Capital Accumulation with Household and Producer Heterogeneity

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Abstract

We study the determinants of long-run outcomes in economies with capital accumulation, financial frictions, and rich consumer and producer heterogeneity. We show that balanced growth paths are isomorphic to equilibria of distorted static economies in which capital goods are treated as intermediate goods sold at a markup reflecting a "Golden Rule" wedge. This wedge, which equals the ratio of capital compensation to investment costs, reflects the fact that the dynamic equilibrium does not maximize long-run consumption. This wedge, whose value depends on the difference between the return on capital and the growth rate, is endogenously determined to clear asset markets. Our isomorphism allows us to use tools developed for distorted static economies to study the long-run behavior of dynamic ones. In particular, we highlight that when the golden rule wedge is large, then response of long-run consumption to shocks depends on whether the shock causes more or less resources to be directed towards capital intensive activities.

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1 Introduction

How are long-run outcomes determined? For example, how important are differences in human capital, productivity, trade barriers, or financial frictions in explaining income differences across countries and regions? To answer such questions, we must account for the endogeneity of the capital stock in the long run. For example, changes in productivity, human capital, or trade costs have direct effects on output, holding capital fixed, but they also have indirect effects through capital accumulation. It is well known that these indirect effects can be quantitatively important. For example, in the neoclassical growth model, a doubling of total factor productivity leads to a doubling of output in the short-run, but almost a tripling of output in the long-run due to induced capital accumulation.

In this paper, we develop a general framework with capital accumulation to characterize how various economic factors affect long-run outcomes. Our framework allows for multiple countries with heterogeneous households and producers in each country. On the production side, we allow for an arbitrary number of perishable goods, capital goods, factor endowments, input-output linkages, elasticities of substitution, and distortions. On the household side, our results can be applied to a wide variety of heterogeneous agent environments including overlapping generations featuring idiosyncratic uninsurable labor and capital income risk.

We show that there is an isomorphism between the balanced growth path of dynamic models and equilibria of distorted static economies. This isomorphism is based on two observations. First, from a technological perspective, in the long-run capital behaves as-if it is an intermediate input produced by an investment sector. In particular, each unit of a capital good i on the balanced growth path requires the production of $g + \delta_i$ units of investment goods, where g is the economy's growth rate and δ_i is the depreciation rate. Second, from an allocational perspective, time discounting and risk premia drive a wedge between the marginal revenue product and marginal cost of capital. In particular, whenever the net return r_i of a capital good i exceeds the economy's growth rate, then the marginal product of that capital stock, $r_i + \delta_i$, exceeds the physical cost of maintaining it, $g + \delta_i$.

These two observations mean that the dynamic steady state is the equilibrium of a particular static economy where capital goods are produced from investment goods and resold at a markup $(r_i + \delta_i)/(g + \delta_i)$, with profits rebated as net capital income to households. Intuitively, the markups reflect that an economy operating away from the Golden Rule ($r_i = g$) does not maximize steady-state consumption given its technological constraints. This is true even if the first welfare theorem holds. Thus, the markup does not capture an inefficiency, but is a static representation of the compensation required by capital owners for postponing consumption and bearing risk, with the size of the resulting profits coinciding with capital returns net of investment costs along the balanced growth path.

Accordingly, the balanced-growth path can be obtained by solving the corresponding static economy provided we know net capital returns r_i and the distribution of net capital income. These returns and their distribution are endogenously determined from household savings behavior combined with asset market clearing conditions. Equilibrium requires that the capital stocks implied by the corresponding static economy are consistent with the desired capital holdings of the households.

Our framework provides a dynamic counterpart to the static results in Hulten (1978), Costinot and Rodriguez-Clare (2014), and Baqaee and Farhi (2020). We use this framework to study the balanced growth path of the economy, asking how long-run outcomes are affected by changes in primitives such as financial frictions, trade costs, taxes, factor supplies, and sectoral productivities.

A key insight that carries over from static results is the distinct role of direct productivity effects and reallocation effects. Using Baqaee and Farhi (2020), we structurally decompose primitive shocks into two different effects: an exogenous technology effect, holding the allocation resources fixed, and an endogenous reallocation effect, holding technologies fixed.

For example, consider the response of long-run consumption to a productivity shock. First, holding the initial allocation of resources fixed, an increase in the productivity of one producer not only raises their output, but also the output of all their customers, customers' customers, and so on. This amplification effect is a generalization of the capital multiplier in the neoclassical growth model and is formally equivalent to how intermediate inputs amplify productivity shocks.

Second, holding technologies constant, the shock can also change the allocation of resources across different uses, raising some producers' outputs and reducing that of others. When $r_i = g$, an envelope argument implies that these reallocation effects have zero effect on long-run consumption to a first-order. However, when $r_i \neq g$, then reallocation effects have first-order effects on long-run consumption. Specifically, if $r_i > g$, then shocks that reallocate resources towards more capital-intensive activities raise aggregate long-run consumption.

This also means that when $r_i = g$, long-run consumption obeys a version of Hulten (1978) where capital goods are treated as intermediates. Namely, the elasticity of long-run consumption to the productivity of each producer is equal to that producer's total sales relative to aggregate consumption (which is larger than the Domar weight if investment is nonzero). That is, when $r_i = g$, the elasticity of long-run consumption to a productivity shock is the Domar weight multiplied by the ratio of GDP to consumption. Away from $r_i = g$, however, there are reallocation effects that must be reckoned with.

To illustrate the importance of these reallocative effects, we use a quantitative version of our model to study the long-run consequences of increases in trade barriers, financial frictions, and schooling, focusing on Mexico as a typical middle income country for illustration. On the production side, our quantitative model consists of a world economy with a nested-CES production network (à la Costinot and Rodriguez-Clare, 2014; Baqaee and Farhi, 2024) with disaggregated capital goods and investment, (à la Ding, 2022). On the household side, our quantitative model features an overlapping generation of households in each country (à la Blanchard, 1985) that face uninsurable idiosyncratic capital risk (à la Angeletos, 2007) and can borrow and lend an international riskfree asset (à la Angeletos and Panousi, 2011).

The response of Mexican consumption to an increase in Mexican exporter's iceberg trade costs is around five times larger when compared to a standard static trade model. The response of world consumption is around 3 times larger compared to a standard static trade model. A significant fraction of this amplification is due to aforementioned reallocation effects, whereby an increase iceberg trade costs to Mexican exporters reallocates resources away from capital, and since capital is already subject to a significant Golden Rule wedge, these reallocation effects depress both Mexican and world income. The effects of accounting for capital accumulation would be much more muted if either the Golden Rule wedge or the trade elasticities were close to zero.

We find a similar result for schooling. Increased schooling is three times more impactful for long-run consumption once we account for capital accumulation. However, a large portion of this is due to the reallocation effects it induces, boosting net capital income for Mexico, and reallocating resources towards capital intensive activities for the world. Both of these effects would once again disappear if the Golden Rule wedge or trade elasticities were zero.

Finally, we also consider how a reducing Mexican financial frictions to US levels would affect long-run Mexican consumption. Lower financial frictions, by lowering the amount of idiosyncratic risk borne by capital owners, result in lower required returns on capital in equilibrium and hence, a lower Golden Rule wedge in Mexico. Despite this fact, we find that a strong increase in Mexican net capital income — that is, we find that Mexico is on the wrong side of its capital income Laffer curve. Furthermore, Mexican wages also strongly rise in response to a lowering of financial frictions.

Overall, our quantitative results show that accounting for capital accumulation, calibrating the size of the Golden Rule wedge correctly, and modeling substitution elasticities are all very important for understanding how country-level or world consumption respond to changes in technologies and policies.

Roadmap. Section 2 illustrates the isomorphism between balanced growth and distorted static equilibria using a simple example. Section 3 generalizes this basic example and shows the basic idea can be extended to much richer environments and shows how intuitions and tools typically applied to study distorted static equilibria can be repurposed to understand balanced growth paths. Section 4 specializes our abstract framework to a more specific parameteric model. Section 5 calibrates the quantitative model and illustrates the importance of reallocation effects for understanding long-run consumption responses to shocks.

Related Literature. Our paper is related to Baqaee and Farhi (2020) and Baqaee and Farhi (2024), which consider production networks with exogenous wedges and exogenous trade imbalances. We extend these frameworks to account for capital accumulation. We relate wedges to rates of return on capital and transfers between countries to international borrowing and lending, all of which are determined endogenously.[1]

Our paper is also closely related to Foerster et al. (2022) and Ding (2022) who study balanced-growth paths of multi-sector models. Foerster et al. (2022) work with a closed Cobb-Douglas models with an infinitely-lived representative agent. In such a model, there are no reallocation effects in response to productivity shocks. Our analysis relaxes these assumptions and shows that the absence of reallocation effects is a knife-edge result. Ding (2022) studies steady-state counterfactuals in open economies with infinitely-lived representative agents and financial autarky. We relax the assumption of infinitely-lived agents, financial autarky, and consider balanced growth (rather than steady-state) equilibria. We focus on the importance of reallocation effects, which depend on the Golden Rule wedge and elasticities of substitution.

Our approach to modelling trade imbalances is based on the intertemporal approach to the current account from international macroeconomics (Obstfeld and Rogoff, 1995). By using a model where demand for savings is not infinitely elastic in steady-state, we are able to solve for the long-run outcomes without having to solve transition dynamics. By allowing for financial frictions, we are able to relate global imbalances in the current

[1] Since we conceptualize the steady-state of a dynamic model with $r \neq g$ using a distorted static economy, our paper is also related to the literature on static misallocation with input-output linkages, like Jones (2011), Bigio and La'O (2016), and Liu (2017).

account to financial development, similar to Caballero et al. (2008) and Mendoza et al. (2009).

Our paper is also related to quantitative dynamic disaggregated and international general equilibrium models, pioneered by Long and Plosser (1983) and Backus et al. (1992). Some recent contributions include Alvarez (2017), Ravikumar et al. (2019), Dix-Carneiro et al. (2023), Lyon and Waugh (2019), Vom Lehn and Winberry (2022), and Kleinman et al. (2023).[2] Whereas this literature uses computational methods to study transition dynamics, our focus is instead on analytical results for understanding and dissecting steady-state outcomes. Second, in contrast to our paper, this literature tends to work with infinitely-lived representative agents which results in a capital supply curve that is infinitely elastic and the rate of return is equal to the discount rate in steady-state.

Our paper is also related to the sufficient statistics literature in trade, like Costinot and Rodriguez-Clare (2014), which we extend to include capital. We show that static trade models can be used to study the steady-state of dynamic economies, where capital is treated as an intermediate input. Our framework supplies additional equations that endogenously pin down trade imbalances and wedges associated with net capital income, which in a static trade model, are exogenous.

2 Illustrative Example

Before stating the general result, we use the neoclassical growth model to show the basic logic of the isomorphism between balanced growth paths and equilibria of distorted static economies. In Section 3, we show that the basic logic behind this illustrative example applies very generally.

Consider an aggregate production function

$$Y(t) = F(L(t), K(t)),$$

where $L(t)$ is a labor endowment and $K(t)$ is capital. Capital is accumulated according to

$$\dot{K}(t) = X(t) - \delta K(t),$$

where $X(t) = Y(t) - C(t)$ is investment. Suppose that the labor endowment grows at

[2] In this paper, we abstract from fixed costs and entry/exit decisions of firms, for example, as in Hopenhayn (1992) or Melitz (2003). See Alessandria et al. (2021) for a review of this literature as it pertains to international trade. At the aggregate level, firm dynamics models can behave similarly to models with capital accumulation. However, this topic is beyond the scope of our current paper.

rate g and that F has constant returns to scale. Then, along a balanced growth path, suppressing time subscripts, the following conditions hold:

$$P(\partial F/\partial L) = W, \quad P(\partial F/\partial K) = P(r+\delta), \quad Y = C + X = C + (g+\delta)K,$$

where P, W, and r are the price of output, the wage, and the net return on capital. The first two conditions follow from cost-minization by the representative firm and the third is the resource constraint, once we impose that investment is such that the capital-output ratio remains constant. These equations collectively pin down the balanced-growth path of the model.

Now consider a static economy instead where K, rather than being a capital stock that is accumulated over time, is an intermediate input produced linearly from output with production technology $K = A_K X$. Suppose that this intermediate is sold at a markup μ_K over marginal cost. The equilibrium conditions of this static economy are:

$$P(\partial F/\partial L) = W, \quad P(\partial F/\partial K) = \mu_H P/A_K, \quad Y = C + X = C + K/A_K.$$

The first two conditions are cost-minimization by the representative firm, and the final equation is the resource constraint.

A key observation of this paper is to note that the two sets of equations coincide if

$$A_K = 1/(g+\delta), \quad \text{and ,} \quad \mu_K = 1 + (r-g)/(g+\delta).$$

That is, long-run outcomes in the dynamic model coincide with outcomes in a static model where capital is treated as-if it is an intermediate input produced from investment with a productivity shifter $1/(g+\delta)$ and sold at a markup over marginal cost $1 + (r-g)/(g+\delta)$. The productivity shifter reflects the fact that, along the balanced growth path, each unit of capital requires $(g+\delta)$ units of investment (otherwise, the capital stock does not grow at the rate g). The markup, μ_K, reflects the fact that, in a dynamic model, payments to capital typically differ from investment costs, since investors are compensated for deferring consumption to the future (in the full-blown model, there is also compensation for bearing risk).

In this simple model, if $r = g$, then $\mu_K = 1$, then the equivalent static economy is efficient. In other words, the decentralized equilibrium of the dynamic model maximizes long-run consumption. This is, of course, the "Golden Rule" for savings (Phelps, 1961).

In this case, the equivalent static economy is efficient since capital is sold at marginal cost. This means that, when $r = g$, long-run consumption obeys a version of Hulten (1978)

— the elasticity of long-run output to a permanent productivity shock to any quantity is equal to steady-state expenditures on that quantity relative to steady-state consumption.

If $r \neq g$, then long-run consumption is not maximized by the decentralized dynamic economy. This means that the equivalent static economy is distorted. Accordingly, the response of long-run consumption to shocks is not described by Hulten (1978). Instead, long-run consumption's response to shocks depends changes in allocative efficiency due to reallocation, characterized by Baqaee and Farhi (2020).

The importance of these reallocation effects depends on the size of the distorting wedge and the extent of reallocation shocks induce. If the equivalent static economy is "almost" efficient, or if shocks do not cause resources to be reallocated across activities, then reallocation effects are small.

The size of the wedge can be gauged by comparing payments to capital, around 40% of GDP in the United States, to investment, around 20% of GDP in the United States. The ratio of these two numbers gives the effective wedge in the static economy. Hence, for the United States, the aggregate markup on capital in the equivalent static economy is roughly 2. The magnitude of this markup hints at the importance of accounting for reallocation effects when analyzing long-run outcomes. Moreover, this effective markup varies greatly across different types of capital — for instance, for housing, the wedge is closer to 3.

In the rest of the paper, we show that the isomorphism between dynamic and static models is remarkably general. This enables tools developed for analyzing distorted static economies, like Baqaee and Farhi (2020) and Baqaee and Farhi (2024), to be deployed to analyze and understand long-run outcomes in dynamic economies.

This is more than simply relabeling variables in a static model however. The reason is that, in general, the return on capital, r, and hence the effective markup, $1 + (r - g)/(g + \delta)$, are endogenously determined by equilibrium in asset markets. Furthermore, if there are multiple countries, transfer payments between countries, which are typically treated as exogenous nuisance parameters in static trade models (Dekle et al., 2007), are also endogenously determined by the gap between country-level investment and savings decisions. Hence, balanced-growth equilibria are a relabeling of an equivalent static model with wedges and transfers where auxiliary equations, from asset markets, determine the wedges and transfers.

3 Model

This section sets up a general model to study balanced growth paths of multi-country economies with capital accumulation. We purposely work with a fairly general and abstract model to demonstrate the broad applicability of the isomorphism between balanced growth paths of dynamic economies and distorted static economies. Accordingly, our model nests many commonly used models with heterogeneous households and disaggregated production. In later sections, when we use this isomorphism to study specific counterfactual questions, we specialize the environment further.

Section 3.1 shows that balanced growth paths can be represented using equivalent static economies, which are defined up to rates of return and profit distributions. Rates of return and profit distributions are determined by asset market equilibrium. To express asset market equilibrium, Section 3.2 provides a general definition of asset demand across a wide range of models, and section 3.3 fully characterizes the balanced growth path given this definition of asset demand.

3.1 Production and capital accumulation

Consider a world economy with C countries. We begin by describing the technological environment before discussing accumulation decisions of households.

Production and capital accumulation. Consider a world economy with C countries, a set of industries N and a set of primary factors F, with each country c having a subset of these industries N_c and of factors F_c. In this paper, the set of factors, F, only includes factor endowments (land, labor, etc.) and does not include accumulable capital.

Each producer $i \in N$ produces a distinct good using the production function

$$Y_i(t) = A_i G_i \left[\{A_f(t) L_{if}(t)\}_{f \in F_c}, \{Y_{ij}(t)\}_{j \in N}, \{K_{ij}(t)\}_{j \in N} \right], \tag{1}$$

where Y_i is output i, $\{L_{if}\}$ is a set of local primary factor inputs, $\{Y_{ij}\}$ is a set of intermediate inputs and $\{K_{ij}\}$ is a set of capital services. Without loss of generality, we assume that production has constant returns to scale (decreasing returns can be captured using producer-specific fixed factors).

Intermediate inputs and capital services purchased by i are sourced from other producers $j \in N$. Hence, each input is associated with precisely one producer. The term A_i is an industry productivity shifter, and $A_f(t)$ is a factor-specific technology term. To ensure the existence of a balanced growth path, we assume that $A_f(t)$ grows at constant rate g_A across all factors and countries. In our main exposition, we assume that $A_f(t)$ is deterministic.[3]

In each country, one industry is designated as the "consumption good" industry, which combines inputs from other producers to produce a final consumption good. This producer does not directly employ factors or capital.[4]

Producers purchase intermediate inputs from other producers, hire primary factors (labor and land) and capital from households, and minimize costs taking input prices as given. This implies a static problem where each producer i minimize total costs

$$\min_{\{L_f(t)\},\{Y_{ij}(t)\},\{K_{ij}(t)\}} \sum_{f\in F_c} w_f(t)L_f(t) + \sum_{j\in N} p_j(t)Y_{ij}(t) + \sum_{j\in N} p_j(t)K_{ij}(t)R_j(t), \tag{2}$$

subject to production technology (1). In (2), w_f is the wage of factor f, p_j is the price of input j and R_j is the shadow price of capital good j. Producer i sells its output at price p_i. We assume that each good i is subject to an exogenous tax, τ_i, that represents either an explicit output tax or an implicit distortion. The price of i net of taxes, $p_i(1-\tau_i)$, equals marginal cost. We assume that tax revenues are rebated to households in proportion to to their primary factor income.[5]

Each good can either be used immediately or accumulated into a stock through investment. When used immediately, the good is either consumed or used as an intermediate input by other producers. When allocated to investment, the good contributes to the stock of capital of that good i, whose accumulation is governed by:

$$\dot{K}_i(t) = X_i(t) - \delta_i K_i(t), \tag{3}$$

where $X_i(t)$ is the investment in capital stock of type i, and δ_i is the depreciation rate. A fully perishable good, that cannot be accumulated at all, will have $\delta_i = \infty$.

The resource constraints for goods are

$$Y_i = C_i + X_i + \sum_{j\in N} Y_{ji}, \qquad i \in N, \tag{4}$$

[3] Our results could be extended to a setup with aggregate risk (similar to Farhi and Gourio, 2018).

[4] This is equivalent to assuming that households within a country have the same homothetic preferences over goods. Heterogeneous preferences within countries can be accommodated by treating households with different preferences as belonging to different countries. We do not pursue this further, but accommodating preference heterogeneity within countries is a straightforward extension.

[5] As in Baqaee and Farhi (2020), output taxes can also be used to represent input-specific distortions through a relabeling. To capture a wedge on i's use of some input j, we can create a fictitious middleman producer that purchases j and sells it to i and place an output tax on this fictitious middleman.

where C_i is the output used for consumption (and is zero for all industries except the designated consumption good industries), Y_{ji} is the output used as an intermediate input by industry j, and X_i is the output used for investment into industry-i capital. Note that since N is the set of all goods producers in the world, (4) is an international resource constraint and accounts for trade between countries.[6]

The resource constraints for primary factors and capital stocks are:

$$L_f = \sum_j L_{jf}, \tag{5}$$

$$K_i = \sum_j K_{ji}. \tag{6}$$

A balanced growth path in this model features constant rates of return $r_i \equiv R_i - \delta_i$ for each capital type, constant prices p_i and wages w_f growing at rate g_A across all factors. All quantities grow at rate $g = g_A + g_L$, where g_L is the growth rate of factor endowments. For any balanced growth path, we define world net capital income as

$$\Pi \equiv \sum_{i \in N} \tau_i p_i Y_i + \sum_{i \in N} R_i p_i K_i - \sum_{i \in N} p_i X_i = \tau_i p_i Y_i + \sum_i (r_i - g) p_i K_i,$$

where $\sum_i \tau_i p_i Y_i$ is wedge income, and the final equality uses the fact that investment, X_i, must equal $(\delta_i + g_i) K_i$ along the balanced growth path. Define the share of net capital income going to country c as

$$\pi_c \equiv \frac{p_c C_c - \sum_{f \in \mathcal{F}_c} w_f L_f}{\Pi},$$

where p_c and C_c are the price and quantity of country's c's consumption good. That is, along the balanced growth path, consumption expenditures in each country are equal to primary factor income of that country plus country c's share of total net capital and wedge income.

The isomorphism between balanced growth paths and equivalent static economies introduced in Section 2 generalizes to our richer setup. Using only that a balanced growth path requires cost minimization of firms and market clearing, we can prove the following proposition that characterizes balanced growth paths up to rates of returns and the profit distribution. To simplify expressions, we state the result where there are no exogenous taxes τ_i. See the appendix for a statement including taxes.

[6]Technological trade costs are nested by our setup since we can formally introduce transportation industries that buy goods in one country and sell them in another country with some TFP shifters capture iceberg trade costs.

Proposition 1 (Isomorphism Between BGP and Distorted Static Economies)**.** *Suppose that we have a balanced growth path with rates of return r_i where the share of net capital income paid to country c is π_c. Then the quantities and prices of the balanced growth path form an equilibrium of an equivalent static economy were*

1. *The production functions of goods are the same as in the dynamic economy.*
2. *Capital goods are intermediates produced with linear technology from investment*

$$K_i = A_{K_i} X_i,$$

   with productivity shifter $A_{K_i} = 1/(g + \delta_i)$.

3. *Capital goods are sold at a markup*

$$\mu_i = \frac{r_i + \delta_i}{g + \delta_i}$$

   with profits distributed to households in proportion to π_c.

We refer to μ_i when i is a capital good as the Golden Rule wedge for i.

3.2 Asset demand

Proposition 1 characterizes balanced growth paths up to a vector of returns $\mathbf{r}$ and the distribution of net capital income π_c. In a dynamic equilibrium, these are pinned down by households' accumulation and portfolio choices. While the specifics vary across models, many macroeconomic frameworks share a common *asset demand structure* in terms of how household accumulation choices shape aggregate outcomes. By identifying and formalizing this shared structure, we can prove results that apply to a wide range of models without specifying all the details of household behavior. Moreover, when presented with a particular model, our definitions let us test whether the model satisfies our assumptions, in which case our general results apply. Below, we provide our formal assumptions followed by examples of models that satisfy them.

Definition 1 (Asset demand correspondence)**.** Consider a dynamic model with production parameters $\Theta_{prod} = \{A_i, G_i, \delta_i\}$ and household parameters by country Θ^c_{hh}. We say that a collection of vector-valued correspondes $A^c(t) = (B^c(t), p_i(t)K^c_i(t))$ are *asset demand correspondences* if the following hold.

1. The correspondences $\{A^c(t)\}_{c \in C}$ do not depend directly on Θ_{prod} or aggregate output quantities (but can depend on factor endowments, domestic household parameters and sequences of returns and prices).

2. For all production parameters, the following equations hold in equilibrium:

$$\begin{pmatrix} 0 \\ p_1(t)K_1(t) \\ \dots \\ p_N(t)K_N(t) \end{pmatrix} = \sum_c A^c(t), \tag{7}$$

   and

$$p_c(t)C_c(t) + \sum_i \dot{A}_i^c(t) \leq \sum_{f_c} w_f(t)L_f(t)(1 + d_c(t)) + \mathbf{r}(t) \cdot A^c(t) \qquad \forall c, \tag{8}$$

   where $\mathbf{r}(\mathbf{t}) = (r_0(t), r_1(t), \dots, r_N(t))$ is a vector consisting of the risk-free rate $r_0(t)$ and capital returns $r_i(t)$, and $d_c(t)$ is government's transfers as a share of primary factor income.

3. For all price sequences, there exists some finite multiple κ of current factor income that $\sum_c A^c(t)$ never goes below

$$\sum_c A^c(t) \geq -\kappa \times \sum_f w_f(t)L_f(t)(1 + d_c(t))$$

The definition distills the macroeconomic role of asset demand across many models into a set of defining characteristics. In standard models, these characteristics generally hold since asset demand is obtained by integrating across households' optimal asset choices. Given such a definition, condition (7) holds due to asset market clearing, while (8) follows from integrating over household budget constraints. The correspondences A^c are independent of production parameters whenever asset choices of atomistic households can be expressed in terms of household parameters, prices, and initial conditions. Last, the lower bound on assets hold under weak restrictions on borrowing. By recasting common properties of asset demand into defining characteristics, Definition 1 makes it possible to implicitly characterize a wide class of models, while simultaneously providing a simple criterion to test if any particular model belongs to the class.

To characterize balanced growth paths, we are interested in the properties of asset demand when capital returns $\mathbf{r}(t)$ and prices $\mathbf{p}(t)$ are constant, and all factor incomes grow at a constant rate g.

Definition 2 (Balanced Growth in Asset Markets)**.** A balanced growth asset demand $\bar{A}^c(\mathbf{r}, \mathbf{p}, \{w_f\}, \{L_f\})$ exists for a set of returns, good prices, factor prices, and factor endowments if there is an initial condition Θ_{init} such that normalized asset demand

$$e^{-gt} A^c(t; \{\mathbf{r}\}, \{\mathbf{p}\}, \{e^{g_A t} w_f\}, \{e^{g_L t} L_f\}, \Theta_{init})$$

is constant over time.

Definition 2 lets us define balanced growth asset demand for any model featuring an asset demand function. The definition is designed to capture the standard property of balanced growth paths that when households face fixed prices, there is some initial state position such that if you start in that state, aggregates do not subsequently change in normalized terms. Note that the formal definition requires us to distinguish between parameters that are initial conditions and those that are not. In most cases there is a natural choice, such as letting Θ_{init} be the initial distribution of households' states and assets.

Given these definitions, we can now state the assumption on the household side of our model.

Assumption 1. *We assume that our model contains the production parameters $\Theta_{prod} = \{A_i, G_i, \delta_i\}$ and the outcomes from Section 3.1, as well has household parameters Θ_{hh} and household initial conditions Θ_{init}. The model admits an asset demand representation as in Definition 1, with a balanced growth asset demand satisfying*

$$\bar{A}^c(\mathbf{r}, \{w_f\}, \{L_f\}) = \sum_{f_c} w_f L_f \times a^d(\mathbf{r}), \tag{9}$$

where $a^d(\mathbf{r})$ is a vector valued function. Furthermore, if there are multiple countries, we assume that $a^d(\mathbf{r}) \cdot \mathbf{r}$ is single-valued for all $\mathbf{r}$.

The assumptions about production parameters, aggregate outcomes, and the existence of asset demand are not particularly restrictive. They obtain in any model which is rich enough to speak to our aggregate questions, and where production parameters only affect household asset accumulation through prices and wedge income. The non-trivial assumption is (9). By requiring that the connection between factor prices and asset demand is fully mediated by aggregate factor income, it for example excludes models where owners of different factors have different savings behaviors. To build intuition for Assumption 1, we present a set of models that satisfy the assumption, and one which does not.

Below, we outline the main features of each model and state the main result together with intuition. The appendix provides more complete model specifications and formal derivations, including a rigorous treatment of asset demand functions being multi-valued.

Neoclassical growth model household. Consider a model with a single country without population growth that has an infinitely lived lived representative household earning all factor income and with preferences as in the neoclassical growth model. The balanced growth asset demand is given by:

$$\frac{B+K}{\sum_{f\in\mathcal{F}} w_f L_f} = \begin{cases} -\frac{1}{r-g} & \text{if } r < \rho + \sigma g_A \\ \left[-\frac{1}{r-g}, \infty\right) & \text{if } r = \rho + \sigma g_A \\ \varnothing & \text{if } r > \rho + \sigma g_A \end{cases}$$

where σ is the inverse of the intertemporal elasticity of substitution and ρ is the time preference rate. The function captures that if the interest rate is consistent with time preferences, assets are constant over time, and any asset level larger than the natural borrowing constraint is a balanced growth asset demand. If the interest rate is lower than that, the only balanced growth path is to start and stay at the natural borrowing limit. If the interest rate is larger than its natural rate, we will always increase assets regardless of the starting point, so there is no balanced growth asset demand. Note that Assumption 1 rules out such a setup for the multi-country case. Indeed, because every initial asset holding is a balanced growth asset holdings, this means that there are multiple total return $\mathbf{r} \cdot a^d(\mathbf{r})$.

Aiyagari model. Suppose that each country is populated by a mass N_c of Aiyagari households earning factor income $\epsilon(t)\sum_{f\in\mathcal{F}_\rfloor} w_f(t)\ell_f(t)$, where ϵ is an idiosyncratic income risk term. Further, assume that household factor endowments grow with rate g_L over time and that they discount the future by ρ. In that case, we can solve a household problem normalized by $\sum_f w_f(t)\ell_f(t)$. The average asset holdings in that problem will be a function $a^c(r)$ for each country with $\lim_{r\to\rho+\sigma g} a^c(r) = \infty$. Provided that households start in that asset distribution, we obtain

$$B^c + K^c = \sum_{f\in F_c} w_f L_f \times a^c(r) \text{ for } r < \rho + \sigma g,$$

where B^c and K^c are normalized by the economy's growth rate. Provided that $a^c(r)$ is strictly increasing in r, this function satisfies Assumption 1.

3.3 Balanced growth path characterization

Provided that our economy satisfies the conditions set out in Assumption 1, we can combine Proposition 1 with the asset demand structure to fully characterized the balanced growth path. We obtain the following theorem.

Theorem 1 (Comparative Statics of Balanced Growth Paths)**.** *Suppose that there is a balanced growth path with constant portfolio shares in an economy satisfying Assumption 1 with balanced growth demand functions $a^c(\mathbf{r})$. The quantities and prices of the balanced growth path form an equilibrium of the equivalent static economy described in Proposition 1, given the return vector $\mathbf{r}$ and profit distribution π_c on the balanced growth path. Furthermore, the return vector and the profit distribution satisfy the following two equations*

$$\frac{1}{\sum_c (1+d_c) \sum_{f \in \mathcal{F}_c} w_f L_f} \begin{pmatrix} K_i \\ 0 \end{pmatrix} = \sum_c \pi_c^F \times a^c(\mathbf{r}) \tag{10}$$

$$\pi_c = \frac{\pi_c^F \times [(\mathbf{r} - g) \cdot a^c(\mathbf{r})]}{\sum_{c'} \pi_{c'}^F \times \left[(\mathbf{r} - g) \cdot a^{c'}(\mathbf{r})\right]} \qquad \forall c \in \mathcal{C}, \tag{11}$$

where $\pi_c^F \equiv \frac{\sum_{f \in \mathcal{F}_c} w_f L_f (1+d_c)}{\sum_{c'} (1+d_{c'}) \sum_{f' \in \mathcal{F}_c} w_{f'} L_{f'}}$ is the share of factor income received by country c.

The key part of the theorem is that it endogenizes the vector of returns $\mathbf{r}$ and the distribution of net capital income π_c, which were left unspecified in the representation in Proposition 1. Equation (10) gives the market clearing condition. The left-hand side gives aggregate asset supply relative to factor income. The right-hand side asserts that this equals asset demand, which is given by country-level asset demand functions weighted by their share of world factor income. Asset market clearing lets us solve for the equilibrium return vector, and (11) expresses the distribution of net capital income given equilibrium returns.

3.4 Long-Run Comparative Statics

If Theorem 1 applies, then long-run comparative statics for all prices and quantities can be derived from Baqaee and Farhi (2024), with the proviso that wedges and transfers between households are endogenously determined by (10) and (11).

We show how to apply these results to obtain long-run comparative statics of aggregate consumption. However, every price and quantity can be similarly characterized.

Definition 3 (Aggregate real consumption)**.** The change in aggregate real consumption is defined as the weighted average of country-level consumptions:

$$d\log C = \sum_{c\in\mathcal{C}} \frac{P_c C_c}{\sum_{c'\in\mathcal{C}} P_{c'} C_{c'}} d\log C_c = \sum_{c\in\mathcal{C}} \Phi_c C_c,$$

where Φ_c is country c's share of total nominal consumption along the balanced growth path.

Let N, K, and F denote the set of perishable goods, capital goods, and factors. Define Ω to be the balanced-growth input-output matrix of the economy. This is a square matrix with dimension $(N+K+F)^2$, where the columns and rows correspond to goods, capital operators, and primary factors:

$$\Omega = \left(\begin{array}{c|c|c} \Omega^{NN} & \Omega^{NK} & \Omega^{NF} \\ \hline \Omega^{KN} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right).$$

The ijth element of Ω is the share of i's revenues spent on j. More precisely, for the rows corresponding to goods, $i \in \mathcal{N}$, we define

$$\Omega_{ij} = \frac{p_j Y_{ij}}{p_i Y_i}\mathbf{1}(j\in\mathcal{N}) + \frac{R_j p_j K_{ij}}{p_i Y_i}\mathbf{1}(j\in\mathcal{K}) + \frac{w_j L_{ij}}{p_i Y_i}\mathbf{1}(j).$$

In contrast to a standard input-output matrix, where investment is treated as part of final demand, here we treat investment as an intermediate input for capital services. Without loss of generality, we can assume that every good can potentially be accumualted as a capital good: $N = K$.[7] Then, for for the capital operators, $i \in \{N+1, \ldots N+K\}$, the submatrix Ω^{KN} is a diagonal matrix where the ith element is investment of capital type i relative to rental payments:

$$\Omega^{KN}_{ii} = \frac{p_i X_i}{R_i p_i K_i} = \frac{g+\delta_i}{r_i+\delta_i} = \frac{1}{\mu_i}.$$

The Leontief inverse associated with the balanced-growth input-output matrix, Ω, is

$$\Psi = (I-\Omega)^{-1} = I + \Omega + \Omega^2 + \ldots.$$

[7]Of course, in equilibrium, the capital stock of some goods may be zero (for example, if the depreciation rate is infinite or if there is no demand for capital services provided by this type of capital).

Let Φ be the consumption expenditures of each country as a share of total consumption. Define the balanced-growth Domar weights for each $i \in \mathcal{N} + F$ to be

$$\lambda_i = \sum_{c \in C} \Phi_c \Psi_{ci}.$$

These balanced-growth Domar weights are related to expenditures in a straightforward way. For each good, $i \in \mathcal{N}$, the balanced-growth Domar weight is the revenue of i divided by total world consumption. For each capital operator $i \in \mathcal{K}$, the balanced-growth Domar weight is total rental payments divided by world consumption. Finally, for primary factors $f \in F$, the balanced-growth Domar weights are factor payments divided by world consumption. Since primary factors play a special role in equilibrium, we denote their Domar weights using capital letters Λ_f for each $f \in F$.

We also define the cost-based input-output matrix, $\tilde{\Omega}$, to be the same as Ω except where we swap the identity matrix in place of Ω^{KN}. That is, the difference between $\tilde{\Omega}$ and Ω is that the former eliminates the "markups" associated with capital goods. We then define the cost-based Leontief inverse, $\tilde{\Omega}$, and the cost-based Domar weights, $\tilde{\lambda}$, analogously. Note that if the Golden Rule is satisfied, the cost-based and revenue-based input-output matrices, Leontief inverses, and Domar weights coincide.

The following proposition characterizes the response of aggregate consumption to changes in primitives.

Proposition 2 (Aggregate Consumption Response). *The change in steady-state aggregate consumption, to a first-order, is given by*

$$d \log C = \underbrace{\sum_{i \in \mathcal{N}} \tilde{\lambda}_i d \log A_i - \sum_{i \in \mathcal{K}} \tilde{\lambda}_i \frac{\delta_i}{\delta_i + g} d\delta_i}_{\text{technology}} \underbrace{- \sum_{m \in \mathcal{K}} \tilde{\lambda}_i d \log \frac{r_i + \delta_i}{g + \delta_i} - \sum_{f \in F} \tilde{\Lambda}_f d \log \Lambda_f}_{\text{reallocation}}.$$

Proposition 2 is an application of Baqaee and Farhi (2020) and we refer readers to that paper for a detailed description. Briefly, Proposition 2 decomposes consumption responses into two effects: a pure technology effect reflecting the mechanical increase in consumption from changes in technological primitives, holding fixed the allocation of resources, and a reallocation effect, reflecting changes in consumption due to reallocations of resources holding technological primitives constant.

If the decentralized equilibrium maxmizes long-run aggregate consumption, then the reallocation effect must be zero, which happens at the Golden Rule. This is the content of the following corollary.

Corollary 1 (Aggregate Consumption Response at Golden Rule)**.** *If $r_i = g$ for every $i \in \mathcal{K}$ in the initial equilibrium, then the change in steady-state aggregate consumption, to a first-order, is given by*

$$d \log C = \sum_{i \in \mathcal{N}} \lambda_i d \log A_i - \sum_{i \in \mathcal{K}} \lambda_i \frac{\delta_i}{\delta_i + g} d \log \delta_i.$$

Corollary 1 is an application of Hulten (1978). If the economy is at the Golden Rule, then the elasticity of long-run aggregate consumption to technological change is just the balanced-growth Domar weight. Although rather than using the Domar weights, sales relative to GDP, we use balanced-growth Domar weights, sales relative to consumption.

Importantly, at the Golden Rule, the response of long-run consumption to permanent shocks does not directly depend on the microeconomic details of production and consumptions — sales relative to consumption are sufficient statistics. This is the consequence of the envelope theorem, which implies that reallocations can be ignored. Away from the Golden Rule, there is no envelope theorem for long-run consumption, and comparative statics become more challenging due to reallocation effects. This brings us back to Proposition 2. To better understand it, first focus on the pure technology effect, before considering the reallocation effect.

Technology effect. The technology term reflects how changes in physical primitives alter aggregate consumption holding the allocation of resources constant. The technology shocks are weighted by cost-based Domar weights instead of revenue-based ones.[8] These cost-based Domar weights, $\tilde{\lambda}$ take into account the roundabout nature of production, where technological improvements are amplified by both intermediate input and investment linkages.

As long as $r_i > g$, the cost-based $\tilde{\lambda}$ are larger than λ. This is because the long-run multiplier effect of a technology shock to i depends on how important i is as a share of the costs of production, $\tilde{\lambda}_i$, rather than as a share of revenues i receives λ_i.

The neoclassical growth model provides a good example. Let α be the elasticity of aggregate output with respect to capital. The balanced-growth cost-based input-output matrix is

$$\tilde{\Omega} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1-\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

[8]The cost-based balanced-growth Domar weights, $\tilde{\lambda}$, in this paper generalize the "sectoral multipliers" described by Foerster et al. (2022).

where the first row is the consumption good, the second good is the output good, the third row is the capital good, the final row is labor.

Consider an increase in the TFP of the production function, corresponding to $d\log A_2 > 0$. The cost-based Domar weight is $\tilde{\lambda}_2 = 1/(1-\alpha)$. Since the long-run interest rate in the neoclassical growth model does not respond to TFP shocks, $d\log r = 0$. Furthermore, if the aggregate production function is Cobb-Douglas, then the labor share also does not respond to TFP shocks $d\log\Lambda = 0$. Hence, Proposition 2 implies that the elasticity of steady-consumption with respect to aggregate productivity is just

$$\frac{\partial \log C}{\partial \log A_2} = \tilde{\lambda}_2 = \frac{1}{1-\alpha} = \frac{1}{1-RK/GDP} \geq \lambda_2 = \frac{1}{1-\alpha(g+\delta)/(r+\delta)} = \frac{1}{1-X/GDP},$$

where RK are rental payments and X is investment. This is the familiar capital multiplier formula of the neoclassical growth model. An increase in technology improves consumption more than one-for-one in steady state because increased technology raises consumption directly, but also indirectly through additional investment, which raises the steady-state capital stock, boosting production, and hence consumption.

The fact that $\tilde{\lambda}_2$ is equal to the reciprocal of the labor share of GDP is not general. A different production structure would give a different value of $\tilde{\lambda}$. However, if $r = g$, then $\tilde{\lambda} = \lambda$, and the capital multiplier is (equivalently) given by the reciprocal of consumption to GDP. By Corollary 1, this latter fact must always be true if $r_i = g$ for every $i \in \mathcal{K}$, regardless of the production structure.

Reallocation effect. Having understood the technology effect, we now turn our attention to the reallocation effect. If the economy is not at the Golden Rule to begin with, and steady-state consumption is not being maximized, then any shock that reallocates resources can have first order effects on steady-state consumption. According to Proposition 2, these reallocation effects are summarized by

$$-\sum_{m\in K} \tilde{\lambda}_i d\log\frac{r_i+\delta_i}{g+\delta_i} - \sum_{f\in F} \tilde{\Lambda}_f d\log\Lambda_f.$$

Intuitively, if $r_i > g$, then steady-state consumption rises if resources are reallocated to capital intensive activities, and vice versa if $r_i < g$. To understand why this expression captures the effects of resource reallocation, suppose that $r_i > g$ and consider a shock that does not change the profit margin of capital operators: $(r_i+\delta)/(g+\delta_i)$ (for example a productivity shock with an infinitely lived representative agent). In this special case, reallocations raise steady-state consumption if primary factor income shares decline:

$\sum_{f\in F}\tilde{\Lambda}_f d\log\Lambda_f < 0$. Intuitively, if capital operators' margins are constant, then primary factor shares decline if, and only if, capital operators scale up their operations (increasing their profit share of net income and reducing the share of primary factors).

Now, consider what happens if the profit margin of capital operators also rises: $d\log(r_i+\delta_i)/(g+\delta_i) > 0$. In this case, there is a mechanical increase in net capital income and (a commensurate reduction in primary factor income shares) even holding the allocation of resources fixed. This mechanical effect is $\sum_{m\in K}\tilde{\lambda}_i d\log\frac{r_i+\delta_i}{g+\delta_i}$ and must be subtracted from the change in primary factor income shares.[9]

For an example, consider again the neoclassical growth model but suppose that the aggregate elasticity of substitution between labor and capital is $\theta \neq 1$. The balanced-growth input-output matrix is exactly the same as before. In response to an aggregate productivity shock $d\log A_2 > 0$, we now have

$$\frac{d\log C}{d\log A_2} = \tilde{\lambda}_2 - \frac{d\log\Lambda_L}{d\log A_2} = \frac{1}{1-\alpha} + (\theta-1)\frac{RK}{P_c C}\frac{r-g}{r+\delta}.$$

The pure technology effect is unaffected. However, the reallocation effect is now nonzero. If capital and labor are substitutes, $\theta > 1$, and $r > g$, then an increase in aggregate productivity lowers the labor share and boosts aggregate consumption. This is because production substitutes towards using more capital, and if $r > g$, then an increase in the capital stock boosts steady state consumption. This effect is absent from the Cobb-Douglas model because no reallocations take place in response to aggregate productivity shocks in that model.

4 A Quantitative Model of the World Economy

In this section, we put Theorem 1 to work by providing an explicit formulation of our model, with fully specified production and asset demand structures. Our model fe atures a rich production network and overlapping generations of households facing idiosyncratic risk in each country. We use Theorem 1 to characterize long-run comparative statics of this economy.

In the next section, we calibrate this quantitative model and use it to illustrate the importance of accounting correctly for reallocation effects.

[9]In an economy where all production and consumption functions are Leontief, so that the allocation of resources is fixed, the reduction in the weighted sum of primary factor incomes is exactly equal to the increase in the weighted sum of pseudo-markups charged by capital owners: $\sum_{f\in F}\tilde{\Lambda}_f d\log\Lambda_f = -\sum_{m\in K}\tilde{\lambda}_i d\log\frac{r_i+\delta_i}{g+\delta_i}$.

We provide a condensed summary of the main ingredients of the model in the main text, and leave the details in the appendix.

4.1 Production side

Specifying the production side of the model simply requires specifying production functions. We consider an Armington model of trade where each country produces differentiated varieties of the same set of products. We index producers in each country by two indices: (c, i), where $c \in C$ is the country of origin and $i \in N_c$ is the industry-type of the producer (e.g. agriculture, mining, and so on).

The production function of (c, i) is Cobb-Douglas composite of labor, capital, and intermediate inputs from other industries. Intermediate inputs from industry type j purchased by (c, i) are aggregated using a CES aggregator with elasticity θ across different origins (c', j).

We assume that each industry in each country, (c, i), has a specialized capital stock produced by a specialized capital goods producing sector. All other goods in the economy are perishable (infinite depreciation). With some abuse of notation, we use $K_{c,i}$ to denote the quantity of the capital good used by sector (c, i). This capital stock is accumulated through a specialized composite investment good. The investment good of (c, i) is is Cobb-Douglas composite of inputs from different industries. Once again, investment inputs from industry type j from different origin countries are aggregated with CES aggregator with elasticity θ.

Following common terminology in the trade literature, we refer to $\theta - 1$ as the trade elasticity.

4.2 Household side

Each country has an overlapping generation of households. Households in country c die at a constant rate $\nu_c \geq 0$ and there is constant population growth in every country $g_L \geq 0$.

Households born at date t_b in country c select a capital line $i \in N_c$ to invest in and mixmize their utility, given by

$$V_c(t_b) = \max_{i \in N_c} V_i(t_b),$$

where the utility of operating capital line i is given by

$$V_i(t_b) = \max_{n(t,t_b),c(t,t_b),b(t,t_b),k_i(t,t_b)} \mathbb{E} \int_{t_b}^{\infty} e^{-(\rho_c+\nu_c)(t-t_b)} \frac{c_i(t_b,t)^{1-1/\gamma}}{1-1/\gamma} dz$$

subject to

$$\begin{aligned}
n(t_b,t) &= b(t_b,t) + p_i(t)k_i(t_b,t), \\
dn(t_b,t) &= \Big\{ w(t)\ell - p_c(t)c(t_b,t) + \nu_c n(t_b,t) + (1-\tau_c^k)r(t)b(t,t_b) \\
&\quad + (1-\tau_c^k)p_i(t)k_i(t_b,t)[R_i - \delta_i] \Big\} dt + \sigma_i \psi_i p_i k_i(t_b,t) dZ, \\
n(t_b,t) &> -\frac{w(t)\ell}{r+\nu_c-g_A}, \qquad k_i(t_b,t) \geq 0, \qquad n(t_b,t_b) = 0.
\end{aligned}$$

The parameters ρ_c and ν_c are the discount and mortality rate, γ is the intertemporal elasticity of substitution, and $c_i(t_b,t)$ is consumption at t. The first constraint states that household networth is the sum of bonds and physical capital. The second constraint is the budget constraint. It states that changes in household net worth $dn(t_b,t)$, over time, are equal to labor income and capital income minus consumption. Labor income is $w(t)\ell$, consumption is $p_c(t)c(t_b,t)$, and capital income has three sources: annuity payments, bond holdings, and physical capital.

Since households can die with positive assets, we must specify what happens to financial assets after death. Following Blanchard (1985), we assume that households have access to an actuarially fair annuity and optimally choose to annuitize all their wealth, yielding a flow payment $\nu_c n(t_b,t)$ of survival benefits from the annuity, where $n(t_b,t)$ is the networth of the household. Bond holdings, $b(t_b,t)$, pay the riskfree rate $r(t)$. Physical capital pays the net return $R_i - \delta_i$. We allow for bond and physical capital returns to be taxed at the rate τ_c^k. As in Angeletos (2007), households bear idiosyncratic capital income risk $\sigma_i\psi_i dZ$ on their capital holdings, where dZ is an increment of a standard idiosyncratic Brownian motion. The term σ_i is the volatility of idiosyncratic risk in owning capital of type i, and ψ_i encodes a skin-in-the-game condition, due to financial frictions, dictating the share of risk in the Brownian motion to which the capital owner must be exposed.[10]

The remaining constraints are: the natural borrowing limit; that physical capital holdings must be non-negative at all times; and that households' networth is zero at birth.

[10]The financial frictions, and volatility of capital income, come from the fact that capital owners can steal capital by misreporting their idiosyncratic shocks. Capital taxes do not apply to the volatility term because we assume that the capital owner's benefits from stealing capital are not taxable.

4.3 Characterizing Asset Demand

In the appendix, we show aggregating across household's portfolio choices yields an asset demand correspondence that satisfies Assumption 1. In particular, the balanced growth asset demand of each country's for physical capital and bonds scales proportionally with primary factor income in that country and grows at the economy's growth rate of g. This means that we can apply Theorem 1. The following propositions pin down the functional form of the asset demand system.

Proposition 3 (Equalization of effective Sharpe ratios.)**.** *The effective Sharpe ratio across all active capital lines in a country are equated. That is, there exists a scalar $S_c \geq 0$ satisfying $S_c = \frac{r_i - r}{\sigma_i \psi_i}$, for every $i \in K_c$ if $K_i \neq 0$. The scalar S_c is the effective Sharpe ratio for country c.*

Proposition 3 implies that the required return on capital i in country c is

$$r_i = r + \sigma_i \psi_i S_c, \tag{12}$$

where r is the global riskfree rate, $\sigma_i \psi_i$ is the idiosyncratic risk in operating capital i, and S_c is the country-level effective Sharpe ratio. That is, the country-level price of risk h_c is a linear function of volatility, and the slope is given by the effective Sharpe ratio.

To pin down the asset demand equations, define the total wealth of an individual at time t in country c born at date t_b operating capital line $i \in N_c$ to be

$$\omega_i(t_b, t) \equiv n_i(t_b, t) + \frac{\sum_{f \in \mathcal{F}_c} w_f(t) \ell_f}{r + \nu_c - g_A},$$

the sum of financial wealth and human wealth (discounted present value of primary factor income). As in Blanchard (1985), human wealth is discounted by the risk-free return the individual gets on financial wealth (the risk free rate r plus annuity payments ν_c).

To characterize equilibrium in asset markets, we make use of the following consequence of Proposition 3.

Proposition 4 (Total wealth.)**.** *Along the balanced growth path, the growth of expected total wealth for an individual in country c operating capital line $i \in N_c$ is born at date t_b is*

$$\mathbb{E}\left[\frac{\dot{\omega}_i(t_b, t)}{\omega_i(t_b, t)}\right] \equiv g_{\omega,c} - \nu_c = \gamma\left[(r - \rho) + \left(\frac{\gamma + 1}{2}\right) S_c^2\right] - \nu_c.$$

Total wealth in each country, denoted by $\mathcal{W}_c(t)$, is given by

$$\mathcal{W}_c(t) = \sum_{i \in \mathcal{K}_c} \int_{-\infty}^{\infty} \omega_i(t_b, t) dt_b = \chi_c \frac{\sum_{f \in \mathcal{F}_c} w_f(t) L_f}{r + \nu_c - g_A},$$

where $\chi_c = \frac{g_L + \nu_c}{g_A + g_L + \nu_c - g_{\omega,c}}$.

The first part of the proposition implies that the expected growth rate of each individual's wealth is equated across all capital lines in a country. The expected growth rate of each individual's wealth is the expected growth of their wealth conditional on survival, $g_{\omega,c}$, minus their risk of dying, ν_c. The intuition for $g_{\omega,c}$ is that individuals that operate more risky capital types earn a higher return on their investments but they invest a small share of their total wealth so that expected growth rates are the same across all capital lines in each country. The growth rate of individual wealth is higher the higher is the IES (lower risk aversion), γ, the higher is the risk free rate than the discount rate, and the higher is the Sharpe ratio.

The second part of the proposition shows that total wealth in each country is proportional to human wealth in that country, where the constant of proportionality, χ_c is higher the faster is the growth rate of wealth at the individual level and lower the higher is the growth rate of the economy or mortality rates.

The following proposition explicitly characterizes the normalized asset demand equations needed to apply Theorem 1.

Proposition 5 (Normalized asset demands)**.** *Along the balanced growth path, the normalized capital and bond demand in each country satisfy*

$$\mathbf{K}[r, r_i; \psi_i \sigma_i] = \gamma \frac{S_c}{\psi_i \sigma_i} \frac{\chi_c}{r + \nu - g_A},$$

and

$$\mathbf{B}[r, r_i; \psi_i \sigma_i] = \frac{1}{r + \nu - g_A} \left[(\chi_c - 1) - \gamma \frac{S_c}{\psi_i \sigma_i} \right],$$

where S_c and χ_c are defined as in Propositions 3 and 4.

The households' willingness to hold capital is decreasing in risk-aversion $1/\gamma$, and idiosyncratic risk, $\psi_i \sigma_i$, but increasing in the Sharpe ratio, S_c, and wealth relative to labor income $\chi_c/(r + \nu - g_A)$. The households' willingness to hold bonds is given by the difference between their financial wealth (total wealth net of human capital wealth) and their holdings of capital.

4.4 Asset Market Equilibrium

With Propositions 3 and 4 in hand, we can apply Theorem 1 to characterize asset market equilibrium along the balanced growth path.

Theorem 2 (Asset Market Clearing). *Let $g = g_A + g_L$ be the growth rate of the world economy. Then along the balanced growth path, the risk premium on capital $i \in N_c$ satisfies*

$$r_i = r + S_c \psi_i \sigma_i. \tag{13}$$

Capital markets clear in each country

$$\sum_{i \in \mathcal{K}_c} \psi_i \sigma_i \frac{p_i X_i}{g + \delta_i} = \gamma \chi_c \frac{\sum_{f \in \mathcal{F}_c} w_f L_f}{r + \nu_c - g_A}, \tag{14}$$

The world bond market clears

$$\sum_{i \in \mathcal{K}_c} \frac{p_i X_i}{g + \delta_i} = \sum_{c \in C} (\chi_c - 1) \frac{\sum_{f \in F_c} w_f L_f}{r + \nu_c - g_A}. \tag{15}$$

Net bond holdings satisfy

$$b_c = (\chi_c - 1) \frac{\sum_{f \in F_c} w_f L_f}{r + v_c - g_A} - \sum_{i \in K_c} \frac{p_i X_i}{g + \delta_i}. \tag{16}$$

The scalar χ_c is the ratio of total wealth to human wealth, defined in Proposition 4.

Theorem 2 pins down required returns, r_i, the riskfree rate, r, and net factor payments, $(r - g)b_c$, as functions of wages of primary factors and investment. Wages and investments are are determined in equilibrium as functions of returns, $\{r_i\}$, and trade imbalances, $(r - g)b_c$, from the isomorphic static model. This isomorphic static trade model can be analyzed using the results in Baqaee and Farhi (2024). For brevity, we leave those equations in Appendix B.

We discuss the conditions in Theorem 2 one by one. Equation (14) ensures that the quantity of risk in each country, defined as the value of the capital stock in each country weighted by idiosyncratic risk $\sum_{i \in \mathcal{K}_c} \sigma_i \psi_i p_i X_i / (g + \delta_i)$, is equal to households' willingness to hold this risk. Country c's willingness to hold risk is related to risk preferences, γ, the Sharpe ratio, S_c, and the households' total (financial and human) wealth. Households' total wealth, in turn, depends on the discounted value of their primary factor income, $\sum_{f \in F_c} w_f L_f / (r + \nu_c - g_A)$, as well as growth rates and mortality rates. Specifically, total wealth relative to human wealth is higher if mortality, ν_c, or population growth, g_L, are lower. Total wealth relative to human wealth is also higher if the expected growth rate of individuals' total wealth $g_{\omega,c}$ is higher than growth in per capita primary factor income growth g_A.

Equation (15) ensures that the global value of the capital stock, $\sum_i p_i X_i / (g + \delta_i)$ is equal to the total financial wealth of all households, since bond holdings must add up to zero, and financial wealth is the sum of physical capital and bonds. Finally, Equation (16) states that net bond holdings of country c are equal to the difference between the financial wealth of its households and its capital stock.

5 Quantitative Results

One of our contributions is to bring together two literatures that have long been estranged from one another: development accounting and trade. Although both emphasize long-run outcomes, the development accounting literature traditionally neglects international trade, while the trade literature traditionally neglects capital accumulation.[11]

In this section, we calibrate the model in Section 4 and revisit some classic questions studied in these literatures: the consequences of increased trade costs and the consequence of increased schooling, allowing for trade costs affecting capital accumulation, and allowing for schooling to interact with international trade. We find important interactions between trade and capital accumulation, but that these interactions depend crucially on the size of the golden-rule wedge.

5.1 Data Sources

To calibrate our model, we rely on the 2013 release of the world input-output database (WIOD), Timmer et al. (2015), the external wealth of nations database, Lane and Milesi-Ferretti (2018), investment flow tables from Ding (2022), and depreciation rates by industry from the Bureau of Economic Analysis.

[11]Of course, there are exceptions, especially in the trade literature, where there is renewed interest in investment, for example Ravikumar et al. (2019), Kleinman et al. (2023), and Ding (2022).

5.2 Calibration Strategy

We consider the elasticity of various equilibrium outcomes to shocks.[12] As is common in the sufficient statistics literature, these elasticities depend on pre-shock equilibrium outcomes. Below, we discuss how we calibrate these outcomes.

Balanced-growth input-output tables. The balanced-growth input-output matrix, Ω, has the following form:

$$\Omega = \left(\begin{array}{c|c|c} \Omega^{NN} & \Omega^{NK} & \Omega^{NF} \\ \hline \Omega^{KN} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right),$$

where the matrices Ω^{NN}, Ω^{NK}, and Ω^{NF} track expenditures on intermediates, capital services, and labor relative to revenues for each sector in each country.[13] We populate these using the WIOD and the supplementary social and economic accounts tables.[14]

The final matrix, Ω^{KN}, captures the composition of each industry's capital stock. The capital stock of each industry is produced using a mixture of different capital goods. So, for example, Ω^{KN}_{ij} is i's expenditures on investment goods sourced from j relative to i's payments to capital services. We set total investment by each industry in each country equal to gross fixed capital formation from the social and economic accounts data from the WIOD, and split this total investment expenditure across different origins using the shares from Ding (2022).

The implicit markups associated with capital services in each industry are the ratio of capital compensation to gross fixed capital formation for that industry. With some abuse of notation, let μ be a diagonal matrix whose ith diagonal element is the ratio of capital compensation to gross fixed capital formation (i.e. investment) if i is a capital good ($i \in \mathcal{K}$). Then the cost-based balanced-growth input-output matrix is $\tilde{\Omega} = \mu\Omega$.

Elasticities of substitution. In our benchmark model, we set all elasticities of substitution equal to one, except for the elasticity of substitution across origins within industries, which we set equal to five (broadly in line with estimates from (Caliendo and Parro, 2015)).

[12]Log-linearized equations are in Appendix B. As in Baqaee and Farhi (2024), we can compute exact solutions by iterating on the loglinearized system of equations. However, since we are interested in elasticities of long-run consumption to shocks, we do not pursue this.

[13]Recall our convention that consumption aggregators belong to the set of perishable goods.

[14]To eliminate cyclical variations, we average these expenditure shares across the years in our sample from 1995 to 2009.

Depreciation and growth rates. We calibrate all industry-level depreciation rates according to the estimates for depreciation rates in the United States from the BEA. We then calibrate the growth rate of the world economy, $g = g_A + g_L$, so that the initial capital stock of the US relative to gross national expenditures is 2.7. This implies that that growth rate is 1.88%. For simplicity, we set the population growth rate, $g_L = 0$.

Rates of return. The rate of return, r_i, on each capital stock i can be deduced by solving $\mu_i = (r_i + \delta_i)/(g + \delta_i)$, since the left-hand side is the ratio of capital compensation to investment, and everything on the right-hand side is known except r_i. We set the risk-free rate equal to 2%.

Steady-state consumption shares. To calibrate the consumption share of each country along the balanced growth path, we assume that net foreign asset positions relative to primary factor income are at their steady-state values. We calibrate net factor payments according to their steady-state values: that is, we set net factor payments from abroad equal to the net return on risk-free capital, $(r - g)$, times net foreign assets.[15]

Calibrating determinants of asset demand. We set the IES, γ, to 0.5 following micro estimates that are typically less than one (e.g. Best et al., 2020). We calibrate the remaining parameters, the initial Sharpe ratio, S_c, the mortality rate, ν_c, and the discount rate ρ_c in each country c, by simultaneously matching three targets: (i) the ratio of financial wealth to primary factor income; (ii) the ratio of risk-premium income relative to primary factor income; and, (iii) an estimate of the elasticity of asset demand with respect to the risk free rate, from Auclert et al. (2021), equal to 18.[16]

5.3 Calibration Results

We apply our calibration strategy to a simplified world economy consisting of the United States, Canada, Mexico, China, and an aggregate country we call "Rest of World" (ROW).

[15] This causes a small discrepancy between consumption expenditures in the WIOD and in our calibration. This is because along the balanced growth path, the current account must be balanced, with trade deficits offsetting net capital income from abroad. We impose this in our model by setting trade deficits equal to net capital income implied by net foreign asset positions times $(r - g)$. Since $(r - g) = 0.12\%$ in our calibration, and trade deficits are typically small relative to gross national expenditures, the resulting discrepancy between expenditure shares in our calibration and the WIOD are very small.

[16] Formally, the targets are $\frac{b_c + \sum_{i \in \mathcal{K}_c} \lambda_i/(r_i + \delta_i)}{\sum_{i \in \mathcal{F}_c} \Lambda_f} = \frac{\chi_c - 1}{r + \nu_c - g_A}$, $\frac{\sum_{i \in \mathcal{K}_c} [r_i - r] \lambda_i/(r_i + \delta_i)}{\sum_{i_c} \Lambda_f} = \gamma \frac{\chi_c}{r + \nu_c - g_A} S_c^2$, and $\epsilon^d = 18 = -\frac{1}{r + \nu_c - g_A} + \frac{\chi_c}{g_\omega^c - g_A} \gamma$.

To illustrate our framework, we focus on shocks to Mexico as an archetypal middle-income small export-oriented economy.

Parameter	**Description**	**US**	**Mexico**	**ROW**
r	Risk-free rate	0.020	0.020	0.020
g	Growth rate	0.017	0.017	0.017
$\bar{r}_c$	Average return on capital	0.105	0.168	0.156
$\bar{\mu}_c$	Harmonic average wedge on capital	2.368	3.349	2.792
Φ_c	Share of consumption	0.294	0.018	0.627
ν_c	Mortality rate	0.082	0.062	0.088
ρ_c	Discount rate	-0.024	0.026	-0.008
S_c	Sharpe ratio	0.222	0.347	0.266
b_c	NFA relative to GDP	-0.165	-0.348	0.085
T_c	Trade balance relative to GDP	-0.000	-0.001	0.000
χ_c	Ratio of total to human wealth	1.400	1.688	1.361
$g_{\omega c}$	Wealth growth (conditional on survival)	0.041	0.042	0.041

Table 1: Summary of steady-state calibrated values for US, Mexico, and ROW

Table 1 summarizes some of the results of our calibration exercise for the US, Mexico, and ROW.[17] In our calibration, the risky return is high in all countries, but is higher on average in Mexico and the ROW than in the US. Recall that the risky return for each capital type i satisfies $(r_i + \delta_i)/(g + \delta_i) = \mu_i$. Hence, the higher is the Golden Rule wedge for capital stock i, the higher is the return on capital i. The golden rule wedge, in turn, is given by the ratio of capital compensation to investment: $\mu_i = R_i K_i / X_i$. Hence, r_i and μ_i tend to be higher in countries where capital compensation, measured as value-added minus labor compensation, is larger than investment. Relatedly, Table 1 shows that the average (harmonic) implicit markup on capital services in our calibration is quite high (the average tends to be above 2). This suggests that the initial equilibrium is very far from maximizing long-run consumption, and so reallocation effects can potentially play a big role in equilibrium responses.

Since we have an overlapping generations model where agents have finite lives, the effective discount rate in our model is the sum of the mortality and discount rate (which must always be positive in order for utility to be bounded). Mexico is the most patient country and the US is the least patient country. Sharpe ratios are around 0.3, but lower for the US and higher for Mexico and ROW. In our calibration, the US and Mexico are net borrowers, whereas ROW is a net saver. This implies that the US and Mexico must run

[17] To save on space, we do not report calibration results for Canada and China.

small trade surpluses in steady state, whereas ROW runs small trade deficits. Finally, the calibration implies similar rates of growth in wealth conditional on survival (around 4%).

5.4 Counterfactual Experiments

We consider three types of shocks to Mexico. An increase in iceberg trade costs of exporting, an in increase in schooling, and a decrease in financial frictions.

Iceberg Costs of Exporting. Table 2 reports the elasticity of consumption for Mexico and the world to iceberg trade costs for Mexican exporters. The first column is our benchmark calibration showing that Mexican long-run consumption declines 0.55 log points when iceberg costs of exporting rise by 1 log point. For the world, the reduction in long-run consumption is 0.015 log points.

We can decompose the losses into a technology effect and a reallocation effect along the lines in Baqaee and Farhi (2024). The technology effect is the reduction in consumption if the share of each resource going to each use is held constant, and the reallocation effect is the reduction in consumption from the fact that in equilibrium the share of each resource going to each use changes.[18] The equilibrium outcome is the sum of these two effects.

For Mexico, most of the negative effects are coming from reallocation effects, since Mexican consumers are not heavily exposed to Mexican exports. For the world, more than 1/3 of the effect is due to reallocation effects. Intuitively, an increase in iceberg costs reallocates resources towards less capital intensive uses, and since capital is under-produced in the initial equilibrium (for the purposes of steady-state consumption), this reallocation reduces global consumption.

The second column matches the same data but instead lowers all trade elasticities from their benchmark value, 5, down to 1 as in Cole and Obstfeld (1991). In this case, there is no expenditure-switching, and hence the share of each resource going to each agent is constant in equilibrium. Since there is no reallocation, this simply knocks out the reallocation effect.

The third column sets $\mu_i = (r_i + \delta_i)/(g + \delta_i) = 1$. To achieve this in our calibration, we hold $\tilde{\Omega}$ constant but assume that $\delta_i = \infty$. In this calibration, capital does adjust in response to the shock, and investment goods are still used to produce capital. So the standard capital as an intertemporal intermediate input amplification mechanism does operate. However, there is no "wedge" on capital services — the price of capital services is equal to investment costs and capital income net of investment is zero. The stark dif-

[18] This is an additively linear decomposition since these are elasticities.

ference between the first and third column show that capital per se is not quantitatively important for these elasticities. Instead, one must account for capital accumulation and the golden rule wedge μ_i.

To better understand column three, focus on the Mexican response first. For this calibration, the elasticity of Mexican consumption is -0.12 instead of -0.56. The technology effect is the same as before, but the reallocation effect is now much weaker. The difference is driven by changes in net capital income. In the benchmark model, the iceberg shock drastically reduces capital income earned by Mexico. In the static model, net capital income is zero, and so that effect disappears.

Now consider the world response, which is the same it would be in a Cobb-Douglas model without reallocations. The reason is that reallocations, which do happen in equilibrium, are now irrelevant — the $\mu_i = 1$ model has the same elasticities as the benchmark model. Hence, reallocations *do* take place in equilibrium. However, these reallocations are irrelevant since the initial equilibrium maximies steady-state world consumption. In fact, by 1, we know that the world consumption effect of -0.0096 is just the negative of the revenue of all Mexican exporters divided by world consumption.

The last column is a static calibration as in Baqaee and Farhi (2024) or Costinot and Rodriguez-Clare (2014). The static calibration treats capital as an endowment and adds investment to consumption. Focus on the Mexican response first. Compared to the benchmark model, the elasticity is much smaller. This is because both the technology and reallocation effects are smaller. The technology effect is smaller because a static model does not account for the round-about nature of investment: Mexican exports contribute, indirectly, towards Mexican investment and capital accumulation. The static model neglects this effect, so the direct technology effect is smaller. Since the local technology effect is quantitively small anyway, this effect is not so important. The big differences between columns one and fourth for Mexico are driven by reallocation effects — specifically, Mexican capital income. In the benchmark model, net capital income in Mexico falls by much more than in the static model. In the dynamic setup, a key effect of increasing trade costs was to reduce the size of Mexico's capital stock. This does not happen in the static model where the capital is simply an endowment and investment is a final expenditure, with no accounting of the dynamic connection between the two.

Finally, consider the world response. The response of world consumption is $1/3$ as large as in the benchmark model. There are two reasons for this. First, the direct technology effect is smaller. Intuitively, this is the traditional capital multiplier logic of the neoclassical growth model. The static model neglects the fact that lower investment results in a smaller capital stock, and hence less production. The second reason is that the reallocation effect in the static model is zero. The reason is that, in the static model, world consumption is maximized in the initial equilibrium.

Metric	**Benchmark**	$\theta = 1$	$\mu_i = 1$	**Static**
Mexican consumption	-0.55674	-0.01349	-0.12360	-0.11964
Local technology	-0.01349	-0.01349	-0.01349	-0.00208
Local reallocation	-0.54325	0.00000	-0.11011	-0.11756
World consumption	-0.01598	-0.00960	-0.00960	-0.00435
World technology	-0.00960	-0.00960	-0.00960	-0.00435
World reallocation	-0.00638	0.00000	-0.00000	0.00000

Table 2: Elasticity of consumption, and its decomposition, in response to iceberg shock to Mexican exporters

Overall, these results illustrate several lessons. The first is that accounting for capital accumulation is extremely important for understanding the long-run consumption response of trade shocks. The second is that the importance of capital accumulation depend critically on how far the elasticity of substitution, especially the trade elasticity, is from one, and how large the golden rule wedge is $(r_i - g)/(g + \delta_i)$. The importance of accounting for capital accumulation is much smaller, if either elasticities are all close to one or the golden rule wedge is closer to zero.

Returns to Schooling. We now consider the elasticity of Mexican consumption and world consumption to the supply of high-skilled Mexican labor. The results are reported in Table 3.

Metric	**Benchmark**	**Capital-skill compl.**	$\theta = 1$	**Cobb-Douglas**	$\mu = 1$	**Static**
Mexican consumption	0.20123	0.19680	0.11399	0.11933	0.12370	0.07672
Local technology	0.11933	0.11933	0.11933	0.11933	0.11933	0.06789
Local reallocation	0.08190	0.07747	-0.00534	0.00000	0.00437	0.00882
World consumption	0.00379	0.00407	0.00311	0.00286	0.00286	0.00136
World technology	0.00286	0.00286	0.00286	0.00286	0.00286	0.00136
World reallocation	0.00092	0.00121	0.00024	0.00000	0.00000	0.00000

Table 3: Elasticity of consumption, and its decomposition, in response to increase in Mexican high skilled labor

In the benchmark model, the elasticity of Mexican consumption to high-skilled labor is 0.20 — with roughly half of the effect coming from direct technology effects and half of it coming from reallocation. For comparison, the local Domar weight of high skilled labor (Mexican high skilled labor divided by Mexican GDP) in this calibration is 0.078 and balanced growth Domar weight (the ratio of high skilled labor income to Mexican consumption) is 0.098. Hence, the effect of increasing high-skilled labor on consumption is much larger than the increase in GDP predicted by Hulten (1978), 0.078, or the increase in steady-state consumption in a closed economy at the golden rule, 0.098.

For the world, about 1/3 of the consumption effect is due to reallocation effects, whereby an increase in Mexican high-skilled labor redistributes production towards relatively more capital intensive activities, and this, ipso facto, raises consumption due to the presence of the initial wedge on capital services.

The second column alters the benchmark model to allow for capital-skill complementarity as in Krusell et al. (2000). That is, we assume that for each industry, value-added is a CES aggregate of two bundles: a Cobb-Douglas bundle of low and medium skilled labor, and a CES bundle of capital services and high-skilled labor. Following Krusell et al. (2000), we assume that capital services and high-skilled labor are complements, with elasticity of substitution 2/3, whereas the bundle of skilled labor equipped with capital is substitutes with the bundle of low- and medium-skill labor, with elasticity of substitution 5/3.

This calibration produces results that are similar to the benchmark model. The reallocation effects for the world are modestly stronger since the increase in high-skilled labor further boosts the reallocation towards capital-intensive activities when there is capital-skill complementarity. However, this does not translate into larger gains for Mexico. This happens because in the calibration with capital-skill complementarity, an increase in high-skilled labor causes offsetting effects on Mexican income and Mexican prices. On the one hand, the increase in net capital income is slightly higher with capital-skill complementarity, but on the other hand, the required rates of return on Mexican capital, and hence the user cost of capital services, are also higher. These have offsetting effects on long-run Mexican consumption.

Column three lowers all trade elasticities from their benchmark value, $\theta = 5$, to one. This significantly reduces the extent of reallocation effects, since the only reallocation happens due to capital-skill complementarity. This shows that expenditure-switching through trade is quantitatively much more important expenditure-switching across primary factors for the effects of increasing high-skilled labor in Mexico. The fourth column sets all elasticities equal to one, once again, neutralizing all reallocation effects in the model.

The fourth column sets the golden rule wedge to zero, once again by setting depreciation to infinity, so that capital effectively becomes an intermediate input. This significantly reduces positive reallocation effects for both Mexico and the world as a whole. For Mexico, the reason is that, in the benchmark model, upskilling boosts Mexican net capital income significantly and when $\mu_i = 1$, there is no net capital income. For the world as a whole, the reason is that when $\mu_i = 1$, world steady-state consumption is already being maximized, and so reallocation towards capital has no effects on global consumption to a first order (by the envelope theorem).

Comparing the fourth column to the final column illustrates the importance of modeling capital accumulation per se (abstracting from the golden rule wedge). In a static model, the response of consumption is less elastic because the static model ignores the roundabout nature of capital production. Nevertheless, simply adding capital accumulation is not enough, and once must also account for the presence of the golden rule wedge. Abstracting from the golden rule wedge strongly limits the importance of reallocation effects at the country level and eliminates them altogether at the global level.

Overall, Table 3 illustrates some important points. First, accounting for capital accumulation is important for understanding the long-run consumption response of upskilling. There are several reasons for this. First, more skilled workers contribute to the production of more capital, which contributes to the production of more consumption. This can be seen by comparing the static column to the column where $\mu = 1$. However, this is far from the whole story. If the economy is not operating at the golden rule $r_i > g$, then an increase skilled labor in Mexico will reallocate resources towards capital intensive activities — this generates additional gains for Mexico, through an increase in net capital income — and additional gains for the world, since capital is underproduced relative to what would maximize long-run consumption. These reallocative effects operate primarily through trade elasticities being larger tha one, and are quantitatively even more important than the pure effects of capital accumulation per se.

Financial Frictions. The previous two examples show that capital accumulation and trade have important interactions, but only if the golden-rule wedge is nonzero. Our final example considers how a reduction in financial frictions, which have no mechanical technological effects, affect long-run consumption.

Consider lowering the riskiness of Mexican capital in each industry to its level in the US, if frictions in Mexico are higher.[19] Table 4 reports the results of this experiment. The

[19]For some industries, like tobacco and construction, our calibration gives lower frictions in Mexico than

Mexican consumption	0.523
Change in nominal income	0.536
Change in consumption price index	0.013
Change in labor income	0.188
Change in capital income	0.347
Change in net factor payments from abroad	0.001

Table 4: Reducing financial frictions in Mexico

first row shows that this results in a very large increase in long-run Mexican consumption of 0.523. The remaining rows of Table 4 decompose the consumption effect into the change in Mexican net national income and the change in the price index of consumption (where world nominal consumption is the numeraire).

Almost the entirety of the effect is driven by an increase in net national income in Mexico. The remaining rows of Table 4 further decompose the increase in Mexican net national income into changes in labor income, changes in net capital income, and changes in net factor payments from abroad. We see that the majority of the effect is driven by an increase in net capital income and in wages.

Net capital income in Mexico rises despite the fact that the shock lowers the required return on Mexican capital: the average risk premium in Mexico falls from 0.48 to 0.29. In this sense, Mexico is on the "wrong" side of its net capital income Laffer curve — a reduction in returns raises net capital income because it greatly increases Mexican capital services. The shock also boosts Mexican wages since an increase in demand for Mexican goods raises Mexican wages.

The consumption price index for Mexico changes very little because of two offsetting effects. On the one hand, the increase in Mexican wages pushes up the Mexican consumption price index. Due to home bias, Mexican goods are disproportionately produced using Mexican labor, so an increase in Mexican wages raises the price of Mexican consumption goods. However, this effect is offset by the fact that the user cost of Mexican capital declines due to the reduction in the required return on capital. These two effects roughly offset one another.

in the US.

6 Conclusion

In this paper, we provide a framework for analyzing long-run comparative statics of dynamic disaggregated economies by representing them as distorted static disaggregated economies. This representation not only allow us to use tools from the static literature to study these models, but it also helps provide intuition about why disaggregated details can matter for understanding long-run comparative statics. The details matter because dynamic equilibria typically do not maximize long-run consumption. This is because in a dynamic decentralized equilibrium, capital compensation typically exceeds investment costs.

The fact that dynamic equilibria typically do not maximize long-run consumption implies that reallocation effects matter to a first order for how long-run consumption responds to shocks. In particular, any shock that reallocates resources towards capital intensive activities boosts long-run consumption. Using a quantitative model, we show that these reallocation effects are very important.

References

Alessandria, G., C. Arkolakis, and K. J. Ruhl (2021). Firm dynamics and trade. *Annual Review of Economics 13*, 253–280.

Alvarez, F. (2017). Capital accumulation and international trade. *Journal of Monetary Economics 91*, 1–18.

Angeletos, G.-M. (2007). Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic dynamics 10*(1), 1–30.

Angeletos, G.-M. and V. Panousi (2011). Financial integration, entrepreneurial risk and global dynamics. *Journal of Economic Theory 146*(3), 863–896.

Auclert, A., H. Malmberg, F. Martenet, and M. Rognlie (2021). Demographics, wealth, and global imbalances in the twenty-first century. Technical report, National Bureau of Economic Research.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992). International real business cycles. *Journal of political Economy 100*(4), 745–775.

Baqaee, D. R. and E. Farhi (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics 135*(1), 105–163.

Baqaee, D. R. and E. Farhi (2024). Networks, barriers, and trade. *Econometrica 92*(2), 505–541.

Best, M. C., J. S. Cloyne, E. Ilzetzki, and H. J. Kleven (2020). Estimating the elasticity of intertemporal substitution using mortgage notches. *The Review of Economic Studies 87*(2), 656–690.

Bigio, S. and J. La'O (2016). Financial frictions in production networks. Technical report.

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of political economy 93*(2), 223–247.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008). An equilibrium model of âglobal imbalancesâ and low interest rates. *American economic review 98*(1), 358–393.

Caliendo, L. and F. Parro (2015). Estimates of the trade and welfare effects of nafta. *The Review of Economic Studies 82*(1), 1–44.

Cole, H. L. and M. Obstfeld (1991). Commodity trade and international risk sharing: How much do financial markets matter? *Journal of monetary economics 28*(1), 3–24.

Costinot, A. and A. Rodriguez-Clare (2014). Trade theory with numbers: quantifying the consequences of globalization. *Handbook of International Economics 4*, 197.

Dekle, R., J. Eaton, and S. Kortum (2007). Unbalanced trade. *American Economic Review 97*(2), 351–355.

Ding, X. (2022). Capital services in global value chains.

Dix-Carneiro, R., J. P. Pessoa, R. Reyes-Heroles, and S. Traiberman (2023). Globalization, trade imbalances, and labor market adjustment. *The Quarterly Journal of Economics 138*(2), 1109–1171.

Farhi, E. and F. Gourio (2018). Accounting for macro-finance trends. *Brookings Papers on Economic Activity*, 147–223.

Foerster, A. T., A. Hornstein, P.-D. G. Sarte, and M. W. Watson (2022). Aggregate implications of changing sectoral trends. *Journal of Political Economy 130*(12), 3286–3333.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, 1127–1150.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies*, 511–518.

Jones, C. I. (2011). Intermediate goods and weak links in the theory of economic development. *American Economic Journal: Macroeconomics*, 1–28.

Kleinman, B., E. Liu, S. J. Redding, and M. Yogo (2023). Neoclassical growth in an interdependent world. Technical report, National Bureau of Economic Research.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica 68*(5), 1029–1053.

Lane, P. R. and G. M. Milesi-Ferretti (2018). The external wealth of nations revisited: international financial integration in the aftermath of the global financial crisis. *IMF Economic Review 66*, 189–222.

Liu, E. (2017). Industrial policies and economic development. Technical report.

Long, J. B. and C. I. Plosser (1983). Real business cycles. *The Journal of Political Economy*, 39–69.

Lyon, S. and M. E. Waugh (2019). Quantifying the losses from international trade. *Unpublished manuscript*.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica 71*(6), 1695–1725.

Mendoza, E. G., V. Quadrini, and J.-V. Rios-Rull (2009). Financial integration, financial development, and global imbalances. *Journal of Political economy 117*(3), 371–416.

Obstfeld, M. and K. Rogoff (1995). The intertemporal approach to the current account. *Handbook of international economics 3*, 1731–1799.

Phelps, E. (1961). The golden rule of accumulation: a fable for growthmen. *The American Economic Review 51*(4), 638–643.

Ravikumar, B., A. M. Santacreu, and M. Sposi (2019). Capital accumulation and dynamic gains from trade. *Journal of International Economics 119*, 93–110.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. De Vries (2015). An illustrated user guide to the world input–output database: the case of global automotive production. *Review of International Economics 23*(3), 575–605.

Vom Lehn, C. and T. Winberry (2022). The investment network, sectoral comovement, and the changing us business cycle. *The Quarterly Journal of Economics 137*(1), 387–433.