Product Mix and Firm Productivity Responses to Trade Competition*

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July 2015

PRELIMINARY AND INCOMPLETE DRAFT

*This research has received funding from the European Research Council (ERC) under the Grant Agreement No. 313522

1 Introduction

We document how demand shocks in export markets lead French multi-product exporters to reallocate the product mix sold in those destinations. We develop a theoretical model of multi-product firms and derive the specific demand and cost conditions needed to generate all those empirical predictions. Our theoretical model highlights how the increased competition from demand shocks in export markets – and the induced product mix reallocations – induce productivity changes within the firm. We then empirically test for this connection between the demand shocks and the productivity of multi-product firms exporting to those destinations. We find that the effect of those demand shocks on productivity are substantial – and explain an important share of aggregate productivity fluctuations for French manufacturing.

Recent studies using detailed micro-level datasets on firms, plants, and the products they produce have documented vast differences in all measurable performance metrics across those different units. Those studies have also documented that these performance differences are systematically related to participation in international markets (see, e.g., Mayer and Ottaviano (2008) for Europe, and Bernard, Jensen, Redding and Schott (2012) for the U.S.): Exporting firms and plants are bigger, more productive, more profitable, and less likely to exit than non-exporters. And better performing firms and plants export a larger number of products to a larger number of destinations. Exporters are larger in terms of employment, output, revenue and profit. Similar patterns also emerge across the set of products sold by multi-product firms. There is a stable performance ranking for firms based on the products' performance in any given market, or in worldwide sales. Thus, better performing products in one market are most likely to be the better performing products in any other market (including the global export market). This also applies to the products' selection into a destination, so better performing products are also sold in a larger set of destinations.

Given this heterogeneity, trade shocks induce many different reallocations across firms and products. Some of these reallocations are driven by 'selection effects' that determine which products are sold where (across domestic and export markets), along with firm entry/exit decisions (into/out of any given export market, or overall entry/exit of the firm). Other reallocations are driven by 'skewness effects' whereby – conditional on selection (a given set of products sold in a given market) – trade affects the relative market shares of those products. Both types of reallocations generate (endogenous) productivity changes that are independent of 'technology' (the production function at the product-level). This creates an additional channel for the aggregate gains from trade.

Unfortunately, measuring the direct impact of trade on those reallocations across firms is a very hard task. On one hand, shocks that affect trade are also likely to affect the distribution of market shares across firms. On the other hand, changes in market shares across firms likely reflect many technological factors (not related to reallocations). Looking at reallocations across products within firms obviates many of these problems. Recent theoretical models of multi-product firms highlight how trade induces a similar pattern of reallocations within firms as it does across firms. And measuring reallocations within multi-product firms has several advantages: Trade shocks that are exogenous to individual firms can be identified much more easily than at a higher level of aggregation; Controls for any technology changes at the firm-level are also possible; and reallocations can be measured for the same set of narrowly defined products sold by same firm across destinations or over time. In addition, impediments to factor reallocations are likely to be substantially higher across firms than across product lines within firms. Moreover, multi-product firms dominate world production and trade flows. Hence, reallocations within multi-product firms have the potential to generate large changes in aggregate productivity. Empirically, we find very strong evidence for the effects of trade shocks on those reallocations, and ultimately on the productivity of multi-product firms. The overall impact on aggregate French manufacturing productivity is substantial.

The rest of the paper is organized as follows. Section 2 presents a selective survey of existing empirical works that look at trade-related reallocations from the quantity or the price viewpoints. Section 3 introduces our dataset on French exporters and provides novel evidence on such reallocations with special emphasis on skewness effects. It shows that positive demand shocks in any given destination market induce French exporters to skew their product level export sales to that destination towards their best performing products. These demand shocks also lead to strong positive responses in both the intensive and extensive margins of export sales to that destination. Section 4 introduces a flexible theoretical framework with multi-product firms to rationalize the effects we find in the data. It highlights that the properties of the demand system in terms of the elasticity of demand and marginal revenue are crucial to generate predictions that are consistent with the observed effects. In particular, these empirically relevant properties rule out the CES case. They also imply that within-firm reallocations across products following positive demand shocks should foster firm productivity through skewness effects. Sections 5 and 6.1 bring this prediction to the data by documenting sizeable impacts of demand shocks on the market shares of a firm's products, and thereby on its productivity. These impacts are sizeable not only at the firm level but also on aggregate industry productivity.

2 Previous Evidence on Trade-Induced Reallocations

In a previous paper, Mayer, Melitz and Ottaviano (2014), we investigated, both theoretically and empirically, the mechanics of these reallocations within multi-product firms. We used a comprehensive firm-level data on annual shipments by all French exporters to all countries in the world (not including the French domestic market) for a set of more than 10,000 goods. Firm-level exports are collected by French customs and include export sales for each 8-digit (combined nomenclature or NC8) product by destination country. Our focus then was on the cross-section of firm-product exports across destinations (for a single year, 2003). We presented evidence that French multi-product firms indeed exhibit a stable ranking of products in terms of their shares of export sales across export destinations with 'core' products being sold in a larger number of destinations (and commanding larger market shares across destinations). We used the term 'skewness' to refer to the concentration of these export market shares in any destination and showed that this skewness consistently varied with destination characteristics such as GDP and geography: French firms sold relatively more of their best performing products in bigger, more centrally-located destinations (where competition from other exporters and domestic producers is tougher).

Other research has also documented similar patterns of product reallocations (within multi-product firms) over time following trade liberalization. For the case of CUSFTA/NAFYA, Baldwin and Gu (2009), Bernard, Redding and Schott (2011), and Iacovone and Javorcik (2008) all report that Canadian, U.S., and Mexican multi-product firms reduced the number of products they produce during these trade-liberalization episodes. Baldwin and Gu (2009) and Bernard, Redding and Schott (2011) further report that CUSFTA induced a significant increase in the skewness of production across products. Iacovone and Javorcik (2008) separately measure the skewness of Mexican firms' export sales to the US. They report an increase in this skewness following NAFTA: They show that Mexican firms expanded their exports of their better performing products (higher market shares) significantly more than those for their worse performing exported products during the period of trade expansion from $1994 - 2003$.

As prices are rarely observed, there is little direct evidence on how markups, prices and costs are related across products supplied with different productivity, and how they respond to trade liberalization.[1] A notable exception is the recent paper by DeLoecker, Goldberg, Pavcnik and Khandelwal (2012) who exploit unique information on the prices and quantities of Indian firms'

[1]Prices are typically backed-out as unit values based on reported quantity information, which is extremely noisy.

products over India's trade liberalization period from $1989-2003$. They also document that better performing firms (higher sales and productivity) produce more products. They then focus on markups. Across firms, they document that those better performing firms set higher markups. They also document a similar patter across the products produced by a given multi-product firm, which sets relatively higher markups on their better performing products (lower marginal cost and higher market shares). In addition, they show strong evidence for endogenous markup adjustments via imperfect pass-through from products' marginal costs to their prices: Only a portion of marginal cost decreases are passed on to consumers in the form of lower prices, while the remaining portion goes to higher markups. This is consistent with recent firm-level evidence on exchange rate pass-through. Berman, Martin and Mayer (2012) analyze the heterogeneous reaction of exporters to real exchange rate changes using a rich French firm-level data set with destination specific export values and volumes on the period $1995-2005$. They find that on average firms react to depreciation by increasing their markup. They also find that high-performance firms increases their markup significantly more – implying that the pass-through rate is significantly lower for better performing firms.

Berman, Martin and Mayer (2012) also find very strong evidence that this pass-through rate (the elasticity of price with respect to the exchange rate) is heterogeneous across firms: it sharply decreases with firm performance. (Better performing firms adjust their markups substantially more than worse performing firms in response to the same exchange rate shock.) Li, Ma and Xu (2015) confirm this result of decreasing pass-through (with firm performance) for Chinese exporters. Chatterjee, Dix-Carneiro, and Vichyanond (2013) also confirm this same result for Brazilian exporters. In addition, they find that this property of decreasing pass-through also holds within multi-product Brazilian firms across their set of exported products. That is, they find substantially lower pass-through rates for a firm's better performing products (with relatively higher market shares).

3 Reallocations Over Time

We now document how changes *within* a destination market over time induce a similar pattern of reallocations as the ones we previously described. More specifically, we show that demand shocks in any given destination market induce firms to skew their product level export sales to that destination towards their best performing products. In terms of first moments, we show that these demand shocks also lead to strong positive responses in both the intensive and extensive margins of export sales to that destination.

3.1 Data

We use the same trade data as Mayer, Melitz and Ottaviano (2014), the only difference being multiple years ($1995 - 2005$) instead of a single year (2003). Besides what we already discussed in Section 2, the reporting criteria for all firms operating in the French metropolitan territory are as follows. For within-EU exports, the firm's annual trade value exceeds 100,000 Euros;[2] and for exports outside the EU, the exported value to a destination exceeds 1,000 Euros or a weight of a ton. Despite these limitations, the database is nearly comprehensive. For instance, in 2005, 103,220 firms report exports across 234 destination countries (or territories) for 9873 products. This represents data on over 2.2 million shipments.

We restrict our analysis to export data in manufacturing industries, mostly eliminating firms in the service and wholesale/distribution sector to ensure that firms take part in the production of the goods they export.[3] This leaves us with data on over a million shipments by firms in the whole range of manufacturing sectors.[4]

Matched balance-sheet data provide us with information on variables that are needed to assess firm productivity such as turnover, value added, employment, investment, raw material use and capital. However, we can only measure product reallocations in terms of sales in export markets as the breakdown of sales across products for the domestic market is not available to us. We will have to take this into account in designing our estimation strategy. The balance-sheet data we have access to comes in two sources where the official identification number of the firm can be matched with customs information. The first source is the EAE, produced by the national statistical institute, and exhaustive for manufacturing firms with size exceeding 20 employees. The second is BRN, which comes from tax authorities, and involves a larger coverage of firms, since it is based on a legal tax regime with a sales threshold which is more encompassing than the 20 employees one. Our approach is to give EAE the priority in the relevant balance-sheet data, because it seems to keep a more consistent track of the main activity of the firm.[5] Table 1 provides a certain number

[2] If that threshold is not met, firms can choose to report under a simplified scheme without supplying export destinations. However, in practice, many firms under that threshold report the detailed export destination information. During our period, The threshold went from 38050 Euros (250000 French Francs) to 100000 Euros in 2001. We ran a series of robustness checks over the first period, 1996-2001, over which there are also very little changes in the product classification (Pierce and Schott, 2012). Results are robust to that reduction in sample.

[3] Some large distributors such as Carrefour account for a disproportionate number of annual shipments.

[4] In a robustness check, we also drop observations for firms that the French national statistical institute reports as having an affiliate abroad. This avoids the issue that multinational firms may substitute exports of some of their best performing products with affiliate production in the destination country, thus reducing noise in the product export skewness. Results are quantitatively very similar in all regressions.

[5] The correlation between numbers reported in EAE and in BRN is extremely high: regressions of BRN and EAE values of the relevant variables (value added, employment, exports, capital stock...) in our sample of firms with a

of statistics relevant to the match between customs and balance sheet data. The overall match is not perfect but covers between 88 and 95 percent of the total value of French exports. The match with firms declaring manufacturing as their main activity is still very good although there is a clear trend of declining quality of match, particularly after 2000. This is also visible in the aggregate growth rate of exports in our sample (column 5) that overall provide a quite good match of the overall exports growth rate in column (4), but deteriorating over time. Our investigations suggest that the increasing propensity of large French manufacturers to declare their main activity as retail or some other service activity might provide part of that explanation.[6] Overall our matched dataset is very comparable to recent papers using the same primary sources as in by Eaton et al. (2011), Berman et al. (2012) or di Giovanni et al. (2014) for instance.

Table 1: Descriptive statistics of our sample

Year	Exports					VA	emp.	VA/emp.	
	value	share		growth rate					
	bn euros	matched	mfg	full	mfg	bn euros	mn	000s euros	growth
1995	211.3	94.9	74.0			177.1	2.903	61.0	
1996	219.6	93.5	73.2	3.9	2.8	178.6	2.918	61.2	0.3
1997	252.7	92.8	72.6	15.1	14.2	188.0	2.899	64.8	6.0
1998	267.1	92.0	72.1	5.7	4.9	193.3	2.914	66.3	2.3
1999	277.5	91.1	71.7	3.9	3.3	198.9	2.870	69.3	4.5
2000	319.4	90.8	71.9	15.1	15.4	209.5	2.924	71.6	3.4
2001	324.6	89.9	69.0	1.6	-2.3	199.4	2.932	68.0	-5.1
2002	321.7	90.4	68.4	-0.9	-1.8	198.0	2.865	69.1	1.7
2003	314.3	90.4	65.3	-2.3	-6.7	187.3	2.633	71.1	2.9
2004	335.0	88.4	64.6	6.6	5.4	193.2	2.577	75.0	5.4
2005	350.8	88.0	62.9	4.7	1.9	194.9	2.505	77.8	3.8

Column (1): total value of exports from the full customs data. Column (2): share matched with balance-sheet data from BRN/EAE. Column (3): share matched and with a manufacturing main activity. Columns (4) and (5): growth rates of the full sample and or our matched mfg. sample respectively. Column (6) to (8) give aggregate figures for manufacturing value added employment and the ratio of the two. Column (7) gives the growth rate of value added over employment in the whole of French manufacturing.

3.2 Measuring Trade Shocks

Consider a firm i who exports a number of products s in industry I to destination d in year t. We measure industries (I) at the 3-digit ISIC level (35 different classifications across French

manufacturing main activity yields coefficients extermely close to 1, with fit between .83 and .99.

[6]The robustness of our results when restricting the sample to early years makes us confident that the quality erosion of the data does not endanger our empirics.

manufacturing). We consider several measures of demand shocks that affect this export flow. At the most aggregate level we use the variation in GDP in d, $\log GDP_{d,t}$. At the industry level I, we use total imports into d *excluding* French exports, $\log M^I_{d,t}$. We can also use our detailed product-level shipment data to construct a firm i-specific demand shock:

$$\text{shock}^I_{i,d,t} \equiv \overline{\log M^s_{d,t}} \quad \text{for all products } s \in I \text{ exported by firm } i \text{ to } d \text{ in year } t_0, \tag{1}$$

where $M^s_{d,t}$ represents total imports into d (again, excluding French exports) for product s. For world trade, the finest level of product level of aggregation is the HS-6 level (from UN-COMTRADE and CEPII-BACI), which is more aggregated than our NC8 classification for French exports (roughly 5,300 HS products per year versus 10,000 NC8 products per year). The construction of the last trade shock is very similar to the one for the industry level imports $\log M^I_{d,t}$, except that we only use imports into d for the precise product categories that firm i exports to d.[7] In order to ensure that this demand shock is exogenous to the firm, we use the set of products exported by the firm in its first export year in our sample (1995, or later if the firm starts exporting later on in our sample), and then exclude this year from our subsequent analysis. Note that we use an un-weighted average so that the shocks for all exported products s (within an industry I) are represented proportionately.

For all of these demand shocks $X_t = GDP_{d,t}, M^I_{d,t}, M^s_{d,t}$, we compute the first difference as the Davis-Haltiwanger growth rate: $\tilde{\Delta} X_t \equiv (X_t - X_{t-1}) / (.5X_t + .5X_{t-1})$. This measure of the first difference preserves observations when the shock switches from 0 to a positive number, and has a maximum growth rate of -2 or 2. This is mostly relevant for our measure of the firm-specific trade shock, where the product-level imports into d, $M^s_{d,t}$ can often switch between 0 and positive values. Whenever $X_{t-1}, X_t > 0$, $\tilde{\Delta} X_t$ is monotonic in $\Delta \log X_t$ and approximately linear for typical growth rates ($|\Delta \log X_t| < 2$).[8] We thus obtain our three measures of trade shocks in first differences: $\tilde{\Delta} GDP_{d,t}, \tilde{\Delta} M^I_{d,t}, \overline{\tilde{\Delta} M^s_{d,t}}$. For the firm i-specific shock $\overline{\tilde{\Delta} M^s_{d,t}}$, we take the un-weighted average of the growth rates for all products exported by the firm in $t-1$.

[7]There is a one-to-many matching between the NC8 and HS6 product classifications, so every NC8 product is assigned a unique HS6 classification. We use the same $M^s_{d,t}$ data for any NC8 product s within the same HS6 classification.

[8]Switching to first difference growth rates measured as $\Delta \log X_t$ (and dropping products with zero trade in the trade shock average) does not materially affect any of our results.

3.3 The Impact of Demand Shocks on Trade Margins and Skewness

Before focusing on the effects of the demand shocks on the skewness of export sales, we first show how the demand shocks affect firm export sales at the intensive and extensive margins (the first moments of the distribution of product export sales). Table 2 reports how our three demand shocks (in first differences) affect changes in firm exports to destination d in ISIC I (so each observation represents a firm-destination-ISIC combination). We decompose the firm's export response to each shock into an intensive margin (average exports per product) and an extensive margin (number of exported products). We clearly see how all three demand shocks induce very strong (and highly significant) positive responses for both margins. This confirms that our demand shocks capture important changes in the local demand faced by French exporters.[9]

Table 2: Demand shocks and local exports

Dependent Variable	Δ log Exports per Product			Δ log # Products Exported		
$\tilde{\Delta}$ GDP Shock	0.493^a			0.149^a		
	(0.048)			(0.016)		
$\tilde{\Delta}$ trade shock		0.277^a			0.076^a	
		(0.009)			(0.004)	
$\tilde{\Delta}$ trade shock - ISIC			0.039^a			0.014^a
			(0.005)			(0.002)
Observations	401575	407520	407520	401575	407520	407520

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$. All regressions include year dummies, and standard errors clustered at the level relevant for the variable of interest: destination country for columns (1) and (4), firm-destination for columns (2) and (5) and ISIC-destination for columns (3) and (6).

We now investigate the consequences of those demand shocks for the skewness of export sales (independent of the level of product sales). In Mayer, Melitz and Ottaviano (2014), we focused on those effects in the cross-section across destinations. Here, we examine the response of skewness within a destination over time using our new demand shocks. We rely on the Theil index as our measure of skewness due to its aggregation properties: We will later aggregate the export responses at the destination-ISIC level up to the firm-level – in order to generate predictions for firm-level productivity. Thus, our measure of skewness for the distribution of firm i's exports to destination

[9] Specifications using the log levels of the shocks and firm-destination-ISIC fixed-effects yield similar results. Other specifications including the three covariates show that those demand shocks are different enough to be estimated jointly while each keeping its positive sign and statistical significance.

d in industry I, $x^s_{i,d,t}$, is the Theil index:

$$T^I_{i,d,t} \equiv \sum_{s \in I} \frac{x^s_{i,d,t}}{x^I_{i,d,t}} \log\left(\frac{x^I_{i,d,t}}{x^s_{i,d,t}}\right), \quad x^I_{idt} \equiv \sum_{s \in I} x^s_{i,d,t}. \tag{2}$$

Table 3 reports regressions of this skewness measure on all three demand shocks (jointly) – at the firm-destination-ISIC level. In the first column, we use a specification in (log) levels (FE), and use firm-destination-ISIC fixed effects to isolate the variation over time. In the second column, we return to our specification in first differences (FD). In the third column we add the firm-destination-ISIC fixed effects to this specification in first differences (FD-FE). This controls for any trend growth rate in our demand shocks over time. Across all three specifications, we see that positive demand shocks induce a highly significant increase in the skewness of firm export sales to a destination. The effect of all three shocks are weakened a little bit due to some collinearity.

Table 3: Demand shocks and local skewness

Dependent Variable	$T^I_{i,d,t}$	$\Delta T^I_{i,d,t}$		$\Delta T^{I,\text{const}}_{i,d,t}$	
Specification	FE	FD	FD-FE	FD	FD-FE
log GDP shock	0.075^a				
	(0.016)				
log trade shock	0.048^a				
	(0.005)				
log trade shock - ISIC	0.002^a				
	(0.000)				
$\tilde{\Delta}$ GDP Shock		0.066^a	0.066^a	-0.006	-0.004
		(0.012)	(0.015)	(0.008)	(0.009)
$\tilde{\Delta}$ trade shock		0.037^a	0.033^a	0.012^a	0.012^a
		(0.005)	(0.006)	(0.003)	(0.003)
$\tilde{\Delta}$ trade shock - ISIC		0.006^a	0.004	0.002	0.004^a
		(0.002)	(0.003)	(0.001)	(0.002)
Observations	479387	401575	401575	442800	442800

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$. FE refers to firm-destination-ISIC fixed effects. All regressions include year dummies, and standard errors clustered at the level of the destination country.

Table 4 therefore re-runs regressions for the most demanding—FD-FE—specification. Column (3) reveals that the ISIC-level shock remains significant at the 1% level when entered on its own. Next, we construct a measure of the change in skewness restricted to the subset of products s

Table 4: Demand shocks and local skewness

Dependent Variable	$\Delta T^{I}_{i,d,t}$			$\Delta T^{I,\text{const}}_{i,d,t}$			Δ log ratio core to 2nd		
$\tilde{\Delta}$ GDP Shock	0.090^{a}			0.006			0.157^{b}		
	(0.010)			(0.008)			(0.067)		
$\tilde{\Delta}$ Trade Shock		0.042^{a}			0.012^{a}			0.059^{b}	
		(0.004)			(0.003)			(0.027)	
$\tilde{\Delta}$ Trade Shock - ISIC			0.008^{a}			0.005^{a}			0.008
			(0.002)			(0.002)			(0.013)
Observations	401575	407520	407520	442800	448424	448424	265723	269401	269401

Standard errors in parentheses: $^{c} < 0.1$, $^{b} < 0.05$, $^{a} < 0.01$. FE refers to firm-destination-ISIC fixed effects. All regressions include year dummies, and standard errors clustered at the level of the destination country.

exported in both periods (for the first difference specifications). This alternate skewness measure $\Delta T^{I,\text{const}}_{i,d,t}$ isolates changes that are driven only by the intensive margin of exports. The two first-difference specifications using this alternate skewness measure are reported in the last two columns of Table 3. The effects of our firm-level trade shock is still highly significant (well beyond the 1% level), whereas the effects based on the ISIC-level trade shock are teetering at the 5% significance level. However, those columns show that the GDP shocks tend to change the export sales of 'incumbent' products proportionately. The effect of the GDP shock on skewness thus comes entirely from the extensive margin (new products exported in response to higher GDP levels in a destination).[10] Table 4 replicates the most demanding regression for $\Delta T^{I,\text{const}}_{i,d,t}$ in the three central columns, with the three demand shocks entered separately. The three last columns introduce one additional dependent variable that is neutral to the extensive margin: the change in the (logged) ratio of core to second product of the firm. The effect of the GDP shock recovers statistical significance, as well as the magnitudes found for the simple change in th Theil index from the three first columns.

4 Theoretical Framework

In the previous section we documented the pattern of product reallocations in response to demand shocks in export markets. We now develop a theoretical model of multi-product firms that highlights the specific demand conditions needed to generate this pattern. We show that these demand conditions are consistent with all of the micro-level evidence on firm/product selections, prices,

[10]These newly exported products have substantially smaller market shares than the incumbent products and therefore contribute to an increase in the skewness of export sales.

and markups presented in section 2. In particular, those demand conditions highlight how demand shocks lead to changes in competition for exporters in those markets (which in turn lead to the observed reallocations). We then show how these changes in competition and induced reallocations generate changes in observed firm productivity.

4.1 Closed Economy

To better highlight the role played by the properties of the demand system, we initially start with a closed economy. We will then move to the open economy to discuss the trade shocks and their effects. In both the open and closed economies, we develop both a general equilibrium (with a single differentiated good sector for the whole economy) and a partial equilibrium version focusing on a single sector among many in the economy. In the latter, we also introduce a short-run version where entry is restricted. (General equilibrium is inherently a long-run scenario.) We show how demand shocks induce the same pattern of skewness effects in all of these model versions. In all cases, it is the properties of the demand system that shape the pattern of reallocations.

Multi-Product Production with Additive Separable Utility

Consider a sector in which labor is the only productive factor. We will distinguish between two scenarios. In the 'partial equilibrium' (PE) scenario, we focus on the sector as a small part of the economy. We take the number of consumers L^c as well as their individual expenditures on the sector's output as exogenously given. We also assume that labor supply to the sector is perfectly elastic at an exogenously given wage (which also determines the exogenous expenditure). We choose units so that both this wage and the exogenously given individual expenditures on the sector's output equal 1. This involves a normalization for the measure of consumers L^c and the choice of labor as numeraire. In the 'general equilibrium' (GE) scenario, we consider the sector as a dominant part of the economy, indeed as the only sector in the economy. Each consumers thus works in the sector and aggregate expenditures are fully absorbed by the sector. In particular, each consumer inelastically supplies one unit of labor so that the number of consumers L^c and the number of workers L^w are the same and coincide with labor endowment L ($L^c = L^w = L$), which is also chosen as *numeraire*. We think of a pure 'demand shock' as an increases in L^c (dL^c) in the PE scenario whereas a change in L (dL) in the GE scenario would capture a 'market size shock' compounding both a demand shock (dL^c) and a supply shock (dL^w) of equal amplitude

$(dL^c = dL^w = dL)$.[11]

In both scenarios each consumer's utility is assumed to be additively separable over a continuum of imperfectly substitutable products indexed by $i \in [0, M]$ where M is the measure of products available. The typical consumer then solves the following utility maximization problem:

$$\max_{x_i \geq 0} \int_0^M u(x_i)di \text{ s.t. } \int_0^M p_i x_i di = 1,$$

where $u(x_i)$ is the sub-utility associated with the consumption of x_i units of product i and expenditure equals 1 (which is normalized expenditure in the PE scenario and unit wage in GE one). We assume that this sub-utlity exhibits the following properties:

(A1) $u(x_i) \geq 0$ with equality for $x_i = 0$; $u'(x_i) > 0$ and $u''(x_i) < 0$ for $x_i \geq 0$.

The first order conditions for the consumer's problem determine the *inverse* demand function:

$$p_i = \frac{u'(x_i)}{\lambda}, \text{ with } \lambda = \int_0^M u'(x_i)x_i di, \tag{3}$$

where $\lambda > 0$ is the marginal utility of income. Larger λ shifts inverse demand downwards, reducing the price the consumer is willing to pay for any level of consumption. Concavity of $u(x_i)$ ensures that x_i satisfying (3) also meets the second order condition for the consumer's problem. Note that λ is an increasing function of M and x_i.

Products are supplied by firms that may be single- or multi-product. Market structure is monopolistically competitive as in Mayer, Melitz and Ottaviano (2014) in that each product is supplied by only one firm and each firm supplies a countable number of the continuum of products. Technology exhibits increasing returns to scale associated with a fixed production cost, along with a constant marginal cost. The fixed cost f is the same for all products while the marginal cost v (variety level cost) differs across them. For a given firm, products are indexed in increasing order m of marginal cost from a 'core product' indexed by $m = 0$. Firm entry incurs a sunk cost f^e. Only after this cost is incurred, entrants randomly draw their marginal cost levels for their core products from a common continuous differentiable distribution defined over the support $[0, \infty)$, with density $\gamma(c)$ and cumulative density $\Gamma(c)$. We use $M(c)$ to denote the number of products supplied by

[11] Parenti, Ushchev and Thisse (2014) consider a number of consumers $L_c = L$, each inelastically supplying y efficiency units so that labor supply equals $L^w = yL = yL^c$. However, they then study the comparative statics with respect to changes in L and y. Both changes thus affect labor supply, whose effects we want instead to neutralize in the PE scenario.

a firm with core marginal cost c and $v(m,c)$ to denote the marginal cost of its m^{th} product. We assume $v(m,c) = cz(m)$ with $z(0) = 1$ and $z'(m) > 0$.[12]

An entrant supplying product i with marginal cost v solves the profit maximization problem:

$$\max_{q_i \geq 0} \pi(q_i) = p_i q_i - v q_i - f,$$

subject to the market clearing condition for its output $q_i = x_i L^c$ and inverse demand given by (3).[13] The optimal level of output $q_v = x_v L^c$ satisfies the first order condition:

$$u'(x_v) + u''(x_v)x_v = \lambda v, \tag{4}$$

where $r(x_v) = \phi(x_v)/\lambda$ with $\phi(x_v) \equiv u'(x_v) + u''(x_v)x_v$ is the *marginal* revenue associated with a given variety. Markup pricing is revealed by rewriting (4) as

$$p(x_v) = \frac{v}{1 - \varepsilon_p(x_v)}, \tag{5}$$

where

$$\varepsilon_p(x_v) \equiv -\frac{u''(x_v)x_v}{u'(x_v)} \tag{6}$$

is the elasticity of inverse demand.[14] This is positive under (A1) and measures the concavity of $u(x_v)$.[15] In order for the first order condition to hold for positive prices $p(x_v)$, the elasticity of inverse demand must be smaller than one. This is our second assumption:

(A2) $\varepsilon_p(x_v) < 1$.

It is equivalent to imposing positive marginal revenue $r(x_v) > 0$.

The optimal level of output x_v must also satisfy the second order condition for profit maximization:

$$\phi'(x_v) \equiv 2u''(x_v) + u'''(x_v)x_v < 0, \tag{7}$$

[12] The assumption $z'(m) > 0$ will generate the within-firm ranking of products discussed in Section 2. In the limit case when $z'(m)$ is infinite, all firms are single-product.

[13] This problem is faced by any entrant, no matter whether single- or multi-product, as our assumptions rule out cannibalization within and between entrants' product ranges.

[14] As in Zhelobodko, Kokovin, Parenti and Thisse (2012), we use $\varepsilon_p(x_v)$ to denote the elasticity of $p(x_v)$ with respect to x_v. This is the inverse of the price elasticity of demand that would be denoted $\varepsilon_x(p_v)$.

[15] This elasticity should not be confused with the elasticity of utility $u'(x_v)x_v/u(x_v)$, which is an inverse measure of "love of variety". As discussed by Neary and Mrazova (2014), this elasticity is important for welfare analysis. In Zhelobodko et al (2012) $\varepsilon_p(x_v)$ is called "relative love of variety".

which can be restated in terms of the elasticity of marginal revenue $\varepsilon_r(x_v)$ as:

$$\varepsilon_r(x_v) \equiv -\frac{\phi'(x_v)x_v}{\phi(x_v)} > 0. \tag{8}$$

This requires the inverse demand to be not too convex and implies that, for any given λ, a unique solution $x_v(\lambda v)$ exists for (4).[16] This is our third assumption:

(A3) $\varepsilon_r(x_v) > 0.$

(A1)-(A3) must all hold for the consumers' and firms' optimization problems to be well-defined. When satisfied, they imply a unique output and price level for all varieties $x_v > 0$ and $p(x_v) > 0$, and for any given $\lambda > 0$. Throughout, we assume that these assumptions hold without explicitly mentioning them each time.

Several implications of cost heterogeneity for product performance matching some key findings discussed in Section 2 can be derived from the conditions for utility and profit maximization along with these assumptions. In particular, lower cost firms/products are associated with lower price, larger output, larger revenue and larger profit.[17]

Free Entry

In equilibrium consumers maximize utility, firms maximize profits, and their optimal choices in the product and labor markets are mutually compatible. To characterize this equilibrium outcome, it is useful to make the dependence of maximized operating profit π_v and profit-maximizing output x_v on the endogenous marginal utility of income explicit. In particular, we define $\pi_v = \pi^*(v, \lambda)L^c$ and $x_v = x^*(\lambda v)$ with

$$\pi^*(v, \lambda) = \max_x \left[\frac{u'(x)}{\lambda} - v\right] x,$$
$$x^*(\lambda v) = \arg\max_x \left[\frac{u'(x)}{\lambda} - v\right] x.$$

[16] In Mrazova and Neary (2014), (8) is equivalently stated as $\rho(x_v) < 2$ where $\rho(x_v) \equiv -\left[u'''(x_v)x_v\right]/u''(x_v) = 2 - \varepsilon_r(x_v)\left[1 - \varepsilon_p(x_v)\right]/\varepsilon_p(x_v)$ measures the convexity of inverse demand.

[17] See the Appendix A for a proof.

By the implicit differentiation of (4) and the definition of $\varepsilon_r(x_v)$, we obtain

$$\begin{aligned} \frac{\partial x^*(\lambda v)}{\partial v} &= -\frac{1}{\varepsilon_r(x_v)}\frac{x_v}{v} < 0, \\ \frac{\partial x^*(\lambda v)}{\partial \lambda} &= -\frac{1}{\varepsilon_r(x_v)}\frac{x_v}{\lambda} < 0. \end{aligned} \tag{9}$$

By the envelope theorem, maximized profit is decreasing in both its arguments:

$$\begin{aligned} \frac{\partial \pi^*(v,\lambda)}{\partial v} &= -x^*(\lambda v) < 0, \\ \frac{\partial \pi^*(v,\lambda)}{\partial \lambda} &= -\frac{u'(x^*(\lambda v))x^*(\lambda v)}{\lambda^2} < 0. \end{aligned} \tag{10}$$

The fact that maximized profit is decreasing in marginal cost implies that only products with marginal cost v below some cost cutoff $\widehat{v}$ can be profitably produced. At the same time, entrants that do not find it profitable to sell even their core product do not produce. Thus, the product cutoff level $\widehat{v}$ is also the firm cutoff level $\widehat{c}$ for core competency: entrants drawing a core marginal cost $c > \widehat{c}$ exit immediately without producing.

Given $v(0,c) = c$, the indifference condition for the marginal producer is:

$$\pi^*(\widehat{c},\lambda)L^c = f. \tag{11}$$

Since $\pi^*(c,\lambda)$ is decreasing in both c and λ, this cutoff condition has a unique solution $\widehat{c}(\lambda)$ and is such that $\widehat{c}'(\lambda) < 0$. For a given measure of entrants N_e, $\widehat{c}(\lambda)$ determines the fraction of those entrants that eventually produce: $\Gamma(\widehat{c}(\lambda))$.

All prospective entrants are identical ex-ante. Free entry then requires that expected profit equal the sunk entry cost. Post-entry, an entrant with a core cost draw $c \leq \widehat{c}$ earns profit:

$$\Pi^*(c,\lambda) \equiv \sum_{m=0}^{M(c)-1} \left[\pi^*\left(cz(m),\lambda\right) L^c - f\right],$$

where $M(c)$ is the number of products the entrant supplies with marginal cost $cz(m) \leq \widehat{c}$. Hence, upon entry, the expected profit of an entrant is

$$\int_0^{\widehat{c}} \Pi^*(c,\lambda)\gamma(c)dc = \int_0^{\widehat{c}} \left\{ \sum_{\{m|cz(m)\le \widehat{c}\}} \left[\pi^*\left(cz(m),\lambda\right)L^c - f\right] \right\} \gamma(c)dc$$
$$= \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[\pi^*\left(cz(m),\lambda\right)L^c - f\right]\gamma(c)dc \right].$$

The free entry condition can then be re-stated as

$$\int_0^{\widehat{c}} \Pi^*(c,\lambda)\gamma(c)dc = \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[\pi^*\left(cz(m),\lambda\right)L^c - f\right]\gamma(c)dc \right] = f^e. \tag{12}$$

Equations (11) and (12) jointly determine the equilibrium cost cutoff $\widehat{c}^*$ and the marginal utility of income λ^*. As both $\Pi^*(c,\lambda)$ and $\widehat{c}(\lambda)$ decrease in λ, this solution $(\widehat{c}^*, \lambda^*)$ exists and is unique. They have to hold both in the PE and the GE scenarios together with the consumer's budget constraint

$$N_e \left(\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} p^*\left(cz(m),\lambda\right) x^*\left(cz(m),\lambda\right)\gamma(c)dc \right) = 1 \tag{13}$$

where $p^*\left(cz(m),\lambda\right)$ is the equilibrium price statisfying (5). (Recall that expenditure per consumer is normalized to 1 in both the PE and GE scenario.) Evaluated for $\widehat{c} = \widehat{c}^*$, (13) determines the equilibrium number of entrants N_e^* and producers $N^* = \Gamma\left(\widehat{c}^*\right) N_e^*$.

In the GE scenario, (13) can be equivalently replaced by the labor market clearing condition

$$N_e \left\{ f^e + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[cz(m)x^*\left(cz(m),\lambda\right)L^c + f\right]\gamma(c)dc \right] \right\} = L^w \tag{14}$$

with $L^c = L^w = L$, the reason being that with free entry the income of labor (used in both fixed and per-unit production; and entry) absorbs all revenues and the wage equals 1 by choice of *numeràire*.

4.2 Reconciling Facts with Demand Assumptions

Up to now, we have placed very few restrictions on the shape of the residual demand curves that the firms face. In particular, the rates of change of the elasticities of residual demand and marginal revenue (the signs of $\varepsilon_p'(x_v)$ and $\varepsilon_r'(x_v)$) were left unrestricted. The sign of those rates of change determine the curvature of the underlying curve: $\varepsilon_p'(x_v) > 0$ implies that inverse demand is log-concave in log-quantities (and similarly for $\varepsilon_r'(x_v)$ and the curvature of marginal revenue). Without additional restrictions on the sign of those rates of change, our model would make predictions that contradict the reallocation evidence we presented in the previous section as well as the previous evidence from the literature we reviewed in section (2). We now show how restrictions on the sign of those elasticity changes are sufficient for the model to predict all of the previously mentioned empirical patterns. In the appendix, we show that those assumptions are also necessary ones for the empirical evidence. Specifically, we show that $\varepsilon_p'(x_v) \leq 0$ (log-convex demand) would generate reallocations that go in the opposite direction to those we documented in the previous section. This implies that log-concave demand ($\varepsilon_p'(x_v) > 0$) is a necessary condition for those observed reallocations. This is our first restriction to the shape of demand:[18]

$$\textbf{(B1)}\ \varepsilon_p'(x_v) > 0.$$

This property of demand – that it becomes more inelastic with with consumption is also known as Marshall's Second Law of Demand.[19] This property of demand is also documented by DeLoecker et al (2013) for products sold by multi-product firms; and by many firm-level studies documenting a pattern of increasing markups with firm size and performance. However, our reallocation evidence validates this assumption without the need for product level prices – thus providing an additional source of verification for this important property of demand.

The evidence of decreasing pass-through (with output) that we previously discussed (for French, Brazilian, and Chinese firms) allows us to restrict the shape of demand beyond (B1). The pass-through elasticity from cost to price is given by:

$$\theta(x_v) = \frac{d \ln p(x_v)}{d \ln v} = \frac{\varepsilon_p(x_v)}{\varepsilon_r(x_v)}. \tag{15}$$

In the appendix, we show that $\varepsilon_r'(x_v) > 0$ (the marginal revenue curve is log-convex) is a necessary

[18] In the terminology of Neary and Mrazova (2014) $\varepsilon_p'(x_v) > 0$ defines the "sub-convex" case, with "sub-convexity" of an inverse demand function $p(x)$ at an arbitrary point x_c being equivalent to the function being less convex at that point than a CES demand function with the same elasticity. In the terminology of Zhelobodko, Kokovin, Parenti and Thisse (2012), in this case preferences are said to display increasing "relative love of variety" (RLV) as consumers care less about variety when their consumption level is lower. RLV is, thus, increasing if and only if the demand for a variety becomes more elastic when the price of this variety rises. Also the "Adjustable pass-through" (Apt) class of demand functions proposed by Fabinger and Weyl (2012) satisfies (B1). This assumption is, instead, weaker than the assumption by Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) that the demand function of any product is log-concave in log-prices. Log-concavity implies (B1) but not vice versa and, thus, log-concavity is not necessary to associate lower cost with larger markups (see the Appendix B for proofs).

[19] We think Peter Neary for bringing this reference to our attention.

condition for $\theta(x_v)$ to be decreasing. This yields our second restriction on the shape of demand:

$$\textbf{(B1')}\ \varepsilon_r'(x_v) > 0.$$

Since $\varepsilon_r'(x_v) > 0$ implies $\varepsilon_p'(x_v) > 0$ (see appendix), (B1') represents a strict subset of the preferences satisfying (B1). Whereas (B1) imposes a positive relationship between markups and output (and lower marginal cost), (B1') adds a prediction between *changes* in costs and *changes* in markups: bigger/better performing products (with lower marginal cost) adjust their markups more than less performing products (with higher marginal cost) to a given change in marginal cost. This is then directly connected to lower pass-through rates for the better performing products. Figure 1 depicts a log-log graph of the inverse demand and marginal revenue curves satisfying restriction (B1').[20] If only (B1) is satisfied, then the inverse demand will still have the same shape shown in Figure 1, but the log-marginal revenue curve need no longer be globally concave. However, it must still be steeper than the inverse demand everywhere: $\varepsilon_r(x_v) > \varepsilon_p(x_v)$ whenever $\varepsilon_p'(x_v) > 0$.

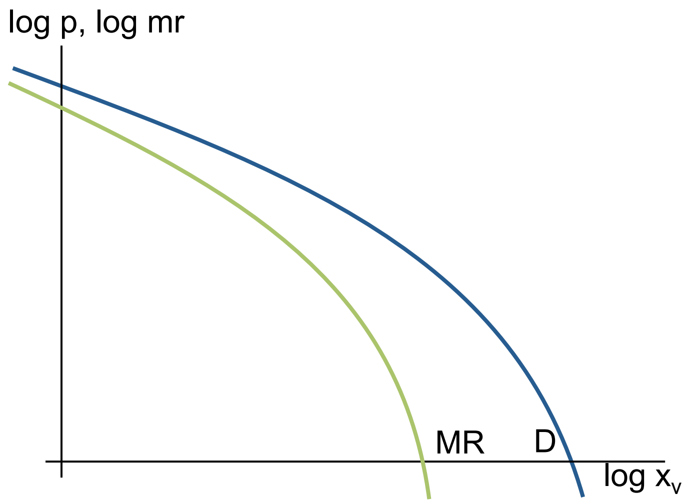


Figure 1: Graphical Representation of Demand Assumptions

[20] Although our results have been derived for an additive separable utility function, they eventually depend on the properties of the associated inverse demand. They thus may hold also for utility or expenditure functions that are not additive separable but still share those properties. In this respect, our results suggest that the taxonomy of demand systems proposed by Mrazova and Neary (2014) in terms of $1/\varepsilon_p(x_v)$ and $\rho(x_v)$ – or equivalently in terms of $\varepsilon_p(x_v)$ and $\varepsilon_r(x_v)$ – could be fruitfully enriched also in terms of the elasticities of $\varepsilon_p(x_v)$ and $\varepsilon_r(x_v)$ to cover additional comparative statics implications that are crucial when products are associated with heterogeneous costs.

4.3 Demand Shock

We now discuss the implications of our model under the two empirically relevant assumptions (B1) and (B1'). Recall that (B1') implies (B1) and therefore represents a strict subset of the preferences satisfying (B1). Throughout, we assume that the firm and consumer problems are well defined so that (A1)-(A3) hold. We focus on the effects of a demand shock (dL^c) on the product range and the product mix. In doing so, we distinguish not only between the PE and the GE scenarios described above but also between a 'long run' scenario with free entry and a 'short run' scenario where entry is restricted (in the PE case).

Long-Run Effects of a Demand Shock The following results hold for $dL^c > 0$:

Lemma 1 *A positive demand shock increases the marginal utility of income both in partial and general equilibrium.*

Proof. See Appendix E.1. ∎

Proposition 2 ***(Extensive margin adjustment)*** *(B1) is necessary and sufficient for a positive demand shock to reduce the cost cutoff and increase (decrease) total profit for low (high) cost products. This holds both in partial and general equilibrium.*

Proof. See Appendix E.2.[21] ∎

Proposition 3 ***(Intensive margin adjustment)*** *(B1') is necessary and sufficient for a positive demand shock to reallocate output from higher to lower cost products. (B1') is also sufficient for a positive demand shock to reallocate revenue from higher to lower cost products. This holds both in partial and general equilibrium.*

Proof. See Appendix E.3. ∎

While these effects of a demand shock are clear cut, since they are accompanied by a changing number of entrants, the impact of the demand shock on the number of producers and products supplied is ambiguous without further assumptions on the distribution of marginal cost $\Gamma(c)$.[22]

In a nutshell, under our assumptions, a positive demand shock induces multi-product incumbents to (weakly) shed some more costly non-core products (and some single-product incumbents to

[21] In the case of non-separable preferences, Parenti, Thisse an Ushchev (2014) characterize general conditions on profits such that larger market size leads to lower cutoff, but point out that general conditions on demand are unavailable due to dependence on the cost distribution.

[22] See Zhelobodko et al (2012).

stop producing altogether). It also induces multi-product incumbents to shift output and revenue towards better-performing products (with lower marginal cost). Since those products already had larger output and revenue before the shock, this leads to an increase in the 'skewness' of output and revenue.

Short-Run Effects in Partial Equilibrium We now consider an alternative short-run situation in which the number of incumbents is fixed at $\overline{N}$ in the PE scenario. In this case (12) no longer holds. The short-run equilibrium is then characterized by *two* conditions. The first is the zero cutoff profit condition (11):

$$\pi^*(\widehat{c}, \lambda)L^c = f.$$

The second is the consumer's budget constraint obtained from (13) after imposing $N_e = \overline{N}$:

$$\overline{N}\left(\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} p\left(x^*\left(cz(m),\lambda\right)\right)x^*\left(cz(m),\lambda\right)\gamma(c)dc\right) = 1. \tag{16}$$

These conditions pin down $\widehat{c}$ and λ for fixed $\overline{N}$. They imply that the results in Lemma 1 and Proposition 3 also hold in the short-run PE scenario. However, (B1) is now a necessary and sufficient condition for a positive demand shock to increase total profits for all firms. Thus, (B1) predicts that a positive demand shock will increase the cost cutoff in the short-run PE scenario.[23] Nonetheless, we show in the next sub-section that in the open economy, all three of our scenarios (GE and PE long-run and PE short-run) predict that the export cutoff decreases in response to a positive demand shock – generating a positive extensive margin response to the demand shock in all three cases.

4.4 Open Economy

With our empirical application in mind, we consider a simplified three-country economy consisting of a Home country (H: France) and a Foreign country (F: RoW) both exporting to a Destination country (D). Analogously to the closed economy, for $l \in \{H, F, D\}$ we use L_l^c to denote country l's number of consumers both in the PE and the GE scenarios and $L_l^w = L_l^c = L_l$ to denote the number of workers in the GE scenario. For $l, h \in \{H, F, D\}$ trade from country l to country h is subject to a variable iceberg friction such that, for any product, $\tau_{lh} > 1$ units have to be shipped for one unit to reach destination. Trade from l to h also faces a fixed export requirement f_{lh}^x incurred

[23]See proof in Appendix F.

in units of labor of the country of origin.

Country D is the focus of the analysis and is assumed to be 'small' from the point of view of both H and F so that changes in destination D-specific variables do not affect equilibrium variables in either H or F (apart from those related to exports to D). Since wages in H and F are fixed, we normalize them to one. Analogously to the closed economy, in the PE scenario we also normalize consumer expenditures in D to one while in the GE scenario we choose labor in D as *numeraire*. The long-run equilibria in the two scenarios are then characterized by country D's zero cutoff profit and free entry conditions with fixed numbers of incumbents as well as fixed domestic cutoffs for countries H and F. The short-run PE equilibrium is, instead, characterized by country D's zero cutoff profit condition with fixed numbers of incumbents for countries D, H and F as well as fixed domestic cutoffs for countries H and F.

To emphasize competition in the destination market D, we focus on a situation in which D does not export (and trade is thus unbalanced); however, the same qualitative results hold when one also allows for exports from D.[24]

Long-Run Effects of a Demand Shock on Exporters

Under the assumption that firms located in D do not export, the long-run GE equilibrium is characterized by the following *four* conditions with $L_l^w = L_l^c = L_l$. The first is the zero cutoff profit condition for domestic sales in country D

$$\pi_{DD}^*(\widehat{c}_{DD}, \lambda_D) L_D^c = f, \tag{17}$$

where λ_D is the marginal utility of income of country D's consumers, $\widehat{c}_{DD}$ is the threshold below which the marginal cost of a firm with marginal cost c located in country D has to fall for the firm to be able to sell in its domestic market, and

$$\pi_{DD}^*(c, \lambda_D) = \max_x \left[\frac{u'(x)}{\lambda_D} - c \right] x \tag{18}$$

is its maximized domestic profit with domestic quantity sold defined as

$$x_{DD}^*(c, \lambda_D) = \arg\max_x \left[\frac{u'(x)}{\lambda_D} - c \right] x. \tag{19}$$

[24]See Appendix G.3 for this extension.

The second condition is the zero cutoff profit condition for export sales from $l \in \{H, F\}$ to country D

$$\pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D)L^c_D = f^x_{lD}, \tag{20}$$

where $\widehat{c}_{lD}$ is the threshold for profitable sales from l to D, and

$$\pi^*_{lD}(\tau_{lD}c, \lambda_D) = \max_x \left[\frac{u'(x)}{\lambda_D} - \tau_{lD}c\right] x \tag{21}$$

is the associated maximized profit. The exported quantity is then

$$x^*_{lD}(\tau_{lD}\lambda_D c) = \max_x \left[\frac{u'(x)}{\lambda_D} - \tau_{lD}c\right] x. \tag{22}$$

The third condition is the the free entry condition for country D

$$\sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DD}/z(m)} \left[\pi^*_{DD}\left(cz(m), \lambda\right) L^c_D - f\right] \gamma(c)dc\right] = f^e. \tag{23}$$

The fourth and last condition is the labor market clearing conditon in country D

$$N^e_D \left\{ f^e + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DD}/z(m)} \left[cz(m)x^*_{DD}\left(cz(m), \lambda\right) L^c_D + f\right] \gamma(c)dc\right]\right\} = L^w_D. \tag{24}$$

As in the closed economy this, can be equivalently restated as the consumer's budget constraint.

Given (18), conditions (17) and (23) determine country D's domestic cost cutoff $\widehat{c}_{DD}$ and its marginal utility of income λ_D. Then, given (19), condition (24) pins down the number of entrants N^e_D and producers $N^p_D = \Gamma\left(\widehat{c}_{DD}\right) N^e_D$. The measure of products sold in D includes also those exported from $l \in \{H, F\}$. These are a fraction $\Gamma\left(\widehat{c}_{lD}\right)$ of the fixed measure $\overline{M}^i_l$ of incumbent products produced in $l \in \{H, F\}$ so that

$$M^x_{lD} = \Gamma\left(\widehat{c}_{lD}\right) \overline{M}^i_l,$$

where the export cutoff $\widehat{c}_{lD}$ is determined by condition (20) given the value of λ_D determined by (17) and (23). It is therefore independent of $\widehat{c}_{DD}$. However, given (10), with $f^X_D > f$ (20) implies $\tau_{lD}\widehat{c}_{lD} < \widehat{c}_{lD}$: the marginal cost of exporters has to be low enough to offset both the variable and the fixed export costs.

Consider now the effects of a rise in L^c_l. By comparing the corresponding expressions for the closed and the open economies, we see that *all closed economy results fully apply to D variables in the open economy.* In particular, given any additive separable utility function satisfying (A1)-(A3): by Lemma 1, a positive demand shock (larger L_D^c) increases country D's marginal utility of income λ_D; by Proposition 2, (B1) is necessary and sufficient for a positive demand shock to reduce country D's domestic cost cutoff $\widehat{c}_{DD}$; by Proposition 3, (B1') is necessary and sufficient for a positive demand shock to reallocate output from higher to lower cost products. (B1') is also sufficient for a positive demand shock to reallocate revenue from higher to lower cost products. As in the closed economy, all these results holding for the GE scenario also apply to the PE scenario.

What does this imply for exporters from H and F? Within-firm reallocations described in Proposition 3 are driven by larger marginal utility of income. As this affects domestic firms and foreign exporters in an analogous way, we can state:

Corollary 4 ***(Intensive margin of exports)*** *(B1') is necessary and sufficient for a positive demand shock in an export destination to reallocate output from higher to lower cost exported products. (B1') is also sufficient for a positive demand shock in an export destination to reallocate revenue from higher to lower cost exported products. All this holds both in partial and general equilibrium.*

In other words, as in the closed economy, under our assumptions a positive demand shock induces multi-product incumbents to shift output and revenue towards better-performing products. As these products already had larger output and revenue before the shock, those reallocations result in an increase of the 'skewness' of output and revenue. All this holds for both domestic firms and foreign exporters inducing parallel intensive margin adjustments.

On the other hand, when it comes to the extensive margin, the adjustments of domestic firms and foreign exporters diverge as long as the fixed export cost is large enough. In particular, when f_D^x is large enough, a positive demand shock in D has opposite effects on the domestic cutoff $\widehat{c}_{DD}$ and the export cutoff $\widehat{c}_{lD}$. The former decreases driving the highest cost products and firms out the market. The latter cutoff, instead, increases allowing new higher cost products to be exported to and new higher cost exporters to sell in country D. This is due to the fact that the positive demand shock, in equilibrium, induces a counterclockwise rotation of the residual demand curve faced by each product. Thus, demand shifts in for worse-performing products while it shifts out for better-performing ones. When the fixed export cost is high enough, the marginal exported product has a relatively larger market share and thus benefits from the outward shift of a portion of the demand curve. Thus, even though competition and selection get tougher in D (λ_D rises) following the positive demand shock, an exporting firm may still respond by increasing the set of products it sells in D, and a firm that was not exporting any product before the shock may still start serving country D's consumers. So long as this condition on the fixed export cost is satisfied, our model is then consistent with the prediction that a positive demand shock in D promotes aggregate exports from H and F. Given the empirical evidence presented in section 3, we assume that this condition is satisfied. We can then state:

Proposition 5 ***(Extensive margin of exports)*** *If the fixed export cost is large enough, (B1) is necessary and sufficient for a positive demand shock in an export destination to raise the export cost cutoff and is sufficient for a positive demand shock to increase aggregate exports.*

Proof. See Appendix G.1. ■

Short-Run Effects on Exporters in Partial Equilibrium

In the short run, the number of incumbent firms in D is fixed at $\overline{N}_D^i$, so (23) does not hold. Then the short-run equilibrium is characterized by the following *four* conditions: the zero cutoff profit condition (17) for domestic sales in country D; the two zero cutoff profit conditions (20) for export sales from $l \in \{H, F\}$ to D; and the consumer's budget constraint in country D (which also takes imports into account):

$$\begin{aligned} 1 = {} & \overline{N}_D^i \left(\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{DD}/z(m)} p_{DD}\left(x_{DD}^*\left(cz(m),\lambda\right)\right) x_{DD}^*\left(cz(m),\lambda\right) \gamma(c) dc \right) \\ & + \overline{N}_H^i \left(\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{HD}/z(m)} p_{HD}\left(x_{HD}^*\left(cz(m),\lambda\right)\right) x_{HD}^*\left(cz(m),\lambda\right) \gamma(c) dc \right) \\ & + \overline{N}_F^i \left(\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{FD}/z(m)} p_{FD}\left(x_{FD}^*\left(cz(m),\lambda\right)\right) x_{FD}^*\left(cz(m),\lambda\right) \gamma(c) dc \right). \end{aligned} \tag{25}$$

Total differentiation of (17), (20) for $l \in \{H, F\}$ and (25) yields the following results.[25] (B1') is necessary and sufficient for a positive demand shock in D (larger L_D^c) to reallocate H's exported output and is sufficient to reallocate revenue from higher to lower cost products (due to larger λ_D). Moreover, as long as the fixed export cost f_{HD}^x is large enough, (B1) is necessary and sufficient for the positive demand shock in D (larger L_D^c) to increase the number of exporters from H to D

[25]See proof in Appendix G.2.

(through higher $\widehat{c}_{HD}$). Finally, as long as the fixed export cost f^x_{HD} is large enough, (B1) is also sufficient for a positive demand shock to increase aggregate exports.

4.5 Multi-Product Productivity

The reallocations we highlight have implications for the productivity of multi-product firms. To see this, define the productivity of a multi-product firm as the employment weighted average productivity across its products

$$\Phi = \sum_{m=0}^{M-1} \frac{s_m}{v_m},$$

where $m = 0, ..., M-1$ is the product index, $1/v_m$ is output per worker for product m and $s_m = v_m x_m / \sum_{m=0}^{M-1} v_m x_m$ is its employment share. M corresponds to the firm's total number of products *supplied* in the closed economy and to the number of its products *exported* to a given destination in the open economy. By Proposition 3, (B1') is necessary and sufficient for a positive demand shock to reallocate output from higher to lower cost products. This also implies that the labor share s_m increases (decreases) for lower (higher) cost products.[26] Thus the intensive margin reallocations (following a positive demand shock) contribute to a productivity increase for the firm.

In the open economy, the contribution of the extensive margin export response is ambiguous. So long as the newly exported products are more productive than the firm average product, the increased production of those products (associated with the new exports) will increase the firm's overall productivity (inclusive of exported and non-exported products). This is likely to be the case if the exported products represent a small selected subset of the high productivity goods produced by the firms (relative to the range of products sold only on the domestic market). In any event, we will directly measure firm productivity and how it is affected by the demand shocks in the firm's export markets in the following sections.

5 Trade Competition and Product Reallocations at the Firm-Level

Our theoretical model highlights how our measured demand shocks induce increases in competition for exporters to those destinations; and how the increased competition generates increases in productivity by shifting market shares and employment towards better performing products. We seek to directly measure this connection between demand shocks and productivity. Since we cannot measure the productivity associated with products sold to a particular destination, we need

[26] See Appendix H for additional details.

to show that the connection between demand shocks and product reallocations aggregates to the firm-level – before examining the link with firm-level productivity changes (which we can directly measure). Our previous results highlighted how demand shocks lead to reallocations towards better performing products at the destination-industry level. In this section, we show how the destination-industry demand shocks can be aggregated to the firm-level – and this firm-level demand shock strongly predicts product reallocations towards better performing products (higher market shares) at the *firm-level*; that is, changes in skewness to the firm's global product mix (the distribution of product sales across all destinations).

Intuitively, since there is a stable ranking of products at the firm level (better performing products in one market are most likely to be the better performing products in other markets – as we previously discussed), then reallocations towards better performing products within destinations should also be reflected in the reallocations of global sales/production towards better performing products; and the strength of this link between the skewness of sales at the destination and global levels should depend on the importance of the destination in the firm's global sales. Our chosen measure of skewness, the Theil index, makes this intuition precise. It is the only measure of skewness that exhibits a stable decomposition from the skewness of global sales into the skewness of destination-level sales (see Jost 2007).[27] Specifically, let $T_{i,t}$ be firm i's Theil index for the skewness of its global exports by product $x^s_{i,t} \equiv \sum_d x^s_{i,d,t}$. (the sum of exports for that product across all destinations).[28] Then this global Theil can be decomposed into a market-share weighted average of the within-destination Theils $T_{i,d,t}$ and a "between-destination" Theil index $T^B_{i,d,t}$ that measures differences in the distribution of product-level market shares across destinations:[29]

$$T_{i,t} = \sum_d \frac{x_{i,d,t}}{x_{i,t}} T_{i,d,t} - \sum_d \frac{x_{i,d,t}}{x_{i,t}} T^B_{i,d,t}, \qquad (26)$$

where $x_{i,d,t} \equiv \sum_s x^s_{i,d,t}$ and $x_{i,t} \equiv \sum_d x_{i,d,t}$ represent firm i's total exports to d, and across destinations to the world (global exports). The between-destination Theil $T^B_{i,d,t}$ is defined as

$$T^B_{i,d,t} = \sum_s \frac{x^s_{i,d,t}}{x_{i,d,t}} \log\left(\frac{x^s_{i,d,t}/x_{i,d,t}}{x^s_{i,t}/x_{i,t}}\right).$$

[27] This decomposition property is similar – but not identical – to the within/between decomposition of Theil indices across populations. In the latter, the sample is split into subsamples. In our case, the same observation (in this case, product sales) is split into "destinations" and the global measure reflects the sum across "destinations".

[28] The Theil index is defined in the same way as the destination level Theil in (2).

[29] For simplicity, we omit the industry referencing I for the destination Theils. The decomposition across industries follows a similar pattern.

Note that the weights used in this decomposition for both the within- and between-destination Theils are the firm's export shares $x_{i,d,t}/x_{i,t}$ across destinations d. The between-destination Theil $T^B_{i,d,t}$ measures the deviation in a product's market share in a destination d, $x^s_{i,d,t}/x_{i,d,t}$, from that product's global market share $x^s_{i,t}/x_{i,t}$ and then averages these deviations across destinations. It is positive and converges to zero as the distributions of product market shares in different destination become increasingly similar.

To better understand the logic behind (26), note that it implies that the average of the within-destination Theil indices can be decomposed into the sum of two positive elements: the global Theil index, and the between-destination Theil index. This decomposition can be interpreted as a decomposition of variance/dispersion. The dispersion observed in the destination level product exports must be explained either by dispersion in global product exports (global Theil index), or by the fact that the distribution of product sales varies across destinations (between-destination Theil index).

A simple example helps to clarify this point. Take a firm with 2 products and 2 destinations. In each destination, exports of one product are x, and exports of the other product are $2x$. This leads to the same value for the within-destination Theil indices of $(1/3)\ln(1/3) + (2/3)\ln(2/3)$, and hence the same value for the average within-destination Theil index. Hence, if the same product is the better performing product in each market (with $2x$ exports), then the distributions will be synchronized across destinations and the between-destination Theil will be zero: all of the dispersion is explained by the global Theil index, whose value is equal to the common value of the two within-destination Theil indices. On the other hand, if the opposite products perform better in each market, global sales are $3x$ for each product. There is thus no variation in global product sales, and the global Theil index is zero. Accordingly, all of the variation in the within-destination Theil indices is explained by the between-destination Theil.

The theoretical model of Bernard, Redding and Schott (2011) with CES demand predicts that the between-destination Theil index would be exactly zero when measured on a common set of exported products across destinations. With linear demand, Mayer, Melitz and Ottaviano (2014) show (theoretically and empirically) that this between-destination index would deviate from zero because skewness varies across destinations. We have shown earlier that this result holds for a larger class of demand systems such that the elasticity and the convexity of inverse demand increase with consumption. Yet, even in these cases, the between-destination Theil is predicted to be small because the ranking of the product sales is very stable across destinations. This leads to a prediction that the market-share weighted average of the destination Theils should be strongly correlated with the firm's global Theil. Empirically, this prediction is strongly confirmed as shown in Figure 2.

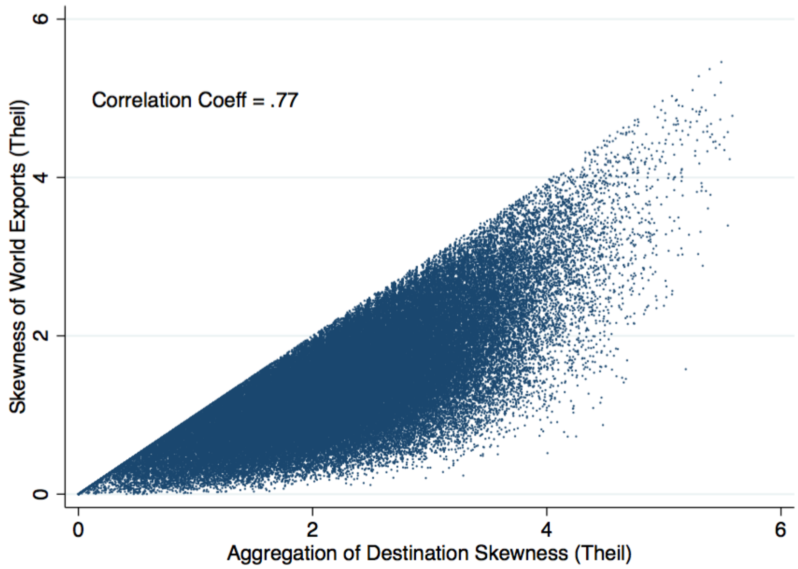


Figure 2: Correlation Between Global Skewness and Average Local Skewness

This high correlation between destination and global skewness of product sales enables us to move from our previous predictions for the effects of the demand shocks on skewness at the destination-level to a new prediction at the firm-level. To do this, we aggregate our destination-industry measures of demand shocks to the firm-level using the same weighing scheme by the firms' export shares across destinations. We thus obtain our firm-level demand shock in (log) levels and first difference:

$$\text{shock}_{i,t} \equiv \sum_{d,I} \frac{x^I_{i,d,t_0}}{x_{i,t_0}} \times \text{shock}^I_{i,d,t}, \qquad \tilde{\Delta}\text{shock}_{i,t} = \sum_{d,I} \frac{x^I_{i,d,t-1}}{x_{i,t-1}} \times \tilde{\Delta}\text{shock}^I_{i,d,t},$$

where $x_{i,t} \equiv \sum_{d,I} x^I_{i,d,t}$ represents firm i's total exports in year t. As was the case for the construction of our firm-level destination shock (see 1), we only use the firm-level information on exported products and market shares in prior years (the year of first export sales t_0 for the demand shock in levels and lagged year $t-1$ for the first difference between t and $t-1$). This ensures the exogeneity of our constructed firm-level demand shocks (exogenous to firm-level actions in year $t > t_0$ for levels, and exogenous to firm-level changes Δ_t for first differences). In particular, changes in the set of exported products or exported market shares are not reflected in the demand shock.[30]

By construction, our firm-level demand shocks will predict changes in the weighted average of destination skewness $T_{i,d,t}$ – and hence will predict changes in the firm's global skewness $T_{i,t}$ (given the high correlation between the two indices). This result is confirmed by our regression of the firms' global Theil on our trade shock measures, reported in the first three columns of Table 5. Our firm-level trade shock has a strong and highly significant (again, well beyond the 1% significance level) impact on the skewness of global exports. The industry level trade shock – which was already substantially weaker than the firm-level trade shock in the destination-level regressions – is no longer significant at the firm level. The GDP shock is not significant in the (log) levels regressions, but is very strong and significant in the two first-difference specifications.

Table 5: The Impact of Demand Shocks on the Global Product Mix (Firm Level)

Dependent Variable	$T_{i,t}$	$\Delta T_{i,t}$		Exp. Intens$_{i,t}$	Δ Exp. Intens$_{i,t}$	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log GDP Shock	-0.001			0.003^a		
	(0.004)			(0.001)		
log trade shock	0.045^a			0.010^a		
	(0.009)			(0.003)		
log trade shock - ISIC	-0.001			0.000		
	(0.001)			(0.000)		
$\tilde{\Delta}$ GDP Shock		0.117^a	0.106^a		0.041^a	0.038^a
		(0.031)	(0.038)		(0.010)	(0.013)
$\tilde{\Delta}$ trade shock		0.057^a	0.050^a		0.016^a	0.014^a
		(0.011)	(0.013)		(0.003)	(0.004)
$\tilde{\Delta}$ trade shock - ISIC		-0.003	-0.009		0.002	0.001
Observations	117981	117981	117981	111860	109049	109049

FE refers to firm-level fixed effects. Standard errors (clustered at the firm level) in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$.

Our global Theil measure $T_{i,t}$ measures the skewness of export sales across all destinations, but it does not entirely reflect the skewness of production levels across the firm's product range. That is because we cannot measure the breakdown of product-level sales on the French domestic market. Ultimately, it is the distribution of labor allocation across products (and the induced

[30]Lileeva and Trefler (2010) and Hummels et al (2014) use a similar strategy to construct firm-level trade-related trade shocks.

distribution of production levels) that determines a firm's labor productivity – conditional on its technology (the production functions for each individual product). As highlighted by our theoretical model, the export market demand shocks generate two different types of reallocations that both contribute to an increased skewness of production levels for the firm: reallocations within the set of exported products, which generate the increased skewness of global exports that we just discussed; but also reallocations from non-exported products towards the better performing exported products (including the extensive margin of newly exported products that we documented at the destination-level). Although we cannot measure the domestic product-level sales, we can measure a single statistic that reflects this reallocation from non-exported to exported goods: the firm's export intensity. We can thus test whether the demand shocks also induce an increase in the firm's export intensity. Those regressions are reported in the last three columns of Table 5, and confirm that our firm-level trade shock has a very strong and highly significant positive impact on a firm's export intensity.[31] The impact of the GDP coefficient is also strong and significant, whereas the industry-level trade shock remains insignificant. Thus, our firm-level trade shock and GDP shock both predict the two types of reallocations towards better performing products that we highlighted in our theoretical model (as a response to increased competition in export markets).

6 Trade Competition and Productivity

We just showed that our firm-level measure of demand shocks (aggregated across destinations) predict increases in the skewness of global exports, and increases in export intensity. Holding firm technology fixed (the productivity of each individual product), this increase in the skewness of global production will generate productivity increases for the firm. We now directly test for this connection from the demand shocks to firm productivity.

We obtain our measure of firm productivity by merging our firm-level trade data with firm-level production data. This latter dataset contains various measures of firm outputs and inputs. As we are interested in picking up productivity fluctuations at a yearly frequency, we focus on labor productivity measured as deflated value added per worker (using sector-specific price deflators). We then separately control for the impact of changes in factor intensities and returns to scale (or variable utilization of labor) on labor productivity. Note that this firm-level productivity measure aggregates (using labor shares) to the overall deflated value-added per worker for manufacturing.

[31] Since the export intensity is a ratio, we do not apply a log-transformation to that variable. However, specifications using the log of export intensity yield very similar results.

So long as our sector-specific price indices are accurately measured, this aggregate productivity measure accurately tracks a welfare-relevant quantity index – even though we do not have access to firm-level prices. In other words, the effect of pure markup changes at the sector level are netted-out of our productivity measure. We will thus report a welfare-relevant aggregate productivity change by aggregating our firm-level productivity changes using the observed changes in labor shares.[32] In addition, we run all of our specifications with sector-time (2 digit NACE) fixed effects, thus eliminating the need for sector-level deflators. Our productivity results therefore capture within-sector effects of the demand shocks, over-and-above any contribution of the sector deflator to a common productivity change across firms.

Our firm-level demand shocks only aggregate across export destinations. It therefore does not incorporate a firm's exposure to demand shocks in its domestic (French) market. This is not possible for two reasons: most importantly, we do not observe the product-level breakdown of the firms' sales in the French market (we only observe total domestic sales across products); in addition, world exports into France would not be exogenous to firm-level technology changes in France. Therefore, we need to adjust our export-specific demand shock using the firm's export intensity to obtain an overall firm-level demand shock relevant for overall production and hence productivity:

$$\text{shock\_intens}_{i,t} = \frac{x_{i,t_0}}{x_{i,t_0} + x_{i,F,t_0}} \text{shock}_{i,t}, \qquad \tilde{\Delta}\text{shock\_intens}_{i,t} = \frac{x_{i,t-1}}{x_{i,t-1} + x_{i,F,t-1}} \tilde{\Delta}\text{shock}_{i,t},$$

where $x_{i,F,t}$ denotes firm i's total (across products) sales to the French domestic market in year t (and the ratio thus measures firm i's export intensity). Once again, we only use prior year's information on firm-level sales to construct this overall demand shock. Note that this adjustment using export intensity is equivalent to assuming a demand shock of zero in the French market and including that market in our aggregation by market share relative to total firm sales $x_{i,t} + x_{i,F,t}$.

6.1 Impact of the Trade Shock on Firm Productivity

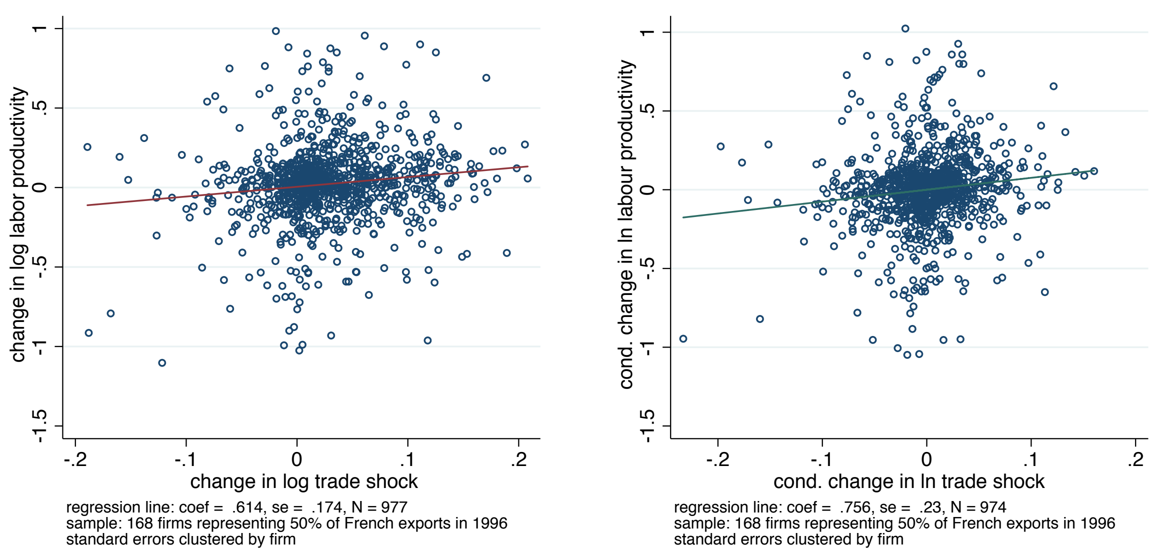
In this section, we investigate the direct link between this firm-level demand shock and firm productivity. Here, we focus exclusively on our firm-specific demand shock from (1), as this is our only shock that exhibits firm-level variation within destinations. Our measure of productivity is the log of value-added per worker. All regressions include industry-year fixed effects, that will capture in

[32] At the firm-level, an increase in markups across all products will be picked-up in our firm productivity measure – even though this does not reflect a welfare-relevant increase in output. But if this is the case, then this firm's labor share will decrease, and its productivity will carry a smaller weight in the aggregate index.

particular different evolutions of price indexes across industries.[33] In order to control for changes in capital intensity, we use the log of capital per worker (K_{it}/L_{it}). We also control for unobserved changes in labor utilization and returns to scale by using the log of raw materials (including energy use), R_{it}. Then, increases in worker effort or higher returns to scale will be reflected in the impact of raw materials use on labor productivity. As there is no issue with zeros for all these firm-level variables, we directly measure the growth rate of those variable using simple first differences of the log levels.

We begin with a graphical representation of the strong positive relationship between firm-level productivity and our constructed demand shock. Figure 3 illustrates the correlation between those variables in first differences for the largest French exporters (representing 50% of French exports in 1996). Panel (a) is the unconditional scatter plot for those variables, while panel (b) shows the added-variable plot for the first-difference regression of productivity on the trade shock, with additional controls for capital intensity, raw materials (both in log first-differences) and time dummies. Those figures clearly highlight the very strong positive response of the large exporters' productivity to changes in trade competition in export markets (captured by the demand shock).

Figure 3: Exporters Representing 50% of French Trade in 1996: First Differences 1996-2005



(a) Unconditional

(b) Conditional

[33] The robustness section in the appendix shows results for an alternative procedure where the baseline table is reproduced using deflated value-added per worker (the value added deflator coming from EUKLEMS dataset for France).

Table 6 shows how this result generalizes to our full sample of firms and our three different specifications (FE, FD, FD-FE). Our theoretical model emphasizes how a multi-product firm's productivity responds to the demand shock via its effect on competition and product reallocations in the firm's export markets. Thus, we assumed that the firm's technology at the product level (the marginal cost $v(m, c)$ for each product m) was exogenous (in particular, in respect to demand fluctuations in export markets). However, there is a substantial literature examining how this technology responds to export market conditions via various forms of innovation or investment choices made by the firm. We feel that the timing dimension of our first difference specifications—especially our FD-FE specifications which nets out any firm-level growth trends–eliminates this technology response channel: It is highly unlikely that a firm's innovations or investment responses to the trade shock in a given year (especially the innovation in the trade shock relative to trend) would be reflected contemporaneously in the firm's productivity. However, we will also show some additional robustness checks that address this potential technology response.

The first three columns of Table 6 show that, across our three timing specifications, there is a stable and very strong response of firm productivity to the trade shock. Since our measure of productivity as value added per worker incorporates neither the impact of changes in input intensities nor the effects of non-constant returns to scale, we directly control for these effects in the next set of regressions. In the last 3 columns of Table 6, we add controls for capital per worker and raw material use (including energy). Both of these controls are highly significant: not surprisingly, increases in capital intensity are reflected in labor productivity; and we find that increases in raw materials use are also associated with higher labor productivity. This would be the case if there are increasing returns to scale in the value-added production function, or if labor utilization/effort increases with scale (in the short-run). However, the very strong effect of the trade shock on firm productivity remains when these controls are added – and they remain highly significant, well beyond the 1% significance level. (From here on out, we will keep those controls in all of our firm-level productivity regressions.)

We now describe several robustness checks that further single-out our theoretical mechanism operating through the demand-side product reallocations for multi-product firms. In the next table we regress our capital intensity measure on our trade shock; the results in Table 7 show that there is no response of investment to the trade shock. This represents another way to show that the short-run timing for the demand shocks precludes a contemporaneous technology response: if this were the case, we would expect to see some of this response reflected in higher investment (along

Table 6: Baseline Results: Impact of Trade Shock on Firm Productivity

Dependent Variable	log prod.	Δ log prod.		log prod.	Δ log prod.	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log (trade shock $\times$ export intens.)	0.061^{a}			0.051^{a}		
	(0.016)			(0.016)		
$\tilde{\Delta}$ (trade shock $\times$ export intens.)		0.106^{a}	0.106^{a}		0.112^{a}	0.113^{a}
		(0.019)	(0.023)		(0.020)	(0.024)
log capital stock per worker				0.117^{a}		
				(0.004)		
log raw materials				0.086^{a}		
				(0.003)		
Δ log capital stock per worker					0.125^{a}	0.133^{a}
					(0.005)	(0.006)
Δ log raw materials					0.092^{a}	0.090^{a}
					(0.003)	(0.003)
Observations	213001	185688	185688	203977	175619	175619

FE refers to firm-level fixed effects. All regressions also include industry-year dummies. Standard errors (clustered at the firm level) in parentheses: $^{c} < 0.1$, $^{b} < 0.05$, $^{a} < 0.01$.

with other responses along the technology dimension).

Next we use a different strategy to control for the effects of non-constant returns to scale or variable labor utilization: in Table 8, we split our sample between year intervals where firms increase/decrease employment. If the effects of the trade shock on productivity were driven by scale effects or higher labor utilization/effort, then we would expect to see the productivity responses concentrated in the split of the sample where firms are expanding employment (and also expanding more generally). Yet, Table 8 shows that this is not the case: the effect of the trade shock on productivity is just as strong (even a bit stronger) in the sub-sample of years where firms are decreasing employment; and in both cases, the coefficients have a similar magnitude to our baseline results in Table 6.[34]

A potential concern with our trade shock variable weighting demand shocks in the destination country by export shares is that it would be correlated with import shocks, i.e. with supply shocks originating from the same foreign countries and affecting directly production costs of French firms through imported intermediate goods. We therefore construct a symmetric set of variables that

[34]Since we are splitting our sample across firms, we no longer rely on the two specifications with firm fixed-effects and only show results for the FD specification.

Table 7: K/L Does Not respond to Trade Shocks

Dependent Variable	ln K/L	Δ ln K/L	Δ ln K/L
Specification	FE	FD	FD-FE
log (trade shock $\times$ export intens.)	0.029		
	(0.021)		
$\tilde{\Delta}$ (trade shock $\times$ export intens.)		0.015	0.002
		(0.022)	(0.027)
Observations	218073	190512	190512

FE refers to firm fixed effects. All regressions also include industry-year dummies. Standard errors (clustered at the firm level) in parentheses:^{c} < 0.1, ^{b} < 0.05, ^{a} < 0.01.

Table 8: Robustness to Scale Effects

Sample	Employment Increase	Employment Decrease
Dependent Variable	Δ log productivity	Δ log productivity
Specification	FD	FD
$\tilde{\Delta}$ (trade shock $\times$ export intens.)	0.128^a	0.170^a
	(0.034)	(0.033)
Δ log capital stock per worker	0.108^a	0.104^a
	(0.006)	(0.006)
Δ log raw materials	0.100^a	0.096^a
	(0.004)	(0.004)
Observations	69881	65739

All regressions include industry-year dummies. Standard errors (clustered at the firm level) in parentheses: ^{c} < 0.1, ^{b} < 0.05, ^{a} < 0.01.

weight changes in exports (to the world except France) from a given product/country by the firm-level import shares from that product/contry. Table 9 introduces those new variables into the baseline specification. Although import shocks have a separate large and very significant effect, it does not affect our main effect of interest in any major way.

In order to further single-out our theoretical mechanism operating through the demand-side product reallocations for multi-product firms, we now report two different types of falsification tests. Our first test highlights that the link between productivity and the trade shocks is only operative for multi-product firms. Table 10 reports the same regression (with controls) as our baseline results from Table 6, but only for single-product exporters. This new table clearly shows that this there is no evidence of this link among this subset of firms. Next, we show that this

Table 9: Robustness to import shocks

Dependent Variable	log prod.	Δ log prod.		log prod.	Δ log prod.	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log (trade shock $\times$ export intens.)	0.068^a			0.056^a		
	(0.020)			(0.019)		
log (trade shock $\times$ import intens.)	0.078^a			0.065^b		
	(0.029)			(0.028)		
$\tilde{\Delta}$ (trade shock $\times$ export intens.)		0.110^a	0.111^a		0.129^a	0.133^a
		(0.022)	(0.026)		(0.022)	(0.027)
$\tilde{\Delta}$ (trade shock $\times$ import intens.)		0.225^a	0.245^a		0.197^a	0.218^a
		(0.034)	(0.041)		(0.033)	(0.041)
log capital stock per worker				0.095^a		
				(0.004)		
log raw materials				0.090^a		
				(0.004)		
Δ log capital stock per worker					0.107^a	0.115^a
					(0.005)	(0.006)
Δ log raw materials					0.091^a	0.090^a
					(0.003)	(0.003)
Observations	133382	152171	152171	129517	144938	144938

FE refers to firm-level fixed effects. All regressions also include industry-year dummies. Standard errors (clustered at the firm level) in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$.

productivity-trade link is only operative for firms with a substantial exposure to export markets (measured by export intensity). Similarly to single-product firms, we would not expect to find a significant productivity-trade link among firms with very low export intensity. This is indeed the case. In Table 11, we re-run our baseline specification using the trade shock before it is interacted with export intensity. The first three columns report the results for the quartile of firms with the lowest export intensity, and highlight that there is no evidence of the productivity-trade link for those firms. On the other hand, we clearly see from the last three columns that this effect is very strong and powerful for the quartile of firms with the highest export intensity.[35]

The firms with high export intensity therefore have a response of productivity to trade shocks

[35] Since the trade shock as not been interacted with export intensity, the coefficients for this top quartile represent significantly higher magnitudes than the average coefficients across the whole sample reported in Table 6 (since export intensity is always below 1). This is also confirmed by a specification with the interacted trade shock restricted to this same top quartile of firms.

Table 10: Robustness: Single Product Firms

Dependent Variable	log prod.	Δ log prod.	
Specification	FE	FD	FD-FE
log (trade shock $\times$ export intens.)	-0.050		
	(0.047)		
log capital stock per worker	0.180^a		
	(0.013)		
log raw materials	0.085^a		
	(0.007)		
$\tilde{\Delta}$ (trade shock $\times$ export intens.)		0.038	0.012
		(0.044)	(0.066)
Δ log capital stock per worker		0.214^a	0.260^a
		(0.013)	(0.019)
Δ log raw materials		0.103^a	0.100^a
		(0.007)	(0.011)
Observations	33198	25519	25519

FE refers to firm fixed effects. All regressions also include industry-year dummies. Standard errors (clustered at the firm level) in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$.

estimated around 10% (columns 5 and 6 of Table 11). How should we interpret this number in terms of the impact of our mechanism on the productivity of the French economy as a whole? Several mechanisms are at play when going from $\beta_{q4} = .092$ for the average marginal effect on firms in the fourth quartile of export intensity, $q4$, to a calculation of the aggregate impact: The distribution of trade shocks across firms, the employment share of the affected firms, and the evolution of this share in the economy.

Table 12 provides a back of the envelope calculation for different sectors in terms of the average yearly change in productivity due to the true trade shocks experienced by firms (the final row providing the aggregate estimate for the whole manufacturing industry). The calculation is done in the following way: i) take the trade shock of each $q4$ firm in t and increase its log productivity in $t-1$ by $\beta_{q4} \times \tilde{\Delta}$trade shock, which gives a predicted productivity in t ii) take the weighted average of those predicted productivities where the weights are employment shares in t, iii) calculate the weighted average of true productivities for the same group of firms in $t-1$, iv) take the ratio of those two average productivities. This provides an aggregate productivity impact for each year that can then be averaged over our period. The result ranges from an average annual impact of 3.4 % for wearing apparel, to a small negative impact for refined petroleum for instance. The average annual

Table 11: Robustness: Low/High export intensity

Sample	exp. intens. quartile # 1			exp. intens. quartile # 4		
Dependent Variable	log prod.	Δ log prod.		log prod.	Δ log prod.	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log trade shock	0.003			0.072^a		
	(0.006)			(0.011)		
log capital stock per worker	0.117^a			0.104^a		
	(0.009)			(0.007)		
log raw materials	0.070^a			0.111^a		
	(0.005)			(0.006)		
$\tilde{\Delta}$ trade shock		0.004	0.006		0.092^a	0.101^a
		(0.008)	(0.009)		(0.014)	(0.016)
Δ log capital stock per worker		0.125^a	0.129^a		0.107^a	0.114^a
		(0.011)	(0.013)		(0.008)	(0.010)
Δ log raw materials		0.084^a	0.081^a		0.108^a	0.104^a
		(0.006)	(0.007)		(0.006)	(0.007)
Observations	38806	30909	30909	57267	48716	48716

Standard errors (clustered at the firm level) in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$.

effect for the whole of manufacturing is about 1.2%, which corresponds to a average annual trade shock of 6.2% reported in the second column, where the same weights are attributed to firm-level $\tilde{\Delta}$trade shock.

7 Conclusion

This paper uses detailed firm-level data to assess the relevance and magnitude of a new channel of gains from trade: the productivity gains associated with demand shocks in export markets (via the induced effect of demand on the product mix of exporters). Our theoretical model predicts that demand shocks generate an endogenous increase in local competition that induces firms to skew their sales towards their better performing products – generating increases in productivity. Empirically, our data matches individual export flows by French firms to each country in the world with balance sheet data needed to evaluate the impact of demand shocks on productivity and to control for confounding factors. The strategy is therefore to look at product mix changes inside the firm, rather than reallocations of market shares across firms. We can therefore control for many alternative explanations that might be correlated with foreign demand shocks – a strategy that

Table 12: Quantification of trade shock effects on productivity

Industry	prod.	trade shock	% high exp.intens.	% mfg. emp.
Wearing Apparel	3.38	5.21	27.36	2.26
Wood	3.37	6.34	20.36	1.7
Tobacco	3.22	43.6	.48	.16
Printing and publishing	2.81	8.48	5.36	3.31
Radio, television and communication	1.8	4.94	59.77	4.31
Leather and footwear	1.79	3.59	26.86	1.21
Textiles	1.69	1.99	33.04	3.29
Motor vehicles, trailers and semi-trailers	1.62	9.8	52.39	7.82
Machinery	1.32	5.54	45.4	9.12
Manufacturing nec	1.19	5.94	22.72	3.56
Pulp and paper	1.18	3.67	30.62	2.82
Chemicals	1.15	6.58	40.55	9.63
Fabricated metal	.94	7.04	17.41	8.81
Medical, precision and optical instruments	.85	5.84	46.82	3.53
Rubber and plastics	.8	5.75	36.97	7.18
Electrical machinery	.73	5.83	53.12	5.17
Basic metals	.7	6.27	58.91	4.06
Food and beverages	.66	6.2	14.12	11.88
Other transport equipment	.65	7.25	69.14	4.3
Office machinery	.64	3.7	42.55	1.09
Other Non-Metallic Mineral	.46	3.89	35.52	3.86
Coke, ref. petr. and nuclear fuel	-.18	5.12	25.54	.93
Total mfg	1.17	6.2	36.66	100

Columns (2) and (3) provide average percentage changes over the 1996-2005 period.

would not be possible when evaluating the effects *across* firms.

Our baseline results shows that the elasticity of labor productivity to trade shocks is between 5 and 11 %. This order of magnitude is very robust to controls for short-run investment by the firm, scale effects, and possibly correlated import shocks. Our measured productivity effect for single product firms is nil, further highlighting the importance of changes in product mix for multi-product firms. We also show that this productivity response is concentrated within the quartile of exporters with the highest export intensities. Taking into account the weight of those firms in the whole economy, we calculate that the average annual increase in French manufacturing productivity – in response to growth in world trade – over our 10 year sample (from 1995-2005) is slightly over 1 percent per year.

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AppendixAppendix

A Higher cost is associated with higher price, lower output, lower revenue and lower profit.

First, implicit differentiation of (4) yields

$$\frac{dx_v}{dv} = \frac{\lambda}{\phi'(x_v)} < 0 \tag{A.1}$$

if and only if (A3) holds. This shows that higher marginal cost leads to smaller output $q_v = x_v L^c$. Second, by (3), smaller output implies higher price as

$$\frac{dp_v}{dx_v} = \frac{u''(x_v)}{\lambda} < 0$$

if and only if (A1) holds. Accordingly, higher marginal cost leads to higher price

$$\frac{dp_v}{dv} = \frac{dp_v}{dx_v}\frac{dx_v}{dv} = \frac{u''(x_v)}{\phi'(x_v)} > 0$$

if and only if (A1) and (A3) hold. Third, if and only if (A2) and (A3) hold, higher marginal cost is associated with lower revenue $R_v = p_v x_v L^c$ as in this case

$$\frac{dR_v}{dv} = [1 - \varepsilon_p(x_v)]\frac{dx_v}{dv}L^c < 0,$$

which shows that the negative effect of cost on output dominates its positive effect on price. Fourth, by the envelope theorem, also profit is a decreasing function of marginal cost as

$$\frac{d\pi_v}{dv} = -x_v L^c < 0 \tag{A.2}$$

where $\pi_v = (p_v - v)x_v L^c$ is maximized operating profit. Hence, (A1), (A2) and (A3) are necessary and sufficient for higher cost products to be associated with higher price, lower output, lower revenue and lower profit.[36]

[36] A necessary and sufficient condition for lower marginal cost to be also associated with larger employment is $\varepsilon_r(x_v) < 1$. To see this note that employment in the supply of a product with marginal cost v equals vx_vL^c. Differentiation yields

$$\frac{dvx_v}{dv} = \left(1 + \frac{\partial x_v}{\partial v}\frac{v}{x_v}\right)x_v = \left(1 - \frac{1}{\varepsilon_r(x_v)}\right)x_v$$

where the second equality is granted by (A1) and (4) together with the definition of $\varepsilon_r(x_v)$.

B Lower cost is associated with higher markup

From (5), the (multiplicative) markup equals

$$m_v = \frac{p_v}{v} = \frac{1}{1 - \varepsilon_p(x_v)} \tag{B.1}$$

Given that in Appendix A we have already shown that under A(3) x_v is a decreasing function of v, markup m_v will be decreasing in v if and only if $\varepsilon_p'(x_v) > 0$. This is our assumption (B1), which can be equivalently stated as $\varepsilon_r(x_v) > \varepsilon_p(x_v)$ noting that we can write

$$\frac{\varepsilon_p'(x_v)x_v}{\varepsilon_p(x_v)} = \frac{1 - \varepsilon_p(x_v)}{\varepsilon_p(x_v)} \left[\varepsilon_r(x_v) - \varepsilon_p(x_v)\right].$$

C Pass-through is incomplete and lower cost is associated smaller pass-through

Define the pass-through as

$$\theta(x_v) \equiv \frac{d \ln p(x_v)}{d \ln v} = \frac{d \ln p(x_v)}{d \ln x_v} \frac{d \ln x_v}{d \ln v}$$

By definition we have

$$\frac{d \ln p(x_v)}{d \ln x_v} = -\varepsilon_p(x_v)$$

while implicit differentiation of (3) gives

$$\frac{dx_v}{dv} = \frac{\lambda}{\phi'(x_v)}$$

so that

$$\frac{d \ln x_v}{d \ln v} = \frac{\phi(x_v)}{\phi'(x_v)x_v} = -\frac{1}{\varepsilon_r(x_v)}$$

where the first equality is granted by (3) and the second equality is granted by definition. Accordingly, the pass-through can be expressed as

$$\theta(x_v) = \frac{\varepsilon_p(x_v)}{\varepsilon_r(x_v)} \tag{C.1}$$

with $\theta(x_v) < 1$ if an only if (B1) holds. Differentiating (C.1) shows that

$$\frac{\varepsilon_p'(x_v)x_v}{\varepsilon_p(x_v)} < \frac{\varepsilon_r'(x_v)x_v}{\varepsilon_r(x_v)}$$

if and only if $\theta'(x_v) < 0$. This is our assumption (B2). Note that, given (B1), $\varepsilon_r'(x_v) > 0$ is necessary (but not sufficient) for $\theta'(x_v) < 0$.

To connect these results on marginal cost pass-through with the evidence on exchange rate pass-through, consider the problem in Berman, Martin and Mayer (2102) of an exporter to destination market D solving

$$\max_{q_i \geq 0} \pi_D(q_i) = \frac{p_i}{\epsilon_D} q_i - v q_i - f$$

subject to

$$p_i = \frac{u'(x_i)}{\lambda}, \text{ with } \lambda = \int_0^M u'(x_i) x_i di,$$

where p_i is the price denominated in the destination's currency, v is the marginal cost denominated in the currency of the firm's country, and ϵ_D is the exchange rate. The FOC for profit maximization is

$$u'(x_v) + u''(x_v)x_v = \epsilon_D \lambda v,$$

which implies that the pass-through from exchange rate to price is isomorphic to the pass-through from marginal cost to price.

D Specific Functional Forms: Bulow-Pfleiderer Demand

A flexible family of inverse demand functions that satisfy not only assumptions (A1)-(A3) but also (B1)-(B3) is the Bulow-Pfleiderer (BP) family recently analyzed by Fabinger and Weyl (2014). In our setup, this family is associated with the sub-utility

$$u(x_i) = \alpha x_i + \frac{\beta}{1-\gamma} (x_i)^{1-\gamma}, \tag{D.1}$$

which has the appealing feature of nesting both CES demand and the linear demand as special cases. The first order condition for the corresponding utility maximization problem

$$\max_{x_i \geq 0} \int_0^M \left[\alpha x_i + \frac{\beta}{1-\gamma} (x_i)^{1-\gamma} \right] di \text{ s.t. } \int_0^M p_i x_i di = 1,$$

is

$$p_i = \frac{\alpha + \beta (x_i)^{-\gamma}}{\lambda}, \text{ with } \lambda = \int_0^M \left[\alpha x_i + \beta (x_i)^{1-\gamma}\right] di, \tag{D.2}$$

where $\lambda > 0$ is the marginal utility of income. As anticipated, this demand paramatrization includes CES demand as a special case for $\alpha = 0$, $\beta = 1 - \gamma$ and $\gamma = 1/\sigma \in (0, 1)$ (where $\sigma > 1$ is the elasticity of demand). It also includes linear demand as another special case for $\alpha > 0$, $\beta < 0$ and $\gamma = -1$.

Given (D.1), these are the conditions on parameters for **(A1)** to hold

$$u'(x_v) = \alpha + \beta (x_v)^{-\gamma} \overset{\textbf{(A1)}}{>} 0 \text{ iff } \frac{\alpha}{\beta} < -(x_v)^{-\gamma} \text{ for } \beta < 0$$

$$u''(x_v) = -\gamma\beta (x_v)^{-\gamma-1} \overset{\textbf{(A1)}}{<} 0 \text{ iff } \gamma\beta > 0 \text{ implied by } \beta < 0 \text{ and } \gamma < 0$$

$$\varepsilon_p(x_v) \equiv -\frac{u''(x_v)x_v}{u'(x_v)} = \frac{\gamma\beta (x_v)^{-\gamma}}{\alpha + \beta (x_v)^{-\gamma}} \overset{\textbf{(A1)}}{>} 0 \text{ implied by } \frac{\alpha}{\beta} < -(x_v)^{-\gamma}, \ \beta < 0 \text{ and } \gamma < 0$$

These are the conditions for **(A2)** to hold

$$\varepsilon_p(x_v) \equiv -\frac{u''(x_v)x_v}{u'(x_v)} = \frac{\gamma\beta (x_v)^{-\gamma}}{\alpha + \beta (x_v)^{-\gamma}} \overset{\textbf{(A2)}}{<} 1 \text{ iff } \gamma\beta (x_v)^{-\gamma} < \alpha + \beta (x_v)^{-\gamma} \text{ for } \frac{\alpha}{\beta} < -(x_v)^{-\gamma}$$

$$\text{i.e. } \frac{\alpha}{\beta} < -(1-\gamma)(x_v)^{-\gamma} = -(x_v)^{-\gamma} + \gamma (x_v)^{-\gamma} \text{ stronger than } \frac{\alpha}{\beta} < -(x_v)^{-\gamma} \text{ for } \gamma < 0$$

These are the conditions for **(A3)** to hold

$$\varepsilon_r(x_v) \equiv -\frac{\phi'(x_v)x_v}{\phi(x_v)} = \frac{\gamma(1-\gamma)\beta (x_v)^{-\gamma}}{\alpha + (1-\gamma)\beta (x_v)^{-\gamma}} \overset{\textbf{(A3)}}{>} 0 \text{ implied by } \frac{\alpha}{\beta} < -(1-\gamma)(x_v)^{-\gamma}, \ \beta < 0 \text{ and } \gamma < 0$$

These are the conditions for **(B1)** to hold

$$\varepsilon_p'(x_v) = -\frac{\alpha\gamma^2\beta (x_v)^{-\gamma-1}}{\left[\alpha + \beta (x_v)^{-\gamma}\right]^2} \overset{\textbf{(B1)}}{>} 0 \text{ implied by } \alpha > 0 \text{ and } \beta < 0$$

These are the conditions for **(B1')** to hold

$$\varepsilon_r'(x_v) = -\frac{\alpha\gamma^2(1-\gamma)\beta (x_v)^{-\gamma-1}}{\left[\alpha + (1-\gamma)\beta (x_v)^{-\gamma}\right]^2} \overset{\textbf{(B1')}}{>} 0 \text{ implied by } \alpha > 0, \ \beta < 0 \text{ and } \gamma < 0$$

Hence, BP demand satisfies assumptions **(A1)-(A3)** and **(B1)-(B1')** for

$$\alpha > 0, \ \beta < 0, \ \gamma < 0, \ \frac{\alpha}{\beta} < -(1-\gamma)(x_v)^{-\gamma}$$

Note that the special case of linear demand ($\alpha = a$, $\beta = -b$ and $\gamma = -1$) satisfies assumptions **(A1)-(A3)** and **(B1)-(B1)** for

$$\alpha = a > 0,\ \beta = -b < 0,\ \gamma = -1 < 0,\ \frac{\alpha}{\beta} = -\frac{a}{b} < -2x_v.$$

Differently, the special case of CES demand ($\alpha = 0$, $\beta = 1-\gamma$, $\gamma = 1/\sigma \in (0,1)$) violates **(B1)-(B1')** as it implies constant elasticities of demand and marginal revenue.

E Long-Run Response to a Demand Shock in Closed Economy

In the long run, the effects of a demand shock (dL^c) on $\widehat{c}^*$ and λ can be characterized extending the analysis by Zhelobodko et al (2012) to the case of multi-product firms emphasizing both extensive and intensive margins adjustments. These effects are fully determined by (11) and (12) both in the PE and the GE scenarios. As L^w does not appear in (11) and (12), all long-run results hold whatever the scenario.

E.1 Lemma 1

Following Zhelobodko et al (2012), rewrite expressions (10) in terms of elasticities as

$$\epsilon_{\pi^*/\lambda}(\lambda v) = \frac{\partial \pi^*(v,\lambda)}{\partial \lambda}\frac{\lambda}{\pi^*(v,\lambda)} = -\frac{1}{1-\frac{c}{\frac{u'(x^*(\lambda v))}{\lambda}}} = -\frac{1}{\varepsilon_p(x^*(\lambda v))} \tag{E.1}$$

$$\epsilon_{\pi^*/v}(\lambda v) = -\frac{\partial \pi^*(v,\lambda)}{\partial v}\frac{v}{\pi^*(v,\lambda)} = -\frac{v}{p(x^*(\lambda v))-v} = -\frac{1-\varepsilon_p(x^*(\lambda v))}{\varepsilon_p(x^*(\lambda v))} \tag{E.2}$$

To obtain $d\lambda^*/dL^c > 0$ divide (12) by L^c to yield

$$\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc = \frac{f_e}{L^c} + \frac{f}{L^c}\sum_{m=0}^{\infty}\Gamma\left(\frac{\widehat{c}}{z(m)}\right) \tag{E.3}$$

where regularity conditions are assumed on $z(m)$ to make the series on the right hand side converge. Then differentiate with respect to L^c to get

$$\begin{aligned}&\frac{\partial\lambda}{\partial L^c}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial\pi^*\left(cz(m),\lambda\right)}{\partial\lambda}\gamma(c)dc + \frac{\partial\widehat{c}}{\partial L^c}\sum_{m=0}^{\infty}\pi^*\left(\widehat{c},\lambda\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)}\\ &= -\frac{f_e}{(L^c)^2} - \frac{f}{(L^c)^2}\sum_{m=0}^{\infty}\Gamma\left(\frac{\widehat{c}}{z(m)}\right) + \frac{\partial\widehat{c}}{\partial L^c}\frac{f}{L^c}\sum_{m=0}^{\infty}\frac{\gamma\left(\widehat{c}/z(m)\right)}{z(m)}\end{aligned}$$

$$\frac{\partial \lambda}{\partial L^c} \sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \frac{\partial \pi^*\left(cz(m),\lambda\right)}{\partial \lambda} \gamma(c)dc + \frac{\partial \widehat{c}}{\partial L^c} \sum_{m=0}^{\infty} \left[\pi^*\left(\widehat{c},\lambda\right) - \frac{f}{L^c}\right] \frac{\gamma(\widehat{c}/z(m))}{z(m)} \tag{E.4}$$
$$= -\frac{f_e}{(L^c)^2} - \frac{f}{(L^c)^2} \sum_{m=0}^{\infty} \Gamma\left(\frac{\widehat{c}}{z(m)}\right)$$

By (11), the term between square brackets equals zero while again (12) implies

$$-\frac{f_e}{(L^c)^2} - \frac{f}{(L^c)^2} \sum_{m=0}^{\infty} \Gamma\left(\frac{\widehat{c}}{z(m)}\right) = -\frac{\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc}{L^c}$$

Exploiting these results, (E.4) can be restated in terms of elasticities as

$$\frac{\partial \lambda}{\partial L^c} \sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \frac{\partial \pi^*\left(cz(m),\lambda\right)}{\partial \lambda} \gamma(c)dc = -\frac{\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc}{L^c}$$

$$\frac{\partial \lambda}{\partial L^c} \frac{L^c}{\lambda} \sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \frac{\partial \pi^*\left(cz(m),\lambda\right)}{\partial \lambda} \frac{\lambda}{\pi^*\left(cz(m),\lambda\right)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc$$
$$= -\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc$$

$$\epsilon_{\lambda/L^c}(\lambda\widehat{c}) \sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \epsilon_{\pi^*/\lambda} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc = -\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc$$

$$\epsilon_{\lambda/L^c}(\lambda\widehat{c}) = -\frac{\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc}{\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\epsilon_{\pi^*/\lambda}\gamma(c)dc} = \frac{\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc}{\sum_{m=0}^{\infty} \int_0^{\widehat{c}/z(m)} \frac{\pi^*(cz(m),\lambda)}{\varepsilon_p(x^*(\lambda cz(m)))}\gamma(c)dc} \tag{E.5}$$

where the last equality is granted by (E.1). Assumption (A2) – according to which $\varepsilon_p(x^*(\lambda v)) \in (0,1)$ holds – implies $\epsilon_{\lambda/L^c} \in (0,1)$. This proves Lemma 1 in the proposition.

E.2 Proposition 2: Extensive Margin Adjustment

Cost Cutoff

To determine the sign of $\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c})$, differentiate (11) with respect to L^c and get

$$\frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \widehat{c}} \frac{\partial \widehat{c}}{\partial L^c} + \frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \lambda} \frac{\partial \lambda}{\partial L^c} = -\frac{f}{(L^c)^2} = -\frac{\pi^*(\widehat{c},\lambda)}{L^c} \tag{E.6}$$

where the last equality is also granted by (11), or in terms of elasticities

$$\epsilon_{\pi^*/\widehat{c}}\epsilon_{\widehat{c}/L^c} + \epsilon_{\pi^*/\lambda}\epsilon_{\lambda/L^c} = -1$$

After substituting for (E.1), (E.2) and (E.5), this equation can be solved for $\epsilon_{\widehat{c}/L^c}$ to yield

$$\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c}) = \frac{\varepsilon_p(x^*(\lambda\widehat{c})) - \epsilon_{\lambda/L^c}}{1-\varepsilon_p(x^*(\lambda\widehat{c}))} = \frac{\varepsilon_p(x^*(\lambda\widehat{c}))}{1-\varepsilon_p(x^*(\lambda\widehat{c}))}\left[1 - \frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \frac{\pi^*(cz(m),\lambda)}{\varepsilon_p(x^*(\lambda\widehat{c}))}\gamma(c)dc}{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \frac{\pi^*(cz(m),\lambda)}{\varepsilon_p(x^*(\lambda cz(m)))}\gamma(c)dc}\right]$$

Given (A2), the sign of this expression is dictated by the sign of the term in square brackets. As under (A2) $x^*(\lambda v) > x^*(\lambda\widehat{c})$ also holds for all $v < \widehat{c}$, then $\varepsilon_p(x^*(\lambda v)) > \varepsilon_p(x^*(\lambda\widehat{c}))$ holds for all $v < \widehat{c}$ if and only if $\varepsilon_p'(x^*(\lambda\widehat{c})) > 0$. In this case, $\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c}) < 0$ follows. This proves that larger L^c reduces the cost cutoff if and only if (B1) holds, which is point (ii) in the proposition.

Profit

To see how changes in L^c affect $\pi^*(v,\lambda)L^c$, differentiate to obtain

$$\begin{aligned}
\frac{\partial\pi^*(v,\lambda)L^c}{\partial L_c} &= \pi^*(v,\lambda) + \frac{\partial\pi^*(v,\lambda)}{\partial\lambda}\frac{\partial\lambda}{\partial L^c}L^c \\
&= \pi^*(v,\lambda)\left(1 + \frac{\partial\pi^*(v,\lambda)}{\partial\lambda}\frac{\lambda}{\pi^*(v,\lambda)}\frac{\partial\lambda}{\partial L^c}\frac{L^c}{\lambda}\right) \\
&= \pi^*(v,\lambda)\left(1 + \epsilon_{\pi^*/\lambda}(\lambda v)\epsilon_{\lambda/L^c}(\lambda\widehat{c})\right)
\end{aligned}$$

which can be rewritten as

$$\frac{\partial\pi^*(v,\lambda)L^c}{\partial L^c}\frac{L^c}{\pi^*(v,\lambda)L^c} = 1 + \epsilon_{\pi^*/\lambda}(\lambda v)\epsilon_{\lambda/L^c}(\lambda\widehat{c})$$

and thus as

$$\epsilon_{\pi^*L^c/L^c}(v,\lambda,\widehat{c}) = 1 + \epsilon_{\pi^*/\lambda}(\lambda v)\epsilon_{\lambda/L^c}(\lambda\widehat{c}) \tag{E.7}$$

where $\epsilon_{\lambda/L^c}(\lambda\widehat{c})$ is common across products while $\epsilon_{\pi^*/\lambda}(\lambda c)$ is product-specific and is given by (E.1) both in the short and in the long run. By (E.1), expression (E.7) can be restated as

$$\epsilon_{\pi^*L^c/L^c}(v,\lambda,\widehat{c}) = 1 - \frac{\epsilon_{\lambda/L^c}(\lambda\widehat{c})}{\varepsilon_p(x^*(\lambda v))}$$

with $\varepsilon_p(x^*(\lambda v)) \in (0,1)$ under (A2).

In the long run, by (E.5), we also have $\epsilon_{\lambda/L^c}(\lambda\widehat{c}) \in (0,1)$ so that $\pi^*(v,\lambda)L^c$ increases with L^c for low v and decreases with λ for high v if and only if $\varepsilon_p^{'}(x_v) > 0$, i.e. if and only if (B1) holds. This applies to both the GE and the PE scenarios.

E.3 Proposition 3: Intensive Margin Adjustment

Output and Revenue

The effects of a demand shock on intensive margin reallocations can be assessed as follows. By Lemma 1, larger L_c (positive demand shock) increases the marginal utility of income λ. Hence, the impact of larger L_c on output and revenue ratios has the same sign as the impact of λ on them.

Consider two products with marginal costs $\underline{v}$ and $\overline{v}$ such that $\underline{v} < \overline{v}$. In Appendix A, we have shown that x_v in a decreasing function of v under (A3) so that $\underline{v} < \overline{v}$ is associated with $x_{\underline{v}} > x_{\overline{v}}$, or equivalently with $x_{\underline{v}}/x_{\overline{v}} > 1$. In elasticity, the impact of λ on the output ratio $x_{\underline{v}}/x_{\overline{v}}$ is

$$\frac{d\left(x_{\underline{v}}/x_{\overline{v}}\right)}{d\lambda}\frac{\lambda}{x_{\underline{v}}/x_{\overline{v}}} = \frac{dx_{\underline{v}}}{d\lambda}\frac{\lambda}{x_{\underline{v}}} - \frac{dx_{\overline{v}}}{d\lambda}\frac{\lambda}{x_{\overline{v}}}.$$

By (9), this impact can be rewritten as

$$\frac{d\left(x_{\underline{v}}/x_{\overline{v}}\right)}{d\lambda}\frac{\lambda}{x_{\underline{v}}/x_{\overline{v}}} = \frac{\varepsilon_r(x_{\underline{v}}) - \varepsilon_r(x_{\overline{v}})}{\varepsilon_r(x_{\underline{v}})\varepsilon_r(x_{\overline{v}})}.$$

which is positive if and only if $\varepsilon_r(x_{\underline{v}}) > \varepsilon_r(x_{\overline{v}})$. Repeating the same argument for any pair of values $x_{\underline{v}}$ and $x_{\overline{v}}$ such that $x_{\underline{v}} > x_{\overline{v}}$ implies that $d\left(x_{\underline{v}}/x_{\overline{v}}\right)/d\lambda > 0$ if and only if $\varepsilon_r'(x_v) > 0$, which is our assumption (B1'). Hence, as $x_{\underline{v}}/x_{\overline{v}} > 1$ held before the positive demand shock increased λ under (B1), (B1') is *necessary and sufficient* for larger L_c to increase the output ratio $x_{\underline{v}}/x_{\overline{v}}$.

Consider now revenue $R_v = p_v x_v L_v$. In elasticity, the impact of λ on revenue is

$$\frac{dR_v}{d\lambda}\frac{\lambda}{R_v} = \left(\frac{dR_v}{dx_v}\frac{x_v}{R_v}\right)\left(\frac{dx_v}{d\lambda}\frac{\lambda}{x_v}\right) = -\frac{1-\varepsilon_p(x_v)}{\varepsilon_r(x_v)},$$

as by (9) we have again

$$\frac{dx_v}{d\lambda}\frac{\lambda}{x_v} = -\frac{1}{\varepsilon_r(x_v)},$$

and, given (4) and (5), by definition we have

$$\frac{dR_v}{dx_v}\frac{x_v}{R_v} = \frac{r_v}{p_v} = 1 - \varepsilon_p(x_v).$$

Under (A2) and (A3), the revenue ratio is such that $R_{\underline{v}}/R_{\overline{v}} > 1$. In elasticity, the impact of λ on this ratio is

$$\frac{d\left(R_{\underline{v}}/R_{\overline{v}}\right)}{d\lambda}\frac{\lambda}{R_{\underline{v}}/R_{\overline{v}}} = -\frac{1-\varepsilon_p(x_{\underline{v}})}{\varepsilon_r(x_{\underline{v}})} + \frac{1-\varepsilon_p(x_{\overline{v}})}{\varepsilon_r(x_{\overline{v}})}.$$

Repeating the same argument for any pair of values $R_{\underline{v}}$ and $R_{\overline{v}}$ such that $R_{\underline{v}} > R_{\overline{v}}$ implies that $d\left(R_{\underline{v}}/R_{\overline{v}}\right)/d\lambda > 0$ if and only if $(1-\varepsilon_p(x_v))/\varepsilon_r(x_v)$ is a decreasing function of x_v, which is implied by (B1) and (B1'). Hence, as $R_{\underline{v}}/R_{\overline{v}} > 1$ held before the positive demand shock increased λ and (B1) together with (B1') imply that $R_{\underline{v}}/R_{\overline{v}}$ rises after the shock, assumptions (B1) and (B1') are *sufficient* for the positive demand shock to increase the revenue ratio.

F Short-Run Adjustment in Closed Economy: Partial Equilibrium

F.1 Cost Cutoff

To see what happens when L^c changes in the short-run PE scenario, differentiate (11) to yield

$$\frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \widehat{c}}\frac{\partial \widehat{c}}{\partial L^c} + \frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \lambda}\frac{\partial \lambda}{\partial L^c} = -\frac{f}{(L^c)^2}$$

Then rewrite (16) as

$$\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} e^*\left(cz(m),\lambda\right)\gamma(c)dc = \frac{1}{\overline{N}}$$

where $e^*\left(cz(m),\lambda\right) = p\left(x^*\left(cz(m),\lambda\right)\right)x^*\left(cz(m),\lambda\right)$ and differentiate to obtain

$$\left[\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial e^*\left(cz(m),\lambda\right)}{\partial \lambda}\gamma(c)dc\right]\frac{\partial \lambda}{\partial L^c} + \sum_{m=0}^{\infty}\frac{e^*\left(\widehat{c}/z(m),\lambda\right)\gamma(\widehat{c}/z(m))}{z(m)}\frac{\partial \widehat{c}}{\partial L^c} = 0.$$

In matrix notation we can rewrite

$$\begin{bmatrix} \sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial e^*(cz(m),\lambda)}{\partial \lambda}\gamma(c)dc & \sum_{m=0}^{\infty}\frac{e^*(\widehat{c}/z(m),\lambda)\gamma(\widehat{c}/z(m))}{z(m)} \\ \frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \lambda} & \frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \widehat{c}} \end{bmatrix}\begin{bmatrix} \frac{\partial \lambda}{\partial L^c} \\ \frac{\partial \widehat{c}}{\partial L^c} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{f}{(L^c)^2} \end{bmatrix}$$

which can be solved by Cramer rule to yield

$$\frac{\partial \lambda}{\partial L^c} = \frac{\frac{f}{(L^c)^2}\sum_{m=0}^{\infty}\frac{e^*(\widehat{c}/z(m),\lambda)\gamma(\widehat{c}/z(m))}{z(m)}}{\frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \widehat{c}}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial e^*(cz(m),\lambda)}{\partial \lambda}\gamma(c)dc - \frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \lambda}\sum_{m=0}^{\infty}\frac{e^*(\widehat{c}/z(m),\lambda)\gamma(\widehat{c}/z(m))}{z(m)}} > 0 \qquad \text{(F.1)}$$

and

$$\frac{\partial \widehat{c}}{\partial L^c} = \frac{-\frac{f}{(L^c)^2}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial e^*(cz(m),\lambda)}{\partial \lambda}\gamma(c)dc}{\frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \widehat{c}}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial e^*(cz(m),\lambda)}{\partial \lambda}\gamma(c)dc - \frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \lambda}\sum_{m=0}^{\infty}\frac{e^*(\widehat{c}/z(m),\lambda)\gamma(\widehat{c}/z(m))}{z(m)}} > 0$$

where the signs are dictated by (9) and (10) as well as by

$$\frac{\partial p^*\left(cz(m),\lambda\right)}{\partial \lambda} = \frac{u''(x^*\left(cz(m),\lambda\right))\lambda - u'(x^*\left(cz(m),\lambda\right))}{\lambda^2} < 0.$$

The fact that the sign of $\partial\lambda/\partial L^c$ is positive implies that the long-run result in Appendix E.1 and Appendix E.3 still hold in the short-run PE scenario. As for the long-run results in Appendix E.2, $\partial\lambda/\partial L^c > 0$ implies that those on profit still hold whereas $\partial\widehat{c}/\partial L^c > 0$ implies that those on the cutoff are reversed in the short-run PE scenario.

F.2 Profit

In Appendix E.2 we have shown that the elasticity of total profit $\pi^* L^c$ to L^c is

$$\epsilon_{\pi^* L^c/L^c}(v,\lambda,\widehat{c}) = 1 - \frac{\epsilon_{\lambda/L^c}(\lambda\widehat{c})}{\varepsilon_p(x^*(\lambda v))}$$

with $\varepsilon_p(x^*(\lambda v)) \in (0,1)$ under (A2). To characterize $\epsilon_{\lambda/L^c}(\lambda\widehat{c})$, we can use (E.6), (E.1) and (E.2) to rewrite (F.1) as

$$\epsilon_{\lambda/L^c}(\lambda\widehat{c}) = \frac{\varepsilon_p(x^*(\lambda\widehat{c}))}{1 - \frac{1-\varepsilon_p(x^*(\lambda\widehat{c}))}{\widehat{c}}\frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial e^*(cz(m),\lambda)}{\partial\lambda}\frac{\lambda}{e^*(cz(m),\lambda)}e^*(cz(m),\lambda)\gamma(c)dc}{\sum_{m=0}^{\infty}\frac{e^*(\widehat{c}/z(m),\lambda)\gamma(\widehat{c}/z(m))}{z(m)}}}$$

where the fact that the denominator is 1 plus a positive value and (B1) ensure

$$\epsilon_{\lambda/L^c}(\lambda\widehat{c}) < \varepsilon_p(x^*(\lambda\widehat{c})) < \varepsilon_p(x^*(\lambda v)) \text{ for all } v < \widehat{c}$$

As this implies $\epsilon_{\lambda/L^c}(\lambda\widehat{c})/\varepsilon_p(x^*(\lambda v)) < 1$, we have

$$\epsilon_{\pi^* L^c/L^c}(v,\lambda,\widehat{c}) = 1 - \frac{\epsilon_{\lambda/L^c}(\lambda\widehat{c})}{\varepsilon_p(x^*(\lambda v))} > 0 \text{ for all } v < \widehat{c}.$$

G Open Economy

G.1 Long-Run Response of the Export Cutoff

To see how the export cutoff $\widehat{c}_{lD}$ changes when L^c_D changes, rewrite (20) as

$$\pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D) = \frac{f^x_D}{L^c_D}$$

with $l \in \{H, F\}$. Differentiation with respect to L^c_D gives

$$\frac{\partial \pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \widehat{c}_{lD}} \frac{\partial \widehat{c}_{lD}}{\partial L^c_D} + \frac{\partial \pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L^c_D} = -\frac{f^x_D}{\left(L^c_D\right)^2},$$

which can be restated as

$$\frac{\partial \widehat{c}_{lD}}{\partial L^c_D} = \frac{-\frac{\partial \pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L^c_D} - \frac{f^x_D}{\left(L^c_D\right)^2}}{\frac{\partial \pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \widehat{c}_{lD}}}. \tag{G.1}$$

Given $\partial \pi^*_{lD}/\partial \lambda_D < 0$, $\partial \pi^*_{lD}/\partial \widehat{c}_{lD} < 0$ (as $\pi^*(v, \lambda)$ is a decreasing function of v) and $\partial \lambda_D/\partial L^c_D > 0$ when (A1)-(A3) hold, (G.1) shows that the export cutoff increases with L^c_D if and only if f^x_D is large enough, leading to a larger number of exported products $M^x_{lD} = \Gamma\left(\widehat{c}_{lD}\right) \overline{M}^i_l$.

Turning to aggregate exports from country $l \in \{H, F\}$, they are defined as

$$EXP_{lD} = \overline{N}^i_l \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{lD}/z(m)} \left[x^*_{lD}\left(cz(m), \lambda\right) L^c_D + f\right] \gamma(c)dc\right]$$

where $\overline{N}^i_l$ is the fixed measure of incumbent firms in l. Dividing by $\overline{N}^i_l$ and the diffentiating gives

$$\begin{aligned}
\frac{1}{\overline{N}^i_l} \frac{\partial EXP_{lD}}{\partial L^c_D} = & \left\{\sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{lD}/z(m)} \frac{\partial x^*_{lD}\left(cz(m), \lambda\right)}{\partial \lambda_D} L^c_D \gamma(c)dc\right]\right\} \frac{\partial \lambda_D}{\partial L^c_D} \\
& + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{lD}/z(m)} x^*_{lD}\left(cz(m), \lambda\right) \gamma(c)dc\right] \\
& + \left\{\sum_{m=0}^{\infty} \left[x^*_{lD}\left(\widehat{c}_{lD}, \lambda\right) L^c_D + f\right] \frac{\gamma(\widehat{c}_{lD}/z(m))}{z(m)}\right\} \frac{\partial \widehat{c}_{lD}}{\partial L^c_D}
\end{aligned}$$

Given $\partial \pi^*_{lD}/\partial \lambda_D < 0$ and $\partial \lambda_D/\partial L^c_D > 0$ when (A1)-(A3) hold, the first term on the right hand side of the last equation is negative. Hence, when (A1)-(A3) hold, a sufficient condition for $\partial EXP_{lD}/\partial L^c_D > 0$ is that f^x_D is large enough to make $\partial \widehat{c}_{lD}/\partial L^c_D$ positive and itself large enough.

G.2 Short-Run Response of the Export Cutoff in Partial Equilibrium

Totally differentiate (25) to obtain

$$\begin{aligned}
&\overline{N}_D^i \left[\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{DD}/z(m)} \frac{\partial e_{DD}^*(cz(m),\lambda_D)}{\partial \lambda_D} \gamma(c)dc\right] \frac{\partial \lambda_D}{\partial L_D^c} + \overline{N}_D^i \sum_{m=0}^{\infty} \frac{e_{DD}^*(\widehat{c}_{DD}/z(m),\lambda)\gamma(\widehat{c}_{DD}/z(m))}{z(m)} \frac{\partial \widehat{c}_{DD}}{\partial L_D^c} \\
&+\overline{N}_H^i \left[\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{HD}/z(m)} \frac{\partial e_{HD}^*(cz(m),\lambda_D)}{\partial \lambda_D} \gamma(c)dc\right] \frac{\partial \lambda_D}{\partial L_D^c} + \overline{N}_H^i \sum_{m=0}^{\infty} \frac{e_{HD}^*(\widehat{c}_{HD}/z(m),\lambda)\gamma(\widehat{c}_{HD}/z(m))}{z(m)} \frac{\partial \widehat{c}_{HD}}{\partial L_D^c} \\
&+\overline{N}_F^i \left[\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{FD}/z(m)} \frac{\partial e_{FD}^*(cz(m),\lambda_D)}{\partial \lambda_D} \gamma(c)dc\right] \frac{\partial \lambda_D}{\partial L_D^c} + \overline{N}_F^i \sum_{m=0}^{\infty} \frac{e_{FD}^*(\widehat{c}_{FD}/z(m),\lambda)\gamma(\widehat{c}_{FD}/z(m))}{z(m)} \frac{\partial \widehat{c}_{FD}}{\partial L_D^c} = 0
\end{aligned}$$

where $e_{lh}^*(cz(m),\lambda_D)$ is the individual expenditure of a consumer in country h on products from country l. Then, rewrite (17) and (20) for $l \in \{H,F\}$as

$$\begin{aligned}
\pi_{DD}^*(\widehat{c}_{DD},\lambda_D) &= \frac{f}{L_D^c}, \\
\pi_{HD}^*(\tau_{HD}\widehat{c}_{HD},\lambda_D) &= \frac{f_{HD}^x}{L_D^c}, \\
\pi_{FD}^*(\tau_{FD}\widehat{c}_{FD},\lambda_D) &= \frac{f_{FD}^x}{L_D^c}.
\end{aligned}$$

Totally differentiating them gives

$$\begin{aligned}
\frac{\partial \pi_{DD}^*(\widehat{c}_{DD},\lambda_D)}{\partial \widehat{c}_{DD}} \frac{\partial \widehat{c}_{DD}}{\partial L_D^c} + \frac{\partial \pi_{DD}^*(\widehat{c}_{DD},\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L_D^c} &= -\frac{f}{\left(L_D^c\right)^2} \\
\frac{\partial \pi_{HD}^*(\tau_{HD}\widehat{c}_{HD},\lambda_D)}{\partial \widehat{c}_{HD}} \frac{\partial \widehat{c}_{HD}}{\partial L_D^c} + \frac{\partial \pi_{HD}^*(\tau_{HD}\widehat{c}_{HD},\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L_D^c} &= -\frac{f_{HD}^x}{\left(L_D^c\right)^2} \\
\frac{\partial \pi_{FD}^*(\tau_{FD}\widehat{c}_{FD},\lambda_D)}{\partial \widehat{c}_{FD}} \frac{\partial \widehat{c}_{FD}}{\partial L_D^c} + \frac{\partial \pi_{FD}^*(\tau_{FD}\widehat{c}_{FD},\lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L_D^c} &= -\frac{f_{FD}^x}{\left(L_D^c\right)^2}
\end{aligned}$$

We can summarize these total differentials in matrix notation as the system of linear equations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda_D}{\partial L_D^c} \\ \frac{\partial \widehat{c}_{DD}}{\partial L_D^c} \\ \frac{\partial \widehat{c}_{HD}}{\partial L_D^c} \\ \frac{\partial \widehat{c}_{FD}}{\partial L_D^c} \end{bmatrix} = \begin{bmatrix} 0 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

with

$$\begin{aligned}
a_{11} = \overline{N}_D^i &\left[\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{DD}/z(m)} \frac{\partial e_{DD}^*(cz(m),\lambda_D)}{\partial \lambda_D} \gamma(c)dc\right] + \\
\overline{N}_H^i &\left[\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{HD}/z(m)} \frac{\partial e_{HD}^*(cz(m),\lambda_D)}{\partial \lambda_D} \gamma(c)dc\right] + \\
\overline{N}_F^i &\left[\sum_{m=0}^{\infty} \int_0^{\widehat{c}_{FD}/z(m)} \frac{\partial e_{FD}^*(cz(m),\lambda_D)}{\partial \lambda_D} \gamma(c)dc\right] < 0
\end{aligned}$$

$$a_{12} = \overline{N}^i_D \sum_{m=0}^{\infty} \frac{e^*_{DD}(\widehat{c}_{DD}/z(m),\lambda)\gamma(\widehat{c}_{DD}/z(m))}{z(m)} > 0$$
$$a_{13} = \overline{N}^i_H \sum_{m=0}^{\infty} \frac{e^*_{HD}(\widehat{c}_{HD}/z(m),\lambda)\gamma(\widehat{c}_{HD}/z(m))}{z(m)} > 0$$
$$a_{14} = \overline{N}^i_F \sum_{m=0}^{\infty} \frac{e^*_{FD}(\widehat{c}_{FD}/z(m),\lambda)\gamma(\widehat{c}_{FD}/z(m))}{z(m)} > 0$$

$$a_{21} = \frac{\partial \pi^*_{DD}(\widehat{c}_{DD},\lambda_D)}{\partial \lambda_D} < 0 \qquad a_{22} = \frac{\partial \pi^*_{DD}(\widehat{c}_{DD},\lambda_D)}{\partial \widehat{c}_{DD}} < 0$$
$$a_{31} = \frac{\partial \pi^*_{HD}(\tau_{HD}\widehat{c}_{HD},\lambda_D)}{\partial \lambda_D} < 0 \quad a_{33} = \frac{\partial \pi^*_{HD}(\tau_{HD}\widehat{c}_{HD},\lambda_D)}{\partial \widehat{c}_{HD}} < 0$$
$$a_{41} = \frac{\partial \pi^*_{FD}(\tau_{FD}\widehat{c}_{FD},\lambda_D)}{\partial \lambda_D} < 0 \quad a_{44} = \frac{\partial \pi^*_{FD}(\tau_{FD}\widehat{c}_{FD},\lambda_D)}{\partial \widehat{c}_{FD}} < 0$$

$$b_2 = -\frac{f}{\left(L^c_D\right)^2} < 0 \quad b_3 = -\frac{f^x_{HD}}{\left(L^c_D\right)^2} < 0 \quad b_4 = -\frac{f^x_{FD}}{\left(L^c_D\right)^2} < 0$$

Given these definitions and their signs, the solution of the system of linear equations gives

$$\frac{\partial \lambda_D}{\partial L^c_D} = \frac{-b_2a_{12}a_{33}a_{44} - b_3a_{13}a_{22}a_{44} - b_4a_{22}a_{14}a_{33}}{a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{13}a_{22}a_{31}a_{44} - a_{22}a_{14}a_{41}a_{33}} > 0$$
$$\frac{\partial \widehat{c}_{DD}}{\partial L^c_D} = \frac{b_2a_{11}a_{33}a_{44} - b_2a_{13}a_{31}a_{44} - b_2a_{14}a_{41}a_{33} + b_3a_{21}a_{13}a_{44} + b_4a_{21}a_{14}a_{33}}{a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{13}a_{22}a_{31}a_{44} - a_{22}a_{14}a_{41}a_{33}} > 0 \text{ iff } |b_2| \text{ large enough}$$
$$\frac{\partial \widehat{c}_{HD}}{\partial L^c_D} = \frac{b_2a_{12}a_{31}a_{44} + b_3a_{11}a_{22}a_{44} - b_3a_{12}a_{21}a_{44} - b_3a_{22}a_{14}a_{41} + b_4a_{22}a_{31}a_{14}}{a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{13}a_{22}a_{31}a_{44} - a_{22}a_{14}a_{41}a_{33}} > 0 \text{ iff } |b_3| \text{ large enough}$$
$$\frac{\partial \widehat{c}_{FD}}{\partial L^c_D} = \frac{b_2a_{12}a_{41}a_{33} + b_3a_{13}a_{22}a_{41} + b_4a_{11}a_{22}a_{33} - b_4a_{12}a_{21}a_{33} - b_4a_{13}a_{22}a_{31}}{a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{13}a_{22}a_{31}a_{44} - a_{22}a_{14}a_{41}a_{33}} > 0 \text{ iff } |b_4| \text{ large enough}$$

where $|b_2|$ is large enough if f is large enough, $|b_3|$ is large enough if f^x_{HD} is large enough, and $|b_4|$ is large enough if f^x_{FD} is large enough.

G.3 Allowing for the Destination Country to Export

In the main text we have analyzed a situation in which D does not export. This assumption is not crucial, either in the long or the short run. Consider the *long run*. With exports from D the free entry condition would become

$$\begin{aligned} f^e = & \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DD}/z(m)} \left[\pi^*_{DD}\left(cz(m),\lambda\right) L^c_D - f\right] \gamma(c)dc \right] \\ & + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DH}/z(m)} \left[\pi^*_{DH}\left(cz(m),\lambda\right) L^c_H - f^x_H\right] \gamma(c)dc \right] \\ & + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DF}/z(m)} \left[\pi^*_{DF}\left(cz(m),\lambda\right) L^c_F - f^x_F\right] \gamma(c)dc \right] \end{aligned}$$

where the second and third terms on the right hand side are constant for our comparative statics analysis with respect to L^c_D. In particular, comparison with (23) reveals that allowing for exports from D is isomorphic to reducing the entry cost f_D^e to

$$f_D^e - \sum_{m=0}^{\infty} \left\{ \int_0^{\widehat{c}_{DH}/z(m)} \left[\pi_{DH}^* \left(cz(m), \lambda\right) L_H^c - f_H^x\right] \gamma(c) dc + \int_0^{\widehat{c}_{DF}/z(m)} \left[\pi_{DF}^* \left(cz(m), \lambda\right) L_F^c - f_F^x\right] \gamma(c) dc \right\}.$$

The assumption of D not exporting is immaterial in the *short run* as it does not affect (17), (20) for $l \in \{H, F\}$ and (25).

H Multi-Product Productivity

Consider employment $\ell_v = v x_v$. As $\underline{v}$ and $\overline{v}$ are given, the employment ratio $\ell_{\underline{v}}/\ell_{\overline{v}}$ increases with λ if and only if the output ratio increases in λ. We have shown in Appendix E.3 that he output ratio $x_{\underline{v}}/x_{\overline{v}}$ increases in λ if and only if $\varepsilon_r'(x_v) > 0$. Hence, given (B1), (B1') is necessary and sufficient for the positive demand shock to increase the employment ratio.

Turning to productivity, we have defined the productivity of a multi-product firm supplying M products indexed $m = 1, ..., M-1$ as the average productivity computed for this set of products $\Phi = \sum_{m=0}^{M-1} s_m/v_m$, where $\ell_m = v_m x_m$ is employment in product m, x_m is its output, and $s_m = \ell_m / \sum_{m=0}^{M-1} \ell_m$ is the product's employment share. To focus on the intensive margin adjustment to a positive demand shock, let $\overline{M}$ be the number of products that the multi-product firm supplies both before and after the shock, and index them by $m = 0, ..., \overline{M}-1$ in increasing order of marginal cost so that $v_0 < v_1 < ... < v_{\overline{M}-1}$. The average productivity computed for these products is then

$$\Phi = \sum_{m=0}^{\overline{M}-1} \frac{s_m}{v_m}.$$

As Φ is the employment share weighted average of the products' productivity levels $1/v_m$'s and these levels are fixed, when the marginal utility of income increases due to a positive demand shock, firm productivity Φ increases if and only if for any pair of products the employment share of the lower marginal cost product rises relative to the share of the higher marginal cost one. Too see when this is the case, consider the employment ratio $\ell_{m'}/\ell_{m''}$ of a lower cost product m' to a higher cost one m'' such that $0 \le m' < m'' \le \overline{M} - 1$. As the employment ratio equals the employment share ratio

$$\frac{\ell_{m'}}{\ell_{m''}} = \frac{\frac{\ell_{m'}}{\sum_{m=0}^{\overline{M}-1} \ell_m}}{\frac{\ell_{m''}}{\sum_{m=0}^{\overline{M}-1} \ell_m}} = \frac{s_{m'}}{s_{m''}},$$

Φ increases if and only if $\ell_{m'}$ increases relative to $\ell_{m''}$ for any $m' < m''$. Hence, as long as (B1) holds, (B1') are also necessary and sufficient for the positive demand shock to increase the productivity of the multi-product firm.[37]

[37] Everything would hold weakly if we relaxed the ranking condition $v_0 < v_1 < ... < v_{M-1}$ to $v_0 \leq v_1 \leq ... \leq v_{M-1}$.