A Theory of Fear of Floating*

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Abstract

Many central banks with flexible exchange rate regimes exhibit a reluctance to allow the exchange rate to fluctuate, a phenomenon known as "fear of floating." We present a simple theory in which fear of floating emerges as an optimal policy outcome. In this environment, an occasionally binding borrowing constraint linked to the real exchange rate creates a feedback loop between aggregate demand and credit conditions. Our analysis shows that under a floating exchange rate regime, the economy is more vulnerable to financial crises with self-fulfilling deleveraging. Moreover, we find that quantitatively, in the presence of self-fulfilling financial crises, a fixed exchange rate regime may welfare dominate a regime with floating exchange rates.

Keywords: Exchange rates, self-fulfilling financial crises, monetary policy

JEL Codes: E44, E52, F33, F34, F36, F41, F45, G01

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1 Introduction

According to the Mundell-Fleming paradigm, a floating exchange rate plays a pivotal role in stabilizing economic fluctuations in open economies. As argued by Friedman (1953), movements in exchange rates help accommodate real shocks in the presence of nominal rigidities, allowing the economy to achieve the allocations that would prevail under flexible prices. In practice, however, many central banks around the world are reluctant to let their exchange rate float. Indeed, the seminal work of Calvo and Reinhart (2002) documented that "countries that say they allow their exchange rate to float mostly do not—there seems to be an epidemic case of fear of floating." Ilzetzki, Reinhart and Rogoff (2019, 2021) highlight that this phenomenon remains pervasive today.[1]

Why are central banks reluctant to let the exchange rate float? Many policy discussions attribute the "fear of floating" phenomenon to concerns that large exchange rate movements can lead to destabilizing effects in financial markets. A large body of literature triggered by the series of financial crises in emerging markets has explored how sudden stops in financial flows can result in depreciations that adversely impact the balance sheets of the corporate and household sectors (e.g., Krugman, 1999). In existing studies, however, a floating exchange rate regime remains optimal, as the flexibility of choosing monetary policy contributes to absorbing external shocks. (e.g., Céspedes, Chang and Velasco, 2004). Moreover, financial factors often call for even higher depreciations to stabilize aggregate demand. Despite much progress in the literature, an important gap therefore remains in our understanding of the link between floating exchange rates and turmoil in financial markets. A theory of fear of floating thus remains elusive.

In this paper, we develop the idea that letting the exchange rate float may expose the economy to a self-fulfilling financial crisis, thereby providing a rationale for the phenomenon of fear of floating.

The model has two key elements: downward nominal wage rigidity in the non-tradable sector and a borrowing constraint on households linked to the value of their income. The first element implies that an exchange rate depreciation helps reduce real wages and potentially offset negative real shocks, thus calling for a flexible exchange rate. The second element implies that negative shocks that lead to deleveraging are amplified through a

[1] Ilzetzki et al. (2019) classify only 10% of the world economy as pure floaters and 51% of the countries as fixed exchange rates. Fukui, Nakamura and Steinsson (2023) find that even currencies that Ilzetzki et al. classify as managed floats exhibit similar comovements as currencies that they classify as very hard pegs.

general equilibrium effect that further tightens the borrowing constraint.[2] Under plausible parameterizations, Schmitt-Grohé and Uribe (2021) have shown that this feature can lead to crises with self-fulfilling deleveraging, in the context of a real environment. Building on their work, we consider an economy in which nominal wages are downwardly rigid and study how the exchange rate regime affects the vulnerability to self-fulfilling financial crises.

In this environment, we first show that the general equilibrium feedback between the value of collateral and the exchange rate depends crucially on the exchange rate regime. On the one hand, a depreciation generates an expansionary effect on employment by shifting demand towards non-traded goods. On the other hand, for a given level of employment, a depreciation reduces the value of non-tradable resources that are used as collateral. Depending on whether price or quantity effects dominate, this second channel could make the nominal depreciation more expansionary or turn the depreciation contractionary, leading to a fall in output.

Our main result is that fixing the nominal exchange rate protects the economy from self-fulfilling financial crises. Specifically, we show that for intermediate values of debt, a flexible exchange rate is vulnerable to a self-fulfilling crisis, whereas a fixed exchange rate uniquely implements the best equilibrium.

To understand this result, consider a situation in which the economy is in a steady state equilibrium and households suddenly become pessimistic and deleverage. The collective reduction in borrowing leads to lower demand for non-tradable goods and for domestic currency. Under a flexible exchange rate, the nominal exchange rate depreciates, leading to a depreciation of the real exchange rate, which feeds into a tighter borrowing constraint. When the tightening of the borrowing constraint is sufficiently strong, the initial panic is validated, and the economy falls into a self-fulfilling crisis with capital outflows and a currency depreciation. In contrast, under a fixed exchange rate, the initial panic affects aggregate demand and output but, not of course ,the exchange rate. If the reduction in output significantly tightens the borrowing constraint, the initial panic is again validated, and a financial crisis would unfold.

These results suggest that the exchange rate regime could potentially have ambiguous effects on the vulnerability to crises. In the flexible exchange rate, the impact of a panic might be less pronounced on employment but more notable on exchange rates. Conversely, a fixed exchange rate might result in a more pronounced effect on employment and a

[2]Both elements have been studied extensively in the literature in emerging markets but, for the most part, in isolation (e.g., Mendoza, 2002; Bianchi, 2011; and Schmitt-Grohé and Uribe, 2016, 2021). An exception is Ottonello (2021), which we discuss more below.

muted impact on exchange rates. Our analytical results show that the possibility of a currency crisis under a flexible exchange rate increases the vulnerability to a financial crisis relative to a fixed exchange rate regime. Specifically, at intermediate values of debt, a fixed exchange rate policy can stabilize the real exchange rate and break the feedback loop that would lead to a self-fulfilling financial crisis under a flexible exchange rate regime.

Our baseline analysis contrasts a flexible exchange rate regime under which money supply is kept constant and a fixed exchange rate regime.[3] We then evaluate optimal policy and establish the importance of commitment in determining the vulnerability to self-fulfilling crises. We first show that when the central bank lacks commitment to the exchange rate policy, a continuum of Markov perfect equilibria exists. Equilibria with higher exchange rate levels exhibit lower output and larger capital outflows. When the central bank has commitment to a fixed exchange rate, we analytically show that this can rule out self-fulfilling crises for low levels of debt. For higher levels of debt, however, an economy with a fixed exchange rate remains exposed to self-fulfilling crises, which can turn out to be more severe than under flexible exchange rates. Following the approach by Bassetto (2005) and Atkeson, Chari and Kehoe (2010), we derive a "sophisticated monetary policy" that can uniquely implement the good equilibrium. Interestingly, the policy resembles a *crawling band*. When households expect deleveraging, the central bank lets the exchange rate move within a band to relax collateral constraints and rules out the equilibrium where households find it individually optimal to deleverage.

The theory helps rationalize the prevalent use of foreign exchange intervention in emerging markets (Calvo, 2006). That is, while a policy of controlling interest rates (or monetary aggregates) can leave the economy vulnerable to crises, a policy commitment to intervene to keep the exchange rate around a peg can help protect the economy from financial crises.

Finally, we extend our analysis to a stochastic environment driven by tradable endowment shocks to quantitatively determine whether fixing the exchange rate remains preferable to letting the exchange rate float. In our calibration, we find that the welfare ranking between fixed and flexible exchange rates depends crucially on the presence of self-fulfilling financial crises. When selecting the good equilibrium in the case of indeterminacy, we find that flexible exchange rates dominate. As in the Mundell-Flemming paradigm, relinquishing the exchange rate as a shock absorber is not desirable. In contrast, when selecting the self-fulfilling crisis equilibrium in the case of indeterminacy, we find that fixing the exchange rate is desirable.

[3] We also consider interest rate rules in the appendix.

Related literature. This paper relates to a vast literature on optimal monetary policy in open economies. As discussed above, a key theme in the literature, going back to Friedman (1953) and Mundell (1960), is that a flexible exchange rate regime can insulate the economy from domestic and external shocks. In Schmitt-Grohé and Uribe (2011), for example, it is optimal to let the exchange rate vary to achieve the flexible price allocation. In the literature there are also many studies in which divine coincidence fails because of terms of trade manipulation motives (e.g., Benigno and Benigno, 2003) or local currency pricing (e.g, Devereux and Engel, 1998). While these factors may downplay the exchange rate as a shock absorber, a flexible exchange rate remains optimal (see, Clarida, Gali and Gertler, 2001).[4] Our paper presents a new perspective by providing a theory in which fixing the exchange rate is desirable because doing so reduces the vulnerability to self-fulfilling financial crises.

Our paper is also related to a large literature on monetary policy with credit frictions in open economies. A central focus in this literature is the examination of how balance sheet constraints can magnify the impact of external shocks and its implications for monetary policy tradeoffs.[5] Most of these studies find that while currency depreciations may have adverse consequences in the presence of currency mismatches, a nominal depreciation by the central bank remains expansionary. A recent exception featuring contractionary depreciation is Cavallino and Sandri (2022). In their model, a depreciation is implemented through a reduction in the interest on reserves, as opposed to conventional monetary policy, as in our paper.[6] As in the other studies in this literature, a flexible exchange rate remains desirable in their framework. Our paper provides, to the best of our knowledge, the first theory of why fixing the exchange rate may reduce an economy's vulnerability to financial crises.

Several studies also consider the interaction between household borrowing constraints linked to income and nominal rigidities (Farhi and Werning, 2016; Ottonello, 2021; Coulibaly, 2023; Basu, Boz, Gopinath, Roch and Unsal, 2023). A key point highlighted in this literature is that the government may choose not to implement the full employment allocation, because this would lead to a higher real depreciation and tighten the borrowing constraint.

[4] Engel (2011) extends the analysis in Clarida et al. (2001) with pricing to market and finds that optimal policy rules under cooperation should target exchange rates, in addition to inflation and the output gap. See Corsetti, Dedola and Leduc (2010) for a review of the literature.

[5] Examples include Aghion, Bacchetta and Banerjee (2000; 2004), Caballero and Krishnamurthy (2001), Gertler, Gilchrist and Natalucci (2007), Lahiri and Végh (2001), Céspedes et al. (2004), Cook (2004), Christiano, Gust and Roldos (2004), Braggion, Christiano and Roldos (2009), Fornaro (2015), Gourinchas (2018), Du and Schreger (2022), Devereux and Yu (2017), Devereux, Young and Yu (2019), **?**, and Cavallino and Sandri (2022).

[6] See also Cook (2004), who provides an interesting model in which a depreciation reduces investment, but output still expands.

In particular, Ottonello (2021) highlights a tradeoff between credit market access and unemployment, and Farhi and Werning (2016) refer to the tradeoff between financial and macroeconomic objectives. While these studies suggest a notion of exchange rate management, in contrast to our work, they suggest that a flexible exchange rate is still desirable. In addition, our analysis also highlights that when the financial channel dominates, a depreciation reduces *at the same time* credit market access and employment, and therefore a depreciation is actually contractionary.[7]

Our paper belongs to the "third-generation" crisis literature. Central to our paper is the idea that general equilibrium feedback that operates through the real exchange rate can lead to multiple equilibria, as in Krugman (1998), Schneider and Tornell (2004), Bocola and Lorenzoni (2020), and Schmitt-Grohé and Uribe (2021). In contrast to these studies, ours considers a monetary model with nominal rigidities. This feature allows us to examine how different exchange rate regimes affect the vulnerability to self-fulfilling financial crises.

Our findings contrast with others in the literature in which a fixed exchange rate increases the vulnerability to crises. Chang and Velasco (2000) show that in an economy with deposits in domestic currency and flexible prices, the ability to depreciate is helpful for reducing the vulnerability to bank runs. Bianchi and Mondragon (2022) show that the ability to use monetary policy helps mitigate the recession in the event of a run on government bonds, thereby making investors less prone to run. These studies examine different sources of multiplicity: the former centers on a coordination problem between creditors and individual banks, while the latter addresses a coordination problem between a government and foreign creditors. In our model, the coordination failure involves instead domestic households, resulting in self-fulfilling deleveraging. Clearly, the mechanisms emphasized in these studies and in our paper can operate simultaneously, which suggests that it may be interesting to study the interaction of these features.

This paper is related to the literature on aggregate demand externalities and macroprudential policy in models with nominal rigidities and monetary policy constraints. In particular, Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2012) consider

[7] This also connects with the unresolved question of whether a depreciation can be contractionary or whether instead recessions induce depreciations, as in standard open economy models (see Frankel, 2005; Edwards, 1985; Calvo, 2005; Uribe and Schmitt-Grohé, 2017; Fukui et al., 2023). The idea that a depreciation can be contractionary goes back to Diaz-Alejandro (1963). See also Tille (2001), Corsetti, Dedola and Leduc (2022), and Auclert, Rognlie, Souchier and Straub (2021) for models of contractionary depreciation through terms of trade channels and Adrian, Erceg, Kolasa, Lindé and Zabczyk (2022) for an adaptive expectation mechanism. See also De Ferra, Mitman and Romei (2020) and Blanchard, Ostry, Ghosh and Chamon (2016) for other related work.

economies with a fixed exchange rate but take as given the choice of the exchange rate regime. Our paper complements these studies by providing a theory of why the central bank finds it optimal to keep the exchange rate fixed.

Our paper is also related to the literature discussing the advantages of joining a monetary union and surrendering monetary independence. Following the work of Mundell (1961), one perspective is that joining a monetary union contributes to reducing transactions and fostering trade integration. Another influential view, spearheaded by Alesina and Barro (2002), is that joining a monetary union or adopting a fixed exchange rate reduces the inflationary bias generated by the time inconsistency problem of monetary policy, as stressed by Barro and Gordon (1983).[8] Our contribution lies in providing a distinct rationale for stabilizing the exchange rate, focusing on the reduced vulnerability to financial crises as a central motivation. This complements the existing literature and sheds light on the importance of exchange rate stability in mitigating financial risks.

Finally, our paper is related to recent work by Itskhoki and Mukhin (2022), which points out a different mechanism that justifies a crawling exchange rate band. In their model, intermediaries are exposed to exchange rate risk. By stabilizing the exchange rate, the central bank can improve access to external capital flows and increase risk-sharing.

Outline. Section 2 presents the model. Section 3 present the theoretical results on how the exchange rate regime affects vulnerability to self-fulfilling financial crises. Section 4 analyzes optimal policy, contrasting Markov equilibrium with sophisticated policies. Section 5 conducts a quantitative analysis. Section 6 concludes.

2 Model

We consider a small open economy with two types of goods: tradables and non-tradables. Time is discrete and infinite. The economy features nominal rigidities and constraints on households' borrowing.

[8] See also Cook and Devereux (2016), Corsetti, Kuester and Müller (2017), Chari, Dovis and Kehoe (2020). Other studies highlight how joining a monetary union may improve risk sharing (Neumeyer, 1998, Arellano and Heathcote, 2010, Fornaro, 2022). An earlier perspective in international macro, following the analysis of Poole (1970), is that a fixed exchange rate dominates a fixed money supply when the economy is subject to large monetary shocks. Nonetheless, as in the findings in other studies, if the central bank can adjust monetary policy (e.g., by raising money supply in response to higher money demand), this option continues to dominate a fixed exchange rate.

2.1 Households

There is a continuum of identical households of measure one. Households have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \chi \log \left(\frac{M_{t+1}}{P_t} \right) \right],$$

where $\chi \geq 0$ and $\beta \in (0, 1)$ is the discount factor. The consumption good c_t is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a constant elasticity of substitution aggregator:

$$c_t = \left[\phi (c_t^T)^{\frac{\gamma-1}{\gamma}} + (1-\phi)(c_t^N)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad \text{where } \phi \in (0, 1).$$

For the most part, we will focus on an elasticity of substitution between tradable and non-tradable consumption, γ, below one, which is the empirically relevant case. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods. The real money holdings, M_{t+1}/P_t, provide liquidity services to households that enter the utility function, where M_{t+1} is the end-of-period money holdings and P_t is the ideal price index in period t. We denote by P_t^N and P_t^T the price of non-tradables and tradables (in terms of the domestic currency), respectively. The ideal price index satisfies

$$P_t = \left[\phi^\gamma \left(P_t^T \right)^{1-\gamma} + (1-\phi)^\gamma \left(P_t^N \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}.$$

We assume that the law of one price holds for the tradable good and normalize the price of the tradable good in units of foreign currency to unity. This implies that $P_t^T = e_t$, where e_t is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency.

Households supply $\bar{h}$ units of labor inelastically. Because of the presence of downward wage rigidity and rationing (to be described below), households' hours worked satisfy $h_t \leq \bar{h}$, which is taken as given by the individual household. Each period households receive a wage rate, W_t, and central bank transfers, T_t, both expressed in terms of domestic currency, which serves as the numeraire. They also receive an endowment y^T of tradable goods and trade one-period non-state-contingent nominal bonds in domestic and foreign currency. The foreign currency bond has an exogenous return R. The domestic currency bond is assumed to be traded only within domestic market and pays a return $\tilde{R}_t$, determined endogenously.[9] The budget constraint of the representative household is therefore given by

$$P_t^T c_t^T + P_t^N c_t^N + M_{t+1} + \tilde{b}_t + e_t b_t = P_t^T y^T + W_t h_t + M_t + \frac{\tilde{b}_{t+1}}{\tilde{R}_t} + \frac{e_t b_{t+1}}{R} + T_t, \tag{1}$$

where $\tilde{b}_t$ and b_t denote respectively the amount of domestic currency debt and foreign currency debt assumed in period $t-1$ and due in period t. The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds, while the right-hand side represents total income, including the returns from bond issuance.

Households face a borrowing constraint that limits foreign currency debt to a fraction κ of their individual current income:

$$\frac{e_t b_{t+1}}{R} \leq \kappa \left(P_t^T y^T + W_t h_t \right). \tag{2}$$

This borrowing constraint captures the idea that current earnings are a critical factor determining credit market access (Jappelli, 1990; Greenwald, 2018 ; Lian and Ma, 2020; Drechsel, 2022) and has been shown to be important for accounting for the dynamics of capital flows in emerging markets (Mendoza, 2002; Bianchi, 2011).[10] To ensure that the borrowing constraint is tighter than the natural debt limit, we assume $0 < \kappa < \frac{R}{R-1}$.

Optimality conditions. First-order conditions with respect to c_t^T and c_t^N imply that

$$\frac{P_t^N}{e_t} = \frac{1-\phi}{\phi} \left(\frac{c_t^N}{c_t^T} \right)^{-\frac{1}{\gamma}} \tag{3}$$

Let $\lambda_t \geq 0$ denote the Lagrange multiplier on the budget constraint (1), $\lambda_t \mu_t \geq 0$ the Lagrange multiplier on the borrowing constraint (2) and u_T the marginal utility of tradable consumption. Households' optimal borrowing choices for foreign currency bonds are

[9] Because there is no uncertainty in the baseline model, portfolio considerations do not play a role (aside from the effect of the exchange rate on the initial real wealth).

[10] The credit constraint can be derived endogenously from a problem of limited enforcement under the assumption that household default occurs at the end of the current period and that upon default, households lose a fraction κ_t of the current income. The borrowing limit could also depend on future income or other variables. What is crucial for our results is that higher current income relaxes the borrowing limit. Moreover, the collateral constraint assumes that only foreign debt can be collateralized. We can show that all the results hold when a fraction of domestic bonds must be collateralized as well.

determined by the following Euler equation and complementary slackness:

$$(1-\mu_t)u_T(c_t^T, c_t^N) = \beta R u_T\left(c_{t+1}^T, c_{t+1}^N\right) \tag{4}$$

$$\mu_t \times \left[\kappa\left(y^T + \frac{W_t}{e_t}h_t\right) - \frac{b_{t+1}}{R}\right] = 0. \tag{5}$$

Similarly, the optimal borrowing choices for domestic currency bonds are determined by

$$u_T(c_t^T, c_t^N) = \beta \tilde{R}_t \frac{e_t}{e_{t+1}} u_T\left(c_{t+1}^T, c_{t+1}^N\right). \tag{6}$$

Households' optimality condition for money balances yields the following money demand equation decreasing in the nominal interest rate:

$$\frac{M_{t+1}}{P_t} = \chi \frac{\tilde{R}_t}{U'(c_t)(\tilde{R}_t - 1)}. \tag{MD}$$

Using the Euler equations for foreign currency bonds and domestic currency bonds and the law of one price, we obtain an interest parity condition, which relates the nominal returns on domestic and foreign currency bonds to the expected currency depreciation:

$$\frac{R}{1-\mu_t} = \tilde{R}_t \frac{e_t}{e_{t+1}}. \tag{7}$$

Notice that when the borrowing constraint binds, we have an endogenous deviation from uncovered interest parity.

2.2 Firms and Nominal Rigidities

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a linear production technology given by $y_t^N = n_t$ and obtains profits given by $\phi_t^N = P_t^N n_t - W_t n_t$. Given the linear production function, we obtain that in equilibrium,

$$P_t^N = W_t. \tag{8}$$

An individual firm is therefore indifferent between any level of employment.[11] We assume there exists a minimum wage in nominal terms. Following Schmitt-Grohé and Uribe

[11] Linearity simplifies the analytical results, but it is possible to extend our analysis for decreasing returns.

(2016), we assume that the current nominal wage is bounded below by the previous period nominal wage; that is, $W_t \geq W_{t-1}$. The labor market is such that aggregate hours worked are the minimum between labor demand and labor supply, following standard disequilibrium models with rationing: $h_t = \min\{n_t, \bar{h}\}$. If the market clearing wage satisfies $W_t > W_{t-1}$, the aggregate number of hours worked equals the aggregate endowment of labor. Otherwise, hours are determined by labor demand, $h_t < \bar{h}$ and $W_t = W_{t-1}$. These conditions can be summarized as

$$(W_t - W_{t-1})(h_t - \bar{h}) = 0. \tag{9}$$

2.3 Monetary Policy

The central bank's budget constraint is given by

$$T_t = M^s_{t+1} - M^s_t.$$

That is, the central bank rebates all revenues from the increase in money supply to the public in the form of lump-sum transfers.

We will mainly focus on two exchange rate regimes. In the case of a flexible exchange rate, the central bank sets the money supply, M^s_t, and lets the exchange rate adjust in equilibrium.[12] In the case of a fixed exchange rate, the central bank sets e_t and lets the money supply adjust to implement that level of the exchange rate. This policy can be implemented through foreign exchange intervention, announcing an exchange rate and promising to exchange domestic currency for foreign currency at the announced exchange rate.[13] Notice that the fact that we allow for lump-sum taxes rules out the need to accumulate ex ante reserves and the possibility of an abandonment triggered by speculative attacks, as studied in Krugman (1979) and the subsequent literature.

2.4 Competitive Equilibrium

An equilibrium requires that the supply of money by the central bank equals the demand for money by households: $M^s_{t+1} = M_{t+1}$. In addition, the aggregate labor demand by

[12]We will also discuss interest rate rules, where the money supply would adjust to implement the desired interest rates.

[13]To the extent that the fixed exchange rate is credible, no actual foreign exchange interventions are needed to implement the peg in equilibrium.

firms equals the units of labor supplied by households:

$$h_t = n_t. \tag{10}$$

Market clearing for the non-tradable good requires that output equal consumption:

$$y_t^N = c_t^N. \tag{11}$$

We assume that the bond denominated in domestic currency is traded only domestically. Market clearing therefore implies

$$\tilde{b}_{t+1} = 0. \tag{12}$$

Combining the budget constraints of households, firms, and the central bank, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - y^T = \frac{b_{t+1}}{R} - b_t, \tag{13}$$

which says that the trade balance must be financed with net bond issuances.

If we combine (8) and (3), we arrive at an equation determining the aggregate demand for non-tradables as a function of the real wage, W_t/e_t, and the level of tradable consumption c_t^T:

$$c_t^N = \left(\frac{1-\phi}{\phi}\frac{e_t}{W_t}\right)^{\gamma} c_t^T. \tag{14}$$

Equations (13) and (14) will play a central role in the model dynamics. In the event of a deleveraging episode triggered by a binding credit constraint, the small open economy will have fewer tradable resources available. For a given relative price of non-tradables, this will lead to a reduction in the demand for non-tradable goods. With flexible wages, W_t would fall until $h_t = c_t^N = \bar{h}$. But if the downward wage rigidity becomes binding, the economy will feature involuntary unemployment, which will in turn feed into consumption and the borrowing capacity.

It is also useful to note that combining (2) with firms' optimality and market clearing, we obtain

$$\frac{e_t b_{t+1}}{R} \leq \kappa \left[P_t^T y^T + P_t^N y_t^N\right],$$

which implies that households in equilibrium can borrow up to a fraction κ of the aggregate income.

We can now define a sticky-wage competitive equilibrium

Definition 1 (Sticky-Wage Equilibrium)**.** Given initial conditions $(B_0.W_{-1})$, a sticky-wage equilibrium is defined by a set of government policies, prices $\{W_t, P_t^N, e_t\}_{t=0}^{\infty}$, and allocations $\{b_{t+1}, h_t, c_t^N, c_t^T\}$ such that

1. households and firms optimize; that is, (1)-(8) hold;
2. non-tradable goods market clears $c_t^N = h_t$;
3. labor market conditions (9) and $W_t \geq W_{t-1}$ hold;
4. the government budget constraint holds;

2.5 Steady-State Equilibrium

We restrict our attention to the case in which $\beta R = 1$. We define a steady-state equilibrium as a competitive equilibrium where all allocations are constant.

Definition 2 (Steady-state equilibrium)**.** A steady-state equilibrium is a competitive equilibrium in which allocations are constant for all $t \geq 0$.

Notice that a constant consumption allocation under $\beta R = 1$ implies that the borrowing constraint is not binding. From the tradable resource constraint, using $B_{t+1} = B_0$, we obtain $c_t^T = y^T - (1-\beta)B_0$.

In the absence of a borrowing constraint, any initial values of debt lower than the natural debt limit would be consistent with a steady-state equilibrium. Our goal next is to define the range of values of initial debt that are consistent with a steady-state equilibrium in the presence of borrowing constraints. Towards this goal, we use households' optimality conditions (3) and (14) and the market clearing condition for non-tradables (11) to define the individual borrowing limit in period t as

$$\bar{b}(B_{t+1}; B_t) = \kappa R \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B_t + \frac{B_{t+1}}{R} \right)^{\frac{1}{\gamma}} (h_t)^{1-\frac{1}{\gamma}} \right]. \tag{15}$$

Equation (15) describes how a household's maximum borrowing capacity $\bar{b}(B_{t+1}; B_t)$ depends on aggregates (B_t, B_{t+1}). We can observe that $\bar{b}(B_{t+1}; B_t)$ is decreasing in initial debt

B_t and increasing in new debt issuances B_{t+1}, reflecting that higher aggregate consumption appreciates the real exchange rate and relaxes individuals' borrowing constraints.

We let $\hat{B}$ denote the unique value of debt such that $\bar{b}(\hat{B}; \hat{B}) = \hat{B}$ when $h_t = \bar{h}$. The lemma below characterizes existence of a steady-state equilibrium and the optimal monetary policy in a steady state.

Lemma 1 (Steady-state equilibrium)**.** *If $B_0 \leq \hat{B}$, we have that the steady-state equilibrium exists. Moreover, at the steady-state equilibrium the optimal allocations satisfy $h_t = \bar{h}$. Moreover, a constant exchange rate where*

$$e_t \geq W_{t-1} \frac{\phi}{1-\phi} \left[\frac{y^T - (1-\beta)B_0}{\bar{h}} \right]^{-\frac{1}{\gamma}}$$

implements the optimal allocations.

Proof. In Appendix A.1 □

By definition of $\hat{B}$, when the initial and end-of-period debt level equals $\hat{B}$, we have that consumption is constant over time and the borrowing constraint holds with equality. It follows, then, that for any level of debt $B_0 < \hat{B}$, the borrowing constraint is satisfied (strictly) and a steady-state equilibrium exists. Moreover, at the steady-state equilibrium, the optimal monetary policy implements full employment.

Given that the borrowing constraint is slack in a steady-state equilibrium, there is only one potential departure from the first-best allocation: the possibility of unemployment. It then follows that the optimal monetary policy achieves full employment. Full employment is achieved in this case by depreciating the currency enough that the nominal wage rigidity is not binding. Clearly, there is a wide range of monetary policies that deliver such an outcome. We focus on a policy that delivers zero inflation for $t = 0, 1, \ldots$ We do this partly for simplicity and partly to capture the traditional price stability objective of central banks. The policy implies that the central bank sets the exchange rate at a constant level given by

$$\bar{e} = W_{-1} \frac{\phi}{1-\phi} \left(\frac{c^T}{\bar{h}} \right)^{-\frac{1}{\gamma}}. \tag{16}$$

To ensure consistency with a constant path for the exchange rate, the central bank needs to set a constant money supply $\bar{M}$. Using (MD), we have that the level of the nominal money supply is given by

$$\frac{\bar{M}}{\chi} = \frac{W_{-1}}{u_N(c^T, \bar{h})} \frac{R}{R-1}. \tag{17}$$

Notice that the value of $\bar{e}$ and $\bar{M}$ depend on B_0. Namely, a higher B_0 implies a lower steady-state level of consumption and therefore requires a higher $\bar{e}$ for a given W_{-1}. Intuitively, when the level of consumption is lower, the real exchange rate is also lower, and achieving a reduction in the real wage requires a higher nominal exchange rate. Notice that by condition (7), it also follows that the interest rates on the two bonds have to be equal, $\tilde{R} = R$.

In the next section, we study how this policy does not guarantee that the steady-state equilibrium is uniquely implemented.

3 Self-Fulfilling Crises

In the model, the amount that households can borrow is increasing in their labor income. Because labor income is linked in equilibrium to the price of non-tradable goods, this implies that the borrowing capacity itself is linked to the price of non-tradables. In turn, because the price of non-tradables is increasing in the aggregate amount of borrowing, the borrowing capacity of an individual agent is increasing in the aggregate amount of borrowing. As shown formally by Schmitt-Grohé and Uribe (2021) in the context of a real model, when this complementarity is strong enough, there is a possibility of multiple equilibria.[14] That is, for a range of initial debt values, a steady-state equilibrium may coexist with another equilibrium in which households reduce their demand for borrowing, the real exchange rate depreciates, and tradable consumption falls.

Our goal in this section is to characterize how the exchange rate regime determines the vulnerability to self-fulfilling crises and how this affects the choice of the optimal monetary policy.

We assume that the economy starts period 0 with an initial debt position $B_0 < \hat{B}$. As shown in Lemma 1, one possible competitive equilibrium in this case is the steady-state equilibrium in which $B_{t+1} = B_0$ for all t, consumption is constant, and the borrowing constraint is slack. In addition, another equilibrium may exist. We refer to a self-fulfilling crisis equilibrium as a competitive equilibrium featuring deleveraging and lower consumption

[14] See also Mendoza (1995) for an early discussion of this possibility and Krugman (1998) for a related analysis.

in period 0. To facilitate the analysis, we focus on a situation in which allocations are constant after period 1.[15]

Definition 3 (Self-Fulfilling Crisis Equilibrium)**.** A self-fulfilling crisis equilibrium is a competitive equilibrium in which $B_1 < B_0$.

The possibility of multiplicity of equilibria depends on the strength of the complementarity between aggregate borrowing decisions and the individual borrowing limit. As we will show formally below, the following assumption will be sufficient to guarantee this possibility.

Assumption 1. *The set of parameters satisfies*

$$\kappa \frac{1-\phi}{\phi} \left[\frac{y^T - \frac{R-1}{R}\hat{B}}{\bar{h}} \right]^{\frac{1}{\gamma}-1} > 1.$$

We assume that Assumption 1 is satisfied in the rest of the paper. This assumption is consistent with a range of plausible parameter values from the data.[16] Even though the model is stylized, it is worth highlighting that it has been shown to be able to replicate important regularities of emerging market business cycles and financial crises (Mendoza, 2002; Bianchi, 2011; Schmitt-Grohé and Uribe, 2021; Ottonello, 2021).

In the next section, we will study how the exchange rate regime affects the vulnerability to self-fulfilling financial crisis equilibria.

3.1 Flexible Exchange Rate Regime

We start by analyzing a flexible exchange rate regime. We focus on the case in which the central bank sets money supply to $M_t = \bar{M}$ and lets the nominal exchange rate fluctuate freely.[17]

We start by showing that in a self-fulfilling crisis equilibrium, the economy experiences a nominal exchange rate depreciation and unemployment; that is, a financial crisis unfolds together with a currency crisis.

[15]This is without loss of generality, in the absence of uncertainty (see Schmitt-Grohé and Uribe, 2021).

[16]For example, if we take $\phi = 0.2$, in line with a 20% share of tradable-output to GDP, $\kappa = 0.3$, in line with observed debt levels, and an annual interest rate of $R = 1.04$, we obtain multiplicity for values of the elasticity γ between 0.5 and 1.

[17]Given our assumption about the separability between consumption and money balances, we can guarantee that under a constant money supply, the steady state equilibrium features a unique price level (Benhabib, Schmitt-Grohé and Uribe, 2001).

Lemma 2 (Unemployment under flexible exchange rate)**.** *In a self-fulfilling crisis equilibrium, the exchange rate depreciates at $t = 0$, and there is unemployment.*

Proof. In Appendix A.4 □

For a given nominal wage and price of non-tradables, a reduction in borrowing implies a reduction in the demand for non-tradable goods and for domestic currency. Given the equilibrium exchange rate and output, the borrowing capacity becomes

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right) \right], \tag{18}$$

where the equilibrium exchange rate e_0 is such that the demand for money (MD) equals the fixed supply of money $\bar{M}$.

Before we formalize the scope for self-fulfilling crises, we present an illustration in Figure 1. The downward-sloping solid line represents the steady-state borrowing limit of a household $\bar{b}(B, B)$—that is, the individual borrowing limit when aggregate debt is constant over time. We can see that at the point where this line intersects the 45-degree line, we reach the point $\hat{B}$. If the initial debt were to start at that point, the borrowing constraint would hold with equality at steady state. The upward-sloping dashed line represents the individual borrowing limit $\bar{b}(B, B_0)$ for a given initial debt level B_0, detailed in the figure. When this line intersects the downward-sloping line, we reach an equilibrium with a level of borrowing equal to the initial level B_0. This is the good equilibrium represented by point G. To see that this is an equilibrium, notice that the borrowing capacity (represented by the intersection between the downward-sloping line and the upward-sloping dashed line) exceeds the actual level of borrowing (represented by the 45-degree line).

When the upward-sloping dashed line intersects the 45-degree line, we have another equilibrium. This occurs at two points, F and F'. At that intersection, the amount of borrowing coincides with the borrowing limit, consumption falls, and households' borrowing IS constrained. Intuitively, when households panic and collectively reduce borrowing, their doing so leads to a reduction in aggregate demand and a currency depreciation. This implies that in equilibrium, households' borrowing constraints becomes tighter, validating the initial panic.

The proposition below characterizes the range of values for initial debt such that the economy can feature self-fulfilling crisis equilibria.

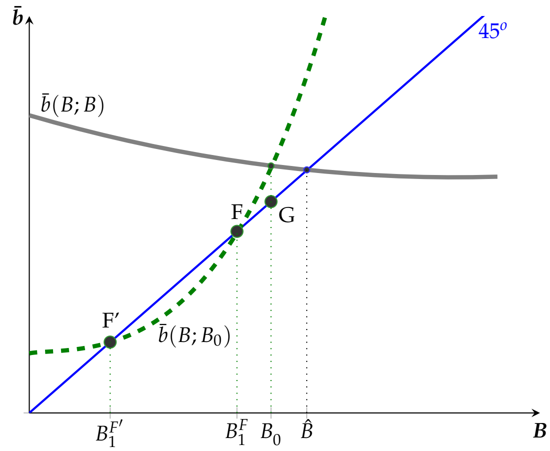

Figure 1: Equilibria under flexible exchange rates

Proposition 1 (Crises under Flexible Exchange Rates)**.** *Suppose Assumption 1 holds and $\gamma < 1$. Under a flexible exchange rate with $\bar{M}$ given by (17),*

i. if $B_0 \in ((1+\kappa)y^T, \hat{B})$, the steady-state equilibrium coexists with a single self-fulfilling crisis equilibrium; moreover, we have that $\hat{B} > (1+\kappa)y^T$, and thus the interval is non-empty;

ii. if $B_0 \in [\underline{B}^m, (1+\kappa)y^T)$, there exist two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, where $\underline{B}^m < (1+\kappa)y^T$ is given by (A.18).

iii. if $B_0 < \underline{B}^m$, we have one and only one equilibrium, which corresponds to the steady-state equilibrium.

Proof. In Appendix A.5 □

When the initial debt level is sufficiently close to $\hat{B}$, we have a single self-fulfilling crisis equilibrium. That is, we have one self-fulfilling crisis equilibrium, in addition to the steady state equilibrium. This occurs when B_0 exceeds the resources available when households are only able to lever on labor income, $(1+\kappa)y^T$.[18] In addition, the proposition shows that there is an interval $[\underline{B}^m, (1+\kappa)y^T)$ for which we have two self-fulfilling crisis equilibria. Finally, when B_0 is sufficiently low, the only equilibrium is the steady-state equilibrium. If households were to panic, an individual household would still be able to

[18] Let us discuss the role of Assumption 1 in the proposition. Assumption 1 says that when evaluated at $\hat{B}$, the derivative of $\bar{b}(B, B_0)$ with respect to B is larger than one (i.e., an increase in aggregate borrowing expands the individual borrowing capacity by more than one unit). By continuity, this implies that the slope of the dashed line evaluated at a B_0 sufficiently close to $\hat{B}$ is larger than one, as illustrated in the figure. Thus, in addition to the equilibrium point G, there exists another equilibrium point F at which the dashed line crosses the 45-degree line, in which case the borrowing constraint becomes binding.

sustain a high enough level of consumption such that the borrowing constraint would not bind, effectively precluding the downward spiral that leads to a crisis.

Our analysis above assumes that under a flexible exchange rate, the central bank sets the money supply. But there are other policies the central bank could follow. For example, the central bank could adjust the money supply to implement full employment or could set the nominal interest rate. We consider these cases in Appendixes B and C, which show similar results as our baseline case.

In our model, a distinct feature of financial crises under flexible exchange rates is that they coincide with currency crises. In the next section, we show that under certain circumstances, a policy of fixed exchange rates can effectively avert the outbreak of a financial crisis.

3.2 Fixed Exchange Rate Regime

We now examine a fixed exchange rate regime in which the central bank sets $e_t = \bar{e}$, where $\bar{e}$ corresponds to the efficient steady-state level given by (16). We first establish that in a self-fulfilling crisis equilibrium, the economy experiences unemployment.

Lemma 3 (Unemployment in Self-Fulfilling Crisis)**.** *In a self-fulfilling crisis, there is involuntary unemployment.*

Proof. In Appendix A.2 □

Under a fixed exchange rate, the downward nominal wage rigidity translates into a downward rigidity on the real wage. When households become unexpectedly pessimistic and increase their savings, the contraction in demand for non-tradables translates one-to-one to a fall in production, causing involuntary unemployment. Given that households work fewer hours than their aggregate endowment of hours, equilibrium in the labor market requires the downward nominal wage rigidity to be binding; that is, $W_0 = W_{-1}$. Notice that the relative price of non-tradables remains fixed at W_{-1}/e_0, and so the borrowing capacity becomes

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right) \right]. \tag{19}$$

Given (19), the upward sloping line in Figure 2 becomes linear. We can now see graphically that the economy displays at most one self-fulfilling financial crisis. Crucially, now the

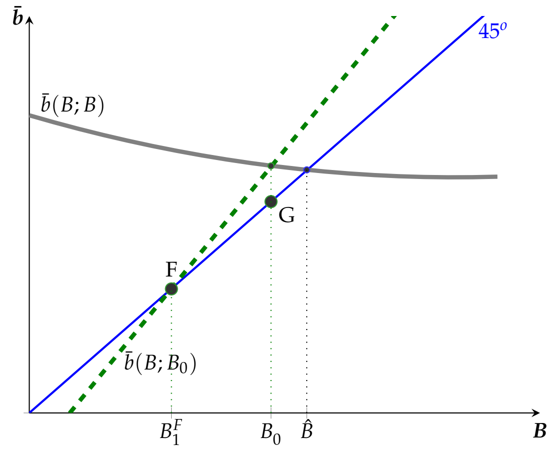

Figure 2: Equilibria under fixed exchange rate

crisis region becomes smaller under a fixed exchange rate. That is, there is a more narrow region of B_0 such that the economy can fall in a financial crisis. The next proposition summarizes these results.

Proposition 2 (Crises under Fixed Exchange Rate)**.** *Suppose Assumption 1 holds and $\gamma < 1$. Under a fixed exchange rate policy with $\bar{e}$ given by* (16)*, we have that*

i. there is a non-empty region of debt levels $B_0 \in ((1+\kappa)y^T, \hat{B})$ for which a single self-fulfilling crisis equilibrium coexists with the steady-state equilibrium;

ii. for $B_0 < (1+\kappa)y^T$, we have a unique equilibrium, and this equilibrium is the steady-state equilibrium.

Proof. In Appendix A.3 □

Given the possibility of multiplicity, it is important to discuss how the central bank is able to uniquely implement the target exchange rate $\bar{e}$. In our model, this is guaranteed by the fact that the central bank has access to lump-sum taxes and transfers. Thus, by accommodating any changes in money demand by injecting or withdrawing currency, it can promise to buy and sell foreign currency at the announced exchange rate and implement the desired level. Notice that to the extent that the fixed exchange rate is credible, no actual foreign exchange intervention is needed to keep the exchange rate at $\bar{e}$.

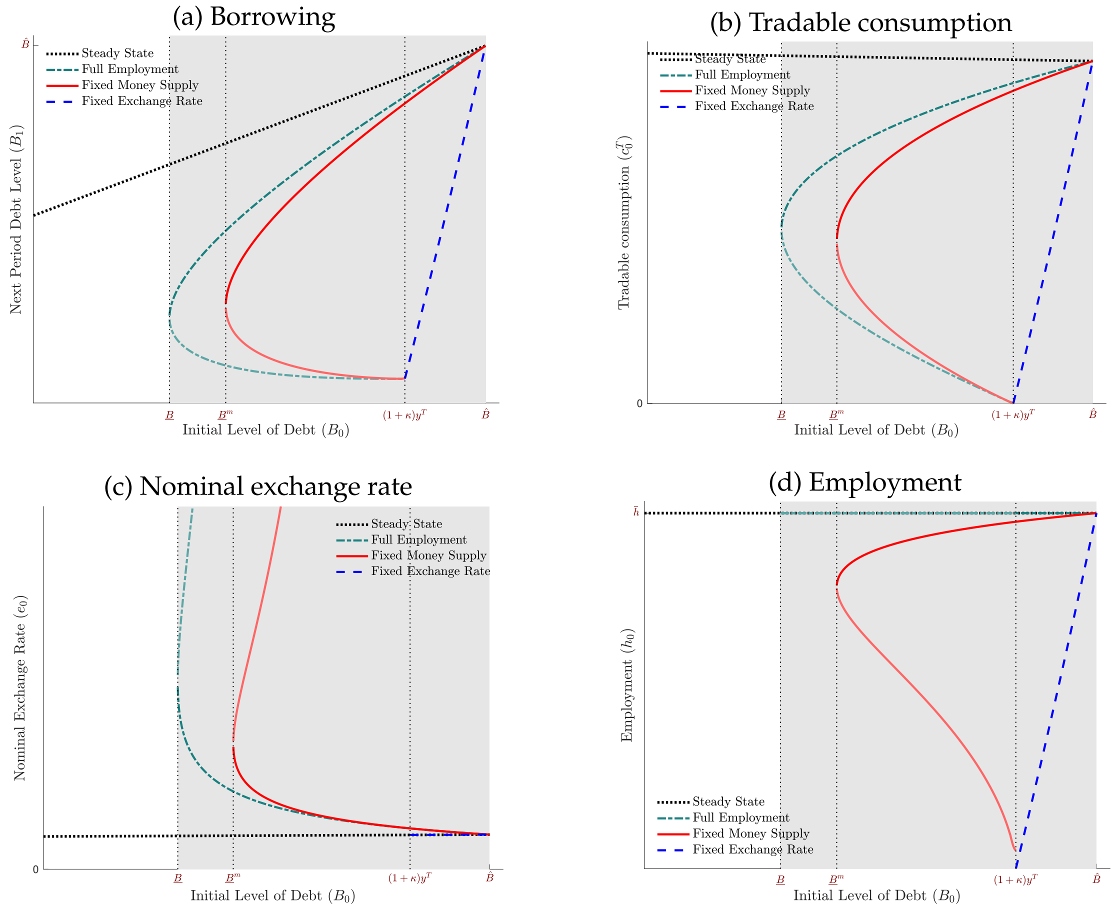

Figure 3: Policy functions

Note: Parameter values are $\phi = 0.2$, $\kappa = 0.3$, $W_{-1} = 1$, $R = 1.04$, $\beta = 1/R$, $\gamma = 0.4$.

3.3 The Costs of Floating

Propositions 1 and 2 make clear that if $B_0 \in [\underline{B}^m, (1+\kappa)y^T)$, a flexible exchange rate regime is vulnerable to a self-fulfilling crisis, whereas a flexible exchange rate regime is not. Thus, it is immediate that if the economy starts from those initial debt levels, a fixed exchange rate dominates a flexible exchange rate in terms of welfare. In this case, letting the nominal exchange rate float leads to perverse movements in prices and output that make the economy vulnerable to a self-fulfilling financial crisis. In this context, rather than serving as a shock absorber, exchange rate fluctuations erode both macroeconomic and financial stability. The central bank therefore suffers from fear of floating.

To shed further light on how the exchange rate regime affects the vulnerability to crises, we numerically solve the model and present the policy functions in Figure 3 under the different regimes for a range of initial values of debt. The dotted line illustrates the steady-state equilibrium, which exists for all debt levels below $\hat{B}$. The blue broken line indicates the self-fulfilling crisis equilibrium under a fixed exchange rate. The red solid line indicates the equilibria under flexible exchange rate (with fixed money supply). For comparison, we also present the case in which the central bank implements full employment, which is represented with the dashed green line.

For sufficiently low debt levels, all regimes feature the steady-state equilibrium. In this region, tradable consumption is decreasing in the debt level, but the economy remains at full employment.[19] For debt levels higher than $(1+\kappa)y^T$ and lower than $\hat{B}$, a self-fulfilling crisis equilibrium emerges for all policy regimes considered.[20] For intermediate debt levels, we have a unique equilibrium under fixed exchange rates, while we have two self-fulfilling crisis equilibria under flexible exchange rates. In this region, we can see that within the two self-fulfilling crisis equilibria under flexible exchange rates, the one with lower borrowing (indicated with the lighter shade) also features lower tradable consumption, lower employment, and a more depreciated exchange rate.

The figure also shows that the nominal exchange can be more appreciated under a full employment policy than under a flexible exchange rate with fixed money supply. This suggests the possibility that a depreciation may be contractionary, a point we examine below.

3.4 Contractionary Depreciations

The higher vulnerability of a floating exchange rate regime is also manifested in the fact that depreciations can be contractionary. To shed further light on why depreciations are contractionary, let us substitute the borrowing constraint with equality in (14) and totally differentiate to obtain

$$\frac{dc_0^N}{de_0} = \frac{c_0^N}{e_0}\left[\gamma + \frac{e_0}{c_0^T}\cdot\frac{1}{R}\frac{d\bar{b}(e_0, B_1; B_0)}{de_0}\right], \tag{20}$$

[19]Notice that even though allocations are the same under the different regimes in this region, exchange rates still differ. However, for reasons of scale,, this is not apparent in the plot.

[20]As the figure shows, in this region, a fixed exchange rate regime experiences more deleveraging than a flexible exchange rate regime, suggesting that floating is preferable in this case. When we turn to optimal policy in Section 4, we will show, however, that a commitment to a sophisticated exchange rate policy would uniquely implement the steady-state equilibrium and dominate a flexible exchange rate regime.

where

$$d\bar{b}_1 = \kappa \frac{W_{-1}}{e_0} \left[dy_0^N - \frac{y_0^N}{e_0} de_0 \right]. \tag{21}$$

Expression (20) spells out two channels by which a nominal exchange depreciation affects demand for non-tradable consumption. First, given a level of resources, a depreciation shifts expenditure toward domestically produced goods. This is the standard expenditure-switching channel that makes depreciations expansionary.

Second, through general equilibrium effects, a depreciation also alters the resources available through a collateral channel, a term characterized in (21). When there is a depreciation, the number of hours that firms demand changes. At the same, given the number of hours (and non-tradable output), depreciation reduces the value measured in units of tradables. Depending on which of these two effects dominates, the collateral channel can be expansionary or contractionary. If it is expansionary (i.e., the number of hours effect dominates the relative price effect), then, the overall effect of a depreciation is to contract output. On the other hand, if the collateral channel is contractionary, the overall effect depends on the strength of this channel relative to that of the expenditure-switching channel. Which effect dominates can be determined by using that in equilibrium $dy_0^N = dc_0^N$ and combining (20) and (21) to solve for dy_0^N and $d\bar{b}$. The proposition uses these relationships to characterize when a depreciation is contractionary.

We summarize these results in the following proposition.

Proposition 3 (Contractionary Depreciations)**.** *Assume a level of debt B_0 such that the borrowing constraint binds. Let $y^N(B_0, e_0)$ be the equilibrium level of output as a function of the initial exchange rate e_0 for an initial debt level B_0. Then, we have that*

i) if $B_0 < (1+\kappa)y^T$ and $\gamma < 1$, $y^N(B_0, e_0)$ is decreasing in e_0 if $e_0 \in [\bar{e}, \underline{e}]$, where $\bar{e}$ is given by (16) and $\underline{e}$ is defined as in (A.22); moreover, it is strictly decreasing if $e_0 \in [\underline{e}\gamma^{\frac{1}{1-\gamma}}, \underline{e}]$;

ii) if $B_0 > (1+\kappa)y^T$ and $\gamma > 1$, $y^N(B_0, e_0)$ is decreasing in e_0 if $e_0 \in [\bar{e}, \underline{e}]$, where $\bar{e}$ is given by (16) and $\underline{e}$ is defined in (A.22); moreover, it is strictly decreasing if $e_0 \in [\underline{e}\gamma^{\frac{1}{1-\gamma}}, \underline{e}]$.

Finally, if (i) and (ii) are not satisfied, a depreciation is expansionary.

Proof. In Appendix A.8 □

Notice that the proposition does not use Assumption 1. That is, while contractionary depreciations are linked to our result on the higher vulnerability under flexible exchange rates, there are also configurations where depreciations are contractionary while the economy displays a unique equilibrium. Given these results, it is useful to connect to the "credit access unemployment tradeoff" analyzed in Ottonello (2021). He considers parameterizations featuring a unique equilibrium and shows quantitative simulations in which the Ramsey optimal policy reduces the volatility of consumption and the real exchange rate, relative to the full-employment allocations. A central theoretical result in his model is that the optimal exchange rate policy does not necessarily implement the full employment allocation. This is because a departure from full employment can be associated with a more appreciated real exchange rate and a more relaxed borrowing limit, *for given tradable consumption*. This can be seen from the fact that the borrowing capacity can be written as $\kappa(y^T + \frac{1-\phi}{\phi}\left(c^T\right)^{\frac{1}{\gamma}} h^{\frac{\gamma-1}{\gamma}})$, which is decreasing in h if $\gamma < 1$ for given c^T.[21] However, the change in employment affects c^T through the borrowing constraint. Our results demonstrate that once we take into account this channel, it is possible that an appreciation is actually expansionary. When an appreciation expands the borrowing capacity, this raises demand for consumption, and this collateral channel can offset the expenditure-switching channel, in line with equations (20) and (21). Therefore, as highlighted in Proposition 3, an appreciation can be expansionary (and a depreciation contractionary). In this case, an appreciation achieves, *at the same time*, an increase in employment and an improvement in credit market access.

4 Optimal Policy: The Role of Commitment

Until now, we have focused on the equilibrium outcomes when the central bank sets an instrument, either the money supply or the exchange rate, at the beginning of time. We now examine optimal policy. Our analysis crucially distinguishes between the case in which the central bank operates under discretion and the one in which it operates under commitment.

4.1 Exchange Rate Policy under Discretion

We start by analyzing the case in which the central bank chooses monetary policy optimally without commitment. As we will see, a key takeaway is that the ability to choose the exchange rate policy under lack of commitment turns out to magnify the vulnerability to

[21]This mechanism is also present in Coulibaly (2023), Basu et al. (2023), and one of the applications in Farhi and Werning (2016).

financial crises.

Because policies can lead to multiple outcomes, analyzing the optimal policy requires being specific about the precise timing of actions. We consider the following timing within the period: (i) households choose b'; (ii) the central bank chooses e; (iii) households choose c^T, c^N and firms choose h.[22]

We solve for the Markov perfect equilibrium (MPE) by backward induction. For any initial value of debt B and any possible B' chosen by households, we can express the problem of the central bank as follows:

$$\max_{c^T,e,h\leq\bar{h},W\geq W_{-1}} u(c^T,h) + \frac{\beta}{1-\beta} u\left[y^T - \frac{R-1}{R}B', \bar{h}\right], \tag{22}$$

$$\text{subject to}$$

$$c^T = y^T - B + \frac{B'}{R}$$

$$h = \left(\frac{1-\phi}{\phi}\frac{e}{W}\right)^{\gamma} c^T$$

$$\frac{B'}{R} \leq \kappa\left[y^T + \left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W}{e}\right)^{1-\gamma} c^T\right],$$

where the continuation value reflects that the economy is in a stationary equilibrium with debt level B'. An inspection of this problem reveals that the central bank must choose a level of employment and associated exchange rate level that induces a feasible level of borrowing for the household. Moreover, the central bank finds it optimal to choose the highest level of employment consistent with a valid continuation equilibrium. Letting $S \equiv (B, W_{-1})$ summarize the aggregate state of the economy at the beginning of the period, we summarize this result in the following proposition.

Proposition 4 (Optimal Policy in a MPE)**.** *For any B', the optimal monetary policy $\mathcal{E}(B';S)$ in a Markov perfect equilibrium implements an employment policy such that*

$$\mathcal{H}\left(B';S\right) = \min\left\{\tilde{h}\left(B';S\right), \bar{h}\right\}, \tag{23}$$

where

$$\tilde{h}\left(B';S\right) = \left[y^T - B + \frac{B'}{R}\right]^{\frac{1}{1-\gamma}} \left[\frac{\kappa(1-\phi)}{\phi(\frac{B'}{R} - \kappa y^T)}\right]^{\frac{\gamma}{1-\gamma}}$$

.

[22] It is equivalent to formulate the problem as the central bank choosing M instead of e.

Proof. In Appendix A.9 □

Given the central bank policy and the aggregate level of debt B', we can formulate the individual household problem as:

$$\max_{c^T,c^N,b'} u(c^T,c^N) + \beta V(b',S'), \tag{24}$$

subject to

$$\mathcal{E}(B';S)c^T + W(B';S)c^N + b = \mathcal{E}(B';S)y^T + \mathcal{W}(B';S)\mathcal{H}(B';S) + \mathcal{E}(B';S)\frac{b'}{R}$$

$$\frac{b'}{R} \leq \kappa\left[y^T + \frac{\mathcal{W}(B';S)}{\mathcal{E}(B';S)}\mathcal{H}(B';S)\right].$$

In a Markov perfect equilibrium as defined below, the conjectured decisions for aggregate debt and exchange rate policy have to be consistent with the actual choices made by households and the central bank.

Definition 4 (Markov Perfect Equilibrium)**.** A Markov perfect equilibrium is defined by central bank policy $\mathcal{E}(B';S)$, policy functions $\mathcal{H}(B';S)$ $\mathcal{C}^T(B';S)$, and $b'(b,B';S)$ such that

1. households' optimization: $b'(b,B';S)$ solves the household's problem (24) given $\mathcal{E}(B';S)$ and $\mathcal{H}(B';S)$;
2. central bank's optimization: $\{\mathcal{E}(B';S),\mathcal{H}(B';S),\mathcal{C}^T(B';S)\}$ solve the central bank's optimal policy problem (22) given B';
3. consistency: the conjectured aggregate debt is consistent with the individual household's optimal borrowing $b'(B,B';S) = B'$.

We showed before that under a fixed money supply, there were at most three equilibria. The next Lemma shows the existence of a continuum of Markov perfect equilibrium; furthermore, the set of equilibria is convex.

Lemma 4 (Convexity)**.** *The set of debt levels B' that constitutes a Markov perfect equilibrium is convex.*

Proof. In Appendix A.10 □

Given two levels of debt that constitute a Markov perfect equilibrium, a convex combination of those debt levels is also a Markov perfect equilibrium. Therefore, determining the lowest and highest debt level suffices to characterize the set of debt levels that constitute a

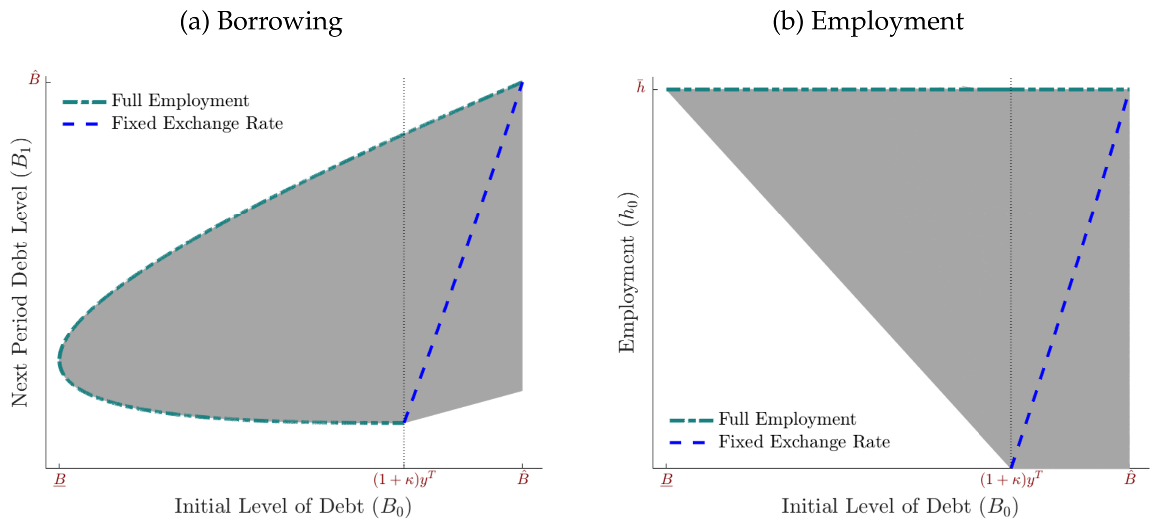

Figure 4: Set of Markov perfect equilibria

Note: Parameter values are $\phi = 0.2$, $\kappa = 0.3$, $W_{-1} = 1$, $R = 1.04$, $\beta = 1/R$, $\gamma = 0.4$.

Markov perfect equilibrium. As it turns out, the bounds of the MPE are determined by the worst and best equilibrium under full employment policy. We illustrate in Figure 4 the set of equilibria and present the formal results in the proposition below.

Proposition 5 (Worst and Best MPE)**.** *The best Markov perfect equilibria corresponds to the best equilibria under full employment policy. If $B_0 < (1+\kappa)y^T$, the worst Markov perfect equilibrium, corresponds to the worst equilibrium under full employment policy.*

Proof. In Appendix A.11 □

4.2 Exchange Rate Policy under Commitment

In the previous section, we found that the ability to choose the exchange rate under the lack of commitment magnifies the vulnerability to self-fulfilling crises. In this section, we show how the ability to commit to monetary policy can help avert self-fulfilling financial crises.

Our approach follows Bassetto (2005) and Atkeson et al. (2010) in that we allow the central bank to commit to a strategy that depends upon the choices of households. We now assume that the central bank announces a commitment to the state contingent exchange rate policy that $e(B_1, B_0)$ before households choose the level of borrowing.[23] Individual households choose their individual level of borrowing b_1 according to their belief about aggregate borrowing B_1, after which the central bank sets M_0 to implement the exchange rate $e(B_1, B_0)$ to which it committed. Finally, households choose their level of consumption, firms choose employment and markets clear.[24]

The next proposition describes the monetary policy strategy that can avert self-fulfilling crises.

Proposition 6 (Unique implementation with sophisticated monetary policy)**.** *There exists an exchange rate rule $e(B_1, B_0)$ that rules out the possibility of self-fulfilling crises equilibria. Given the initial debt-to-tradable-output ratio of the economy, this rule can be described as follows:*

$$e(B_1, B_0) = \begin{cases} \bar{e} & \text{if } B_0 \leq (1+\kappa)y^T \\ \bar{e}\left[\frac{B_1}{B_0} + \left(1 - \frac{B_1}{B_0}\right)\Phi\left(B_1, B_0\right)\right], & \text{otherwise,} \end{cases} \tag{25}$$

where $\bar{e}$ is given by (16) *and*

$$\Phi\left(B_1, B_0\right) \equiv \left[\frac{y^T - (1-\beta)\,B_0}{\bar{h}}\right]^{\frac{1}{\gamma}}\left[\kappa\frac{1-\phi}{\phi}\frac{R(y^T - B_0) + B_1}{B_1 - R\kappa y^T}\right]^{\frac{1}{1-\gamma}}$$

Proof. In Appendix A.12 □

When the economy starts with a relatively low level of debt, $B_0 \leq (1+\kappa)y^T$, an announcement by the central bank to commit to stabilizing the exchange rate at its natural level $\bar{e}$ is sufficient to guarantee the implementation of the steady-state equilibrium. This result is in line with Proposition 2.

When the initial debt exceeds that amount, a non-state-contingent commitment is not enough to uniquely implement the good equilibrium. However, the proposition presents a sophisticated policy that can rule out a self-fulfilling crisis. As shown in (25), the exchange rate turns out to be a combination of the desired exchange rate level and the exchange rate that the government chooses in the Markov perfect equilibrium for a given B_1, with weights that depend on the deviation of the net foreign asset position relative to the efficient one. This policy can therefore be interpreted as a *flotation band*. Notice that this rule implements

[23] Notice that we do not need to specify policies in response to non-degenerate actions by households, because the household optimum is unique and thus government responses to non-degenerate actions are irrelevant for the game (see Bassetto, 2005 for a discussion).

[24] Schmitt-Grohé and Uribe (2016) provide a feedback rule for capital control that can also implement the good equilibrium. Methodologically, we follow more closely Bassetto (2005) and Atkeson et al. (2010).

the first-best allocation $e(B_1, B_0) = \bar{e}$ if aggregate borrowing coincides with the desired level of borrowing $B_1 = B_0$. However, when aggregate borrowing falls below the desirable level, the central bank tolerates exchange rate depreciations but commits to appreciating it below its level under free floating. At the expense of creating involuntary unemployment, the appreciation of the nominal exchange rate relaxes the individual household's borrowing constraint, making $b_1 = B_1$ suboptimal from the individual household's perspective. The policy rule (25) thus ensures the unique implementation of the steady-state equilibrium by making the best response of each household different from the average choice whenever $B_0 < B_1$, and hence discouraging deviations from the desired level of borrowing.

A key takeaway from this section is that the ability to commit to an exchange rate policy is crucial to reducing financial fragility. When the central bank operates without commitment, an active central bank policy can backfire, leading to a wide set of equilibria. When the central bank can commit, it is possible to ensure the unique implementation of the good equilibrium and thus rule out self-fulfilling financial crises.

5 Quantification with Fundamental Shocks

So far, our analysis has not considered fundamental shocks, such as tradable income fluctuations. In scenarios in which fluctuations are driven by these fundamental shocks, existing research, like Schmitt-Grohé and Uribe (2021), has shown that a floating exchange rate is preferable to a fixed exchange rate. We therefore extend our analysis to a stochastic environment to determine whether fixing the exchange rate can outperform floating, or if floating remains the more desirable option, as suggested by previous studies.

We extend the model with shocks to the endowment of tradables y_t^T that follows an AR(1) process and allow nominal wages to fall sluggishly. In particular, the process for y_t^T follows a univariate AR(1) process $\ln y_t^T = 0.53 \ln y_{t-1}^T + \varepsilon_t$, where $\varepsilon_t \sim N(0, 0.058)$ and the downward nominal wage ridigity is such that $W_t \geq \rho_w W_{t-1}$. We set $\rho = 0.96$ in our annual calibration, implying that nominal wages can fall by up to 4% per year.

The rest of the parameters are as follows. The world risk-free interest rate is set at 4%. The discount factor β and the weight on tradable goods in the consumption bundle ϕ are set to match an average net foreign asset position to GDP of -29% and the observed share of tradable output in total output of 26%. This approach leads to $\beta = 0.91$ and $\phi = 0.26$. The collateral coefficient κ is set to 0.32, as in Bianchi (2011). Finally, we set the elasticity of substitution between tradable and nontradable consumption γ to 0.5, as in Schmitt-Grohé and Uribe (2021).

We consider two possible equilibrium selection criteria when the economy is in the self-fulfilling crisis region. In one case, we pick the best equilibrium (that is, the case with the lowest current account reversal). In the other case, we pick the worst equilibrium (i.e., the case with the highest current account reversal). In this context, we compare a fixed exchange rate policy $e = \bar{e}$ (where $\bar{e}$ is normalized to one) to a policy that lets the exchange rate fluctuate to achieve $h_t = \bar{h} = 1$. We focus on the comparison of these two regimes to highlight how the costs/gains from fixing depend on the existence of self-fulfilling crises.

We find that the welfare gains from fixing the exchange rate depend crucially on whether the economy features self-fulfilling financial crises. When we always select the good equilibrium in the area of vulnerability, we find that fixing the exchange delivers an average welfare loss of 0.3% of permanent consumption. That is, starting from the ergodic distribution for a flexible exchange rate, households are willing to give up 0.3% percent of the consumption bundle across all future states to remain in the flexible exchange rate in the absence of self-fulfilling crises. Instead, when we select the bad equilibrium in the area of vulnerability, we find a welfare *gain* of fixing the exchange rate of 1.9% of permanent consumption. This implies that the desirability of fixing the exchange rate remains even in the presence of fundamental shocks driving the economy.

6 Conclusion

We provide a theory of fear of floating, the ubiquitous policy among central banks of preventing large fluctuations in exchange rates. The central mechanism in the paper that gives rise to financial fragility emerges from the interaction between a feedback loop between relative prices and borrowing conditions and the lack of commitment of monetary policy to keeping the exchange rate stable. In our model, an exchange rate depreciation does not play the role of a shock absorber, in contrast to the Mundell-Fleming paradigm. Instead, it makes the economy more vulnerable to a self-fulfilling financial crisis and can be contractionary. While the precise details involve external borrowing and a relative price between tradable goods and non-tradable goods, the key insights can be extended to other frameworks involving domestic borrowing and asset prices. We leave an exploration of these channels for future work.

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APPENDIX TO "A THEORY OF FEAR OF FLOATING"

A Proofs

A.1 Proof of Lemma 1

We start by showing that the steady-state equilibrium exists if $B_0 \leq \hat{B}$. At the steady-state equilibrium $B_{t+1} = B_0$ for all t and by (13) $c^T = y^T - \frac{R-1}{R} B_0$. The equilibrium exists if the collateral constraint is satisfied. That is, if

$$B_0 \leq \bar{b}(B_0; B_0) = \kappa R \left[y_t^T + \frac{1-\phi}{\phi} \left(y_t^T - \frac{R-1}{R} B_0 \right)^{\frac{1}{\gamma}} (h)^{\frac{\gamma-1}{\gamma}} \right].$$

where $h \leq \bar{h}$ is the steady-state level of employment. Because $\bar{b}(\hat{B}; \hat{B}) = \hat{B}$ (by definition of $\hat{B}$) and $\frac{\partial \bar{b}(B_0; B_0)}{\partial B_0} < 0$, it follows that for any $B_0 \leq \hat{B}$ we have $\bar{b}(B_0; B_0) \geq B_0$. Moreover, $\kappa < \frac{R}{R-1}$ ensures that $c^T > 0$.

The second part of the proof requires showing that it is optimal for the government to implement a full-employment allocation. Because allocations are constant at the steady-state equilibrium, from (4) we have

$$(1-\mu) u_T(c^T, h) = u_T(c^T, h) \Rightarrow \mu = 0$$

The borrowing constraint does not bind. The problem of the central bank then reduces to

$$\max_{c^T, h, e, W \geq W_{-1}} \frac{1}{1-\beta} u(c^T, h),$$

subject to

$$c^T = y^T - \frac{R-1}{R} B_0 \tag{A.1}$$

$$h = \left(\frac{1-\phi}{\phi} \frac{e}{W} \right)^{\gamma} c^T \tag{A.2}$$

$$h \leq \bar{h} \tag{A.3}$$

Because e only appears in (A.2), it is immediate that (A.2) does not bind. Since the objective is strictly increasing in h, it must be that (A.3) binds, and thus $h = \bar{h}$.

Finally plugging $h = \bar{h}$ and (A.1) into (A.2) and using use $W \geq W_{-1}$ we get

$$e \geq W_{-1} \frac{\phi}{1-\phi} \left[\frac{y^T - (1-\beta) B_0}{\bar{h}} \right]^{-\frac{1}{\gamma}} \tag{A.4}$$

□

A.2 Proof of Lemma 3

The proof is by contradiction. Suppose that $h_0 = \bar{h}$. From (14) , we have

$$W_0 = \bar{e}\,\frac{1-\phi}{\phi}\left(\frac{y^T - B_0 + \frac{B_1}{R}}{\bar{h}}\right)^{\frac{1}{\gamma}} \tag{A.5}$$

By definition of $\bar{e}$, we also have that wages in a steady state equilibrium satisfies

$$W_{-1} = \bar{e}\,\frac{1-\phi}{\phi}\left(\frac{y^T - \frac{R-1}{R}B_0}{\bar{h}}\right)^{\frac{1}{\gamma}} \tag{A.6}$$

Because $B_1 < B_0$, (A.5) and (A.6) imply that $W_0 < W_{-1}$ which violates downward wage rigidity. Therefore, $h_0 < \bar{h}$ in a self-fulfilling crisis equilibrium. □

A.3 Proof of Proposition 2

The maximum borrowing of an individual household under fixed exchange rates is

$$\bar{b}(B_1; B_0) = \kappa R\left[y^T + \left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{\bar{e}}\right)^{1-\gamma}\left(y^T - B_0 + \frac{B_1}{R}\right)\right]$$

and we have that

$$\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} = \left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{\bar{e}}\right)^{1-\gamma}$$

Notice that B_1 is part of an equilibrium if $\bar{b}(B_1; B_0) = B_1$, $B_1 < B_0$, and $\frac{B_1}{R} > B_0 - y^T$. The first condition is such that the constraint holds with equality. The second condition ensures that $\mu > 0$ and the last condition ensures that $c_0^T > 0$. Because $\bar{b}(B_0; B_0) > B_0$, that is the borrowing constraint does not bind in the stationary equilibrium, a sufficient condition for non-existence B_1 that satisfies the first two conditions is $\partial\bar{b}(B_1; B_0)/\partial B_1 < 1$.

Using (16) to substitute for $W_{-1}/\bar{e}$ leads to

$$\frac{\partial \bar{b}(B_1; B_0)}{\partial B_1} = \kappa\frac{1-\phi}{\phi}\left(\frac{y^T - \frac{R-1}{R}B_0}{\bar{h}}\right)^{\frac{1-\gamma}{\gamma}} > \kappa\frac{1-\phi}{\phi}\left(\frac{y^T - \frac{R-1}{R}\hat{B}}{\bar{h}}\right)^{\frac{1-\gamma}{\gamma}} > 1$$

where the first inequality uses $B_0 < \hat{B}$ and the last inequality uses Assumption 1. Given that $\bar{b}(B_0; B_0) > B_0$ and $\frac{\partial \bar{b}(B_1;B_0)}{\partial B_1} > 1$, it follows by continuity of the function $\bar{b}(B_1; B_0) - B_1$ that there exists $B_1 < B_0$ such that $\bar{b}(B_1; B_0) - B_1 = 0$. Next, we need to check condition under which $c_0^T > 0$ in the self-fulfilling crises equilibrium. Using $\bar{b}(B_1; B_0) = B_1$ and the resource constraint (13) we get

$$c_0^T = \frac{B_0 - (1+\kappa)y^T}{\kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{W_{-1}}{\bar{e}}\right)^{1-\gamma} - 1}$$

Hence, $c_0^T > 0$ if and only if $B_0 > (1+\kappa)y^T$. Moreover, because $B_0 < \hat{B}$ it follows that a self-fulfilling crisis equilibrium coexists with the stationary equilibrium under $e_0 = \bar{e}$ for any $B_0 \in ((1+\kappa)y^T, \hat{B})$. It remains to show that $((1+\kappa)y^T, \hat{B})$ is non-empty. Recall that $\bar{B}(\hat{B}, \hat{B}) = 0$, that is

$$\begin{aligned}
\frac{\hat{B}}{R} &= \kappa y^T + \kappa \frac{1-\phi}{\phi} \left(y^T - \frac{R-1}{R}\hat{B} \right)^{\frac{1}{\gamma}} \left(\bar{h}\right)^{\frac{\gamma-1}{\gamma}} \\
&= \kappa y^T + \hat{c}^T \kappa \frac{1-\phi}{\phi} \left[\frac{y^T - (1-\beta)\hat{B}}{\bar{h}} \right]^{\frac{1-\gamma}{\gamma}} \qquad \text{(A.7)}
\end{aligned}$$

Using the resource constraint $\hat{c}^T = y^T - \hat{B} + \frac{\hat{B}}{R}$ and substituting (A.7), we get

$$\left[1 - \kappa \frac{1-\phi}{\phi} \left(\frac{y^T - (1-\beta)\hat{B}}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} \right] \hat{c}^T = (1+\kappa)y^T - \hat{B} \qquad \text{(A.8)}$$

Assumption 1 implies that the left-hand side of equation (A.8) is negative. Therefore, $(1+\kappa)y^T < \hat{B}$. The interval $((1+\kappa)y^T, \hat{B})$ is thus non-empty. $\square$

A.4 Proof of Lemma 2

Unemployment. Assume by contradiction that $h_0 = \bar{h}$. The demand for money in the steady state equilibrium and in period 0 are given by

$$\frac{\chi W_{-1}}{\bar{M}} = \left[1 - \frac{1}{R}\right] u_N \left(y^T - \frac{R-1}{R} B_0, \bar{h} \right) \qquad \text{(A.9a)}$$

$$\frac{\chi W_0}{\bar{M}} = \left[1 - \frac{1}{\tilde{R}_0}\right] u_N \left(y^T - B_0 + \frac{B_1}{R}, \bar{h} \right) \qquad \text{(A.9b)}$$

it must be that $\tilde{R}_0 \geq R$. This is because if it were that $\tilde{R}_0 < R$ then by (A.9a) and (A.9b) we have that $W_0 < W_{-1}$ which violates the constraint on the nominal wage. Given that

$\tilde{R}_0 \geq R$, from the (7) condition $e_0 \leq \frac{e_1}{1-\mu_0}$. Then, using (14) we get

$$h_0 = \left[\frac{1-\phi}{\phi}\frac{e_0}{W_0}\right]^\gamma c_0^T \leq \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\frac{1}{1-\mu_0}\right]^\gamma c_0^T$$

Using (4) to substitute for μ_0 we arrive to

$$h_0 \leq \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^\gamma c_1^T \left(\frac{c_0}{c_1}\right)^{1-\gamma} < \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^\gamma c_1^T = \bar{h} \tag{A.10}$$

where the second inequality uses $c_0 < c_1$ by $c_0^T < c_1^T$. (A.10) contradicts $h = \bar{h}$. Therefore, $h_0 < \bar{h}$ in a self-fulfilling crises equilibrium when it exists.

Exchange depreciation. Using (7) and (4) to substitute for $\tilde{R}_0$ and μ_0, (MD) becomes

$$\frac{\chi e_0}{\bar{M}} = \left[1 - \frac{e_0}{Re_1}\frac{u_T(c_1^T, \bar{h})}{u_T\left(c_0^T, c_0^N\right)}\right] u_T(c_0^T, c_0^N) \tag{A.11}$$

Using $\frac{e_1}{W_0} = \frac{u_T(c_1^T,\bar{h})}{u_N(c_1^T,\bar{h})}$ and plugging it into (A.11) we get

$$\frac{\chi}{\bar{M}} = \left[\frac{1}{e_0}u_T\left(c_0^T, \left(\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right)^\gamma c_0^T\right) - \frac{1}{RW_{-1}}u_N(c_1^T, \bar{h})\right] \tag{A.12}$$

Note that $W_0 = W_{-1}$ because $h_0 < \bar{h}$. We use $c_0^T = y^T - B_0 + \frac{B_1}{R}$ and $c_1^T = y^T + (1-\beta)B_1$ and totally differentiate (A.12) with respect to B_1 to obtain

$$[\gamma + (1-\gamma)\tilde{\phi}_0]\frac{Rc_0^T}{e_0}\frac{de_0}{dB_1} = -\left[1 + \tilde{\phi}_0\frac{(1-\gamma)(R-1)}{\gamma(1-\mu_0)\tilde{R}_0}\right] < 0 \tag{A.13}$$

where $\tilde{\phi}_0 \equiv e_0 c_0^T / (e_0 c_0^T + W_0 c_0^N) \in (0,1)$. It follows that in a self-fulfilling crisis equilibrium ($B_1 < B_0$), the exchange rate depreciates. $\square$

A.5 Proof of Proposition 1

Under flexible exchange rates with fixed money supply, the maximum borrowing of an individual household is given by

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi}\right)^\gamma \left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\left(y^T - B_0 + \frac{B_1}{R}\right)\right]$$

where e_0 is determined by the implicit function (A.12). Letting $\xi_{B_1} \equiv \frac{Rc_0^T}{e_0}\frac{de_0}{dB_1}$ denote the elasticity of the nominal exchange rate with respect to B_1 (A.13), we have

$$\frac{\partial \bar{b}(B_1;B_0)}{dB_1} = \kappa\frac{1-\phi}{\phi}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}[1-(1-\gamma)\xi_{B_1}]$$
$$\frac{\partial^2 \bar{b}(B_1;B_0)}{dB_1^2} = (1-\gamma)\left[-\frac{\partial \bar{b}(B_1;B_0)}{dB_1}\xi_{B_1} - \kappa\frac{1-\phi}{\phi}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\frac{d\xi_{B_1}}{dB_1}\right]$$

Owing to $\xi_{B_1} < 0$, we have that $\frac{\partial \bar{b}(B_1;B_0)}{dB_1} > 0$. Differentiating (A.13), we obtain after some algebraic manipulation,

$$\frac{d\xi_{B_1}}{dB_1} = -\frac{(1-\gamma)^2(1-\tilde{\phi}_0)^2}{1-(1-\gamma)(1-\tilde{\phi}_0)}\frac{1}{Rc_0^T}(\xi_{B_1})^2$$
$$-(1-\gamma)\frac{(1-\beta)\tilde{\phi}_1}{\gamma\tilde{R}_0c_1^T}\left[\frac{2+(R-1)\,c_0^T/c_1^T}{\gamma+(1-\gamma)\tilde{\phi}_0} - \xi_{B_1}\right] < 0 \tag{A.14}$$

It follows from (A.14) and $\xi_{B_1} < 0$ that $\frac{\partial^2 \bar{b}(B_1;B_0)}{dB_1^2} > 0$.

Note again that B_1 is part of a self-fulfilling crises equilibrium if the following conditions are satisfied $\bar{b}(B_1;B_0) = B_1$, $B_1 < B_0$, and $\frac{B_1}{R} > B_0 - y^T$. Because $\bar{b}(B_1;B_0)$ is an increasing and convex function in B_1 with $\bar{b}(B_0;B_0) > B_0$, the equation $\bar{b}(B_1;B_0) = B_1$ has at most two solutions with one solution featuring $\frac{\partial \bar{b}(B_1;\tilde{B}_0)}{dB_1} \geq 1$. Moreover, owing to

$$\frac{\partial \bar{b}(B_1;B_0)}{dB_0} = \kappa R\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\left[-1-(1-\gamma)\frac{c_0^T}{e_0}\frac{\partial e_0}{\partial B_0}\right]$$
$$= \kappa R\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\left[-1-\frac{1-\gamma}{\gamma+(1-\gamma)\tilde{\phi}_0}\right] < 0$$

at the minimum level of initial debt level, $\underline{B}^m$, for which a crises equilibrium exists, we have $\frac{\partial \bar{b}(B_1;\underline{B}^m)}{dB_1} = 1$. To simplify the algebra, let define $\psi_0 \equiv 1-(1-\gamma)\xi_{B_1} > 1$. We have

$$\frac{\partial \bar{b}(B_1;\underline{B}^m)}{dB_1} = 1 \Leftrightarrow \psi_0\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma} = 1 \tag{A.15}$$
$$\Leftrightarrow \psi_0\kappa\frac{1-\phi}{\phi}\left(\frac{c_0^T}{h_0}\right)^{\frac{1-\gamma}{\gamma}} = 1. \tag{A.16}$$

Using $\underline{B}^m = y^T + \frac{B_1}{R} - c_0^T$ and plugging in $B_1 = \bar{b}(B_1; \underline{B}^m)$, we obtain by (A.15),

$$\underline{B}^m = (1+\kappa)y^T + \left(\frac{1}{\psi_0} - 1\right) c_0^T \tag{A.17}$$

Using (A.16), one can solve for c_0^T and obtain (recall that $\psi_0 > 1$)

$$\underline{B}^m = (1+\kappa)y^T - \frac{\psi_0 - 1}{\psi_0} \left[\psi_0 \kappa \frac{1-\phi}{\phi}\right]^{\frac{-\gamma}{1-\gamma}} h_0 \tag{A.18}$$

Thus, at least one self-fulfilling crises equilibrium coexists with the steady state equilibrium for $B_0 \in (\underline{B}^m, \hat{B})$ where $\underline{B}^m$ is given by (A.18) and we use $B_0 < \hat{B}$ by Lemma 1. It can also be shown that $\psi_0 > \frac{1}{\gamma}$. Using $\frac{1}{\psi_0} < \gamma$ and $h_0 < \bar{h}$, it follows from (A.18) that

$$\underline{B}^m > (1+\kappa)y^T - (1-\gamma) \left[\frac{1}{\gamma} \kappa \frac{1-\phi}{\phi}\right]^{\frac{-\gamma}{1-\gamma}} \bar{h} \tag{A.19}$$

Moreover, since $\bar{b}(B_1; B_0)$ is convex in its first argument, the equation $\bar{b}(B_1; B_0) = B_1$ has two solutions if and only if $\bar{b}(B_1; B_0) = B_1$ has a solution and at $\tilde{B}_1$ such that $\frac{\partial \bar{b}(B_1;B_0)}{\partial B_1}|_{\tilde{B}_1} = 0$ we have $\bar{b}(\tilde{B}_1; B_0) > B_0$. Because $\partial \bar{b}(B_1; B_0)/\partial B_1 = 0$ implies that $c_0^T = 0$, it follows that $\tilde{B}_1$ lowest value in the feasible domain of B_1, i.e. $\tilde{B}_1 = R(B_0 - y^T)$, and we have

$$\bar{b}(R(B_0 - y^T); B_0) = (1+\kappa)\kappa y^T > B_0 \;\Leftrightarrow\; B_0 < (1+\kappa)\kappa y^T$$

Therefore, $\bar{b}(B_1; B_0) = B_1$ has two solutions for $B_0 \in \left[\underline{B}^m, (1+\kappa)y^T\right)$ and a unique solution for $B_0 \in \left[(1+\kappa)y^T, \hat{B}\right)$. Furthermore, as shown in Appendix A.3, the interval $[(1+\kappa)y^T, \hat{B})$ is non-empty under Assumption 1. $\square$

A.6 Proof of Proposition B.1

Under full employment policy, the maximum borrowing of an individual household is

$$\bar{b}(B_1; B_0) = \kappa R \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B_0 + \frac{B_1}{R}\right)^{\frac{1}{\gamma}} (\bar{h})^{\frac{\gamma-1}{\gamma}}\right]$$

Notice again that B_1 is part of a self-fulfilling crises equilibrium if the following conditions are satisfied: $\bar{b}(B_1; B_0) \geq 0$, $B_1 \leq B_0$, and $\frac{B_1}{R} > B_0 - y^T$. Because $\bar{b}(B_1; B_0)$ is an increasing and convex function in B_1 with $\bar{b}(B_0; B_0) > B_0$, the equation $\bar{b}(B_1; B_0) = B_1$ has at most two solutions with one solution featuring $\frac{\partial \bar{b}(B_1;\tilde{B}_0)}{dB_1} \geq 1$. Moreover, owing to

$$\frac{\partial \bar{b}(B_1; B_0)}{dB_0} = -\kappa R \frac{1-\phi}{\phi\gamma} \left(y^T - B_0 + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} < 0$$

at the minimum level of initial debt level, $\underline{B}$, for which a crises equilibrium exists, we have

$$\frac{\partial \bar{b}(B_1; \underline{B})}{\partial B_1} = 1 \Leftrightarrow \frac{\kappa}{\gamma} \frac{1-\phi}{\phi} \left(\frac{c_0^T}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} = 1 \tag{A.20}$$

Using $\underline{B} = y^T + \frac{B_1}{R} - c_0^T$ and plugging in $\bar{b}(B_1; \underline{B}) = B_1$ to substitute for B_1 yields

$$\underline{B} = (1+\kappa) y^T - (1-\gamma) \left[\frac{\kappa(1-\phi)}{\gamma\phi} \right]^{\frac{\gamma}{\gamma-1}} \bar{h} \tag{A.21}$$

Thus, at least one self-fulfilling crises equilibrium coexists with the steady state equilibrium for $B_0 \in (\underline{B}, \hat{B})$ where $\underline{B}$ is given by (A.21) and we use $B_0 < \hat{B}$ by Lemma 1.

Moreover, since $\bar{b}(B_1; B_0)$ is convex in its first argument, the equation $\bar{b}(B_1; B_0) = B_1$ has two solutions if and only if $\bar{b}(B_1; B_0) = B_1$ has a solution and at $\tilde{B}_1$ where $\frac{\partial \bar{b}(B_1;B_0)}{\partial B_1}|_{\tilde{B}_1} = 0$ we have $\bar{b}(\tilde{B}_1; B_0) > B_0$. Because $\partial \bar{b}(B_1; B_0)/\partial B_1 = 0$ implies that $c_0^T = 0$, it follows that $\tilde{B}_1$ lowest value in the feasible domain of B_1, i.e. $\tilde{B}_1 = R(B_0 - y^T)$, and we have

$$\bar{b}(R(B_0 - y^T); B_0) = (1+\kappa)\kappa y^T - B_0 > 0 \;\Leftrightarrow\; B_0 < (1+\kappa)\kappa y^T$$

Therefore, $\bar{b}(B_1; B_0) = 0$ has two solutions for $B_0 \in \left[\underline{B}, (1+\kappa)y^T\right)$ and a unique solution for $B_0 \in \left[(1+\kappa)y^T, \hat{B}\right)$. Furthermore, as shown above the interval $[(1+\kappa)y^T, \hat{B})$ is non-empty under Assumption 1. By (A.19), we have $\underline{B} < \underline{B}^m$. □

A.7 Proof of Corollary ??

Let W_{ss} and W_{crisis} be the welfare in the steady state equilibrium and in the self-fulfilling crisis equilibrium, respectively. We have $W_{ss} > W_{crisis}$. Suppose now that $B_0 < (1+\kappa)y^T$ and there is non-zero probability $\pi > 0$ that the economy ends in a self-fulfilling crisis equilibrium when the economy is in the vulnerable region. By Proposition 2, under fixed exchange rate there is a unique equilibrium which implies that $W_{fix} = W_{ss}$. By Proposition 1 and B.1, there are two self-fulfilling crisis that coexists with the steady state equilibrium under flexible exchange rate. Thus, $W_{flex} < W_{ss} = W_{fix}$. □

A.8 Proof of Proposition 3

Let us define

$$\underline{e} = W_{-1}\left[\frac{\kappa}{\gamma}\left(\frac{1-\phi}{\phi}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}} \tag{A.22}$$

Combining (14) with market clearing for nontradables, $c_0^N = y_0^N$, we have

$$y_0^N = \left(\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right)^{\gamma}\left[y^T - B_0 + \frac{B_1}{R}\right] \tag{A.23}$$

We also have that if the borrowing constraint holds with equality B_1 is given by (2)

$$B_1 = \kappa y^T + \kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\frac{B_0-(1+\kappa)\,y^T}{\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}-1} \tag{A.24}$$

Substituting b_1 in (A.23) and deriving

$$y_0^N = \left(\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right)^{\gamma}\frac{B_0-(1+\kappa)\,y^T}{\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}-1} \tag{A.25}$$

We then differentiate (A.25) with respect to e_0 to obtain

$$\frac{dy_0^N}{de_0} = \frac{\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}-\gamma}{\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}-1}\cdot\frac{y_0^N}{e_0}. \tag{A.26}$$

Let us denote by $y_0^N(\bar{e})$ the level of output for $e_0 = \bar{e}$. We have

$$y_0^N(\bar{e}) = \left(\frac{1-\phi}{\phi}\frac{\bar{e}}{W_{-1}}\right)^{\gamma}c_0^T < \left(\frac{1-\phi}{\phi}\frac{\bar{e}}{W_{-1}}\right)^{\gamma}\left[y_0^T - \frac{R-1}{R}B_0\right] = \bar{h}$$

Case (i). Consider the case of $\gamma < 1$ and $B_0 < (1+\kappa)y^T$. From (A.25), it follows that $y_0^N > 0$ if and only if

$$\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} < 1 \tag{A.27}$$

Moreover, for any $e_0 \in (\bar{e}, \underline{e})$ we have

$$\kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} > \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{\underline{e}}{W_{-1}}\right)^{\gamma-1} = \gamma \tag{A.28}$$

Using (A.27) and (A.28), it follows from (A.26) that $\frac{dy_0^N}{de_0} < 0$. Moreover, notice by (A.27) that y_0^N is well defined, i.e. $y_0^N > 0$ iff $e_0 > \underline{e}\gamma^{\frac{1}{1-\gamma}}$. □

Case (ii) Consider now that $\gamma > 1$ and $B_0 > (1+\kappa)y^T$. It is then straightforward to see that y_0^N is well defined, that is $y_0^N > 0$, if and only if

$$\kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} > 1 \tag{A.29}$$

Moreover, for any $e_0 \in (\bar{e}, \underline{e})$ we have

$$\kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} < \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{\underline{e}}{W_{-1}}\right)^{\gamma-1} = \gamma \tag{A.30}$$

Using (A.29) and (A.30), it follows from (A.26) that $\frac{dy_0^N}{de_0} < 0$. Moreover, notice by (A.29) that y_0^N is well defined, i.e. $y_0^N > 0$ iff $e_0 > \underline{e}\gamma^{\frac{1}{1-\gamma}}$.

Finally, for the expansionary case, notice that if $\gamma \leq 1$ and $B_0 > (1+\kappa)y^T$ or if $\gamma \geq 1$ and $B_0 < (1+\kappa)y^T$ then we have by (A.25) and (A.26) that $\frac{dy_0^N}{de_0} > 0$.

A.9 Proof of Proposition 4

For any B and any possible B' chosen by households, the central bank solves

$$\max_{c^T, c^N, e, W \geq W_{-1}} u(c^T, h) + \frac{\beta}{1-\beta} u\left[y^T - \frac{R-1}{R}B', \bar{h}\right],$$

subject to

$$c^T = y^T - B + \frac{B'}{R} \tag{A.31}$$

$$h = \left(\frac{1-\phi}{\phi}\frac{e}{W}\right)^{\gamma} c^T \tag{A.32}$$

$$\frac{B'}{R} \leq \kappa \left[y^T + \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{W}{e}\right)^{1-\gamma} c^T\right] \tag{A.33}$$

$$h \leq \bar{h} \tag{A.34}$$

Notice that if the borrowing constraint (A.33) is not binding, then the optimal monetary policy implies $h = \bar{h}$. To prove this, assume by contradiction that the borrowing constraint is not binding. Because in this case e only appears in (A.32), it is immediate that (A.32) does not bind. Letting $\eta \geq 0$ be the Lagrange multiplier on (A.34), the optimality condition for h requires $\eta = u_N(c^T, h) > 0$ which implies that $h = \bar{h}$. Assuming that the borrowing constraint (A.33) binds, it is possible to combine (A.32) and (A.34), and use (A.31) to get

$$h = \left[y^T - B + \frac{B'}{R}\right]^{\frac{1}{1-\gamma}} \left[\frac{\kappa(1-\phi)}{\phi(\frac{B'}{R} - \kappa y^T)}\right]^{\frac{\gamma}{1-\gamma}} \equiv \tilde{h}\left(B'; S\right)$$

If $\tilde{h}\left(B'; S\right) \geq \bar{h}$, then (A.34) binds and $h = \bar{h}$. Otherwise $h = \tilde{h}\left(B'; S\right)$. The employment policy function $\mathcal{H}\left(B'; S\right)$ is therefore given by

$$\mathcal{H}\left(B'; S\right) = \min\{\tilde{h}\left(B'; S\right), \bar{h}\} \tag{A.35}$$

□

A.10 Proof of Lemma 4

$\tilde{B}_1$ is part of a Markov perfect equilibrium if $\tilde{B}_1$ satisfies

$$\tilde{B}_1 = \kappa \left[y^T + \frac{1-\phi}{\phi}\left(y^T - B_0 + \frac{\tilde{B}_1}{R}\right)^{\frac{1}{\gamma}} h(\tilde{B}_1)^{1-\frac{1}{\gamma}}\right] \tag{A.36}$$

where $h(\tilde{B}_1)$ is the solution to

$$\begin{aligned} &\max_{h \leq \bar{h}} h \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A.37) \\ &\text{s.t. } \tilde{B}_1 \leq \kappa \left[y^T + \frac{1-\phi}{\phi}\left(y^T - B_0 + \frac{\tilde{B}_1}{R}\right)^{\frac{1}{\gamma}} h^{1-\frac{1}{\gamma}}\right] \end{aligned}$$

Given $\gamma < 1$, the constraint set is decreasing in h, this means that the constraint is binding and the borrowing constraint holds with equality provided that $h < \bar{h}$.

Let B_1^i and B_1^j be part of a Markov perfect equilibrium (MPE) and $h(B_1^i), h(B_1^j)$ the associated levels of employment. Assume without loss of generality that $B_1^i > B_1^j$ By continuity of the right-hand side of (A.36), we have that for any $B' \in (B_1^j, B_1^i)$, there exists a level of employment such that (A.36) holds with equality. This proves that the set of MPE is convex. □

A.11 Proof of Proposition 5

Consider that the economy starts with $B_0 < (1+\kappa)y^T$, then Proposition B.1 establishes that there exists two self-fulfilling crises equilibria that coexist with the steady state equilibrium.

Lower bound. Let B_1^{FE} be the smallest of the two level of borrowing associated with a full employment policy characterized in Proposition B.1 (point F' in Figure 1). As shown in Appendix A.6, because $\bar{b}(B_1, B_0)$ is convex in B_1 under full employment,

$$\frac{\partial \bar{b}(B_1, B_0)}{\partial B_1}\bigg|_{B_1^{FE}} < 1 \tag{A.38}$$

Suppose now there is a Markov perfect equilibrium with $h < h_0$ and $B_1 = B_1^{FE} - \varepsilon < B_1^{FE}$ where $\varepsilon > 0$ is arbitrarily small. B_1 is a Markov perfect equilibrium implies

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1^{FE}}{R} \right)^{\frac{1}{\gamma}} (h_0)^{1-\frac{1}{\gamma}} \right] - \frac{B_1^{FE}}{R} = 0 \tag{A.39}$$

with $h_0 < \bar{h}$. However, by (A.38) we have that

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{1-\frac{1}{\gamma}} \right] - \frac{B_1}{R} > 0 \tag{A.40}$$

From (A.40) it is straightforward to see that (A.39) holds if and only if for $h_0 > \bar{h}$. Thus, we reach a contradiction. Because the set of MPE is convex and $B_1^{FE} - \varepsilon$ with $\varepsilon > 0$ is not a MPE, any $B_1 < B_1^{FE}$ is not part of a MPE. Therefore B_1^{FE} is the lowest B' in the set of MPE.

Upper bound. Let B_1^{HE} be the largest of the two level of borrowing associated with a full employment policy characterized in Proposition B.1 (point F in Figure 1). As shown in Appendix A.6, because $\bar{b}(B_1, B_0)$ is convex in B_1 under full employment,

$$\frac{\partial \bar{b}(B_1, B_0)}{\partial B_1}\bigg|_{B_1^{HE}} > 1 \tag{A.41}$$

Suppose now there is an Markov perfect equilibrium with $h_0 < \bar{h}$ and $B_1 = B_1^{HE} + \varepsilon > B_1^{HE}$ where $\varepsilon > 0$ is arbitrarily small. B_1 is a Markov perfect equilibrium implies

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1^{HE}}{R} \right)^{\frac{1}{\gamma}} (h_0)^{1-\frac{1}{\gamma}} \right] - \frac{B_1}{R} = 0 \tag{A.42}$$

with $h_0 < \bar{h}$. However, by (A.41) we have that

$$\kappa \left[y^T + \frac{1-\phi}{\phi} \left(y^T - B + \frac{B_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{1-\frac{1}{\gamma}} \right] - \frac{B_1}{R} > 0 \tag{A.43}$$

From (A.43) it is straightforward to see that (A.42) holds if and only if for $h_0 \geq \bar{h}$. Thus, we reach a contradiction. Because the set of MPE is convex and $B_1^{HE} + \varepsilon$ with $\varepsilon > 0$ is not a MPE, any $B_1 > B_1^{HE}$ is not part of a MPE. Therefore B_1^{HE} is the lowest B' in the set of MPE.

A.12 Proof of Proposition 6

$e(B_1, B_0)$ rules out the possibility of self-fulfilling crisis when $B_0 < (1+\kappa)y^T$ follows from Proposition 2. For $B_0 \geq (1+\kappa)y^T$, we start by rewriting the policy rule. We have

$$e(B_1, B_0) = \bar{e} \left[\frac{B_1}{B_0} + \left(1 - \frac{B_1}{B_0} \right) \underbrace{\Phi\left(B_1, B_0\right)}_{\text{depreciation in a MPE}} \right], \tag{A.44}$$

where $\bar{e}\,\Phi\left(B_1, B_0\right)$ is the exchange rate level in the Markov perfect equilibrium with

$$\Phi\left(B_1, B_0\right) \equiv \left[\frac{y^T - (1-\beta)\, B_0}{\bar{h}} \right]^{\frac{1}{\gamma}} \left[\kappa \frac{1-\phi}{\phi} \frac{R(y^T - B_0) + B_1}{B_1 - R\kappa y^T} \right]^{\frac{1}{1-\gamma}}$$

We want to show that under the policy rule (A.44), if a generic household i believes that all other households will choose B_1 that household will find it optimal to choose a different action, that is $b_1 \neq B_1$.

Assume that $B_1 < B_0$ and the household i chooses $b_1 = B_1$. Then, the household's Euler equation for foreign bonds (4) requires $\mu_0 > 0$. Moreover, notice that for $e_0^{MPE} = \bar{e}\Phi\left(B_1, B_0\right)$ that the borrowing constraint is satisfied with equality. Because the government commits to appreciating the exchange rate above this level, $\bar{e} < e(B_1, B_0) < e_0^{MPE}$, the borrowing constraint is relaxed

$$b_1 < \bar{b}(B_1, B_0) = \kappa R \left[y^T + \frac{W_{-1}}{e_0(B_1, B_0)} h_0 \right] \tag{A.45}$$

The complementary slackness condition is not satisfied $\mu_0(b_1 - \bar{b}(B_1, B_0)) > 0$. It is thus not optimal for the household i to choose $b_1 = B_1$. This proves that the set of MPE is convex. $\square$

B Full-employment policy

We now consider a regime in which the central bank adjusts the money supply to implement full employment. We first show that this is indeed feasible for the central bank.

Lemma B.1. *Given a competitive equilibrium with flexible wages, there exists a nominal exchange rate policy under sticky wages that implements the flexible wage allocation.*

Echoing the results of Proposition 1 , we have that:

Proposition B.1 (Crises Under Full Employment Policy)**.** *Suppose Assumption 1 holds and* $\gamma < 1$*. Then, under flexible wages,*

i. if $B_0 \in ((1+\kappa)y^T, \hat{B})$*, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium. Moreover, we have that* $\hat{B} > (1+\kappa)y^T$*, and thus the interval is non-empty;*

ii. if $b_0 \in [\underline{B}, (1+\kappa)y^T)$*, there exist two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, where* $\underline{B}$ *is given by*

$$\underline{B} \equiv (1+\kappa)y^T - (1-\gamma)\left[\frac{1}{\gamma}\frac{\kappa(1-\phi)}{\phi}\right]^{\frac{\gamma}{\gamma-1}} \bar{h} < \underline{B}^m;$$

iii. if $B_0 < \underline{B}$*, we have one and only one equilibrium (which corresponds to the steady-state equilibrium).*

Proof. In Appendix A.6 □

By Proposition 1 and B.1, we can establish that the crisis region under full employment is larger than under fixed money supply (and therefore also larger than under fixed exchange rate).

C Interest Rate Policy

We consider in this section a flexible exchange rate regime where the central bank controls nominal rates. We consider two cases, an interest rate peg and a form of Taylor rule. We will establish that just like in the case of a fixed money supply, we have two self-fulfilling crises equilibria and the crisis region expands relative to the fixed exchange rate regime.

C.1 Crises Region under Interest Rate Peg

Let us focus on a regime where the exchange rate tomorrow is given by $\bar{e}$ and the central bank today sets the nominal rate at $\tilde{R} = R$. The current exchange is then determined by

$$e_0 = \frac{\bar{e}}{1-\mu_0} \tag{B.1}$$

We have the following proposition:

Lemma C.1 (Unemployment under target rate)**.** *In a self-fulfilling crisis, the exchange rate depreciates at $t = 0$ and there is unemployment.*

Proof. To see this why $h_0 < \bar{h}$, combine market clearing $h_0 = y_0^N = c_0^N$ with the demand for non-tradables (14) to obtain

$$h_0 = \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\frac{1}{1-\mu_0}\right]^{\gamma} c_0^T$$

Using (4) and $c_0^T < c_1^T$ we arrive to

$$h_0 < \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^{\gamma} c_1^T = \bar{h} \tag{B.2}$$

Therefore, if a self-fulfilling crisis exists under $\tilde{R}_0 = R$ it has to be that $h_0 < \bar{h}$. □

We turn to showing that the exchange rate depreciates. From (B.1), we have

$$e_0 = W_{-1}\frac{u_T(y^T - B_0 + \frac{B_1}{R}, h_0)}{u_N(y^T - \frac{R-1}{R}B_1, \bar{h})}, \quad \text{with } h_0 = \left(\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right)^{\gamma}\left(y^T - B_0 + \frac{B_1}{R}\right) \tag{B.3}$$

Totally differentiating (B.3) yields

$$[\gamma + (1-\gamma)\tilde{\phi}_0]\,\frac{Rc_0^T}{e_0}\frac{de_0}{dB_1} = -\left[1 + (R-1)\frac{-u_{TT}(c_1^T, \bar{h})}{u_N(c_1^T, \bar{h})}\right] \tag{B.4}$$

where $u_{TT}(c^T, h) \equiv \frac{\partial u(c^T,h)^2}{\partial (c^T)^2} < 0$. The exchange rate depreciates in a crisis equilibrium. □

The next proposition characterizes when an economy under an interest rate peg features multiple equilibria:

Proposition C.1 (Crises under target rate)**.** *Suppose Assumption 1 holds and $\gamma < 1$. Under a flexible exchange rate with a target interest rate,*

i. if $B_0 \in ((1+\kappa)y^T, \hat{B}) \neq \emptyset$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium.

ii. *if* $B_0 \in [\underline{B}^r, (1+\kappa)y^T)$, *there exists two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, with* $\underline{B}^r > \underline{B}$.

iii. *if* $B_0 < \underline{B}^r$, *we have one and only one equilibrium (which corresponds to the steady state equilibrium).*

Proof. Note that the maximum borrowing capacity is given by (18) where e_0 is determined by (B.1). Differentiating (18), we obtain

$$\begin{aligned}\frac{\partial \bar{b}(B_1;B_0)}{dB_1} &= \kappa\frac{1-\phi}{\phi}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}[1-(1-\gamma)\xi_{B_1}] \\ \frac{\partial^2 \bar{b}(B_1;B_0)}{dB_1^2} &= (1-\gamma)\left[-\frac{\partial \bar{b}(B_1;B_0)}{dB_1}\xi_{B_1} - \kappa\frac{1-\phi}{\phi}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\frac{d\xi_{B_1}}{dB_1}\right]\end{aligned}$$

where $\xi_{B_1} \equiv \frac{Rc_0^T}{e_0}\frac{de_0}{dB_1} < 0$ is the elasticity of the exchange rate with respect to B_1 in (A.13). Differentiating (B.4), we obtain

$$\begin{aligned}\frac{d\xi_{B_1}}{dB_1} = &-\frac{(1-\gamma)^2(1-\tilde{\phi}_0)^2}{1-(1-\gamma)(1-\tilde{\phi}_0)}\frac{1}{Rc_0^T}(\xi_{B_1})^2 \\ &-(1-\gamma)\frac{(1-\beta)\tilde{\phi}_1}{\gamma\tilde{R}_0c_1^T}\left[\frac{2+(R-1)\,c_0^T/c_1^T}{1-(1-\gamma)(1-\tilde{\phi}_0)} - \xi_{B_1}\right] < 0 \end{aligned} \tag{B.5}$$

It follows from (B.4) and $\xi_{B_1} < 0$ that $\frac{\partial^2 \bar{b}(B_1;B_0)}{dB_1^2} > 0$. Next, following the same steps as in the proof of Proposition 1, we arrive at

$$\underline{B}^r = (1+\kappa)y^T - \frac{\psi_0 - 1}{\psi_0}\left[\psi_0\kappa\frac{1-\phi}{\phi}\right]^{\frac{-\gamma}{1-\gamma}} h_0 \tag{B.6}$$

Thus, at least one self-fulfilling crises equilibrium coexists with the steady state equilibrium for $B_0 \in (\underline{B}^R, \hat{B})$ where $\underline{B}^R$ is given by (B.6) and we use $B_0 < \hat{B}$ by Lemma 1. Using $\frac{1}{\psi_0} < \gamma$ and $h_0 < \bar{h}$, we have follows from (B.6) that

$$\underline{B}^r > (1+\kappa)y^T - (1-\gamma)\left[\frac{1}{\gamma}\kappa\frac{1-\phi}{\phi}\right]^{\frac{-\gamma}{1-\gamma}} \bar{h} \tag{B.7}$$

Moreover, since $\bar{b}(B_1;B_0)$ is convex in its first argument, the equation $\bar{b}(B_1;B_0) = B_1$ has two solutions if and only if $\bar{b}(B_1;B_0) = B_1$ has a solution and at $\tilde{B}_1$ such that $\frac{\partial \bar{b}(B_1;B_0)}{\partial B_1}|_{\tilde{B}_1} = 0$ we have $\bar{b}(\tilde{B}_1;B_0) > B_0$. Because $\partial \bar{b}(B_1;B_0)/\partial B_1 = 0$ implies that $c_0^T = 0$, it follows that $\tilde{B}_1$

lowest value in the feasible domain of B_1, i.e. $\tilde{B}_1 = R(B_0 - y^T)$, and we have

$$\bar{b}(R(B_0 - y^T); B_0) = (1+\kappa)\kappa y^T > B_0 \;\Leftrightarrow\; B_0 < (1+\kappa)\kappa y^T$$

Therefore, $\bar{b}(B_1; B_0) = B_1$ has two solutions for $B_0 \in \left[\underline{B}^r, (1+\kappa)y^T\right)$ and a unique solution for $B_0 \in \left[(1+\kappa)y^T, \hat{B}\right)$ which is non-empty under Assumption 1. $\square$

C.2 Crises Region under an Interest Rate Rule

We consider a Taylor rule that depends only on output gap for simplicity. In particular,

$$\tilde{R}_0 = R\left(\frac{h_0}{\bar{h}}\right)^{-\phi_h}, \tag{B.8}$$

where $\phi_h \geq 0$ is a non-negative coefficient that describes the strength of the interest rate response to deviations of output from its efficient level. For $\phi_h \to \infty$, the rule (B.8) corresponds to the full employment policy where monetary policy ensures $h_0 = \bar{h}$ and for $\phi_h = 0$ the rule (B.8) reduces to $\tilde{R}_0 = R$, i.e. the interest rate target policy. Using (7) and substituting for μ_0 using (4) we get

$$\tilde{R}_0 e_0 = e_1 \frac{u_T(y^T - B_0 + \frac{B_1}{R}, h_0)}{u_T(y^T - \frac{R-1}{R}B_1, \bar{h})} \tag{B.9}$$

and totally differentiating it we arrive to

$$\left[(1-\phi_h)\gamma + (1-\gamma)\tilde{\phi}_0\right]\frac{Rc_0^T}{e_0}\frac{de_0}{dB_1} = -\left[1 + \phi_h + (R-1)\frac{-u_{TT}(c_1^T, \bar{h})}{u_N(c_1^T, \bar{h})}\right] \tag{B.10}$$

We characterize the crisis region in the proposition below.

Proposition C.2 (Crises under Taylor rules)**.** *Suppose Assumption 1 holds and $\gamma < 1$. Under a flexible exchange rate where monetary policy is set according to the Taylor rule* (B.8)*,*

i. if $B_0 \in ((1+\kappa)y^T, \hat{B}) \neq \emptyset$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium.

ii. if $B_0 \in [\underline{B}^T, (1+\kappa)y^T)$, there exists two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, with $\underline{B}^T > \underline{B}$.

iii. if $B_0 < \underline{B}^T$, we have one and only one equilibrium (which corresponds to the steady state equilibrium).

Proof. The proof follows the same steps as the proof of Proposition C.1 with $\xi_{B_1} \equiv \frac{Rc_0^T}{e_0}\frac{de_0}{dB_1}$ now given by (B.10). $\square$