Screening Using a Menu of Contracts in Imperfectly Competitive Markets with Adverse Selection

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Abstract

I develop a quantitative model of screening with menus using a demand system that nests various degrees of competition, from perfect competition to monopoly. I show the existence of a pure strategy Nash equilibrium and characterise it in closed form. I then analyse a contractual externality and build a sufficient statistic to test for its empirical relevance. I also use the model to study credit market policies in the presence of screening. I show that contrary to conventional wisdom, increasing capital requirements, increasing the Federal Reserve rate, or decreasing competition can increase lending. I provide an empirical application in the context of consumer credit and show that, due to the externality, the menus contain too many maturity options. Model parameters are recovered using a linear regression of prices on quantities controlling for contract market shares.

1 Introduction

In many selection markets,[1] firms use menus of contracts to make their customers reveal their private information through their choices (i.e., screening). Examples include the insurance market (Handel, Hendel, and Whinston 2015; Einav, Finkelstein, and Tebaldi 2019); the mortgage market (Taburet, Polo, and Vo 2024); the market for credit cards (Nelson 2018); and the housing rental market (Humphries, Nelson, Nguyen, Dijk, and Waldinger 2024, Calder-Wang and Kim 2024).

To quantify frictions in selection markets and perform counterfactual analysis, the empirical literature relies on quantitative models (see Einav, Finkelstein, and Mahoney 2021 for a survey and Veiga and Weyl (2016) or Mahoney and Weyl (2017) for recent examples). Yet, existing frameworks abstract from screening, which can lead to wrong conclusions by neglecting the interaction between contracts. This omission largely stems from screening models' being unsuitable for empirical work. Existing models of screening in selection market consider extreme market structures and face issues of equilibrium non-existence (e.g., monopoly for Stiglitz 1977, and perfect competition for Rothschild and Stiglitz 1976 or Azevedo and Gottlieb 2017). As such, their predictions heavily depend on the choice of market structure or equilibrium refinement used to solve the model.[2]

This paper bridges theory and empirical research on adverse selection. I incorporate imperfect competition in an otherwise standard screening model à la Rothschild and Stiglitz (1976). Following the empirical industrial organization literature, I model competition using a discrete choice approach, as McFadden (1981). This approach has two benefits. First, it makes the model more realistic thanks to demand functions that nest various degrees of competition, from perfect competition to monopoly. Second, it gives the model convenient analytical properties under very mild conditions (existence of the equilibrium, with analytical,

1. Following Einav, Finkelstein, and Mahoney 2021, we refer to selection markets as markets in which consumers vary in how much they are willing to pay for a product and how costly they are to the seller.

2. See, Riley 1979, Wilson 1980 for equilibrium refinements and Handel, Hendel, and Whinston (2015) for and empirical application.

empirical and computational tractability), allowing it to cater to various empirical set-ups (multiple customer types, lenders and product characteristics).

I use the framework to analyze theoretically how incentives to screen and welfare vary with competition or policies affecting firms' marginal costs. I also discuss the identification and estimation of model parameters for a situation in which an econometrician has access to data on the menus of contracts and choices for one firm (e.g., Hertzberg, Liberman, and Paravisini 2018); or when they only observe the contracts chosen for the full market (e.g., credit register data, as in Crawford, Pavanini, and Schivardi 2018). In particular, I discuss identification issues that are specific to finance and such, have not been addressed by the industrial organisation literature.[3] Finally, I provide a working example in the context of consumer credit using data from Hertzberg, Liberman, and Paravisini (2018).

Allowing for imperfect competition has three main effects on the analysis. First, it solves the equilibrium non-existence problem. The model delivers a pure strategy Nash equilibrium, which I characterize in closed form for some special cases. The equilibrium existence relies on the demand functions being continuous and concave. The result holds for all degrees of competition and in the context of multidimensional screening. The closed-form characterization is important for empirical applications because it allows me to derive sharp moment conditions and renders the exclusion restrictions transparent.

Second, the competition between firms creates a contractual externality, for which I derive a sufficient statistic. I show that the market equilibria can be inefficient in the second-best sense, so an informationally constrained social planner can create a menu that makes both firms and their clients better off. This externality is costly only when the model parameters are such that the equilibrium in Rothschild and Stiglitz (1976) does not exist. The inefficiency results from the fact that when lenders screen borrowers using a menu—in particular when they cream-skim—, they change the composition of their competitors' borrowers, and thus

3. For instance, the fact that willignness to pay for contract features are endogeneous objects as they depend on borrowers' default probabilities or the fact that marginal cost are easier to estimate directly than demand elasticities in banking markets.

their screening cost.

Focusing on the second best is relevant when the policies that restore the first best are too costly (e.g., ban of some products coupled with re-distributive taxes). I show that when achieving the first best outcome is not possible, government interventions designed to prevent cream-skimming — such as, for instance, several rules in the 2010 U.S. Patient Protection and Affordable Care Act or the 2010 Dodd-Frank reforms on mortgages— can make market participants worse off.

Third, I show that policies that affect competition or the cost of originating a contract have unintended consequences due to the presence of competition and screening. For instance, in the context of credit markets, increasing capital requirements, increasing the Federal Reserve rate, or decreasing competition can increase lending.

To be close to the empirical application, I present my model in the context of credit markets. However, the framework is general enough to capture labour markets, insurance markets or other financial markets.[4] I provide micro-foundations for the functional forms used and develop a unifying framework for screening in credit markets.

In the baseline model, a contract comprises a loan size and an interest rate. Borrowers have quasilinear utility over the present value of the loan. They have private information about their willingness to pay (WTP) for each dollar borrowed and their default probabilities. To study selection markets, I allow borrowers' WTP to be positively correlated with their default probabilities. In that context, screening is achieved by setting the interest rate above the low-default borrower's WTP for larger loans. That way, high-default borrowers self-select into higher-rate contracts. Screening can, therefore, restore perfect information pricing. However, it also requires that low-default borrowers get a smaller loan than high-default borrowers, which is not necessarily what would happen in the first best.

To model borrowers' choice of lender, I assume that part of borrowers' utility contains

4. A key requirement is that contracts are effectively exclusive; see, for instance, Attar, Mariotti, and Salanié 2011 for non-exclusive contracts under perfect competition. Search costs may prevent borrowers from getting multiple contracts in practice, which allows firms to screen.

a borrower-bank-specific random demand shock with a continuous probability distribution function, which enters the utility either multiplicatively or additively. This shock can be interpreted as borrowers having search costs or being imperfectly informed about the contracts offered by each bank (as in Varian 1980, or Rochet and Stole 2002). I consider the situation in which demand shocks are uncorrelated with borrowers' preferences for contract characteristics, so banks cannot use their menus to sort borrowers on their random shock realization. In particular, when the shocks follow an extreme value distribution, its variance parameterizes the product demand elasticity. In the limit case in which the variance of the shock tends to infinity, each lender behaves like a monopolist. When the variance tends to zero, borrowers' demand elasticity becomes infinite, as in the perfect competition case.[5]

I show that when demand elasticities are high, the equilibrium menu consists of the Rothschild Stiglitz separating contracts in which lenders break even on each contract. However, contrarily to Rothschild and Stiglitz (1976), offering separating contracts is an equilibrium no matter the relative number of high versus low default borrowers. In Rothschild and Stiglitz, separating contracts cannot be offered when the relative number of low-default borrowers is high. This is because, in that case, being pooled with high-default borrowers is not too costly for low-default borrowers, so a competitor can offer a pooling contract that makes all borrowers better off. In my model, Rothschild and Stiglitz deviations from screening attract relatively too many high-default borrowers to be profitable. This is because those deviations are more valuable to high-default borrowers, and with concave demand functions, there is always a mass of borrowers that are harder to attract.

I discuss how to accommodate the model to various empirical setups by including endogenous default (moral hazard, burden of payment), maturity, loan size and interest rate and an arbitrary number of borrowers and lenders. I also extend my theoretical results—existence of the equilibrium, and convergence to separating contracts equilibrium when demand elas-

5. Multiplicative shocks imply that our demand functions are concave no matter the degree of competition, leading to better analytical results than additive ones.

ticities tend to infinity[6]—to the multidimensional screening case with an arbitrary number of contract terms.

The identification and estimation of the model parameters rely on using equilibrium conditions linking price to quantities and contract shares. It enables me to recover the marginal cost, the information rent, and the demand semi-elasticity as a constant and a slope of linear regression. My approach does not require estimating default elasticities, nor does it require data on menus of contracts offered by other lenders to recover demand elasticities. This is convenient because those elasticities are difficult to estimate under adverse selection, given that borrowers who chose different contracts are not comparable.

To illustrate the approach, I provide an application using data from an online lending platform (Lending Club) as in Hertzberg, Liberman, and Paravisini (2018).[7] I use the estimates to decompose the equilibrium interest rate into a break-even price, a perfect information markup and an information premium or discount. The information premium divided by the perfect information markup of the contract designed for high-default borrowers is a sufficient statistic for the existence of the contractual externality.

Interpreting the regression error term through the lens of the theoretical model, I show that the independent variables need to be uncorrelated with unobserved demand heterogeneity. As a result, I use the variation coming from the borrower's observable risk category. The identifying assumption is that the average demand elasticity to the subjective loan present value is uncorrelated with the borrower risk category assigned by the lender conditional on contract choice.

As in Hertzberg, Liberman, and Paravisini (2018), I find that high-default borrowers self-select into higher-maturity loans. Using the structural model, I can also characterize the distortions caused by adverse selection and imperfect competition. I show that prices

6. As long as separation is possible while pricing at marginal cost.

7. Although contracts are not formally exclusive in this market, the fact that Hertzberg, Liberman, and Paravisini (2018) finds that screening with menus happens in practice shows that using Rothschild and Stiglitz (1976) framework (exclusive contracts) instead of Attar, Mariotti, and Salanié (2011) (non-exclusive contracts) is a valid approach in this context.

are close to the first best case, but maturities are not. Borrowers who choose high-maturity contracts are less price elastic but get an asymmetric information discount that mitigates the competition channel. The information rent is equal to 15 percent of the loan's present value. Using sufficient statistics, I show that both borrowers and lenders would be better off if lenders could cross-subsidize customers in order to increase the maturity of their lower-maturity contracts while maintaining incentives to self-select. The threat of cream skimming prevents this situation from emerging.

The rest of the paper is structured as follows. Section 2 provides a literature review. I describe the model setup in section 3. Section 4 analyzes incentives to screen and characterizes the market equilibrium. Section 6 discusses extensions and how to bring the model to the data. Section 7 presents the empirical application. Section 8 concludes.

2 Literature Review

This paper contributes to the theoretical literature on adverse selection. It is more closely related to the strand of the literature studying screening with menus in selection markets. Most papers assume either perfect competition (Rothschild and Stiglitz 1976) or monopoly (à la Stiglitz 1977), and rely on equilibrium refinements to analyse the perfect competition case. The use of equilibrium refinements emerged as pure strategy equilibriums may not exist, and mixed strategy equilibriums are difficult to compute even in stylized frameworks (see Farinha Luz (2017) for the first numerical characterization in a Rothschild and Stiglitz (1976) model). Recent examples of perfect competition models are those of Azevedo and Gottlieb (2017)—which proposes an equilibrium refinement micro-founded using a capacity constraint— and Guerrieri, Shimer, and Wright (2010)—with a pure strategy characterization of the equilibrium using a model in which each principal can match with at most one agent.

My paper extends Azevedo and Gottlieb (2017) to allow for imperfect competition. I also show that concave demand functions (micro-founded by multiplicative demand shocks) disci-

pline the model under perfect competition without the need to impose a capacity constraint on lenders.

My paper is closely related to Chade and Swinkels (2021) and Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), who use a framework with imperfect competition and menus of contracts. Chade and Swinkels (2021) focus, similarly to Rochet and Stole 2002, on a situation without adverse selection: the cost of serving customers is homogeneous so menus are used to screen agents with heterogeneous willingness to pay.[8] As a result, the model does not describe a selection market, and the perfect competition equilibrium always exists and replicates the first best outcome. In contrast, this paper focuses on a situation closer to the banking market (i.e., a selection market), where the cost heterogeneity is on the agent, not the principal.

Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019) is closer to my framework. They use a search model à la Burdett and Judd (1983) to model imperfect competition in an otherwise standard screening model with goods of different quality. Contrarily to Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), I focus on building an empirically tractable model, and on a theory of the second best for my welfare analysis. To do so, I use a continuous demand system instead of a discrete one. It allows me to prove the existence of the equilibrium under more general conditions and guarantees the existence of a pure strategies equilibrium. Those two conditions are crucial for empirical tractability, to make the source of variation used to recover the model parameters transparent, and to derive sufficient statistics.

My paper also contributes to the growing empirical literature on adverse selection. A first strand of the literature developed tests for adverse selection using reduced-form approaches (Chiappori and Salanie 2000, Karlan and Zinman 2009, Hertzberg, Liberman, and Paravisini 2018). Following Einav, Finkelstein, and Cullen (2010), a second strand of the literature developed models to estimate the cost of adverse selection. Those papers are based on models with a single contract such as Akerlof (1978) and generally use a sufficient statistic approach. Examples include Veiga and Weyl (2016) for a model with one contract and two

8. Unlike Rochet and Stole 2002, the principal in Chade and Swinkels (2021) has heterogeneous costs of producing a good of a given quality.

characteristics (price and quality) offered by a monopolist, and Mahoney and Weyl (2017) for a framework using Bresnahan (1989) conduct approach to model competition. In credit markets, the literature is less based on the sufficient statistic approach and has focused on quantitative models in which lenders can only choose interest rates (e.g., Crawford, Pavanini, and Schivardi 2018, Nelson 2018, Darmouni 2020, Yannelis and Zhang 2021).

My paper differs from previous empirical works in that it looks at a situation where lenders can screen their customers by designing a menu of contracts. I also focus on the second-best equilibrium for my welfare analysis.[9] My paper can thus provide insight into the range of products offered and can be used for welfare analysis and to perform counterfactual simulations. Furthermore, my modelling of competition is based on a discrete choice approach rather than Bresnahan (1989). I can thus speak to a wider variety of setups, even when moving away from the perfect competition and the monopoly cases.

The closest paper is Handel, Hendel, and Whinston (2015), which uses Rothschild and Stiglitz (1976) in the context of health insurance. In contrast, my paper focuses on credit markets, and the empirical predictions do not rely on the use of equilibrium refinement or the assumption that the market is perfectly competitive, which may not hold in credit markets.

3 Model Set-up

3.1 General considerations

Economic Agents: The economy is populated with two groups of agents: borrowers and lenders. I also refer to the second group as banks. There is a finite number of banks B indexed by b. There is a discrete number of borrowers indexed by j. Borrowers can be of different types, indexed by $i \in \Gamma$. The number of type i borrowers is denoted n_i. The number

9. The closest paper is by Taburet, Polo, and Vo (2024), who use a structural model of screening for default probability. I make different modelling assumptions to analyse the theoretical properties of the model and develop a tractable model that allows studying a wider range of counterfactual experiments. I also propose a different research design that can be applied when only data on an individual firm are available.

of agents is exogenous.

Timing and information structure: There are two periods. In the first period, borrowers choose at most one loan contract from one lender. After the loan is originated, borrowers may experience an income shock that triggers default. The loan matures in the second period.

Lenders cannot observe borrowers' willingness to pay for contract terms, their default probabilities and how likely they are to move to a competitor. However, they know the distribution of those variables and can use menus of contracts to make borrowers reveal their type.

3.2 Borrowers

In this section, I model borrowers' decision to participate in the credit market as well as their choice of loan contract and repayment behaviour. All parameters defined in this section are part of borrower i information set at the time they choose their contract and bank.

3.2.1 Choice of contract and bank

Utility derived from contracts: The utility of type i borrowers is additively separable in contract characteristics and prices. Formally, we have:

$$u_i(X, R) := \alpha_i[\overbrace{\sum_{d=1}^{D} \theta_{id} v_d(X_d)}^{v(X;\theta_i)} - R] \tag{1}$$

$(X := (X_d)_d, R)$ are contract terms. R can be interpreted as the price of a contract with characteristics X. $D \geqslant 1$ is the number of contract characteristics.

$(\alpha_i, \theta_i := (\theta_{id})_d)$ parameterize the borrower utility. I loosely refer to θ_i as the borrower's willingness to pay. $\alpha_i \theta_{id} v_d(X_d)$ is the part of the utility that derives from contract term X_d.

$v_d(X_d)$ is a one-to-one continuous function. $\alpha_i R_i$ is interpreted as the loan present value.

Example of utility: To fix ideas, let us interpret the borrower as a firm that uses an NPV rule to make their choice of investments. The firm uses a constant discount rate $\delta_i := (1 - d_i)$, with $d_i \in [0, 1)$ being the per period exogenous default probability. The firm borrows an amount L to generate cash flows in each period equal to $Z_i F(L)$ for T years. The costs of the loan are the expected monthly payments (m) conditional on survival $(1-d_i)$, and the potential collateral loss ($\gamma_i K$) conditional on default (d_i). The loan maturity is denoted M. Formally, the NPV of the project is:

$$u_i(L, K, m, T) = \sum_{t=0}^{T} \delta_i^t Z_i F(L) - \sum_{t=0}^{M} \delta_i^t [d_i \gamma_i K + (1 - d_i)m] \tag{2}$$

When $d_i \to 1$, we have:

$$u_i(L, K, m, T) \approx (T + 1)[1 - T d_i] \cdot Z_i F(L) - M \cdot [d_i \gamma_i K + (1 - d_i)m] \tag{3}$$

Denoting $R = Mm$ and $X = (L, MK)$, the utility (3) fits into the general formula (1), with $(1 - d_i)Mm$ being the loan present value and α defined as the survival probability $(1 - d_i)$. The assumption that the per-period default probabilities are close to one is reasonable in most economic environments[10] and allows the utility to have the right functional form for the model to be well behaved.

Choice of bank b contract c: Borrower j of type i chooses the bank b that offers them the best deal c. Formally:

$$b_{ij} = \arg\max_{b \in \mathbf{B}} \{\max_{c \in \mathcal{M}_b} \{V(u_i(X_c, R_c), \mu_{ib} + \sigma_i^{-1} \epsilon_{jb})\}\} \tag{4}$$

B is the set of banks the borrower can borrow from.

10. For instance, the delinquency Rate on all Loans in the United States is around 2 per cent (see, for instance, the Federal Reserve Bank of St. Louis database[11]).

$\mathcal{M}_b$ is the set of contracts bank b offers. Borrowers always have the option to get the contract $u_i(0,0) \in \mathcal{M}_b$ (i.e., not to borrow).

$V(u_i(X_c, R_c), \mu_{ib} + \sigma_i^{-1}\epsilon_{jb})$ is the utility of getting a contract c from bank b given that borrower j has the type i. I assume that the utility function V is additively or multiplicatively separable. The function's first argument captures the utility derived from the contract. The second argument is a borrower-bank-specific demand shock (σ_i^{-1} parametrizes its variance and μ_{ib} its mean).

The demand shocks $(\epsilon_{jb})_b$ are known to borrower j when they choose their bank and contract. The index j refers to the individual borrower instead of its type. This distinction between borrower and type is made for conciseness of the notation, as I show in Lemma 1 below that additive or multiplicative separability prevents lenders from directly screening borrowers on their ϵ_{jb} draws.

Demand shocks are the main departure from the classic principal-agent model. It allows for imperfect substitution between contracts from different lenders and, thus, introduces some degree of market power. The demand shock can be interpreted as product differentiation—for instance, the distance between the borrower and the closest bank branches in Hoteling (1929) or Rochet and Stole (2002)—or imperfect information about the products available as in Varian (1980).

Example of demand function: Equation (4) implies a demand function faced by lenders. To fix ideas, let us look at the demand function faced by a bank b for borrowers of type i when the lender does not know the demand shocks $(\epsilon_{jb})_j$. If the shocks are independent, extreme value distributed conditional on borrower preferences ($\Theta_i := (\theta_i, (\mu_{ib})_b, \sigma_i, \alpha_i)$), and

enter the utility multiplicatively, equation (4) implies:

$$D_{ib}(u_i) := E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i] \tag{5}$$

$$= \overbrace{n_i}^{market\ size} \overbrace{\frac{exp(\sigma_i\{ln(u_{ib}) + \mu_{ib})\})}{\sum_{x\in\mathbf{B}} exp(\sigma_i\{ln(u_{ix}) + \mu_{ib})\}}}^{Probability\ i\ chooses\ b}, \text{ if } V = u_{ib} \cdot exp(\mu_{ib} + \sigma_i^{-1}\epsilon_{jb}) \tag{6}$$

$$\implies \frac{\partial D_{ib}(u_i)}{\partial u_{ib}} \frac{u_{ib}}{D_{ib}(u_i)} = \sigma_i(1 - D_{ib}(u_i)) \tag{7}$$

$u_{ib} := \arg\max_{c\in\mathcal{M}_b}\{u_i(X_c, R_c)\}$ is the borrower i utility when choosing the best contract offered by bank b. $u_i := (u_{ib})_{b\in\mathbf{B}}$ denotes the vector of utilities.

In that example, σ drives the demand elasticity and is thus a convenient way to model competition. When σ tends to infinity, holding utilities constant, the demand becomes close to perfectly elastic. I refer to this situation as the perfect competition case. When σ tends to zero, the demand is inelastic. I refer to this as the monopoly case.

3.2.2 Default probabilities

The market is adversely selected as the borrower's willingness to pay (θ_i) provides information about their default probabilities. Formally, the expected default probability conditional on the borrower information set ($\mathcal{I}_{ij} := \{(\epsilon_{jb}), \Theta_i := (\theta_i, (\mu_{ib})_b, \sigma_i, \alpha_i)\}$) when they choose their loan contract is:

$$E[default_i|\mathcal{I}_{ij}] = d(\Theta_i) \tag{8}$$

$default_i$ is a dummy equal to 1 if the borrower defaults in period 2. $d(\cdot) \in [0, 1]$ is the borrower i default probability. I assume that the default probability is independent of the realization of the demand shock (ϵ_{jb}). Hence, the fact that high default borrowers may tend to go to certain types of banks or search with lower intensity is captured by the mean and the variance of the demand shocks (i.e., μ_{ib} and σ_i respectively).

Example of micro-foundation of default: To fix ideas, let us consider that a borrower defaults if their wage W_i is lower than the loan monthly payment $m_i > 0$. A wage process that satisfies equation (8) is:

$$Pr(m_i > W_i) = d_i, \text{ with } W_i = \begin{cases} \bar{W}_i \text{ with probability } 1 - d_i \\ 0 \text{ with probability } d_i \end{cases} \tag{9}$$

The modelling is consistent with the empirical evidence in Hertzberg, Liberman, and Paravisini (2018) showing that the unobserved heterogeneity driving borrowers' behaviour in the consumer lending market is due to private information about the income process rather than a difference in risk aversion or interest rate risk. It is also consistent with the Bank of England NMG household survey that documents that the main reasons for defaulting are an increase in other bills or unexpected expenses/costs (21 percent), being made redundant, unemployed or getting a cut in wage (67 percent), or an increase in loan payment (10 percent).

The example from section 3.2.1 provides a micro-foundation of the correlation between default d_i and borrowers' willingness to pay (θ_i). Indeed, from equation (3), we have: $\theta_i = \frac{\partial_L u(L,R)}{\partial_R u(L,R)} = \frac{(T+1)[1-Td_i]}{1-d_i}$. This example highlights that firms with a higher default probability are less sensitive to interest rates, making them more likely to have a higher θ. However, those firms are also less likely to benefit from the increase in production from their loan size, which makes them less likely to have a higher θ. A market can thus feature positive or adverse selection depending on which effect dominates.

Endogenous default: Equation (8) neglects the impact of contract terms on default. This assumption is made for tractability reasons but is a good approximation in environments where default sensitivities are low around the equilibrium interest rate (see, for instance Nelson 2018). Examples of such situations are cases where defaults are triggered by large income shocks such as being made redundant[12].

12. It is a main trigger of default according to BoE NGM survey

The model can allow for endogenous default as long as the resulting utility derived from contracts is additively separable, as in equation (1). There are different ways to achieve this. For instance, one can linearize the utility function $u(X, R)$ defined in equation (2) around the equilibrium contract terms, taking into account that default is also a function of contract terms. Alternatively, one can assume that default is piece-wise linear with one kink, defined as functions of monthly payments as in equation (9). Finally, if the borrower's default probability does not directly impact the utility derived from the loan (as in the case of consumer credit so that $T = 0$ in equation (3)), one can use the following functional form for default: $d(\theta_i, m) = 1 - f(\theta_i) - \beta_i \cdot m^{-1}$ with $\beta_i \in \mathbf{R}$ and f being a function of θ_i.

3.3 Lenders

I first describe how each lender designs their menu of contracts. Then, I rewrite the lender program in terms of promised utilities to separate the competition channel from the screening one.

3.3.1 Menu Design

Lenders do not observe individual borrower characteristics $(\Theta_i, d_i, \epsilon_{jb})$, but they know the population distribution. As such, they can use a menu of contracts to make borrower reveal their private information through their choices. I use the following Lemma to derive the incentive compatibility constraints in terms of $u_i(X_c, R_c)$ only, and ensure that lenders cannot screen on demand shocks (ϵ_{jb}):

Lemma 1: *When V is either multiplicatively or additively separable, invertible in its first argument, and demand shocks (ϵ_{jb}) are uncorrelated with willingness to pay (θ_i), lenders cannot screen on demand shocks.*

Proof: See Appendix C.

Using Lemma 1, lender b designs a menu of contracts to maximize their expected profits subject to the following incentive compatibility and participation constraints:

$$\max_{\{(X_{ib},R_{ib})\}} E[\sum_{i\in\Gamma} \overbrace{E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i]}^{Demand} \overbrace{NPV_i}^{Net\ Present\ Value\ of\ lending} |\mathcal{I}_b] \tag{10}$$

$$s.t.\ (IC_{ib}):\ u_i(X_{ib},R_{ib}) \geqslant u_i(X_{\tilde{i}b},R_{\tilde{i}b})\ \forall i,\tilde{i}\in\Gamma$$

$$(IR_{ib}):\ u_i(X_{ib},R_{ib}) \geqslant u_i(0,0)$$

With b_{ij} defined in eq. (4)

$\mathcal{I}_b$ is lender b information set; it contains the distributions of (Θ, d, ϵ) for each borrower j conditional on their choice of contract, and the menus of contracts of all lenders in the market.

The incentive compatibility constraint (IC_{ib}) ensures that type i borrowers choose the contract designed for them. The index ib used for contract terms highlights that this contract is offered by bank b and designed for type i borrowers. Under the assumptions of Lemma 1, lenders can only separate borrowers based on their parameter θ as only they enter the (IC) constraints.[13] Thus, screening borrowers based on their other characteristics $(\alpha_i, d_i, (\mu_{ib})_b, \sigma_i)$ is only possible through their correlations with θ.

The individual rationality constraint (IR_{ib}) ensures that the borrower i contract gives them at least as much utility as the one they would get without borrowing.

Net Present Value of Lending: Lender b net present value of lending to borrower i via a contract ib is denoted NPV_{ib}:

$$NPV_{ib} := \alpha_i R_{ib} - mc_b(X_{ib}) \tag{11}$$

13. The α terms enter the left and right-hand side separately and thus cancel each other.

mc is the marginal cost of lending via contract ib. It is a convex function that increases in X. $mc_b(0)$ equals zero so that the contract $(0,0)$ can be interpreted as rejecting the borrower's application.

α_i is the same parameter as the one in the borrowers' utility function (1), so that $\alpha_i R_{ib}$ can be interpreted as the present value of the loan. To fix ideas, when d is the monthly default probability, m is the monthly payment, the lender is risk neutral, with a time discount rate of one and a maturity of M, the PV function is: $m\frac{1-(1-d_i)^M}{d_i} \approx (1-d_i)Mm$, when $d \to 1$. In that case, we can interpret α_i as the monthly survival probability $(1-d_i)$ and R as the maturity times the monthly payment Mm.

3.3.2 Utility Representation

Given that the incentive compatibility and individual rationality constraints are stated in terms of utilities and quantities, the literature usually transforms the lender's program from the price-characteristic space to the utility-characteristic space (see, for instance, Rochet 2024). This formulation helps to separate the effect of competition—through utilities—from the effect of imperfect information on product characteristics (X). It also makes the program similar to the workhorse monopoly model. I thus use the change of variable $\tilde{u}_{ib} = v(X;\theta_i) - R_i$ and rewrite the bank maximization the following way:

$$\max_{\{(X_{ib},u_{ib})\in\mathcal{F}\}} E[\sum_{i\in\Gamma} \overbrace{D_{ib}(\alpha_i\tilde{u}_{ib})}^{Demand}\{ \overbrace{S_{ib}(X)}^{Social\ Surplus} - \alpha_i\tilde{u}_i\}|\mathcal{I}_b] \tag{12}$$

$$s.t.\ (IC_{ib}):\ \tilde{u}_{ib} \geqslant \tilde{u}_{\tilde{i}b} + \underbrace{\Delta\theta_{i\tilde{i}} v(X_{\tilde{i}})}_{Information\ rent}\ ,\ \forall i,\tilde{i}\in\Gamma$$

The demand function $D_{ib}(\theta_i\tilde{u}_{ib}) := E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i]$ is equal to the probability that borrower i chooses bank b conditional on the menus offered in the market. Abusing the notation, I use D_i and $D_{ib}(u_{ib})$ when unambiguous instead of $D_{ib}(u_i)$, where $u_i := (u_{ib})_b$ is the vector of promised utilities offered by the lenders to borrower i.

$S_{ib}(X) := \alpha_i v(X;\theta_i) - mc_b(X)$ is the social surplus generated by the loan.

$\mathcal{F}$ is the set of feasible contract. The contract in the feasible set must satisfy the individual rationality constraint $\tilde{u}_{ib} \geqslant \frac{u_i(0,0)}{\alpha_i}$.

The incentive compatibility constraint (IC_{ib}) can be rewritten as in the monopoly case. It highlights that if the lender wants to provide utility $\tilde{u}_{\tilde{i}b}$ to borrower $\tilde{i}$ with a contract with characteristic $X_{\tilde{i}}$, it has to provide at least $\tilde{u}_{\tilde{i}b} + \Delta\theta_{i\tilde{i}}v(X_{\tilde{i}})$ to borrower i. The term $\Delta\theta_{i\tilde{i}}v(X_{\tilde{i}}) := (\theta_i - \theta_{\tilde{i}})v(X_{\tilde{i}})$ is called an information rent in the literature.

This formulation of problem 12 highlights that lenders use contract characteristics (X) to maximize the social surplus ($S(X)$) and use interest rates (R or equivalently $\tilde{u}$) to split the surplus. Due to asymmetric information, lenders have to take into account that contract characteristics affect the information rent ($\Delta\theta_{i\tilde{i}}v(X_{\tilde{i}})$) and thus the interest rate that they have to provide to borrowers to maintain incentives to self-select.

4 Optimal Contracts

In this section, I first list the conditions for the lender program to be well-behaved and the equilibrium to exist. Then, I analyze the perfect information and the imperfect information cases.

4.1 Optima and Equilibrium Existence

The following conditions ensure that the first-order conditions are necessary and sufficient conditions for an optimum and that an equilibrium exists. Economic intuition for why the equilibrium exists is provided in section 5.1, which analyses the equilibrium contracts:

Lemma 2: Optima and Equilibrium Existence

Optima — *There exist a unique solution to problem (12) as long as the demand functions* $D_{ib}(u_i) := E_\epsilon[\mathbf{1}_{\{b_{ij}=b\}}|\Theta_i]$ *are concave in* u_{ib}*, and the surplus functions* $S_{ib}(X) := \alpha_i \sum_c \theta_{id} v_d(X_d) - mc_b(X)$ *are increasing and concave in* X *and* v_d *are one-to-one functions.*

Equilibrium Existence — *There exists an equilibrium as long as demand functions* (D_i) *are continuous in utilities* u_i*, marginal costs* mc *and utilities functions* v *are continuous in contract characteristics* X *and the feasible set of contract* $\mathcal{F}$ *is compact.*

Proof: See Appendix C.1.1.

Lemma 2 is derived using a standard analysis tool for convex problems. This is the key advantage of setting the demand system as a discrete choice problem to get continuous demand functions.

Lemma 2 guarantees the existence of a pure strategy Nash equilibrium. This result contrasts with, for instance, Rothschild and Stiglitz (1976), or Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), which use discontinuous demand function. Having a pure strategy equilibrium is more realistic as, in most empirical applications, lenders do not randomize their menu in each period. It is also more analytically tractable, even in situations with an arbitrary number of lenders, borrowers and product characteristics. Furthermore, it allows the model's first-order conditions to be used to derive clean moment conditions that make the source of variation required to recover the model parameters transparent. Those generalizations are essential for catering to a wide variety of empirical applications.

Example of demand functions: When demand shocks are extreme value distributed and the V functions are multiplicatively separable, as in equation (6), the demand functions are continuous and concave. It is a convenient functional form as it makes the demand function logit-like and thus close to the one used in most empirical applications (see, for instance, Crawford, Pavanini, and Schivardi 2018 or Benetton, Gavazza, and Surico 2021).

When the V function is additively separable, the demand function is concave, but only for low values of σ. To make the model well-behaved when σ is high, one has to add a capacity constraint as shown in Appendix C.1.3. The capacity constraint is used in Azevedo

and Gottlieb (2017), in their perfect competition model.

4.2 Characterization of the Contracts: Observable type

To gain intuition about the economic forces driving lenders' decisions, I first start by solving the perfect information case. Formally, I assume that lenders observe all borrower characteristics except for the demand shocks (i.e., the parameters (Θ_i, d_i) are observable). Not observing the demand shocks allows the demand functions to be continuous. I use this modelling to study the effect of competition in this setup.

Formally, using the first order conditions of model 12 without the incentive compatibility constraints, the optimal contract terms are characterized by:

$$Contract\ Terms: \ \partial_X S_{ib}(X_i) = 0 \iff \overbrace{\alpha_i \partial_{X_i} v(X_i;\theta_i)}^{Marginal\ benefits} = \overbrace{\partial_{X_i} mc_b(X_i)}^{Marginal\ costs} \tag{13}$$

$$Pricing: \quad \underbrace{\alpha_i R_i}_{\substack{Loan\ Present\ value}} = \underbrace{mc(X_i)}_{Break\ even\ price} + \underbrace{\frac{D_i}{\partial_{u_i} D_i}}_{Markup} \tag{14}$$

The above equations state that lenders set contract characteristics (X) to maximize the social surplus (equation (13)), and use the interest rates to split the surplus between borrowers and lenders (equation (14)). How the surplus is split depends on the demand semi-elasticity ($\frac{D_i}{\partial_{u_i} D_i^b}$): When lenders offer products that are close substitutes (e.g., σ high in the logit case, so that $\frac{D_i}{\partial_{u_i} D_i^b}$ is low), lenders have low market power and thus equate the loan present value to the contract marginal cost ($mc(X_i)$).

4.3 Characterization of the Contracts: Unobservable Type

I now consider that lenders cannot observe any of the borrowers' characteristics $(\Theta_i, d_i, \epsilon_{jb})$ and thus solve problem 12 with the incentive compatibility constraints.

4.3.1 Multidimensional Screening: $dim(\theta) > 1$ and $dim(\Gamma) > 2$

Characterizing the equilibrium is challenging when the number of contract terms and the number of agents' types are greater than two (i.e., $dim(\theta) > 1$ and $dim(\Gamma) > 2$).[14] In that case, the set of incentive compatibility constraints that are binding is endogenous to contract terms, which leads to tractability issues (see, for instance, Rochet 2024 or Carroll 2017).

As a result, the vast majority of the empirical and theoretical literature focuses on the situation in which $dim(\theta_i) = 1$ and endogenizes prices and at most one contract characteristic.[15] This assumption does not restrict other parameters (i.e., $(\Theta_i \backslash \{\theta_i\}, d_i,)$ to be heterogeneous conditional on θ_i.

The focus on one dimension is not as restrictive as it seems. Indeed, the recent theoretical literature on robust contract theory (see, for instance, Carroll 2017) showed that when the principal does not know the joint distribution of preferences and evaluates mechanisms according to the worst-case scenario, the optimum for the principal is to screen each component separately.

When utilities are linear, one can show that only one characteristic is used to screen between two borrowers. The characteristic is the one that is less costly to distort (i.e., the one that has the minimum shadow cost: $\lambda_{ij} := \frac{\partial_X S_j(X)}{\theta_i - \theta_j}$ where $\theta_i > \theta_j$ and $S_j(X)$ is the social surplus). When the characteristic that has the lowest shadow cost is the same for each pair of borrowers, then the problem boils down to a unidimensional screening situation as well.

4.3.2 Screening when $dim(\theta_i) = 1$

Motivated by example (3) and the empirical application, I focus here on a situation where lenders can choose interest rates, maturity and loan size so that θ_i is of dimension one. I interpret X as the loan size and R as the monthly payments times the maturity.

The assumption that $dim(\theta_i) = 1$ is relaxed in section 5.1, which derives general results

14. With each element of θ_i being random variables that are not perfectly correlated

15. See, for instance, Veiga and Weyl (2016) for an empirical model without menus, and Azevedo and Gottlieb (2017) for empirical applications, and Einav, Finkelstein, and Mahoney (2021) for a review.

for the multidimensional screening case (i.e., $dim(\theta_i) > 1$) under perfect competition.

When $dim(\theta_i) = 1$, screening is achieved by making borrowers with a high willingness-to-pay (high θ) self-select into high loan high interest rate contracts. The set of incentive compatibility constraints can be simplified: Only the incentive compatibility constraints of the borrowers with close θ are relevant. I thus order borrowers according to their θ value such that borrower i has a higher θ than borrower $i-1$. I index by I the borrower with the highest θ and 0 the borrower with the lowest. The relevant incentive compatibility constraints for borrower i are only the one of borrower $i+1$ ($i+1$ potentially wants to pretend to be of the i type) and the one of borrower $i-1$ (i potentially wants to pretend to be of the $i-1$ type).

For exposition purposes, I focus here on an interior solution and consider that $(\alpha_i, d_i, (\mu_{ib})_b, \sigma_i)$ are the average values of $(\alpha_j, d_j, (\mu_{jb})_b, \sigma_j)$ conditional on θ_i, where i index the different values of (θ_i) and j the borrower. The latter only affects equation (16) below and can be generalized by multiplying both sides by $\partial_{ui} D_i$ and taking the expectation conditional on the lender information set.

The optimal contracts is characterized by:

$$Contract\ Terms:\ X_i = \begin{cases} X_i^{II} := v^{-1}(v(X_{i+1}) - \frac{R_{i+1}-R_i}{\theta_{i+1}}) \text{ if } X_i^{II} \leqslant X_i^{PI} \\ X_i^{PI} \text{ if i=I or if } X_i^{II} > X_i^{PI} \end{cases} \tag{15}$$

$$Pricing: \overbrace{\alpha_i R_i}^{Loan\ Present\ Value} = \overbrace{mc(X_i)}^{break\ even\ price} + \overbrace{\frac{D_i}{\partial_{u_i} D_i}}^{Perfect\ information\ markup} \overbrace{+ \frac{D_i}{\partial_{u_i} D_i} IR_i(X_i, X_{i-1}, \frac{D_{i-1}}{D_i})]}^{Information\ discount/premium} \tag{16}$$

$IR_i(X_i, X_{i-1}, \frac{D_{i-1}}{D_i}) := \alpha_i^{-1}[\frac{\overbrace{\partial_{X_i} S_i(X_i)}^{Marginal\ social\ surplus}}{\underbrace{\Delta\theta_{i+1}\partial_{X_i} v(X_i)}_{Marginal\ information\ rent}} \mathbf{1}_{i\neq I} - \frac{D_{i-1}}{D_i} \frac{\partial_{X_{i-1}} S_{i-1}(X_{i-1})}{\Delta\theta_{i,i-1}\partial_{X_{i-1}} v(X_{i-1})} \mathbf{1}_{i\neq 0}]$ is an information distortion.

X^{PI} which solves equation (13)

Equation (15) highlights the tools that lenders can use to maintain borrowers' incentives to

self-select: credit rationing (X_i^{II}) and cross-subsidies ($R_{i+1} - R_i$). It states that the contract characteristics are equal to their first best value (X^{PI}) when the pricing of contracts—characterized by equation (16)— makes them incentive-compatible. When the contract prices (R) are insufficient to maintain borrowers' incentives to self-select, the lender sets the contract characteristics such that the incentive compatibility constraint is binding (i.e., $X_i = X_i^{II}$).

As in Rothschild and Stiglitz (1976), there is no distortion at the top: The borrower with the highest willingness to pay (θ) gets the first best loan size. Other borrowers get a lower loan size, and potentially credit rationed. How credit rationed the borrower is (X_i^{II}) depends on the interest rate spread ($\frac{R_{i+1}-R_i}{\theta_{i+1}}$) and the loan size of the borrower's willingness to pay just above (X_{i+1}).

Equation (16) is the optimal pricing equation. It is a key novelty of the paper as the literature on selection markets does not use continuous demand systems and thus cannot rely on first-order conditions.

I rewrite the optimal pricing equation as the sum of three terms: a break-even price, a perfect information markup, and an information distortion (i.e., the cross-subsidy term). This decomposition has two benefits. First, it provides intuition about the economic forces at play. Second, as discussed in section 6.1, the equation can directly be used as a moment condition to recover the model parameters.

The first two terms of equation (16) can be interpreted as the best response of bank b to its competitors' contracts if lender b had perfect information about its customer. Those components have already been analyzed in section 4.2 as they appear in equation (14). The last term (IR_i) captures the impact of adverse selection. It provides intuition about the lender incentives to use credit rationing or cross-subsidy to maintain borrowers' incentives to self-select.

The first element of IR_i in equation (16) (i.e., $\frac{\partial_{X_i} S_i(X_i)}{\Delta\theta_{i+1}\partial_{X_i} v(X_i)}$) is positive and affect the rate of low default borrowers when $Corr(\theta_i, d_i) > 0$. I call that term the information premium. It prevents the higher default borrowers ($i + 1$) from choosing the contract designed for

the lower default borrower (i). The information premium formula captures the intensive margin costs and benefits of increasing interest rates above the perfect information ones. The numerator represents the benefits of increasing the loan face value R: It relaxes the incentive compatibility constraint and thus increases the surplus generated by the loan by $\partial_X S$ for each dollar additional dollar lent. However, for a given increase in the loan face value R, there is a limit to how large the loan of the low-default borrower can increase without attracting the high-default borrower. This amount is exactly equal to one over the information rent.

The second term of IR_i (i.e., $\frac{D_{i-1}}{D_i}\frac{\partial_{X_{i-1}} S_{i-1}(X_{i-1})}{\Delta\theta_{i,i-1}\partial_{X_{i-1}} v(X_{i-1})}$) is negative and affects the rate of high default borrower when $Corr(\theta_i, d_i) > 0$. I call that term the information discount. This discount ensures high default borrowers have lower incentives to select the contract designed for the low default borrowers $i-1$. The intuition is the following. By providing a discount (i.e., lowering the interest rate of the i contract by $\Delta\theta_{i,i-1}\partial_{X_{i-1}} v(X_{i-1})$), the lender lower their profit margin on each i borrower they attracted (i.e., D_i). However, the information rent allows lenders to lower the amount of credit rationing and thus their profit margin on each $i-1$ borrower by $\partial_{X_i} S_i(X_i)$. This effect is multiplied by the number of customers they have (i.e., D_{i-1}).

The pricing distortion IR_i is multiplied by the demand semi-elasticity, which captures the extensive margin cost of playing with interest rates R (i.e., changing the promised utility u_i level). When the demand semi-elasticity is high, increasing rates too much compared to the perfect information pricing is costly due to the effect on the number of customers. Thus, the lender has incentives to keep the demand unchanged and use more credit rationing.

4.3.3 Graphical Intuition: Linear Model

To provide intuition about equation (16)—and later on the equilibrium existence result under perfect competition—let us focus on the two borrower-type case and represent the lender problem b in a graph. To make the model close to the empirical application setup in sec-

tion 7, I assume utilities and marginal costs are linear, and the feasible set is $X \in [0, \bar{X}]$. I consider two borrowers (G and B). B has a higher willingness to pay (θ) and a higher default probability. I plot borrower indifference curves (blue for B borrowers and green for G borrowers) and lenders' break-even conditions for each market segment (light grey horizontal lines) in figures (1) and (2). To render the graph more legible, I consider the situation in which lenders must pay a fixed cost of originating contracts (i.e., $mc(X) = f \in \mathbf{R}^+$) so the grey lines are horizontal. I denote $(C_1, \tilde{C}_1)$ the perfect information contracts of bank b. C_2 and C_3 are the imperfect information contracts.

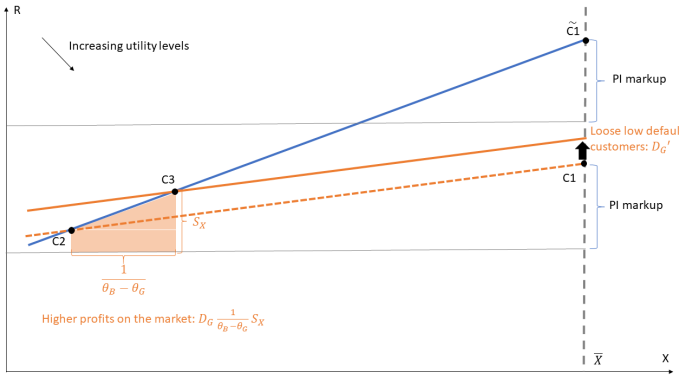

Figure 1: Effect of information premium: More profits per G customer (C_2 to C_3), loose G customers with respect to perfect information utility levels (C_1).

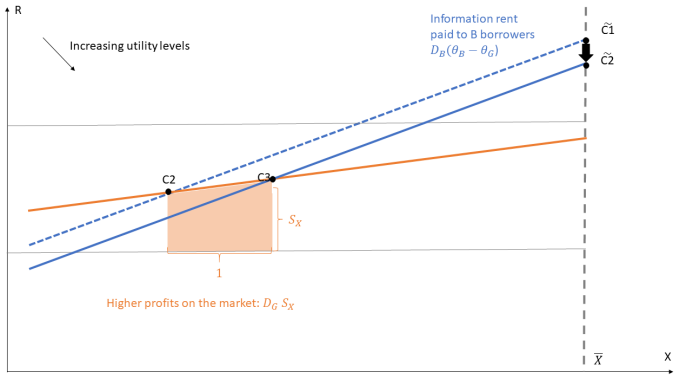

Figure 2: Effect of information discount: More profits per G customer (C_2 to C_3), loose B customers with respect to perfect information ($\tilde{C}_1$ to $\tilde{C}_2$).

Figure 1 shows the effect cost associated with providing an information premium ($\frac{D_G}{\partial_R D_G} \cdot \frac{\partial_{X_i} S_i(X_i)}{\Delta\theta_{i+1}\partial_{X_i} v(X_i)}$ in equation (16)). It relaxes the incentive compatibility constraints and allows the firm to offer contract C_3 instead of C_2 to low default borrowers. This increases lender's profits by $\frac{\partial_{X_i} S_i(X_i)}{\Delta\theta_{i+1}\partial_{X_i} v(X_i)}$. However, this comes at the cost of losing customers ($\frac{D_G}{\partial_R D_G}$).

Figure 2 shows the effect cost associated with providing an information premium ($\frac{D_B}{\partial_R D_B} \cdot \frac{D_G \partial_{X_i} S_i(X_i)}{D_B \Delta\theta_{i+1}\partial_{X_i} v(X_i)}$ in equation (16)). It relaxes the incentive compatibility constraints and allows the firm to offer contract C_3 instead of C_2 to low default borrowers. This increases the lender's profits by $\partial_{X_i} S_i(X_i)$ to each low default borrower. However, this comes at the cost of having to provide an information rent ($\Delta\theta_{i+1}\partial_{X_i} v(X_i)$) to bad borrowers. The rent has both an

intensive and an extensive margin effect: it is paid to existing borrowers and will attract new customers as well. The extensive margin is captured by $(\frac{D_B}{\partial_R D_B})$.

5 Analysis of the Equilibrium Contracts

In this section, I analyse the equilibrium contract for the multidimensional screening case under perfect competition. I then do a positive and normative analysis of the equilibrium contracts for the model used in the empirical application.

5.1 Perfect Competition and Multidimensional Screening

In the textbook perfect competition model of screening (i.e., Rothschild and Stiglitz 1976), pooling contracts cannot be offered in equilibrium. This is because of cream-skimming: a lender can take advantage of the pooling contracts offered by its competitors and introduce a new contract with a smaller loan size and price it so that it only steals the most profitable borrowers.

Yet, separating contracts cannot be an equilibrium when the number of low-default borrowers is large enough. In that case, both low and high-default borrowers prefer to be pooled, so a competitor can make profits by offering pooling contracts. The low-default borrowers get a lower credit constraint at the cost of a slight interest rate increase, and the high-default borrowers get a lower rate. The above intuition is summarized in Figures 10 and 11 in the appendix for the linear model presented in section 4.3.3.

The following theorem shows that the separating contracts constitute an equilibrium in my model even when demands are almost perfectly elastic. The intuition is provided below and in Figure 3.

Theorem 1: Perfect competition equilibrium.

Under the conditions of Lemma 2, if it is possible to design a menu of separating contracts in

which lender price each contract at their marginal cost, then perfect competition equilibrium's (i.e. $\partial_{u_i} D_i(u_i) \to \infty \ \forall (u_i)_i$) will feature no cross-subsidy and full separation.

Corollary: Rothschild and Stiglitz case (i.e.,** $dim(\theta_i) = 1$ **and** $corr(\theta_i, d_i) > 0$**).
As $\partial_{u_i} D_i(u_i) \to \infty \ \forall u_i$, the equilibrium tends to the Rothschild and Stiglitz (1976) separating contracts even in the region in which the equilibrium does not exist in Rothschild and Stiglitz (1976).

***Corollary: Rochet and Stole case (i.e.,** $dim(\theta_i) = 1$ **and** $corr(\theta_i, d_i) \leqslant 0$**).** *As $\partial_{u_i} D_i(u_i) \to \infty \ \forall u_i$, the perfect competition equilibrium tend to the first best contract.*

Proof: See Appendix C.2.

For intuition, let us focus on the Rothschild and Stiglitz case as in section 4.3.3. In my model, the deviations from separating contracts—represented by contract (C_p) in Figure 3—are not profitable, so the equilibrium with separating contracts always exists.

The reason is that deviations from screening attract too many high default borrowers to be profitable. This is because they do not attract the whole market—because of concave demand functions—and are relatively more valuable to costly borrowers. The intuition behind why deviation from screening is more valuable to high-default borrowers is the following. Pooling deviations (i.e., using cross-subsidies) require increasing the G contract loan size, which is relatively more valuable to the high-default borrowers.

A visual representation of this argument is provided in Figure 3. The intuition relies on two points. First, the most profitable perfect competition pooling deviation implies moving along the G borrower indifference curve. This deviation is more attractive to B borrowers as it relies on increasing the loan size and pricing each additional loan dollar lent at the low default borrower's willingness to pay. Second, because of concave demand functions, there is always a mass of borrowers that are harder to attract, even when the demand elasticities are close to infinity at equilibrium. Taken together, those two points imply that the deviation

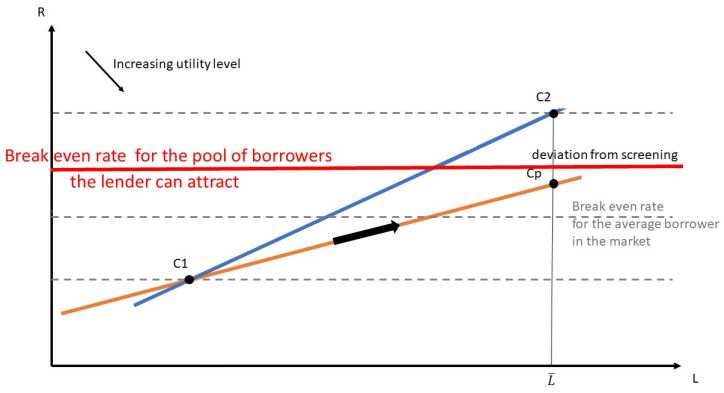

Figure 3: Deviation (C_p) from screening contracts (C_1, C_2)

attracts too many high-default borrowers to be profitable. The break-even rate for the average borrower the lender can attract —represented by the red line in Figure 3— is above the low default borrower indifference curve. This is true even when the break-even rate for the average borrower participating in the market is below the deviation (C_p).

Contrarily to Azevedo and Gottlieb (2017), this result does not require a capacity constraint and is derived for any demand function and dimension of contracts.

5.2 Positive Analysis of the Equilibrium Contracts

In this section, I analyze the properties of the equilibrium contract. For simplicity of the exposition and to analyze the model that is close to my empirical application, I consider the model presented in section 4.3.3 with demand shocks are additive and extreme value distributed so that the demand has a logit form. Additional technical assumptions are described in Appendix D.

Results: Equilibrium and comparative statics.

Equilibrium Characterization: *The model can be solved in closed form, by replacing the demand elasticities by* $\frac{1}{\sigma_i(1-\frac{1}{B})}$ *and the demand* D_i *by* $\frac{n_i}{B}$ *in equation (16).*

Competition: *As long as no* (IR_i) *constraints are binding, a decrease in competition*

decreases the amount of credit rationing.

Marginal cost of lending: *Increasing the marginal cost of lending of low loan contracts decreases the credit rationing level and interest rate spread under high competition (i.e., $\frac{1}{\sigma_i(1-\frac{1}{B})}$ low). The effect goes in an opposite direction under low competition.*

Increasing the marginal cost of lending of high loan contracts increases the amount of credit rationing.

Proof: See Appendix L

The intuition for the result on competition is the following. Theorem 1 shows that borrowers are credit-rationed under low competition levels if the market is adversely selected. When competition decreases, lenders have incentives to increase interest rates R for both contracts to increase their profits. However, increasing rates relatively less for high-default borrowers (i.e., B borrowers) alleviates the credit constraint, which allows lenders to make more profits from the low-default market segment. With logit demand, the second channel dominates. When the individual rationality constraint for the low-default borrower (i.e., G borrowers) is binding, the lender cannot increase the rate on the low-default borrower contract, so the credit rationing increases.

To analyse the result on the marginal costs, I decompose the effect into a perfect competition and an imperfect competition channel. Let us consider a situation in which the marginal cost of the G contract increases.

Under perfect competition, increasing the marginal cost of the G contract (i.e., the one designed for G borrowers) makes it less attractive to B borrowers. As a result, the incentive compatibility constraint is relaxed and lenders can increase the loan size of the G contract.[16]

Under imperfect competition, there is an additional effect. The increase in the marginal

16. Increasing the marginal cost of B contracts has the opposite effect, as it tightens the incentive compatibility constraints. When both marginal costs increase, the second effect dominates as interest rates increase relatively more in the B contract because they are scaled up by default probabilities.

cost decreases the surplus of lending via the G contract. As the potential surplus for G contracts decreases, lenders have less incentive to provide an information rent to B borrowers in order to lower the credit constraint on G contracts.

I summarize the result in Figure 4. When the marginal cost increases relatively more for the G contract, the equilibrium region moves from the dotted lines to the solid lines in figure 4.

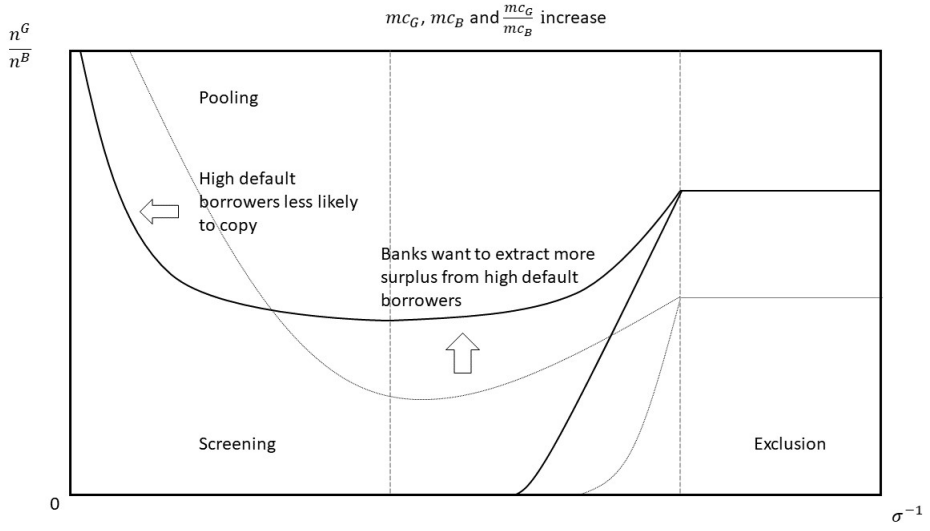

Figure 4: Equilibrium regions when marginal costs increases

Implication for banking: Banking policies often affect the marginal cost of lending but are not analyzed through the lens of screening models. Different policies can be affected depending on the contract characteristics used to screen. For instance, in markets where lenders screen with maturity (see Hertzberg, Liberman, and Paravisini 2018 for empirical evidence), then the effect of quantitative easing (QE) likely depends on the degree of competition. This is because maturity plays the same role as interest rate in the model, and that QE affect marginal costs of originating contracts with different maturity. In markets where lenders screen with Loan-to-Values (see Taburet, Polo, and Vo 2024 for empirical evidence), then capital requirements are the relevant policy. LTV plays the same role as loan size in

the model, and capital requirements are often LTV-specific.

5.3 Normative Analysis: Screening Externality and Pareto Improvement

In this subsection, I show that a contractual externality may cause excessive screening. This implies that there is room for policy interventions.

Contractual externality: To get a lower bound on the contractual externality, I show that there exists a menu of contracts that makes all borrowers and lenders better off. I do so by solving for the optimal contract of an informationally constrained social planner. Formally, the social planner maximizes lenders' profits subject to the same incentives compatibility constraints as the lender and subject to providing at least as much utility to borrowers as the market contracts for a given competition level (denoted (u_B^*, u_G^*)). That is, the problem is the same as (12) but with constant market shares (n_i) and the participation constraints being equation to u_i^* instead of $\bar{V}$. The constant market shares assumption shuts down the contractual externality.

Using the first-order conditions, I derive the following Proposition:

Proposition 4: Sufficient Statistic for the Contractual Externality. *Banks should pool borrowers together at $\bar{X}$ if and only if:*

$$\frac{\theta_G - \frac{mc}{\alpha_G}}{(\theta_B - \theta_G)} \frac{n_G}{n_B} > 1 \tag{17}$$

However, except under monopoly, the competitive equilibrium switching point is different than the social planner one (17).

Proposition 4 shows the conditions under which the equilibrium is inefficient in the second-best sense.

The friction emerges for the following reasons. Maintaining customers' incentives to self-select is costly as it may require distorting contracts relative to the first best. When the distortions are too high, it is more efficient to pool borrowers instead. Yet, if pooling contracts are offered and competition is high, a competitor can take advantage of a pooling contract by introducing a product that steals the most profitable customers only (cream skimming). The friction arises because lenders do not internalize how their screening strategies (e.g., cream skimming) change the types of borrowers selecting competitors' products — and thus the cost of lending via those products.

This friction exists only when pooling is a Pareto improvement over screening so that cream-skimming deviations are ex-post-inefficient. Equation (17) captures the condition for the latter to be true. The numerator is the benefit of increasing X_G while the denominator represents the costs. Increasing the loan size allows the social planner to generate more profits $(\theta_i - \frac{mc}{\alpha_i})$ per G borrowers. However, to maintain incentives to self-select, lenders have to decrease the interest rate on the B market segment, leading to a loss of $(\theta_B - \theta_G)$ per B borrower. In this model, the utilitarian welfare function, used in, for instance, Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019), is equivalent to decreasing the level of credit rationing (i.e., considering only the numerator of equation (17)).

Equation (17) can be used to test for the existence of the contractual externality. One can use a revealed preference approach to recover borrowers' willingness to pay and relative contracts' shares to get the relative numbers of borrowers. The marginal cost can be recovered either using accounting data or by estimating it using the interest rate first order conditions as standard in the industrial organisation literature.

Effect of Policy interventions: In appendix P, I show that decreasing competition can lead to a Pareto improvement. The intuition is that it lowers' incentives to implement

cream-skimming deviations. The policy can make G Borrowers better off by lowering their credit constraint at the cost of increasing the higher markup. B Borrowers get a higher information rent but also a higher markup. Finally, a decrease in competition allows lenders to extract more surplus from their customers. Figure 5 illustrate the set of parameters for which decreasing competition is a Pareto improvement. This is the case when cream skimming happens and is inefficient. That is when the number of high borrowers is relatively large, and competition is high.

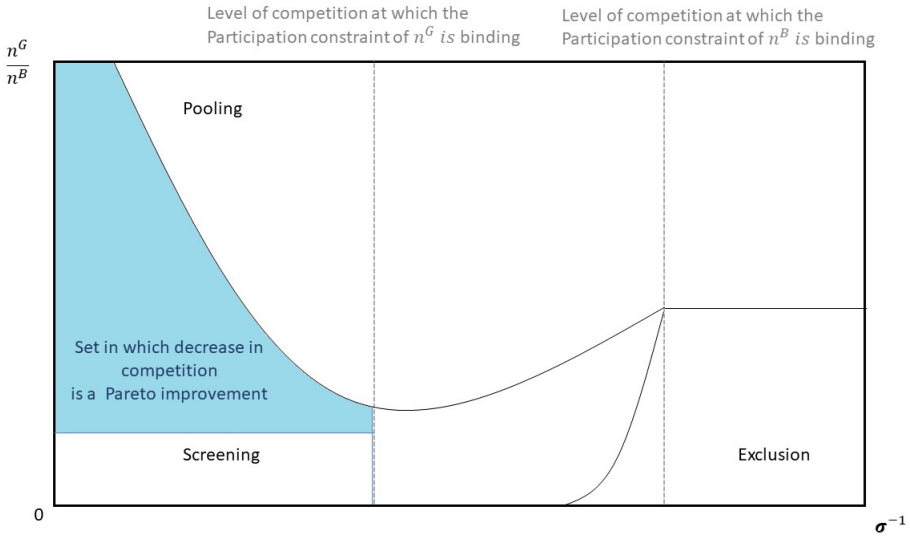

Figure 5: Pareto set

6 Bringing the Model to the Data: A Guidebook

This section develops a guide on how to bring the model to the data.

I discuss the identification and estimation of the model parameters for types of datasets commonly used in banking or insurance. In the first scenario, I consider that the econometrician has data on menus and customer choices but only for one lender. This is the most common scenario in which menus are observable (see, for instance, Hertzberg, Liberman, and Paravisini (2018) for the credit market or Tebaldi (2024) for the insurance market). I refer

to this as the proprietary data case.

Second, I analyse the situation where the econometrician has data on multiple lenders but can only see the equilibrium contracts chosen by each customer. This is the data constraint faced when using credit register data (see, for instance, Crawford, Pavanini, and Schivardi 2018).

6.1 First Case: Proprietary Data on a Single Firm

The estimation of the model parameters is based on a three-step procedure. Step 1 recovers the default probabilities (d_i), which allows us to get α_i. Step 2 identifies the willingness to pay (θ), and step 3 the marginal costs (mc) as well as the demand elasticities parameter (σ). I use the general utility specification described by equation (3).

Step 1: Default Probabilities (d).

Similarly to equation (9), I assume that borrower i defaults if their disposable income W_{it} at time t is lower than the loan monthly payment m_{it}. I consider that disposable income is observable by the lender at origination and that the probability of it changing over time is exogenous and is the borrower's (partially) private information. In practice, lenders have a good sense of borrowers' disposable income through credit checks.

In addition to its tractability, this modelling assumption has a convenient implication: It allows recovering the key model parameters without estimating default elasticities to monthly payments (see Steps 2 and 3). Identifying default elasticities relies on comparing borrowers who are observationally equivalent but chose different contracts. Because of the selection on unobservables, finding the right control group is problematic when lenders screen.[17]

Formally, denoting T_i the loan maturity in months, the borrower survival probability at maturity is:

17. Alternatively, the default probability can be parametrized as $\theta = \alpha + \frac{\beta}{R}$. This approach also yields closed-form solutions.

$$S_i := E[\mathbf{1}_{\{\exists! t_i : m_{it} > W_{it}\}} | X_i^B, i \text{ chose contract } c] \tag{18}$$

X_i^B are borrowers' characteristics, contracts and menus characteristics or any other variables that are observable by the lender when the contract is originated, that predict the income or the contract selection process, and that are legal to use to make acceptance and rejection or pricing decisions.

For each observationally equivalent borrower choosing contract c (indexed by j $\in [\![1, n_c]\!]$), one can estimate the average survival probability ($\hat{S}_i$) using realised default at maturity:

$$\hat{S}_i := \sum_{j=1}^{n_c} \mathbf{1}_{\{j \text{ did not default}\}} \tag{19}$$

Using a constant hazards rate model, this yields the following constant monthly survival probability:

$$\hat{\alpha}_i := e^{-\hat{\lambda}_i}, \text{ with } \lambda_i : \ \hat{S}_i = e^{-\hat{\lambda}_i T_i} \tag{20}$$

The present value of a loan is then $[1 - e^{-\lambda T}]m \approx \lambda T m$. The constant hazards rate model is convenient when screening with maturity (see empirical application). In that case, λ plays the same role as α and Tm as R in the theoretical model. Constant hazard rates are consistent with assuming that the income process follows the same Bernoulli distribution in each period.

Step 2: Borrower preferences (θ_i).

I use a revealed preference approach to recover borrowers' preferences. The identification leverages the idea that if a borrower chooses a contract with a loan size of \$1,000 while

they had access to a contract a loan size of \$1,100 for an interest rate increase of, say, 100 bps, it must be that their willingness to pay for loan size is lower than 100 bps. Figure (6) provides a visual representation of this argument. I plot the internet rate of the contract as a function of loan size (i.e., the pricing schedule) and the borrower indifference curve. When a continuum of contracts is offered, the optimality condition implies that the slope of the indifference curve (i.e., the willingness to pay) must be tangent to the pricing schedule at the contract chosen.

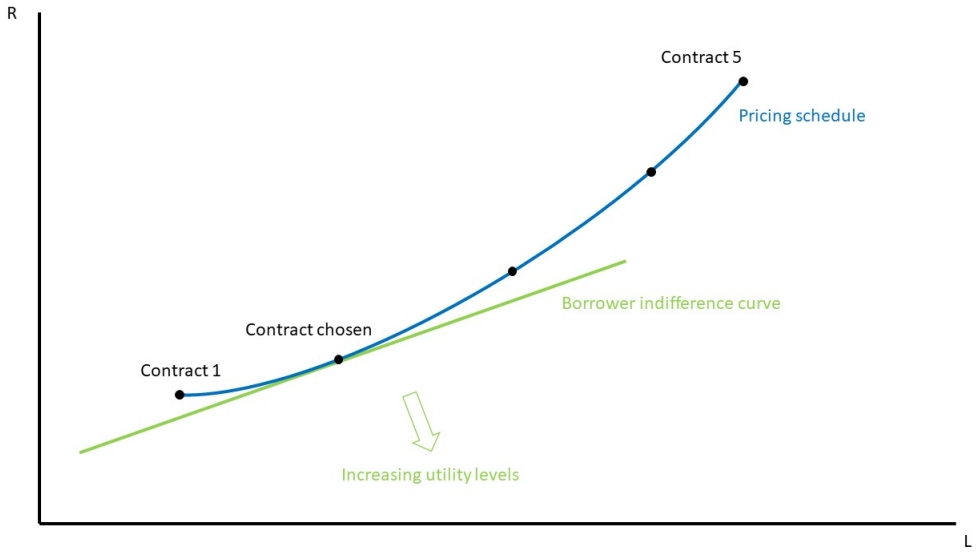

Figure 6: The slope of the pricing schedule at the contract chosen allows us to recover the borrower's willingness to pay θ_i

One challenge with this approach is that willingness to pay θ_i depends on their default probabilities (as highlighted in equation (3)), which are endogenous objects. This implies that the econometrician potentially needs to estimate the default probability if the borrower were to choose another contract. Because of adverse selection, this is problematic.

There is no standard way to address this point as the industrial organisation literature typically focuses on the market without moral hazard and thus estimates willingness to pay as a fundamental parameter of the borrowers' utilities.

As a solution to this problem, I propose to use contracts that yield the same monthly payments (leveraging the variety of maturity and loan size options offered in practice) so that

default probability is plausibly constant for a given individual. Depending on the setting and the data available, it may be preferable to depart from this assumption. The interested reader willing to do so can refer to, for instance, Taburet, Polo, and Vo (2024).

Using contracts with similar monthly payments to recover preferences is plausibly the relevant set of contracts in practice. Indeed, monthly payment targeting is consistent with empirical evidence (see Argyle, Nadauld, and Palmer 2020 for an example using the car loan market, Graham and Harvey 2001 or their CFO Survey). Furthermore, this consideration set also arises naturally in a model with linear utility and inefficient default (see the following footnote[18] and Appendix H). In that case, borrowers max out their maximum monthly payment capacity, so using contracts with the same maturity yields is the relevant set of moments to use.

Formally, I construct the moments using the fact that borrower i chooses the contract designed for them (contract i) if its indirect utility is higher than the one if they were to choose similar contracts (i-1 and i+1). Considering contracts with the same monthly payment and denoting G the probability distribution function of θ conditional on default probabilities, I get the following bounds:

$$u_i(R_i, L_i) \geqslant u_i(R_j, L_j) \ \forall j \implies a := \frac{R_i - R_{i+1}}{L_i - L_{i+1}} \leqslant \theta_i \leqslant b := \frac{R_i - R_{i-1}}{L_i - L_{i-1}} \tag{21}$$

L is the loan size, and R is the maturity times the monthly payments.

Relative contract market shares ($\frac{n_i}{\sum_c n_c}$) and the variables (a, b) are typically observable by the econometrician. An estimate of G is thus:

$$\hat{G}(a \leqslant \theta_i \leqslant b) = \frac{n_i}{\sum_c n_c} \tag{22}$$

Exogenous variation in a or b — i.e. variation uncorrelated with the distribution of θ (i.e.,

18. For instance with the following model: $\max_L \alpha L - p(rL > W)rL - (1-p)c$, $c > r\bar{W}$ and W follows a Bernoulli with its maximum value being $\bar{W}$.

the G function)— allows the recovery of the full distribution non-parametrically. One can use variation in contract terms coming from either cross-sectional or time series variation. For instance, assuming that the distribution of borrower type stays constant over time, exogenous variation in the spread between contracts' interest rates allows the recovery of the distribution of borrowers' willingness to pay.

Parametric assumptions on G can help with power when the number of observations is small. Moral hazard, as long as its effects are homogeneous across types, can be dealt with by looking at variations of contracts other than contract i. For instance, Taburet, Polo, and Vo (2024) uses variation in the contract-specific marginal cost of lending to recover the preference heterogeneity in a model with moral hazard and adverse selection.

Step 3: Demand elasticities (σ_i).

I recover the demand elasticity using the pricing equation (16) as a moment condition. My approach contrasts with the standard industrial organisation approach, which estimates demand elasticities using a discrete choice model (e.g., logit in Berry, Levinsohn, and Pakes 1995) and uses the pricing equation (16) to recover marginal costs. There are two reasons to depart from the approach used in industrial organisation. First, data on choice sets at the individual level is not available; and using comparable borrowers to construct choice set is problematic due to bank specialization and selection on unobservables. Second, data on marginal lending costs at the individual level are less challenging to recover in banking: it is common practice to consider that the deposit rate plus the capital requirements constitute the marginal cost of lending. Central banks record those two elements.

The estimation procedure is as follows: Given an estimate of borrower's parameter (α_i, θ_i), and conditional on observing the cost of lending (mc) one can construct the asymmetric information premium and discount using the formula for IR from equation (16). In the case of linear preferences, we have:

$$\hat{\alpha}_i \hat{IR}_i := [\frac{1}{n_i} \frac{\frac{\theta_i}{\alpha_i} - mc}{\Delta\theta_i} 1_{i>1}] - [\frac{1}{n_{i-1}} \frac{\frac{\theta_{i+1}}{\alpha_{i+1}} - mc}{\Delta\theta_{i+1}}] 1_{i<x} \tag{23}$$

Abusing the notation, I use σ_i for the equilibrium demand semi-elasticity instead of $\frac{1}{\sigma(1-\frac{n_i}{\mathbf{B}})}$. The demand elasticity of each group is recovered using the pricing equation (16):

$$\alpha_i R_i = \hat{m}c\text{Ł}_i + \sigma_i[1 + IR_i] \implies E[\hat{\sigma}|c] := R_i - \frac{\hat{m}c\text{Ł}_i}{E[\hat{\theta}_i|c]}[1 + \hat{IR}_i]^{-1} \tag{24}$$

Equation (24) is derived using the first-order condition of the lender problem with respect to maturity (T), holding the monthly payment constant (m).[19] That way, thanks to the assumption that only monthly payments affect defaults, I can recover the demand parameters without estimating default semi-elasticities, a problematic parameter to estimate in selection markets (see step 1).

By assuming that semi-elasticities are borrower-specific parameters (σ_i), I avoid estimating demand elasticities using choice data as in for instance Crawford, Pavanini, and Schivardi (2018).[20] The constant semi-elasticity assumption can be justified in two ways. First, it arises naturally in the symmetric equilibrium model (the semi-elasticity is $\frac{1}{\sigma(1-\frac{n_i}{\mathbf{B}})}$ which are parameters in my model). Second, it can be interpreted as lenders considering the local borrowers' semi-elasticity as an approximation when setting contract terms.

Alternative to Steps 2 and 3.

Instead of estimating the demand parameters and the demand elasticity separately, one can

19. In the model described in Appendix H, the model features a corner solution for monthly payments so borrowers max out their maximum monthly payment capacity.

20. Similarly to the default elasticities, demand elasticities are problematic to estimate when there is a selection of unobservables, even with credit register type of data. This is because the econometrician only observes the contract chosen and thus has to estimate the contract offered by competitors using observational equivalent borrowers.

run a linear regression based on the pricing equation (16). The variation in the loan size allows the recovery of the marginal cost (mc), and the variation in information rents identifies the demand semi-elasticity (σ). I illustrate this approach and discuss the exclusion restrictions in the empirical application (section 7).

6.2 Second Case: Credit Register Data

When data on menus are unavailable, one has to predict the menus using observational equivalent borrowers (see, for instance, Crawford, Pavanini, and Schivardi 2018).

In the context of my model, I need a parameterization flexible enough to feature random coefficients and separate choices of lender (i.e., competition) and contracts (i.e., screening). Using a nested logit with random coefficient is not empirically tractable. As a result, I propose to estimate the pricing schedule for each borrower and characteristics at the lender level and use it to recover the willingness to pay (θ) as in step 2. Then use a logit model with linear utilities to recover the demand elasticity using choice data. A standard parameterization of the utility is: $u^*_{i,b} + \mu_b + \sigma\varepsilon_{ib}$, with ε_{ib}, with $u^*_{i,b}$ estimated from step 2, μ_b being a bank fixed effect and ε_{ib} being extreme value distributed. This procedure is similar to the nested logit approach with random coefficients but is more flexible and tractable.

Alternatively, once can use the approach developed in the section called "Alternative to Steps 2 and 3" and implemented in the empirical section of the paper. This approach has the advantage compared to the logit model that one does not have to take a stand on the choice set available to the borrower.

7 Empirical Application

To illustrate how to bring the model to the data, I provide an empirical application using data from a key paper on screening in credit markets: Hertzberg, Liberman, and Paravisini (2018).

7.1 Data and Setting

The data concerns an online lending company, Lending Club (henceforth LC) in 2013. This is an ideal institutional setting for studying screening with menus of contracts. The dataset contains all the information observed by lenders at the time of origination. The menu offered, borrowers' choice, and defaults are also observable.

LC is an online lending platform that offers unsecured loans for amounts between $1,000 and $35,000. LC was the largest online lending platform in the U.S. in that period. It originated $4.4B in consumer loans across 45 states. Prosper Marketplace, its nearest rival, originated $1.6B in the same year.

The borrowing process is the following. Prior to the borrower selecting a loan amount or maturity, LC assigns one of 25 risk categories to them using a proprietary credit risk assessment algorithm. The algorithm uses the hard information in a borrower's credit report (e.g., FICO score, outstanding debt, repayment status) and income. Conditional on the risk category, borrowers get offered the same menu. They can chose how much to borrow and chose a maturity of 36 or 60 months. The interest rate only depends on the risk category, not on the loan size or maturity. For low loan size amounts, only the low-maturity contract is available. I observe the risk category, the menu available to the borrower and their choice. Figure 7 plots the menu available to a borrower categorised as C1 in February 2013.

Model set-up: Hertzberg, Liberman, and Paravisini (2018) shows that LC can screen borrowers with maturity. To capture the LC set-up, I thus allow lenders to choose monthly payments, maturity and loan size. I use a flexible model that allows for moral hazard and adverse selection: default probabilities depend on borrower characteristics and monthly payments. I also allow for an arbitrary number of types.[21] Maturity plays the same role as interest rate in the baseline model. I derive the pricing equation by taking the first-order

21. Appendix (S) shows that the equilibrium conditions of the model with an arbitrary number of borrower types but with LC choosing to offer two optimally, and the optimality conditions with two borrower types are identical.

conditions with respect to maturity. Appendix R derives sufficient conditions for the model to match the menu patterns (i.e., fixed interest rates conditional on the risk category, and a continuum of loan size offered).

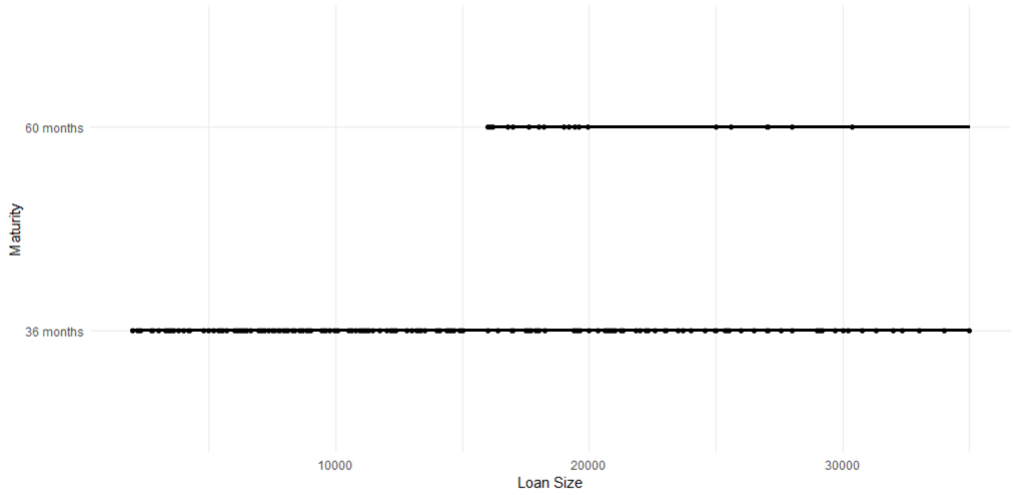

Figure 7: Maturity as a function of loan size (investment grade C1 in February 2013)

7.2 Identification and Estimation

I apply the approach discussed in section 6.1. Given the variation in menus generated by risk categories, I can use only step 1 and a generalized version of step 3 to recover the key model parameters. Nonetheless, I present the results of step 2 at the end of the section to show how they can be used to get information about the distribution of willingness to pay and that the results are plausible.

Default Probability Estimates (Step 1): I start by calculating the average default probability on long and short-maturity conditional on the borrower's investment grade and

the monthly payment-to-income ratio (Denoted PI_i). I run the following regression:

$$1 - Default_i = \delta_i^{gt} + \beta \frac{Montly\ Payment_i}{Income_i} + \beta^M \mathbf{1}_{\{T_i=60\ Month\}} + e_i \tag{25}$$

$Default_i$ is a dummy equal to 1 if the loan is reported as charged off after the loan should have been completely repaid. δ_i^{gt} are investment grade-time fixed effects. The superscript g indexes the grade and t the time. $\frac{Montly\ Payment_i}{Income_i}$ is the monthly payment to yearly income ratio. I normalize the standard deviation to one. β^M is the coefficient of interest. This coefficient captures whether high default borrower self-selects into different loans.

Denoting λ_i the hazard rate, I can recover the model parameter ($\alpha_i \approx \lambda_i$) as the hazard rate using a linear approximation of equation (20) with:

$$\lambda_i := -\frac{log(E[1 - Default_i | g_i, T_i, \frac{Montly\ Payment_i}{Income_i}])}{T_i} \tag{26}$$

Marginal costs, Demand elasticities, asymmetric information premium and discounts (Generalized Step 3):

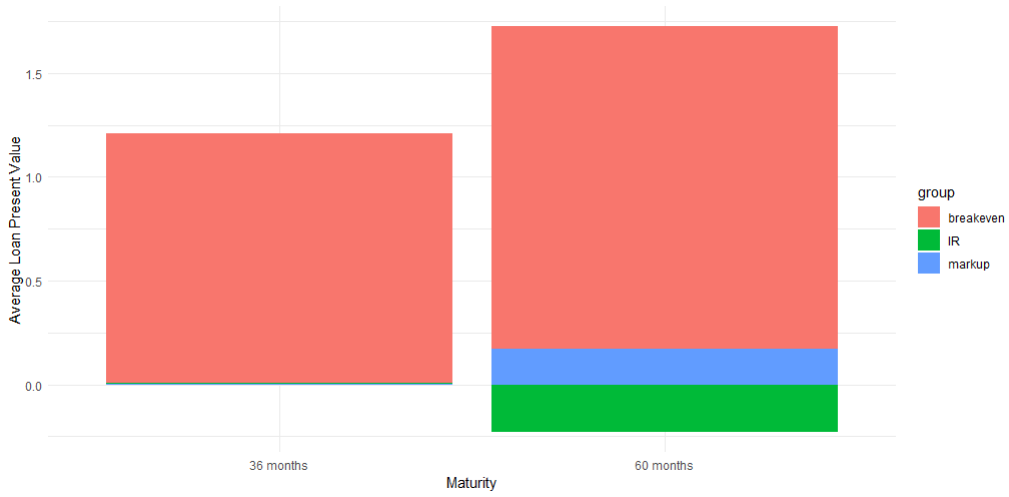

Figure 8: Decomposition of the present value of the average loan (normalized by loan size)

I use the optimal pricing condition (16) to recover the marginal cost, the asymmetric

information premium and discounts and the demand elasticity.

Instead of inverting the equation like suggested in section (6), I leverage the variation in the relative market share of the contracts for each investment grade. I do so because I do not observe the marginal cost.

In practice, I run the following two linear regressions. For the high maturity contract, regress the present value of the loan on loan size, a constant, and the relative contract shares ($\frac{n_{36}}{n_{60}}$ where 36 stands for the low maturity contract). For the high-maturity contract, I regress the present value of the loan on the loan size and a constant. I parametrize the principal of the loan R as a linear approximation of the continuous time annuity formula ($\rho R := M[1-e^{\rho T}] \approx \rho m T$ where m is the monthly payment T is the maturity so that $\alpha M \rho T$ is the approximate loan present value). All coefficients on the right and side should thus be interpreted as divided by ρ. This implies that the equation identifies the relative value of each component of the pricing equation. Formally, I run the two regressions for each month:

$$\overbrace{E[\alpha_i|g_i, T_i = 60]R_i}^{Present\ Value} = \overbrace{mc_{T_i}\text{Ł}_i}^{Break\ even} + \overbrace{\sigma_{T_i}}^{\text{“}mark\ up\text{”}} + \overbrace{\beta\frac{n_{36g}}{n_{60g}}}^{Information\ rent:\ -\sigma_{T_i}\frac{n_{36g}}{n_{60g}}IR_g} + e_i \tag{27}$$

$$\underbrace{E[\alpha_i|g_i, T_i = 36]R_i}_{Present\ value} = \underbrace{mc_{T_i}\text{Ł}_i}_{Break\ even} + \underbrace{\tilde{\alpha}}_{Total\ markup:\ \sigma_{T_i}(1+IR_g)} + e_i \tag{28}$$

$(mc_{T_i}, \sigma_{T_i}, \beta, \alpha)$ are the parameters to estimate. I interpret them through the lens of the structural model with linear preferences, and demand semi-elasticities being borrower specific parameters. Abusing the notation, I denote σ the inverse demand semi-elasticity with respect to the loan present value. For concision, I refer to it in this section as the demand sensitivities.

The error term e_i captures the heterogeneity in demand sensitivities ($\sigma_i - E[\sigma_i]$) and information rent ($IR_g\sigma_i - E[IR_i\sigma_i]$ where $IR_g := \frac{\frac{\theta_{36g}}{\alpha_{36g}} - mc_T}{\theta 60g - \theta_{36g}}$). I allow for the marginal cost to vary at the contract-month level, so it is unlikely that the error term contains heterogeneity

in costs.

The coefficient β is the information rent $\sigma_{60}IR$. I recover $\hat{IR}$ as $\frac{\hat{\beta}}{\hat{\sigma}_{60}}$.

The coefficient $\tilde{\alpha}$ captures the perfect information markup and the asymmetric information premium $\sigma_{36}[1 + IR]$. I recover $\hat{\sigma}_{36}$ as $\frac{\tilde{\alpha}}{1+\hat{IR}}$.

Equations (27) and (28) come from the theoretical model first order condition. Consequently, they should thus not be interpreted as capturing the causal impact of contract term on the loan present value. However, the use of first order condition as moment does not imply that exclusion restrictions are irrelevant. Indeed, allowing for the error terms to come from borrower unobservable heterogeneity implies some exclusion restrictions, discussed in the next two paragraphs.

The variation in relative market shares across investment grades allows us to identify β. The identifying assumption is that the relative market share variations $(\frac{n_{36g}}{n_{60g}})$ are uncorrelated with preference heterogeneity $\theta_{60g} - \theta_{36g}$ and uncorrelated with the average demand sensitivity σ_B of borrowers choosing 60-month maturity contracts. As the investment grade goal is to predict default (i.e., the borrower cost), not demand sensitivities, this assumption is plausible. Notice that, according to the model, we recover the (inverse) demand semi-elasticity with respect to the loan present value (σ). This is less problematic that recovering the the sensitivity to interest rate: The argument that the exclusion restrictions are not satisfied because high default borrowers are less sensitive to interest rate changes because they expect to repay it less often is thus not a concern here.

The variation in loan size within a group allows for recovery of the marginal cost mc. Conditional on the market segment, the loan size variation should be uncorrelated with deviation from the average demand elasticities σ_{T_i} (i.e., search costs). To mitigate the endogeneity of concerts, I use a granular instrument approach. That is, I instrument loan size using the residual from a loan size regression controlling for contract characteristics and observable borrowers' characteristics. The underlying assumption is that the search cost heterogeneity is uncorrelated with the residual demand. The results are very similar with and without

the instrument. This is likely because the variation in loan size are uncorrelated with lender choices. This is expected if, for instance, all lenders would offer the same borrower similar loan size.

Although not exploited in this empirical application, the linearity of the model allows the use of the full set of tools developed in the reduced-form literature, such as difference in difference. This is one main advantage of this model compared to a theory-oriented framework such as Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2019). The simplicity of the model also makes the exclusion restrictions transparent, as the error term has a structural interpretation. A caveat of the approach is that results depend on the interpretation of the coefficient of the linear regression. Given that markup and adverse selection are not directly observable, putting some structure is however necessary.

7.3 Results

Table 1 reports the regression results of the default regressions. The average default probability is 15 percent. Borrowers that chose high maturity contracts are α is 4% more likely to see their loan charged off. The implied average hazard rate is $4 \cdot 10^{-3}$ and the one of borrowers with high maturity contract is $2.7 \cdot 10^{-3}$. The hazard rate allows taking into account the fact that with constant default probabilities, longer loan are more likely to be charged-off.

The results of the present value regressions are reported in Tables 2 and 3. All coefficients are significant and have the sign predicted by the theory. Figure (8) provides a visual decomposition of the present value of the loan. All else equal, high-maturity contracts are more expensive to originate (the marginal cost is 1.5 per dollar lent instead of instead of 1.2). Borrowers that self-select into low maturity contracts are less likely to default (from step 1). They are also very elastic, so they do not pay any markups. This is consistent with the fact that searching costs are very low in the online lending market (as in Agarwal, Grigsby, Hortaçsu, Matvos, Seru, and Yao 2024). Borrowers that self-select into high-maturity contracts are more likely to default (from step 1). They are less price elastic. Their perfect

information markup would be 0.2 dollars per dollar borrowed. However, because of adverse selection they get an information rent that cancels this markup.

Overall, Due to screening, the price are close the the perfect competition perfect information case. However, low-default borrowers get a lower maturity compared to what they should get under perfect information. To understand whether there is too much screening, I use the social planner framework described in section 5.3.

Sufficient Statistic. Using proposition 4, I find that: $\frac{n_{36g}}{n_{60g}} I = 1.32 > E[\theta_i | g_i, T_i = 60] = 0.99$. This result implies that there is excessive screening. Due to the contractual externality, low default borrowers get a lower maturity than what they should get under perfect information but also lower maturity than what they should get in the second best scenario (informationally constrained social planner). Decreasing competition or increasing the marginal cost of lending via low-maturity contracts can thus lead to a Pareto improvement. The exact cost of the distortion requires extrapolating preferences away from their local estimates. Using the linear model assumption, low-default borrowers would prefer to go all the way to 60-month contracts.

Distribution of willingness to pay: To test if the inequality constraint assumption delivers reasonable results, I implement step 2 of section 6. I use equation (21) to recover the distribution of borrowers' willingness to pay. For each borrower, I calculate the implied loan size that would give them the same monthly payment if they were to change maturity. I consider only borrowers for whom the alternative combination of loan size and maturity is feasible, given the menus offered by LC.

For a given investment grade, the inequality allows me to recover one lower bound for the willingness to pay off high-maturity borrowers. I use the interest rate variation across borrower investment grades to recover the shape of the distribution of willingness to pay. The identifying assumption is that the distribution of marginal utility of loan size ($\alpha \sim G$)

does not change with investment grades.

This result also implies that increasing the marginal cost of originating low-maturity contracts can lead to higher maturities being offered and lower levels of credit rationing. It is a welfare-improving policy.

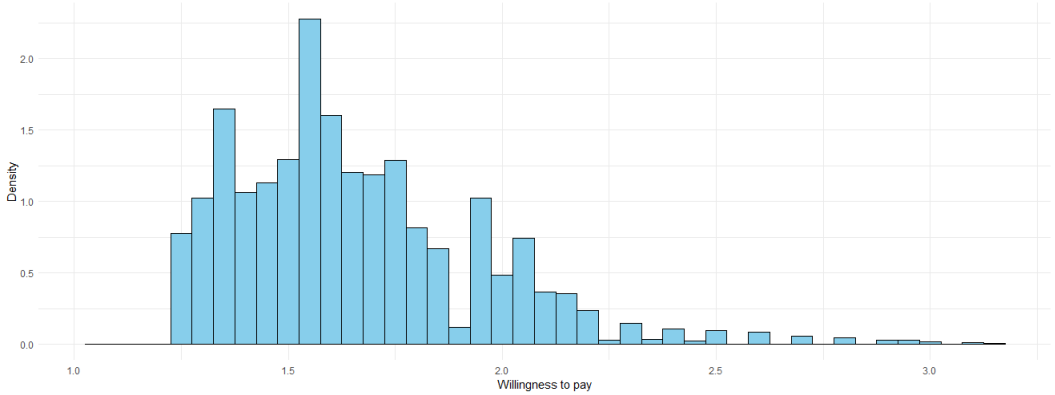

Figure 9: Borrowers' willingness to pay (all market segment pooled)

Figure (9) report the distribution of willingness to pay. The average willingness to pay is 1.6. Interpreting the value in terms of a two-period model, the results imply that for each dollar borrower in period 0, a borrower is willing to pay up to 1.5 dollars in period 2.

8 Conclusion

This paper develops and analyses the properties of a new screening framework that can be brought to the data to study the contract term distortions that stem from screening.

The paper carefully analyses the existence of equilibrium, one of the central theoretical issues of screening models. I show that the equilibrium always exists and characterize it in closed form.

The paper then brings the model to the data. The model's tractability allows for discussion of exclusion restriction and provides a structural interpretation of a linear regression of prices on quantity. I propose an approach that is robust to adverse selection and does not rely on estimating borrowers' choice set and default elasticities, which is problematic in the context of adverse selection.

The tractability of the model also opens the way to study a wide set of counterfactuals with endogenous menus of contracts. Existing models with endogenous menus, such as Wollmann (2018), use numerical methods that are computationally demanding and are often not robust to the starting point.

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A Figures

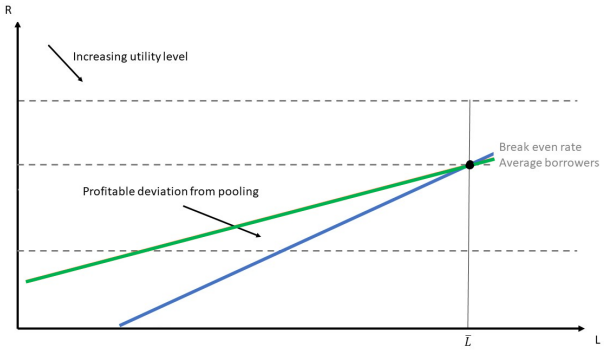

Figure 10: Perfect competition case: Deviation from pooling always exists

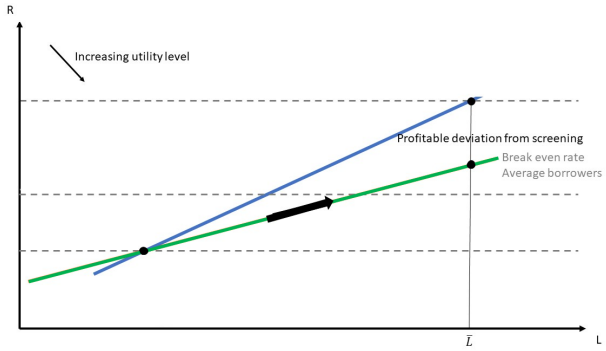

Figure 11: Perfect competition case: Deviation from screening always when the share of good borrower is high

B Tables

Table 1: Survival Probabilities

	Loan is fully paid at maturity
Term: 60 months	-4***
	(0.0118)
Monthly payment to income ratio	-2***
	(0.01445)
Subgrade fixed effect	Yes
Average number of defaults at maturity	15

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p $<$ 0.01, **p $<$ 0.05, *p $<$ 0.1

Table 2: Present value regression (60-month maturity)

	Present Value
Intercept	$4e^{3***}$
	$(1.5e^{2})$
Loan Size	1.5^{***}
	$(5.5e^{-3})$
Market share ratio	-9.3 e^{3***}
	$(1.3e^{2})$
Average present value	34,438
R^2	0.98

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

Table 3: Present value regression (36-month maturity)

	Present Value
Intercept	82^{**}
	(25.4)
Loan Size	1.2^{***}
	$(1.5e^{-3})$
Average present value	16,940
R^2	0.98

Note: Coefficients are reported in percent. Standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1

C Proofs

Proof Lemma 1: Starting from $V(u_i(X_c, R_c), \sigma_i^{-1}\epsilon_{jb}) = f(u_i(X_c, R_c)) \cdot g(\sigma_i^{-1}\epsilon_{jb})$, the condition to induce borrower to chose contract c over $\tilde{c}$ is: $f(u_i(X_c, R_c)) \cdot g(\sigma_i^{-1}\epsilon_{jb}) \geqslant f(u_i(X_{\tilde{c}}, R_{\tilde{c}})) \cdot g(\sigma_i^{-1}\epsilon_{jb})$ which in independent of $g(\sigma_i^{-1}\epsilon_{jb})$. A similar argument can be made for additive separability. The lender can thus only recover θ_i using contract terms. The expected the demand function $E[b_{ij}|\theta_i]$ with b_{ij} defined in eq. (4) is thus independent of the demand shocks.

C.1 Proofs Existence and Uniqueness

We use the following change of variable:

$$u := v(X;\theta) - F(R) \Longleftrightarrow R = F^{-1}(v(X;\theta) - u). \tag{29}$$

The bank's objective function for a particular borrower type is thus:

$$\Pi(X_i, u_i) := D(u_i)\pi(X_i, u_i) \tag{30}$$

$$with\ \pi(X_i, u_i) := PV(d_i, X_i, u_i) - mc(X_i, u_i). \tag{31}$$

$$and\ D : \text{is a continuous demand function} \tag{32}$$

The borrower incentive compatibility constraints can be rewritten:

$$g(u_i, u_j, X_i) := u_i - [u_j + \Delta v(X_j; WTP_{ij})] \geqslant 0 \tag{33}$$

The subscript i denotes the borrower type and contract designed for that type. $\Delta v(X_j;\theta_{ij}) := v(X_j;\theta_i) - v(X_j;\theta_j)$.

C.1.1 Optima: Proof of Lemma 1 for Concave Demand

We use the Karush–Kuhn–Tucker conditions to show that the first-order conditions are necessary and sufficient for a global maximum. We need (i) g functions defined in equation (33) to be concave and (ii) the objective function (Π) defined in equation (30) to be concave.

(i): Denoting (X:= $(x_c)_c$) and assuming that $F(X) = \sum_c f(x_c)$, where f is a one-to-one mapping, we denote $Z_c := f(X_c)$ so that $S(Z) := \sum_c Z_c - mc((f^{-1}(Z_c))_c)$.

(ii): We use the fact that the sum of two concave functions is concave and that concavity is preserved by log. The objective function is thus concave when $log(D(u)) + log(S(X) - u)$ is concave. Sufficient conditions are that $D(u)$ and $S(Z)$ are concave.

Using (i) and (ii), we have that $S(Z)$ increasing and concave is a sufficient condition.

Examples: The functions $\phi(u) := \frac{e^{\sigma ln(u)}}{A+e^{\sigma ln(u)}}$, $A \in \mathbf{R}^+$ and $S(X) = \sum_c \alpha_c X_c - mcX_c$ defined on $[0, \bar{X}_c]^C$ with C is the cardinality of the vector X, satisfy this property.

C.1.2 Existence of the Equilibrium

The Lagrangian is:

$$L := \sum_{i \in \Gamma} D(\theta_i u_{ib})[S_{ib}(X) - \theta_i u_i] + \sum_{ij} \lambda_{ij}\{u_{ib} - u_{jb} - \Delta_{ij} v(X_j)\} \tag{34}$$

Dropping the b index for notational convenience, the first-order conditions are:

$$\theta_i\{D_i'[S(X) - \theta_i u_i] - D_i\} = \sum_j \{\lambda_{ji} - \lambda_{ij}\} \tag{35}$$

$$D_i(\theta u_i) S'(X) = \sum_j \lambda_{ji} \Delta\theta_{ij} v'(X) \tag{36}$$

$$\lambda_{ij}\{u_{ib} - u_{jb} - \Delta_{ij} v(X_j)\} = 0 \tag{37}$$

I use Brouwer fixed-point theorem with the function $\psi := (\psi^b(U))_b$ with $U := (u_{ib})_b$ and $\psi^b(U) := S_b(X^*(U)) - \frac{D_i^b(U) + \sum_{ij}\{\lambda_{ji}(U) - \lambda_{ij}(U)\}}{\theta_i^2 D_i^b(U)}$ where $X^*(U)$ and $\lambda(U)$ are recovered using equations (36) and (37). When the objective and constraints are continuous in a compact space, the functions $X^*(U)$ and $\lambda(U)$ are continuous.

C.1.3 Proof with Linear Logit Demand

TO DO: EXTEND THE PROOF WHEN F(X)= $\sum F_c(X_c)$.

For the lender's maximization problem to be well-behaved, the Hessian matrix of $\Pi(L, u)$ needs to be negative definite. Using Sylverster's criterion, we need to show that the leading principal minors of the Hessian matrix of $-\Pi(L, u)$ have positive determinants:

The conditions to be satisfied are

$$\partial_{u^2}\Phi = \Phi_{uu}\pi + 2\Phi_u\pi_u < 0 \; and \; \Phi\pi_{LL} \cdot \partial_{u^2}(\Phi\pi) - 2\partial_{uL}(\Phi\pi) > 0 \tag{38}$$

$$\iff \pi\{(1-2\phi)\} < \frac{2}{\sigma} \tag{39}$$

$$and \;\; WTP \cdot F_{LL} \cdot [\sigma\pi(1-2\Phi) - 2] - \sigma(1-\Phi)[WTP \cdot F_L - mc]^2 > 0 \tag{40}$$

We separate the case in which competition is low enough (a) from the one in which competition is high (b). If (b) is satisfied, we do not need the assumption on (a) for the Nash equilibrium to exist.

(a) **Condition when competition is low enough:** The left-hand side is bounded above by $\pi\{(1-2\phi)\} < \{WTP \cdot F(\bar{L}) - mc \cdot \bar{L} - u\}(1-2\phi) < WTP \cdot F(\bar{L}) - mc\bar{L} - \bar{u} < WTP \cdot F(\bar{L}) - mc\bar{L}$. Where $\bar{F}(\bar{L})$ is the loan size that maximizes the social surplus. For the last inequality, we use the fact that utilities cannot be negative because of a participation constraint ($i.e., u \geqslant 0$). The inequality implies that there exists a level of competition $\sigma < \bar{\sigma} := \frac{1}{WTPF(\bar{L}) - mc \cdot \bar{L} - 2}$ such that condition (39) is satisfied.

Similarly, equation (40) can be written:

$$-WTPF_{LL}[2 - \sigma\pi(1-2\phi)] > \sigma(1-\phi)[WTPF_L - mc]^2 > 0 \tag{41}$$

$$\implies -F_{LL}(L) > \frac{\sigma\bar{A}}{2 - \sigma[WTPF(\bar{L}) - mc\bar{L}]} > 0, \forall L \in [0, \tilde{L}] \tag{42}$$

Condition (42) is derived assuming that F in increasing and denoting: $WTPF_L - mc < \bar{A} := max\{[F_L(0) - mc], [F_L(\tilde{L}) - mc]\}, \; \forall L \in [0, \tilde{L}]$ and $(1-\phi) \geqslant 1$. Condition (42) is thus satisfied when the utility function F is increasing and sufficiently concave.

When condition (42) is satisfied for all $\sigma < \bar{\sigma}$, the equilibrium is unique and exists.

Condition when competition is high:

Because of the participation constraint, individual profits are bounded by $c := [WTP \cdot F(\bar{L}) - mc\bar{L}$. Then we have:

$$\partial_u \Pi(u) < \partial_u \Phi c \implies \pi < c + \frac{1}{\sigma(1-\phi)}. \tag{43}$$

If there is a capacity constraint k, then

$$\pi < c + \frac{1}{\sigma(1-k)}. \tag{44}$$

The market share is bounded by one, so that $((1-2\phi) < 1)$.

We thus have $\pi\{(1-2\phi)\} < \frac{1}{\sigma(1-k)} + c < \frac{2}{\sigma} - \varepsilon$ with $\varepsilon > 0$.

Hence, k should be lower than $\frac{1-c\sigma}{2-c\sigma}$; this is decreasing for σ large enough, so we have $k < \frac{1}{2}$.

Replacing the bound in equation (40), we have:

$$-WTPF_{LL} > \frac{[WTPF_L - mc]^2}{\varepsilon} > 0 \tag{45}$$

Similarly to case (a), this expression is satisfied when F is increasing and sufficiently concave.

Intuition: Why the Capacity Constraint Works

Small deviations from screening to pooling attract a high proportion of high-default borrowers ($\bar{n}$). Indeed, starting from the screening candidate, we get

$$\frac{\Delta\bar{u}}{\Delta u} \cdot \frac{\Phi(\bar{u})(1-\Phi(\bar{u}))}{\Phi(u)(1-\Phi(u))} \tag{46}$$

$$\implies \frac{u^n - u^o + (\frac{\bar{WTP}}{\bar{\theta}} - \frac{WTP}{\theta})L}{u^n - u^o} \cdot \frac{\bar{n}}{n}\frac{(1-\bar{n})}{(1-n)} \tag{47}$$

$$\implies \frac{\bar{n}}{n} < \frac{u^n - u^o + (\frac{\bar{WTP}}{\bar{\theta}} - \frac{WTP}{\theta})L}{u^n - u^o} \cdot \frac{\bar{n}}{n} < \frac{u^n - u^o + \overbrace{(\frac{\bar{WTP}}{\bar{\theta}} - \frac{WTP}{\theta})L}^{>0}}{u^n - u^o} \cdot \frac{\bar{n}}{n}\frac{(1-\bar{n})}{(1-n)} \tag{48}$$

Local deviations are not profitable, but large ones are. The capacity constraint limits the ability to do non-local deviations and thus restores pure strategy equilibrium existence.

Condition for screening inefficient:

$$\frac{n_G}{n_B} IR_B := \frac{n_G \theta_G}{n_B \theta_B} \frac{WTP_G - \frac{mc}{\theta_G}}{WTP_B - WTP_G} > 1 \tag{49}$$

Equilibrium L_G amount:

$$L_G = \delta[\bar{L} + \frac{1}{\sigma}\tilde{\delta}\frac{1}{WTP_B - WTP_G}(1 + \frac{\theta_G n_G}{\theta_B n_B})] = \delta[\bar{L} + \frac{1}{\sigma}\frac{\tilde{\delta}}{WTP_B - WTP_G} + \frac{IR}{\sigma[WTP_B - \frac{mc}{\theta_B}]})] < \bar{L} \text{ for } \sigma \text{ h} \tag{50}$$

with: $\delta := \frac{WTP_B - \frac{mc}{\theta_B}}{WTP_B - \frac{mc}{\theta_G}}$ is the strength of the AS problem. $\tilde{\delta} := \frac{WTP_G - \frac{mc}{\theta_G}}{WTP_B - \frac{mc}{\theta_B}}$ is the relative profit measure.

When $\sigma = \frac{2}{A_B \bar{L}}$, $IR = 1^+$:

$$L_G = \delta[1 + \frac{\tilde{\delta}}{WTP_B - WTP_G}\frac{A_B}{2} + \frac{1^+}{2})] < \bar{L} \text{ For } WTP_B - WTP_G \text{ large enough or } A_B \text{ low enough (this i} \tag{51}$$

Given the condition for the function to be well behaved and screening to be inefficient, there exists a zone in which banks screen. We can thus analyse the interior solution to understand the screening property trade-off. The level of inefficiency should be low enough so that the bound on the degree of competition is low enough that banks to not have enough freedom to pool borrowers.

C.2 Proof Theorem 2

(i)

(ii) Denoting the lagrange multiplier by λ_{ij} writing the first order conditions of problem 12, we get that:

$$\alpha_i R_i = mc(X) + \frac{D_i + \sum_j [\lambda_{ij} - \lambda_{ji}]}{D_i'} \tag{52}$$

$$D_i S'(X_i) = \sum_j [\lambda_{ji} v'(X_i)] \implies 0 \leqslant \sum_j \lambda_{ji} \leqslant \frac{D_i S'(X_i)}{min_j\{\Delta_{ij} v'(X_j)\}} \tag{53}$$

Using the bound, when $D_i' \to \infty$, and $\exists \bar{B} : S'(X_i) < \bar{B} \forall X$ and $v'(X_j) \neq 0 \ \forall X$, we must have the lender price each borrower at marginal costs.

D Model for the positive analysis

I consider two contract characteristics (loan size L and the loan present value R) and two borrowers (G and B). B has a higher willingness to pay θ and a higher default probability d. Demand shocks are additive and extreme value distributed so that the demand has a logit form. In that case, I use a capacity constraint to ensure the concavity of the demand. Utilities are linear in contract terms. The banks all have the same marginal cost of lending, which is a picewise linear function. I assume the following technical conditions:

$$mc(L) := \begin{cases} \underline{mc} \text{ if } L \in [0, \underline{L}) \\ \underline{mc} \cdot \underline{L} + \bar{mc} \cdot L \text{ if } L \in (\underline{L}, \bar{L}) \\ \infty \text{ Otherwise} \end{cases} \tag{54}$$

$$with\ 0 < \underline{mc} < \bar{mc},\ and\ \theta_B > \frac{\bar{mc}}{\alpha_B},\ \frac{\bar{mc}}{\alpha_G} > \theta_G > \frac{\underline{mc}}{\alpha_G} \tag{55}$$

The conditions ensure that the marginal cost of lending is always different for the two contracts, which is required for our second result.

E Model with CES demand function

Example: Perfect information equilibrium contracts. Let us consider the case in which demands have a logit form as in equation (6) and lenders are homogeneous. The unique symmetric equilibrium is characterized by:

$$X_i^{PI} : \partial_X S_{ib}(X_i^{PI}) = 0 \tag{56}$$

$$\theta_i R_i = mc(X_i^{PI}) + \frac{\varepsilon_{ib}^{-1}}{1+\varepsilon_{ib}^{-1}} S_{ib}(X_i^{PI}) \tag{57}$$

Where $\varepsilon_{ib} := \sigma_i(1 - D_{ib})$ is the semi-elasticity with respect to the utility ($u_{ib} := \theta_i \tilde{u}_{ib}$). When normalizing the outside option of not borrowing to zero, $D_{ib} = \frac{1}{B}$ so the semi-elasticity is equal to $\sigma_i(1 - \frac{1}{B})$ in a symmetric equilibrium.

Equation (56) gives the optimal contract characteristics, and equation (57) gives the equilibrium pricing as a function of contract characteristics. When the parameter driving the demand elasticity tends to infinity (i.e. when $\sigma \to \infty$), lenders price at marginal costs $\frac{mc(X_i)}{\theta_i}$. When demand elasticity decreases, the markup increases.

Example: Imperfect information equilibrium contracts. When $dim(\theta_i) = 1$, the demand has a logit form as in equation (6) and lenders are homogeneous, there is a unique symmetric equilibrium. To simplify the equilibrium expressions, let us assume that utilities are linear and marginal costs is piecewise linear such that $\partial_X u_i(X, R) = \alpha_i \theta_i > \partial_X mc(X) = mc$ on $[0, \bar{X}]$. In that case, R is linear in X (i.e., $R = rX$), and the equilibrium contrast are characterized by:

$$Contract\ Terms: \ X_i = \begin{cases} \bar{X}, \text{ if } i = I \\ X_{i+1} \frac{\theta_{i+1} - r_{i+1}}{\theta_{i+1} - r_i}, \text{ Otherwise} \end{cases} \tag{58}$$

$$Pricing: \ \theta_i r_i = mc + \frac{\varepsilon_{ib}^{-1}(1 + IR_i)}{1 + \varepsilon_{ib}^{-1}(1 + IR_i)} [\theta_i \alpha_i - mc] \tag{59}$$

With $IR_i := \theta_i^{-1}[\frac{\alpha_i\theta_i - mc}{\Delta\theta_{i+1,i}}\mathbf{1}_{i\neq I} - \frac{n_{i-1}}{n_i}\frac{\alpha_{i-1}\theta_{i-1} - mc}{\Delta\theta_{i,i-1}}\mathbf{1}_{i\neq 0}]$ and $\varepsilon_{ib} := \sigma_i(1 - \frac{1}{B})$.

F Extension: Loan-to-value fees, Maturity, and Collateral

The loan size L and the repayment R can also be interpreted as LTV and maturity, respectively. The following paragraph presents a formal argument for the LTV and its maturity.

Loan-to-value and fees: The model presented in Section **??** can be extended by allowing borrowers to have income (A) in the first period of the contract to allow for down payments (D) and fees (f). The utility thus becomes

$$u_i = [\delta\theta + 1]F(L + D) + u(A - D - f) - \delta\theta u(W - R) \tag{60}$$

In that case, we get that high-default borrowers are less willing to put their own wealth into their houses. This justifies the fact that, under screening, high-LTV loans will be selected by borrowers with unobservably high default probabilities.

When the marginal cost of lending mc is greater than 1, the optimal contract is such that the lender requires much down payment as possible (i.e., $D = A$). Loan size L in equation (1) can thus also be interpreted as loan-to-value.

Maturity: The full model with maturity is presented in Appendix H. This section focuses solely on the main equation. When the default rate is constant in each period, the borrower's value function can be written as

$$u_i = \frac{F(L+D) - \theta C[1 - e^{-\rho T}]}{\rho} - D - f. \tag{61}$$

$\frac{F(L)}{\rho}$ is the present value of the loan. $\frac{C}{\rho}[1 - e^{-\rho T}]$ is the present value of the debt for borrowers. ρ is the discount rate. θ is the per-period survival probability. T is the maturity. $\theta C := \theta r + (1-\theta)[\gamma K + \Delta]$ is the expected per period loan cost for borrowers, with r as the monthly payment, γK the collateral value, and Δ as a cost of defaulting beyond the loss of collateral.

The supply side is identical to the one presented in the following section but set in continuous time. Lenders offer differentiated loan products and maximize their expected profits. I show in Appendix H that it is optimal for the lender to set very high monthly payments (infinite) and minimise the contract's maturity so that the borrower's probability of paying the non monetary cost is minimal. If there is a limit on how high the monthly payment can be, for instance, repayment cannot be higher than disposable monthly income, it is optimal to set the monthly payment at the limit and extract surplus with the longer maturity.

Collateral: Appendix J states the conditions required for collateralized debt to arise as the optimal contract. There are two critical assumptions. The first states that lenders cannot observe the cash flow (which is costly to do so as in Townsend 1979), so borrowers can lie about their income. The second assumption is that the bank can use collateral and seize it upon default if this is more efficient than spending the verification cost. For the collateral to be seized only upon default, the bank must value it less than the borrower. This assumption is adapted from Lacker (2001) to fit mortgage markets (see Appendix J). Papoutsi, Paravisini, Rappoport, and Taburet (2024) show that empirically, this assumption holds in the market for corporate loans.

G Intuition credit rationing

Figure (12) summarizes the model equilibrium under perfect information when utilities and marginal costs are linear, and the feasible set is $L \in [0, \bar{L}]$.

I consider two borrowers (G and B). B has a higher θ and a higher default probability.

I plot borrower indifference curves (blue for B borrowers and green for G borrowers) and lenders' break-even conditions for each market segment. To render the graph more legible, I consider the situation in which lenders have to pay a fixed cost of originating contracts (i.e., $mcL = f \in \mathbf{R}^+$).

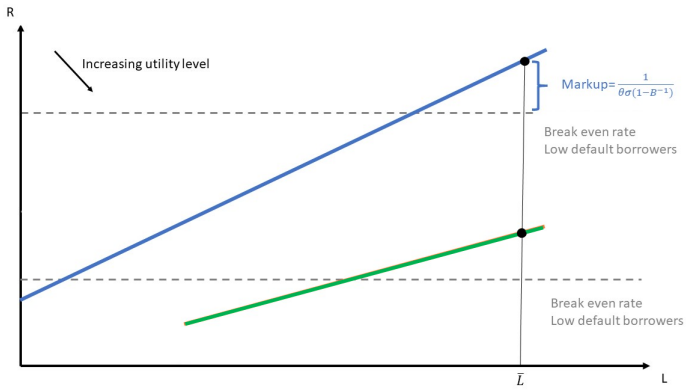

Figure 12: Equilibrium contracts under perfect information

In that set-up, the perfect information contract and the perfect information contracts in best contracts are not incentive-compatible. If they were both offered in a menu, all borrowers would choose the cheaper product (see Figure 13).

Thus, under imperfect information, banks must distort the first best contracts to maintain borrowers' incentives to self-select. To do so, lenders can use two tools: interest rates (via cross-subsidies) and loan size (via credit rationing). Figures 14 and 15 give a visual representation of how incentives are maintained using each of these margins.

I analyse in the bulk of the paper which market segment/contract characteristic lender distort. I show that high competition forces lenders to set prices close to their perfect information values and maintain incentives to self-select using loan size. Otherwise, a competitor

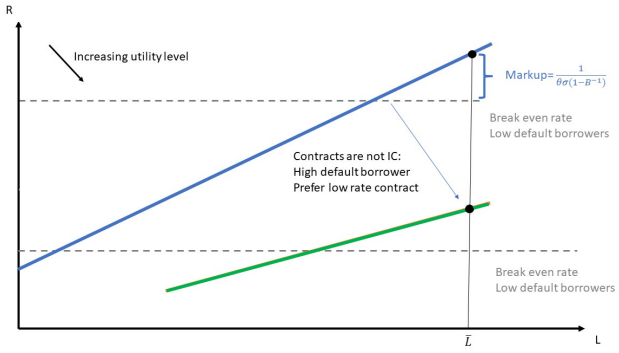

Figure 13: First best contracts are not incentive compatible

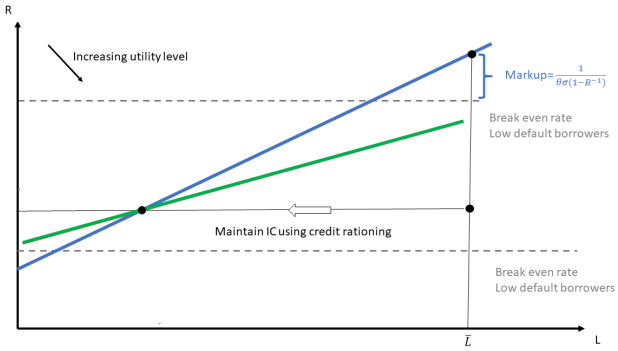

Figure 14: Tool 1 to maintain IC: Use credit rationing

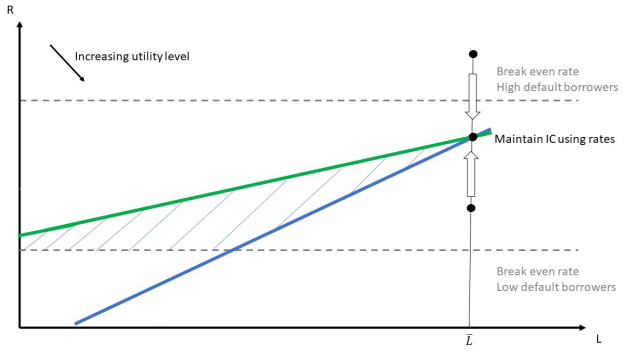

Figure 15: Tool 2 to maintain IC: Use interest rates

can offer a contract that steals only the most profitable customers by offering a contract in the dashed region of Figure 14. I also show that low competition allows the use of cross-subsidies and that lenders choose to distort the contracts of the market segment that generates the lower surplus.

H Model with Maturity

H.1 Borrower

A borrower derives utility $F(z)$ during the time interval Δt. With probability $(1 - \theta\Delta t)$ the borrower can repay the monthly loan payment rL during that period. With probability

$(1-\theta)\Delta t$, the borrower defaults and loses collateral cK.

Denoting $V(t)$ the value function of the borrower at time t when he did not default and $\tilde{V}(t)$ when he defaults, denoting the discount factor by ρ, the value function is

$$V(t) = F(z)\Delta t + \frac{\Delta t\lambda(t)}{1+\rho\Delta t}\{[\tilde{V}(t+\Delta t) - cK]\} + \frac{1-\lambda(t)\Delta t}{1+\rho\Delta t}[V(t+\Delta t) - rL]. \tag{62}$$

Assuming constant default probability, constant hazard rate λ and outside option, taking the limit when $\Delta t \to 0$ and using the boundary condition $V(T) = \bar{V}(L) = V(T) = F(L)\int_0^\infty e^{-\rho t}dt = \frac{1}{\rho}F(L)$ and using the discount factor ρ for the period below the maturity, we get for $t < T$:

$$V = \frac{F(L)}{\rho}e^{-\rho T} + \frac{F(L)-R}{\rho}[1-e^{-\rho T}] \tag{63}$$

$\frac{F(L)}{\rho}$ is the present value of lending. $e^{-\rho T}$ is the probability of survival until maturity times the discount rate. $R := \theta rL + \theta[cK + \Delta V]$ is the expected per period cost of the loan for borrowers.

I parameterize $\tilde{V} = V - \delta$ so that ΔV is a constant equal to δ. This implies that lenders expect next-period value functions $\tilde{V}(t+\Delta t)$ and $V(t+\Delta t)$ to satisfy an equilibrium condition $\tilde{V} = V - \delta$. This can be interpreted as borrowers who renegotiate the debt upon default and get their previous value function minus some amount δ.

H.2 Lender

Each lender maximizes expected profits:

$$\max_{\{r,L,K,T\}} \Phi(V(t))\{\frac{(1-\lambda)rL+\lambda cK}{\rho}[1-e^{-\rho(T-t)}]-mcL\} \tag{64}$$

Using equation (61) to express the problem in terms of promised utility yields

$$\max_{\{V,K,T,L\}} \Phi(V)\{\frac{F(L)}{\rho}-V-\lambda\frac{\Delta V}{\rho}[1-e^{-\rho(T-t)}]-mcL\} \tag{65}$$

So, lenders want to minimize borrowers' default to prevent borrowers from paying the cost of defaulting ΔV. With exogenous default, lenders want to set maturity to zero and monthly payment to infinity.

If there is a limit to how high the monthly payment can be (for instance, if monthly payment rL cannot be above monthly income W), then the lender sets rL to W and extracts surplus using the interest rate rather than maturities.

I Extensions: Many Borrowers, Many Contract Features

The model can be extended to feature an arbitrary number of characteristics X, as long as they enter linearly:[22]

$$\beta'(L,X)-\theta R \tag{66}$$

where R is the present value of the loan and X a set of characteristics such as fees or financial advice, and β is a vector of preferences.

22. Interaction terms are possible, since $\beta_1 L+\beta_2 L\cdot C=\beta_1 L+\beta_2 X$.

Using the fact that $R = WTP(L, X) - u$, the lender problem is thus

$$\max_{\{u_i, X_i, L_i\}} \sum_i N_i(\theta_i u_i)[\beta'(L_i, X_i) - \theta_i u_i - mc(X, L)] \tag{67}$$

$$s.t.\ u_i \geqslant u_j + (WTP_i - WTP_j)'(X_j, L_j)$$

I.1 Many borrower types, unidimensional case

Let us consider x borrower types. I denote them with the index $i \in \mathcal{N}^*$, with $WTP_i > WTP_j,\ \forall i < j$. The incentive compatibility constraints can be written as a set of IC and a monotonicity constraint:

$$(ICs):\ u_1 \geqslant u_2 + \Delta WTP_1 L_2 \geqslant u_3 + \Delta WTP_2 L_3 ... \geqslant u_x + \Delta WTP_{x-1} L_x \tag{68}$$

$$(Ms): L_i \geqslant L_j, \forall i > j \tag{69}$$

$\Delta WTP_i := WTP_i - WTP_{i+1}$

Let us focus on a situation in which competition is high so that we have an interior solution (i.e., $L_i < \bar{L},\ \forall i > 1$). At equilibrium, the optimal contract is

$$L_1 = \bar{L} \tag{70}$$

$$L_{i-1} - L_i = \frac{R_{i-1} - R_i}{WTP_{i-1}} \in [0, \bar{L}],\ i > 1 \tag{71}$$

$$R_i = \frac{mcL_i}{\theta_i} + \frac{1}{\sigma_i \theta_i}\{[1 + \frac{1}{n_i}\frac{\alpha_i - mc}{\Delta WTP_i} 1_{i>1}] - [\frac{1}{n_{i-1}}\frac{\alpha_{i+1} - mc}{\Delta WTP_{i+1}}] 1_{i<x}\} \tag{72}$$

I.2 Many borrower types, multidimensional case

Let us denote X_i a vector of non-interest-rate characteristics of the contract designed for borrower i. The incentive compatibility constraints are

$$(ICs): \ u_1 \geqslant u_2 + \Delta WTP_1(L_2, X_2) \geqslant u_3 + \Delta WTP_2(L_3, X_3)... \geqslant u_x + \Delta WTP_{x-1}(L_x, X_x) \tag{73}$$

$$(Ms): WTP_{ij}L_i, X_i) \geqslant WTP_{ij}(L_j, X_j), \forall i > j \tag{74}$$

$\Delta WTP_{ij} := WTP_i - WTP_j$

The difficulty comes from having to guess and track which ICC is binding. The simplest case is when the ordering of borrowers and willingness to pay is the same along each characteristic X. This is a natural case when the only source of unobservable heterogeneity comes from default probabilities. In that case, the lender distorts the characteristics that have the lowest shadow cost.

An alternative solution is to assume that lenders maximize the worst-case scenario, as in Carroll (2017). In that case, the optimum for the principal is to screen along each contract term separately.

Appendix J discusses the necessary conditions for collateralized debt to be the optimal contract.

I.3 Endogeneous default probabilities

The model can be extended to allow for endogenous default probabilities.

The analysis stays exactly the same, and closed form solution are available when survival probabilities have the form:

$$\theta_i = \delta_i + \frac{\beta_i}{R_i} \tag{75}$$

Where δ_i and β_i are borrower-specific parameters. β_i drives moral hazard or burden of payment and δ_i is the adverse selection. R_i is the monthly payment.

J Conditions for Collateralized Debt to Be the Optimal Contract

The assumptions are:

ASSUMPTION 1: *Ex post private information.* In this paper, I model this assuming that banks cannot observe the second period cash flow (W) of the borrower,[23] borrowers can thus lie about their income and hide it from the bank. The bank can spend some amount to verify it.

ASSUMPTION 2: *The house can be used as collateral (i.e., housing is observable). Using collateral to deal with ex post private information is less costly than verifying cash flows. The borrower values the house more than the bank.* This assumption ensures that in the optimal mortgage contract, the bank asks for cash instead of housing when possible. In this paper, the reason the borrower prefers the house more than the bank is that borrower values the house more than its selling price and the bank has a utility over cash rather than house.

At t=2, borrowers privately observe their income realization ($\tilde{W}$) and choose whether to file for default or not. Borrowers default when they cannot repay (i.e., $\delta H + W - R < 0$) or when it's better for them to strategically default (i.e., $H + u(W - R) < u(K + W)$ or $u(\delta H - R + W) < u(K + W)$. K is the amount the bank gives back (or asks for) after seizing the house and selling it. K thus has to be lower than or equal to $\delta H - R$ so that all inequalities are satisfied. In my model, since borrowers value cash more than upon default than banks ($\alpha > 1$), the constraint is binding $K = \delta H - R$. Notice that the bank wants to prevent strategic default when the borrower is in negative equity ($\delta H < R$), so it punishes

23. Or it is costly to do so.

the the borrower by seizing more than the house (i.e. $K = \delta H - R$ can be negative).

K Lemmas

Proofs of Lemmas 1 and 2: The two ICCs can be written $(WTP_B - WTP_G)'L_B \geqslant u_B - u_G \geqslant (WTP_B - WTP_G)'L_G$. or $IC_B : \ (WTP_B - WTP_G)'L_B \geqslant u_B - u_G$ and $MC : \ L_B \geqslant L_G$. When $L_B = L_G$, as in the FB, this implies that $\bar{u}_B - u_G = (WTP_B - WTP_G)'\bar{L}$, which is the case when $R_G = R_B$. This implies that both IC constraints are binding. If $L_B > L_G$, $(WTP_B - WTP_G)'L_B \geqslant u_B - u_G$ must be binding.

Assuming that the constraint (MC) is not binding and solving for problem 67, we notice that (IC_B) does not depend on L_B. Thus $L_B = \bar{L}$.

Intuitively, the self-selection problem comes from high-default (high-WTP) borrowers who want to pretend to be low-default (low-WTP) borrowers to get a cheaper loan. If the bank offers a contract with distorted B loan size ($L_B < \bar{L}$), there is a Pareto improvement that leads to either more profits in the B market segment or relaxes the IC constraint. For instance, increasing L_B up to $\bar{L}$ and increasing the interest rate by $WTP_B(L_B - \bar{L})$ increases bank profits without affecting the incentive compatibility constraints.

The fact that the participation constraint is not binding is shown by solving for the perfect competition model to get the equilibrium conditions from Proposition 3 and then using comparative statics by varying the competition level.

L Proof of Results

Result 2: (i)

$$\partial_{\theta_G^{-1}} L_G < 0 \iff mcL_G < \frac{1}{\sigma}[-\partial_{\theta_G^{-1}} CS - \partial_{\theta_G^{-1}} MC] \tag{76}$$

$$\iff mc\sigma \frac{(WTP_B - \frac{mc}{\theta_B})\bar{L}}{WTP_B - \frac{mc}{\theta_G}} < [\underbrace{-\partial CS}_{cross-subsidy\ channel} - \overbrace{mc\frac{CS}{WTP_B - \frac{mc}{\theta_G}}}^{perfect\ competition\ channel} - \underbrace{\partial MU}_{competition\ channel} - \overbrace{mc\frac{MU}{WTP_B -}}^{perfect\ competition} \tag{77}$$

$-\partial MU - mc\frac{MU}{WTP_B - \frac{mc}{\theta_G}}$ is negative. $-\partial CS$ is positive $-mc\frac{CS}{WTP_B - \frac{mc}{\theta_G}}$ is negative, so the difference can be positive or negative. It is negative when the no-cross-subsidy effect is stronger than the cross-subsidy effect.

To focus on the cross-subsidy channel, let us consider that borrowers have the same elasticity (i.e., $\sigma := \sigma_B \theta_B = \sigma_G \theta_G$), so the competition channel is absent. In that case, we have

$$\frac{\partial CS}{CS} = -\frac{WTP_B}{WTP_B - WTP_G} \tag{78}$$

$$\implies \frac{\partial CS}{CS} - \frac{mc}{WTP_B - \frac{mc}{\theta_G}} \geqslant 0 \iff \frac{1}{1 + \theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant \frac{mc}{WTP_B} \tag{79}$$

This implies that a decrease in θ_G has a positive impact when competition is low. It has a negative impact when competition is high if $\frac{WTP_B}{1+\theta_G^{-1} - \frac{WTP_G}{WTP_B}} \geqslant mc$.

The right-hand side captures the perfect competition channel. When the marginal costs are high, changes in survival probabilities have a high impact on the perfect competition spread between rates. When WTP_B is high, the high-default borrower is less likely to copy the low-default contract, so the perfect competition channel is relatively less important.

The left-hand side captures the strength of the cross-subsidy channel. When the spread between preferences is high ($\frac{WTP_G}{WTP_B}$) or the survival probability of the low-default borrower is low (θ_G), the cross-subsidy channel is low. This renders the channel less likely to dominate the marginal cost effect.

(ii)

$$\partial_{\theta_B} L_G > 0 \Longleftrightarrow 0 < \frac{mc}{\theta_G} L_G + \frac{1}{\sigma}\{\partial_{\theta_B} CS + \partial_{\theta_B} MC\} \tag{80}$$

We have: $\partial_{\theta_B^{-1}}(\theta_B MC) > 0$ $\partial_{\theta_B^{-1}}(\theta_B CS) < 0$. To focus on the cross-subsidy channel, let us consider that borrowers have the same elasticity (i.e., $\sigma := \sigma_B \theta_B = \sigma_G \theta_G$), so the competition channel is absent. In that case, we have

$$\frac{\partial_{\theta_B} CS}{CS} = -\frac{WTP_B}{WTP_B - WTP_G}\frac{1}{\theta_B} \tag{81}$$

$$\Longrightarrow -\frac{\partial_{\theta_B} CS}{CS} - \frac{mc}{\theta_G}\frac{1}{WTP_B - \frac{mc}{\theta_G}} > 0 \Longleftrightarrow \frac{WTP_B}{1 + \frac{1}{\theta_B} - \frac{WTP_G}{WTP_B}}\frac{\theta_G}{\theta_B} \gtrless \frac{mc}{WTP_B} \tag{82}$$

Result 3: (i) $\partial_\alpha L_G > 0 \Longleftrightarrow L_G < \bar{L} + \underbrace{\frac{mc}{\alpha^2}\frac{1}{\sigma}\frac{\alpha}{WTP_B - WTP_G}[\frac{\theta_B}{\theta_G} + \frac{n_B}{n_G}]}_{>0}$

(ii) $\partial_{\alpha_G} L_G > 0 \Longleftrightarrow \frac{\alpha_G}{\theta_B} L_G < \frac{1}{\sigma} CS \implies \bar{\sigma}_\alpha := \frac{CS[\frac{mc}{\alpha_G} - 1] - MU}{PC}$, with $PC := S_B \bar{L}$

Result 4: (i) $\partial_{mc} L_G > 0 \Longleftrightarrow \frac{1}{\theta_G} L_G - \frac{1}{\theta_B}\bar{L} - \frac{1}{\theta_G}\frac{CS}{S_G} > 0$ $\sigma_{mc} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC - \frac{\theta_G}{\theta_B}\bar{L} \cdot S_{BG}} < 0$ if $CS[\frac{S_{BG}}{S_G} - 1] - MU > 0$ with $S_{BG} := WTP_B - \frac{mc_G}{\theta_G}$ and $S_G := WTP_G - \frac{mc_G}{\theta_G}$.

(ii) $\partial_{mc_G} L_G > 0 \Longleftrightarrow \frac{1}{\theta_G} L_G - \frac{1}{\theta_G \sigma}\frac{CS}{S_G} > 0$ so $\sigma_{mc_G} := \frac{CS[\frac{S_{BG}}{S_G} - 1] - MU}{PC}$ with $S_{BG} := WTP_B - \frac{mc_G}{\theta_G}$ and $S_G := WTP_G - \frac{mc_G}{\theta_G}$.

(iii) $\partial_{mc_B} L_G < 0 \Longleftrightarrow \frac{-\frac{\bar{L}}{\theta_B}}{WTP_B - \frac{\theta_G}{mc_G}} < 0$ which always holds.

M Proof of Propositions

Proposition 4 $\partial_\delta L_G = \underbrace{\frac{\frac{mc_2}{(\theta_B + \delta_B)^2}}{WTP_B - \frac{mc_2}{\theta_G}} L_{90}}_{\text{“perfect competition” effect}} - \underbrace{IR_B \frac{1}{\theta_B + \delta_B}}_{\text{“monopoly” competition effect}}$

Proof (ii): $u_i = S(L_i) - \frac{1}{\sigma} - IR_G 1_{i=G} + IR_B 1_{i=B}$. For G borrowers, $S(L_G)$ increases when

L_G increases. In the other market segment, $\partial_\delta u_B = -\partial_\delta R_B = \frac{1}{\theta_B+\delta_B}[\underbrace{\frac{mcL_{95}}{\theta_B+\delta_B}}_{fair\ price} - \underbrace{IR_B}_{Information\ rent}]$

N Model with Heterogeneous Marginal Costs

In this section, I introduce heterogeneous marginal costs to analyze the impact of marginal cost policies (i.e. capital requirements).

For simplicity, since they do not add anything to the policy analysis, I assume that the price elasticity of both borrowers is the same ($\sigma := \sigma_G(1-\frac{n_G}{B}) = \sigma_B(1-\frac{n_B}{B})$).

The key point of those policies is that they are designed differently for each specific product. For tractability, it is useful to model those policies as a piecewise linear function over some product characteristics. As a result, it is also convenient to introduce discontinuities in the marginal costs at the same thresholds so that the first-best products are different for each borrower. This assumption ensures that demand is continuous.

Formally, I consider a situation in which the marginal cost of lending via a contract with a characteristic X (e.g. LTV) is higher above a certain threshold $\bar{T}$:

$$mc(X) = mc_1 + 1_{X>\bar{T}}(mc_2 - mc_1) \tag{83}$$

Borrowers have preferences such that

$$WTP_G \geqslant \frac{mc_1}{\theta_G},\ WTP_G < \frac{mc_1}{\theta_G} \tag{84}$$

$$WTP_B > \frac{mc_2}{\theta_B},\ WTP_B \geqslant \frac{mc_2}{\theta_B} \tag{85}$$

For tractability, I consider that the policies are not large enough to change the above ordering.

N.0.1 Policy experiment 1: Effect of changes in capital requirements

Capital requirements are often based on loan-to-values ratios (for instance, in Basel III). For this reason, I model capital requirements based on LTV. I consider that the marginal cost now takes the form

$$mc_c = mc - \omega^l \frac{1}{LTV} 1_{LTV<L\bar{T}V} - (\omega^h - \omega^l) \frac{1}{LTV} 1_{LTV \geqslant L\bar{T}V} \tag{86}$$

with $\omega \frac{1}{LTV}$ being the capital requirements and ω capturing how the capital requirements vary with the loan leverage. A positive ω^l (or ω^h) implies that capital requirements are increasing in LTV when LTV is below (above) a threshold.

Assumptions 5: Let us assume that the increase in capital requirements is not high enough to render the NPV of lending negative.

For simplicity of exposition, let us consider the case in which the optional contract is such that both borrowers put the same level of down payment (D) (as in F)[24] This assumption does not impact the results.

Using Proposition 2 and Lemma 4, the equilibrium distortions can be written:

24. That is, $\frac{1}{\epsilon} < \frac{mc}{\theta}$ so that the optimal contract features screening on loan size and a maximum amount of deposit D.

$$L_G = \overbrace{\frac{1}{S_{BG}}[S_B(\bar{L} - D)}^{perfect\ competition\ distortion\ level} + \underbrace{(\frac{1}{\sigma_G\theta_G} - \frac{1}{\sigma_B\theta_B})}_{competition\ channel} + \overbrace{\frac{I_G}{\sigma_G\theta_G} + \frac{|I_B|}{\sigma_B\theta_B})}^{cross-subsidy\ channel} \tag{87}$$

$$S_{BG} := WTP_B - \frac{mc + \omega^G}{\theta_G} \tag{88}$$

$$S_i := WTP_i - \frac{mc + \omega^i}{\theta_i},\ i \in \{G, B\} \tag{89}$$

$$I_B := -\frac{n_G}{n_B}\frac{\overbrace{\alpha_G - mc - \omega^G}^{Benefit\ of\ pooling:\ More\ surplus}}{\underbrace{WTP_B - WTP_G}_{Cost\ of\ pooling:\ Information\ rent}} > 0 \tag{90}$$

$$-I_G := \frac{\alpha_G - mc - \omega^G}{WTP_B - WTP_G} > 0 \tag{91}$$

O More Comparative statics

Result 2: Default probabilities (Adverse Selection). *An increase in adverse selection, measured as the survival probability spread ($\frac{\theta_G}{\theta_B}$), has different effects depending on whether the changes come from default probability in the B or G market segment.*

(i) An increase in $\frac{\theta_G}{\theta_B}$ caused by a decrease in G default probability increases credit rationing under high competition levels. Under low competition levels, it relaxes credit rationing if the cross-subsidy channel is higher than the perfect competition channel and the competition chanel. When demand elasticities are the same (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$), so that the competition channel is absent, the cross-subsidy channel dominates if and only if $\frac{1}{1+\theta_G^{-1}-\frac{WTP_G}{WTP_B}} \geqslant \frac{mc}{WTP_B}$. The competition switching point is $\sigma = \frac{CS(\varepsilon_{CS}-mc)}{mc\cdot L^{PC}}$, where L^{PC} is the perfect competition loan size and ε is the cross-subsidy semi-elasticity to θ_G^{-1}.

(ii) An increase in $\frac{\theta_G}{\theta_B}$ caused by an increase in B default probability decreases credit rationing under high competition levels. Under low competition levels, it relaxes credit rationing if the cross-subsidy channel is higher than the perfect competition channel and the competition chan-

nel. When demand elasticities are the same (i.e., $\sigma := \sigma_B\theta_B = \sigma_G\theta_G$), so that the competition channel is absent, the cross-subsidy channel dominates if and only if $\frac{WTP_B}{1+\frac{1}{\theta_B}-\frac{WTP_G}{WTP_B}}\frac{\theta_G}{\theta_B} \geqslant \frac{mc}{WTP_B}$.

Proof: See Appendix L.

Three forces are at play. To illustrate, let us consider a situation in which the default probability of the B market segment increases.

First, under a high level of competition, the default probability increase must be fully passed through B contract interest rates. In turn, the loan size of the G contract must be lowered to maintain incentives to self-select. If borrowers' willingness to pay is an increasing function of default probabilities, an additional force pushes the credit constraint in the opposite direction. The increase in default probability increases borrowers' willingness to pay for loan size. As a result, for a given level of credit rationing, the low-default contract thus becomes relatively less attractive. This effect allows lenders to decrease the credit constraint. When borrowers' willingness to pay is $WTP_i = \frac{\alpha_i}{\theta_i}$, the latter effect dominates.

Second, the decrease in the default probability of B borrowers renders them more sensitive to interest rate changes. Thus, the perfect information markup must decrease. As a result, the spread between interest rates and credit constraints decreases. This allows lenders to lower the credit constraint level.

Third, the decrease in the default probability of B borrowers increases the potential profits in this market. This creates incentives to distort market segment G to extract more surplus from contracts designed for market B.

I summarize Result 2 in Figure 16 for the case in which the cross-subsidy channel dominates. When adverse selection decreases due to a decrease in market B default probability, the equilibrium region moves from the dotted lines to the solid lines in Figure 16.

Result 3: Willingness to pay (House Prices and Help-to-Buy).

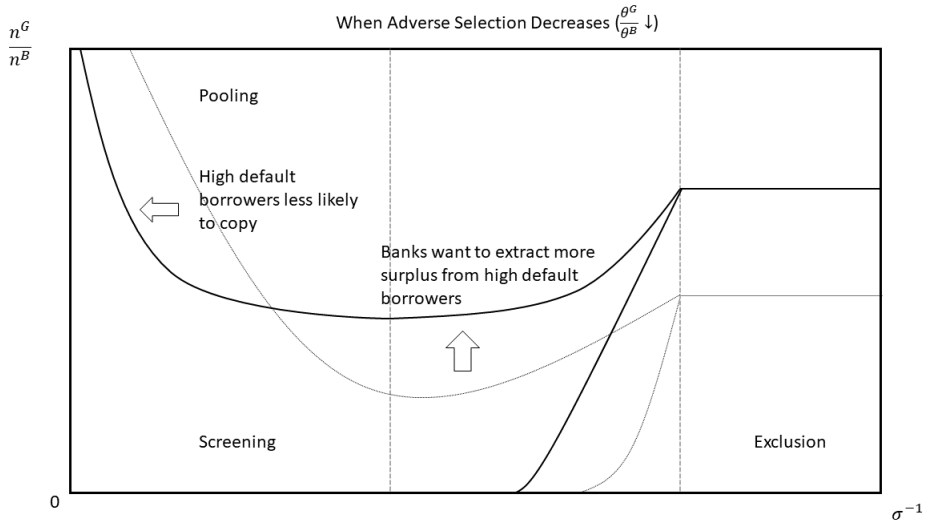

Figure 16: Equilibrium regions when adverse selection decreases

(i) A decrease in house prices (i.e., $\alpha_G = \alpha_B$ increases) decreases the interest rate and loan size spread between contracts. This effect is stronger under low levels of competition as long as the lender is not pooling (i.e., $\sigma > \bar{\sigma}$).

(ii) A government intervention to increase the value of small-size loans via a help-to-buy scheme (i.e., α_G increases) increases the credit rationing and the interest rate spread under high competition. It decreases it under low competition (i.e., $\sigma < \bar{\sigma}_\alpha$) if and only if its $CS\frac{mc}{\alpha_G} > CS + MC$. The competition threshold is then $\bar{\sigma}_\alpha := \frac{-MU - CS[1-\frac{mc}{\alpha_G}]}{PC} > 0$, where $PC := S_B\bar{L}$ denotes the utility of B borrowers when getting the perfect competition B contract.

Proof: See Appendix L

The intuition for the result (i) is the following. Decreasing house prices increase borrowers' willingness to pay for each dollar lent. This lowers credit constraints via two channels. First, the price change renders credit rationing more costly for high-default borrowers. All else constant, the G contract becomes less attractive, so the perfect competition credit rationing level decreases. Second, credit rationing becomes more costly for lenders as the G market

segment generates more surplus. This creates incentives to use asymmetric information discounts and premiums more intensively to lower credit rationing.

Two channels drive the help-to-buy result. First, holding contract terms constant, the G contract becomes more attractive to B borrowers. This renders the level of cross-subsidy required to completely undo credit rationing higher. Second, the help-to-buy policy renders the G market segment more profitable, which creates incentives to increase the cross-subsidy level. In the formula $CS\frac{mc}{\alpha_G} > CS + MC$, $CS + MC$ captures the first channel while $CS\frac{mc}{\alpha_G}$ captures the second channel. $\frac{mc}{\alpha_G}$ is the strength of the marginal impact of an increase in help to buy on the cross-subsidy.

P Normative Analysis

To analyze whether policy interventions have a positive impact on welfare, I use the following lemma:

***Lemma 5: Equilibrium utility and profit levels (Interior Solution)**:*

$$u_B = (\alpha_B - mc)\bar{L} - \pi_B \tag{92}$$

$$u_G = (\alpha_B - mc)L_G - \pi_G \tag{93}$$

$$\Pi_B := n_B\pi_B = n_B\frac{1}{\sigma_B\theta_B}[1 - \frac{\alpha_G - mc}{WTP_G - WTP_B}\frac{n_G}{n_B}] \tag{94}$$

$$\Pi_G := n_G\pi_G = n_G\frac{1}{\sigma_G\theta_G}[1 + \frac{\alpha_G - mc}{WTP_G - WTP_B}] \tag{95}$$

Equations (92) and (93) state that borrowers' utility is the surplus generated by the loan minus the lender's profits per borrower. Equations (94) and (94) state that the lender's profits are the markup plus or minus the asymmetric information premium or discount.

Changes in competition: I analyse how changes in competition (σ^{-1}) affect the welfare of different market participants. For the welfare results to be independent of the welfare function used, I then examine whether there exists a situation in which a decrease in competition leads to Pareto improvements.

Decreases in competition decrease the interest rate of the B contract and increase the B borrower's utility when the asymmetric information discount is higher than the pure markup effect (i.e., $\frac{\alpha_G - mc}{WTP_G - WTP_B}\frac{n_G}{n_B} > 1$).[25] This happens when the surplus in the B market segment is large (i.e., $\frac{\alpha_G - mc}{WTP_G - WTP_B}$ large) and when the cost of screening is low (i.e., $WTP_G - WTP_B$ low) and the relative number of G borrowers is high.

A decrease in competition increases G borrowers' rate but also relaxes the credit rationing. It increases welfare when the increase in price is compensated by the increase in loan size (i.e., $\frac{S_G}{S_{BG}}[MU + CS] > \frac{1}{\Delta\sigma\theta_G} + \frac{1}{\Delta\sigma\theta_G}[\frac{S_G}{WTP_B - WTP_G}]$). This happens when the relative number of G borrowers is high or if the G market segment is relatively more elastic.

A decrease in competition has an ambiguous effect on lenders' profits. It increases lenders' profits if incentives to provide an information rent are low enough $n_G \frac{S_G}{WTP_B - WTP_G}[\frac{1}{\Delta\sigma\theta_G} - \frac{1}{\theta_B}] + \frac{n_B}{\theta_B} + \frac{n_G}{\Delta\sigma\theta_G} > 0$. For instance, when interest rate elasticities are the same (i.e., $\sigma_G\theta_G = \sigma_B\theta_B$), a decrease in competition always increases profits. When the interest rate elasticity of the G market segment is high and the benefit of screening is large ($\frac{S_G}{WTP_B - WTP_G} > n_B$ and $\Delta\sigma$ large enough), a decrease in competition can decrease profits. This is because a decrease in competition creates incentives to distort the B market segment more to extract surplus on the G market segment. However, since the G market segment is relatively more elastic, lenders begin to compete more on this market. Overall, the latter effect dominates.

As a result, when the number of high-default-borrowers is high enough and the G market's surplus is high enough, a decrease in competition is a Pareto improvement.

Changes in the marginal cost of lending: I analyse how changes in the marginal

25. Because of the linearity assumption of the utility, the condition implies that lenders make negative profits on the B market segment. This is not necessarily true otherwise.

cost of lending (mc) affect the welfare of different market participants. For welfare results to be independent of the welfare function used, I then examine whether there exists a situation in which a decrease in competition leads to Pareto improvements.

Let us denote mc_i the marginal cost of lending via i contract.

An increase in mc_G renders interest rates in the B market higher and the surplus in the G market lower, which renders the surplus in the B market relatively higher. This gives incentives to provide less information rent to B borrowers. A decrease in mc_B is passed through lower interest rates. This increases the welfare of B borrowers.

The effect of an increase in mc_G on G borrower is ambiguous. An increase in mc_G is partially passed through interest rates. It also lowers the relative surplus in the G market segment, which triggers incentives to increase the asymmetric information discount (i.e., the interest rate) so that more profits can be made in the B market segment. The rate increase also decreases cream-skimming incentives, so lenders can increase the cross-subsidy and the amount lent in the G market segment without the threat of losing their customers. Overall, welfare can increase. mc_B changes have no impact.

Increases in mc_G positively impact profits if the interest rate elasticity is higher in the G market segment. This is because an increase in mc_G triggers incentives to provide lower information rents. mc_B changes have no impact on lenders' profits in my model, because the cost is fully passed through borrowers.

As a result, an increase in mc_G coupled with a decrease in mc_B can be a Pareto improvement policy. The extreme-case scenario is equivalent to banning contracts with lower than $\bar{L}$ loan size, which prevents cream-skimming. This is a Pareto-efficient policy when cream-skimming is ex post inefficient.

Q Borrower preference approximation

Using data on menus, I approximate the pricing schedule $(L_c, r_c, T_c)_c$, where r is the interest rate and T is the maturity using a spline function. This interpolation allows to use first-order conditions to derive the moment equation. I discuss alternative options in the paragraph in section 6.1 labelled "Heterogeneity within contracts".

Formally, I first calculate the present value of the loan $R_{i,c} = \sum_{t=1}^{T} \theta_i^t T_i$ for each contract c holding the monthly payment (T_i) constant. Then, I find the coefficients $(\gamma_1, \gamma_2, \gamma_3)$ such that $R_i(L) := \hat{\gamma}_{1,i} + \hat{\gamma}_{2,i} L + \frac{\hat{\gamma}_{3,i}}{2} L^2$ best match the data points $(R_{i,c})$.

I recover borrowers' preferences using a revealed-preference approach. For a given monthly payment and an implied pricing schedule, I have

$$u_i^* := \max_L u(L) := \alpha_i L - \theta_i R_i(L) \implies L_i = \frac{\alpha_i}{\hat{\gamma}_{3,i}\theta_i} - \frac{\hat{\gamma}_{2,i}}{\theta_i} \tag{96}$$

For each contract c, I can thus recover borrowers' average preference by inverting equation (96) and using the average default estimates:

$$E[\hat{\alpha}_i|c] = L_i \hat{\gamma}_{2,i}(1 + E[\hat{\theta}_i|c]) - \hat{\gamma_{2,i}} \tag{97}$$

Equation (97) states that the slope of the pricing schedule for the contract chosen allows us to recover the borrower's willingness to pay $\frac{\alpha_i}{\theta_i}$. Figure 5 provides a visual representation of the identification strategy. An identification threat would be the slope of the pricing schedules, which varies due to unobserved contract characteristics such as covenant. In that case, the econometrician is not identifying the willingness to pay for loan size but rather the willingness to pay for both covenant and loan size. The "Heterogeneity within contracts" paragraph discusses how to deal with this issue.

R Adapting the Model to the Empirical Setting

For the model to be able to deliver a continuum of loan sizes, I consider the following extensions. For a given risk category, I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \frac{\alpha_i}{\theta_i}, \sigma_i)$), where mp_i is the disposable income, $\frac{\alpha_i}{\theta_i}$ is the willingness to pay for one dollar, and σ_i is the demand elasticity.

Borrowers have linear utility on loan size so that, conditional on maturity, they borrow as much as possible until their monthly payment capacity (mp_i) is maxed out. For each borrower, I can thus construct the following incentive compatibility constraint:

$$\alpha_i L_i - PV(mp_i, T_i) \geqslant \alpha_i L_j(mp_i, T_j) - PV_i(mp_i, T_j) \tag{98}$$

$L_j(mp_i, T_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity. $PV(mp_i, T_i)$ is the present value of the loan as a function of maturity T_i.

Using the annuity formula, I can show that the heterogeneity in mp is irrelevant, because it cancels out in the incentive compatibility constraints. This feature allows us to abstract away from mp and focus only on the two contract cases. [26]

The fact that LC offers the same interest rate independent of loan size is consistent with the assumption that the distribution of maximum monthly payment mp_i is independent of the distribution of willingness to pay and $\frac{\alpha_i}{\theta_i}$ and demand elasticities σ_i conditional on risk category. This is the modelling approach I use for this application. I recover the distribution of mp using the observed distribution of monthly payments.

I am agnostic about the number of types $(\frac{\alpha_i}{\theta_i}, \sigma_i)$. Appendix (S) shows that the equilibrium conditions of the model with an arbitrary number of borrower types – but with LC choosing to offer two optimally–and the optimality conditions with two borrower types are identical.

26. In continuous time, the principal is $L = mp \cdot [1 - \frac{e^{-r_i T_i}}{r_i}]$ and the present value is $mp \cdot e^{-\lambda_i M_j}$. λ_i is the hazard rate, so that $e^{-\lambda_i}$ is the monthly survival probability. Formally, I linearize $e^{-\lambda_i M_j}$ and use the change of variable $\tilde{L}_i := [1 - \frac{e^{-r_i T_i}}{r_i}]$.

Since I do not observe maturity above 60 months, I assume that its cost is such that longer than 60-month loans are not profitable. This assumption plays the same role as the maximum loan size $\bar{H}$ in the baseline model.

S Empirical Application Model

I consider that there are N types of borrowers (i.e., N combinations of $(mp_i, \frac{\alpha_i}{\theta_i}, \sigma_i)$), where $\bar{m}p_i$ is the maximum monthly payment the borrower can repay. For each borrower type, I construct an incentive compatibility constraint:

$$L_i - \frac{\theta_i}{\alpha_i} mp_i T_i \geqslant L_j(mp_i, M_j) - \frac{\theta_i}{\alpha_i} mp_i M_j \tag{99}$$

where $L_j(mp_i, M_j)$ is the loan size that would be chosen by the borrower if they were to choose a contract with the other maturity.

$$\sum_{i=1}^{l} n_i \phi(u_i) \theta_i [R_l - \frac{mc}{\theta_i} L_h] + \sum_{i=h}^{n} n_i \phi(u_i) \theta_i [R_h - \frac{mc}{\theta_i} L_h] \tag{100}$$

$$with\ u_i := WTP_i L_i - R_i \tag{101}$$

$$\{h := l+1, l\} : u_h(L_h, R_h) := u_l(L_l, R_l) + (WTP_h - WTP_l) L_i \tag{102}$$

Using the average utility in each market segment ($\bar{u} := u_i - \Delta_i L_i$ with $\bar{\Delta}_i := (\bar{WTP} - WTP_i)$):

$$\sum_{i=1}^{l} n_i \phi(\bar{u} + \bar{\Delta}_i L_h) \theta_i [(W\bar{T}P - \frac{mc}{\theta_i}) L_h - \bar{u}] + \sum_{i=h}^{n} n_i \phi(\underline{u} + \Delta_i L_l) \theta_i [(\underline{WTP} - \frac{mc}{\theta_i}) L_h - \underline{u}] \tag{103}$$

$$\bar{u} + (WTP_h - W\bar{T}P) L_h = \underline{u} + (WTP_l - \underline{WTP}) L_l + (WTP_h - WTP_l) L_l \tag{104}$$

Replacing the constraint and taking the first-order conditions with respect to ($\bar{u}$ and $\underline{u}$), we obtain the same first-order conditions as in the two-borrower models.