Expectations and the Rate of Inflation*

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What is the causal effect of higher expectations of future inflation on current inflation? I compute this passthrough for a series of canonical firm-pricing models, allowing for arbitrary (non-rational) expectations. In the Calvo model, the expectational-passthrough can be made arbitrarily close to zero for sufficiently high stickiness, but in practice I show it is close to 1 for reasonable parameters. In the Taylor model, in contrast, the upper bound for passthrough is ½ instead of 1. For a general time-dependent model I show that: (i) passthrough is given by a simple sufficient statistic: the ratio of the average duration of ongoing price spells to that of completed price spells; (ii) the lowest possible passthrough is ½, attained by Taylor pricing; and (iii) passthrough can be theoretically greater than 1 for hazards that decrease over time (iv) I provide a generalized Phillips curve for current inflation as a function of expectations of future inflation and realized past inflations; (v) the coefficients on future expectations are decreasing with the horizon, indicating that it is expectations in the near future that matters the most and that expectations of long-run inflation are completely irrelevant; (vi) the sum of all coefficients, both past and future, sums to one, so that the long-run Phillips curve is vertical. I then turn to continuous-time state-dependent "menu cost" models and show that passthrough in these models can be extremely low or extremely high on impact, depending on parameters. Responses are more moderate once they are time-aggregated to a quarter. Finally, I suggest a model where firms must pay a fixed cost for changing their sS pricing policy bands. This extension gives a passthrough of 0 for small enough changes in expectations.

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1 Introduction

It is widely believed that inflation is strongly affected by the expectations of future inflation, perhaps in a near one-to-one relationship. This tight reverse feedback mechanism justifies considering the management of expectations a crucial part of monetary policy playbook. As the story goes, if inflation expectations are stable or so-called "well anchored" at low levels, then inflation may remain stable and low; on the other hand, if bad news or high inflation rates raise the specter of future inflation, then inflation inevitably follows from the rise in expectations. The tail wags the dog.[1]

Expectations no doubt matter to some degree, but just how direct and powerful is the causal link or passthrough from inflation expectations to current inflation? What features of the economy determine the magnitude of this passthrough? Is it long-run or short-run inflation expectations that matters the most? Finally, how does past inflation, as opposed to expected future inflation, impact current inflation?

I explore these questions within a set of canonical economic models of firm pricing. I calculate the passthrough for the ubiquitous Calvo-pricing setting first, contrast this to Taylor-pricing and then develop a result for more general time-dependent models. Finally, I also explore a variety of state dependent "sS menu cost" models. Studying the passthrough predicted by this wide class of models is complementary to a few empirical efforts in this direction. As is well appreciated, it is quite challenging to credibly identify and isolate the effect of inflation expectations empirically.

Although the backbone of the models I employ are standard, the way I solve them is less standard. I solve for the impact of a change in inflation expectations, holding all other relevant variables fixed. Changing expectations in this way requires parting ways with rational expectations or any other particular model of expectations formation. Thus, a key element of my analysis is flexibly allowing for any arbitrary set of expectations and solving for the "temporary equilibrium" at that point in time.[2]

My goal is to shed light on the short-run impact of inflation expectations on current inflation: to go from inflation expectations to current inflation. This complements an extensive and important body of theoretical and empirical work going in the opposite direction and studying the formation of expectations from past inflation.

Allowing for arbitrary expectations has an additional advantage: the solution for in-

[1]Theoretically, early rational expectations models relating inflation and output such as Lucas (1972) justified such a belief and captured the ideas from Friedman (1968) and Phelps (1967). As pointed out below, a cursory approach to the Calvo model also reinforces such a belief.

[2]The term "temporary equilibrium" dates back to Marshall and was adapted by Hicks, Patinkin, Grandmont, Phelps and others, and has been employed more recently in the learning literature and other departures from rational expectations.

flation as a function of expectations can always be combined, if desired, with any model of expectations formation, including rational expectations as a special case. Even if this were the ultimate goal, it is beneficial not to leap to such a combination of assumptions in a single step. Indeed, the temporary equilibrium that allows for arbitrary expectations can help shed light into the economic mechanisms at play even under rational expectations.[3]

The metric I focus on is the impact effect on inflation of a change in expectations of future inflation. To keep things simple, I first start by assuming that firms expect a constant inflation rate. This allows me to focus on a single *passthrough* coefficient that measures the current impact on inflation of a change in expectations. I later relax this assumption to disentangle short- vs long-run expectations of inflation.

My results cast doubt on the firmly held view that the passthrough from inflation expectations to inflation is nearly one-for-one. On the one hand, I show that this passthrough can take on a wide range of values and is plausibly much lower than unity. On the other hand, the passthrough is greater or equal to ½ in time-dependent setups so these theories put a non-trivial lower bound on the impact of expectations. However, I also show that state-dependent "menu cost" pricing setups are capable of lower passthroughs.

I start with the widely adopted Calvo pricing model, where firms have a constant probability of getting a chance to reset their price. For this model, it is extremely tempting and common to read the passthrough off the linearized "Phillips curve" equation as being equal to the discount factor, which then leads one to conclude that passthrough is close to one. The logic is correct for computing the "long run slope of the Phillips curve", i.e. the steady solution with constant inflation, but is not correct for deriving the independent role of expectations themselves.[4] One way to see this is that solving forward we can also write inflation as a function of current and future output gaps. Should we conclude that the passthrough is zero or one?

The paradox dissolves by noting that both expressions impose rational expectations. As a result, inflation expectations are not free but instead tied down to the future evolution of output. With rational expectations one simply cannot consider the thought experiment of modifying expectations without also manipulating future output. In contrast, my goal is to express inflation as a function of inflation expectations, treating these as free variables, as well as current and expected future real marginal costs (which, in turn, could

[3]This general idea is embraced in many contexts by economists. To take a microeconomic example, the notions of supply and demand curves are beloved concepts, and nobody insist on leaping instead to only solving for the equilibrium. Instead, one often pauses first to think about each curve's determinants, how elastic they are, their shape, etc. This is especially useful if we then wish to move from a competitive analysis to, say, that of a monopolist—we now use the demand curve differently. This is akin to considering different models of expectations formation.

[4]This point is well appreciated in the learning literature.

be related to output gaps) or any other variables affecting pricing decisions.

Solving for the passthrough from expected inflation to inflation in the Calvo model, I show that it has the potential for being very low. Indeed, for any given discount factor, this passthrough goes to zero as prices become fully rigid. This theoretical possibility notwithstanding, I show that, in practice, for plausible parameter values, the passthrough is close to one.

We find ourselves back with the same conclusion of a near one-for-one passthrough from inflation expectations to realized inflation, but this time on better logical footing. However, my next results show that this conclusion is special to the Calvo setup, it is not a robust economic conclusion.

I turn first to the Taylor-pricing case where firms set prices every fixed number of periods N. This form of price rigidity introduced by Fischer (1977) and Taylor (1980) for wages and prices is a natural benchmark and was initially popular in influential studies of monetary policy and nominal rigidities. However, it fell out of favor when Calvo (1983) provided a more tractable setup with a constant probability of price changes, simplifying aggregation and dynamics.[5] I find that passthrough in the Taylor setting is well approximated in practice by ½ rather than 1.

What is the economic mechanism behind these results and what explains this difference? Intuitively, when firm plan to have prices fixed for some time they want to set them so that they average out to an ideal price. When firms anticipate positive inflation this ideal price is rising. As a result, to get things right on average, they set their price initially above the ideal price and over time their price ends up below their ideal price. The greater the expected inflation, the greater must be this initial price "overshoot".

This overshooting mechanism is at the heart of the transmission mechanism from expectations of future inflation to current inflation. Overshooting is more aggressive in the Calvo-pricing setup because the constant probability of a price change creates a right-tailed risk of prices remaining unchanged for very long periods of time. Indeed, the median price spell duration is lower than the mean. In contrast, in the Taylor setup there is no such right-tailed risk (median equals average duration) and overshooting is less aggressive.

In both Calvo and Taylor, the passthrough coefficient does not depend on the frequency of price changes. Lower frequency makes firms overshoot inflation more, but this is exactly offset by the having fewer firms changing prices at any point in time.

The Calvo and Taylor model constitute two historically important benchmarks, yet

[5] Models with nominal rigidities have a large state variable: the entire distribution of prices, but this state can be dispensed with in the special Calvo-pricing model.

they are both special. To dive deeper I study a general time-dependent pricing model, with an arbitrary hazard rate schedule. The hazard gives the probability of a price change as a function of the time elapsed since the last price change.

For this relatively general setup, I obtain a surprisingly simple result: passthrough equals the ratio of two duration measures. The numerator is the average duration of ongoing price spells. The denominator is the average duration of completed spells. Revisiting Taylor, it is easy to see that this ratio is ½ since the average age of ongoing prices is N/2. In the Calvo case, both averages are equal to each other. More generally, if one thinks of the average duration of ongoing prices as a proxy for that of completed prices there are two sources of "bias". On the one hand, for a given spell, the age of an ongoing price is lower than its eventual duration. On the other hand, short spells are underrepresented relative to longer spells. From the duration literature, it is well known that with an exponential distribution these two effects offset each other, which is why passthrough equals one in Calvo.

I then show that across all time-dependent hazard rates (in the limit with no discounting) the lowest possible passthrough is ½ attained by Taylor. Intuitively, all other specifications have greater right-tail risk for firms. Passthroughs above 1 are also possible for distributions with fatter tails than the exponential, such as a Pareto distribution, obtained with falling hazard rates.

I then extend the analysis to allow for general expectations of future inflation, that depend on the horizon of the forecast, as well for nonzero past realized inflation. I produce a general Phillips curve, with two sets of coefficients, those on expected future inflation rates and on past realized inflation rates. I show that the coefficients on expectations are largest at shorter horizons. Thus, expectations of short run inflation dominate. Indeed, expectations of inflation for the very long run are shown to be irrelevant.

The coefficients on past inflation are generally non-zero, except in the Calvo case. Indeed, the sum of both sets of coefficients, past and future, add up to one. In this sense, the "long-run Phillips curve" can be said to be vertical: any steady state inflation is possible (at a "natural" real marginal cost). Although long-run neutrality is sometimes taken for granted, it is not immediately apparent in this general setup.

This more general analysis highlights the spirit of my analysis, as well as its advantage and limits. I am purposefully seeking to elaborate on the determination of inflation, focusing on price setting firms and allowing for flexible expectations of future inflation as well as past inflation, and other real determinants. In this way, my analysis can be seen as characterizing a more flexible Phillips curve. The spirit of the analysis is *not* to "close the model" and combine this condition with a theory of expectation formation, or with other pieces of a greater macroeconomic model and a specification of policy. Any such exercise would be special or open up a plethora of options and can be carried out in other studies. Put starkly, I seek to further our understanding of one important equilibrium condition and leave using it in combination with other equilibrium conditions for another day.

Finally, I turn to state dependent "menu cost" models. I first show that the standard set of these models produce extreme results: inflation can jump discontinuously up or down when expected inflation rises! This has the potential for making passthrough very small (even negative) or very large. However, behind this result is the prediction that the frequency of price changes dramatically in the very short run. In my view, this is an unrealistic feature of these basic models.

Thus, I consider two variations. In the first, the frequency of price changes is assumed fixed in the very short run. The motivation is that the resources of goods and time to change prices is difficult to adjust in the short run. I show that this can produce a more reasonable passthrough, potentially below ½.

In the second extension, I elevate the main feature of menu cost models to another level: I consider fixed costs of changing the sS pricing bands. I argue that it is difficult to entertain costs to changing prices, for given pricing rules, and not also consider the costs of changing the bands themselves. If the change in expectations is not too large, the firm will not find it profitable enough to pay the fixed cost to change the bands. If the bands do not change, then there is no change in inflation and the passthrough is zero. Numerical explorations show that reasonable values of the fixed cost produce relatively wide ranges of inaction. For example, if the fixed cost of changing bands is 5 times that of a price change then it takes a 12% rise in expected inflation to trigger a change in the pricing bands.

This paper makes contact with several strands of research. There is a vast empirical literature on the determinants of inflation related to the estimation or testing of so-called Phillips curve. In the context of a hybrid New Keynesian Phillips Curve Galí and Gertler (1999) estimate time-series regressions coefficients on future inflation and past inflation and find a non trivial role for future inflation (about 0.6–0.7). These findings and their interpretation have been debated; see example Rudd and Whelan, 2005 and a response by Gali, Gertler and David Lopez-Salido (2005).[6] More important, however, for the purposes of the present paper, is that these estimates do not use data on actual expectations, but instead use future realized inflation, as justified under the rational-expectations model they lay out. Thus, they cannot separate the role of expectations of inflation from that of other determinants.

[6] Rudd (2021) is a recent paper collecting arguments against the impact of expectations of inflation.

There is a large body of empirical and theoretical work on the formation of expectations, as well as a growing recent body of work on the implications of these expectations.[7] Among the later, the most pertinent attempt to estimate causal impacts of inflation expectation based on surveys of firms. These papers exploit randomized information provision and find relatively small passthrough (Coibion et al., 2020, 2018b) or zero (Rosolia, 2021).

There is a large literature studying departures from rational expectations of various particular kinds, such as learning or other models of expectations. The notion of temporary equilibrium is routinely within the learning literature. It was first put to use in a Calvo sticky price model by Preston (2005). Outside the learning literature such as García-Schmidt and Woodford (2019) and Farhi and Werning (2019) employ the notion of a temporary equilibrium and explore particular departures from rational expectations. These common thread of these papers and others is that they combine Calvo pricing and consumption decisions to study the response to various shocks. By comparison, the present paper is focused on the pricing process or Phillips curve, abstracting from other components of the economy and policy as well as not adopting a particular model of expectations formation. This allows me to explore in detail the passthrough of inflation expectations to inflation and its dependence on the pricing model away from Calvo. These two approaches are complementary.

The literature on time-dependent pricing has developed important results and concepts under rational expectations. Sheedy (2010) derives a Phillips curve and introduces a notion of duration selection that ensures that past inflation raises current inflation.[8] I also derive a Phillips curve but without imposing rational expectations, so the coefficients in both representations are very different. Carvalho and Schwartzman (2015) study a permanent shock to money and show how the cumulative response of output is related to a measure of duration selection.[9] Although the questions and the nature of the experiment are quite different from mine there are some interesting parallels in the results.

The literature on state-dependent "menu cost" pricing is vast. Especially relevant for this paper are Sheshinski and Weiss (1977) and Alvarez et al. (2019) who study steady state effects of inflation. Out of steady state, the literature has focuses on permanent monetary shocks or other related shocks, e.g. Golosov and Lucas (2007), Auclert et al. (2022), and the type of shock to inflation expectations that I focus on here has not been

[7]For the former, see For examples, see the surveys by D'acunto, Malmendier and Weber (2022) and Coibion, Gorodnichenko and Kamdar (2018a).

[8]Whelan (2007) derives a Phillips curve with general time dependent pricing, but includes past expectations of real marginal costs. As Sheedy (2010) shows this affects the coefficients on past inflation.

[9]Carvalho and Kryvtsov (2021) implement an empirical approach to selection, based on the coefficient in a regression of inflation on price resetting.

studied.

Akerlof (1979) advanced an idea related to my mc^2 extension for an inventory model of money demand.[10] On difference is that, in the money demand context, specific assumptions for the way idiosyncratic income and spending respond to aggregate income are required. These have no counterpart in my analysis.

2 Calvo 1 vs. Taylor ½

I begin by starting with the most familiar pricing models. This is useful to understand the general spirit of my exercise. The Calvo model features a constant hazard probability of a price changes each period; it is very tractable and for this reason often used as part of the textbook New Keynesian model. The Taylor model, introduced before Calvo, has a constant interval of time between price changes, which is quite natural for some goods and labor, but it is somewhat less tractable. For both these models we shall reach very simple and stark conclusions.

I will present the models in discrete time, but take the continuous-time limit.

2.1 Calvo Pricing

I start with the most familiar form for price stickiness, the Calvo-pricing setup. Before jumping into the calculations it is useful to define what the question is and is not.

The Passthrough Question. The goal is to compute the passthrough going from expectations of future inflation to current actual inflation. I hold fixed current and future real marginal costs and do not impose rational expectations or any other particular form of expectations (adaptive, learning, inattentive, level-k, imperfect information, etc.). I will also start by imposing that firms expect the inflation rate to be constant over time. This allows me to focus on a single passthrough coefficient. I later extend the analysis relaxing this assumption.

Taking expectations as given, at any point in time, I study the best response of firms and aggregate them to compute a "temporary equilibrium". This allows one to contemplate the impact effect at that point in time of a change in expectations, a comparative static exercise.

Studying a dynamic response may require adopting a particular model of expectations formation, as well as specifying the household and policy side of the model. But these

[10] I thank John Cochrane for bring this relevant paper to my attention.

are not required to study the impact effect which is my focus. In any case the Phillips curve I develop must hold at any point in time, even as expectations evolve. Thus, in characterizing this object, I am providing an essential input into any dynamic analysis that must be carried out with a fuller model.

NK Phillips Curve Cannot Provide Answer. To see why this distinction is important, let us review standard practice, which does not separately condition on expectations and other determinants of inflation. With Calvo pricing in a rational-expectations equilibrium inflation satisfies

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}.$$

Here π_t is aggregate inflation and x_t is the "output gap" (departure from the flexible-price value of output), κ is a parameter summarizing price stickiness, and β is the time discount factor. This equilibrium condition is often labeled the "New Keynesian Phillips Curve". From this equation, it is tempting to conclude the the discount factor $\beta \in (0,1)$ is the sought after passthrough going from inflation expectations to inflation. Furthermore, assuming β near 1 one is then led to conclude a near one-for-one passthrough.

However, this reasoning is misleading and does not answer the passthrough question stated earlier. The reason is that the Phillips curve equilibrium condition is derived under the assumption of rational expectations. Thus, the term $\mathbb{E}_t \pi_{t+1}$ is doing double duty: it is capturing expected inflation but also the effect of future output gaps and these two must be related. Indeed, one can solve forward to express inflation as being proportional to the discounted sum ofs expected output gaps

$$\pi_t = \kappa \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t [x_{t+s}].$$

which might now, equally misleadingly, suggest a zero passthrough from expected inflation to inflation.[11]

[11] Hazell et al. (2022) use the NK Phillips curve, with rational expectations, and decompose it in a way that a casual reader may interpret as a passthrough of 1, but it is important to understand why this is not the case. They assume inflation and output gaps are stationary and define long run expected inflation and output gaps as $\bar{\pi} = \mathbb{E}_t \pi_{t+s}$ and $\bar{x} = \mathbb{E}_t x_{t+s}$ (which are independent of t because of stationarity). They then write the demeaned condition $\pi_t - \bar{\pi} = \kappa(x_t - \bar{x}) + \beta \mathbb{E}_t(\pi_{t+1} - \bar{\pi})$ and solve it forward $\pi_t = \bar{\pi} + \kappa \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t[\hat{x}_{t+s}]$ where $\hat{x}_t = x_{t+s} - \bar{x}$ is the deviation of the output gap from its long run average. This suggests a zero passthrough for short- or medium-run expectations, but a one-to-one passthrough for long-run inflation. However, here once again a change in expected inflation $\bar{\pi}$ requires a change in future output gaps $\bar{x}$. In other words, this is a correct decomposition that is useful for some purposes, but it does not isolate the expectational passthrough.

In the present paper I explore the impact of changes in expectations even when the expectation of very long-run inflation is constant. Indeed, I find that expectations for the very long-run are entirely irrelevant, but that expectations for the short run do matter.

These observations underscore the spirit of our exercise: to separate expectation of future inflation and expectations of other variables, such as the output gap or real marginal costs. To do so requires abandoning rational expectations and deriving equilibrium conditions for inflation that allow for any expectation of inflation and other determinants. These points are well appreciated in the learning literature.

Optimal Pricing. In the Calvo model firms have a constant probability $1-\lambda$ of a price reset opportunity each period. Following standard practice, we approximate around a zero inflation steady state with a constant (nominal and real) interest rate of $\frac{1}{1+r}=\beta$. The log linearized reset price is

$$p_t^* = \mu + (1-\beta\lambda)\mathbb{E}_{t-1}\sum_{s=0}^{\infty}(\beta\lambda)^s(P_{t+s}+mc_{t+s})$$

That is, firms reset their price at a weighted average of nominal marginal costs plus a markup. I have jumped directly to this relatively standard condition. In Section 3.2 I derive a similar condition for a much more general model that also allows for general technology and preferences, permits general shocks (e.g. markup shocks), as well as strategic complementarities in prices.

To focus on the contribution from inflation expectations I rewrite the condition as

$$p_t^* - P_{t-1} = (1-\beta\lambda)\mathbb{E}_{t-1}\sum_{s=0}^{\infty}(\beta\lambda)^s(P_{t+s}-P_{t-1}) + a_t$$

where $a_t \equiv \mu + \mathbb{E}_{t-1}\sum_{s=0}^{\infty}(\beta\lambda)^s mc_{t+s}$ collects all the non inflation expectation items, i.e. real marginal costs and markup.

Expectations. In what follows, expectations should not be interpreted as objective, but rather as firms' own subjective expectations. As discussed above, I will purposefully stay away from placing restrictions on expectations or modeling their evolution. Taking expectations as given, at any point in time, I study the best response of firms and aggregate them to compute a "temporary equilibrium". This allows one to contemplate the impact effect at that point in time of a change in expectations, a comparative static exercise.

A brief word about the presence of $\mathbb{E}_{t-1}$ instead of $\mathbb{E}_t$ in the above expression: firms that reset their price do so simultaneously and do not have the price level P_t in their information set. Conceptually, firms choosing p_t^* form expectations about P_t just as they do for P_{t+1}, P_{t+2} etc. It helps to think of price reset p_t^* being chosen at the end of period $t-1$. None of this will play an important role: if we take a period to be a short amount of time, such as a day or week, then these distinctions becomes trivial and vanish in the continuous-time limit.

Inflation Expectations. I first focus on a simple situation with where future inflation is expected to remain constant, so that

$$\mathbb{E}_{t-1}P_{t+s} - P_{t-1} = \pi^e(1+s)$$

for some constant π^e. The assumption is about current expectations of inflation at different horizons, not about expectations remaining constant over time. I relax this simplifying assumption later and consider general expectations for inflation that depend on the horizon.[12]

Substituting this constant expectation into the price setting condition gives

$$p_t^* - P_{t-1} = (1-\beta\lambda)\sum_{s=0}^{\infty}(\beta\lambda)^s(1+s)\pi^e + a_t = \frac{1}{1-\beta\lambda}\pi^e + a_t.$$

The focus in this paper is on the term involving expected inflation π^e.

Overshooting. Note that as long as prices are sticky ($\lambda > 0$) the coefficient on the previous expression on π^e is greater than one: so that with $\pi^e > 0$ the firm *overshoots* its price relative to the static optimum. The economic rationale for overshooting is that future inflation erodes its price p_t^* relative to the rising ideal price. In the flexible price limit as $\lambda \to 0$ or as firms become myopic $\beta \to 0$ we see the coefficient becomes one: firms do not overshoot their price, and simply set it at the static optimum $p_t^* = P_t^e = P_{t-1} + \pi^e$. Conversely, as $\beta\lambda \to 1$ overshooting becomes infinitely large.

Inflation. Inflation is the weighted average of inflation across firms that cannot change their prices 0 with weight λ and those that do reset prices $p_t^* - P_{t-1}$ with weight $1-\lambda$,

$$\pi_t \equiv P_t - P_{t-1} = (1-\lambda)(p_t^* - P_{t-1})$$

Combining the two previous equations then gives

$$\pi_t = \phi\pi^e + (1-\lambda)a_t$$

[12]Note that firms expect inflation π^e between $t-1$ and t. This is in keeping with the discussion above regarding the fact that firms do not know P_t when they set p_t^*.

where

$$\phi = \frac{1-\lambda}{1-\beta\lambda}$$

As is well known, inflation is entirely forward-looking in the Calvo model, so that past inflation does not appear in the above expression. This reflects the fact that reset-pricing behavior is forward looking *and* that firms that get to reset prices are randomly drawn from the pool of all firms, so there is no selection effect—a special property of the constant hazard assumption.

Note that in the extreme case where prices are fully flexible $\lambda = 0$ then $\phi = 1$. This makes sense, when prices are not sticky then expectations of inflation are expectations about the price level; firms naturally react one-for-one to expectations about the price level. Moreover, since all firms can change prices, this gives a passthrough of one. This is similar to the logic underlying the model in Lucas (1972).

Away from this flexible-price limit other possibilities arise as the next proposition summarizes. All the results follow directly from the above expression for passthrough.

Proposition 1. *In the Calvo model passthrough ϕ satisfies*

$$\phi \in (0, 1)$$

and any value can be attained in this interval for some parameters. Indeed, in the limit of no discounting or when prices are full flexible passthrough attains its upper bound $\phi \to 1$; conversely, with positive discounting in the limit of rigid prices then passthrough attains its lower bound $\phi \to 0$.

Any value of ϕ between zero and one can be obtained by varying the degree of price stickiness for any level of discounting. This dispels the notion that passthrough equals β or 1 in the New Keynesian Phillips curve. Theoretically, much lower values are possible.

However, we next argue that in practice, for reasonable parameter values, passthrough ϕ is quite close to its upper bound of 1. This is simpler to see in the continuous time limit, to which we now turn.

Continuous Time. The expression for passthrough becomes particularly simple in the continuous-time version of the model. This can be done by setting $\beta = e^{-\rho\Delta}$ and $\lambda = e^{-\delta\Delta}$ and taking the limit as the period length shrinks $\Delta \to 0$, or by setting up the model in continuous time directly, we arrives at

$$\pi_t = \phi\pi^e + \delta a_t$$

where

$$\phi = \frac{1}{\rho/\delta + 1},$$

and $a_t \equiv \mu + \mathbb{E}_t \int e^{-(\rho+\delta)s} mc_{t+s} ds$. We see now that passthrough only depends on the ratio of parameters ρ/δ. Proposition 1 applies to this continuous time formula.

Using the continuous time condition we easily see that for reasonable values of parameters ϕ will be relatively close to 1. For example, for $\rho \leq 0.05$ and $\delta \geq 1$ (i.e. average duration of a year or less) then $\phi \geq 0.95$. Although theoretically ϕ can be very low, in practice this requires significant amounts of impatience or stickiness. We conclude that the limit with no discounting giving $\phi = 1$ is a good approximation in the Calvo model. This conclusion also applies to the discrete time version of the model.[13]

2.2 Taylor Pricing

In the Taylor setting prices are changed every N periods, with firms staggered over time so that a fraction $1/N$ changes prices each period. Once again, we write the reset price as a weighted average of the expected prices

$$p_t^* = \frac{\sum_{s=0}^{N-1} \beta^s P_{t+s}^e}{\sum_{s=0}^{N-1} \beta^s} + a_t = P_{t-1} + \frac{\sum_{s=0}^{N-1} \beta^s (s+1)}{\sum_{s=0}^{N-1} \beta^s} \pi^e + a_t$$

where $a_t \equiv \mu + \mathbb{E}_{t-1} \sum_{s=0}^{N-1} \beta^s mc_{t+s} / \sum_{s=0}^{N-1} \beta^s$. Combining this with

$$\pi_t = \frac{1}{N}(p_t^* - P_{t-1})$$

we arrive at

$$\pi_t = \phi \pi^e + \frac{1}{N} a_t$$

with

$$\phi = \frac{1}{N} \frac{\sum_{s=0}^{N-1} \beta^s (s+1)}{\sum_{s=0}^{N-1} \beta^s}$$

and ϕ is increasing in β.[14] Then in the extreme case with $N = 1$ we find $\phi = 1$. Expectations of inflation are expectations about the price level, which affect firm prices one for one. This is just as in the Calvo case with $\lambda = 0$ and, again, similar in spirit to Lucas

[13] To see this, set a period to a year and use $\beta = 0.95$ and $\lambda = 1/2$ (giving a relatively large average duration of 2 years): then $\phi = \frac{1-0.5}{1-0.95\times 0.5} = 0.5/0.525 = 0.952$. Setting a period to a quarter instead with $\beta = 0.99$ and $\lambda = 0.75$ (i.e. average duration of 4 quarters) gives $\phi = \frac{1-0.75}{1-0.99\times 0.75} = 0.25/0.2575 = 0.97$.

[14] The expression $\frac{\sum_{s=0}^{N-1} \beta^s (s+1)}{\sum_{s=0}^{N-1} \beta^s}$ is a weighted average of the sequence $0, 1, 2, \ldots, N-1$ and an increase in β puts relative more weight on higher values.

(1972).

Away from this special case this is no longer the case. In the no discounting limit $\beta \to 1$

$$\phi = \frac{1}{2}(1 + \frac{1}{N})$$

Indeed, if prices as fixed for some given calendar time but we treat periods as very short then we approach the continuous-time limit $N \to \infty$ with $\phi \to$ ½. This is the value of ϕ one obtains in the continuous-time version of the model without discounting.

To derive the continuous-time limit with discounting set $\beta = e^{-\rho\Delta}$ and $N = \frac{1}{\delta\Delta}$ and take the limit as $\Delta \to 0$ (see Appendix A) gives

$$\pi_t = \phi\pi^e + \delta a_t$$

where

$$\phi = \frac{1}{\rho/\delta} - \frac{1}{e^{\rho/\delta} - 1}$$

is decreasing in ρ/δ. Again only the ratio ρ/δ matters.

Proposition 2. *For the continuos-time Taylor-pricing model*

$$\phi \in (0, \tfrac{1}{2})$$

and

$$\phi \to \frac{1}{2}$$

in the limit of no discounting or flexible prices $\rho/\delta \to 0$; whereas $\phi \to 0$ in the limit of rigid prices $\rho/\delta \to \infty$.

Although the result shows that ϕ near zero is a theoretical possibility and that ½ is an upper bound, for reasonable values of parameters, the passthrough ϕ once again lies close to this upper bound. For example, if $\rho < 0.05$ and $\delta \leq 1$ then one calculates that $\phi \geq 0.495$. Once again, as with Calvo, we conclude that the no-discounting case is a good approximation for reasonable parameters.

The coefficient of ½ on future inflation stands in contrast with some well-known derivations for the Taylor model in the New Keynesian literature. In particular, for the special case of $N = 2$ Roberts (1995) works out a condition for inflation under rational expectations that features the expectation of inflation in the next period with a unit coefficient. However, this derivation is carried out under rational expectations, so it cannot be directly compared to my analysis nor interpreted as the effect of expectations.[15]

[15]In addition, the coefficient in Roberts (1995) cannot be easily interpreted to trade out the reaction

2.3 Intuition: Calvo vs Taylor

Why is expectational passthrough lower in the Taylor price setting relative to Calvo?

As discussed earlier, when firms expect positive inflation they reset their price above their ideal static price. This "overshooting" mechanism is at the heart of the transmission mechanism from expectations of future inflation to current inflation.

Intuitively, price overshooting is more aggressive

with Calvo pricing because, for a given average duration, duration is uncertain. The risk of prices remaining unchanged for very long periods of time induces firms to overshoot their price. In contrast, in the Taylor setup where firms change their prices every fixed number of periods, there is no such right-tailed risk. Firms overshoots their ideal price, but do so less aggressively. Indeed, the price will be too high exactly half the time, too low the rest. This leads to a lower passthrough, that exactly equals 1/2.

It is interesting to note that in both cases the passthrough coefficient does not depend on the frequency of price changes. Lower frequency makes firms overshoot inflation proportionally more, but this is exactly offset by the fact that there are fewer firms changing prices.

3 Sufficient Statistics for General Time Dependent Pricing

I now consider a general time dependent model. As we shall see, the more general results I obtain can still be stated rather simply in terms of sufficient statistics. The more general formulation also helps sheds further light on the two previous special cases.

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of inflation to shocks because it contains an expectation error term that will systematically co-move with inflation. Roberts (1995) equation (8) can be written as

$$\pi_t = \mathbb{E}_t \pi_{t+1} + a_t + \eta_t$$

where a_t collects terms related to the real economy and η_t is an expectations error given by $\eta_t = \mathbb{E}_{t-1} P_t - P_t = \mathbb{E}_{t-1}\pi_t - \pi_t$. The problem with this expression is that information shocks at t that move $\mathbb{E}_t \pi_{t+1}$ and hence π_t also affect η_t (by definition). Substituting, we see that

$$\pi_t = \frac{1}{2}\mathbb{E}_t \pi_{t+1} + \frac{1}{2} a_t + \frac{1}{2}\mathbb{E}_{t-1}\pi_t.$$

In this expression the coefficient on $\mathbb{E}_t \pi_{t+1}$ is ½ rather than unity.

3.1 Preliminaries

We take as given a hazard function h_s giving the probability of getting a price reset $s+1$ periods since the previous reset (e.g. h_0 denotes the probability of resetting a price if the price was also reset in the previous period). It is useful to imagine price reseting happening at the end of a period and selecting the price that will be in place the at the beginning of the next period.

The hazard rate determines the survival probability S_s for each age $s = 0, 1, \ldots$

$$S_{s+1} = S_s(1 - h_s)$$

with $S_0 = 1$. Note that $F_s = 1 - S_{s+1}$ ($F_{-1} = 0$) represents the cumulative distribution function for the duration of *completed* spells i.e. the probability a spell will be s or less is given by F_s. The associated density is $f_s = F_s - F_{s-1} = S_s - S_{s+1} = S_s h_s$.

Rescale the survival probability so that it adds up to one defines the distribution of *ongoing* spells

$$\omega_s = \frac{S_s}{\sum_{s=0}^{\infty} S_s},$$

where I assume $\sum_{s=0}^{\infty} S_s < \infty$. This distribution has two economic interpretations. First, it represents the unique invariant distribution under the Markov process for age s, defined by $s' = s + 1$ with probability $1 - h_s$ and $s' = 0$ with probability h_s.[16] Under this interpretation ω_s represents the fraction of firms with age s in a cross-section of firms as well as the "long run" average time spent at age s for a given firm. A second interpretation is also possible, one that holds for a single firm and a single price spell: ω_s represents the expected amount of time spent at age s divided by the expected time spent at all other ages. In this way, it captures the relative importance of age s relative to all other ages for a given spell.

Using the distributions of completed and ongoing spells f_s and ω_s I define the average hazard $\bar{h} \equiv \sum_s h_s \omega_s$, the average duration of completed spells $\bar{d} \equiv \sum_{s=0}^{\infty} f_s(s+1)$ and the average duration of ongoing spells $\hat{d} \equiv \sum_{s=0}^{\infty} \omega_s(s+1)$.[17] I assume all these averages are

[16] This follows because $\omega_{s+1} = \omega_s(1 - h_s)$ and $\omega_0 = \sum_{s=0}^{\infty} h_s \omega_s$. The invariant distribution is unique because $s' = 0$ is a recurrent point. Under relatively weak conditions, so that $S_s \in (0, 1)$ for some s, the unique invariant distribution is also stable, i.e. we converge to it starting from any other distribution. The Taylor case is a knife-edged case lacking stability, starting from any distribution we cycle endlessly every N periods.

[17] We take the expectation of $s + 1$ not s because a spell that is reset at $s = 0$ is a spell of duration 1.

finite valued. One can show that[18]

$$\bar{h} = \omega_0 = \frac{1}{\bar{d}}.$$

These relations are intuitive. The average frequency of price changes $\bar{h}$ must equal the density of firms that are resetting their price ω_0. Likewise, the average duration of completed price spells $\bar{d}$ equals the reciprocal of the frequency of price changes $1/\bar{h}$, a relation familiar in the special Calvo case with $h_s = \bar{h}$.

Calvo and Taylor Again. For reference, in the Calvo model the probability of a price change is constant so that $h_s = \bar{h}$ and $S_s = (1-\bar{h})^s$, yielding $\omega_s = \bar{h}(1-\bar{h})^s$. In the Taylor model, instead, prices are stuck for N periods (over $t = 0, 1, \ldots N-1$) so that $h_s = 0$ for $s < N-1$ and $h_s = 1$ for $s \geq N-1$ so that $S_s = 1$ for $s \leq N-1$ and $S_s = 0$ for $s \geq N$, yielding $\omega_s = \frac{1}{N}$ for $s \leq N-1$ and $\bar{h} = \frac{1}{N}$.

Note that in the Calvo case the distribution of completed spells is the same as that of ongoing spells, since both are exponential. In contrast, in the Taylor case the distribution of ongoing spells is uniform, whereas that of completed spells has full mass at N. In particular, the average duration of completed spells is greater than that of ongoing price spells in Taylor but identical in Calvo. This will play a role in interpreting our previous results on expectational passthrough.

Prices and Inflation. The price level (in logs) is defined as the average across firms, which equals the weighted average of past reset prices

$$P_t = \sum_{s=0}^{\infty} \omega_s p^*_{t-s}.$$

Inflation is then given by

$$\pi_t = P_t - P_{t-1}.$$

[18]Using the definitions above we have that $\omega_s = h_s\omega_s + \omega_{s+1}$ implying $\omega_0 = \sum_{s=0}^{T} h_s\omega_s + \omega_{T+1} = \sum_{s=0}^{\infty} h_s\omega_s = \bar{h}$ where I have used that $\omega_{T+1} \to 0$ because $S_{T+1} \to 0$ is implied by the assumption that $\sum_{s=0}^{\infty} S_s < \infty$. Next

$$\bar{d} \equiv \sum_{s=0}^{\infty} f_s(s+1) = \sum_{s=0}^{\infty} (S_s - S_{s+1})(s+1) = \sum_{s=0}^{\infty} S_s(s+1) - \sum_{s=1}^{\infty} S_s s = \sum_{s=0}^{\infty} S_s = \frac{1}{\omega_0}.$$

Note that these same calculations also justify the second interpretation for ω_s mentioned above, since $\omega_s = S_s/\bar{d}$.

A bit of algebra shows that

$$\pi_t = \sum_{s=0}^{\infty} \omega_s h_s (p_t^* - p_{t-1-s}^*)$$

inflation is a weighted average of the inflation rate associated with each firm (note that, mechanically, a fraction $1 - \bar{h}$ firms produce zero inflation).

3.2 Price Setting Approximation

This section provides a simple result justifying and generalizing the type of log-linearized calculations used in the Calvo and Taylor settings. Apart from considering a general time-dependent model. Perhaps less well appreciated is the fact that the linearized condition obtains even allowing for a general profit function, obtained from general production functions and demand functions, as well as allowing for shocks to both. In particular, the analysis allows for strategic price complementarities, whereby the optimal price is not simply a constant markup over nominal marginal cost, but instead depends on the prices set by other firms.

The firm faces a path of interest rates q_{t+s} and a path of θ_t shocks to its profit function. These shocks can capture changes in their production functions or the demand firms face—leading to variations in desired prices and markups. A firm resetting its price in period t then solves

$$\max_{p_t^*} \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s} \omega_s \Pi(p_t^* - P_{t+s}, \theta_{t+s})$$

with first-order condition

$$\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s} \omega_s \Pi_p(p_t^* - P_{t+s}, \theta_{t+s}) = 0$$

Consider a small variation in the firms' problem $\{P_{t+s}, \theta_{t+s}, q_{t+s}, \omega_s\}$ around a perfect-foresight equilibrium with zero inflation $P_{t+s} = P_t$ and constant primitives $\theta_t = \bar{\theta}$.

Proposition 3. *To a first-order approximation around a zero inflation steady state the reset price satisfies*

$$p_t^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} \beta^s \omega_s \, P_t^e}{\sum_{s=0}^{\infty} \beta^s \omega_s} + a_t$$

where

$$a_t = \frac{\Pi_{p\theta}}{\Pi_{pp}} \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} \beta^s \omega_s \, \theta_{t+s}}{\sum_{s=0}^{\infty} \beta^s \omega_s}.$$

My analysis is focused on the effects of inflation expectations affecting $\{P_t^e\}$, so it takes the se-

quence for $\{\theta_t\}$ and hence $\{a_t\}$ as given. However, it is apparent that one could analyze the passthrough of shocks $\{\theta_t\}$ analogously.

3.3 Sufficient Statistics for Passthrough

I now calculate the impact of a sudden change in inflation expectations. I do so approximating around a zero inflation rate steady state and also focus on the no discounting limit $\beta \to 1$.

Without discounting the optimal reset price is a weighted average of the marginal cost. Thus, abstracting from shifters to real marginal costs to focus on expected inflation, we have:

$$p_t^* = \sum_{s=0}^{\infty} \omega_s P_{t+s}^e = \sum_{s=0}^{\infty} \omega_s \pi^e (1+s) + P_{t-1}$$

This gap between the reset price and the price level is one element that is needed to calculate inflation.

We assume inflation has been zero in the past (we later study the effects of past inflation) and that all firms have the initial price $p_{t-s}^* = P_{t-1}$ for all $s = 1, 2, \dots$ and change their price to p_t^* at $t = 0$ when the shock to expectations occurs. Inflation, then is simply $\pi_t = \bar{h}(p_t^* - P_{t-1})$. Combining the above expression then gives the following result.

Proposition 4. *Up to a first-order approximation around a zero inflation steady state with zero past inflation*

$$\pi_t = \phi \pi^e + a_t$$

where

$$\phi = \bar{h} \sum_{s=0}^{\infty} \omega_s (1+s) = \frac{\sum_{s=0}^{\infty} \omega_s (1+s)}{\sum_{s=0}^{\infty} f_s (1+s)} = \frac{\hat{d}}{\bar{d}}$$

The passthrough ϕ equals the ratio of the duration of ongoing spells $\hat{d}$ to that of completed spells $\bar{d}$. This is a very simple formula in terms of two sufficient statistics that are in principle directly observable in the data.

The economic intuition is that the duration of ongoing spells controls the incentive firms have to overshoot their price, relative to their current ideal price, in the face of expected inflation. Indeed, the weights ω_s for ongoing spells captures the average time firms their price will be at different durations. This average duration determines how much expected inflation will impact its pricing decision today. In contrast, the duration of completed spells is not relevant for this decision, but appears in the numerator because its captures the frequency of price changes.

Proposition 4 highlights the role played by the pattern of time dependence. In contrast, the degree of price stickiness itself is irrelevant: if we scale up both ongoing and completed durations proportionally the ratio is unaffected.

The literature on time-dependent pricing has noted how the shape of hazard rates, not just price frequency, matters for the dynamics of price and output adjustment following a monetary shock. Carvalho and Schwartzman (2015) discuss the "selection effect" present in a continuous-time time-dependent model (see also Alvarez et al. 2011). In particular, they study the cumulative effect on output of a permanent shock to nominal spending and show that it is linked to a measure of selection that can be related to the frequency of price changes and the variance of price durations.

Two important cases: Calvo and Taylor. We now use this general result to revisit the two special cases considered earlier. In Calvo $\bar{d} = \hat{d} = \frac{1}{\bar{h}}$ while in Taylor $\bar{d} = N \neq \hat{d} = \frac{N+1}{2}$ implying

$$\phi_{Calvo} = 1$$

$$\phi_{Taylor} \to \frac{1}{2} \qquad N \to \infty$$

Extension with Heterogeneity. The result extends easily with heterogeneity in the hazard rates. Let firms of type i have hazard $h(s;i)$ with associated $\bar{h}(i)$,$\hat{d}(i)$ and $\bar{d}(i) = 1/\bar{h}(i)$, then one can show that

$$\pi = \phi \pi^e + \bar{a}_t$$

$$\phi = \int \frac{\hat{d}(i)}{\bar{d}(i)}\, di = \int \bar{h}(i)\hat{d}(i)\, di$$

Note that ϕ is generally different from $\frac{\int \hat{d}(i)di}{\int \bar{d}(i)di}$ as well as different from $\int \frac{1}{\bar{d}(i)}di \cdot \int \hat{d}(i)\, di = \int \bar{h}(i)di \cdot \int \hat{d}(i)\, di$. Indeed $\bar{h}$ and $\hat{d}$ may be correlated in the population of firms. Indeed, this correlation may be negative so that heterogeneity of $\bar{h}$ and $\hat{d}$ cancels out. For example, heterogeneity in the hazard $\bar{h}$ in Calvo is irrelevant, we always have $\phi = 1$. Likewise, heterogeneity in the length of rigidity N within Taylor is irrelevant and always gives $\phi = 1/2$. So in these cases heterogeneity of the frequency of price changes does not affect ϕ. However, other forms of heterogeneity in the hazard function may matter: for example, if a fraction of firms have a Calvo hazard and another have a Taylor hazard.

How Low Can We Go? How High? What is the range of possible passthrough ϕ? In particular, how low can we make ϕ by choice of the hazard function? I now show that the lowest possible passthrough is $1/2$ achieved by the Taylor pricing case.

Proposition 5. *Let ϕ be given by Proposition 4 then for any $\{h_s\}$ we have*

$$\phi \geq \frac{1}{2}.$$

Moreover, any value of ϕ can be attained by some choice of the hazard function. In particular, $\phi > 1$ and arbitrarily large is possible.

Proof. One can show that

$$\hat{d} = \sum_{s=0}^{\infty} \left(1 - \frac{\sum_{n=0}^{s}(1 - F_n)}{\bar{d}} \right)$$

Next we show that for any duration $\bar{d} \in \{0, 1, 2, \dots\}$ the distribution $\{F_n\}$ that minimizes $\hat{d}$ subject to $\bar{d} = \sum_{n=0}^{\infty}(1 - F_s)$ is the Dirac distribution $\{F_n^*\}$ with full mass at $\bar{d}$. Any alternative distribution $\{\tilde{F}_n\}$ with $\sum(1 - \tilde{F}_n)ds = \bar{d}$ second order dominates F_n^* implying that $\sum_{n=0}^{s}(\tilde{F}_n - F_n^*) \geq 0$ for all s. This then implies

$$\phi_{\tilde{F}} - \phi_{F^*} = \frac{1}{\bar{d}} \sum_{s=0}^{\infty} \sum_{n=0}^{s} (\tilde{F}_n - F_n^*) \geq 0$$

Moreover $\phi_{F^*} = 1/2$. The Dirac F^* corresponds to the Taylor case. For $\bar{d} \notin \{0, 1, 2, \dots\}$ a similar result holds but with a distribution with mass only at the two closest values $s \in \{0, 1, 2, \dots\}$.

Next I show that arbitrarily large value $\phi > 1$ are possible. Fix the average completed duration $\bar{d}$. Now pick any whole number $\tilde{s} \geq \bar{d}$ and set $f_{\tilde{s}} = \bar{d}/\tilde{s}$ and $f_0 = 1 - \bar{d}/\tilde{s}$ and $f_s = 0$ otherwise; this gives a bimodal distribution with average duration of completed spells $\bar{d}$. Next, we compute the average duration of ongoing spells. Then $S_0 = 1$, $S_s = \bar{d}/\tilde{s}$ for $s = 1, 2, \dots, \tilde{s}$ and

$$\omega_s = \frac{S_s}{\sum S_s} = \frac{\bar{d}/\tilde{s}}{1 + \bar{d}} > 0$$

so that

$$\hat{d} = 1 + \frac{\bar{d}}{1 + \bar{d}} \frac{\tilde{s} + 1}{2}.$$

The result then follows by choosing $\tilde{s}$ large enough. Note that using the same construction we have

$$\phi = \frac{\hat{d}}{\bar{d}} = \frac{1}{\bar{d}} + \frac{1}{2} \frac{1 + \tilde{s}}{1 + \bar{d}}.$$

This implies that we can attain any value for ϕ in a limiting sense. Choosing any desired value for ϕ we can send $\bar{d} \to \infty$ and $\bar{s} \to \infty$ so that $\frac{1}{2}\frac{1+\tilde{s}}{1+\bar{d}} \to \phi$. Small perturbations of this construction can attain any value of ϕ without taking limits. □

The intuition for this result is as follows. The duration of ongoing spells suffers from two "biases" that make it generally different from that of completed spells. Firstly, for any given spell the age at which we sample an ongoing spell is by definition below that of the completed spell; a downward bias. Secondly, unless spells are all of the same duration there is also an upward bias because we oversample relatively longer spells. Intuitively, Taylor minimizes the ratio at $1/2$ because it has the downward bias, but not the upward bias.

The proposition not only rules out $\phi < 1/2$, but shows that any value $\phi \geq 1/2$ is possible. Since Taylor attains ½ and Calvo 1, it should be intuitive that we can get anything in between by mixing these two models. But $\phi > 1$ arbitrarily large is perhaps less obvious. The intuitive idea is that we can make average duration of ongoing spells very large relative to the completed ones by having most spells end as soon as they start, but have a small fraction end after a very long time. Ongoing spells are then the selected sample of "survivors" with very long durations. Effectively, we can make the second positive "bias" described in the previous paragraph arbitrarily large.

Discounting. What happens away from the no discounting case, when $\beta < 1$? With discounting, firms set prices that are a weighted average including the discounting. As a result, discounting lowers passthrough.

Proposition 6. *The passthrough $\phi_\beta$as a function of β is increasing in β and*

$$\phi_\beta = \frac{\hat{d}_\beta}{\hat{d}} \phi_{\beta=1}$$

where $\hat{d}_\beta = \sum_{s=0}^{\infty} \hat{\omega}_{s,\beta} s$ and $\hat{\omega}_{s,\beta} = \beta^s \omega_s / \sum_{s=0}^{\infty} \beta^s \omega_s$ is a weight with the property that $\hat{\omega}_{s,\beta}$ is increasing in a first order stochastic dominance (FOSD) sense. Thus, $\hat{d}_\beta$ is increasing and $\hat{d}_1 = \hat{d}$.

For Calvo and Taylor we saw that the no-discounting case provided a very good approximation. Based on this proposition, we now see that this conclusion is more general as long as $\hat{d}_\beta / \hat{d}$ lies close to 1. For reasonable discount rates and hazard rates this must be the case: for price stickiness lasting a year or so, discounting within that year does not change the relative weights $\hat{\omega}_{s,\beta}$ significantly, thus it will not change $\hat{d}_\beta$ and ϕ_β significantly.

4 Short vs Long Run Expectations and Past Inflation

I now develop the equilibrium condition for current inflation while relaxing the simplifying assumptions imposed previously. This equilibrium condition is informally termed a "Phillips curve". Thus, this section develops a Phillips curve for a general-time dependent model.

I relax two assumptions from the previous section. First, expectations for inflation at different horizons are no longer assumed to be flat: expectations are given by an arbitrary sequence $\{\pi^e_{t+s}\}_{s=0}^{\infty}$ where π^e_{t+s} represents the expectations of inflation for period $t+s$ held at time t; previously $\pi^e_t = \pi^e$ constant. Second, I study the impact of past realized inflation taking any given any sequence $\{\pi_{t-s}\}_{s=0}^{\infty}$; previously $\pi_{t-s} = 0$ for $s > 0$.

The next proposition writes current inflation as a linear function of expectations of future inflation rates and past realized inflation rates.

Proposition 7. *Suppose the hazard function is strictly positive $h_s > 0$ for all s then to a first-order approximation in the limit of no discounting we have*

$$\pi_t = \sum_{s=0}^{\infty} \phi_s \pi^e_{t+s} + \sum_{s=-1}^{-\infty} \phi_s \pi_{t+s} + a_t$$

where $a_t = \alpha \sum_{s=0}^{\infty} \omega_s \theta^e_{t+s}$ is the expectation of real variables. For $s > 0$ we have $\phi_s \geq 0$ and decreasing. Finally,

$$\sum_{s=-\infty}^{\infty} \phi_s = 1.$$

The previous section characterized the sum of the coefficients on expectations of future inflation $\phi = \sum_{s=0}^{\infty} \phi_s$. This proposition generalizes this result, breaking down ϕ over different horizons. In particular, the result shows that expectations of inflation have a positive effect on inflation, that is declining with the horizon.

The result also includes the effect of past inflation. These backward coefficients are zero in the Calvo case, but are generally not zero away from Calvo pricing. Sheedy (2010) solves for a Phillips curve under rational expectations. Both the coefficients on past inflation and the coefficients on expectation of future inflation are different.[19] For example, under rational expectations (in the limit of $\beta \to 1$) the coefficient on π^e_{t+1} is equal to 1, just as in the New Keynesian model and the other coefficient on expectations take the opposite sign as the coefficients on past inflation. Thus, if the coefficients on past inflation are positive, then the coefficients on expected future inflation (other than the one-period-ahead inflation) are negative. In contrast, in my formulation the coefficients on future inflation are always positive, and decline towards zero. Indeed, they are zero outside the range of price stickiness: if $\omega_N = 1$ then $\phi_n = 0$ for $n \geq N$.

[19]Relatedly, my formulation includes expectations of future real variables (marginal costs, markups etc.), whereas the formulation in Sheedy (2010) only includes the current real marginal cost. This is precisely why the expectations of future inflation plays a dual role in his formulation, capturing both inflation and future real marginal costs, just as I discussed is the case in the standard New Keynesian model with Calvo pricing.

The sum of both sets of coefficients, backwards and forwards, is one. This implies that with $a_t = 0$ for any $\bar{\pi}$ with $\pi^e_{t+s} = \pi_{t-s} = \bar{\pi}$ for $s \neq 1$ gives $\pi_t = \bar{\pi}$. In this way, any constant level of inflation is a steady state under rational expectations. Informally, one might say that the long-run Phillips curve is vertical. Note that by the same reasoning if $a_t > 0$ then there is no steady state inflation, although informally one might say that the unique steady state has $\bar{\pi} = \infty$ (likewise if $a_t < 0$ then informally $\bar{\pi} = -\infty$).

Proof and Intuition. To see how this result is derived we first compute the reset price. This satisfies the usual condition but this time we do not impose $P^e_{t+s} = \pi^e(1+s)$ (setting $a_t = 0$ to simplify expressions)

$$p^*_t = \sum_{s=0}^{\infty} \omega_s P^e_{t+s} = \sum_{s=0}^{\infty} \omega_s \sum_{j=0}^{s} \pi^e_{t+j} + P_{t-1} = \sum_{s=0}^{\infty} (1 - \Omega_{s-1}) \pi^e_{t+s} + P_{t-1}$$

where the cumulative distribution is $\Omega_s = \sum_{n=0}^{s} \omega_n$ with the convention that $\Omega_{-1} = 0$. Using $\pi_t = \sum_{s=0}^{\infty} \omega_s h_s (p^*_t - p^*_{t-1-s})$ we arrive a

$$\begin{aligned} \pi_t &= \sum_{s=0}^{\infty} \omega_s h_s (p^*_t - P_{t-1}) + \sum_{s=0}^{\infty} \omega_s h_s (P_{t-1} - p^*_{t-1-s}) \\ &= \bar{h} \sum_{s=0}^{\infty} (1 - \Omega_{s-1}) \pi^e_{t+s} + \sum_{s=0}^{\infty} \omega_s (\bar{h} - h_s) p^*_{t-1-s} \end{aligned}$$

The first set of terms provides the required coefficients $\phi_s = \bar{h}(1 - \Omega_{s-1})$ for expectations of future inflation $s \geq 0$. We see that that ϕ_s are decreasing since the cumulative distribution Ω_{s-1} is increasing. Intuitively, when resetting their prices, firms care relatively more about earlier inflation because it affects the price level over more periods. In fact, if $S_s = 0$ for $s \geq N$ for some N then inflation expectations beyond N are completely irrelevant.

The second backward-looking set of terms depend on past reset prices $\{p^*_{t-s-1}\}$. Using ω_s as probabilities we have $\sum_{s=0}^{\infty} \omega_s h_s = \mathbb{E} h_s = \bar{h}$, so that

$$\sum_{s=0}^{\infty} \omega_s (\bar{h} - h_s) p^*_{t-1-s} = -\mathrm{Cov}\left(h_s, p^*_{t-1-s}\right)$$

We see immediately that if $h_s = \bar{h}$ is constant as in Calvo then this backward-looking term is zero. It is also zero if one starts at a zero inflation steady state with constant past reset prices $p^*_{t-1-s} = p^*$; our working assumption in the previous sections. In contrast, if the reset price p^*_t has been rising in the past and the hazard rate h_s is increasing then the covariance is negative, and the new term contributes towards positive current inflation.

To next step in the proof is to convert the expression in terms of past reset prices into one involving past inflation rates. The price level is a weighted average of past reset prices

$$P_t = \sum_{s=0}^{\infty} \omega_s p^*_{t-s}.$$

The key idea is to prove that this can be inverted to solve for

$$p^*_t = \sum_{s=0}^{\infty} \alpha_s P_{t-s}$$

with $\sum_{s=0}^{\infty} \alpha_s = 1$. This is possible by applying the Eneström–Kakeya Theorem (Gardner and Govil, 2014), noting that the hypothesis are satisfied because $\omega_s \geq 0$ and $\omega_{s+1} \leq \omega_s$. Substituting this expression for p^* and rearranging then provides a linear expression in terms of past inflation. I omit the details here, but collect the relevant expressions in an Appendix.

Finally, to see that $\sum_{s=-\infty}^{\infty} \phi_s = 1$ we can work through the effects of constant past inflation $\pi \neq 0$, noting that this can only emerge from constant inflation in the reset price $p^*_{t-s} = p^*_t - s\pi$.[20] Evaluating the second term then gives

$$\pi \sum_{s=-1}^{\infty} \phi_s = \sum_{s=0}^{\infty} \omega_s(\bar{h} - h_s) p^*_{t-1-s} = \pi \sum_{s=0}^{\infty} \omega_s(\bar{h} - h_s)s = \pi\bar{h} \sum_{s=0}^{\infty} (\omega_s s - f_s s) = \pi(1 - \phi).$$

Cancelling π we obtain $\sum_{s=-1}^{\infty} \phi_s = 1 - \phi$ and the result follows.

Economically this result can be interpreted as saying that the "long-run Phillips curve is vertical". That is, if $a_t = 0$ for all t then any constant solution for $\pi_t = \pi$ is possible; if instead $a_t = \bar{a} > 0$ then $\pi = \infty$ and if $\bar{a}_t = \bar{a} < 0$ then $\pi = -\infty$. .

5 Menu Costs: State Dependent Models

I now explore the implications of "menu cost" state-dependent models. The key difference between these models and the ones studied earlier is that the frequency of price adjustments is endogenous. In particular, firms can be seen as using so-called "sS bands" characterizing regions of inaction where the price is left unchanged.

[20] To see this note that if $p^*_{t-s} - p^*_{t-s-1} = \pi$ for all $s = 0, 1, \ldots$ then

$$P_{t+1} - P_t = \sum_{s=0}^{\infty} \omega_s (p^*_{t+1-s} - p^*_{t-s}) = \sum_{s=0}^{\infty} \omega_s \pi = \pi$$

The same is true for all earlier dates. Thus, constant reset price inflation produces constant past inflation. But since we can invert p^* from P this must be the only possible sequence of p^* consistent with constant inflation in P.

I first explore the simplest setting introduced by Sheshinski and Weiss (1977), a deterministic economy with positive inflation, where there are only price increases. As I will show the predictions of this model are quite extreme. In particular, an increase in expected inflation can actually *reduce* inflation all the way to zero. This is due to extreme movements in the frequency of price adjustments brought about by discrete changes in the sS bands. This may not be realistic, so I study a version of the model where the frequency of price changes cannot be changed in the very short run. This model produces a passthrough that is positive but lower than the Taylor one of ½.

5.1 Setup

It is useful to move to a continuous time setup. At any point in time t a firm has price p_t^i and aggregate price $P_t = \int p_t^i dt$. It useful to define the price gap $x_t^i = p_t^i - P_t$.

Firms profits are a function of their price gap $f(x)$. They discounted profits at a constant rate ρ, although we often take the limit as $\rho \to 0$. Firms must pay a fixed cost c each time they change their price, as a result they change their price only at discrete intervals of time and use an (s, S) rule keeping x in the interval $[s, S]$ with $s < 0 < S$ adjusting the price up to $x = S$ when $x = s$. We are interested in characterizing the (s, S) rule and deriving its implications for the passthrough from expected inflation to realized inflation.

Consider a steady state where expected inflation is given by $\pi^e > 0$. Let V^* denote the stationary value of a firm that has just optimally changed its price. Then the firm problem at time t is

$$V(x_t; \pi^e) \equiv \max_T \int_0^T e^{-\rho s} f(x_{t+s}) dt + e^{-\rho T}(V^*(\pi^e) - c)$$
$$x_{t+s} = x_t - \pi^e t$$

with x_t given. The first order condition for T gives

$$f(\underline{x}) = \rho(V^*(\pi) - c)$$

a condition for the lower bound on the sS policy $\underline{x} = x_t e^{-\pi T}$. The upper bound of the sS policy is the optimal reset price $x^* \in \arg\max_x V(x; \pi)$. This reset price x^* actually satisfies a condition akin to the Taylor model: the price is set so that the average marginal profit is zero

$$\frac{1}{\rho} \int_0^{T^*} e^{-\rho s} f'(x^* - \pi s) dt = 0.$$

Taking a quadratic approximation of $f(x)$ around the maximal value $\hat{x} = \arg\max_x f(x)$ gives f' linear, and in the limit $\rho \to 0$ gives $x^* = \frac{1}{T} \int_0^{T^*} \pi t + \hat{x} = \frac{1}{2} T^* + \hat{x}$. The $\hat{x}$ shifter plays the same role as a_t did in the time-dependent model; we set $\hat{x} = 0$. The bands are then symmetric around zero: $\underline{x} = -x^*$.

Sheshinski and Weiss (1977) proved that firms will expand their bands, so that $x^*(\pi^e) = -\underline{x}(\pi^e)$ is increasing in π^e. Moreover, firms anticipate that prices will be set for a shorter amount of time $T = \frac{\pi^e}{2x^*(\pi^e)}$ is decreasing in π^e.[21]

5.2 A Shock To Expectations with Full Adjustment

Now suppose we are at a steady state with constant expected and actual inflation that coincide $\pi_0^e = \pi_0$. At a steady state we have an invariant cross-sectional distribution of firms distributed with density $\omega(x)$.[22] In equilibrium we must have $\int x\omega(x) = 0$. The invariant distribution is uniform between $[\underline{x}, x^*]$ with constant density $\omega = \frac{1}{x^*-\underline{x}}$.[23]

The rate of inflation pushes firms down to the boundary $\underline{x}$ making them change prices; the higher is inflation the greater the flow of firms hitting the boundary. The density of firms, or frequency of price changes, equals $\bar{h} = \omega\pi$. These firms change their price by a discrete positive amount $\Delta_+ = x^* - \underline{x}$. Thus, inflation is given by the product

$$\pi = \bar{h}\Delta.$$

One then observes that $\pi = \bar{h}\Delta = \omega\pi\frac{1}{\omega} = \pi$ a consistency condition.[24]

Now from this steady state position, imagine firms anticipate higher inflation $\pi^e > \pi_0$. Thus, when expectations change firms will expand their bands immediately. As a result the distribution of firms is strictly away from the bands and no firms adjust prices, resulting in zero inflation. A decrease in the inflation expectations shrinks the optimal bands, calling on a mass of firms to adjust prices.

Proposition 8. *In the Sheshinski-Weiss menu cost model, starting from a steady state with π_0, an increase in inflation expectations at $t = 0$ $\pi^e > \pi_0$ lowers realized inflation on impact to zero $\pi = 0$.*

Conversely, a decrease in inflation expectations $\pi^e < \pi_0$ induces at a mass of firms to change their price immediately. This results in an upward jump in the price level that is greater the larger the change in expectations $\pi_0 - \pi^e$.

[21] This latter result requires imposing a condition that is satisfied with our quadratic approximation for f.

[22] In equilibrium we require the consistency condition that $\int \omega(s)xds = 0$

[23] This is the unique invariant distribution and one can ensure stability by perturbing this model slightly, e.g. allowing some small Poisson arrival of free price changes.

[24] For given bounds inflation is indeterminate, but this conclusion is knife-edged and dependent on the simplifying assumptions we have adopted; thus, we will not be concerned with it.

For an increase in π^e the passthrough is initially negative infinity. Once inflation is at zero further increases in π^e have no effect, so the marginal passthrough becomes zero. On the other hand, for a decrease in inflation expectations the immediate passthrough is also negative infinity, but even more extreme since it is the price level, not inflation, that jumps up. These results are obviously extreme, but they illustrate the possibilities when the fraction of firms changing their price becomes endogenous.

5.3 Adjustment Frictions for Price Frequency

The results above are extreme and probably not realistic. Across steady states one can imagine firms adjusting the frequency of price changes, taking as given a "menu cost" of each price change, i.e. the cost is linear in frequency. But in the short run the resources devoted to changing prices may not be perfectly adjustable. For example, if we think of a retail store, managing many products and prices, then changing prices requires employee time devoted to this task. Staffing and training of these employees cannot be immediately adjusted so the retail store cannot suddenly increase the rate at which they change prices arbitrarily; nor will they want to stop adjusting prices altogether since that would leave idle time and resources devoted to that task.

On the other hand, given enough time the frequency of price changes can be changed, staffing rearrangements or hiring can be done.

Now let us revisit the result from the previous section, focusing on a rise in inflation expectations. If inflation expectations rise, the bands are widened, and the frequency of price adjustments was predicted to fall to zero. However, let us now instead entertain that in the very short run this frequency is held constant at its previous value, due to the notion that resources devoted to price changes are fixed in the very short run. On the other hand, the firm anticipates that it will be able to adjust the frequency of price changes over the medium term. Indeed, suppose the firm anticipates that if it is resetting its price today, then by the time it has to reset it again, it will have been able to freely adjust its price adjustment frequency. The next result shows that under these assumptions the passthrough is positive, but below 1/2 the value with Taylor pricing.

Proposition 9. *In a Sheshinski-Weiss menu-cost model where frequency of price adjustment is fixed in the very short run, the marginal passthrough from inflation expectations to inflation satisfies*

$$0 < \phi < 1/2.$$

The calculations behind this result are as follows. If the frequency of price adjustment is fixed then inflation on impact satisfies

$$\pi = \bar{h}(\pi_0)(x^*(\pi^e) - \underline{x}(\pi_0))$$

with $\bar{h}$ and $\underline{x}$ held fixed at its previous value. Now this is the same calculation for the Taylor model except for the value that $x^*(\pi^e) = p^*(\pi^e) - P$. In the Taylor case we have

$$p^* - P = \frac{1}{2}T\pi^e$$

with $T = T^*(\pi_0)$ fixed at its original value. Instead, we now have the same formula but with $\pi^e > \pi_0$ the anticipated value of $T^*(\pi^e)$ is lower. Intuitively, firms anticipate that they do not need to overshoot due to inflation as much because they will increase the frequency of price adjustment in the near future. Thus, the price spell that is just starting is anticipated to be of lower duration. Since the Taylor case gave 1/2 we now get a passthrough below 1/2.

5.4 Idiosyncratic Uncertainty

In the simple Sheshinski and Weiss (1977) menu cost model, all price changes are prompted by inflation. The desired relative prices are constant and in the absence of inflation there are no price changes. These models have been extended in various ways to incorporate idiosyncratic uncertainty at the product level, so that prices are changed up and down even when aggregate inflation is zero, as in Golosov and Lucas (2007), Midrigan (2011) and many others. We now consider these extensions.

Departing from the basic benchmark opens a host of opportunities. These models have been extended not just to include uncertainty, but also to allow for free opportunities to change prices, to consider firms managing multiple products and prices, etc. Here we shall explore a simple setting, following Alvarez et al. (2019).

The model is cast is once again cast in continuous time, except that now due to shocks to marginal costs we postulate that firms keep track of $x_t = -\pi_t dt + \sigma dW_t$ where W_t is a Brownian motion process, so that dW_t can be interpreted as an iid shock across periods, permanently impacting x_t with standard deviation σ. Once again the firm sets up pricing bands, except that now they are characterized by three numbers: $\underline{x}$, x^* and a new upper bound $\bar{x}$. the firm adjusts prices whenever x_t hits $\underline{x}$ or $\bar{x}$, in which case it resets x to x^*.

This model is more general than the one we studied earlier. The Sheshinski-Weiss case sets $\sigma = 0$, but, intuitively, similar results obtain when σ is small enough relative to inflation π_0. However, when $\pi_0 = 0$ and $\sigma > 0$ or, more generally, when σ is large and

π is small then we get different results. First, the distribution is no longer uniform, but it peaks at x^* instead. Secondly and most importantly, an increase in π^e does not widen the sS bands, instead: it shifts them to the right (see the proof of Proposition 1 in Alvarez et al., 2019). This induces an increase in price increases from the bottom at $\underline{x}$—indeed a mass of firms instead of a flow—and a drop to zero in the flow of price adjustments downward, at the upper bound $\bar{x}$. As a result, one obtains an extreme result: a discrete upwards jump in the price level.

Proposition 10. *In the symmetric Golosov-Lucas case with idiosyncratic shocks $\sigma > 0$ and zero initial inflation $\pi_0 = 0$ an increase in expectations of inflation produces an upward jump in the price level.*

These extreme results can once again be arrested by slowing the adjustment in the frequency of price adjustments in the short run. If we assume that both the frequency of price increases *and* the frequency of price decreases must remain constant, in the short run, then the passthrough is below $1/2$, as before. However, if the total frequency must remain unchanged in the short run, but that the firm can optimally reallocate this frequency between price increases and decreases then it the firm may wish to reallocate all price changes to the price increases, leading to a discrete jump in inflation in response to a small increase in π^e.

5.5 Time Aggregation

Up to now we have explored the effects "on impact" in continuous time. As we shall see, this skews the findings towards extreme results, in both directions depending on the parameters. In this section, we consider a less extreme "short run" by numerically simulating the dynamics beyond the immediate impact and tracing them out over one quarter.[25]

The calibration behind this exercises is done at a steady state with zero inflation, choosing the menu cost and the size of shocks σ so as to obtain a price frequency of 24% per quarter and an average adjustment of 8.5%, following Alvarez et al. (2019). We then vary the baseline inflation rate $\pi_0 > 0$ holding all parameters fixed. Recall that when π_0 is large relative to σ we approach the Sheshinski-Weiss case. In each case we compute the

[25]Recall that in this paper I abstain from exploring dynamics and focuses on the impact effects, for two reasons. First, it is an input into any further analysis into the dynamics; second, it allows us to take a temporary equilibrium approach and not model the updating of expectations. Aggregating up to a quarter is in keeping with this spirit of focusing on impact effects because most macroeconomic models are set in discrete time with a period calibrated to a quarter.

response of a sudden change in inflation expectations, which for concreteness we set as a rise in expectations of 4%.

Figure 1 shows the response of the price level (normalized relative to the shock) to a sudden change in inflation expectations. The results are in line with the previous theoretical results but also uncover that extreme outcomes are limited to a short interval around $t = 0$. The effects over one quarter are more moderate and intermediate.

In the immediate aftershock near $t = 0$ one observes rather extreme outcomes for low or high π_0. This is in line with the theoretical observations made earlier. For example, for $\pi_0 = 0$ at $t = 0$ there is a small upwards jump and the slope is very high. This echos the results above for the Golosov-Lucas case with $\pi_0 = 0$. Conversely, for high π_0 we see that the response is negative and sharply so; a similar result holds for smaller π_0 inflation rates if σ is made lower. This mirrors the theoretical results in the Sheshinski-Weiss case.

However, in all cases, after a short time the response revert to something more moderate. Indeed, for $\pi_0 = 0$ over the course of one quarter when $\sigma = 0$ we find a passthrough close to 1, just as in the Calvo model. This echos findings in Auclert et al. (2022) and others showing that menu-cost models close to Golosov-Lucas behave qualitatively much like to a Calvo model, but with a higher degree of price flexibility. On the other hand, this simulation shows that with higher inflation rates the cumulative one-quarter response lies just above $1/2$, the benchmark in the Taylor model.

6 mc^2

The spirit of state dependent models is that firms are often inactive and do not adjust prices frequently because there is a fixed "menu" cost. Sometimes this cost is taken literally and narrowly in terms of the goods and time cost of printing physical menus, catalogs or relabeling sticker prices on physical goods in supermarkets. These costs are relatively small and are reduced considerably by automation or online pricing. However, it is often argued that a more relevant and unavoidable cost is the managerial time involved in changing prices.

In this section I take this idea of managerial costs seriously and push it to the next level. During normal times the firm may have converged on certain sS policies that are optimal for some steady state inflation rate. For example, a manager may have decided to keep prices in a band ±5% of some desired markup over marginal costs. It seems natural to think that there are managerial costs to reconsidering and changing these bands to

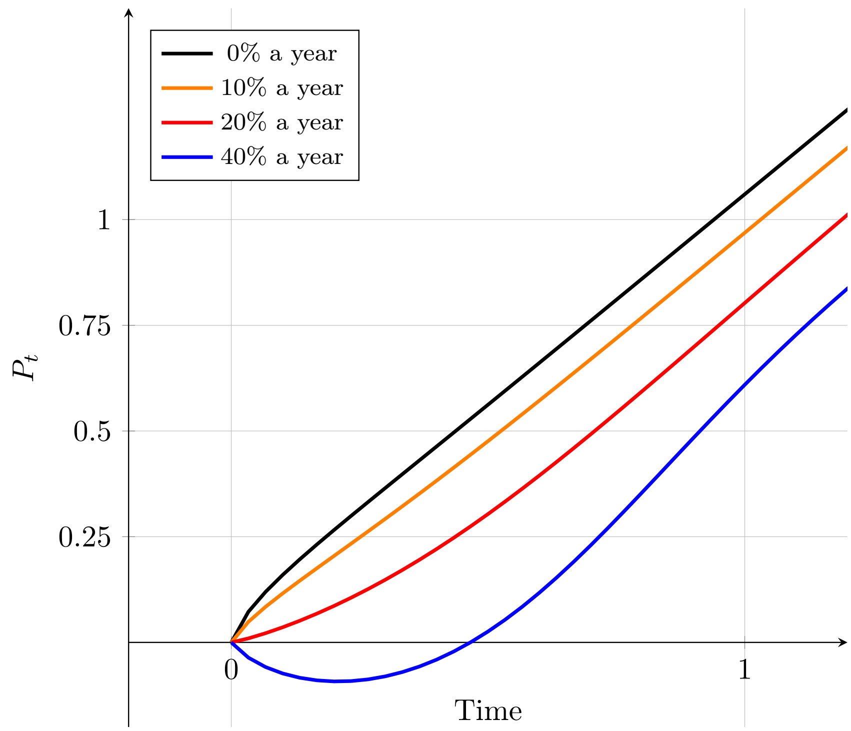

Figure 1: Impulse response of a change in inflation expectations.

a new situation, especially if this situation is temporary. Doing so requires reviewing the available information, weighing the tradeoffs, holding meetings, making decisions, communicating and implementing them.

Following the menu cost literature, we formalize these ideas in a stylized way, assuming there is a fixed cost $c_B > 0$ that must be paid to modify some preexisting pricing bands. If the cost is not paid, the firm can continue using its previous pricing bands.

I present the ideas in the simplest context, the Sheshinski and Weiss (1977), but the central ideas go through more generally for other state dependent models. For any arbitrary bands $\underline{x}$ and x^* let us denote by $V(\underline{x}, x^*, \pi^e)$ the anticipated value obtained by a firm with expectations π^e. Let $V^*(\pi^e)$ denote the value function using optimal bands $\underline{x}(\pi^e)$ and $x^*(\pi^e)$ given π^e. Then if

$$V(\underline{x}(\pi_0), x^*(\pi_0), \pi^e) \geq V^*(\pi^e) - c_B$$

the firm will choose to maintain its old bands. If the inequality is violated then the new bands are implemented. For given π_0 this induces a region of inaction for π^e around π_0.

Proposition 11. *Consider the Sheshinski-Weiss menu cost model extended so that, in addition to*

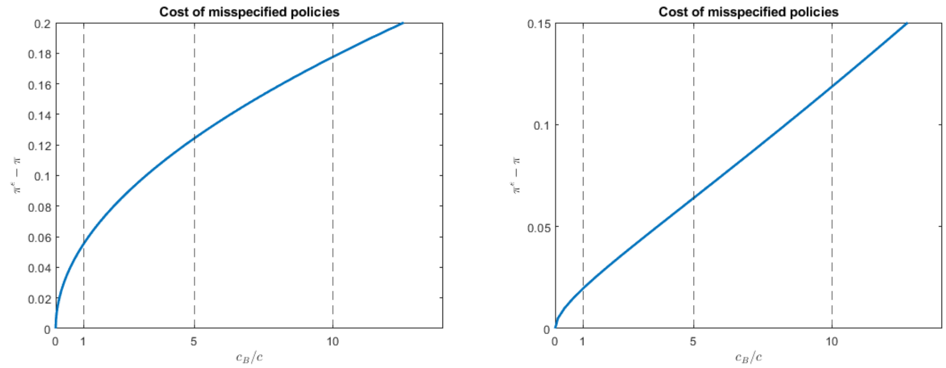

Figure 2: Upper inaction region $\bar{\pi} - \pi_0$ as a function of relative fixed costs c_B/c. Left panel $\sigma = 0.1641$; right panel $\sigma = 0$.

menu costs for changing prices, there are also fixed costs c_B for changing the pricing bands $(\underline{x}, x^*)$.

Then starting from an steady-state with inflation and expected inflation equal to π_0 and associated optimized bands $\underline{x}(\pi_0)$ and $x^*(\pi_0)$, there is an interval of inaction $[\underline{\pi}, \bar{\pi}]$ with $\underline{\pi} < \pi_0 < \bar{\pi}$ such that the the firm maintains its bands unchanged. Then for any $\pi^e \in I_0$ inflation remains unchanged, so the passthrough from inflation expectations to inflation is zero.

Moreover, the inaction region increases in c_B and $\frac{\partial}{\partial c_B}\underline{\pi} \to -\infty$ and $\frac{\partial}{\partial c_B}\bar{\pi} \to \infty$ as $c_b \to 0$.

If the bands do not changed then inflation expectations has no effect on firm behavior and hence no effect on inflation. The last part of the proposition suggests that the bands can be quite significant even for small fixed costs. The reasoning is the same as in Akerlof and Yellen (1985) and Mankiw (1985): if the bands were initially optimal the loses from not changing them are only second order.

Next, I perform a quantitative exploration. Annual inflation is initially 2% and the real discount rate is 2%. The size of the menu cost and idiosyncratic shocks is calibrated to match the observed size and frequency of price adjustments (in their sample from Argentina, those values are 10% and 2.7 adjustments per year during low inflation times).

I benchmark the costs of changing the pricing bands relative to the menu costs. It seems natural to imagine that the costs of changing the bands may be significantly higher than changing a single price, following a pre-established rule. Thus, I compute the upper inaction region $\bar{\pi} - \pi_0$ as a function of c_B/c between 1–10. The result with and without idiosyncratic shocks is plotted in Figure (2).

For example, with idiosyncratic uncertainty, if costs are five times greater than menu costs, then the change in inflation expectations must be upwards of 12% for the firm to find it worthwhile to re-optimize the pricing bands. The implied costs of following a sub-optimal policy are small so that large changes in expectations are required to trigger changes in the bands for these range of costs.

7 Conclusions

In this paper I take a step towards understanding the effect of inflation expectations on pricing and inflation, holding all other determinants of these fixed. I show that the common perception that optimizing models imply a 1-to-1 passthrough is generally misleading. My results uncover that this passthrough depends quite a bit on the pricing model. Exploring a wide range of cases I try to make the case that much lower values of the passthrough are possible and plausible.

Casting aside these results, I believe that a side-product of the analysis is that it lends a greater economic intuition and understanding for the transmission mechanism of the inflationary process, often ignored in formal analyses or the subject of speculation. In particular, expectations matters to the extent that individual firms "overshoot" their ideal relative price or if the frequency of price increases rises in the short run (the overall frequency of price changes is not relevant, as I show). Understanding this mechanism suggests new empirical or theoretical directions. Can we measure this overshooting directly at the microeconomic level? Theoretically, are there other important economic considerations shaping the degree of overshooting such as price complementarities or the shape of demand?

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A Continuous Time Taylor Model

In continuous time price setting is given by

$$p_t^* - P_t = \frac{\int_0^{1/\delta} e^{-\rho s} s ds}{\int_0^{1/\delta} e^{-\rho s} ds} \pi^e + a_t$$

Using that

$$\int_0^{\Delta} e^{-\rho s} s ds = -\frac{1}{\rho} e^{-\rho \Delta} \Delta + \frac{1}{\rho^2}(1 - e^{-\rho\Delta}) = \frac{1}{\rho^2}(1 - e^{-\rho\Delta}(1 + \rho\Delta))$$

and $\int_0^{\Delta} e^{-\rho s} ds = \frac{1}{\rho}(1 - e^{-\rho\Delta})$ we arrive at

$$p^* - P = \frac{1}{\rho} \frac{(1 - e^{-\rho/\delta}(1 + \rho/\delta))}{1 - e^{-\rho/\delta}} \pi^e$$

Using that $\pi = \delta(p^* - P)$ gives the desired expression.

B First Order Approximation for General Time Dependent Model

Consider a general time-dependent model defined by the hazard rate function $\{h_s\}$. Let $\omega_{s+1} = \omega_s(1 - h_s)$ (the value of ω_0 will be inconsequential). The firm faces a path of interest rates q_{t+s}. And a path of θ_t shocks to its profit function.

The firm then solves

$$\max_{p^*} \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi(p_t^* - P_{t+s}, \theta_{t+s})$$

with first-order condition

$$\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_p(p_t^* - P_{t+s}, \theta_{t+s}) = 0$$

We consider a small variation in the firms' problem $\{P_{t+s}, \theta_{t+s}, q_{t+s}, \omega_s\}$.

Totally differentiating gives

$$0 = \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_{pp,t} \left(dp_t^* - dP_{t+s}^e\right) + \mathbb{E}_{t-1} \sum_{s=0}^{\infty} \Pi_{p,t}(dq_{t+s}\omega_s + q_{t+s}d\omega_s) + \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_{p\theta,t}\, d\theta_{t+s}$$

Around a steady state with zero inflation $\Pi_{pp,t}$ and $\Pi_{p\theta,t}$ are constant over time and $\Pi_{p,t} = 0$ from the first order condition. Thus, the middle term cancels and rearranging the remaining terms and setting gives

$$dp_t^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, dP_{t+s}^e}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s} + da_t$$

At a zero inflation steady state we also have that the constants $p^* = P^e$ and $a = 0$ satisfy

$$p^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, P^e}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s} + a.$$

Adding these two conditions gives the desired result since $p_t = p^* + dp_t^*$, $P_t^e = P^* + dP_t^e$ and $a_t = a + da_t$ up to first order. Finally, setting $q_{t+s} = \beta^{t+s}$ gives the desired result.

C Coefficients on Past Inflation

The inverse has coefficients satisfying the recursion

$$\alpha_s = \begin{cases} \frac{1}{\omega_0} & s = 0 \\ -\sum_{j=1}^{s} S_j \alpha_{s-j} & s \geq 1 \end{cases}$$

Writing p_t^* in terms of past inflation rates gives

$$\begin{aligned}
p_t^* &= \sum_{s=0}^{\infty} \alpha_s P_{t-s} \\
&= P_t - \sum_{s=1}^{\infty} \alpha_s \left(P_t - P_{t-s}\right) \\
&= P_t - \sum_{s=1}^{\infty} \alpha_s \left(\sum_{j=0}^{s-1} \pi_{t-j}\right) \\
&= P_t - \sum_{s=1}^{\infty} \left(\sum_{j=s}^{\infty} \alpha_j\right) \pi_{t-s+1}
\end{aligned}$$

Define $\gamma_s = \sum_{j=s}^{\infty} \alpha_j$.

$$p_t^* = P_t - \sum_{s=1}^{\infty} \gamma_s \pi_{t-s+1} \tag{1}$$

Equivalently

$$p_{t-s-1}^* - P_{t-1} = p_{t-s-1}^* - P_{t-s-1} + P_{t-s-1} - P_t = -\sum_{z=1}^{\infty} \gamma_z \pi_{t-s-z} - \left(P_{t-1} - P_{t-s-1}\right)$$

Since $\sum_{s=0}^{\infty} \omega_s \left(\bar{h} - h_s\right) = 0$ then

$$\sum_{s=0}^{\infty} \omega_s \left(\bar{h} - h_s\right) p_{t-s-1}^* = \sum_{s=0}^{\infty} \omega_s \left(\bar{h} - h_s\right) \left(p_{t-s-1}^* - P_{t-1}\right) = \sum_{s=0}^{\infty} \delta_s \left(p_{t-s-1}^* - P_{t-1}\right)$$

some calculations

$$\begin{aligned}
\sum_{s=0}^{\infty} \delta_s \left(p_{t-s-1}^* - P_t\right) &= \sum_{s=0}^{\infty} \delta_s \left[-\sum_{z=1}^{\infty} \gamma_z \pi_{t-s-z} + P_{t-s-1} - P_{t-1}\right] \\
&= -\sum_{s=0}^{\infty} \sum_{z=1}^{\infty} \delta_s \gamma_z \pi_{t-s-z} - \sum_{s=0}^{\infty} \delta_s \left(P_{t-1} - P_{t-s-1}\right)
\end{aligned}$$

define $\theta_s = -\sum_{z=1}^{s} \delta_{s-z} \gamma_z$ and $\mu_s = \sum_{z=s}^{\infty} \delta_z$, then

$$\sum_{s=0}^{\infty} \delta_s \left(p_{t-s-1}^* - P_t\right) = \sum_{s=1}^{\infty} \theta_s \pi_{t-s} - \sum_{s=1}^{\infty} \mu_s \pi_{t-s}$$

thus

$$\sum_{s=0} \omega_s \left(\bar{h} - h_s\right) p_{t-s-1}^* = \sum_{s=1}^{\infty} \phi_{-s} \pi_{t-s} \tag{2}$$

with

$$\phi_{-s} = \theta_s - \mu_s.$$

This provides the desired expressions to compute ϕ_s for all $s < 0$.