, C. I. and Rebelo, S. T. (1988). Production, growth and business cycles. *Journal of Monetary Economics*, **21** (2-3), 195–232.

Klette, T. J. and Kortum, S. (2004). Innovating Firms and Aggregate Innovation. *Journal of Political Economy*, **112** (5).

Kortum, S. (1997). Research, Patenting, and Technological Change. *Econometrica*, **65** (6), 1389–1419.

Lambin, J. J. (1976). *Advertising, Competition and Market Conduct in Oligopoly Over Time*. Amsterdam: North-Holland.

Lewis, R. A. and Rao, J. M. (2015). The unfavorable economics of measuring the returns to advertising. *Quarterly Journal of Economics*, **130** (4), 1941–1973.

— and Reiley, D. H. (2014). Online ads and offline sales: Measuring the effect of retail advertising via a controlled experiment on Yahoo! *Quantitative Marketing and Economics*, **12** (3), 235–266.

Michel, C., Sovinsky, M., Proto, E. and Oswald, A. J. (2019). Advertising as a major source of human dissatisfaction: Cross-national evidence on one million europeans. *The Economics of Happiness: How the Easterlin Paradox Transformed Our Understanding of Well-Being and Progress*, **i** (November), 217–239.

Milgrom, P. and Roberts, J. (1986). Price and Advertising Signals of Product Quality. *Journal of Political Economy*, **94** (4), 796–821.

Molinari, B. and Turino, F. (2009). Advertising and Business Cycle Fluctuations.

Nakamura, L., Samuels, J. and Soloveichik, R. (2017). Measuring the "Free" Digital Economy within the GDP and Productivity Accounts.

Nelson, P. (1974). Advertising as Information. *Journal of Political Economy*, **82** (4), 729–754.

Ngai, L. R. and Pissarides, C. A. (2008). Trends in hours and economic growth. *Review of Economic Dynamics*, **11** (2), 239–256.

Nordhaus, W. D. and Tobin, J. (1972). *Is Growth Obsolete?*, vol. 5.

OfCOM (2019). *Online Nation 2019 Report*. Tech. Rep. May.

Ramey, V. A. and Francis, N. (2009). A century of work and leisure. *American Economic Journal: Macroeconomics*, **1** (2), 189–224.

Rauch, F. (2013). Advertising expenditure and consumer prices. *International Journal of Industrial Organization*, **31** (4), 331–341.

Reis, F. (2015). Patterns of Substitution between Internet and Television in the Era of Media Streaming-Evidence from a Randomized Experiment.

Rochet, J.-C. and Tirole, J. (2003). Platform Competition in two-sided markets. *Journal of the european economic association 1 (4), 990-1029*.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy*, **98** (5), S71–S102.

Royal Society for Public Health (2017). Social media and young people' s mental health and wellbeing. *Royal Society for Public Health*, , (May), 32.

Scanlon, P. (2018). *Why Do People Work So Hard?* 2018 Meeting Papers 1206, Society for Economic Dynamics.

Segerstrom, P. S. (1998). Endogenous Growth without Scale Effects. *American Economic Review*, **88** (5), 1290–1310.

Solow, R. M. (1987). We'd Better Watch out...

Stahl, D. O. (1989). Oligopolistic Pricing with Sequential Consumer Search. *The American Economic Review*, **79** (4), 700–712.

Steger, T. M. (2005). Welfare Implications of Non-Scale R&D-Based Growth Models. *Source: The Scandinavian Journal of Economics*, **107** (4), 737–757.

Stiglitz, J. E., Sen, A. and Fitoussi, J.-P. (2009). *Report by the Commission on the Measurement of Economic Performance and Social Progress*. Tech. rep.

Syverson, C. (2017). Challenges to Mismeasurement Explanations for the US Productivity Slowdown. *Journal of Economic Perspectives*, **31** (2), 165–186.

Trimborn, T., Koch, K. J. and Steger, T. M. (2008). Multidimensional transitional dynamics: A simple numerical procedure. *Macroeconomic Dynamics*, **12** (3), 301–319.

Tucker, C. E. (2012). The economics of advertising and privacy. *International Journal of Industrial Organization*, **30** (3), 326–329.

Vandenbroucke, G. (2009). Trends in hours: The U.S. from 1900 to 1950. *Journal of Economic Dynamics and Control*, **33** (1), 237–249.

Veblen, T. (1899). *The Theory of the Leisure Class*. MacMillan and Co. Ltd.

Wallsten, S. (2013). *What Are We Not Doing When We're Online*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

Appendices (for online publication only)

A Illustrative evidence

This Appendix further motivates the focus of this paper and forms a background to the analysis.

Evidence on leisure-enhancing innovations

Figure 2 in the main text illustrated the increased importance of the digital sub-sector of the attention economy since the mid-1990s. The available industry statistics reinforce this message. For example, Figure A.1 shows the dramatic rise in the number of smartphone apps, with the majority available free of charge to the consumer. The fact that millions of apps have been created over the past decade is a testament to the innovative efforts of firms in the attention economy.[68]

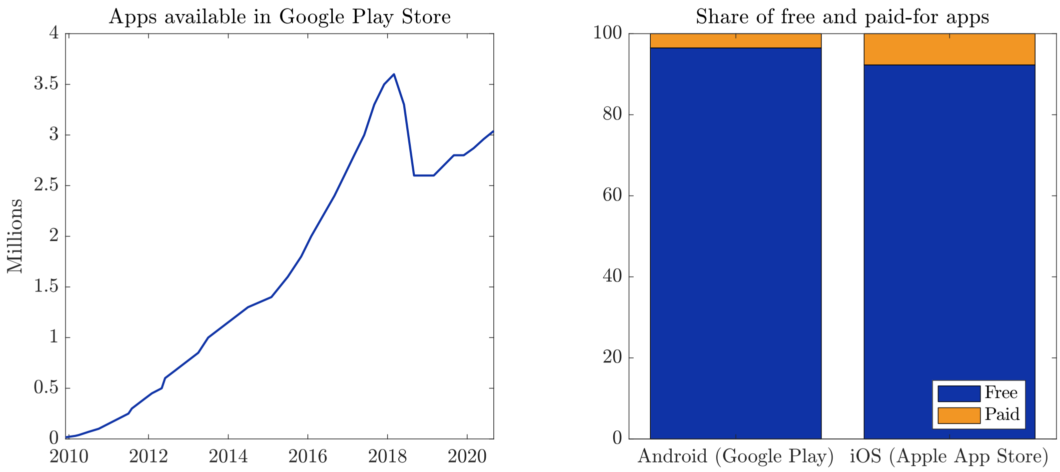

Figure A.1
Smartphone Apps

Source: The number of apps in Google Play Store is from Google, App Annie and AppBrain. The paid vs free apps breakdown is from 42matters, an app analytics company.

Consistent with the rapid technological progress within it, the leisure sector appears to be an increasingly important driver of the overall R&D spending. No exact measure for the share

[68] The market structure in the app market is more complex than in the model presented in the main text. Apps are available on platforms such as Google Play Store or Apple App Store, but are produced by many firms, not just Google or Apple. This additional layer of intermediation does not change the economics of the paper though: the incentives to capture the time and attention of the end-user remains. Future work could usefully explore the competition, business stealing and firm dynamics aspects of the app producers or other firms within the broadly defined leisure sector.

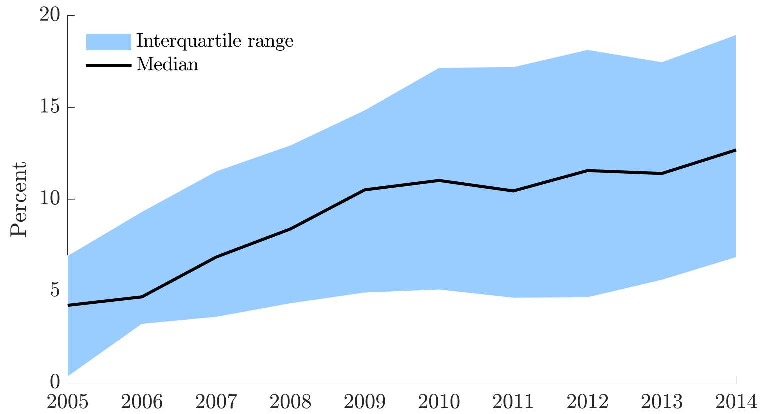

Figure A.2

R&D Expenditure Share of the (Proxy for the) Leisure Economy

Source: OECD. Includes data for an unbalanced panel of 39 countries. The figure shows the median and the interquartile range of the country-level share of R&D spending in the following sectors: publishing; motion picture, video and television program production; sound recording; programming and broadcasting activities; telecommunications services; computer programming, consultancy and related activities; information service activities; data processing, hosting and related activities; and web portals.

of attention economy in overall R&D spending is available; but it is possible to construct rough proxies by considering a subset of industries which are most likely engaged in leisure-enhancing innovations. Figure A.2 shows that the share of R&D spending accounted for by the sectors such as video and TV program production, sound recording, broadcasting and web portals has been rising over time.

Recent changes in time allocation patterns

While hours worked in the United States have fallen by less than in other countries (recall the middle panel of Figure 2), the trend in leisure time has been clearly upwards. Data from the annual American Time Use Survey, available from 2003, show that the largest increase in any category has been recorded in the "relaxing and leisure" category. A breakdown of the increase reveals that this rise is more-than-accounted-for by changes in the categories most directly related to leisure technologies, such as watching TV or using a computer (Figure A.3).

There are reasons why the time use survey data may underestimate the time that actually spent on modern leisure technologies, and perhaps overestimate the time spent working (or at least working attentively). First, the survey aims to uncover a person's main activity at any given point in time during a day, and so if some of the leisure technologies are used during other activities (for example during work hours), their use will not be recorded. This is important since the evidence (which I discuss below) suggests that smartphones in particular are being used with a constant frequency throughout the day, including during work hours, and some of that use is likely to be related to leisure. For the same reason the BLS acknowledges that ATUS is not a good

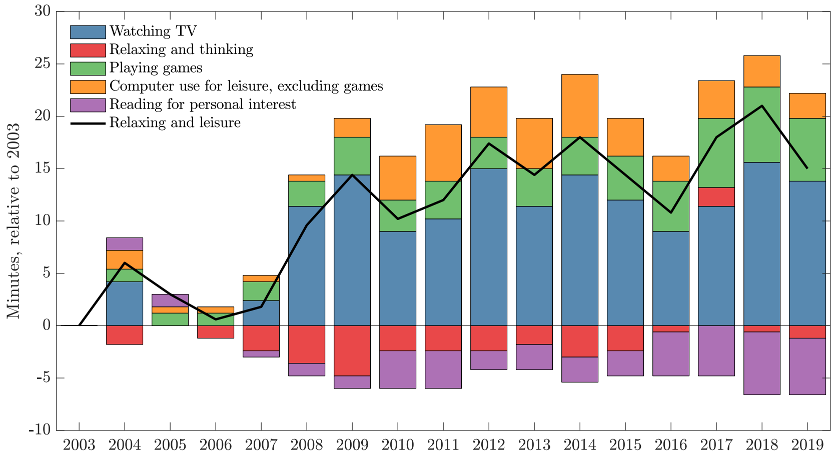

Figure A.3

Decomposition of the Increase in Relaxing and Leisure – the Category in the American Time Use Survey that has Experienced the Largest Increase Since 2003

Source: American Time Use Survey.

source of information on time spent online and/or using a computer or a smartphone: the survey is designed in such a way that time is split across many traditional categories such as working, socializing, etc.[69] This could give a misleading steer on the use of the leisure technologies if, for example, socializing today is different to socializing in the past (in particular if socializing today involved the use of leisure technologies). A related point is that, since people tend to check their phone very frequently (numerous estimates available online suggest that we pick-up our phones between 50 and 100 times a day), it is likely that the responders under-report usage when they fill in the survey. Consistent with that, some anecdotal evidence and the popularity of screen-time-tracking software suggests that users may find it hard to control the frequency of use and overall amount of time they spend on their devices. That could suggest possible underreporting in the surveys.

Given these possible shortcomings of the time-use survey data, the device-tracking data from Nielsen offers useful cross-check (even as it is not without drawbacks). The data paints a picture of a much more dramatic changes in time-use linked to technology (Figure A.4). For example, the data suggest that the amount of time spent on a smartphone *more than quadrupled* over the last 7 years, reaching over 3 hours daily. One of the limitations of these data is that they are not additive: a person can engage in multiple activities at once (e.g. watching TV and engaging on social media on the smartphone). Another is that the time spent on the devices could be productive time. Nonetheless, these data are a useful complement to the traditional time use surveys which naturally struggle to capture the short-but-frequent spells of usage.

The evidence on how people spend time at work (and indeed how much work is being carried out at home) is imperfect. For that reason it is useful to consider experimental tracking data on

[69] See https://www.bls.gov/tus/atusfaqs.htm#24 for the discussion of this point.

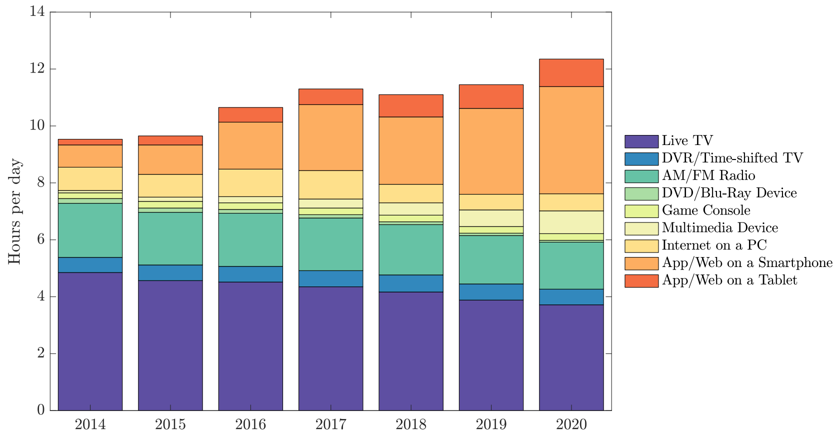

Figure A.4

Average Time Spent on Media Consumption per Adult in the US

Source: Nielsen. Note: Figures for representative samples of total US population (whether or not have the technology). More than one technology may be used at any given time, thus the total is indicative only. Data on TV and internet usage, and the usage of TV-connected devices are based on 248,095 individuals in 2016 and similar sample sizes in other years. Data on radio are based on a sample of around 400,000 individuals. There are approximately 9,000 smartphone and 1,300 tablet panelists in the U.S. across both iOS and Android smartphone devices.

the frequency of use of technology throughout the day. In one such study, Christensen *et al.* (2016) measured smartphone screen time over the course of an average day among a sample of 653 people in 2014 in the United States (Figure A.5). Time spent on the phone averaged 1 hour and 29 minutes per day, a little higher than what the Nielsen data suggest (which makes sense since the study included only users, while Nielsen aim to weight their results to capture non-adopters). Most strikingly, the mobile phone usage appears to be uniformly distributed throughout the day, suggesting that leisure time is, in part, substituting for time spent working. In a different study, Wallsten (2013) uses time use surveys to estimate that each minute spent on the internet is associated with loss of work-time of about 20 seconds.

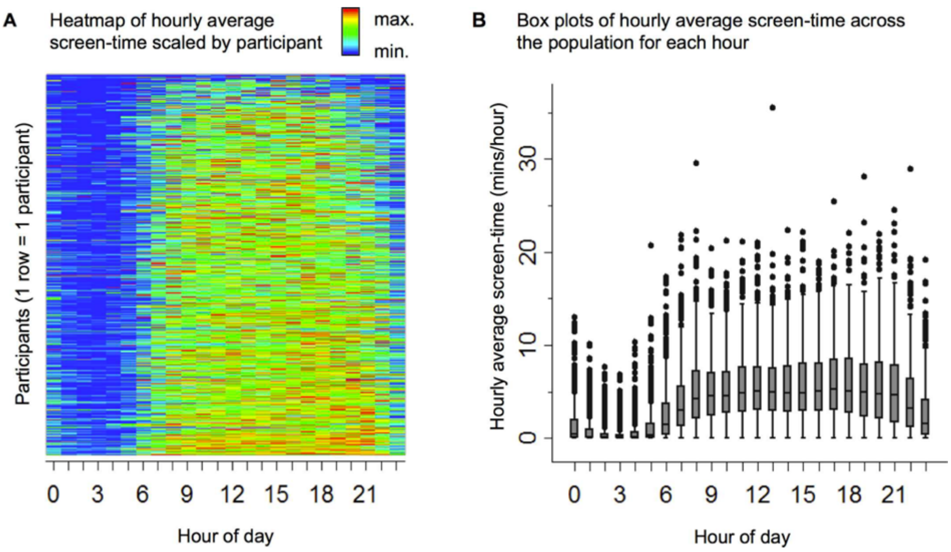

Figure A.5
Mobile Phone Use Over the Course of an Average Day

Source: Christensen *et al.* (2016).

Indeed, one feature of the latest technology is that it allowed leisure to "compete" with work much more directly than has been the case in the past. While it may not have been possible to watch TV at work, online entertainment is available during the work hours. This is, at least in part, balanced by the possibility of accessing work emails at home (see Footnote **??**). Future research should consider ways of measuring in more detail how people spend time at work and how much work is done at home.

B Proofs

Proof of Lemma 1

Proof. The Hamiltonian associated with household problem (1) is:

$$\mathcal{H}(K, C, h; \mu_K) = \log(C/N) + M^{\frac{1}{\nu-1}}(1-h) + \mu\left(whN + rK - C - V\dot{A} + A\Pi + J\Pi_B\right).$$

The necessary conditions for an interior optimum are:

$$C^{-1} = \mu \tag{71}$$

$$M^{\frac{1}{\nu-1}} = \mu w N \tag{72}$$

$$\rho\mu_K - \dot{\mu}_K = \mu r. \tag{73}$$

The transversality condition is:

$$\lim_{t\to\infty} e^{-\rho t}\mu \cdot \left(K(t) + \int_0^{A(t)} V(i,t)di \right) = 0. \tag{74}$$

Equation (73) implies $\frac{\dot{\mu}}{\mu} = -(r-\rho)$. Integrating this equation we obtain $\mu(t) = u'(c_0) \exp\left(-\int_0^t (r(s)-\rho)\, ds\right)$. Substituting in (74) we can write the the transversality condition as

$$\lim_{t\to\infty} \left[\exp\left(-\int_0^t r(s)ds \right) \cdot \left(K(t) + \int_0^{A(t)} V(i,t)di \right) \right] = 0. \tag{75}$$

Combining (71) and (72) with the labor demand equation (79) derived below yields:

$$hM^{\frac{1}{\nu-1}} = \frac{(1-\alpha)Y}{(1-s_A-s_M)C}$$

where $s_A := \frac{L_A}{L_Y+L_A+L_M}$ and $s_M := \frac{L_M}{L_Y+L_A+L_M}$ are the shares of labor employed in the two R&D sectors. Letting $\Phi := \frac{1-\alpha}{1-s_A-s_M}\frac{Y}{C}$, we obtain the (interior) solution:

$$h = \Phi M^{\frac{1}{1-\nu}}.$$

Since hours worked are bounded from above by 1, the complete solution is:

$$h = \min\{1, \Phi M^{\frac{1}{1-\nu}}\}, \tag{76}$$

and since $\ell = 1 - h$:

$$\ell = \max\{0, 1 - \Phi M^{\frac{1}{1-\nu}}\}. \tag{77}$$

□

Proof of Lemma 2

Proof. Suppose that the platform sector is active: $\Omega = 1$. The maximization problem of the final good producer is:

$$\max_{x_i, L_Y} \int_0^A \left(\left(\frac{b_i}{\bar{b}} \right)^{\chi} x_i \right)^{\alpha} L_Y^{1-\alpha} di - \int_0^A x_i p_i di - wL_Y \tag{78}$$

The first order conditions are:

$$(1-\alpha)\frac{Y}{L_Y} = w \tag{79}$$

$$\alpha\left(\frac{b_i}{\bar{\bar{b}}}\right)^{\alpha\chi} x_i^{\alpha-1} L_Y^{1-\alpha} = p_i \tag{80}$$

where p_i is the price of variety i of the intermediate good. In a symmetric equilibrium $x_i = x\forall i$, $k_i = k\forall i$ and $p_i = p\forall i$. Furthermore, since the linear production technology implies that each intermediate's capital is equal to its output, we can define the aggregate capital stock as $K := Ax$. The final output can be written as

$$Y = \left(\frac{b}{\bar{\bar{b}}}\right)^{\alpha\chi} K^{\alpha}\left((1-s_A-s_M)hNA\right)^{1-\alpha}. \tag{81}$$

Condition (80) can then be re-written as:

$$\alpha\frac{Y}{K} = p. \tag{82}$$

The problem of each intermediate producer is (dropping the i subscript):

$$\max_{x,b} px - (r+\delta)x - p_B b \tag{83}$$

subject to (80) and technology $x = k$. The first order conditions are:

$$\alpha p = r + \delta \tag{84}$$

$$p_B = \alpha^2\chi\frac{b^{\alpha\chi-1}}{\bar{\bar{b}}^{\alpha\chi}} x^{\alpha} L_Y^{1-\alpha} \tag{85}$$

Together with equation (82), the first of these conditions gives the familiar capital demand condition:

$$\alpha^2\frac{Y}{K} = r + \delta. \tag{86}$$

The optimal output of each producer can be derived from plugging the first order condition into the demand curve:

$$x = \left(\frac{\alpha^2}{r+\delta}\left(\frac{b}{\bar{\bar{b}}}\right)^{\alpha\chi}\right)^{\frac{1}{1-\alpha}} L_Y \tag{87}$$

which gives the following expression for capital stock:

$$K = Ax = \left(\frac{\alpha^2}{r+\delta}\right)^{\frac{1}{1-\alpha}} AL_Y. \tag{88}$$

Equations (87), (85) and (88) yield:

$$p_B = \alpha^2 \chi \frac{Y}{A} \frac{1}{\bar{b}} \left(\frac{b}{\bar{b}} \right)^{\frac{\alpha}{1-\alpha}\chi - 1}. \tag{89}$$

In equilibrium $b_i = \bar{b}$ and the results in the Lemma follow immediately from (82), (87) and (89). Equation (89) also implies

$$p_B b = \alpha^2 \chi \frac{Y}{A}. \tag{90}$$

Equilibrium profits of intermediate firms are thus:

$$\Pi = \alpha \frac{Y}{A} (1 - \alpha - \alpha\chi). \tag{91}$$

□

Proof of Lemma 3

Proof. The optimality condition that characterizes the solution to problem (27) is:

$$p_B - \left(1 - \frac{\alpha}{1-\alpha}\chi\right) p_B \frac{A\bar{b}}{B_j + \sum_{k \neq j} B_k} \frac{B_j}{A\bar{b}} = \mathcal{M}^B.$$

In a symmetric equilibrium $B_j = B_k \forall j, k$ hence:

$$p_B \left(1 - \left(1 - \frac{\alpha}{1-\alpha}\chi\right) \frac{1}{J}\right) = \mathcal{M}^B.$$

□

Proof of Lemma 4

Proof. In equilibrium price equals markup over marginal cost:

$$p_B = \Psi w \frac{M}{\ell A^{\phi}}. \tag{92}$$

Using (23) we get:

$$\alpha^2 \chi \frac{Y}{B} = \Psi w \frac{M}{\ell A^{\phi}} \tag{93}$$

Since $B = \ell$ by the platforms' technology and $w = (1-\alpha)\frac{Y}{(1-s_A-s_M)Nh}$ we have:

$$M = \frac{\alpha^2 \chi}{(1-\alpha)\Psi} (1 - s_A - s_M) h N A^{\phi}. \tag{94}$$

Noting that (20) implies $M = s_M hNA^{\phi}$ yields the result.

From $M = A^{\phi} L_M$ and equation (94) we get

$$L_M = \frac{\alpha^2 \chi}{\Psi(1-\alpha)} L_Y. \tag{95}$$

Profits of each platform are equal to

$$\Pi_B^J = \left(p_B - \mathcal{M}^B\right) B_j = \left(\Psi - 1\right) \mathcal{M}^B B_j = \left(\Psi - 1\right) w L_M^j$$

The wage bill paid by the platforms is, by equation (95), equal to:

$$wL_M = (1-\alpha) Y \frac{L_M}{L_Y} = Y \frac{\alpha^2 \chi}{\Psi}$$

and therefore we have

$$\Pi_B = \sum_J \Pi_B^j = \left(\Psi - 1\right) \frac{\alpha^2 \chi}{\Psi} Y.$$

□

Proof of Proposition 3

Proof. Suppose that $\ell > 0$. Combining (76) and (94) and solving for M:

$$M = \left(s_M \Phi N A^{\phi}\right)^{\frac{\nu-1}{\nu}}. \tag{96}$$

Under Assumption 2, for $t < \hat{t}$, $A(t) = A_0 e^{\frac{n}{1-\phi} t} = A_0 \left(\frac{N}{N_0}\right)^{\frac{1}{1-\phi}}$. Equation (77) implies that $\ell > 0$ if and only if $M > \Phi^{\nu-1}$or equivalently that $\left(s_M \Phi N^{\frac{2-\phi}{1-\phi}} \left(\frac{A_0}{N_0^{\frac{1}{1-\phi}}}\right)^{\phi}\right)^{\frac{1}{\nu}} > \Phi$. So there is leisure-enhancing technical change if$N(t) > \left(\frac{\Phi(\hat{t})^{\nu-1}}{s_M(\hat{t})} \left(\frac{N_0^{\frac{1}{1-\phi}}}{A_0}\right)^{\phi}\right)^{\frac{1-\phi}{2-\phi}}$ where Φ and Υ are evaluated at $\hat{t}$ once platforms have entered. Letting $\Gamma := \left(\frac{\Phi(\hat{t})^{\nu-1}}{s_M(\hat{t})} \left(\frac{N_0^{\frac{1}{1-\phi}}}{A_0}\right)^{\phi}\right)^{\frac{1-\phi}{2-\phi}}$ completes the proof. □

Proof of Proposition 4

Proof. The first part of the Proposition follows directly from Proposition 3.

In segment 1, platforms are inactive: $\Omega = 0$, $s_M = 0$ and $h = 1$. Differentiating equation

(4) with respect to time gives the formula for g. Equation (81) implies that output per capita is given by:

$$\bar{y} = \bar{k}^{\alpha}((1 - s_A)A)^{1-\alpha}$$

where $\bar{k} := \frac{K}{N}$ and $\bar{y} := \frac{Y}{N}$. $\bar{k}$ and $\bar{y}$ both grow at equal rate $\gamma_{\bar{y}}$ on the BGP, and thus $\gamma_{\bar{y}} = \alpha\gamma_{\bar{y}} + (1 - \alpha)g$ which implies $\gamma_{\bar{y}} = g$.

Turning to segment 2, equation (76) implies that asymptotically:

$$\gamma_h = \frac{1}{1 - \nu}\gamma_M \tag{97}$$

Differentiating the ideas production functions (4) and (20) with respect to time and assuming balanced growth gives the following two equations

$$0 = (\phi - 1)\gamma_A + n + \gamma_h \tag{98}$$

$$0 = \phi\gamma_A - \gamma_M + n + \gamma_h \tag{99}$$

which imply $\gamma_A = \gamma_M$ as well as the formulas in the proposition. Per capita output growth rate follows from $\gamma_{\bar{y}} = \alpha\gamma_{\bar{y}} + (1 - \alpha)\left(\gamma_A + \frac{1}{1-\nu}\gamma_A\right)$.

Equation (19) implies

$$\frac{\dot{M}}{M} = L_M \frac{A^{\phi}}{M}.$$

Differentiating this expression with respect to time gives, along the balanced growth path:

$$0 = \phi\gamma_A - \gamma_M + n + \gamma_h$$

which is identical to (99) above and thus yields the same long-run conclusions. □

Proof of Proposition 5

Proof. The share of workers in the R&D sector is pinned down by the expected zero-profit condition $wL_A = V\dot{A}$ where V is given by (16). Differentiating equation (16) with respect to time yields a Bellman equation:

$$\dot{V} = Vr - \Pi. \tag{100}$$

Thus we can write $r = \frac{\Pi}{V} + \frac{\dot{V}}{V}$ and $V = \left(r - \frac{\dot{V}}{V}\right)^{-1}\Pi$. On the balanced growth path, r and $\frac{\dot{V}}{V}$ are constant and so V and Π must grow at the same rate. Equation (24) implies that the growth rate of Π and V is equal to $\gamma_Y - \gamma_A$. Plugging this into the expected zero profit condition above we get:

$$(1 - \alpha)\frac{s_A}{1 - s_A - s_M} = \frac{\alpha\,(1 - \alpha - \alpha\chi)}{r - (\gamma_Y - \gamma_A)}\gamma_A. \tag{101}$$

We also have:

$$s_M = \frac{\alpha^2 \chi}{\Psi(1-\alpha)}\left(1 - s_A - s_M\right).$$

Solving for s_M:

$$s_M = \frac{\frac{\alpha^2}{\Psi(1-\alpha)}\chi\left(1 - s_A\right)}{1 + \frac{\alpha^2}{\Psi(1-\alpha)}\chi} = \frac{1 - s_A}{1 + \frac{\Psi(1-\alpha)}{\alpha^2\chi}} \tag{102}$$

Plugging this into (101) and solving for s_A yields the result. □

Proof of Proposition 6

The resource constraint of the economy is $\dot{K} = Y - C - \delta K$. On the BGP, capital and output grow at the same rate so that the capital to output ratio and the interest rate are constant. Taking logs and differentiating the expression for k with respect to time gives $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \gamma_Y$, so that $\dot{K} = \frac{\dot{k}}{k}K + \gamma_Y K = \dot{k}N^{\beta_Y} + \gamma_Y K$ and

$$\dot{k}N^{\beta_Y} + \gamma_Y K = Y - C - \delta K.$$

Dividing through by N^{β_Y} and rearranging yields equation (39).

To obtain equation (40), differentiate a with respect to time to obtain $\dot{A} = \dot{a}N^{\beta_A} + \gamma_A A$. Solving for $\dot{a}$ gives:

$$\dot{a} = \frac{A^{\phi}L_A}{N^{\beta_A}} - \gamma_A a = a^{\phi}N^{\beta_A\phi}s_A\tilde{h}N^{\frac{1}{1-\nu}\beta_A}N^{1-\beta_A} - \gamma_A a.$$

Noting that $\beta_A\phi + \frac{1}{1-\nu}\beta_A + 1 - \beta_A = 0$ we obtain equation (40).

Differentiating c with respect to time we get so that $\frac{\dot{C}}{C} = \frac{\dot{c}}{c} + \beta_Y n$. Optimality conditions (71) and (73) give $\frac{\dot{C}}{C} = r - \rho$. Together these yield (41).

Taking logs and differentiating the expression for v gives $\frac{\dot{v}}{v} = \frac{\dot{V}}{V} - (\gamma_Y - \gamma_A)$. Thus$\dot{V} = \dot{v}(N^{\beta_Y - \beta_A}) + (\gamma_Y - \gamma_A)V$. Plugging this into equation (100) yields $\dot{v}N^{\beta_Y - \beta_A} + (\gamma_Y - \gamma_A)V = Vr - \Pi$. Dividing by $N^{\beta_Y - \beta_A}$ yields the result.

Wages in the final goods sector and in the R&D sector are equal in equilibrium: $(1-\alpha)\frac{Y}{L_Y} = \frac{V\dot{A}}{L_A}$. By definition of the stationary variables, this equation can be written as:

$$(1-\alpha)\frac{yN^{\beta_Y}}{1 - s_A} = vN^{\beta_Y - \beta_A}\frac{a^{\phi}\left(s\tilde{h}\right)^{\lambda}N^{\beta_A}}{s_A}$$

which simplifies to equation (43).

Equilibrium output is $Y = K^{\alpha}((1 - s_A - s_M)hNA)^{1-\alpha}$. Dividing through by N^{β_Y} and noting that $\alpha\beta_Y + (1-\alpha)(\frac{1}{1-\nu}\beta_A + 1 + \beta_A) = \beta_Y$ we obtain the expression for normalized output.

Equation (94) implies

$$M = \left(\frac{\alpha^2\chi}{(1-\alpha)\Psi}(1-s_A-s_M)\Phi N A^{\phi}\right)^{\frac{\nu-1}{\nu}}$$

Thus

$$h = \Phi M^{\frac{1}{1-\nu}} = \Phi\left(\frac{\alpha^2}{\Psi(1-\alpha)}\chi(1-s_A-s_M)\Phi N A^{\phi}\right)^{-\frac{1}{\nu}}$$

and, using the definitions of stationary variables,

$$\tilde{h} = \left(\Phi^{1-\nu}\frac{\alpha^2}{\Psi(1-\alpha)}\chi(1-s_A-s_M)a^{\phi}\right)^{-\frac{1}{\nu}}.$$

Since in equilibrium there is no jump in hours worked at $\hat{t}$, we need $\lim_{t\to\hat{t}^-}\tilde{h} = \lim_{t\to\hat{t}^+}\tilde{h}$, from which equation (45) follows.

Equations (46) and (47) follow immediately from equations (86) and (24), respectively. Equation (48) follows from equation (102).

Proof of Proposition 7

Proof. V_{2a} grows faster than output because $\frac{1-h}{h}$ grows over time. The growth rate of $V_{2b} = (\exp(M^{\frac{1}{\nu-1}}-\Phi)-1)C$ is

$$\frac{\dot{V}_{2b}}{V_{2b}} = \gamma_Y + \frac{\exp(M^{\frac{1}{\nu-1}}-\Phi)\frac{1}{\nu-1}M^{\frac{2-\nu}{\nu-1}}\dot{M}}{\exp(M^{\frac{1}{\nu-1}}-\Phi)-1}$$

The result follows because the final term on the right-hand side is positive. □

Planning problem and decentralized equilibrium with exogenously given M

The current-value Hamiltonian that corresponds to the planner's problem is:

$$\mathcal{H}(K,A,c,h,s,\mu_A,\mu_K) = \log c + M(1-h) + \mu_K\left((A(1-s)hN)^{\alpha}K^{1-\alpha} - C\right) + \mu_A\left(shNA^{\phi}\right)$$

The conditions that characterize the optimal growth path are:

$$c^{-1} = \mu_K N \tag{103}$$

$$\mu_K(1-\alpha)\frac{Y}{1-s} = \mu_A\frac{\dot{A}}{s} \tag{104}$$

$$M \leq \mu_K(1-\alpha)\frac{Y}{h} + \mu_A\frac{\dot{A}}{h} \tag{105}$$

$$\rho - \frac{\dot{\mu}_A}{\mu_A} = \phi\frac{\dot{A}}{A} + \frac{\mu_K}{\mu_A}(1-\alpha)\frac{Y}{A} \tag{106}$$

$$\rho - \frac{\dot{\mu}_K}{\mu_K} = \alpha\frac{Y}{K} \tag{107}$$

$$\lim_{t\to\infty}[\exp(-\rho t)\mu_K K] = 0 \tag{108}$$

$$\lim_{t\to\infty}[\exp(-\rho t)\mu_A A] = 0 \tag{109}$$

where (105) holds with equality when $h < 1$. On the optimal path, hours worked will be stationary and the R&D production function implies that the growth rate of A is given by $g \equiv \frac{\dot{A}}{A} = \frac{n}{1-\phi}$. Equation (103) implies that $-g = \frac{\dot{\mu}_K}{\mu_K} + n$. Equation (106) implies that on BGP $\frac{\mu_K}{\mu_A}(1-\alpha)\frac{Y}{A}$ is constant, thus $\frac{\dot{\mu}_K}{\mu_K} - \frac{\dot{\mu}_A}{\mu_A} + n = 0$ and so $-g = \frac{\dot{\mu}_A}{\mu_A}$. Then equations (106) and (107) give:

$$\rho + g = \phi g + \frac{\mu_K}{\mu_A}(1-\alpha)\frac{Y}{A} \tag{110}$$

$$\rho + g + n = \alpha\frac{Y}{K} \tag{111}$$

Equations (104) and (110) combine to give:

$$(\rho + g(1-\phi))\frac{s^{SP}}{1-s^{SP}} = g.$$

Solving for the optimal share of labor in R&D:

$$s^{SP} = \frac{1}{1 + \left(\frac{\rho}{g} + 1 - \phi\right)}.$$

Equation (105) gives

$$h^{SP} = \min\left\{1, \frac{\mu_K(1-\alpha)Y + \mu_A\dot{A}}{M}\right\} = \min\left\{1, \frac{(1-\alpha)\left(\frac{Y}{C}\right)^{SP}\left(1 + \frac{s^{SP}}{1-s^{SP}}\right)}{M}\right\}$$

where the second equality follows from (103) and (104). The resource constraint implies that the consumption to output ratio in the optimal allocation is given by:

$$\left(\frac{Y}{C}\right)^{SP} = \left(1-(g+n)\frac{K}{Y}\right)^{-1} = \left(1-(g+n)\left(\frac{\alpha}{\rho+n+g}\right)\right)^{-1}$$

where the second equality follows from (111).

Equilibrium

Equilibrium growth rate is the same as on the optimal path. The usual equilibrium conditions yield the equilibrium share of labor employed in the R&D sector:

$$s^{DC} = \frac{1}{1+\frac{1}{\alpha}\left(\frac{\rho}{g}+1\right)} < s^{SP}.$$

Household's intratemporal optimality condition implies:

$$M \leq \frac{w_Y(1-s^{DC})+w_A s^{DC}}{c} = \frac{(1-\alpha)\frac{Y}{h}\frac{1}{N}+V\frac{\dot{A}}{h}\frac{1}{N}}{c} \tag{112}$$

where the second equality follows from labor demands in the final- and the R&D sectors. Together with the expected zero profit condition in the R&D sector, equation (112) implies:

$$h^{DC} = \min\left\{1, \frac{(1-\alpha)\left(\frac{Y}{C}\right)^{DC}\left(1+\frac{s^{DC}}{1-s^{DC}}\right)}{M}\right\},$$

where, by the budget constraint and the Euler Equation, $\left(\frac{Y}{C}\right)^{DC} = \left(1-(g+n)\frac{\alpha^2}{\rho+n+g}\right)^{-1} < \left(\frac{Y}{C}\right)^{SP}$.

Since $s^{SP} > s^{DC}$ and $\left(\frac{Y}{C}\right)^{SP} > \left(\frac{Y}{C}\right)^{DC}$, we have $h^{SP} \geq h^{DC}$ which holds strictly if M is such that h^{DC} is interior.

C Leisure-consumption complementarities

Consider a more general model where each leisure activity requires leisure time $\ell(\iota)$, free leisure services $m(\iota)$ and leisure consumption goods $c(\iota)$. For simplicity, assume that elasticity of substitution between time or leisure services and leisure consumption within activity is equal to one, so that:

$$l := \left(\int_0^M \left[\left(\left(\min\{\ell(\iota), m(\iota)\}\right)^{\varphi}\left(c(\iota)\right)^{1-\varphi}\right)\right]^{\frac{\nu-1}{\nu}} d\iota\right)^{\frac{\nu}{\nu-1}}$$

where $\varphi \in (0,1]$. We recover the formulation in the main text by setting $\varphi = 1$. A symmetric allocation of time and consumption across activities implies that

$$l = \left(M \left(\left(\frac{\ell}{M} \right)^{\varphi} \left(\frac{C_L}{M} \right)^{1-\varphi} \right)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} = M^{\frac{1}{\nu-1}} \ell^{\varphi} C_L^{1-\varphi}$$

To see the consequences of this formulation for the leisure supply of the household, consider the simple static time allocation problem:

$$\max_{C_L,h} \log(wh - p_L C_L) + M^{\frac{1}{\nu-1}} (1-h)^{\varphi} (C_L)^{1-\varphi}$$

The first order conditions are:

$$\frac{1}{C} p_L = (1-\varphi) M^{\frac{1}{\nu-1}} (1-h)^{\varphi} (C_L)^{-\varphi} \tag{113}$$

$$\frac{1}{C} w = \varphi M^{\frac{1}{\nu-1}} (1-h)^{\varphi-1} (C_L)^{1-\varphi} \tag{114}$$

Thus the expenditure shares are constant and:

$$C_L = \frac{1-\varphi}{\varphi} \frac{w}{p_L} (1-h).$$

Plugging this into (114):

$$C = \frac{w}{\varphi M^{\frac{1}{\nu-1}} \left(\frac{1-\varphi}{\varphi} \frac{w}{p_L} \right)^{1-\varphi}}. \tag{115}$$

Combining 115 with the budget constraint we obtain:

$$h = \min \left\{ 1, 1-\varphi + M^{\frac{1}{1-\nu}} \left(\frac{\varphi}{1-\varphi} \frac{p_L}{w} \right)^{1-\varphi} \right\} \tag{116}$$

$$C = \varphi^{\varphi} (1-\varphi)^{\varphi-1} M^{\frac{1}{1-\nu}} w^{\varphi} p_L^{1-\varphi} \tag{117}$$

$$C_L = \frac{1-\varphi}{\varphi} \frac{w}{p_L} (1-h) \tag{118}$$

Equation (116) shows that the time that households allocate to leisure continues to depend positively on leisure technologies M, so that the main mechanisms and hence the implications of the paper go through with that more general formulation. It also shows that in presence of leisure consumption goods, hours worked do not converge to zero but instead to a lower bound of $1-\varphi$. This is intuitive: in the limit, households must afford to buy leisure consumption goods therefore they work more than in the baseline model. Moreover, equations (117) and (118) show that an expansion in leisure technologies acts as a relative demand shifter, boosting demand for consumption goods that are complimentary with leisure and reducing demand for traditional consumption.

D The platform pricing decision

The model developed in this Appendix builds on Rochet and Tirole (2003) and Armstrong (2006). The environment is simpler than the problem considered in the main text; it serves to highlight the important issues when it comes to the optimal pricing strategy of platforms operating in two-sided leisure markets. In particular, it shows what kind of considerations may be important in driving low or zero prices. In short, high elasticities of consumer demand and substantial benefits to the other side of the market (advertisers) can lead to the optimal pricing strategy that features zero-price leisure services in equilibrium. These basic insights extend beyond the simple monopoly structure to models of platform competition.

Suppose there are two groups: a unit measure of consumers (group 1) and measure-A of firms / advertisers (group 2), interested in interacting with each other through a monopoly platform. In particular, suppose that the platform provides consumers with leisure technologies of value M and charges them price p_1 for accessing the service. Furthermore, consumers may care about how many firms advertise on the platform (with ambiguous sign). The platform charges firms price p_2 for accessing the platform. Since firms use the platform to build brand equity capital, their benefit from using the platform depends on the total time that consumers spend on the platform. Consistent with this description, assume that the utilities of consumers and firms are, respectively:

$$u_1 = \alpha_1 A - p_1 + M + \epsilon \qquad u_2 = \alpha_2 \ell - p_2$$

where $\alpha_2 > 0$, ϵ mean-zero random component, and ℓ is the number / share of consumers that end up using the service. I assume that all agents for whom utility is non-negative participate.

The sign of α_1 is ambiguous as consumers could derive benefits from greater visibility of brands and extra information about their products, but could also find advertising tiresome. To maintain a neutral stance and to make the assumption consistent with the rest of the text, suppose that $\alpha_1 = 0$.

The share of consumers using the platform is a non-decreasing function of utility:

$$\ell = f(u_1) = \phi(\underset{-}{p_1}, \underset{+}{M}).$$

Suppose it costs the platform $\mathbb{C}(M)$ to produce leisure services and brand equity. The platform then chooses prices and quantity M to maximize profits:

$$\max_{p_1,p_2,M} \Pi_B = p_1\ell + p_2 A - \mathbb{C}(M).$$

Given no random component in the utility of the firms, the platform extracts all surplus from the firm side by charging:

$$p_2 = \alpha_2 \ell.$$

We thus have:

$$\Pi_B = p_1\phi(p_1, M) + A\alpha_2\phi(p_1, M) - \mathbb{C}(M).$$

Profit maximization implies the following optimality conditions:

$$\phi + p_1\phi_{p_1} + A\alpha_2\phi_{p_1} = 0$$

$$p_1\phi_M + A\alpha_2\phi_M - \mathbb{C}'(M) = 0.$$

Together these imply:

$$p_1 = \frac{\phi + \mathbb{C}'(M)}{\phi_M - \phi_{p_1}} - A\alpha_2 \tag{119}$$

Equation (119) pins down the optimal price that the platform charges the consumers. Derivatives ϕ_{p_1} and ϕ_M are negative and positive respectively, so the first term on the right hand side is positive. The optimal price can be zero or negative if the term $A\alpha_2$ is larger than $\frac{\phi+\mathbb{C}'(M)}{\phi_M-\phi_{p_1}}$. This is more likely when: (i) demand for platform services is low (low ϕ); (ii) it is cheap to produce leisure services (low $\mathbb{C}'(M)$); (iii) consumer demand is highly elastic to prices and leisure technologies (high $\phi_M - \phi_{p_1}$); and (iv) when there are many advertisers whose utility is highly sensitive to the number of consumers using the service (high A and α_2, respectively). Many of these conditions are likely to be satisfied in the context of leisure platforms, hence the proliferation of zero-price services that we observe in the real world. This analysis underlies the logic of focusing on free leisure services in the rest of the paper. See Appendix C for how to incorporate paid-for leisure consumption goods into the model.

E Schumpeterian economy with leisure-enhancing technological change

Consider the basic Schumpeterian growth model with constant population of size N. Each household works h hours, with $h = \min\{1, \Phi M^{\frac{1}{1-\nu}}\}$, as in the model in the main text. Final output is given by:

$$Y = \int_0^1 A_i^{1-\alpha}\left(\left(\frac{b_i}{\bar{b}}\right)^{\chi} x_i\right)^{\alpha} di \cdot L_Y^{1-\alpha}$$

where x_i are the intermediate inputs and A_i is the input-specific productivity. Intermediate product demand is:

$$p_i = \alpha(A_i L_Y)^{1-\alpha}\left(\frac{b(i)}{\bar{b}}\right)^{\alpha\chi} x_i^{\alpha-1}.$$

Thus intermediate producer's problem is to

$$\max_{x_i,b_i}\ \alpha(A_i L_Y)^{1-\alpha}\left(\frac{b_i}{\bar{b}}\right)^{\alpha\chi} x_i^{\alpha} - (r+\delta)x_i - p_B b_i$$

which implies equilibrium quantity:

$$x_i = \left(\frac{\alpha^2}{r+\delta}\right)^{\frac{1}{1-\alpha}} A_i L_Y$$

We can thus write the final output as:

$$Y = \left(\frac{\alpha^2}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}\left(\int_0^1 A_i di\right) L_Y$$

Equilibrium spend on ads is the same as in the main text. Equilibrium profits are therefore:

$$\Pi_i = x_i(p-(r+\delta)-\chi(r+\delta)) = \left(\frac{1}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} A_i L_Y(1-\alpha-\alpha\chi) = \pi A_i L_Y.$$

Research

Assume that research costs R_i of final output every period. Research is risky. Denote by μ the probability that research succeeds, and by $A^* := \gamma A$ the target productivity level of the successful innovation. Finally, define $n := R/A^*$ as the productivity adjusted expenditure. Then assume that the success function follows:

$$\mu_i = \lambda n_i^{\sigma} = \lambda\left(\frac{R_i}{A_i^*}\right)^{\sigma}$$

Note that $\mu_i' = \lambda\sigma n_i^{\sigma-1}$. Assume for simplicity that a successful innovator operates the technology for one period, and is subsequently removed either by another innovator or, if no innovator succeeds, by a randomly chosen individual. Thus the reward from pursuing research is $\mu_i\Pi_i$ and the entrepreneur maximizes

$$\max_{R_i} \lambda\left(\frac{R_i}{A_i^*}\right)^{\sigma}\Pi_i - R_i$$

The optimality condition yields:

$$\lambda \sigma n_i^{\sigma-1} \frac{\Pi_i}{A_i^*} = \lambda \sigma n_i^{\sigma-1} \pi L_Y = 1$$

Solving for n_i gives

$$n_i = (\lambda \sigma \pi L_Y)^{\frac{1}{1-\sigma}}$$

and the optimal frequency of success is $\mu_i = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L_Y)^{\frac{\sigma}{1-\sigma}}$.

Growth

Growth rate of A is computed as follows:

$$A_{t+1} = \mu A_{t,success} + (1-\mu) A_{t,failure} = \mu \gamma A_t + (1-\mu) A_t$$

Thus

$$\gamma_A = \mu(\gamma - 1) = \lambda^{\frac{1}{1-\sigma}} (\sigma \pi L_Y)^{\frac{\sigma}{1-\sigma}} (\gamma - 1).$$

Clearly, there are two channels through which leisure-enhancing technologies affect γ_A. First, since $L_Y = hN$ by labor market clearing, declining hours worked lead to a declining growth rate of traditional TFP through a market-size effect. Second, since π is diminished by $\alpha^2 \chi$ per unit sold, this also lowers the incentives to R&D and thus lowers economic growth. This latter effect is analogous to the level effect working through the lower share of R&D workers in the baseline model.

F Dynamic ideas production function

Suppose the leisure ideas production function is dynamic, that is:

$$\dot{M}_j = L_j^M \cdot A^\phi. \tag{120}$$

Aggregate new leisure technologies are

$$\dot{M} = \sum \dot{M}_j = \sum \left(L_j^M \cdot A^\phi \right) = A^\phi \sum L_M^j = A^\phi L_M$$

Each platform solves a dynamic optimal control problem:

$$\max_{L_M^j} \int_0^\infty e^{-rt} \left(p_B \cdot M_j \frac{\ell}{M} - w L_j^M \right) dt$$

subject to (120) and

$$p_B\left(B_j+\sum_{k\neq j}B_k\right)=\alpha^2\chi\frac{Y}{B_j+\sum_{k\neq j}B_k}\left(\frac{B_j+\sum_{k\neq j}B_k}{A\bar{b}}\right)^{\frac{\alpha}{1-\alpha}\chi}.$$

The Hamiltonian of this problem is:

$$\mathcal{H}=p_B\cdot M_j\frac{\ell}{M}-wL_j^M+\mu\left[L_M^j\cdot A^\phi\right]$$

By the Maximum Principle, the optimality conditions are:

$$\mathcal{H}_{L_j^M}=-w+\mu A^\phi=0 \tag{121}$$

$$\mathcal{H}_{M_j}=p_B\frac{\ell}{M}+M_j\frac{\ell}{M}\left(\frac{\alpha}{1-\alpha}\chi-1\right)p_B\frac{1}{\sum M_{j'}\frac{\ell}{M}}\frac{\ell}{M}=r\mu-\dot{\mu} \tag{122}$$

Equation (122) yields:

$$p_B\frac{\ell}{M}\left(1+\left(\frac{\alpha}{1-\alpha}\chi-1\right)\frac{M_j}{M}\right)=r\mu-\dot{\mu}$$

In a symmetric equilibrium $\frac{M_j}{M}=\frac{1}{J}$ so that

$$p_B\frac{\ell}{M}\left(1+\left(\frac{\alpha}{1-\alpha}\chi-1\right)\frac{1}{J}\right)=r\mu-\dot{\mu}.$$

In equilibrium, $p_B=\alpha^2\chi\frac{Y}{B}$ and $\ell=B$ so:

$$\alpha^2\chi\frac{Y}{M}\left(1+\left(\frac{\alpha}{1-\alpha}\chi-1\right)\frac{1}{J}\right)=r\mu-\dot{\mu}.$$

Equilibrium requires wages are equalized across sectors thus

$$(1-\alpha)\frac{Y}{L_Y}=V\frac{\dot{A}}{L_A}=\mu\frac{\dot{M}}{L_M}$$

To sum up, relative to the case with the static formulation considered in the main text, we now have:

$$\dot{M} = A^{\phi} L_M \tag{123}$$

$$\alpha^2 \chi \frac{Y}{M} \left(1 + \left(\frac{\alpha}{1-\alpha}\chi - 1\right)\frac{1}{J}\right) = r\mu - \dot{\mu} \tag{124}$$

$$\mu \frac{\dot{M}}{s_M} = V \frac{\dot{A}}{s_A} \tag{125}$$

where the final equation replaces (48).

Clearly $\gamma_M = \gamma_A$ so that $m := \frac{M}{N^{\beta_A}}$ is stationary on the balanced growth path. Thus $\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \beta_A n$. Therefore $\dot{M} = \frac{\dot{m}}{m} M - \beta_A n M = \dot{m} N^{\beta_A} - \gamma_A m N^{\beta_A}$. Using these results in equation (123) yields

$$\dot{m} N^{\beta_A} - \gamma_A m N^{\beta_A} = a^{\phi} N^{\phi \beta_A} s_M h N = a^{\phi} N^{\phi \beta_A} s_M \tilde{h} N^{\frac{1}{1-\nu}\beta_A} N,$$

which simplifies to

$$\dot{m} = a^{\phi} s_M \tilde{h} - \gamma_A m.$$

Define $\tilde{\mu} := \frac{\mu}{N^{\beta_\mu}}$ to be the normalized level of the costate. We have $\frac{\dot{\tilde{\mu}}}{\tilde{\mu}} = \frac{\dot{\mu}}{\mu} - \beta_\mu n$ and so $\dot{\mu} = \frac{\dot{\tilde{\mu}}}{\tilde{\mu}} \mu + \beta_\mu n \mu = \dot{\tilde{\mu}} N^{\beta_\mu} + \beta_\mu n \tilde{\mu} N^{\beta_\mu}$. Therefore:

$$\alpha^2 \chi \frac{y N^{\beta_Y}}{m N^{\beta_A}} \left(1 + \left(\frac{\alpha}{1-\alpha}\chi - 1\right)\frac{1}{J}\right) = r\tilde{\mu} N^{\beta_\mu} - \dot{\tilde{\mu}} N^{\beta_\mu} - \beta_\mu n \tilde{\mu} N^{\beta_\mu}$$

Thus

$$\beta_Y - \beta_A = \beta_\mu$$

and therefore equation (124) in stationary form is:

$$\dot{\tilde{\mu}} = \frac{y}{m}\Psi + (r - (\beta_Y - \beta_A)n)\tilde{\mu}.$$

Equation (125) yields

$$\tilde{\mu} m^{\phi} = v a^{\phi}.$$

To summarize, the system of equations that pins down the equilibrium with a dynamic leisure production function in (19) is:

$$\dot{k} = y - c - \delta k - \gamma_Y k \tag{126}$$

$$\dot{a} = a^{\phi} s_A \tilde{h} - \gamma_A a \tag{127}$$

$$\dot{m} = a^{\phi} s_M \tilde{h} - \gamma_A m \tag{128}$$

$$\dot{c} = c\left(r - \rho - \gamma_Y\right) \tag{129}$$

$$\dot{v} = v\left(r - (\gamma_Y - \gamma_A)\right) - \pi \tag{130}$$

$$\dot{\tilde{\mu}} = \frac{y}{m}\Psi + (r - (\beta_Y - \beta_A)n)\tilde{\mu} \tag{131}$$

$$(1-\alpha)\frac{y}{1 - s_A - s_M} = v a^{\phi} \tilde{h} \tag{132}$$

$$y = k^{\alpha}\left((1 - s_A - s_M)\tilde{h} a\right)^{1-\alpha} \tag{133}$$

$$\tilde{h} = \left(h^{\hat{t}}\right)^{\frac{\Omega-1}{1-\nu}} (\Phi m)^{\frac{\Omega}{1-\nu}} \tag{134}$$

$$r = \alpha^2 \frac{y}{k} - \delta \tag{135}$$

$$\pi = \alpha \frac{y}{a}\left(1 - \alpha - \alpha\chi\Omega\right) \tag{136}$$

$$\tilde{\mu} m^{\phi} = v a^{\phi} \tag{137}$$

where $h^{\hat{t}} := \Phi(\hat{t}) m(\hat{t})$.

G Non-marketable leisure

Suppose leisure output is a combination of marketable and non-marketable leisure, such as hiking or walking in the park. For simplicity, assume that the elasticity of substitution between marketable and non-marketable leisure is one so that:

$$l = l_M^{\eta} \ell_N^{1-\eta}$$

where ℓ_N is time spent hiking, $l_M := \left(\int_0^M \underbrace{[\min\{\ell(\iota), m(\iota)\}]}_{\text{activity}(\iota)}^{\frac{\nu-1}{\nu}} d\iota\right)^{\frac{\nu}{\nu-1}}$ and ℓ_M is total marketable leisure time as before. Since $\frac{\ell_N}{\ell_M} = \frac{1-\eta}{\eta}$ and $l_M = \ell_M M^{\frac{1}{\nu-1}}$ we get

$$l = \ell_M M^{\frac{\eta}{\nu-1}} \left(\frac{1-\eta}{\eta}\right)^{1-\eta}$$

which is similar to before ($l = \ell_M M^{\frac{1}{\nu-1}}$). Labor supply is in this case

$$h = M^{\frac{\eta}{1-\nu}} \eta^{-\eta} (1-\eta)^{\eta-1}.$$

Thus all the results of the benchmark framework go through after parameter ν is recalibrated to reflect the fact that leisure technologies crowd out not just time at work but also time spent on non-marketable leisure.

H Alternative ways of modeling advertising

This Appendix sketches out two alternative ways to incorporate brand equity competition into the monopolistic competition framework. Note first that the final good production function (imposing symmetry in advertising) can be written as

$$Y = \left(\left(\int_0^A x_i^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}\right)^{\alpha} L_Y^{1-\alpha} di$$

where $\epsilon := \frac{1}{1-\alpha}$ is the elasticity of substitution across the intermediate goods. Here I consider two alternatives to the combative advertising assumption outlined in the main text: that advertising shifts the intensity of tastes towards consumption goods (equivalently raises total factor productivity in the final sector); and that advertising makes products more differentiated.

Non-combative advertising

Consider first a formulation where advertising is not combative, but instead shifts the intensity of preferences as follows:

$$Y = \left(\left(\int_0^A (b_i^{\chi} x_i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}\right)^{\alpha} L_Y^{1-\alpha} di = \int_0^A (b_i^{\chi} x_i)^{\alpha} di L_Y^{1-\alpha}$$

In a symmetric equilibrium with $K = Ax$ and $B = Ab$ we have:

$$Y = (B^{\chi} K)^{\alpha} A^{1-\alpha-\alpha\chi} L_Y{}^{1-\alpha}.$$

This equation shows that this alternative formulation will have important implications for the growth rate of output. Recall that by equation (18) $B = \ell$, so that output growth will be fastest at low levels of B, when ad spending and leisure hours are growing the fastest. Over time ads cease to be a source of growth; instead, the formulation suggests that output will be growing more slowly (the exponent on A is $1-\alpha-\alpha\chi$ instead of $1-\alpha$). Demand for good i is

$$\alpha b_i^{\alpha\chi} x_i^{\alpha-1} L_Y^{1-\alpha} = p_i. \tag{138}$$

The demand for brand equity is:

$$p_B = \alpha^2 \chi \frac{Y}{A} b^{\frac{\alpha}{1-\alpha}\chi - 1}. \tag{139}$$

Equation (93) becomes

$$\alpha^2 \chi \frac{Y}{A}\frac{A}{B}\left(\frac{B}{A}\right)^{\frac{\alpha}{1-\alpha}\chi} = \Psi w \frac{M}{\ell A^{\phi}},$$

which yields the supply of leisure technologies:

$$M = \frac{\alpha^2 \chi}{(1-\alpha)\Psi}(1-s_A-s_M)h(1-h)^{\frac{\alpha}{1-\alpha}\chi} N A^{\phi - \frac{\alpha}{1-\alpha}\chi}$$

Assuming that $\phi - \frac{\alpha}{1-\alpha}\chi > 0$, it is clear that the structure of equilibrium is similar to the model in the main text.

Advertising that alters elasticity of substitution across goods

Consider now the following formulation:

$$Y = \int_0^A (x_i - b_i)^{\alpha}\, di L_Y^{1-\alpha}$$

Demand is:

$$x_i = \left(\frac{\alpha}{p_i}\right)^{\frac{1}{1-\alpha}} L_Y + b_i$$

Clearly, advertising shifts demand. But now it also makes demand more inelastic. The elasticity of demand is

$$\left|\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}\right| = \frac{1}{1-\alpha}\left(1 - \frac{b_i}{x_i}\right)$$

In this economy brand equity competition exacerbates the monopoly power of firms, raising prices and lowering output, moving the economy further away from the competitive benchmark.

I Alternative calibration of elasticity ν

To illustrate robustness of the main findings to alternative values of elasticity across leisure varieties ν, Figure A.6 shows the paths for hours worked and traditional TFP growth for the calibration in the main text, as well as a lower and a higher value of ν. While the qualitative conclusions are unchanged, the different calibrations do matter for the quantitative implications. The parameter enters non-linearly, so that a lower value changes the results considerably more.

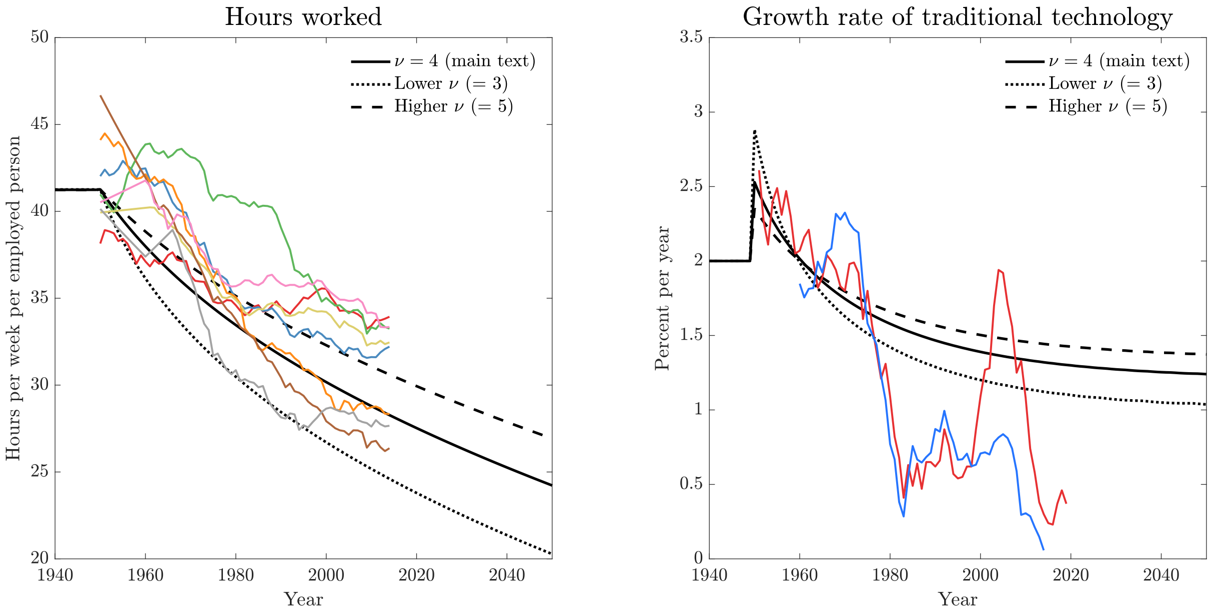

Figure A.6

Robustness to Higher and Lower Values of Elasticity ν.

J Anticipated entry of the leisure platforms

The equilibrium concept in Definition 1 incorporates the assumption that the entry to leisure platforms is unanticipated. Figure A.7 presents the solution to the model when the platform entry is instead anticipated. Naturally, segment 1 no longer features exact balanced growth. But these effects are relatively minor, underlying the focus of the main text on the simpler case with unanticipated platform entry.

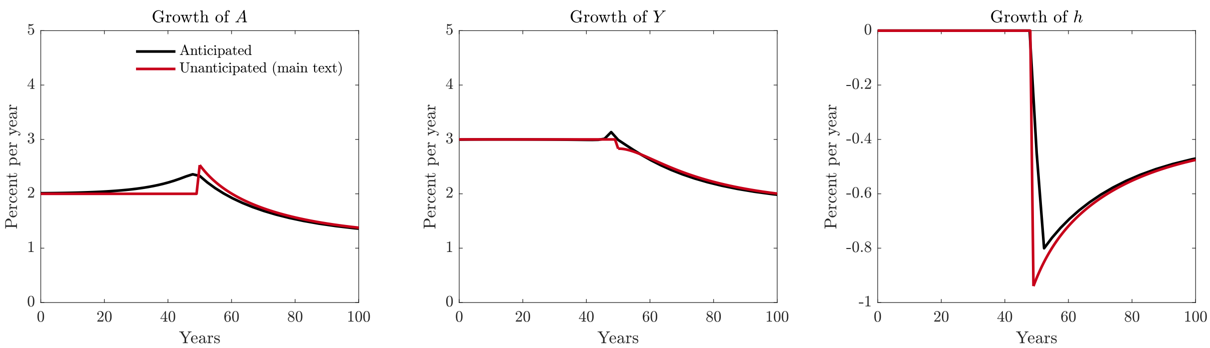

Figure A.7

Transition Dynamics When the Entry of the Platforms is Anticipated