Regulating Dynamic Contracts*

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Abstract

We study optimal regulation of dynamic contracts in a market in which firms and agents meet sequentially, and the firms, who have private information about the match productivity, can offer long-term contracts. Left unregulated, firms use contracts to inefficiently restrict agent movement, and extract all surplus from agents. The regulator can restrict the set of permitted contracts which firms can offer. We derive an optimal regulatory policy in which all contracts take a simple structure, comprising a signing bonus, a flat wage, and a termination fee if the agent wants to leave the relationship. Regulation links the permitted termination fee to the wage, to incentivize firms to offer higher wages to agents with better match productivity, in exchange for more protection from poaching. Our results provide insights into the debate on the regulation of non-compete clauses in employment contracts.

1 Introduction

Dynamic markets with search and matching frictions, asymmetric information, and lopsided bargaining power are known to suffer from inefficiencies and unequal distribution of surplus. Policymakers frequently regulate the use of contracts in these markets to mitigate inefficiencies and improve equity between contracting parties. For

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instance, firms with bargaining power in labor markets use sophisticated contracting tools such as non-compete clauses and training repayment fees to restrict worker mobility and extract surplus from the employment relationship. Such concerns have led regulators to use wage regulation and place restrictions on the permitted structure of employment contracts. For example, in 2024, the Federal Trade Commission (FTC) proposed a ban on the use of non-compete clauses in employment contracts:

> "Research shows that employers' use of noncompetes to restrict workers' mobility significantly suppresses workers' wages—even for those not subject to noncompetes, or subject to noncompetes that are unenforceable under state law...The proposed rule would ensure that employers can't exploit their outsized bargaining power to limit workers' opportunities and stifle competition."
>
> —Elizabeth Wilkins, FTC

Analogous concerns exist in contracting between firms and suppliers—as highlighted in Aghion and Bolton [1987], incumbent firms have historically used exclusivity clauses in contracts to prevent their suppliers from contracting with their competition to exercise monopoly power. In this paper, we study the optimal design of contract regulation in a dynamic matching market with search frictions and asymmetric information. We pose the regulation problem as one of decentralized delegation: the regulator restricts the set of permitted contracts between firms and agents.

We consider a large, infinite horizon market in which firms and agents meet sequentially over time and can form a match to begin bilateral relationships. We assume firms have the bargaining power: when an agent meets a firm, the firm privately observes the match productivity, and can offer the agent contract.[1] A contract specifies payments, and relationship termination decision over time, as a function of the firm-agent bilateral history, and can involve full, partial, or no commitment from either side. Agents continue to meet other firms while matched, and can receive external offers. A regulatory policy specifies a set of bilateral contracts, which firms are restricted to making offers from. The regulator's objective is to maximize a weighted sum of agent and firm utilities.

[1] There is growing recognition that wage setting in labor markets is monopsonistic. Card [2022] makes the case that "many—or even most—firms have some wage-setting power."

Left unregulated, we show the market outcome is inefficient. There is an essentially unique equilibrium outcome and distribution over agent movement between firms. Firms offer contracts that impose a penalty—a termination fee—if the agent wants to join another firm. This effectively sets a price at which the firm "sells" the agent to firms that wish to hire him in the future, as they must provide compensation for the termination fee. In equilibrium, firms optimally set a price which restrict efficient movement. For instance, a firm that meets an unemployed agent could make a take-it-or-leave-it contract offer that extracts all the surplus from the relationship and imposes a high fee that prevents the agent from ever leaving. Alternatively, the firm could insist the agent pays the value of the lost surplus to the firm if he wants to leave. This leaves surplus on the table for the agent to pay the fee and join any firm that is more productive. However, an intermediate price larger than the feasible surplus in the current match but lower than that in some future matches, captures a chunk of surplus from future (more productive) firms, and is more profitable for the firm making the offer.

The outcome fails to maximize total surplus: agent movement to more productive matches is restricted by termination fees. Firms behave like monopolists when setting termination fees, but cannot observe agents' productivity with firms arriving in the future. This means firms can do no better than setting a fixed price that is inefficiently high. These actions impose an externality on the market, including other firms, as they create barriers to entry for future firms to contract with agents. In fact, even from the perspective of a regulator who only cares about firms, the unregulated outcome is flawed, as it fails to maximize total firm profits. If, in addition, the regulator cares about agents' share of surplus and places more weight on agents than firms, the unregulated outcome is problematic, as it gives rise to extreme inequality: unregulated firms use their bargaining power to extract *all* surplus from agents.

These market failures highlight the need for regulatory intervention, regardless of the regulator's objective. While efficient career paths require agents to move whenever more productive firms arrive, increasing agent surplus requires firms to pay agents more when match productivity is higher. We show that firms' private information results in a regulatory trade-off between efficiency and optimal surplus distribution: providing firms incentives to share more surplus with agents comes at the cost of distortions to agent movement under regulation.

Our main result shows that there is an optimal regulatory policy which takes a particularly simple form, consisting of *stationary contracts with an exit clause*. These contracts consist of an upfront signing bonus, a constant wage, and a termination fee which the agent must pay to end the relationship. The termination fee acts as a non-compete with an exit clause to buy out the non-compete. Crucially, regulation sets a permitted schedule of terms: it imposes a minimum wage, and ties the permitted termination fee to the wage that is set. This regulatory policy implements an equilibrium in which unemployed agents receive and accept offers from firms with match productivity above a certain threshold. In equilibrium, employed agents receive and accept new offers whenever they meet more productive firms, and firms offer a signing bonus that depends on the contract terms in the agent's existing match. The terms of employment change based on match productivity, with more productive firms setting higher wages. The regulated contracts allow firms to only partially commit to the offer: a firm can commit to any signing bonus, but the *contract terms*—the wage and termination fee—are set only after the agent accepts the offer. This effectively prevents agents from signing a new contract until after they pay the termination fee to end an existing relationship.[2]

The regulated schedule of terms forces firms to use wages as well as termination fees as a retention tool to prevent the agent being poached—in contrast to the unregulated outcome where firms used termination fees alone. It creates a trade-off for firms: better wages are tied to higher termination fees, with more protection from poaching, and longer expected employment at the firm. Firms with higher match productivity get more value, and set better wages to induce a longer expected tenure at the firm. The regulated link between wages and permitted termination fees makes it costly for firms to use termination fees and limits the inefficiencies associated with them, while increasing the share of surplus going to agents. Permitted termination fees are key to ensuring that firms pay large signing bonuses and wages. They protect firms from having the agent poached by less productive firms. The bonus therefore plays two roles. It compensates agents for termination fees, but also provides a signal of productivity with a new firm; in equilibrium, agents correctly expect a wage increase under a new contract.

[2]Here, the non-compete prevents the agent from signing a contract with a new firm until after it has been bought out (i.e., termination fee is paid). A new firm can offer to pay a share of the termination fee through the bonus but the agent cannot sign the terms with the new firm until after the termination fee is paid.

If the regulator only cares about total surplus (or places more weight on firms), the regulated outcome maximizes total surplus: agents are employed immediately in the market, and always move to more productive firms. However, achieving efficiency imposes a cap on agent surplus due to firm information rents. If the regulator places more weight on agents, regulation imposes a binding minimum wage and low productivity firms make no offers to an unemployed agent even though a match can generate positive surplus. This distortion creates early career agent unemployment, but raises agent surplus over the career as it reduces information rents gained by high productivity firms. Perhaps surprisingly, this is the only allocative distortion. Once employed, agents move efficiently between firms whenever gains in surplus exist. Over their career, agents only enter more productive relationships, and the expected tenure increases at each subsequent firm. The more weight the regulator places on agents, the higher the minimum wage and agent surplus under optimal regulation.

Contracts with limited commitment arise endogenously through optimal regulation, which prevents firms from committing to the wage at the same time as the signing bonus is chosen. This is because it is cheaper for firms to pay the agent in wages rather than signing bonus—a higher wage rewards the firm with a larger termination fee (improves firm payoffs on the intensive margin), while the bonus affects the extensive margin (whether the agent accepts) but has no effect on the relationship once the agent has joined. Given the opportunity, firms would like to substitute wages for bonuses, while still making the agent willing to accept the contract. This can be distortionary, as firms will prevent workers from leaving for more productive matches. Structuring contracts so that firms cannot commit to other terms than the signing bonus at the offer stage prevents such deviations.

To prove our main result, we analyze a relaxed problem in which the regulator designs a direct centralized mechanism: firms send messages to the regulator as they arrive, and the mechanism specifies which firm agents are allocated and payments. We characterize the solution to the centralized problem, which provides an upper bound on the regulator's payoff from decentralized regulation. The solution to the centralized mechanism does not pin down payments exactly and not all solutions to the centralized problem are implementable through decentralized regulation. Nevertheless, we show that our decentralized regulatory policy and the corresponding equilibrium implement the allocation rule and net transfers of the centralized problem, and are therefore optimal. While we impose that the set of contracts available in the regulatory policy is the same to all firms, this is without loss: that is, the regulator does just as well as if he could use a more sophisticated centralized mechanism.

Our model and results have relevance for a number of applications. The merits of non-compete clauses in employment contracts and their permitted use or banning are widely debated by regulators. In our setting, contracts with termination fees play an important role both in the unregulated and regulated market. We can interpret these termination fees as non-compete clause which can be bought out by workers, or as contractual clawback which imposes penalties such as loss of vested stock options, or paying back of salary. This highlights a novel use of non-competes that can arise even in the absence of common motivation for their use (e.g., protecting firm human capital investment or intellectual property). While unregulated firms can use non-competes to inefficiently restrict worker mobility, our results suggest that an outright ban on non-competes may fail to achieve the desired result. First, firms may sidestep a ban through less direct contracting mechanisms (such as clawback or deferred bonuses) to impose analogous penalties on workers who wish to exit. Moreover, even if such penalties can be prevented, the resulting outcome may suffer from different inefficiencies because firms are completely unprotected from poaching of employees.

Our result suggests that instead of a ban, it may be optimal for regulators to permit the use of *regulated non-competes*: workers are allowed to buy out the non-compete, and the fee for doing so is linked to the wage paid by the firm. While unregulated use of non-compete clauses may lead to lower productivity and worker surplus extraction, a well regulated market can benefit from these frictions in a way that improves efficiency and benefits workers. Prior to the FTC's proposed ban on non-competes, regulatory policies have tied permitted non-competes and termination fees to wages. For instance, some states in the U.S. have legislation to limit the use of non-competes specifically for low-wage workers, while still allowing their use for workers with higher wages.[3] The Texas Business and Commerce Code stipulates that physicians with non-competes be permitted to buy out non-competes at a "reasonable price."

Our baseline model is intentionally simple to focus on the driving forces behind our result, and omits a number of variables which may be relevant in focal applications. We analyze a number of extensions to the model, and show that our main insights about the structure of optimal regulation is robust to these changes. Our extensions

[3] The cutoff for what constitutes low-wage varies across states (see Waisbord [2022]).

augment the model to allow for costly effort by agents, agent private information about outside options, human capital investment by firms, and break-up costs suffered by firms when agents leave. The last two are particularly relevant for application to skilled labor markets and the debate on non-competes. The primary justifications provided by proponents of non-competes invoke the protection they provide firms from poaching given the need to incentivize firms to invest in human capital, or the costs to firms if workers with access to proprietary information join competitors. Our baseline model highlights a use for regulated non-competes that is independent of such considerations, and adding these forces only strengthens our main insight.

Our paper studies regulation as optimal delegation of dynamic contracts in a market with idiosyncratic match productivity. Regulation features a novel feedback effect, as it changes agents' expected outside options from future matches. Agent outside options are endogenous to the regulatory policy, which in turn affect offered contracts and incentives in equilibrium. Despite the complex structure of the delegation space and game, we show it is optimal to restrict permitted contracts to take a simple structure with a strong form of stationarity. Regulated contracts exhibit limited commitment for firms, and may impose a binding minimum wage that causes early-career unemployment. Crucially, introducing regulated frictions (termination fees) improves market efficiency and surplus distribution.

2 Related Literature

Our paper contributes to the literature on delegated contracting in multiagent games, which has primarily studied static settings (e.g., Melumad et al. [1995], Hiriart and Martimort [2012], Malladi [2022], Bhaskar et al. [2023]), by studying delegation of contracts in a dynamic environment with a rich contract space. These papers study delegation spaces that explicitly model the underlying contracting problem. Bhaskar et al. [2023] is related in spirit, allowing regulators to design the space of permitted contracts but focuses static insurance markets where the types of contracts used is substantially different.

A large literature building on Baron and Myerson [1982] has studied optimal monopoly regulation in static settings with a monopolist with private information. Other papers, including some in dynamic settings, have considered the impact of specific regulatory interventions such as minimal deferral periods (Hoffmann et al. [2022]), caps on total compensation (Thanassoulis [2012]), changing the transparency of contracts for consumers (Inderst and Ottaviani [2009]), changing minimum wage (Loertscher and Muir [2022]) and limiting the length of non-compete clauses (Shi [2023]). Our contribution is to study optimal contract regulation in a dynamic setting with multiple active players with full discretion for the regulator to restrict contracts in any way.

We also contribute to the extensive literature on dynamic contracts. Much of this literature has focused on a single principal-agent interaction subject either to moral hazard or dynamically arriving private information (e.g., Pavan et al. [2014]), with some allowing for agent turnover (e.g., Garrett and Pavan [2012]). The impact of agent movement across firms has been studied in the search and matching literature with on-the-job search—a large market of firms and agents are randomly matched, and agents keep matching with firms while employed (e.g., Harris and Holmstrom [1982], Pissarides [1994], Postel-Vinay and Robin [2002], Cahuc et al. [2006], Shi [2023]). A key feature of our model relative to this literature is that we study how the regulator restricts the contract space, while in much of the previous literature, the contract space is exogenously fixed. In our setting, regulation has a feedback effect which endogenously changes the value of search and agent outside options.

Our results emphasize a potential use for non-compete clauses and contract termination fees. Aghion and Bolton [1987] study the impact of termination fees in unregulated markets and show how incumbent firms can extract surplus from more efficient entrants, using high termination fees that prevent the consumers from moving to more efficient firms. Similar inefficiencies arise in our model in the unregulated market—though our regulatory design problem and dynamic setting requires us to consider a richer setting with a more complicated game. Other work has explored potential benefits of break-up fees and non-compete clauses in reducing the diffusion of proprietary knowledge (Franco and Filson [2006]), encouraging human capital investment (Rubin and Shedd [1981], Shi [2023]), inducing additional firm entry in contests (Che and Lewis [2007]) and increasing efficient job assignment when firms are worried about workers being poached, and promotions are viewed as a signal about worker quality (Mukherjee and Vasconcelos [2018]). Our results show that, even absent these considerations, the optimal regulatory policy allows *regulated* termination fees (which can be interpreted as buyouts of a non-compete agreement).

3 Model

We study contracting between an infinitely-lived agent (A) and a sequence of infinitely lived firms. Time is discrete, with dates $t \in \{0, 1, ...\}$. Firms and the agent discount the future by a common discount factor $\delta \in (0, 1)$.

Firm-agent relationships and bilateral contracts: Firms are characterized by a private match productivity (type) k. If a firm with type k forms a match (enters into a relationship) with the agent, then in every period until the relationship ends: the firm sends a message $m \in M$ to the agent (where M is a sufficiently rich message space), the agent sends a message $r \in M$ to the firm, the firm realizes a payoff of k and there is a payment p between the firm and agent. At the end of each period, the relationship is either continued ($a = 0$) or terminated ($a = 1$).

We denote by $h_s := \{m_s, r_s, p_s, a_s\}$, a *firm-agent history* in period s of the relationship, and by $h^s := \{h_\tau\}_{\tau=0}^{s-1}$, a firm-agent history at the beginning of period s of a relationship that has not yet been terminated. Let $\mathcal{H}^s$ be the set of firm-agent histories in period s of the relationship.

A *firm-agent bilateral contract* x consists of a sequence of functions $\{(p_s, a_s)\}_{s=0}^{\infty}$ where $(p_s, a_s) : \mathcal{H}^s \times M \times M \to \mathbb{R} \times \{0, 1\}$ are functions of the firm-agent history and the messages in period s of the relationship, p_s is a payment between the firm and agent, and a_s determines whether the relationship is terminated.[4] Firms leave the game once the relationship is terminated. We allow payments to be negative, meaning a payment from the agent to the firm—we impose no limited liability constraints. The agent's utility is linear in payments and the firm's payoff is the sum of produced output and payments. If a firm and agent agree to a contract, they commit to the continuation decisions and payments specified. We assume the agent starts the game unemployed, receiving zero payments in each period until he first accepts a contract.

Contract regulation: There is a regulator (R) who restricts the set of permitted contracts. Let $\mathcal{C}$ be the set of all bilateral contracts. A *regulatory policy* is a set $\mathcal{R} \subseteq \mathcal{C}$, such that all firms are restricted to making contract offers from $\mathcal{R}$ or to offer no contract, denoted by $x_\emptyset$; we assume $x_\emptyset \in \mathcal{R}$ for all choices of $\mathcal{R}$. The regulator's objective is to maximize a weighted sum of agent utility and firms' profits, with weight

[4] While the definition of contracts as a function of messages sent by players may seem somewhat abstract, we define them this way as we wish to study optimal regulation of the space of contracts, and this general formulation places no a priori restriction on the types of contract that can be used.

$\lambda \in [0,1]$ on the agent utility and $1-\lambda$ on the sum of firms' profits.

Firm-agent matching and regulated equilibrium: The agent randomly meets a new firm at the start of each date t, which privately observes its type $k_t \in \mathcal{K} \subset \mathbb{R}_+$, with $\overline{k} = \max \mathcal{K}$.[5] We will assume $\mathcal{K}$ is either finite[6] or equal to $[0, \overline{k}]$ and each k_t drawn independently and identically from a distribution F with pdf (or pmf) f; for expositional ease, we will focus on the case of $\mathcal{K} = [0, \overline{k}]$ unless otherwise stated. The arriving firm makes a take-it-or-leave-it contract offer $x_t \in \mathcal{R}$ to the agent, which is not observed by any existing firm the agent is contracting with. Then, if the agent is under an existing contract, the contracting relationship proceeds during date t, and at the end of the period, the agent accepts ($d_t = 1$) or rejects ($d_t = 0$) the offered contract at the end of the period. The agent cannot contract with two firms simultaneously, so can only accept a new contract if the existing contract has been terminated. The timing is summarized in Figure 1.[7]

We denote the *public history* during date t as $h_t^P := \{m_t, r_t, p_t, a_t, \hat{x}_t\}$, where $\hat{x}_t$ is the contract accepted by the agent at date t and by $h_t^P := \{h_s^P\}_{s=0}^{t-1}$ the entire public history up to date t.[8] We denote the *agent history* during date t by $h_t^A := \{m_t, x_t, r_t, p_t, a_t, d_t\}$, and by $h_A^t := \{h_s^A\}_{s=0}^{t-1}$, the entire agent history up to the date t.[9] Let $\mathcal{H}_P^t$ and $\mathcal{H}_A^t$ be the set of all public and agent histories respectively, at date t.

A strategy for the agent is a sequence of functions $\sigma^A = \{(r_t, d_t)\}_{t=0}^{\infty}$. The function $r_t : \mathcal{H}_A^t \times M \times \mathcal{R} \to M$ is the message sent by the agent in any current contract as a function of the history, the current firm's message, and new contract offer. The decision $d_t : \mathcal{H}_A^t \times M \times \mathcal{R} \times M \to \{0,1\}$ is the agent's decision to accept or reject the new contract offer as a function of the history, the current firm's message, the new contract offered and the agent's message in the current contract. The strategy of a firm arriving at date t, is $\sigma_t^F = \{x_t, \{m_t^s\}_{s=0}^{\infty}\}$ where $x_t : \mathcal{K} \times \mathcal{H}_P^t \to \mathcal{R}$ determines the contract the firm offers and $m_t^s : \mathcal{K} \times \mathcal{H}_P^{t+s} \to M$ is the message the firm sends in

[5] We can easily extend the model and results to allow for the possibility that no firm arrives in a period.

[6] We will focus on the case of generic values of $\mathcal{K}$ when finite for expositional ease.

[7] If the agent accepts a new contract in period t, he cannot receive new payments from the contract until $t+1$. This timing is for notational convenience and does not impact results. Richer communication protocols (e.g., multiple rounds of communication) or information disclosure protocols (e.g., including a mediator or allowing the incumbent firm to observe and/or contract on any new contract offers) can be added without changing the results.

[8] $\hat{x}_t = x_t$ if x_t was offered and the agent accepted, and we write $\hat{x}_t = 0$ if x_t was not accepted.

[9] Note that agent histories include rejected contract offers, which are not publicly observed.

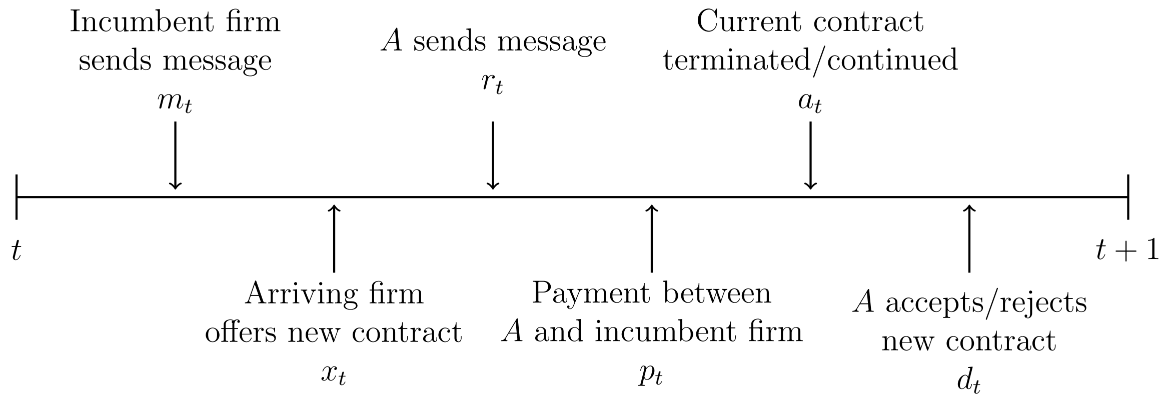

Figure 1: Timing during date t.

period s of the relationship with the agent.

A Perfect Bayesian Equilibrium (hereafter equilibrium) is a collection of strategies $\sigma = \left(\sigma^A, \{\sigma_t^F\}_{t=0}^{\infty}\right)$ and belief systems for each player, such that all strategies entail best responses at every history, given the players' beliefs, and each player updates their beliefs according to Bayes' rule wherever possible. Any equilibrium induces a distribution over the matches and productivities at each date t, which we denote $\hat{k}_t$. Given an equilibrium with strategies σ, the regulator's expected payoff is

$$\mathbb{E}^{\sigma}\Big[\sum_{t=0}^{\infty} \delta^t \Big(\lambda p_t + (1-\lambda)(\hat{k}_t - p_t)\Big)\Big]. \tag{1}$$

We restrict attention to regulatory policies $\mathcal{R}$ for which an equilibrium exists and, as is standard, assume players play the regulator's preferred equilibrium.[10] A regulatory policy $\mathcal{R}$ is *optimal* if there exists an equilibrium σ such that $\mathcal{R}, \sigma$ maximize (1).

Finally, we impose a standard regularity assumption on the distribution F, namely monotonicity of the virtual value function.[11]

Assumption 1 (Regularity)**.** $k - \frac{1-F(k)}{f(k)}$ *is strictly increasing in* k*.*

[10] Individual rationality is captured for both sides: firms have the option of making no offer, and the agent has the option of turning down offers and remaining unemployed.

[11] This assumption is stated for continuous types. For finite types we impose the analogous condition that $k - \frac{1-F(k)}{f(k)}\Delta_k$ is increasing where Δ_k is the distance between k and the next highest type (and is 0 if $k = \overline{k}$).

Comments on the model:

Decentralized bilateral contracting: We pose regulation as delegation—the regulator cannot run a centralized mechanism, but sets the rules of what contracts are permitted. The regulatory policy is *static*: the set of permitted contracts is the same for all firms (and after all histories). This restriction is motivated by the fact that regulators do not typically have the resources to coordinate and arbitrate matching and contracting between firms and agents in an economy. Contracts can condition only the on the bilateral history of the parties, and not on past or future histories of either side that may or may not be verifiable for these parties. This is a natural in practice, and shuts down the possibility of history-dependent regulatory policies.[12]

Commitment: The definition of a contract (effectively) embeds all possible forms of commitment, on one or both sides—we can capture full, partial, or no commitment for either party (relational contracts). For instance, if the contract does not respond to either firm or agent messages, then the contract entails two-sided full commitment. If the contract responds non-trivially to the messages sent by one party at some history, this entails limited commitment for that party at that history—they can change the contracting outcome using messages which cannot be committed to.[13]

Firm private information: Match productivity is private information to the firm (and not contractible). In general, the contribution of one agent's output to a firm's profits is hard to disentangle. While the firm may know its value for the agent, this is not publicly verifiable and there is no way to formally contract on it.

Individual Rationality: Ex-ante individual rationality must hold for the agent—as he can decline all offers, the agent's ex-ante expected payoff must be weakly positive. However, once in a contract, no limited liability constraints are imposed. This allows for contracts that prevent the agent from leaving a relationship by stipulating large payments from the agent upon contract termination. In reality, one might expect there to be moral or legal constraints which prevent such forced work, entailing individual rationality constraints at all histories. However, the optimal regulatory policy we construct satisfies interim agent individual rationality, so we leave such constraints out for simplicity and to emphasize that they are in fact non-binding.

[12] This would be a strange feature and unlikely to be feasible in practice: for instance, if a regulator permitted only former employees of a certain firm to sign a particular type of contract.

[13] E.g., we can capture a sequence of short-term contracts if the message m_s determines the wage today, r_s is the accept/reject decision, and both messages determine if the relationship continues.

Applications: Our setup is general enough to capture a number of economic settings. A focal application is a skilled labor market, in which the agent is a worker meeting firms with idiosyncratic value of employing the worker. The model also captures interactions between upstream and downstream firms contracting over the supply of an input, or consumer-firm interactions, where the agent is the seller meeting a sequence of firms with heterogeneous value for the good, or alternatively, a buyer with a known value for the good and meets a sequence of firms with heterogeneous costs of supplying the good.[14] For ease of exposition, we use terminology throughout the paper which fits the focal labor market application.

4 Unregulated Benchmark

First, we address the important question of why this market might need to be regulated. As a benchmark, total surplus is maximized by the **efficient path**: the agent joins the first firm he meets, and subsequently joins a new firm whenever a firm with a higher match productivity than the current match arrives. As we have transferable utility, any **first-best** outcome must maximize total surplus by having the agent follow the efficient path, and net transfers redistribute surplus optimally between the agent and firms according to λ: if $\lambda = \frac{1}{2}$, surplus can be distributed in any way; if $\lambda > \frac{1}{2}$ it is optimal to transfer all surplus to the agent, while if $\lambda < \frac{1}{2}$, all surplus optimally goes to firms. To assess the need for regulation, we analyze the unregulated benchmark in which firms are unrestricted in the contracts they may offer, so $\mathcal{R} = \mathcal{C}$.

The unregulated market gives rise to a dynamic signaling game in which firms can offer arbitrarily complex contracts. In principle, there could be many equilibria. When a firm and agent meet, the continuation value to each from forming a match depends on how the contract splits surplus generated in the bilateral relationship, but also the contracts offered by firms that arrive in the future. Expected future offers determine not only the agent's outside options, but also the payoff from accepting a contract for both agent and the proposing firm. For instance, contracts can impose a termination fee if the agent leaves, allowing the incumbent to extract surplus from relationships with new firms. The value from this depends on the contracts new firms will offer (which determines the agent's willingness to leave his current contractt) and the agent's beliefs about arriving firms. Thus in general, a firm's value from

[14] We can nest this in our framework setting k to be the agent value minus the cost for a firm.

contracting with the agent is dependent on equilibrium beliefs and the behavior of other firms.

One might expect that this makes it possible to sustain different kinds of equilibria (with varying distribution of surplus) by punishing firms that deviate through changes in the behavior of future firms. For example, future firms and the agent may change their behavior after deviations in a way that lowers the surplus and agent value of future matches, either reducing the deviating firm's value from a contract, or lowering the agent's continuation value so that he rejects the contract after a deviation. Despite this rich set of possibilities, our first result shows there is a unique equilibrium outcome.[15] When looking at the limit as the type space becomes fine for a fixed $\overline{k}$, we say $|\mathcal{K}| \to \infty$, by which we mean a set of type spaces $\{\mathcal{K}_n\}_{n=1}^{\infty}$ (with corresponding cdfs F_n) such that $\lim_{n\to\infty} |\mathcal{K}_n| = \infty$ and F_n converging to a strictly increasing cdf on $[0, \overline{k}]$.

Proposition 1. *Suppose $\mathcal{R} = \mathcal{C}$ and $\mathcal{K}$ finite. There exists a function $\theta(k)$ such that in any equilibrium,*

i) The agent receives zero expected utility,

ii) For $k < \overline{k}$, an agent employed by a firm with productivity k leaves for a firm of productivity k' if and only if $k' \geq \theta(k)$ and never leaves a type $\overline{k}$ firm for type $k' < \overline{k}$.

Moreover, for all $k < \overline{k}$, $\theta(k) > k$ and is strictly bounded away from k as $|\mathcal{K}| \to \infty$.

The proposition pins down an essentially unique (stochastic) path of agent movement between firms. The threshold function $\theta(k)$ determines transitions: if the agent is employed by a type k firm, he leaves whenever type $k' \geq \theta(k)$ arrives, and stays otherwise. As $\theta(k) \geq k$, the agent only moves to new firms that are more productive than the current employer. As part of the proof, we construct an equilibrium in simple contracts which implement these outcomes.[16] Firms offer contracts that pay

[15] For this result we assume $\mathcal{K}$ is finite. The assumption of finite types simplifies the arguments. The continuous case can be approximated by a fine grid of types in the interval. We conjecture that the result holds for the continuous case, but analyzing the model is significantly more difficult. However, analogous results can be realized when $\mathcal{K} = [0, \overline{k}]$ under other natural perturbations of the model—e.g., adding a small fixed cost for firms when the agent joins a new firm.

[16] Equilibrium outcomes and net payments are pinned down across equilibria, but exact timing of payments is not.

an upfront bonus and charge the agent a termination fee to leave. If the agent is employed, arriving firms adjust their upfront bonus in order to compensate the agent for any termination fee needed to end the current relationship. The offered termination fee determines the threshold type that will offer a contract.

A key part of the proof is showing equilibrium uniqueness—other stationary or non-stationary outcomes cannot be sustained. We inductively construct an upper-bound on the joint surplus that can be generated and find a contract that guarantees this upper bound regardless of what other firms do. The scope to sustain other outcomes is unraveled by the behavior of the most productive firms. Type $\overline{k}$ firms can generate more surplus with the agent than others, so gain nothing from the agent's search. They offer contracts which generate maximal surplus, under which the agent receives the same value as in his previous contract (plus any termination fee), and set a termination fee which which prevents the agent from ever leaving. These firms induce the agent to leave his current firm if and only if the agent's continuation value of remaining in the current contract plus any termination fees for leaving is below $\frac{\delta\overline{k}}{1-\delta}$. Thus, because (conditional on the agent's current contract and a $\overline{k}$ type arriving) both the agent and type $\overline{k}$'s continuation values are fixed across all equilibrium, type $\overline{k}$ firms cannot be incentivized to punish deviations by earlier firms.

We then show that no type wants to "sell" to a lower type, and the best that the type just below $\overline{k}$ can do is to sell to type $\overline{k}$, extracting all surplus from type $\overline{k}$. Moreover, the second-highest type can achieve this upper-bound by setting a termination fee of $\frac{\delta\overline{k}}{1-\delta}$, as type $\overline{k}$ will offer a contract that induces the agent to pay this fee. We show this is an upper-bound on the firm's payoffs across all equilibria, and construct a contract which guarantees this payoff in any equilibrium, uniquely pinning down outcomes for the top two types. We extend this construction to lower types: the best the third-highest type can do is decide whether to sell to only $\overline{k}$, or to both the types above, extracting all surplus from the second-highest type, whichever is more profitable, and can guarantee this payoff in any equilibrium. Proceeding in this way, any arriving firm anticipates that if it offers its best possible contract, the agent will still move to the desired high types in future. Thus, all types can guarantee their best possible payoff across equilibria, net the agent's continuation value when the firm and the agent meet. Firms optimally make offers such that the agent's participation constraint binds, so the agent's payoff is equal to his outside option, zero.[17]

Market failures: The unregulated outcome fails to maximize total market surplus, and is suboptimal for all λ. While efficiency requires the agent to move to more productive firms, in equilibrium the agent only moves from a firm of type k to types above $\theta(k) > k$. As $|\mathcal{K}| \to \infty$, there is an interval $(k, \theta(k))$ for some k, containing types that are more productive than k, but are shut out of hiring the agent—equilibrium transitions are inefficient. In the following discussion we focus on this limit case.

The driving force behind inefficiency is firm private information. Take an arriving firm with productivity $k \in (0, \overline{k})$ that arrives when the agent is unemployed. That firm is the only buyer of the agent's services at that time, and makes a take-it-or-leave-it offer which extracts all of the surplus. At the same time, the firm can set a termination fee to gain rights to the agent's future productivity with other firms—a barrier to entry for future firms. How does the firm choose this fee given that future firms' willingness to pay is private information? If the firm sets a sufficiently large termination fee (above $\frac{\delta \overline{k}}{1-\delta}$) there is a complete barrier to entry, the agent never leaves, and the firm's continuation value is $\frac{\delta k}{1-\delta}$. If the firm sets a fee of $\frac{\delta k}{1-\delta}$, it is no better off than setting the high termination fee.[18] If all firms with type k set a termination fee of $\frac{\delta k}{1-\delta}$, the resulting outcome is efficient as arriving firms compensate the agent for the fee if and only if they have a higher type. However, firms can do better by setting a fee of $z \in (\frac{\delta k}{1-\delta}, \frac{\delta \overline{k}}{1-\delta})$, as a set of types above k are willing to make an offer with a bonus which compensates the agent, and moreover, the initial firm's payoff is strictly larger than its value of continuing to employ the agent.

This "price setting" is analogous to inefficient monopoly pricing.[19] The initial firm sets an inefficiently high price–firms with types in an interval immediately above k are deterred from offering a contract as the required bonus is too high. Firms use contracts to create inefficient barriers to agent mobility between jobs. Proposition 1 thus provides a normative justification for the role of regulators from a pure efficiency

[17]Once we establish equilibrium uniqueness, the intuition for full extraction of agent surplus is similar to the Diamond paradox (Diamond [1971]) as only one firm arrives at a time. If multiple firms could arrive simultaneously, the agent could earn positive surplus. However, even in this case, the agent may still earn less than the regulator desires (depending on the value of λ).

[18]The firm's payoff for a termination fee of $\frac{\delta k}{1-\delta}$ when the agent does leave are equal to the value to that firm of continuing to employ the agent forever.

[19]This contract maximizes joint surplus between the agent and incumbent firm. Inefficiency is driven by the incomplete information about future firms' productivity, not firms' bargaining power.

stand-point, even putting aside distributional concerns.

The unregulated outcome even fails to maximize total firm profits, so is suboptimal even if the regulator only cares about firms ($\lambda = 0$). To see this, note that since the agent receives zero surplus, the distribution of surplus is regulator-optimal in this case, but maximizing firm profits requires maximizing total surplus. Even though firms have all the bargaining power, they impose an externality on each other through their use of termination fees to restrict efficient agent movement. If the regulator places more weight on agent welfare ($\lambda > \frac{1}{2}$), even the distribution of surplus is suboptimal. Firms exploit their bargaining power to extract all surplus, and the agent's participation constraint binds. While distributional concerns are driven by firm bargaining power, unregulated market inefficiency is not. The baseline model has firms making offers, but even if the agent has the bargaining power and makes offers to arriving firms, firm private information will still lead to inefficient agent movement: the agent will capture surplus from more productive firms by demanding compensation that shuts out some types above the current match.

Standard regulatory policies: Given that firms use termination fees—which we can interpret as non-competes with a buyout, and offer zero surplus to agents—two relevant policies that regulators commonly use in employment markets come to mind as potential solutions to these problems—minimum wages and a ban on noncompete clauses in contracts (penalties for contract termination). Can either of these policies help or improve on the unregulated market outcome?

Unfortunately, such simple policies may be insufficient to properly address the failures that arise in the unregulated market. A minimum wage can potentially increase agent surplus when employed, but as usual, comes at the risk that a set of firms will make no offers even when positive match surplus is feasible. Moreover, a minimum wage cannot achieve optimal surplus division. A ban on penalties for contract termination non-competes may be difficult to enforce without placing additional restrictions on the space of contracts, as firms can use complex contract structures which impose effective penalties on agents if they wish to exit—for instance, by backloading payments which can be foregone. Theoretically, even if the regulator can ban termination fees, this can have unintended distortionary effects on the market because a lack of termination fees affords firms too little protection from poaching. Agents may even move from more productive firms to less productive firms in equilibrium, so job transitions can become even more inefficient than without any regulation.[20] The limitations of these policies suggests that a more general approach is needed.

5 Optimal Regulation

We now turn to the design of an optimal regulatory policy. For each permitted contract, we can think of the regulator as choosing subsets of M^A and M^F of M, of permitted messages for the agent and the firm respectively.[21] Henceforth we will abuse notation slightly and refer to contracts that are permitted by the regulator as including permitted message spaces M^A and M^F. We start by defining a focal class of contracts from which we will construct an optimal policy.

Definition 1 (Stationary Contract with Exit Clause)**.** *A stationary contract with an exit clause (SCEC) is a contract $x = (\{(p_s, a_s)\}_{s=0}^{\infty}, M^A, M^F)$, for which $M^A = \{quit,\ stay\}$ and $M^F \subset \mathbb{R}^2_+$, and there exists a signing bonus b, such that for all s, $h \in \mathcal{H}^s$, and*

$$\textit{(i)}\ \ p_s(h,r,m) = \begin{cases} w+b & \textit{if } s=0, m_0=(w,z), r=stay \\ w+b-z & \textit{if } s=0, m_0=(w,z), r=quit \\ w & \textit{if } s>0, m_0=(w,z), r=stay \\ w-z & \textit{if } s>0, m_0=(w,z), r=quit \end{cases}$$

$$\textit{(ii)}\ \ a_s(h,r,m) = \begin{cases} 0 & \textit{if } r=stay \\ 1 & \textit{if } r=quit \end{cases}$$

The contract commits to pay the agent a signing bonus b, which is paid to the agent in the first period of the relationship. The contract is stationary: after the relationship starts, the agent receives the same wage w in every period until the relationship ends. If the agent ever wishes to end the relationship, he sends the message "*quit*". The contract includes an exit clause with a termination fee z—if the agent quits,

[20] Given the richness of the contract space, is not clear how we should define a ban on termination fees in the general space of contracts. The example we discuss here is in the context of a restricted space in which firms can offer any fixed wage contracts but cannot impose a termination fee which requires the agent to make a payment to the firm upon exit.

[21] Other messages can be ruled out either by triggering punishments, or the contract can map M into M^A and M^F, such that all messages by the agent or firm are interpreted by the contract as coming from these sets. Formally, this means there exist functions $\phi^A : M \to M^A$ and $\phi^F : M \to M^F$ such that the contract functions depend only on the $\phi^A(r)$ and $\phi^F(m)$ for any $r, m \in M$.

he pays the termination fee.[22] One can also think of this as a non-compete clause, which can be bought out by paying the termination fee. The contract entails partial commitment from the firm, as the firm sets the wage and termination fee, (w, z), in the first period of the relationship—only after the agent has accepted, and the relationship has begun. The choice of M^F limits the set of wage and termination fees that the firm can choose among. We will show that optimal regulation will implement an equilibrium in which firms offer only SCECs. Therefore, a natural way to interpret this limited commitment is that this class of contracts entails a strong form of non-compete clause: the agent cannot sign a contract with a new firm while still under a non-compete (i.e., before the termination fee has been paid). Arriving firms are allowed to pay a share of the fee to buy out the non-compete (the bonus), but parties cannot signed the contract (i.e., commit to the terms) until after the buyout.[23]

Our main result shows there is an optimal regulatory policy in which the regulator restricts contracts to be of this form, but imposes a minimum wage, and specifically restricts the permitted schedule of termination fees to be a function of the wage.[24]

Theorem 1. *There exists a minimum wage $\underline{w}^*$ and termination fee function $z^*(\cdot)$ of the wage that is set, and a schedule $S = \{(w, z^*(w))\}_{w \geq \underline{w}^*}$, such that*

$$\mathcal{R}^* = \{x \text{ an SCEC } | M^F = S\}$$

is an optimal regulatory policy.

The permitted schedule of wage and termination fee are constructed to implement an outcome that splits the surplus between firms and agents optimally, subject to providing firms incentives to offer wages commensurate with the match productivity.

Optimal equilibrium under $\mathcal{R}^*$: The optimal regulatory policy implements an equilibrium σ^*, in which each arriving firm's contract offer depends only on the firm's type and the terms of the agent's current contract, (w', z'). The agent's strategy depends on (w', z'), the offered contract, and his belief about the type of the new firm and does not rely on complex equilibrium strategies by the players: that is, the

[22]We refer to these penalties as termination fees throughout. These final payments can alternatively be interpreted as deferred payments with a clawback provision if the agent leaves the firm.

[23]Formally, the model does not permit the agent to walk away from the contract and enter unemployment after seeing the wage. This is for notational convenience: we could allow the agent to enter unemployment without paying the termination fee without changing any results.

[24]For ease of exposition and interpretation, we state this formally as a restriction on the contract and messages the firm can send. This could equivalently be stated as a restriction on only the contract functions, with no restriction on the message space.

equilibrium is Markovian in the state variables of the arriving firm's productivity and the current contract the agent is in.

Let $\underline{k}$ be the lowest productivity firm that offers a contract which is accepted by the agent. There exist functions $\xi(k)$, $w^*(k)$ and $b(k)$, with $\xi(0) = \underline{k}$ and $\xi(k) \geq k$ for all k, such that the equilibrium has the following structure:

- At any date, suppose the agent is under contract with terms (w', z') with a firm with productivity k'.[25]
- An arriving firm with type k observes the agent's current wage w', which corresponds to the equilibrium wage offered by some type k'. If $k < \xi(k')$, the firm offers no contract—therefore if the agent is unemployed, firms with $k < \underline{k}$ make no offer. If $\xi(k) \geq k'$, the firm offers a SCEC with bonus $b(k')$, and if the agent accepts, sends the message $m_0 = (w, z^*(w))$ with $w = w^*(k)$, to set the wage and termination fee.
- On the equilibrium path, if the agent receives an offer with bonus $b(k')$ or more, he infers that the type of the arriving firm is larger than $\xi(k')$, sends the message "*quit*" if under an existing contract, and accepts the offer.[26]

Incentives: Optimal regulation links the permitted terms of the contract with the offered wage to facilitate more efficient agent movement and distribute surplus to the agent. Firms with higher productivity are willing to pay more to employ the agent for longer. Broadly, there are two ways they can increase the expected duration of the relationship: pay higher wages (making the current relationship more attractive) and using higher termination fees (making leaving less attractive). Using wages is costly for the firm, while termination fees compensate the firm whenever the agent does leave; in the unregulated economy, firms exclusively use termination fees. In contrast, optimal regulation forces firms to use a combination of both, tying the permitted termination fee to the wage. This induces higher type firms to pay better wages to use stronger termination fees—increasing the share of surplus going to the

[25] If the agent is not under contract, these are all zero.

[26] Off the equilibrium path, there are multiple natural ways to define the agent's beliefs when he receives an offer with a signing bonus b other than $b(k')$. For instance, say the agent has "passive beliefs"—that is, for all offers, he believes that the type of the new firm is larger than $\xi(k')$. In this case, if $b < b(k')$, the agent finds it optimal to reject the offer, and if $b > b(k')$, he accepts the offer. As a result, no firm finds it optimal to deviate from the equilibrium.

agent. At the same time, higher termination fees are now costly in terms of wages, which can deter firms from setting inefficiently high termination fees. We view the regulatory requirement that firms use both wages and termination fees, and the tying together of the permitted regulation fees with wages, as our key economic takeaway.

The signing bonus plays two incentive roles. It compensates the agent for paying the termination fee to the previous firm, but also provides a signal of the arriving firm's type—and the agent's expected terms at that firm, which are only selected after the offer is accepted. In equilibrium, firms with productivity below the threshold $\xi(k)$ do not find it optimal to offer a signing bonus of $b(k)$, so when offered this, the agent infers the firm has a type $\tilde{k} \geq \xi(k)$ and will choose terms $(w, z^*(w))$, with $w = w^*(\tilde{k})$.[27] The partial commitment allowed by regulated contracts plays an important incentive role, which we discuss in a few paragraphs, in the context of the optimal allocation properties described in Proposition 2.

Efficiency and Surplus Distribution: Adjustments to the permitted schedule can induce more or less efficient agent movement. Joint surplus is maximized only if the agent transitions follow the efficient path—working for the first arriving firm, and subsequently moves to any firm more productive than his current one. The following proposition states that the optimal regulated outcome is efficient if the regulator doesn't care about the distribution of surplus or cares more about firms, but there is a trade-off between efficiency and better distribution of surplus if the regulator places strictly more weight on the agent.

Proposition 2. *Under the optimal regulatory policy,*

i) Total surplus is maximized and $\underline{k} = 0$ if and only if $\lambda \leq \frac{1}{2}$.

ii) For $\lambda > \frac{1}{2}$, we have $\underline{k} > 0$ and total surplus is decreasing in λ.

iii) Agent transitions are efficient after first employment: $\xi(k) = k$ for all $k \geq \underline{k}$.

iv) The agent's ex-ante utility is zero for $\lambda < \frac{1}{2}$, and increasing in λ for $\lambda > \frac{1}{2}$.

Regulation can implement the efficient path by setting the termination fee function $z^*(w)$ to ensure that any arriving firm with a higher type than the current match

[27]The use of signing bonuses as a signaling device has been explored in Van Wesep [2010] to signal employee fit with a firm when firms are exogenously fixed to use short-term contracts. Our results suggest this can arise as an outcome of optimal regulation.

makes an offer to the agent. In particular, this means all types make an offer to an unemployed agent, and once employed, all firms with a higher type than the current match make an offer with a positive signing bonus. If $\lambda = \frac{1}{2}$ the regulator is indifferent to the distribution of surplus, and optimal regulation implements the efficient path. Fixing attention to efficient regulatory policies, the distribution of surplus in the outcome can vary, depending on the equilibrium signing bonus offered at time zero. This must be weakly negative—as otherwise the lowest types of firm will not make an offer—and is essentially an entry fee for the agent if strictly negative. It's possible to increase the entry fee up to the point that agent's equilibrium surplus is zero, which distributes all surplus to firms and maximizes total firm profits. In fact, this is the optimal outcome if $\lambda < \frac{1}{2}$. At the other extreme, agent surplus is highest if the entry fee is zero, and type zero firms receive an expected payoff of zero. However, firms must receive information rents, so there is a cap on the share of total surplus that can go to the agent. To increase agent surplus further, the regulation must incentivize firms to pay higher wages and share more surplus, but this requires the regulation to distort agent movement and reduce firm information rents.

Therefore, for $\lambda > \frac{1}{2}$, there is a trade-off between maximizing total surplus and distributing more surplus to the agent. In principle, optimal distortions to the agent's career path could be complex: say the agent is employed by a firm with type k. Efficiency requires movement to any type $k' > k$, but allowing the movement to a larger set of types increases information rents for arriving firms. Shutting out movement to a set of types in $[k, k+\epsilon)$ lowers information rents. Perhaps surprisingly, the optimal distortions occur only through early-career unemployment, and agent transitions are efficient after the agent's first match: there is a binding minimum wage which excludes firms below $\underline{k} > 0$ from the market. This removes information rents for these types, and reduces rents for more productive firms by decreasing the set of types in the market. The agent may remain unemployed for some time at the start of his career, but once employment commences, so does efficient movement between firms. As the regulator's weight on agents increases, so do the distortions to total surplus, and a larger set of firm types at the bottom of the distribution are shut out of the market.

The signing bonus is strictly positive whenever the agent is already employed. If $\lambda > \frac{1}{2}$, it is optimal to have no entry fee, while for $\lambda < \frac{1}{2}$, the agent pays an entry fee under optimal regulation. While an entry fee might seem somewhat unusual in a labor market application, one might naturally expect that the regulator places at least equal weight on workers in such a setting. On the other hand, $\lambda < \frac{1}{2}$ may be relevant in applications where the agent is a firm, such as vertical contracting, in which case an entry fees may be reasonable.

Coming back to the role of partial commitment: under our optimal regulation, firms are not allowed to commit to the wage and termination fee at the time of offer, only the bonus.[28] To see why, note that efficient movement after first employment means that if the agent is employed by a firm of type k and moves to type $k' > k$, then whether the firm moves to $k'' > k$ is independent of k. That is, the agent's transition from one firm to the next does not depend on the agent's past employers. The bonus offer by k' has to ensure that the agent leaves firm k, and the wage and termination fee ensure the agent only leaves k' for higher types. As the regulated termination fee function is the same across the market, the bonus and wage choice must be independent of each other in the implemented equilibrium. However, some firms will prefer to substitute wages for bonuses if they could, because a higher wage entails a larger termination fee, while the agent only cares about expected payments, not their form. In other words, the bonus helps on the extensive margin of recruiting the agent, while the wage helps the extensive, but also the intensive margin of the value from recruitment. If firms could commit to all terms at the time of the offer, they could profit by lowering the bonus and raising the wage and termination fee, distorting the market allocation.

Career path: Optimal regulation generates an upward trajectory to the agent's career path—new jobs are with more productive firms, with higher wages and are expected to last longer. We also show that the agents' continuation value is increasing over time, a property not implied by the fact that wages are increasing.[29] Given that an ex-ante individual rationality constraint holds for the agent (as he can always

[28] There is a large literature on firm-agent contracts with partial or no commitment, such as relational, or self-enforcing contracts (e.g., Levin [2003] among many others). The contracting space from which the regulator specifies permitted contracts embeds all possible forms of commitment, for either party. Our optimal regulatory policy permits contracts which only allow the firm partial commitment to some contracting terms at the time that the contract is offered. Limited commitment arises endogenously—it can be optimal for regulators to allow contracts that prohibit full commitment, and are "relational" in some aspects.

[29] Although wages increase in type, so does the length of time before the agent joins a new firm. Though we might think monotonicity is implied by the fact that the agent can reject offers, when the agent accepts, he does not know the firm's type. In principle, he may match with a type lower than the expected type, with a lower continuation value relative to the previous firm.

turn down all contract offers), this result immediately implies that interim individual rationality constraints requiring the agent to have a weakly positive continuation value in each period can be added without changing the optimal regulatory policy. Thus, we can allow the agent to return to unemployment with a one-period delay without paying the termination fee, which can be interpreted as a one-period non-compete clause in the absence of a buyout. Thus, optimal regulation does not require the use of long-term non-compete clauses.

6 Analysis

6.1 Upper Bound: Centralized Mechanism

First, we derive an upper-bound on the value of optimal regulation,[30] in which we analyze a centralized direct mechanism. We define a centralized direct mechanism as follows: a firm arriving at date t—henceforth referred to as firm t—reports its type, and if the agent is currently at a firm, the mechanism specifies A actions and payments, p_t ,between A and the incumbent firm, as well as the termination decision, a_t and whether the agent forms a match, d_t, with firm t, conditioning on the *entire history* of play. We assume firm t knows that they arrived in period t but does not observe the history leading up to time t. By the revelation principle, we focus on mechanisms which induce truthful revelation of firm types. We also assume that the mechanism can use a public randomization device in each period.[31] This is an upper bound on our regulatory problem, since a centralized mechanism can induce a larger set of outcomes than decentralized bilateral contracting.

Let $h_t^C = \{(p_s, d_s, a_s, m_s)\}_{s=0}^t$ be a period t centralized history, where m_s is the reported productivity of the firm meeting A in period s, and let $\mathcal{H}_t^C$ be the set of all period t centralized histories. To simplify notation, we will suppress the dependence of these functions on the history and realized variables, though sometimes we will include the dependence on the reported type in period t. Note that we ignore A incentives in the centralized mechanism.

The contract termination functions $\{a_s\}_{s=t+1}^{\infty}$ induce a stopping time $\psi_t := \inf\{s \geq t+1 : \ a_s = 1\}$ for the last period in which period t firm employs A (conditional on

[30] We state everything for the case of $\mathcal{K} = [0, \bar{k}]$.

[31] For expositional ease, we drop dependence of the mechanism on this public randomization device.

$d_t = 1$). For each t, firm t's incentive constraint to report truthfully is:[32]

$$\mathbb{E}[d_t\{\sum_{s=t+1}^{\psi_t} \delta^{s-t}(k - p_s)\}|m_t = k] \geq \mathbb{E}[d_t\{\sum_{s=t+1}^{\psi_t} \delta^{s-t}(k - p_s)\}|m_t = k'] \quad \forall k, k'. \quad (IC_t)$$

For all t, the mechanism must also satisfy individual rationality for firm t:

$$\mathbb{E}[d_t\{\sum_{s=t+1}^{\psi_t} \delta^{s-t}(k - p_s)\}|m_t = k] \geq 0 \quad \forall k \in \mathcal{K}, \quad (IR_t)$$

as well as ex-ante individual rationality for A at time zero:

$$\mathbb{E}[\sum_{t=0}^{\infty} \delta^t p_t] \geq 0. \quad (IR_0^A)$$

R's centralized design problem can then be written as

$$\sup_{(p_t, d_t, a_t)_{t=0}^{\infty}} \mathbb{E}[\sum_{t=0}^{\infty} \delta^t(\lambda p_t + (1-\lambda)(\hat{k}_t - p_t))] \quad (2)$$

$$\text{subject to } IR_0^A,\ IR_t,\ IC_t\ \forall t \in \{0, 1, ...\}.$$

We construct a Lagrangian which incorporates IR_0^A into the objective.[33] Let $\lambda_0 \geq 0$ be the multiplier on IR_0^A and set $\alpha = \frac{2\lambda+\lambda_0-1}{\lambda+\lambda_0}$ and $\beta = \frac{1-\lambda}{\lambda+\lambda_0}$. Normalizing by $\lambda + \lambda_0$, the optimal centralized mechanism solves

$$\sup_{(p_t, d_t, a_t)_{t=0}^{\infty}} \mathbb{E}[\sum_{t=0}^{\infty} \delta^t(\alpha p_t + \beta \hat{k}_t)]$$

$$\text{subject to } \ IR_t,\ IC_t\ \forall t \in \{0, 1, ...\}.$$

It is clear that $\alpha \geq 0$: otherwise the optimal policy requires arbitrarily large transfers from A to the firms, violating IR_0^A.

Let $\{Q_t, T_t\}_{t=0}^{\infty}$ be a reduced form centralized mechanism, where we define $Q_t(k) := \mathbb{E}[d_t \sum_{s=t+1}^{\psi_t} \delta^{s-t}|m_t = k]$ and $T_t(k) := \mathbb{E}[d_t \sum_{s=t+1}^{\psi_t} \delta^{s-t} p_s|m_t = k]$ to denote, respectively, the expected discounted length of the match and expected discounted transfers of a firm t reporting type k. The choice of $\{Q_t\}_{t=0}^{\infty}$ must be feasible—there exist feasible policies $\{a_t\}_{t=0}^{\infty}$ that generate $\{Q_t\}_{t=0}^{\infty}$. This places limits on the set of $\{Q_t\}_{t=0}^{\infty}$ that

[32] As firms do not observe the history, payoffs take expectations over histories.

[33] The validity of this Lagrangian approach follows from Theorem 13 of Piunovskiy [2012]. To apply the result, we must check the existence of a finite upper-bound on the value of the Lagrangian. As transfers are unbounded, this is not immediate from our formulation. To derive the upper-bound, we note it is wlog to restrict attention to $(p_t, a_t, d_t)_{t=0}^{\infty}$ in (2) such that $\mathbb{E}[\sum_{t=0}^{\infty} \delta^t p_t]$ is bounded below by 0 (otherwise IR_0^A would be violated) and bounded above by $\frac{\bar{k}}{1-\delta}$ (otherwise, IR_t would be violated for some firm as the total transfers made by all firms cannot exceed the surplus generated without violating some individual rationality constraint). The Lagrangian is then bounded.

can be implemented. For example, setting $Q_t(\cdot) = \frac{\delta}{1-\delta}$ for some t requires $Q_s(\cdot) = 0$ for all $s > t$ and $Q_{s'}(\cdot) \leq \frac{\delta-\delta^{t-s'}}{1-\delta}$ for all $s' < t$.

By the law of iterated expectations, the objective function becomes:

$$\mathbb{E}[\sum_{t=0}^{\infty}\delta^t(\alpha T_t(k_t) + \beta k_t Q_t(k_t))],$$

and firm incentive and individual rationality constraints become:

$$Q_t(k)k - T_t(k) \geq Q_t(k')k - T_t(k')\ \forall k, k' \in \mathcal{K}, \quad (IC_t)$$

$$Q_t(k)k - T_t(k) \geq 0\ \forall k \in \mathcal{K}. \quad (IR_t)$$

These now take the form familiar from static mechanism design, and we can derive the standard Myersonian characterization.[34]

Lemma 1. *Q_t, T_t satisfies IC_t and IR_t if and only if, for some $\underline{U}_t^F \geq 0$, Q_t is increasing in k and $T_t(k) = Q_t(k)k - \int_0^k Q_t(k')dk' - \underline{U}_t^F$.*

$\underline{U}_t^F$ is the utility of the lowest type firm. It is clearly optimal to set $\underline{U}_t^F = 0$ whenever $\alpha > 0$. When $\alpha = 0$, $\underline{U}_t^F$ does not enter the objective function.[35] Substituting for transfers, using the fact that $\alpha + \beta = 1$, and changing the order of integration, we write R's problem as

$$\sup_{\{Q_t\}_{t=0}^{\infty}} \mathbb{E}\big[\sum_{t=0}^{\infty}\delta^t(k_t - \alpha\frac{1-F(k_t)}{f(k_t)})Q_t(k_t)\big] \quad (3)$$

$$\text{subject to } \{Q_t\}_{t=0}^{\infty} \text{ feasible,}$$

$$Q_t(\cdot) \text{ increasing for all } t.$$

The feasibility constraints for $\{Q_t\}_{t=0}^{\infty}$ prevent us maximizing the objective pointwise. We first decompose $Q_t(k)$ into two components. Let $q_t(k) := \mathbb{E}[\sum_{s=t+1}^{\psi_t(k)}\delta^{s-t}|m_t = k, d_t(k) = 1]$ be the expected discounted length of relationship with a firm t of type k, conditional on A moving to that firm.[36] We then have $Q_t(k) = \mathbb{E}[d_t(k)q_t(k)]$.

We can now equivalently define the centralized mechanism via a sequence of functions $\{d_t, a_t, q_t\}_{t=0}^{\infty}$ from $[0, \overline{k}] \times \mathcal{H}_{t-1}^C \times [\underline{z}, \overline{z}]$ into $[0,1]$, $[0,1]$, $[\delta, \frac{\delta}{1-\delta}]$ subject to the feasibility constraint that $\{a_s\}_{s=t+1}^{\infty}$ generate expected discounted length of employment q_t.[37]

[34] The proof is standard and thus omitted. For expositional ease, we write all the results below for the case when $\mathcal{K} = [0, \overline{k}]$; analogous results hold for the case when $\mathcal{K}$ is finite.

[35] When $\lambda < \frac{1}{2}$, it may be optimal to set $\underline{U}_t^F > 0$. We discuss how to deal with this case in the Appendix. For $\lambda = \frac{1}{2}$, $\underline{U}_t^F = 0$ will be optimal, but not uniquely so.

[36] We again simplify notation by dropping dependence on h_{t-1}^C.

[37] We allow $d_t \in (0,1)$, permitting randomized transitions between firms; such randomized rules

We now drop the monotonicity constraint that Q_t is increasing, and formulate (3) recursively. Consider some period t in which a new match begins, subject to a promise-keeping constraint (PK) that firm t employs A for an expected discounted time of q. The value of R's problem at t, when $d_t(k_t) = 1$, can be decomposed into the surplus and transfers generated from the current match plus a continuation value $V(q_t(k_t))$:

$$(k_t - \alpha\frac{1-F(k_t)}{f(k_t)})q_t(k_t) + \underbrace{\mathbb{E}\big[\sum_{s=t+1}^{\infty}\delta^s(k_s - \alpha\frac{1-F(k_s)}{f(k_s)})Q_s(k_s)|q_t(k_t) = q\big]}_{\equiv V(q)}. \quad (4)$$

That is, $V(q)$ is the maximized value of R's objective at the start of a firm-A match when the firm is promised q expected discounted length of employment, net all payments and output from that relationship.

Expression (4) captures R's tradeoffs. We will show that $V(q)$ is decreasing in q for the values of q that will be delivered in any optimal mechanism. As a result, when $k_t - \alpha\frac{1-F(k_t)}{f(k_t)} > 0$, promising higher $q_t(k_t)$ increases R's payoff today, but comes at the cost of lowering the continuation value $V(q)$: if the current firm is promised a longer expected match, then the mechanism must lower future A transitions to arriving firms. Mathematically, this resembles a Mussa-Rosen problem of providing quality q at cost $-V(q)$. Crucially, in our problem, the "cost" function $V(q)$ is endogenous and depends on the future choices of the mechanism. We label the time zero problem, when A is unemployed, as the case $V(q_0)$, in which we drop promise-keeping.

We now characterize an optimal centralized mechanism. There is a strong stationarity in the solution—the mechanism is the same across dates, depending only on the type of the arriving firm and promised value to the incumbent firm. Conditional on matching, the value promised to the new match is history-independent.

Lemma 2. *There exist increasing functions $\phi^*(q)$ and $q^*(k)$, such that if $q \in \{q_0\} \cup [\delta, \frac{\delta}{1-\delta}]$, any solution to $V(q)$ satisfies (a.s.) $d_s(k) = a_s(k) = \mathbb{1}(k \geq \phi^*(q))$ and $q_s(k) = q^*(k)$ for all s. Moreover, $\phi^*(q_0) = \underline{k} := \min\{k|k - \alpha\frac{1-F(k)}{f(k)} \geq 0\}$.*

An arriving firm of type k is promised "quality" $q^*(k)$, conditional on being matched with A. The promised quality determines a cutoff type $\xi(k) := \phi^*(q^*(k))$—with $\xi(0) := \phi^*(q_0)$—such that A leaves for any arriving firm with type $k' \geq \xi(k)$.[38] Therefore, the terms of any relationship that begins depend only on the type of the

can be generated using the public randomization device.

[38]Note that since $a_s(k) = d_s(k)$, A never leaves a firm to go into unemployment.

matching firm, independent of the history, while formation of a match depends on the types of both the arriving firm and the incumbent firm. An unemployed A only joins arriving firms with type above $\underline{k}$. Given this allocation rule, the transfers are pinned down by Lemma 1. As $d_s(k)q_s(k) = \mathbb{1}(k \geq \xi(k))q^*(k)$ and $q^*(k)$ is increasing in k, we have that $Q_s(k) = \mathbb{E}[d_s(k)q_s(k)]$ is increasing in k, so it was without loss to drop the monotonicity constraint.

What is the duration of employment and the efficiency of transitions? It is clear that $\xi(k)$ is increasing in k, and the expected length of employment can be calculated to be $\frac{1}{1-F(\xi(k))}$ which is increasing in $\xi(k)$. To consider the efficiency of transitions, we compare $\xi(k)$ to k. If $\xi(k) = k$, A joins new firms with a higher type. If $\xi(k) > k$, A leaves only when the arriving firm has a sufficiently high productivity relative to the current firm. $\xi(k) < k$ means A can move to lower type firms.

The latter case, while ex-post inefficient, is plausible when viewed through the lens of q as a choice of "quality" in a revenue maximization problem with cost $V(q)$—inefficient downward distortions are often optimal in the presence of dynamic incentive constraints. The case of $\xi(k) > k$ also reflects a known distortion from mechanism design—the exclusion of some efficient types. Such exclusion decreases the information rents of new firms. The next lemma summarizes the above properties, and shows that there are no downward distortions, and upward distortions only occur when A is unemployed, shutting out a set of types below $\underline{k}$ when $\alpha > 0$.

Lemma 3. *Match productivity increases over time: $\xi(k) \geq k$. Moreover, for all $k \geq \underline{k}$, we have $\xi(k) = k$, and $\underline{k} > 0$ if and only if $\alpha > 0$.*

This implies the allocation is efficient once A is first employed, and we can see that the expected length of employment at a firm increases with each subsequent move.

6.2 Decentralized Implementation

We now construct a decentralized regulatory policy $\mathcal{R}$ which induces an equilibrium that implements the optimal centralized outcome. This requires: 1) on-path equilibrium A transitions between firms agree with the optimal centralized allocation: if A is employed by a firm of type k' (with unemployment corresponding to $k' = 0$), he moves to a new firm of type k if and only $k \geq \xi(k')$; and 2) the net expected discounted transfers between A and each firm agree with those in the centralized mechanism to ensure that the distribution of surplus is the same.

In contrast to the centralized problem, the delegation problem crucially includes A incentives to accept and reject contracts.[39] As a result, *the timing of transfers matters*, unlike in the centralized mechanism, and not all centralized implementations of the transfers can be achieved in a decentralized manner. Firms also observe the public history in the decentralized market, so there are more firm incentive constraints than in the centralized problem. We construct a schedule of permitted terms for SCEC contracts, $\{(w^*(k), z^*(k))\}_{k\in[\underline{k},\overline{k}]}$, and a bonus function $b(k')$ that implement the optimal centralized allocation.[40] The schedule and bonuses ensure that, when A is employed by a type k' firm, only arriving firms with $k \geq \xi(k')$ find it profitable to make an offer with bonus $b(k')$, and this offer is accepted by A.

The timing of SCEC contracts plays an important role in ensuring that A's expected tenure at a firm depends only on the firm's type, and not the previous match(es): the contract terms are chosen once the relationship begins, and depend only on k, while the bonus, which incentivizes A to pay the termination fee $z^*(k')$ and leave the current firm, also depends on k'. This means we can define the equilibrium incentive constraints for type k when selecting terms from the permitted schedule independent of the incentives to offer an equilibrium bonus. Choosing terms corresponding to k means the firm employs A for expected discounted time of $q^*(k)$, and receives an expected payoff of

$$kq^*(k) - b(k') - \underbrace{(q^*(k)w^*(k) - (1 - q^*(k)\frac{1-\delta}{\delta})z^*(k))}_{\equiv T(k)},$$

where $T(k)$ represents the expected discounted transfers in the relationship minus the bonus. When the firm chooses terms, the bonus is given, so by a standard Myersonian formula, it is incentive compatible for the firm to choose terms $(w^*(k), z^*(k))$ from the permitted schedule if $T(k) = kq^*(k) - \int_{\xi(0)}^{k} q^*(\tilde{k})d\tilde{k}$, so we have

$$(q^*(k)w^*(k) - (1 - q^*(k)\frac{1-\delta}{\delta})z^*(k)) = kq^*(k) - \int_{\xi(0)}^{k} q^*(\tilde{k})d\tilde{k}. \tag{5}$$

[39]Without A incentives, the net transfers can be distributed in any way. For instance, they can be paid as a lump sum at the time each firm arrives—these transfers are by construction incentive compatible for each type of firm given the allocation rule. However, a decentralized regulatory policy which uses contracts with lump sum transfers at the beginning of the relationship cannot implement the transitions we want in the decentralized problem—if employed by a type k firm, A may accept offers from future firms with type $k' < \xi(k)$, and this unravels firm incentives.

[40]Note that we later show (in Lemma 6) that $w^*(k)$ is strictly increasing in k, which means the wage uniquely pins down match productivity. As a result, we can re-label the above schedule to be a function of w as in the statement of Theorem 1. We set $z^*(w) = z^*(\overline{k})$ for any $w > \overline{w}$.

To ensure A only accepts offers with a bonus of at least $b(k')$, we set terms so that A is indifferent between the equilibrium payoff from accepting an SCEC with bonus $b(k')$ and staying at the current firm with type k' forever:

$$-z^*(k') + b(k') + \mathbb{E}[U(k)|k \geq \xi(k')] = \frac{\delta}{1-\delta} w^*(k'), \tag{6}$$

where $U(k)$ is A's continuation value at this history from moving to a firm of type k under the optimal centralized mechanism, minus $b(k')$. Notice that if we set $b(k') = \int_{\xi(0)}^{\xi(k')} q^*(\tilde{k})d\tilde{k}$, then $b(k') + T(k) = q^*(k)k - \int_{\xi(k')}^{k} q^*(\tilde{k})d\tilde{k}$ over all histories given behavior in the target equilibrium, which is the expected discounted sum of transfers under the optimal centralized mechanism and deters firms with $k < \xi(k')$ from offering a contract A will accept. We then solve (5) and (6) for $w^*(k)$ and $z^*(k)$.

Finally, we show that this construction is an equilibrium: At the start of a contract, type k firms are incentivized to choose terms $(w^*(k), z^*(k))$ by construction; A only accepts offers with a bonus larger than $b(k')$;[41] given this, only firms with type $k \geq \xi(k')$ make an offer, as otherwise, their payoff is negative, and they offer the lowest possible bonus, $b(k')$. We show that A's expected utility from joining more productive firms is increasing, which allows us to verify that A has no incentive to reject a contract he is called to accept or quit a contract early. The formal proof is in the appendix.

7 Discussion

7.1 Extensions

Our main results provide a rationale for a regulated use of termination fees in sequential matching markets. While our baseline model omits some features of contracting relationships to focus on the driving forces behind our results, it is natural to ask how robust the insights are if the model is enriched to capture additional aspects of such relationships. We discuss a number of extensions below, *under which the qualitative structure of optimal regulation remains the same*—regulated SCECs. We defer formal descriptions and results to the Appendix and Online Appendix.

Costly effort. In many settings, output may depend on effort choices by the agent. We augment the model to assume the firm receives a payoff of ke when the agent exerts costly effort e that is observable and contractible.[42] SCEC terms specify a

[41] Assuming natural off-path beliefs as described in section 5.

[42] The assumption that effort is observable is not crucial. We could allow for hidden effort but given risk-neutrality and the lack of limited liability, under mild conditions on the signal structure, the contract can induce any effort profile with the same expected utilities as when effort is contractible.

constant effort choice by the agent (for the duration of the contract), and the regulator permitted schedule of terms also ties the effort choice to the wage. While post-employment agent transitions remain efficient (i.e., the agent always moves to a more productive firm), regulated effort choices may be distorted away from the surplus maximizing level.[43] In particular, if $\lambda > \frac{1}{2}$, effort is continually distorted downwards. Effort distortion over the career path are driven by the fact that effort is costly for the agent, while transitions are not, and there is a benefit to decreasing effort away from the surplus maximizing level due to the additional agent benefit. Effort is decreasing in λ, though still increasing the productivity k of the match.

Private Agent Outside Option. We extend the model to incorporate private agent outside options that can be taken by the agent at any time. For instance, workers may have private information about the value of employment in another industry, or the cost of staying in the market.

Contract Information Disclosure. We assume the agent's history of accepted contracts and relationships are publicly observable, while rejected offers are not observable to future firms. For instance, employment history is typically observable on resumes, but job search history is not. The assumed information structure is not important for our results on optimal regulation, which hold under general informational assumptions. In this extension, we allow contracts to include general public information disclosure as part of contract terms—arriving firms only observe information that is permitted to be disclosed by the contract. This can capture contracts that include restrictions on information disclosure, such as NDAs. In principle, this generalization allows regulators to use sophisticated information disclosure policies. However, the centralized problem assumes that firms do not observe the history of play before they arrive. Therefore, the regulator cannot do better under arbitrary contract information disclosure relative to public histories, but can also implement the same outcome under alternative requirements such as mandatory wage disclosure.

Firm Investments and Proprietary Information. Some aspects of contracting have traditionally been used to justify the use of non-competes, such as firm incentives to invest in costly human capital acquisition, and protection of firm proprietary information. As our baseline illustrates that regulated non-competes can be beneficial even in the absence of such forces, adding these only strengthens our main point.

[43] See Lemma 5 in the Appendix.

Human Capital Investment: Human capital acquisition is a necessary part of some industries, which require workers to acquire either firm-specific or general skills, to fulfill their roles. As training requires firms to invest in new hires, non-compete or clawback clauses have been used by firms to prevent workers from leaving after receiving costly training. Proponents of non-competes argue this incentivize firms to invest more in human capital, which improves productivity. Our baseline model provides a justification for (regulated) non-competes and break-up clauses, even absent such concerns. We explore both general and firm-specific human capital, and find their addition only strengthens our main point.

General human capital investment increases the value of the worker to all firms in the market. This is easily incorporated into the model when publicly observable and contractible. We also consider the case of *firm-specific human capital*, where the firm's investment increases match productivity but is not observed by the agent.[44] In contrast to general human capital, firm-specific human capital has no external value, so cannot be incorporated into the "price" at which the agent leaves for new jobs. Therefore, if firms cannot expect to employ the agent for a sufficient duration, they will not make significant investments, which will lower firms' willingness to pay the agent. In the context of other applications, these investments can be interpreted as firm-specific investments that must be made in vertical contracting relationships.

Firm Break-up Costs: In some industries, firms have valuable proprietary information, and workers require access to this to fulfill their roles. If workers with knowledge of this information leave, there can be direct or indirect information leakage to other firms in the industry.[45] To model this, we add a cost for firms if the agent joins another firm. This can capture information leakage of proprietary information, but also more general organizational disruption upon employee exit. We allow the cost to depend on k, to capture the fact that workers in more productive positions may have access to more sensitive information and create more disruptions upon departure.

7.2 Applications and Implications for Policy

Our model can be applied to a number of markets with matching and long-term contracting. Policymakers often regulate these markets to mitigate market failures

[44] That the firm's choice is private information rules out complicated contracts exploiting common knowledge to effectively make the choice contactable. General human capital, which is typically common knowledge, is assumed to be contractable for similar reasons.

[45] In principle, information leakage can be banned by NDAs, but this can be difficult to enforce.

such as inefficiencies or disparate distribution of surplus. How can we interpret our results in the context of such applications, and what do they imply for policymakers?

A focal application is employment contracting. In both the unregulated benchmark and regulated outcome, firms use contracts with termination fees to restrict worker movement. Termination fees impose a cost on workers for moving jobs. We can interpret these fees as various common features of contracting in markets for skilled labor: First, as a non-compete clause which prevents workers from moving jobs, but can be bought out for a fee. We can also interpret the fee as a clawback clause (common in the financial services industry) in an employment contract which requires workers to pay back salary, or give up vested stock options or employer-matched 401(k) contributions. Prospective employers offer signing bonuses to incentivize workers who are subject to contractual clauses, a common feature of financial labor markets.

Our results have policy implications, particularly in light of the recent debate around, and proposed ban by the FTC of, non-compete clauses. While firms exploit termination fees in an unregulated market to inefficiently restrict worker movement and extract surplus, our results suggest that optimal regulation can improve on an outright ban on non-competes. A ban may be counterproductive and increase inefficiency fue to insufficient protection from poaching for firms, or otherwise, firms may find alternative ways to impose costs on workers leaving, through schemes such as contractual clawback. Instead, we find that regulating the structure of permitted contracts, and specifically tying salaries to the penalties firms can impose on workers who leave, aligns firm incentives better with the regulator's objective. Our central economic takeaway is the benefit of forcing firms to use wages in conjunction with termination fees to retain workers. This imposes a cost on firms—higher wages—for setting higher termination fees, and leads to more efficient worker movement, as well as better surplus division by providing incentives for firms to pay workers more.

There exist examples of regulatory policies in the spirit of our result. Prior to the FTC's proposed ban, states such as Maryland, Oregon, and Illinois permit non-competes for high-wage workers, but not low-wage. In Texas, physicians may be subject to non-competes, but must be permitted to buy out the non-compete at a reasonable price. Another example is the "Pelé law", introduced in Brazil in 1998, which stipulates that professional soccer teams that sign junior players can set transfer fees (which must be paid if the player wishes to join another team within a certain time frame) can be at most 100 times the player's salary.

Another application of particular interest is contracting between a firm selling a good to another, such as an upstream supplier and a downstream firm. We can view the use of termination fees as an exclusivity (sole supplier) clause which prevents the buyer from purchasing from other firms without paying a penalty. The legality of such clauses is subject to scrutiny by the FTC, and unreasonable restriction of trade that inhibit competition may violate antitrust laws such as the Sherman Act or Clayton Act.[46] Our results suggest that optimally regulated contracts should permit exclusivity clauses that can be dissolved with a penalty payment. Moreover, the penalty should be tied to the price at which the good is supplied—firms that offer lower prices are permitted to set higher penalties if the buyer switches supplier.

References

Philippe Aghion and Patrick Bolton. Contracts as a barrier to entry. *The American Economic Review*, pages 388–401, 1987.

David P. Baron and Roger B. Myerson. Regulating a Monopolist with Unknown Costs. *Econometrica*, 50(4):911–930, 1982.

Dhruva Bhaskar, Andrew McClellan, and Evan Sadler. Regulation design in insurance markets. *American Economic Review*, 113(10):2546–2580, 2023.

Pierre Cahuc, Fabien Postel-Vinay, and Jean-Marc Robin. Wage bargaining with on-the-job search: Theory and evidence. *Econometrica*, 74(2):323–364, 2006.

David Card. Who set *Your* wage? *American Economic Review*, 112(4):1075–1090, 2022.

Yeon-Koo Che and Tracy R Lewis. The role of lockups in takeover contests. *The RAND Journal of Economics*, 38(3):648–669, 2007.

Peter A Diamond. A model of price adjustment. *Journal of economic theory*, 3(2): 156–168, 1971.

April Mitchell Franco and Darren Filson. Spin-outs: knowledge diffusion through employee mobility. *The Rand journal of economics*, 37(4):841–860, 2006.

Daniel F Garrett and Alessandro Pavan. Managerial turnover in a changing world. *Journal of Political Economy*, 120(5):879–925, 2012.

Alex Gershkov, Benny Moldovanu, Philipp Strack, and Mengxi Zhang. A theory of

[46]See Aghion and Bolton [1987] for a case in which exclusivity clauses were ruled unlawful.

auctions with endogenous valuations. *Journal of Political Economy*, 129(4):1011–1051, 2021.

Milton Harris and Bengt Holmstrom. A theory of wage dynamics. *The Review of Economic Studies*, 49(3):315–333, 1982.

Yolande Hiriart and David Martimort. How much discretion for risk regulators? *The RAND Journal of Economics*, 43(2):283–314, 2012.

Florian Hoffmann, Roman Inderst, and Marcus Opp. The economics of deferral and clawback requirements. *The Journal of Finance*, 77(4):2423–2470, 2022.

Roman Inderst and Marco Ottaviani. Misselling through agents. *American Economic Review*, 99(3):883–908, 2009.

Jonathan Levin. Relational incentive contracts. *American Economic Review*, 93(3): 835–857, 2003.

Simon Loertscher and Ellen V Muir. Wage dispersion, involuntary unemployment and minimum wages under monopsony and oligopsony. 2022.

Suraj Malladi. Delegated costly screening. 2022. Working paper.

Nahum D Melumad, Dilip Mookherjee, and Stefan Reichelstein. Hierarchical decentralization of incentive contracts. *The RAND Journal of Economics*, pages 654–672, 1995.

Paul Milgrom and Ilya Segal. Envelope theorems for arbitrary choice sets. *Econometrica*, 70(2):583–601, 2002.

Paul Milgrom and Chris Shannon. Monotone comparative statics. *Econometrica*, pages 157–180, 1994.

Arijit Mukherjee and Luís Vasconcelos. On the trade-off between efficiency in job assignment and turnover: The role of breakup fees. *The Journal of Law, Economics, and Organization*, 34(2):230–271, 2018.

Alessandro Pavan, Ilya Segal, and Juuso Toikka. Dynamic Mechanism Design: A Myersonian Approach. *Econometrica*, 82(2):601–653, 2014.

Christopher A. Pissarides. Search unemployment with on-the-job search. *Review of Economic Studies*, 61(3):457–475, 1994.

AB Piunovskiy. *Optimal control of random sequences in problems with constraints*, volume 410. Springer Science & Business Media, 2012.

Fabien Postel-Vinay and Jean-Marc Robin. Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6):2295–2350, 2002.

Paul H Rubin and Peter Shedd. Human capital and covenants not to compete. *The*

Journal of Legal Studies, 10(1):93–110, 1981.

Liyan Shi. Optimal Regulation of Noncompete Contracts. *Econometrica*, 91(2):425–463, 2023.

John Thanassoulis. The case for intervening in bankers' pay. *The Journal of Finance*, 67(3):849–895, 2012.

Edward Dickersin Van Wesep. Pay (be) for (e) performance: the signing bonus as an incentive device. *The Review of Financial Studies*, 23(10):3812–3848, 2010.

Ivy Waisbord. Prohibitions on Non-Compete Agreements for Low-Wage Workers. https://www.americanbar.org/groups/litigation/committees/business-torts-unfair-competition/practice/2022/prohibitions-on-non-compete-agreements-low-wage-workers, 2022. [].

Appendix A Main Proofs

We will prove all our results for the baseline version of the model extended to allow for effort choice. In each period, of a relationship, the agent exerts observable and contractible effort $e \in E := [\underline{e}, \overline{e}] \subset \mathbb{R}_+$ with $\overline{e} > 0$, and which generates output ke. The cost of effort is $c(\cdot)$, which is strictly increasing, strictly convex, and differentiable with $c(\underline{e}) = c'(\underline{e}) = 0$. Theorem 1 follow from the more general results below by taking $\underline{e} = \overline{e} = 1$. The proofs of all other results can be found in the Online Appendix.

We state all proofs for the case $\mathcal{K} = [0, \overline{k}]$; the proof for $\mathcal{K}$ finite is analogous. We now prove a version of Theorem 1 for the model with costly effort. Proposition 2 follows immediately from the properties derived in the proof of this more general result.

A.1 Centralized Mechanism

We now redefine $Q_t(k) = \mathbb{E}[d_t \sum_{s=t+1}^{\psi_t} \delta^{s-t} e_s | m_t = k]$ and $q_t(k) = \mathbb{E}[\sum_{s=t+1}^{\psi_t} \delta^{s-t} e_s | m_t = k, d_t = 1]$. Under this new definition of Q_t, the analysis of Lemma 1 is identical and we can rewrite R's objective in the centralized mechanism as

$$\sup_{\{(Q_t, e_t)\}_{t=0}^{\infty}} \mathbb{E}\Big[\sum_{t=0}^{\infty} \delta^t \big((k_t - \alpha \frac{1 - F(k_t)}{f(k_t)}) Q_t(k_t) - c(e_t)\big)\Big] \tag{7}$$

subject to $\{Q_t, e_t\}_{t=0}^{\infty}$ feasible, $Q_t(\cdot)$ increasing for all t.

We use an analogous definition of $q_t, V(q)$ to the baseline model, with effort in the definition of $V(q)$. Let $g(k,q) := (k - \alpha\frac{1-F(k)}{f(k)})q$. We write $V(q)$ recursively:

$$V(q) = \sup_{\{(d_s,q_s,a_s,e_s)\}_{s=t+1}^{\infty}} \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t}\Pi_{s'=t+1}^{s-1}(1-d_{s'}(k_{s'}))\big\{-c(e_s) + d_s(k_s)\big(g(k_s,q_s(k_s)) + V(q_s(k_s))\big)\big\}] \tag{8}$$

$$\text{subject to } PK: \ q = \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t}\Pi_{s'=t+1}^{s-1}(1-a_{s'}(k_{s'}))e_s].$$

We consider a relaxed version of (8) with value function V_+, replacing the equality in the PK constraint with an inequality and replacing the permanent decision a_s to terminate the contract with a sequence of one-period variables $\tilde{a}_s \in [0,1]$ such that surplus is generated in period s with probability $\tilde{a}_s = 0$:[47]

$$V_+(q) = \sup_{\{(d_s,q_s,\tilde{a}_s,e_s)\}_{s=t+1}^{\infty}} \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t}\Pi_{s'=t+1}^{s-1}(1-d_{s'}(k_{s'}))\{-c(e_s) + d_s(k_s)(g(k_s,q_s(k_s)) + V_+(q_s(k_s)))\}\big)] \tag{9}$$

$$\text{subject to } PK': \ q \leq \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t}\Pi_{s'=t+1}^{s-1}(1-d_{s'}(k_{s'}))(1-\tilde{a}_s)e_s].$$

The use of $1-d_s$ (rather than $1-a_s$) in the objective and PK' constraint reflects the fact that it without loss to have the contract continue until matching with a new firm (as the $\tilde{a}_s$ variable allows for free disposal of effort).[48]

It is clear that $V_+(q)$ is an upper-bound on $V(q)$ and is weakly decreasing in q. Using public randomization, standard results imply that V_+ is concave. Thus, there exists q_+ such that V_+ is constant on $[0, q_+]$ and strictly decreasing on $[q_+, \frac{\delta\bar{e}}{1-\delta}]$. If the solution to $V_+(q)$ has PK' binding for each date after first matching with a firm, then the value to R generated is the same from the optimal policy for V_+ as V.

If $q = \frac{\delta\bar{e}}{1-\delta}$, then the only feasible solution to $V(q)$ and $V_+(q)$ sets $d_s(k_s) = 0$, $\tilde{a}_s(k) = 0$, $e_s(k_s) = \bar{e}$ for a probability one set of histories (the choice of $q_s(k_s)$ is immaterial

[47] By replacing a_s with $\tilde{a}_s$ is a relaxation because the payoff from a_s can be replicated by a choice of $\tilde{a}_s$ such that $\tilde{a}_s = 1$ at histories after $a_s = 1$ until the next match is formed. This relaxation is useful as we do not need to keep track of whether or not the contract has been terminated prior to the next match being formed.

[48] It might seem odd to allow for free disposal of effort at this point, given the PK' constraint only slackens when effort is added. However, we allow for such a possibility as we want to show that effort is never disposed of on-path under any optimal policy, which will then allow us to argue that, in our original problem for V, A never re-enters unemployment (i.e., $a_s = 1, d_s = 0$).

if $d_s(k_s) = 0$). Therefore, let us focus on the case of $q < \frac{\delta \overline{e}}{1-\delta}$.

We can strictly satisfy PK' by setting $d_s(k) = 0,\ \tilde{a}_s(k) = 0,\ e_s(k) = \overline{e}\ \forall s, k$, so a Slater condition holds for (9). By Theorem 14 of Piunovskiy [2012], a solution to this dynamic constrained maximization problem exists. By Theorem 13 of Piunovskiy [2012], there exists $\eta \geq 0$ such that any solution $(d_s^*, \tilde{a}_s^*, q_s^*, e_s^*)$ to (9) solves the following Lagrangian:

$$\sup_{\{(d_s,\tilde{a}_s,q_s,e_s)\}_{s=t+1}^{\infty}} \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t}\Pi_{s'=t+1}^{s-1}(1-d_{s'}(k_{s'}))\{\eta(1-\tilde{a}_s)e_s - c(e_s) \qquad (10)$$
$$+ d_s(k_s)(g(k_s, q_s(k_s)) + V_+(q_s(k_s)))\}\big)] - \eta q.$$

Our next Lemma provides properties of the solution to $V_+(q)$ (which gives most of Lemma 2) as well as the Lagrangian.

Lemma 4. *Take any solution to* (9).

(a) There exist increasing functions $\phi^*(q), e^*(q)$ and $q^*(k)$ such that for all $q \in [\delta, \frac{\delta}{1-\delta}]$, we must have (a.s.) $d_s(k) = \mathbb{1}(k \geq \phi^*(q))$, $q_s(k) = q^*(k) > q_+$ for a probability one set of $k \geq \phi^*(q)$ and $e_s = e^*(q)$ for all s.

(b) If $q > q_+$, then $\tilde{a}_s(k) = 0$ for all s is uniquely optimal and PK' binds.

(c) $V_+'(q) = \eta$.

Proof. We now solve the Lagrangian (10) for arbitrary $\eta \geq 0$. It is clear that $(e_s, \tilde{a}_s) \in \arg\max_{e,a}\ \eta(1-a)e - c(e)$ for all s, so $\tilde{a}_s = 0$ is uniquely optimal when $\eta > 0$ and effort is unique and increasing in η.

Let $\tilde{V}$ be the value of the Lagrangian at the beginning of a period $s \geq t+1$ prior to receiving a report about k_s.[49] If A does not move to the firm arriving in period s, namely $d_s(k_s) = 0$, then the continuation value is $\delta\tilde{V}$. If A does switch firms, then the payoff is $g(k, q_s(k)) + V(q_s(k))$. Thus, $\tilde{V}$ can be written recursively as

$$\tilde{V} = \sup_{(d_s,q_s)} \mathbb{E}\big[\{\max_{e,a}\ \eta(1-\tilde{a}_s)e - c(e)\} + (1-d_s(k))\delta\tilde{V} + d_s(k)\big(g(k, q_s(k)) + V_+(q_s(k))\big)\big].$$

[49] In the Lagrangian form, q is no longer a state variable, and we can write the value function for this relaxed problem for periods $s \geq t+1$ in a recursive manner with no state variable—there is strong stationarity across time as there is no changing state to keep track of.

Removing the term $\max_{e,a}\ \eta(1-\tilde{a}_s)e - c(e)$ and multiplying the expectation by δ yields the recursive equation for V_+ if PK' is dropped, which has a value $V_+(q_+)$. Because $\max_{e,a}\ \eta(1-\tilde{a}_s)e - c(e) \geq 0$, we then have $\delta\tilde{V} \geq V_+(q_+)$.

Without loss, $q_s(k) > 0$ whenever $d_s(k) = 1$: the only way to deliver $q_s(k) = 0$ is to set $\tilde{a}_{s'}(k)e_{s'} = 0,\ s' \geq s+1$ for the length of the match, for which an equivalent payoff can be generated by keeping the current match with the same match policy (as if A has joined the period s firm) and setting $\tilde{a}_{s'}(k) = 0,\ s' \geq s+1$ until the next match. The optimal $q_s(k)$ can be chosen in pointwise manner:

$$q_s(k) \in \arg\max_{q\in[0,\frac{\delta\bar{e}}{1-\delta}]}\ g(k,q) + V_+(q).$$

Because $g(k,q) + V_+(q)$ is supermodular in q and k, Theorem 4' of Milgrom and Shannon [1994] implies that any selection from this arg max above is increasing in k; moreover, the arg max is independent of s and η and is unique for almost all k. Let $q^*(k)$ be the maximal selection from $\arg\max_{q\in[0,\frac{\delta\bar{e}}{1-\delta}]}\ g(k,q) + V_+(q)$. Because $g(k,q)$ is strictly increasing in k, $g(k,q^*(k)) + V_+(q^*(k))$ is strictly increasing in k as well.

The switching decision $d_s(k)$ is also pinned down pointwise:

$$d_s(k) \in \arg\max_{d\in[0,1]}\ (1-d)\delta\tilde{V} + d[g(k,q^*(k)) + V_+(q^*(k))].$$

As $g(k,q^*(k)) + V_+(q^*(k))$ is strictly increasing in k, the solution has a bang-bang structure, with $d_s(k) = \mathbb{1}(k \geq \phi^*(q))$ where $\phi^*(q) \in (0,\bar{k})$ is the unique solution k' to

$$g(k',q^*(k')) + V_+(q^*(k')) = \delta\tilde{V}. \tag{11}$$

The LHS of (11) is strictly increasing in k while the RHS is strictly increasing in η. Therefore, the cutoff type (when interior) is uniquely pinned down by η and is strictly increasing in η; moreover, $\phi^*(q)$ must be increasing in q. Thus, η must increase in q, which implies effort is increasing in q (as the optimal effort is increasing in η).

By the arguments above, the optimal policy in the Lagrangian is unique for $\eta > 0$, and is the unique solution to $V_+(q)$. We now argue that $\eta = 0$ if and only if $q \leq q_+$. When $\eta = 0$, there is an optimal solution that generates expected discounted effort of at most q_+ (as the Lagrangian with $\eta = 0$ is equivalent to the unconstrained value of V_+, which is maximized only at $q' \leq q_+$), so $\eta = 0$ only if $q \leq q_+$. Suppose $q \leq q_+$ and $\eta > 0$. As we have argue above, the threshold for allowing A to join a new contract is strictly increasing in η, which implies that the expected discounted effort is strictly increasing in η. Thus, the expected discounted effort will be strictly above q_+. Because

V_+ is strictly decreasing above q_+, this corresponds to an optimal policy that delivers a value less than $V_+(q_+) = V_+(q)$, a contradiction. Because the cutoff type $\phi^*(q)$ is strictly increasing in η, there is a unique value for the multiplier associated with $V_+(q)$. Given the uniqueness of the multiplier, by the envelope theorem of Milgrom and Segal [2002], we then have $V_+'(q) = -\eta$. We note that, by the arguments above, we have $V_+'(q_+) = 0$ and $\tilde{a}_s(k) = 0$ whenever $q > q_+$.

We now argue that $q^*(k) > q_+$ whenever $d^*(k) = 1$ and $k - \alpha\frac{1-F(k)}{f(k)} \neq 0$ (this last condition holds for a probability one set of k by Assumption 1). Suppose not, so there exists k such that $d^*(k) = 1$ and $q^*(k) \leq q_+$. Then, because $q^*(k) \in \arg\max_{q'}\ g(k,q') + V_+(q')$ and $V_+'(q') = 0$ for $q' \in [0, q_+]$ it must be that $g(k,q)$ is decreasing in q at $q^*(k)$. Because $g(k,q)$ is convex in q and $g(k,0) = 0$, for $g(k,q)$ to be decreasing in q at $q^*(k)$, it must be that $g(k, q^*(k)) < 0$.[50] For $d^*(k) = 1$, it must be that $g(k,q^*(k)) + V_+(q^*(k)) \geq \delta\tilde{V}$, which then implies that $V_+(q^*(k)) > \delta\tilde{V}$, a contradiction of $\delta\tilde{V} \geq V_+(q^*(k)) = V_+(q_+)$. Therefore, $q^*(k) > q_+$ whenever $d_s(k) = 1$ and $k - \alpha\frac{1-F(k)}{f(k)} \neq 0$.

Next, we argue the constraint PK' must hold with equality if $q > q_+$. Dropping the PK' constraint in V_+, the optimal solution must implement discounted effort weakly below q_+; But such a solution would violate PK' as $q > q_+$. Therefore, PK' is essential and, by Corollary 4 of Piunovskiy [2012], PK' must hold with equality. ■

Note that R's problem when A is unemployed corresponds to $V_+(0)$. By Lemma 4, we know that each subsequent choice will lead to, with probability one, a $q_s > q_+$. Because the optimal policy only chooses $q^*(k)$ for which the PK' constraint in V_+ is binding, the optimal policy will also satisfy the PK constraint in $V(q)$ once A is employed. Therefore, the optimal policy generating V_+ (which is an upper-bound on V) must also be a solution to V. The results of Lemma 2 then follows from Lemma 4 along with the observation that because $\tilde{a}_s = 0$ is always optimal, it is never optimal to set $a_s(k) = 1$ when $d_s(k) = 0$; because A must end the relationship with an incumbent firm before starting a new relationship, $d_s(k) = 1$ implies $a_s(k) = 1$. Therefore, $a_s(k) = 1 \iff d_s(k) = 1$.

This policy then gives us our optimal centralized mechanism in the relaxed problem

[50] We invoke the convexity of $g(k,q)$ in q rather than just linearity (which would be sufficient for the arguments here) so that the arguments here are immediately applicable in other extensions to the baseline model.

in which we dropped the monotonicity constraint on Q_t. It is easy to see from the form of optimal policy that Q_t will be increasing in k, so dropping monotonicity was without loss.

Finally, we argue that an unemployed A matches with a type k firm if $k-\alpha\frac{1-F(k)}{f(k)}>0$ and only if $k-\alpha\frac{1-F(k)}{f(k)}\geq 0$. Consider the problem for $V_+(0)$. It is clear that matching with any firm $k-\alpha\frac{1-F(k)}{f(k)}<0$ is suboptimal as $g(k,q)<0$ for all $q>0$ for such firms. For firms with $k-\alpha\frac{1-F(k)}{f(k)}>0$, we can do better than remaining unmatched, setting $q=\delta\underline{e}$, having A generate effort $\underline{e}$ for a single period and setting $\tilde{a}_s=0$ for all future periods of the match. This generates an immediate positive payoff in the first period of the relationship with no impact on future firms, so does not affect incentives.

Proof of Lemma 3

Proof. Take a match of productivity k. Because $\xi(0)\geq 0$ follows trivially, we focus on the case of $k\geq\xi(0)$. From the equation for $\phi^*(q)$, we get that $\xi(k)=k$ if and only if $\delta\tilde{V}_\eta=g(k,q^*(k))+V_+(q^*(k))$, where η and $\tilde{V}_\eta$ are the multiplier and corresponding $\tilde{V}$ (as in the proof of Lemma 4) associated with $V_+(q^*(k))$. By strong duality and the fact that the Lagrangian is equal to $\delta\tilde{V}_\eta-\eta q^*(k)$, we have $V_+(q^*(k))=\delta\tilde{V}_\eta-\eta q^*(k)$. Plugging this into our desired expression for $\delta\tilde{V}_\eta$ and simplifying, we want to show $(k-\alpha\frac{1-F(k)}{f(k)}-\eta)q^*(k)=0$. This equality holds by $\eta=-V'(q^*(k))$ (Lemma 4) and that the first-order condition for $q^*(k)$ is $(k-\alpha\frac{1-F(k)}{f(k)})=-V_+'(q^*(k))$. Finally, the statement regarding $\underline{k}$ is an immediate corollary of Lemma 2. ■

Next, we characterize the optimal effort level; the proof is in the Online Appendix.

Lemma 5. *Effort is given by* $e^*(k)\in\arg\max_{e\in E}\ (k-\alpha\frac{1-F(k)}{f(k)})e-c(e)$, *which is increasing in* k.

A.2 Decentralized Implementation

We now turn to implementing the centralized mechanism in a decentralized regulatory policy. We begin by redefining the class of SCEC contracts, allowing for effort:

Definition 2. *A stationary contract with an exit clause (SCEC) is a contract* $x=(\{(p_s,e_s,a_s)\}_{s=0}^{\infty},M^A,M^F)$, *for which* $M^A=\{quit,\ stay\}$ *and* $M^F\subset\mathbb{R}^3_+$, *and there exists a signing bonus* b, *such that for all* s, $h\in\mathcal{H}^s$, *and*

(i) $e_s(h,r,m)=e\quad$ *if* $m_0=(w,e,z)$.

(ii) $p_s(h,r,m) = \begin{cases} w + b\mathbb{1}(s=0) & \text{if } m_0 = (w,e,z), r = stay \\ w + b\mathbb{1}(s=0) - z & \text{if } m_0 = (w,e,z), r = quit \end{cases}$

(iii) $a_s(h,r,m) = \begin{cases} 1 & \text{if } r = stay \\ 0 & \text{if } r = quit \end{cases}$

Define $T(k)$ and $b(k)$ as in our baseline model. We will specify an equilibrium in which a firm of productivity k encountering A employed by a firm with productivity $k' < k$ chooses a contract that delivers an expected discounted transfer by the k firm of $b(k') + T(k) = q^*(k)k - \int_{k'}^{k} q^*(k'')dk''$. Taking the expectation over all histories, this will generate an expected discounted transfer by a period t firm of productivity k of $T_t(k)$ when $U_t^F = 0$, the same as in the centralized mechanism.

To generate these transfers, we specify that the period k firm arriving to A matched with a firm of productivity k' offer a bonus of $\frac{b(k')}{\delta}$ we will choose $z^*(k')$ so that A only leaves for bonuses of at least $\frac{b(k')}{\delta}$. Define the expected discounted length of the relationship for a firm reporting k as $\gamma(k) := \mathbb{E}[\sum_{t=1}^{\psi_0(k)} \delta^t] = \frac{q^*(k)}{e^*(k)}$. If $k = \overline{k}$, then $\gamma(k) = \frac{\delta}{1-\delta}$, then A can never leave the firm. We can set z^* to be sufficiently high that no individually rational offer can induce A to leave. Going forward, we focus on $k \in [\underline{k}, \overline{k})$, for which $\gamma(k) < \frac{\delta}{1-\delta}$. It is clear that $\psi_t(k)$ is independent of t, so we drop dependence on time. The net payment to A from contract $(b, w^*(k), z^*(k))$ is

$$\delta b + \mathbb{E}[\sum_{t=1}^{\psi(k)} \delta^t w^*(k)] - \mathbb{E}[\delta^{\psi(k)} z^*(k)] = \delta b + \gamma(k)w^*(k) - (1 - \gamma(k)\frac{1-\delta}{\delta})z^*(k).$$

To ensure this implements net transfers $b(k') + T(k)$ when $b = \frac{b(k')}{\delta}$ and $k \geq \xi(k')$ we need

$$T(k) = \gamma(k)w^*(k) - (1 - \gamma(k)\frac{1-\delta}{\delta})z^*(k).$$

Solving for $w^*(k)$, we have $w^*(k) = \frac{T(k) + (1-\gamma(k)\frac{1-\delta}{\delta})z^*(k)}{\gamma(k)}$.

Next, we choose $z^*(k)$ so that A is indifferent between staying with his current firm forever and leaving to take a new contract when offered a bonus of $\frac{b(k)}{\delta}$. Let $U(k)$ be A's continuation value in the centralized mechanism, from moving to a firm of productivity k when currently matched with a firm of productivity k', net the bonus, when a firm of productivity k makes net payments $T(k)$ after the bonus. We construct an equilibrium in which all firms with $k_t \geq \xi(k)$ offer $b = \frac{b(k)}{\delta}$. Thus, A's payoff from accepting this offer is $-z^*(k)+\delta b+\mathbb{E}[U(k'')|k''\geq \xi(k)]$. Because A is indifferent between leaving and staying at this bonus, A's continuation value when employed at a firm is equal to staying with his current firm forever. The continuation value of staying at his current firm forever $\frac{\delta}{1-\delta}(w^*(k)-c(e^*(k)))$ (and so is equal to $U(k)$). Setting these equal, we have

$$-z^*(k)+b(k)+\mathbb{E}[U(k'')|k''\geq \xi(k)]=\frac{\delta}{1-\delta}(w^*(k)-c(e^*(k))). \tag{12}$$

Solving these equations for $w^*(k)$ and $z^*(k)$ yields

$$\begin{aligned} w^*(k) &= \frac{1-\delta}{\delta}\Big[T(k)+(\frac{\delta}{1-\delta}-\gamma(k))c(e^*(k)) \\ &\qquad +(1-\gamma(k)\frac{1-\delta}{\delta})\big[b(k)+\mathbb{E}[U(k'')|k''\geq \xi(k)]\big]\Big], \\ z^*(k) &= -T(k)+c(e^*(k))\gamma(k)+\big(b(k)+\mathbb{E}[U(k'')|k''\geq \xi(k)]\big)\gamma(k)\frac{1-\delta}{\delta}. \end{aligned} \tag{13}$$

Our next Lemma derives some properties of A's per-period utility $w^*(k)-c(e^*(k))$ when not transitioning firms. Given the strategies above, this immediately implies the same properties hold for $U(k)$.

Lemma 6. *$w^*(k)-c(e^*(k))$ is positive and increasing in k.*

Lemma 5 and the above lemma imply that $w^*(k)$ is strictly increasing in k. Therefore, there is a one-to-one map between wages and other terms of the SCEC, so the wage determines all terms. We now prove that these strategies constitute an equilibrium.

Theorem 2. *There exists a $\underline{w}^*$, $e^*(\cdot)$, $z^*(\cdot)$ and a schedule $S=\{(w,e^*(w),z^*(w))\}_{w\geq \underline{w}^*}$, such that*

$$\mathcal{R}^*=\{x \text{ an SCEC } | M^F=S\}$$

is an optimal regulatory policy. Moreover, $e^*(w)\in \arg\max_{e\in E}\ e(k_w-\alpha\frac{1-F(k_w)}{f(k_w)})-c(e)$ where k_w is the type that chooses wage w.

Proof. We specify equilibrium strategies as follows. First, consider an employed A currently matched with a firm who chose wage $w^*(k')$ for $k'>0$. A will accept a contract from any arriving firm who offers a SCEC contract with bonus b if and only if $b\geq \frac{b(k')}{\delta}$ and A believes the firm's productivity conditional on any offer $b\geq 0$ is distributed according to the conditional probability of $k\geq \xi(k')$ and any off-path $b\leq 0$ is expected to have come from $k=0$. A firm with productivity k who matches with A currently in a contract with $(w^*(k'),e^*(k'),z^*(k'))$ (the value of k' can be inferred from the wages) will offer a contract with bonus $\frac{b(k')}{\delta}$ and then set terms $(w^*(k), e^*(k), z^*(k))$ if $k \geq \xi(k')$ and offer no contract otherwise. If their contract is accepted, on- or off-path, a type k firm reports $m_0 = (w^*(k), e^*(k), z^*(k))$. Next, we consider an unemployed A. Equilibrium strategies are identical when $\alpha > 0$ or $\lambda \geq \frac{1}{2}$ with new firms offering a bonus b_0 of 0; accepting such a contract generates a payoff for A of $\mathbb{E}[U(k)|k \geq \underline{k}]$, which is positive by Lemma 6. If A is unemployed and $\alpha = 0, \lambda < \frac{1}{2}$, then new firms offer a bonus of $b_0 = -\mathbb{E}[U(k)]$ (this ensures that A's time zero individual rationality constraint binds) and equilibrium strategies are otherwise the same.

It is clear that above strategies lead to the same outcomes as in the centralized mechanism. Because the centralized mechanism represents an upper-bound on the utility that R can achieve, we conclude that $\mathcal{R}^*$ is optimal as long as the above strategies are an equilibrium, which we now verify.

We first show A has no profitable one-shot deviations. Consider A currently in a contract with terms $(w^*(k), e^*(k), z^*(k))$ for some k (the case when A is unemployed is analogous). If they accept a new contract with bonus b, their expected continuation value is $-z^*(k) + b + \mathbb{E}[U(k'')|k'' \geq \xi(k)]$. If they decline, then their expected continuation value is $U(k) = \frac{\delta(w^*(k) - c(e^*(k)))}{1-\delta}$. Thus, A finds it optimal to accept the contract if and only if $b \geq \frac{b(k)}{\delta}$.

Next, we argue that A has no incentive to unilaterally quit and reject an arriving firm's offer—namely, to leave a firm to rejoin unemployment. We first show that $U(\underline{k})$ must be weakly above A's value of being unemployed. If $\lambda < \frac{1}{2}$, the value of unemployment is zero[51] so this follows immediately from the fact that $U(\underline{k}) \geq 0$ by Lemma 6. If $\lambda \geq \frac{1}{2}$, then this follows from the fact that type $\underline{k}$ firm receives zero surplus; if A was better off being unmatched than being matched with $\underline{k}$, R could improve the centralized mechanism by not matching A with the $\underline{k}$ firm, a contradiction of the optimality of the centralized mechanism. Thus, quitting and rejecting the firm's offer yields at most

$$U(\underline{k}) - z^*(k) = U(\underline{k}) + U(k) - (b(k) + \mathbb{E}[U(k'')|k'' \geq \xi(k)]),$$

where the equality follows from (12) (and $U(k) = \frac{\delta}{1-\delta}(w^*(k) - z^*(k))$). Remaining at his current firm yields $U(k)$, which is then better than quitting if $U(\underline{k}) - (b(k) + \mathbb{E}[U(k'')|k'' \geq \xi(k)] \leq 0$. This inequality follows from the fact that, by Lemma 6,

[51] IR_0^A binds and the continuation game when unemployed is the same as at $t = 0$.

$U(k') = \frac{\delta}{1-\delta}(w^*(k') - c(e^*(k')))$ is increasing in k' and $b(k) \geq 0$. We conclude that A has no profitable deviations.

Finally, we consider the firm's incentives. A firm matching with A in a contract using $(w(k'), z(k'))$ will never find it optimal to offer any $b > \frac{b(k')}{\delta}$ as they could lower the bonus and still have A accept the contract. Conditional on the offer being accepted, the firm's continuation value is their value maximized over messages k'':

$$\begin{aligned}
&\max_{k''} \; q^*(k'')k - \gamma(k'')w^*(k'') + (1 - \gamma(k'')(1-\delta))z^*(k'') - \delta\frac{b(k')}{\delta} \\
&= \max_{k''} \; q^*(k'')k - T(k'') - b(k') \\
&= q^*(k)k - T(k) - b(k').
\end{aligned}$$

where the final equality follows from the fact that truthful reporting is incentive compatible given the form of $T(k)$. Thus, no type k firm has an incentive to choose $m_0 \neq (w^*(k), e^*(k), z^*(k))$. Moving back to the choice of a bonus, if the firm offers $b = \frac{b(k')}{\delta}$, the contract will be accepted and their expected utility is

$$\begin{aligned}
q^*(k)k - T(k) - b(k') &= q^*(k)k - (q^*(k'')k - \int_{\xi(0)}^{k} q^*(k'')dk'') - \int_{\xi(0)}^{\xi(k')} q^*(k'')dk'' \\
&= \int_{\xi(k')}^{k} q^*(k'')dk''.
\end{aligned}$$

This utility is positive if and only $k \geq \xi(k')$. Thus, the firm finds it optimal to offer a contract with $b = \frac{b(k')}{\delta}$ if and only if $k \geq \xi(k')$; firms with productivity $k < \xi(k')$ will choose to offer no contract. Therefore, no firm has a profitable deviation at the contract offer stage. Because no player has an incentive to deviate, our proposed strategies form an equilibrium.

We next argue that this above construction is optimal for R. The above strategies with $\alpha = b_0 = 0$ (which implies $\underline{k} = 0$) implement the efficient outcome and so are optimal for $\lambda = \frac{1}{2}$. If $\lambda < 0.5$, then any outcome that is efficient and for which IR_0^A binds is optimal; this is satisfied for the case when $\alpha = \underline{k} = 0, b_0 = -\mathbb{E}[U(k)]$.[52] If $\lambda > \frac{1}{2}$, then $\alpha > 0$, in which case A's payoff is strictly positive and IR_0^A is slack (i.e., $\lambda_0 = 0$). Because the centralized mechanism is characterized for each α and represents an upper-bound on what R can achieve, the above strategies are optimal as the implement the centralized outcome for the corresponding α. ■

[52]This choice of bonus corresponds to the case of setting $U_0^F > 0$ in the centralized mechanism.

Online Appendix

Appendix B Optimal Regulation: Omitted Proofs

Proof of Lemma 5

Proof. To show that the expected length of employment is increasing in productivity, it suffices to show that $\xi(\cdot)$ is increasing. Let η_k be the multiplier associated with $V_+(q^*(k))$. By Lemma 4, $V_+'(q^*(k)) = -\eta_k = -(k - \alpha\frac{1-F(k)}{f(k)})$. Note that $k - \alpha\frac{1-F(k)}{f(k)}$ is increasing in k by Assumption 1 and the fact that $\alpha \leq 1$. Effort $e^*(k) = \arg\max_{e\in E}\ \eta_k e - c(e)$; because $\eta_k e - c(e)$ is supermodular in e and k, Topkis's theorem implies $e^*(k)$ is (weakly) increasing in k. ■

Proof of Lemma 6

Proof. Rearranging (13) yields

$$\frac{\delta}{1-\delta}(w^*(k) - c(e^*(k))) = T(k) - \gamma(k)c(e^*(k)) + (1-\gamma(k)\frac{1-\delta}{\delta})\big(b(k) + \mathbb{E}[U(k')|k' \geq \xi(k)]\big). \tag{14}$$

We first argue that $T(k) - \gamma(k)c(e^*(k)) \geq 0$ for all $k \in [\underline{k}, \overline{k}]$, strictly so for $k > \underline{k}$. At $\underline{k}$, we have $T(\underline{k}) - \gamma(\underline{k})c(e^*(\underline{k})) = \gamma(\underline{k})(\underline{k}e^*(\underline{k}) - c(e^*(\underline{k})))$. This is weakly positive if $0 \leq ke^*(k) - c(e^*(k))$ for all k. This inequality follows if $e^*(k) = \underline{e}$ by $c(\underline{e}) = 0$ and, if $e^*(k) > \underline{e}$, then by $e^*(k) \in \arg\max_{e\in E}\ (k - \alpha\frac{1-F(k)}{f(k)})e - c(e)$, we have $0 \leq k\underline{e} \leq (k - \alpha\frac{1-F(k)}{f(k)})e^*(k) - c(e^*(k)) \leq ke^*(k) - c(e^*(k))$.

Next, we take the derivative of $T(k) - \gamma(k)c(e^*(k))$ with respect to k, which yields $\gamma'(k)(e^*(k)k - c(e^*(k))) - \gamma(k)\frac{de^*(k)}{dk}(k - c'(e^*(k)))$. By Lemma 5, $\gamma'(k) \geq 0$ and $\frac{de^*(k)}{dk} \geq 0$. By the first-order condition of $e^*(k)$, $c'(e^*(k)) = k - \alpha\frac{1-F(k)}{f(k)}$ if $\frac{de^*(k)}{dk} > 0$, so $\frac{de^*(k)}{dk}(k - c'(e^*(k))) \geq 0$. Moreover, $e^*(k)k - c(e^*(k)) \geq 0$, strictly so if $k > \underline{k}$, as it is clearly only optimal to match A and a firm for whom the surplus generated will be positive. We conclude that $T(k) - \gamma(k)c(e^*(k))$ is strictly increasing in k on $[\underline{k}, \overline{k}]$ and so is strictly positive for all $k > \underline{k}$.

Next, we note that $\lim_{k\to\overline{k}}\ 1 - \gamma(k)\frac{1-\delta}{\delta} = 0$. Therefore, $w^*(k) - c(e^*(k)) > 0$ for all k sufficiently close to $\overline{k}$ by $T(k) - \gamma(k)c(e^*(k)) > 0$. Moreover, because $1 - \gamma(k)\frac{1-\delta}{\delta} \geq 0$ and $b(k) \geq 0$, we have $\frac{\delta}{1-\delta}(w^*(k) - c(e^*(k))) \geq T(k) - \gamma(k)c(e^*(k)) \geq 0$ whenever

$U(k') = \frac{\delta}{1-\delta}(w^*(k') - c(e^*(k'))) \geq 0$ for all $k' > k$. Working down from $\overline{k}$, we conclude that $w^*(k) - c(e^*(k)) \geq 0$ for all $k \in [\underline{k}, \overline{k}]$.

Next, we show that $w^*(k) - c(e^*(k))$ is increasing in k. From (14), the derivative of $\frac{\delta}{1-\delta}(w^*(k) - c(e^*(k)))$ with respect to k is (after some simplification)

$$\frac{\delta(e^*(k)k - c(e^*(k)))f(k)}{(1-\delta F(k))^2} + \frac{d}{dk}\left[(1-\gamma(k)\frac{1-\delta}{\delta})\big(b(k) + \mathbb{E}[U(k')|k' \geq k]\big)\right] \qquad (15)$$
$$+ \frac{de^*(k)}{dk}(k - c'(e^*(k)))\gamma^*(k).$$

The last line is positive by previous arguments. Therefore, to show that $\frac{\delta}{1-\delta}(w^*(k) - c(e^*(k)))$ is increasing in k, it suffices to show that the first line above is positive.

Consider the centralized design problem. Rather than thinking of $q^*(k)$, we can equivalently think of R as choosing a constant effort level $e^*(k)$ and a cutoff $\xi(k)$ such that a firm reporting k employs A as long as no firm arrives with productivity above $\xi(k)$; we then have $\gamma(k) = \frac{1}{1-\delta F(\xi(k))}$ and $q(k) = e^*(k)\gamma(k)$; let $\xi^*(\cdot)$ be the optimal such cutoff, which by our earlier analysis we know is $\xi^*(k) = \max\{k, \underline{k}\}$.

Consider the choice of a cutoff function ξ_0 while A is unemployed while reverting to ξ^* in all future matches (by our previous analysis, we know that $\xi_0 = \xi^*$ will be optimal; to derive necessary conditions on ξ^*, it is useful to consider the choice of ξ_0 separately). We restrict attention to $\xi_0(k) \geq \xi^*(0)$ and specify that no firms with $k \leq \xi^*(0)$ enter.

R's payoff under the choice of ξ_0 is given by A's payoff, the payoff of the first firm to match with A, and the payoff of all subsequent firms. We calculate each of these for an arbitrary choice of ξ_0 (with expected transfers given by the same formula for $T_0(k)$ after replacing $q^*(k)$ with $\frac{e^*(k)}{1-\delta F(\xi_0(k))}$), beginning with the payoff of the first firm to match with A. If this firm has a productivity k', they receive an expected utility of

$$\mathbb{1}(k' \geq \xi_0(0)) \int_{\xi^*(0)}^{k'} \frac{e^*(k'')}{1-\delta F(\xi_0(k''))} dk''.$$

We now consider A. Under ξ_0 (with reversion to ξ^* after the first match), A's expected utility when unemployed and a firm of $k' \geq \xi_0(0)$ arrives is

$$U(k';\xi_0) := \frac{k'e^*(k') - c(e^*(k'))}{1-\delta F(\xi_0(k'))} - \int_{\xi_0(0)}^{k'} \frac{e^*(k'')}{1-\delta F(\xi_0(k''))} dk'' \\ + (1 - \frac{1-\delta}{\delta(1-\delta F(\xi_0(k')))})(\int_{\xi^*(0)}^{\xi_0(k')} q^*(k'')dk'' + \mathbb{E}[U(k'')|k'' \geq \xi_0(k')])$$

Finally, we turn to the expected utility of future arriving firms. Let $J(\ell)$ be the expected utility of firms arriving at dates $s \geq t+1$ when A is still matched with a firm of productivity k given a cutoff of $\xi(k) = \ell$. Given that optimal cutoffs ξ^* will be used in each period after the first one, we can write J as

$$J(\ell) = \frac{\int_\ell^{\overline{k}} \big(\int_\ell^{k'} q^*(k'')dk'' + \delta J(\xi^*(k'))\big) f(k')dk'}{1-\delta F(\ell)}.$$

R's utility when A is unemployed, only matches with $k' \geq \xi^*(0)$, and initial cutoffs $\xi_0(\cdot)$ upon first matching are used is

$$\int_{\xi^*(0)}^{\overline{k}} \left[\alpha U(k';\xi_0) + \int_{\xi^*(0)}^{k'} \frac{e^*(k'')}{1-\delta F(\xi_0(k''))} dk'' + \delta J(\xi_0(k))\right] \frac{f(k')}{1-\delta F(\xi^*(0))} dk'.$$

We note J is decreasing above $\xi^*(0)$ as

$$J'(\ell) = \frac{\delta f(\ell)\big(J(\ell) - J(\xi^*(\ell))\big) - (1-F(\ell))q^*(\ell)}{1-\delta F(\ell)} = \frac{-(1-F(\ell))q^*(\ell)}{1-\delta F(\ell)} \leq 0,$$

where the last equality holds because $\xi^*(\ell) = \ell$ for $\ell \geq \xi^*(0)$.

For $k' \geq \xi^*(0)$, the first-order condition for $\xi_0(k') = \ell$ (which must hold at $\ell = \xi^*(k') = k'$) then yields (after normalizing by $\frac{\lambda+\lambda_0}{1-\delta F(\xi^*(0))}$)

$$0 = \Bigg[\frac{\delta(k'e^*(k') - c(e^*(k')))f(k')}{(1-\delta F(k'))^2} \\ + \frac{d}{d\ell}|_{\ell=k'} \Bigg\{ (1 - \frac{1-\delta}{\delta(1-\delta F(\ell))})(\int_{\xi^*(0)}^{\ell} q^*(k'')dk'' + \mathbb{E}[U(k'')|k'' \geq \ell]) \Bigg\} \Bigg] f(k') \\ - \alpha \frac{\delta f(k')e^*(k')}{(1-\delta F(k'))^2}(1-F(k')) + \beta\delta J'(k')f(k').$$

Notice that the first two lines are equal to the first line is (15). Therefore, it suffices to show the last line is weakly negative, which follows from $\alpha \geq 0$ and $J'(k') \leq 0$. ■

Proof of Proposition 2

Proof. i), ii), iii) and the first part of iv) follow from the proof of Theorem 2 and Lemma 2 (the definition of $\underline{k}$). That A's ex-ante utility is strictly increasing in λ for $\lambda > \frac{1}{2}$ follows from the fact that in this case $\alpha = \frac{2\lambda-1}{\lambda}$, which is strictly increasing in λ, and therefore so is $\underline{k}$. Given that total surplus strictly decreases as λ increases above $\frac{1}{2}$, it must be that A utility must also increase—otherwise we could find an improvement by implementing an outcome with more surplus.

■

Appendix C Unregulated Benchmark: Proofs

We prove a more general version of Proposition 1 in the model with effort choice.

Proof of Proposition 1

We start by defining some useful notation. The maximal per period surplus generated when A is employed by a firm with productivity k is $\Sigma(k) := \max_{e \in E} \; ke - c(e)$. It is easy to see that $\Sigma(k)$ is strictly increasing and continuous. We also let $\overline{F}(k) = \sum_{k' \geq k} f(k')$ denote the tail distribution of productivity (including the probability of type k).

Take an arbitrary equilibrium. Let $v_t(k;x)$ be the equilibrium continuation value to A if he accepts a contract x offered by a period t type k firm (we suppress dependence on the equilibrium and history up to date t for notational simplicity); in case no contract is offered, i.e., $x = x_\emptyset$, we take $v_t(k;x_\emptyset) = -\infty$. Without loss, we assume that no firm offers a contract $x \neq x_\emptyset$ after any history that is rejected by A with probability one. Let $\underline{v}_t$ be A's continuation value from turning down the offer in period t after some history h_t, minus any payments required by the current contract between the current firm and A at the end of period t if A terminates the relationship.[53] Thus,

[53]For instance, if this is negative because A has to make a payment to the current firm for leaving, $\underline{v}_t$ will be larger than A's value of staying in the current contract. Because contracts and histories are public, an arriving firm has access to the same information as A and so in equilibrium can calculate this value $\underline{v}_t$.

A will accept a contract x from a firm arriving in period t if $\mathbb{E}[v_t(k_t;x)|x_t = x] > \underline{v}_t$ and reject x if $\underline{v}_t > \mathbb{E}[v_t(k_t;x)|x_t = x]$.

Let $\overline{U}(k)$ denote the supremum of joint (sum of) firm and A continuation values that can be achieved in any equilibrium in a relationship between A and a firm with productivity k, after a contract is accepted by A at some history and after all payments are made to any firm A is leaving.[54] We calculate this starting in the period the contract is accepted (that is, the period before the relationship begins).[55]

Our first Lemma provides equilibrium properties for type $\overline{k}$ firms.

Lemma 7. $\overline{U}(\overline{k}) = \frac{\delta\Sigma(\overline{k})}{1-\delta}$, *and*

(a) For any $v \in \mathbb{R}$, there exists a contract such that if a firm of type $\overline{k}$ arrives in any period in any equilibrium, if A accepts the contract, A's continuation value is v and the firm's continuation value is $\overline{U}(\overline{k}) - v$.

(b) In any equilibrium, if a firm of type $\overline{k}$ arrives in period t, then

(i) If $\overline{U}(\overline{k}) > \underline{v}_t$, the firm will offer a contract that A accepts.

(ii) The firm never offers a contract x with $v_t(\overline{k};x) > \underline{v}_t$.

Proof. First, we show that for any $v \in \mathbb{R}$, there exists a contract, in any equilibrium, that delivers v utility to A and $\frac{\delta\Sigma(\overline{k})}{1-\delta} - v$ to the firm. Let $e' = \arg\max_{e\in E}\ ke - c(e)$. We specify the contract to never allow A to leave,[56] paying a wage of $\frac{v}{\delta} + c(e')$ in the first period and $c(e')$ in every future period. This generates a value to A of v and a total joint surplus of $\frac{\delta\Sigma(\overline{k})}{1-\delta}$, so the firm's surplus is $\frac{\delta\Sigma(\overline{k})}{1-\delta} - v$. Moreover, this is true in any equilibrium as payments and surplus generated are deterministic and do not depend on the actions of any players. Now, since $\overline{k}$ is the highest type and $\Sigma(k)$ is increasing in k, joint surplus in any relationship is bounded above by $\frac{\delta\Sigma(\overline{k})}{1-\delta}$. Therefore, $\overline{U}(\overline{k}) = \frac{\delta\Sigma(\overline{k})}{1-\delta}$, and (a) holds.

Next, we establish (b). Suppose $\underline{v}_t < \overline{U}(\overline{k})$. We argue that the firm arriving in period t with productivity $\overline{k}$ will hire A; it suffices to show that there exists a contract that

[54] Note that we place no restrictions on A's equilibrium belief about the firm, only that the firm's type is k, but A may not know this for sure (say if types pool on the same contract).

[55] Specifically, if a contract x generating $\overline{U}(k)$ is offered by a period t firm and generates joint value U from once the relationship starts in period $t+1$, then $\overline{U}(k) = \delta U$.

[56] This can always be implemented by setting a sufficiently large termination fee

generates a positive expected utility for the firm. By property a), for any $\epsilon > 0$, there exists a contract the delivers $\overline{U}(\overline{k}) - (\underline{v}_t + \epsilon)$ to the firm and $\underline{v}_t + \epsilon$ to A (and so will be accepted by A). For $\epsilon \in (0, \overline{U}(\overline{k}) - \underline{v}_t)$, this contract generates a strictly positive expected utility for the firm.

Finally, we show that the firm will never offer a contract which gives A a continuation value above $\underline{v}_t$. Suppose in some equilibrium the firm finds it profitable to offer a contract giving A a continuation value of $v > \underline{v}_t$. Then the firm's continuation value is at most $\overline{U}(\overline{k}) - v$. But by the above arguments, there exists a contract guaranteeing the firm a payoff arbitrarily close $\overline{U}(\overline{k}) - \underline{v}_t$, which is strictly higher than the firm's equilibrium payoff, a contradiction. ■

We will characterize the set of equilibrium outcomes via an inductive argument using the properties in Lemma 7. Let $\mathcal{K} = \{k^0, k^1, ..., k^N\}$ with $0 = k^0 < k^1 < ... < k^N = \overline{k}$. Define the following function: let $J(\overline{k}) := \frac{\Sigma(\overline{k})}{1-\delta}$, and for $k < \overline{k}$,

$$J(k^n) := \max_{k' \geq k^{n+1}} \Sigma(k^n) + \overline{F}(k')\overline{U}(k') + \delta(1 - \overline{F}(k'))J(k^n) \tag{16}$$
$$= \max_{k' \geq k^{n+1}} \frac{\Sigma(k^n) + \overline{F}(k')\overline{U}(k')}{1 - \delta(1 - \overline{F}(k'))}.$$

This corresponds to the value for a firm of type $k^n < k^N$ of optimally setting a "price" of $\overline{U}(k')$ for A to leave, assuming it will be paid if and only if a firm with type at least k' arrives, and receiving all the surplus in the relationship in the meantime.[57]

The following lemma describes equilibrium properties for all firm types.

Lemma 8. *For all $k \in \mathcal{K}$, $\overline{U}(k) = \delta J(k)$, and*

(a) For any $v \in \mathbb{R}$ and $\epsilon > 0$, there exists a contract such that if a firm of type k arrives in any period in any equilibrium, if A accepts the contract, A's continuation value is v and the firm's continuation value at least $\overline{U}(k) - v - \epsilon$.

(b) In any equilibrium, if a firm of type k arrives in period t, then

(i) If $\overline{U}(k) > \underline{v}_t$, the firm will offer a contract A accepts.

[57] It is useful for the induction argument to restrict the cutoff type a firm for productivity k sells to (with corresponding price given by $\overline{U}$) to be strictly greater than k. This constraint will not be binding.

(ii) The firm never offers A a contract x with $v_t(k;x) > \underline{v}_t$.[58]

(c) $\overline{U}(k)$ is strictly increasing in k on $[k^c, \overline{k}]$ and $\overline{U}(k^c) > \overline{U}(k')$ for all $k' < k^c$.

To prove the lemma, we proceed by induction. For our inductive step, let k^c be such that the properties of Lemma 8 hold for all $k \geq k^c$. As we have shown above, these properties hold for $k = \overline{k}$, so the existence of such a k^c is not vacuous.[59] We will proceed to show that these properties hold for $k = k^{c-1}$ in the subsequent lemmas.

Our next lemma considers $k \geq k^{c-1}$ and places a bound on $\overline{U}(k')$ for $k' \leq k$, which we will then use to find a contract that generates $\overline{U}(k)$ joint continuation value in any equilibrium.

Lemma 9. *For $k \in [k^{c-1}, \overline{k}]$ and $k' \leq k$, $\overline{U}(k') \leq \delta J(k)$, strictly so if $k' < k$.*

Proof. Let $k^m \in [k^{c-1}, \overline{k}]$ and $k' < k^m$—the case $k' = k^m$ follows from the same arguments as below, taking strict inequalities to be weak. Let $\epsilon \in \big(0, \min_{n\in\{1,\dots,N\}} \delta(\Sigma(k^n) - \Sigma(k^{n-1}))\big)$. Take a contract x that is offered (and then accepted) in some equilibrium by a firm arriving in some period t with productivity k', that generates joint continuation value at least $\overline{U}(k') - \epsilon$. Let $\{e_s\}_{s=t+1}^{\infty}$ be the effort profiles used in this contract.

Let τ be the first time after t at which A accepts an offer from a firm with productivity above $k^{m+1} \geq k^c$ (with corresponding contract x_τ). By property (b)(ii) of Lemma 8, $v_\tau(k_\tau; x_\tau) \leq \underline{v}_\tau$.

Let $\hat{\tau}_1$ be stopping time for the first date $s > t$ at which A leaves the current firm for a firm with productivity $k'' \leq k^m$ (with $x_{\hat{\tau}_1}$ being the offered contract). The joint continuation value from this contract is then

$$\mathbb{E}\Big[\sum_{s=t+1}^{\hat{\tau}_1 \wedge \tau} \delta^{s-t}(k' e_s - c(e_s)) + \delta^{\hat{\tau}_1 - t} v_{\hat{\tau}_1}(k_{\hat{\tau}_1}; x_{\hat{\tau}_1}) \mathbb{1}(\hat{\tau}_1 < \tau) + \delta^{\tau - t} v_\tau(k_\tau; x_\tau) \mathbb{1}(\hat{\tau}_1 > \tau)\Big].$$

By our selection of contract, this value is in $(\overline{U}(k') - \epsilon, \overline{U}(k')]$.

[58] We note that, while A only accepts contracts that give him an expected continuation value weakly above $\underline{v}_t$, these contracts could in principle involve pooling multiple firms types, in which case the continuation value with some types could be strictly below $\underline{v}_t$. While we will show that this is never the case, a priori the possibility remains.

[59] The last part of property c) clearly holds when $k^c = \overline{k}$ as $\overline{U}(\overline{k})$ generates the maximal possible surplus.

The per-period surplus $k'e_s - c(e_s)$ prior to $\hat{\tau}_1 \wedge \tau + 1$ is bounded above by $\Sigma(k')$, which is strictly below $\Sigma(k^m)$. Individual rationality for the firm offering a contract in $\hat{\tau}_1 \neq \tau$ implies that $\overline{U}(k_{\hat{\tau}_1}) \geq v_{\hat{\tau}_1}(k_{\hat{\tau}_1}; x_{\hat{\tau}_1})$. Thus, also using $v_\tau(k_\tau; x_\tau) \leq \underline{v}_\tau$, we have

$$\begin{aligned}
\overline{U}(k') &< \mathbb{E}[\sum_{s=t+1}^{\hat{\tau}_1 \wedge \tau} \delta^{s-t}(k'e_s - c(e_s)) + \delta^{\hat{\tau}_1 - t} v_{\hat{\tau}_1}(k_{\hat{\tau}_1}; x_{\hat{\tau}_1}) \mathbb{1}(\hat{\tau}_1 < \tau) + \delta^{\tau - t} v_\tau(k_\tau; x_\tau) \mathbb{1}(\hat{\tau}_1 > \tau)] + \epsilon \\
&< \mathbb{E}[\sum_{s=t+1}^{\hat{\tau}_1} \delta^{s-t}\Sigma(k^m) + \delta^{\hat{\tau}_1 - t}\overline{U}(k_{\hat{\tau}_1})\mathbb{1}(\tau > \hat{\tau}_1) + \delta^{\tau-t}\underline{v}_\tau \mathbb{1}(\hat{\tau}_1 > \tau)].
\end{aligned}$$

For $n \geq 2$, let $\hat{\tau}_n$ be the stopping time for the first date after $\hat{\tau}_{n-1}$ at which A moves to an nth new firm (after time t) with productivity $k'' \leq k^m$. Take $\hat{\tau}_0 = t$ and $k_{\hat{\tau}_0} = k^m$. Repeating the argument above, we get that

$$\begin{aligned}
\overline{U}(k') &< \mathbb{E}[\sum_{n=0}^{\infty}\{\sum_{s=\hat{\tau}_n+1}^{\hat{\tau}_{n+1}\wedge\tau} \delta^{s-t}\Sigma(k_{\hat{\tau}_n})\}\mathbb{1}(\tau > \hat{\tau}_n) + \delta^{\tau-t}\underline{v}_\tau] \qquad (17) \\
&\leq \mathbb{E}[\sum_{s=t+1}^{\tau} \delta^{s-t}\Sigma(k^m) + \delta^{\tau-t}\underline{v}_\tau].
\end{aligned}$$

where the second inequality follows from $k_{\hat{\tau}_n} \leq k^m$ for all $n \geq 1$.

Let $\underline{k}_s$ be the lowest type $k \geq k^c$ that makes an offer accepted in period s if $s = \tau$. We now show $\underline{v}_\tau \leq \overline{U}(\underline{k}_\tau)$. For the sake of contradiction, suppose $\overline{U}(\underline{k}_\tau) < \underline{v}_\tau$. Individual rationality for $\underline{k}_\tau$ then implies $v_\tau(\underline{k}_\tau; x_\tau) < \underline{v}_\tau$ (as the $\underline{k}_\tau$ firm's payoff is at most $\overline{U}(\underline{k}_\tau) - v_\tau(\underline{k}_\tau; x_\tau) < \underline{v}_\tau - v_\tau(\underline{k}_\tau; x_\tau)$). For $\tau = s$, in order for A to accept the contract $x = x_s$ offered by $\underline{k}_s$, we must have $\mathbb{E}[v_s(k_s; x)|x_s = x] \geq \underline{v}_s$, so there must exist a k'' that offers the same contract x as $\underline{k}_\tau$ and with $v_s(k''; x) > \underline{v}_s$; by property (c) of Lemma 8, $k'' < k^c$. The continuation value for k'' from such a contract is at most $\overline{U}(k'') - v_s(k''; x) \leq \overline{U}(k'') - \overline{U}(\underline{k}_s) < 0$, where the last inequality follows from property (c) of the inductive step. But this contradicts individual rationality for k''. Therefore, $\underline{v}_\tau \leq \overline{U}(\underline{k}_\tau)$. Using this bound in (17) then yields.

$$\overline{U}(k') < \mathbb{E}[\sum_{s=t+1}^{\tau} \delta^{s-t}\Sigma(k^m) + \delta^{\tau-t}\overline{U}(\underline{k}_\tau)]. \qquad (18)$$

All firms with productivity above $\underline{k}_s$ will also make a contract offer that is accepted by A (a higher productivity can always imitate a lower productivity firm and earn strictly higher surplus than the lower firm). Thus, $\mathbb{P}(\tau = s | \tau > s-1, h_s) = \overline{F}(\underline{k}_s)$, so the RHS of the above inequality becomes

$$\mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t} \Pi_{s'=t+1}^{s-1} (1 - \overline{F}(\underline{k}_{s'}))(\Sigma(k^m) + \overline{F}(\underline{k}_{s'})\overline{U}(\underline{k}_s))].$$

We can then further bound the above expression by maximizing over the cutoffs:

$$\max_{\{k''_s\}_{s=t+1}^{\infty}:\ k''_s \geq k^{m+1}\ \forall s} \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t} \Pi_{s'=t+1}^{s-1} (1 - \overline{F}(k''_{s'}))(\Sigma(k^m) + \overline{F}(k''_s)\overline{U}(k'_s))].$$

Written recursively, the above optimization problem has a value $\delta \tilde{J}$ (the extra δ coming from the fact that we start the above sum at $s = t+1$) where

$$\tilde{J} = \max_{k'' \geq k^{m+1}} \Sigma(k^m) + \overline{F}(k'')\overline{U}(k'') + \delta(1 - \overline{F}(k''))\tilde{J},$$

which implies that $\tilde{J} = J(k^m)$. That $\overline{U}(k') < \delta J(k^m)$ then follows from (18). ■

Our next lemma shows that there exists a contract such that a firm with type k^{c-1} can generate a joint surplus arbitrarily close to $\delta J(k^{c-1})$ in any equilibrium.

Lemma 10. *For each $v \in \mathbb{R}$ and sufficiently small $\epsilon > 0$, there exists a contract that, in any equilibrium and at any history at which the arriving firm has type k^{c-1}, if accepted, delivers continuation value of v to A and $\delta J(k^{c-1}) - v - \epsilon$ to the firm.*

Proof. Let $k^* \in \arg\max_{k' \geq k^c} \frac{\Sigma(k^{c-1}) + \overline{F}(k')\overline{U}(k')}{1 - \delta(1 - \overline{F}(k'))}$ and take any sufficiently small $\epsilon > 0$. Take an arbitrary equilibrium, and a firm of type k^{c-1} arriving in period t. Consider the following contract: The firm's messages have no impact on p_s, a_s while A has a binary message that either terminates the contract or continues it. A exerts effort $e' \in \arg\max_{e \in E}\ k^{c-1}e - c(e)$ in each period he is employed. A receives a transfer of $\frac{v}{\delta} + c(e')$ in the first period of the contract, $c(e')$ in each future period and an additional transfer of $-(\overline{U}(k^*) - \epsilon)$ in any period in which the contract is terminated. Notice that the incumbent firm cannot impact the future in this contract, so the set of possible A payoffs do not depend on the firm's type.

Because A has the option of turning down all future contracts, his continuation value of accepting this contract is at least $\delta[\frac{v}{\delta} + \mathbb{E}[\sum_{t=0}^{\infty} \delta^t (c(e') - c(e'))] = v$. Moreover, by the same logic, we have $\underline{v}_s \geq \overline{U}(k^*) - \epsilon$ for all $s > t$.[60]

Next, we argue that if A accepts the contract, firms arriving in periods $s > t$ with type below k^* will not offer $x \neq x_\emptyset$ when ϵ is sufficiently small. Suppose that some type $k' < k^*$ did at some period $s > t$. By the inductive step, each $k > k^c$ offers x with $v_s(k; x) \leq \underline{v}_s$, so for A to accept x in period s, some type k'' must also offer x, and we must have $v_s(k''; x) \geq \underline{v}_s$. Without loss, assume k' is such a firm. Thus, type k' has continuation value at most

$$\overline{U}(k') - v_s(k'; x) \leq \overline{U}(k') - \underline{v}_s \leq \overline{U}(k') - \overline{U}(k^*) + \epsilon.$$

By (c) in Lemma 8, we have $\overline{U}(k') < \overline{U}(k^*)$, so for ϵ sufficiently small, this upper bound is strictly negative. Therefore, k' has a profitable deviation to making no offer, a contradiction, and no type below k^* offers $x \neq x_\emptyset$.

We then argue that $\underline{v}_s = \overline{U}(k^*) - \epsilon$. By the above argument, only firms with productivity above $k^* \geq k^c$ will offer a contract that is accepted, so by property (b)(ii) of Lemma 8, for all on-path x, $v_s(k_s; x) \leq \underline{v}_t$. Take an on-path $x \neq x_\emptyset$. Because $\mathbb{E}[v_s(k_s; x) | x_s = x] \geq \underline{v}_s$ for A to accept x, $v_s(k'; x) = \underline{v}_s$ for each $k' \geq k^*$ offering x. This implies that A has an optimal strategy of turning down all offers. Hence, A's continuation value when accepting the contract is v and his continuation value at all future dates in the contract is exactly equal to 0. Because $\underline{v}_s$ also includes the payments made upon terminating the contract, $\underline{v}_s = \overline{U}(k^*) - \epsilon$.

As $k^* \geq k^c$, by properties (b) and (c) of Lemma 8, each firm with $k' \geq k^*$ will offer a contract that is accepted by A, in which case the incumbent firm's payoff is equal to $\overline{U}(k^*) - \epsilon$. Writing this recursively, the firm's continuation value in the first-period of the contract is $\tilde{J} = \Sigma(k) + \overline{F}(k^*)(\overline{U}(k^*) - \epsilon) + (1 - \overline{F}(k^*))\delta\tilde{J}$, which implies $\tilde{J}$ converges to $J(k)$ as $\epsilon \to 0$ and the contract delivers the firm a value arbitrarily close to $\delta J(k) - v$. ■

This lemma and Lemma 9 imply that $\overline{U}(k^{c-1}) = \delta J(k^{c-1})$, since the contract described above yields an equilibrium joint surplus arbitrarily close to $\delta J(k^{c-1})$ which is an

[60] From period $t+1$, A can turn down all future contracts and receive 0 in each period, and if he ever wants to leave, the payment is $-\overline{U}(k^*) + \epsilon$. Therefore, $\underline{v}_s$ is at least $\overline{U}(k^*) - \epsilon$.

upper bound on $\overline{U}(k^{c-1})$. This and Lemma 10 show that properties (a) and (b)(i) of Lemma 8 hold for type k^{c-1}. Next, we verify that property (b)(ii) also holds for k^{c-1}.

Lemma 11. *In any equilibrium, a firm arriving in period t with type k^{c-1} never offers a contract x with $v_t(k^{c-1}; x) > \underline{v}_t$.*

Proof. Suppose, for the sake of contradiction, that type k^{c-1} offers a contract x in period t with $v_t(k^{c-1}; x) > \underline{v}_t$. Then the firm's payoff is at most $\overline{U}(k^{c-1}) - v_t(k^{c-1}; x)$. By Lemma 10, for $\epsilon \in (0, v_t(k^{c-1}; x) - \underline{v}_t)$ the firm could offer a contract that guarantees it a payoff of at least $\delta J(k^{c-1}) - \underline{v}_t - \epsilon = \overline{U}(k^{c-1}) - \underline{v}_t - \epsilon > \overline{U}(k^{c-1}) - v_t(k^{c-1}; x)$ and this contract will be accepted by A as it gives him a payoff of at least $\underline{v}_t + \epsilon$), making it a profitable deviation from x. ■

It remains to show that $\overline{U}(k^{c-1}) < \overline{U}(k^c)$ and $\overline{U}(k^{c-1}) > \overline{U}(k')$ for all $k' < k^{c-1}$ (to establish property (c)). Take $k' < k^{c-1} < k^c$. By Lemma 9, we have $\overline{U}(k') < \delta J(k^{c-1})$ and $\overline{U}(k^{c-1}) < \delta J(k^c)$. Because $\delta J(k) = \overline{U}(k)$ for all $k \geq k^{c-1}$, we then have $\overline{U}(k') < \overline{U}(k^{c-1}) < \overline{U}(k^c)$. This completes the induction argument, and the proof of Lemma 8.

We now show that A's expected utility is zero in any equilibrium. By property (c) of Lemma 8, we know that $v_t(k_t; x) \leq \underline{v}_t$ for all on-path x offered, for all t, so A weakly prefers to turn down all contracts offered in all periods—namely, A's equilibrium payoff is the same as from the strategy under which he accepts no contracts, which delivers him an expected utility of zero as he begins unemployed.

Next, we show there exists a function $\theta(k)$ such that, in any equilibrium, A leaves a firm of productivity k if and only if A encounters a firm of productivity $k' \geq \theta(k)$. Take an arbitrary equilibrium and assume A accepts a contract offer from type k in some period t. For the sake of contradiction, suppose the relationship generates joint continuation value U, and after some history, A leaves the relationship to join a firm with productivity $k' < k$. By the arguments in the proof of Lemma 9, this implies $U < \overline{U}(k)$. By the equilibrium properties we have proven, the continuation value for type k under this contract is $U - \underline{v}_t$, but we also know that k can guarantee a payoff arbitrarily close to $\overline{U}(k) - \underline{v}_t$ for a strictly higher payoff, a contradiction. Therefore, it must be that in equilibrium, if A is employed by a firm of type k, he will never join a firm of with type strictly less than k.

Let $\underline{k}_s$ be the lowest type in period $s > t$ that A will accept a contract from and leave type k. By the same arguments as in the proof of Lemma 9, we have

$$\overline{U}(k) \leq \max_{\{k''_s\}_{s=t+1}^{\infty}:\ k''_s > k\ \forall s} \mathbb{E}[\sum_{s=t+1}^{\infty} \delta^{s-t} \Pi_{s'=t+1}^{s-1} (1 - \overline{F}(k''_{s'}))(\Sigma(k) + \overline{F}(k''_s)\overline{U}(k'_s))].$$

Written recursively, the above optimization problem on the RHS has a value $\delta J(k)$ (the extra δ coming from the fact that we start the above sum at $s = t + 1$). Thus, there is a unique type $\underline{k}_s$ for each period (and equal across all periods s) if the $J(k)$ has a unique arg max. This is easily seen to be the case for generic values of $\overline{k}$.[61] Let $\theta(k)$ be the (generically) unique value of $\underline{k}_s$.

Finally, we show that $\theta(k)$ is bounded away from k as $|\mathcal{K}| \to \infty$. Fix any $k \in (0, \overline{k})$ and take some sufficiently small $\epsilon > 0$. For the sake of contradiction, suppose we can find arbitrarily large N and type space $\mathcal{K}$ with $|\mathcal{K}| = N$ (we suppress dependence of the cdf on $N, \mathcal{K}$) such that the corresponding $\theta(k)$ has $\theta(k) - k < \epsilon$. Let $\theta^2(k) = \theta(\theta(k))$. Note that, by definition of $\theta(k)$, we have $J(k) = \frac{\Sigma(k)+\overline{F}(\theta(k))\overline{U}(\theta(k))}{1-\delta(1-\overline{F}(\theta(k)))} \geq \frac{\Sigma(k)+\overline{F}(\theta^2(k))\overline{U}(\theta^2(k))}{1-\delta(1-\overline{F}(\theta^2(k)))}$. Because $J(\theta(k)) = \frac{\Sigma(\theta(k))+\overline{F}(\theta^2(k))\overline{U}(\theta^2(k))}{1-\delta(1-\overline{F}(\theta^2(k)))}$, we have

$$J(\theta(k)) - J(k) \leq \frac{\Sigma(\theta(k)) - \Sigma(k)}{1 - \delta(1 - \overline{F}(\theta^2(k)))} \leq \frac{\epsilon \overline{e}}{1 - \delta}.$$

From (16) and $\overline{U}(\theta(k)) = \delta J(\theta(k))$, we then have

$$\begin{aligned} J(k) &= \Sigma(k) + \overline{F}(\theta(k)))\delta J(\theta(k)) + \delta(1 - \overline{F}(\theta(k)))J(k) \\ &\leq \Sigma(k) + \overline{F}(\theta(k)))\delta(\frac{\epsilon \overline{e}}{1 - \delta} + J(k)) + \delta(1 - \overline{F}(\theta(k)))J(k), \end{aligned}$$

which simplifies to $J(k) \leq \frac{\delta \epsilon \overline{e}}{(1-\delta)^2} + \frac{\Sigma(k)}{1-\delta}$.

Take $k' \in \mathcal{K}$ such that $\frac{\delta \overline{F}(k')(\Sigma(k')-\Sigma(k))}{1-\delta(1-\overline{F}(k'))} > \frac{\delta \epsilon \overline{e}}{(1-\delta)^2}$ (we have taken ϵ sufficiently small, so such a $k' > k$ exists). Note that $J(k') \geq \frac{\Sigma(k')}{1-\delta}$ and so

$$J(k) \geq \frac{\Sigma(k) + \overline{F}(k')\overline{U}(k')}{1 - \delta(1 - \overline{F}(k'))} \geq \frac{\Sigma(k) + \overline{F}(k')\frac{\Sigma(k')}{1-\delta}}{1 - \delta(1 - \overline{F}(k'))}.$$

[61] By perturbing $\overline{k}$, we can perturb all values of $\overline{U}(k'')$, and for generic values of $\{\overline{U}(k'')\}_{k'' \geq k}$, $J(k)$ has a unique maximizer.

Combining this inequality with $J(k) \leq \frac{\delta\epsilon\bar{e}}{(1-\delta)^2} + \frac{\Sigma(k)}{1-\delta}$ and simplifying yields

$$\frac{\delta\bar{e}\epsilon}{1-\delta} \geq \frac{\delta\overline{F}(k')(\Sigma(k') - \Sigma(k))}{1 - \delta(1 - \overline{F}(k'))},$$

a contradiction of the definition of k'. Thus, $\theta(k)$ is bounded away from k as $|\mathcal{K}| \to \infty$.

Appendix D Extensions: Formal Results and Proofs

Throughout these extensions, we assume there is no cost of effort as in the baseline model (i.e., $E = \{1\}$).

D.1 Agent Private Outside Option

We now extend the baseline model to allow the agent to have a private outside option $\omega \in \Omega \subset \mathbb{R}_+$ that he can take any time. For simplicity, we assume Ω is finite.

Proposition 3. *There exists a schedule S such that $\mathcal{R}^* = \{x \text{ an } SCEC | M^F = S\}$ is an optimal regulatory policy. There exists ω_R such that A with outside option $\omega > \omega_R$ immediately takes his outside option, and never takes his outside option if $\omega \leq \omega_R$.*

Proof. Take any regulatory policy $\mathcal{R}$ and equilibrium σ that generates a strictly higher utility for R than having all ω types take their outside option immediately. Let $\widehat{U}(\omega) = \mathbb{E}[\sum_{t=0}^{\tau} \delta^t p_t | \omega]$ be the expected utility from type ω until taking the outside option at some time τ. Type ω's utility from the regulatory policy is then $\widehat{U}(\omega) + \mathbb{E}[\delta^\tau \omega | \omega]$. Take any $\omega > \omega'$. Incentive compatibility of each types equilibrium strategy implies

$$\widehat{U}(\omega) + \mathbb{E}[\delta^\tau | \omega]\omega \geq \widehat{U}(\omega') + \mathbb{E}[\delta^\tau | \omega']\omega$$
$$\widehat{U}(\omega') + \mathbb{E}[\delta^\tau | \omega']\omega' \geq \widehat{U}(\omega) + \mathbb{E}[\delta^\tau | \omega]\omega'.$$

Adding these together and simplifying yields $\mathbb{E}[\delta^\tau | \omega] \geq \mathbb{E}[\delta^\tau | \omega']$, which then implies that $\widehat{U}(\omega) \leq \widehat{U}(\omega')$. Let ω_R be the lowest ω that does not immediately take his outside option with probability one in σ. Individual rationality for ω_R, namely $\widehat{U}(\omega_R) + \mathbb{E}[\delta^\tau | \omega_R]\omega_R \geq \omega_R$ then implies that $\widehat{U}(\omega_R) \geq \omega_R$.

Consider a version of our centralized problem in which R (but not firms) can directly observe ω (and can condition on it) but is constrained to deliver at least ω_R expected utility (in payments net effort costs) to each type $\omega \leq \omega_R$ and each type greater than ω_R takes their outside option immediately. The mechanism specifies, for each $\omega \leq \omega_R$, $(d_t^\omega, a_t^\omega, p_t^\omega, \tau^\omega)$ that map histories into decision variables as in our baseline model where τ^ω is a stopping time for the date at which type ω taking their outside option. The incentive constraint for a firm of type k arriving in period t is

$$\begin{aligned} IC_t : \mathbb{E}[d_t^\omega \sum_{s=t+1}^{\psi_t^\omega} \delta^{s-t}(k - p_s^\omega)|m_t = k, \omega \leq \omega_R] \\ \geq \mathbb{E}[d_t^\omega \sum_{s=t+1}^{\psi_t^\omega} \delta^{s-t}(k - p_s^\omega)|m_t = k', \omega \leq \omega_R] \ \forall k, k' \in [0, \overline{k}]. \end{aligned}$$

where the expectation is taken over all histories and types ω conditional on $\omega \leq \omega_R$ and ψ_t^ω is the last date at which ω is matched with the date t firm. In addition, the mechanism must satisfy an individual rationality constraint for each firm t:

$$IR_t : \ \mathbb{E}[d_t^\omega \sum_{s=t+1}^{\psi_t^\omega} \delta^{s-t}(k - p_s^\omega)|m_t = k, \omega \leq \omega_R] \geq 0 \ \forall k \in [0, \overline{k}].$$

For each the $\omega \leq \omega_R$, we add the promise-keeping constraint (where $\mathbb{E}^\omega$ is the expectation conditional on ω)

$$PK_0^\omega : \ \mathbb{E}^\omega[\sum_{t=0}^{\tau^\omega} \delta^s p_s^\omega] \geq \omega_R.$$

R's centralized design problem can then be written as

$$\sup_{\{(d_t^\omega, a_t^\omega, p_t^\omega)_{t=0}^\infty, \tau^\omega\}_{\omega \leq \omega_R}} \mathbb{E}[\sum_{t=0}^{\tau^\omega} \delta^t \big(\lambda p_s^\omega + (1-\lambda)(\hat{k}_s - p_s^\omega)\big) + \lambda \delta^{\tau^\omega} \omega | \omega \leq \omega_R] \tag{19}$$
$$\text{subject to } PK_0^\omega, \ IR_t, \ IC_t \ \forall t \in \{0, 1, ...\}, \ \omega \leq \omega_R.$$

Because $\widehat{U}(\omega') \geq \widehat{U}(\omega_R) \geq \omega_R$ for all $\omega' \leq \omega_R$, the strategies under $\mathcal{R}, \sigma$ are feasible in the above problem, which implies the value of this centralized problem is weakly higher than the regulatory policy $\mathcal{R}$ and equilibrium σ. Let λ_0^ω be the multiplier on PK_0^ω (normalized by the probability of type ω) and $\alpha_0^\omega = \frac{2\lambda + \lambda_0^\omega - 1}{\lambda + \lambda_0^\omega}$ and q^ω, d^ω be analogous to q, d in our baseline model for type ω. By the same arguments as in our baseline model, we can write the design problem as

$$\sup_{\{(p^{\omega}_{s},,d^{\omega}_{s},\tau^{\omega})_{t=0}^{\infty}\}_{\omega\leq\omega_R}} \mathbb{E}[\sum_{t=0}^{\tau^{\omega}} \delta^t \big(d_t^{\omega}(k_t)q_t^{\omega}(k_t)(k-\alpha_0^{\omega}\frac{1-F(k_t)}{f(k_t)})\big)|\omega\leq\omega_R] \tag{20}$$
$$\text{subject to } \mathbb{E}[d_t^{\omega}(k_t)q_t^{\omega}(k_t)] \text{ increasing in } k\ \forall t$$

Analysis of the centralized problem for each type is identical to that in our baseline model. The relaxed centralized problem is stationary (for each ω) and so, for each ω, it is either optimal to take the outside option immediately or never take it; because the continuation value is weakly increasing in k, it is never optimal to take the outside option once ω has been employed (namely, it is only potentially optimal for ω to take the outside option whenever unemployed). Moreover, we can select transfers such that IR_t, IC_t hold even if firms can observe ω: defining the expected discounted effort of k employing type ω to be $q^{\omega}(k)$, transfers conditional on type ω are $q^{\omega}(k)k - \int_{\xi^{\omega}(k')}^{k} q^{\omega}(k'')dk''$ whenever matching with k from k' (where $\xi^{\omega}(k')$ is the lowest type k that ω will leave k' to join). Suppose it is weakly optimal for type $\omega \leq \omega_R$ to take their outside option when unemployed. Because PK_0^{ω} is satisfied, then, because the mechanism is stationary while unemployed, it will still be satisfied under the mechanism in which ω never takes his outside option. Moreover, the mechanism in which ω never takes their outside option weakly increases all firms' expected utility as well as A's (since his continuation value of remaining in the mechanism is at least $\omega_R \geq \omega$), so switching to the mechanism that never takes the outside option weakly increases R's utility. Therefore, it is without loss to take the solution to the problem in which no $\omega \leq \omega_R$ ever take their outside option.

We can then solve the mechanism type-by-type with a multiplier λ_0^{ω} corresponding to each PK_0^{ω} (in place of the multiplier λ_0 on A's IR constraint in the baseline model). The optimal mechanism for each type ω has the same structure as in our baseline model. We next argue that the mechanism is the same for each type (which need not be the case if $\lambda_0^{\omega} \neq \lambda_0^{\omega'}$ for some $\omega, \omega' \leq \omega_R$). As shown in the baseline model, the only place the value of λ_0 plays a role is the cutoff type for matching with firms (namely, what is the lowest type k firm to match with ω). Conditional on the constraint binding, the cutoff (and structure of the centralized payments) conditional on ω is identical. If PK_0^{ω} is slack for some ω, then R's expected utility from that type must be higher than the mechanism under the binding constraint. But then R could use this non-binding mechanism for all $\omega' \leq \omega_R$ (this still satisfies all constraints as it delivers a utility strictly greater than ω_R by PK_0^ω slack) and be weakly better off. Thus, it is without loss to assume that all $\omega \leq \omega_R$ receive the same mechanism. From this, we can apply the same arguments as in our baseline model to construct a regulatory policy $\mathcal{R}'$ and equilibrium σ' that delivers a weakly higher utility than $\mathcal{R}$ and σ. This argument shows that, by maximizing over the cutoff type ω_R in this relaxed centralized mechanism, we can find an upper-bound on R's expected utility in any regulatory policy. ■

D.2 Human Capital Investment

D.2.1 General Human Capital

We assume that agent's level of general human capital is publicly observable and contractible. We done the general human capital of the agent in period t by $y_t \in \mathbb{R}_+$ and assume the match productivity is $r(k, y)$, which is increasing in k, y. Human capital can be increased by the firm at a cost at the beginning of the relationship: we assume there is a cost C such that, if the human capital of the agent is y', the cost of increasing it to $y > y'$ is $C(y, y')$. We impose the following assumption on r, F analogous to Assumption 1:

Assumption 2. *For any $\alpha \in [0, 1]$, $r(k, y) - \alpha \frac{\partial r(k,y)}{\partial k} \frac{1-F(k)}{f(k)}$ is strictly increasing in k and y.*

This holds, for example, the case of multiplicative value of human capital (i.e., $r(k, y) = ky$) and Assumption 1 holds. Because human capital level y is contractible, contracts can specify how much investment in human capital firms must make and can condition payments and termination decisions upon the incoming level of human capital. We then modify the definition of an SCEC contract so that the choice of terms at the start of a relationship includes a level of investment in general human capital, y', by the firm.

Our main result then generalizes to show that there exists a schedule of permitted terms $S(y)$ indexed by y, such that if A currently has human capital level y, an optimal regulatory policy allows firms to offer SCECs, and to set terms from $S(y)$ within the contract.

Proposition 4. *Under Assumption 2, for each y, there exists a minimum wage $\underline{w}_y^*$, functions z_y^*, y_y^* and a schedule $S(y) := \{(w, z_y^*(w), y^*(w))\}_{w \geq \underline{w}_y^*}$, such that $\mathcal{R} =$*

$\{x \text{ an } SCEC | M^F = S(y)\}$ *is an optimal regulatory policy.*

Including y as a state variable, the proof is analogous to the proof of the main result.

D.2.2 Firm-Specific Human Capital

We incorporate this into the model by allowing firms to choose an investment level $\iota \in \mathbb{R}_+$ at the beginning of the relationship, incurring a one-time cost $c^F(\iota)$, resulting in match productivity of $v(k, \iota)$. We assume c^F is convex and increasing, with $c^F(0) = 0$, and that $v(k, \iota)$ is increasing and differentiable in both arguments, non-negative and supermodular.[62]

Define $\zeta^*(k, q) = \max_{\iota \in \mathbb{R}_+} v(k, \iota)q - c^F(\iota)$. By Lemma 1 of Gershkov et al. [2021], $\zeta^*(k, q)$ is supermodular and increasing in k.[63] We next state an assumption analogous to Assumption 1.

Assumption 3. *For any* $\alpha \in [0, 1]$, $\zeta^*(k, q) - \alpha \frac{\partial \zeta^*(k,q)}{\partial k} \frac{1-F(k)}{f(k)}$ *is strictly increasing in* k*, convex in* q *and strictly supermodular in* q, k.

Gershkov et al. [2021] provide a number of sufficient conditions for Assumption 3 for v, c^F that satisfy this when $\alpha = 1$ (and can easily be adapted for the case when $\alpha < 1$), including cases when investments are

(a) additively separable from types (i.e., $v(k, \iota) = k + \iota$) and Assumption 1 holds,

(b) multiplicative (i.e., $v(k, \iota) = \iota$) and $c^F(\iota) = a\frac{\iota^\ell}{\ell}$ for some $\ell > 1, a > 0$ and $\frac{1-F(\theta)}{f(\theta)}$ is decreasing,

(c) the firm must pay a fixed cost (i.e., $c^F(\iota) = \nu \mathbb{1}(\iota > 0)$ for some $\nu > 0$ and $v(k, \iota) = k\mathbb{1}(\iota > 0)$) and Assumption 1 holds.

Under this assumption, the qualitative structure of optimal regulations remains unchanged when adding human capital investment.

Proposition 5. *Under Assumption 3, there exists a schedule* S *such that* $\mathcal{R}^* = \{x \text{ an } SCEC | M^F = S\}$ *is an optimal regulatory policy.*

[62] This captures many natural setups, such as multiplicative investments (i.e., $v(k, \iota) = k\iota$), additive investments (i.e., $v(k, \iota) = k + \iota$) and fixed costs of hiring the agent (i.e., $v(k, \iota) = k\mathbb{1}(\iota > 0)$ with $c^F(\iota) = \nu\mathbb{1}(\iota > 0)$ for some $\nu > 0$).

[63] Mathematically, the analysis is similar to that in Gershkov et al. [2021] who study the design of optimal auctions when participants make costly investments that impact their valuation for the good being auctioned.

Proof. The basic structure of the proof matches that in our baseline model. We first derive the optimal centralized mechanism. Redefine $g(k,q) = \zeta^*(k,q) - \alpha \frac{\partial \zeta^*(k,q)}{\partial q} \frac{1-F(k)}{f(k)}$ (where α again depends on the value of λ and the multiplier on A's individual rationality constraint). Assumption 3 implies that $g(k,q)$ is supermodular and increasing in k for any q. By the arguments in Proposition 1 of Gershkov et al. [2021], expected net transfers are given by $T_t(k) = \mathbb{E}[d_t(k)\zeta^*(k, q_t(k)) - \int_0^k d_t(k') \frac{\partial \zeta^*(k', q_t(k'))}{\partial k} dk' - \underline{U}_t^F]$ and R's objective can be written as

$$\sup_{\{(d_t,a_t,q_t)\}_{t=0}^{\infty}} \mathbb{E}\big[d_t \sum_{t=0}^{\infty} \delta^t g(k_t, q_t)\big] \tag{21}$$

subject to $\{d_t, a_t, q_t\}_{t=0}^{\infty}$ feasible, $\mathbb{E}[d_t q_t | m_t = k]$ increasing in k.

By the same arguments as in the baseline model, the identical results as in Lemma 2 apply.[64] We can then define w^*, z^* analogously to in our baseline model, where (for $k > \xi(0)$) $T(k) = \zeta^*(k, q^*(k)) - \int_{\xi(0)}^k \frac{\partial \zeta^*(k', q_t(k'))}{\partial k} dk'$.

We now argue that $w^*(k) \geq 0$ for all k. Note that $T(\xi(0)) = \zeta^*(\xi(0), q^*(\xi(0))) \geq 0$ and $\frac{dT(k)}{dk} = \frac{\partial \zeta^*(k, q_t(k))}{dq} \geq 0$; thus, $T(k) \geq 0$ for all $k \geq \underline{k}$. By the same arguments as in Lemma 6, we have that $w^*(k) \geq 0$ for all $k \geq \xi(0)$. This implies $U(k) \geq 0$ for all $k \geq \xi(0)$ and, by (12), that $-z^*(k) \leq \frac{\delta}{1-\delta}(w^*(k) - c(e^*(k))) = U(k)$.

We then specify an identical equilibrium (as in our baseline model) in the decentralized problem except for after the off-path action in which A exits an incumbent contract and reenters unemployment (i.e., turns down the new contract after exiting). We specify in equilibrium that all firms, when arriving after A has quit a contract and become unemployed, offer a bonus $\underline{b}$ sufficiently low that A's value of accepting the contract is 0 given that A believes the set of firms to offer said bonus to be those for which $\underline{b}$ (followed by reporting their type to be $\min\{\underline{k}, k\}$) is individually rational. Type $k < \underline{k}$ receives an expected utility under this strategy of $-\underline{b} + (ke^*(\underline{k}) - w^*(\underline{k}))\gamma(\underline{k}) + (1 - \gamma(\underline{k})\frac{1-\delta}{\delta})z^*(\underline{k})$. The cutoff type k_c that offers such contract to A will then be pinned down by $\underline{b} = (k_c e^*(\underline{k}) - w^*(\underline{k}))\gamma(\underline{k}) + (1 - \gamma(\underline{k})\frac{1-\delta}{\delta})z^*(\underline{k})$. A receives an expected utility of $\underline{b} + \mathbb{E}[U(\min\{k, \underline{k}\}) | k \geq k_c]$, so $\underline{b}$ is defined by

[64] Note that the arguments for Lemma 2 only relied only that $g(k,q)$, properties which also hold in our redefined g.

$\underline{b} = -\mathbb{E}[U(\min\{k, \underline{k}\})|k \geq k_c]$. Therefore, k_c must satisfy

$$k_c e^*(\underline{k}) - w^*(\underline{k}))\gamma(\underline{k}) + (1 - \gamma(\underline{k})\frac{1-\delta}{\delta})z^*(\underline{k}) = \mathbb{E}[U(\min\{k, \underline{k}\})|k \geq k_c].$$

If no such k_c exists, then set $k_c = 0$. In this off-path history, A assigns a belief to k equal to the prior belief conditional on $k \geq k_c$ and is called to accept any bonus $b > \underline{b}$, after which we return to equilibrium play as if the deviation had not occurred.

Verifying this is an equilibrium follows from the same arguments as in the proof of Theorem 2 except for the history in which A has reentered unemployment. After such a history, no firm has any incentive to deviate (as they would never prefer to set a higher bonus) and A has no incentive to exit a contract with type k and reenter unemployment as his payoff from doing so $-z^*(k)$ while his payoff to remaining in his current contract is $U(k)$. ■

D.3 Firm Break-up Costs

Assume that firms incur a cost of $c_b(k)$ when the agent exits the relationship. The expected utility of a productivity k firm expecting to employ A for q (discounted) expected length of time is given by $m(k,q) \equiv kq - c_b(k)(1 - q\frac{1-\delta}{\delta})$. The next assumption plays an analogous role to that of Assumption 1.

Assumption 4. *For any $\alpha \in [0,1]$, $m(k,q) - \alpha\frac{\partial m(k,q)}{dk}\frac{1-F(k)}{f(k)}$ is strictly increasing in k for any q such that $m(k,q) - \alpha\frac{\partial m(k,q)}{dk}\frac{1-F(k)}{f(k)} > 0$ and supermodular in q, k.*

A sufficient condition for Assumption 4 is that $c_b(k)$ is not increasing too fast in k[65] and Assumption 1 holds. With this assumption, we can again show that the qualitative structure of the optimal regulatory policy is the same as in our baseline model.

Proposition 6. *Under Assumption 4, there exists a schedule S such that $\mathcal{R}^* = \{x \text{ an } SCEC | M^F = S\}$ is an optimal regulatory policy.*

Proof. Consider the centralized mechanism. By standard arguments, we can characterize the transfers for a type k arriving in period t as $T_t(k) = \mathbb{E}[d_t(k)(m(k, q_t(k)) - \int_0^k d_t(k)\frac{\partial m(k', q_t(k'))}{\partial k}dk' - \underline{U}_t^F]$ where $\underline{U}_t^F$ is the expected utility of $k' = 0$ in period t. Let

[65] Having costs increasingly rapidly in k could lead to a higher productivity type being less efficient from a surplus maximizing perspective. Keeping the increase in costs low ensures that higher productivity types are more efficient.

$g(k,q) = m(k,q) - \alpha \frac{\partial m(k, q_t(k))}{\partial k} \frac{1-F(k)}{f(k)}$ (where α is as in the baseline model). Plugging the transfers into R's objective and simplifying, we get

$$\sup_{\{(d_t, a_t, q_t)\}_{t=0}^{\infty}} \mathbb{E}\big[d_t \sum_{t=0}^{\infty} \delta^t g(k_t, q_t)\big] \tag{22}$$

$$\text{subject to } \{d_t, a_t, q_t\}_{t=0}^{\infty} \text{ feasible, } \mathbb{E}[d_t q_t | m_t = k] \text{ increasing in } k.$$

The rest of the proof follows by the same arguments as in the proof of Proposition 6 once we note that $g(k,q)$ possesses the same properties in both. ■